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THE GOLDEN SECTION: THERE WAS ALWAYS A FIRST TIME - BUT WHEN WAS IT?

ALBERT VAN DER SCHOOT

Name: Albert van der Schoot (b. Groningen, 1949)
Profession: Musicologist, Philosopher
Fields of interest: Aesthetics, Philosophy of Culture, Philosophy of Music
Affiliation: University of Amsterdam
E-mail: a.vanderschoot2@contact.uva.nl
Major publications and/or exhibitions:
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Abstract: *Since 1951, there has been an ongoing search for the first occurrence of the notion 'golden section' (henceforth: the GS). In that year, George Sarton wondered how it was that this now so well-known name for such a famous proportion could not be found in an earlier text than in Martin Ohm's textbook of mathematics from 1835, and he invited researchers to find earlier occurrences of the term (Sarton 1951).*
Many authors who have not seriously studied the history of the GS claim that it was Leonardo da Vinci who coined the name sectio aurea *for this proportion, and that it was widely in use among artists in the Renaissance. The one and only book devoted to it during the Renaissance shows the opposite. The concept was in use only in a mathematical context until Adolf Zeising published his book in 1854; and purposeful application of the GS by artists would only become possible in an era that felt free to break with a solid Vitruvian tradition.*

Keywords: Luca Pacioli; Adolf Zeising; Franz Liszt; Béla Bartók.

INTRODUCTION

The question 'when does the golden section make its appearance in history?' can be subdivided as follows:

1) When was the first time in history that scholars (or anyone else) showed any awareness of this particular proportion?
2) When was the first time in history that this proportion was coined the 'golden section', in any language?
3) When was the first time in history that an artist showed aesthetic interest in the GS, with the *intention* of applying it in his own work?
4) When was the first time in history that an artist effectively *did* 'make use of' the GS?

All these questions, especially the last one, are less unambiguous than they sound. We must set crite-ria for what counts as 'being aware of' and for 'making use of'. Is it enough to 'find' the GS in a work for us to say that the artist 'applied' the GS?

KNOWLEDGE DOES NOT IMPLY IMPLEMENTATION

1) If we find a drawing of a pentagram from the Chalcolithic period, some 5,000 to 3,000 years BC (Herz-Fischler 1987, p. 58), does that imply that someone so long ago was aware of the GS? Obviously not: that would be the same as claiming that anyone drawing a rectangular triangle is aware of the Pythagorean Theorem. We need some proof of actual consciousness of this particular proportion, expressed in words (or in formulae, but that is a rather modern way of expressing mathematical insights). The same goes for the supposed use of the pentagram as a recognition symbol among the Pythagoreans. In the writings of Plato and Aristotle we can observe how problematic the notion of *incommensurability* still was for the most learned representatives of their generation; if understanding the relationship between the side and the diagonal of the same square constitutes a problem, then understanding the irrational value of the GS is definitively a bridge too far.

Knowing the respect that quite a few Greek scholars (including Plato) had for Egyptian art, it seems worthwhile to take a look into the most famous mathematical document from the Egyptian heritage, the *Rhind Papyrus* (dating from ± 1500 BC). Fractals and proportions are abundant in this text, and there are instructions for determining the slope of a pyramid, but there is no trace of the golden ratio. We have no idea where it was first developed, but the first known source that unmistakably proves consciousness of this ratio plus many of its remarkable properties is found in Euclid's *Elements.* Euclid will not have been the first to notice these tasty mathematical bites, since his *Elements* contain a compilation of Greek mathematical knowledge from around 300 BC, but thanks to this compilation, he is considered the founding father of geometry in the western hemisphere. As opposed to the mathematical parables we find in Plato, Euclid's theorems are pure mathematics, without any deeper significance than just the mathematical insights themselves. Euclid uses the GS in the geometry of areas in Book II of the *Elements* and applies it in Book VI for a further understanding of proportions. This is also the book in which he provides us with the classical definition that remained in use for over 2,000 years: *a straight line is said to have been divided in extreme and mean ratio if the ratio of the larger to the smaller part equals the ratio of the whole to the larger part*. And as opposed to Plato and the Pythagoreans, there is nothing ambiguous about this definition: it is mathematical and nothing else. In the European tradition, no earlier definition is known.

