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## **MONETARY POLICY SHOCKS IN A DSGE MODEL WITH A SHADOW BANKING SYSTEM**

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# Monetary policy shocks in a DSGE model with a shadow banking system\*

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## Abstract

This paper is motivated by the recent financial crisis and addresses whether a “too low for too long” interest rate policy may generate a boom-bust cycle. We suggest a model in which a microfounded shadow banking sector is included in an otherwise state-of-the-art DSGE model. When faced with perverse incentives, financial intermediaries within the shadow banking sector can divert a fraction of stockholders’ profits for their own benefits and extend credit at a discounted rate. The model predicts that long periods of accommodative monetary policy do create the preconditions for, but do not cause *per se*, a boom-bust cycle. Rather, it is the combination of a persistent monetary ease with microeconomic distortions in the financial system that causes a boom-bust.

Keywords: monetary policy, DSGE model, shadow banking system, boom-bust

*JEL* codes: E32, E44, E52, G24

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# 1 Introduction

The Fed funds rate: too low for too long? Some observers have recently criticized the Fed for helping fuel the credit/house-price boom and thereby the subprime crisis by keeping interest rates too low for too long. If correct, this criticism would have important implications for the future conduct of monetary policy. Unfortunately, conventional dynamic stochastic general equilibrium (DSGE) models are not well suited to address this issue because of their rather simple modelling structure of the financial system and of its relation with the real economy. In particular, although financial intermediaries have been at the center of the subprime crisis, they have played so far a relatively passive role in macroeconomic models.

This article aims to determine whether long periods of loose monetary policy may have a part to play in generating a boom-bust cycle. We do so by building a DSGE model with a two-sector financial system – a retail banking sector and a “shadow” banking sector in which there may exist optimism and perverse incentives. Such a model, in which we explicitly model the behavior of financial intermediaries within the shadow banking system, aims at providing further insights into the transmission of monetary policy.

The paper is organized as follows. Section 2 briefly reviews the two strands of literature on which our work builds, one on the causes of the subprime crisis and one on the role of the financial sector in DSGE models. Section 3 provides a general descriptive overview of our model. Section 4 describes the financial system and, in particular, the modelling of the shadow banking system based on microeconomic foundations. Section 5 details the calibration of the model and presents the impulse responses in both a) a one-period expansionary monetary policy shock and b) a “persistently low interest rate” scenario. Section 6 concludes.<sup>1</sup>

The central result of the paper is that a “too low for too long” interest rate policy does create the preconditions for