2) People tend to attribute interesting renovations to well-known names; numerous are the books claiming that it was Leonardo da Vinci who introduced the term *sectio aurea* for our proportion.

The truth is that Leonardo did not use that term even once. A term he did use was *divina proportione*, the title of the book by Luca Pacioli which Leonardo illustrated (Pacioli 1509). But alas: he never used it with the significance that Pacioli gave to this term. Pacioli used it as a synonym for Euclid's *division in extreme and mean ratio.* He chose the term to express the similitude which he believed existed between this proportion and God Almighty: both are characterised by unicity, trinity, irrationality, and continuity. For Leonardo, however, *divina* is merely an *epitheton ornans*, glorifying the *proportionalità* that he was of course interested in, and that he had studied from the earlier work by the mathematician (Pacioli 1494).

It took almost half a century since Sarton's call was answered: Dénes Nagy found the term in a geometry textbook from 1830, by Ferdinand Wolff (Nagy 1997, 2007). After that, earlier occurrences were discovered from the 1820s. Six years ago, I had the good fortune of discovering another *Lehrbuch der Geometrie*, from 1824, by Georg Winkler. This book requests the reader to divide a line according to the middle and extreme ratio, and Winkler adds, with the same casualty that we find with Ohm and with Wolff: in older geometrical writings, this proportion used to be called the *goldener Schnitt* (Winkler 1824, p.80). In the first edition (1814), he had not yet mentioned this term.

But the hunt was not over. By the time that everyone who had seriously studied the question was convinced that the term GS originated in Germany in the early 19th century, Roger Herz-Fischler published Otfried Lieberknecht's discovery of an earlier occurrence in Gehler's *Physikalisches Wörterbuch* from ... 1789 (Herz-Fischler 2019). This text presents several synonyms: *güldener Schnitt, media et extrema ratio,* and *sectio aurea* or *divina*. Moreover, the context is not purely mathematical, but physical: some people claimed, erroneously, that in equal time lapses, the acceleration of a falling object increases according to the GS (Gehler 1789, p. 120).

Before the journal in which Herz-Fischler's article was to appear was printed, its editors received another contribution (Becker 2019), which they included in the same issue: 'An even earlier (1717) usage of the expression *golden section*'. This concerns the *Cursus Mathematicus*, another mathematical textbook, by Johann Wentzel Kaschube. To the Euclidean definition he added: 'The ancients called this section the golden one' (Becker 2019, p. 83).

Again, the ancients! All these German authors seem to be convinced of this classical origin, but no-one has provided us with a reference, or even just a name. Apart from that: the gap between 1717 and 1789 is gigantic – what happened in the meantime? And before 1717? If it were true (as opposed to our expectations so far) that the GS *is* really a concept from antiquity, then it cannot be an originally German term. Yet: so far, we have only met that claim in German texts ...

But whatever may have been the very first mentioning, the decisive breakthrough of the GS among non-mathematicians was a German publication: Adolf Zeising's *Neue Lehre* (Zeising 1854), in

which the author claims that the GS was constitutive for both natural and artistic proportions. The follow-up of this book in France was provided by naval officer Matila Ghyka, who influenced a lot of Frensch artists, such as Le Corbusier (Ghyka 1927, and later books).

3) In his bestseller *The Da Vinci Code*, Dan Brown frequently refers to Vitruvius as an authority who recommended the GS. In reality, Vitruvius does not even mention the GS – not under any name. So, what about the Renaissance – had not Pacioli addressed his book especially to 'all who studied philosophy, perspective, painting, sculpture, architecture or music' (Pacioli 1509)? Yes, he had. But not with the intention of making them proportionate their own creations according to the GS. Pacioli is clear about the reason for calling this the *divine* proportion: its stupendous effects are '*non naturali ma divini'* (Pacioli 1509, cap. 6). They are the work of God: no human architect could realize them.

The *Divina Proportione* is not the evidence that the GS enjoyed a widespread application by Renaissance painters, but rather the evidence of the opposite: nowhere does Pacioli speak about the divine proportion in the sense of *applying* it to one's own work; and whenever he speaks about how to compose one's own work, his guide is the widely venerated Vitruvius. This becomes even clearer in the treatise on architecture, which he added to the printed edition of the *Divina Proportione*. More than that: it is impossible for Pacioli and his contemporaries to imagine a man-made work of art as being composed 'according to the divine proportion'. The *proportio divina* is not a human measure: humans must stick to a proportion which is *rationale e sempre per numero se po explicare* (Pacioli 1509, appendix on architecture, cap. 20). Pacioli does not *promote* the artistic application of the GS, he *excludes* it.

The first artist who showed an interest in the GS with the purpose of implementing it in his own artistic toolbox was neither an architect, nor a painter. It was the composer Franz Liszt, who, having heard of the book that Adolf Zeising had published in 1854, asked his stepdaughter Marie, in his letter from May 6, 1859, to initiate him in 'les secrets du "goldene Schnitt" que je voudrais bien savoir appliquer dans mes compositions' (Pocknell *et al*., 2010, p. 166). He had heard of Zeising's book but had not read it – and if he had, he would have noticed how clumsily Zeising had phrased the thirty pages he devoted to music. Liszt's desire to break new grounds in the musical arsenal could never be satisfied by such a primitive theory as Zeising's. In the year after he wrote this letter to Marie, he made another, more radical choice: the choice for the full symmetry of the whole-tone-scale, in *Der traurige Mönch*, for piano and declamation. With hindsight, we may recognize the preparatory run-up to *Der traurige Mönch* in two earlier compositions, or rather their opening themes: the first theme of the first piano concerto (1849) consists exclusively of half-tone steps; the first theme of the *Faust Symphony* (1857) is built up from four consecutive augmented triads, which

together fill up the tonal space. What the two themes have in common is that they lack the tension caused by the difference between whole tones and semi-tones, which is the landmark of diatonic tonality; however, both are encompassed in a larger whole which conforms to that demand. And that is the revolutionary difference with *Der traurige Mönch*: this piece is in no tonality whatsoever. It is symmetrical throughout: it makes use of the whole-tone scale, without any semi-tones. This implies that the dynamics of harmonic tension and resolution towards a root tone is completely absent.

That was a revolutionary choice, which was hardly noticed at the time. It would take until 1923 when Arnold Schönberg systematically released all his compositions from tonal functionality and declared that from now on the twelve tones shall only be related to each other, and not to any tonal or harmonic structure.

4) So, if Liszt never realized his plan to apply the GS to his own work, then who was the first to do so? The expression 'making use of the GS' is ambiguous. Should a painter consciously proportionate his painting according to the GS, in order for us to recognize his *application* of this ratio? Around 1900, the *notion* of the '*section d'or*' (or *nombre d'or*) became very popular among artists in France. In their search for new ways in painting, the GS seemed the magic promise of a new realm of artistic revelation. About Paul Sérusier, Herz-Fischler notes: 'le nombre d'or était surtout un idéal philosophique, plutôt qu'une quantité mathématique à employer pour la composition de ses oeuvres' (Herz-Fischler 1997, p.11).

This attitude is especially characteristic for a whole group of cubist painters known as 'the Puteaux group', who united under the banner of the 'Section d'or' and chose this as the covering title of their exposition in 1912. Core members were Marcel Duchamp and his two brothers; poet Guillaume Apollinaire acted as their mentor. It is beyond doubt that these painters were under the spell of the *conception* of the GS, and that this spell did not concern the GS itself, but rather the lofty world of antiquity, philosophy, and mathematics they associated with it. However, none of the works exposed at the exposition in 1912 betrayed any connection with the GS.

Later interviews with them as well as writings by or about them confirm this picture. But they also show that some French artists, by 1920, used 'golden section triangles' to *verify* the proportions in their works. These may have been Kepler triangles (Herz-Fischler 1997). Such triangles have side lengths $1 : \sqrt{\Phi} : \Phi$; in decimal approach 1 : 1,272 : 1,618. Would this count as 'applying the GS in one's work'? Arguably: yes. And so it seems that the veneration of the conception of the GS in the circle around the Duchamp brothers led, a decade later, to taking into account the actual value of this proportion. It is difficult to point to one artist in particular, since they discussed and practiced these matters together. Yet, in this context, Gino Severini should be singled out. He formed part of

the Italian futurist movement of Marinetti and Russolo, but also felt attracted to French cubism. He settled in Paris and became part of the avant-garde circles there. Like the group around the Duchamp brothers, he came under the spell of the veneration of the GS but did not remain satisfied with vague references to its classical status. More than the others, he was searching for a solid mathematical understanding of his own work, considering himself a 'scientific cubist'. About the earlier period, he said later: 'j'étais content d'en comprendre, même de façon rudimentaire, la valeur géométrique, pour pouvoir m'en servir dans mes compositions' (Severini 1995, p. 213).

Severini does not know that he is repeating the words that Liszt wrote in his letter to Marie, sixty years earlier. Both artists, originally not francophone, use the French language to express their wish to 'apply' the GS in their own creations. There are differences, though. The first difference is that Liszt left it at that, whereas Severini is one of the painters who used this ratio in setting up the proportions of his work. The second is that Severini seriously studied its mathematical background and published *Du cubisme au classicisme*, with the telling subtitle *Esthétique du compas et du nombre* (Severini 1921). In this booklet, he gives the correct mathematical information about both the GS and the Fibonacci series (Severini 1921, p. 30), and explains how, in his own work, he combines this proportion with other, rational proportions. This results, in his view, not in *symmetry* but in *eurythmy* – and that term became current in the interbellum, also in pictorial art, in dance and in architecture (Rudolf Steiner!). The term *eurythmy* does not call for mathematical precision; it evokes the same feelings of awe and admiration that the *section d'or* did for the Puteaux group.

The Puteaux group must be considered rather as being in the same situation as Liszt (whose letter to Marie, of course, they knew nothing about) than as predecessors of Le Corbusier, whose intimation with the GS, both theoretically and practically, is beyond doubt. His well-known measuring system of the *Modulor* is indeed based on the GS, and his architectural work after World War II follows that system.

And in music? Roy Howat tried, not very successfully, to make the case for Claude Debussy as a composer who applied the GS (Howat 1983). Much more convincing are Ernő Lendvai's writings about the Fibonacci structures in the works of Béla Bartók, especially his works from the 1930s. Even if these are not always so exact as Lendvai suggests, the overall structure of a number of Bartók's compositions conforms pretty well to a *continuous division* according to our celebrated ratio; and since continuity is a basic property of the divine proportion (Pacioli 1509, cap. 5 and 7), we might state that Bartók's oeuvre is a much better testimony of the use of the GS in art than that of any of the artists mentioned before. However! If we add the claim that in order to speak of an artist's 'use' of the GS we must be sure that this artist had the GS in mind when creating the work,

then we are at a loss. According to all testimonies, never in his life has Bartók spoken, let alone written about the GS. In interviews he detested the idea that his music should meet a mathematical structure. Neither in his library, nor in his notes has anything been found to suggest that he ever consciously dealt with this proportion.

Does this mean Bartók kept the secret to himself? Or did he acquire his knowledge in another way?

Let me answer this question with a quote from a biography of the last years of his life:

> *And the way he'd hold a pinecone to his ear as if it were a fancy seashell – what on earth could he have heard in it? And they ask me if I know why he should have wanted to pull the pinecone apart and stare at each bit of it, as if he were going to discover some new wonder in every piece ...* (Fassett, 1970, p. 2).

CONCLUSION

In order to understand that not any individual painter in the Renaissance was ever concerned with 'applying' the GS in his own work, there is no need to investigate each painter's oeuvre. It suffices to understand that the very *conception* of 'applying the golden section' to any artefact was unthinkable in a world dominated by rational composition. The measurement schemes of Leonardo, Michelangelo, Dürer and others testify to this. Only in the 19th century did it become possible to look at both nature and art with a non-rational structure in mind. And it was not before the 20th century that the GS struck root as an instrument in the artist's toolkit.

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Albert van der SCHOOT

Dr. Albert van der Schoot studied musicology and philosophy at the University of Amsterdam, and music pedagogy at the Ferenc Liszt Academy in Budapest. From 1979 to 2014 he was a research lecturer in Aesthetics and Philosophy of Culture at the University of Amsterdam, and from 2006 to 2014 he taught Philosophy of Music at the University of Antwerp. In 1991, he was a visiting professor in Aesthetics at Indiana University (Bloomington, US), and from 2004 to 2008 he was a professor in 'Art and Reflection' at the ArtEZ Academy of Arts. His dissertation on the history of the golden section was translated into German: *Die Geschichte des goldenen Schnitts – Aufstieg und Fall der göttlichen Proportion*, Stuttgart 2016 (2nd ed.). Since his retirement, he continues publishing and lecturing.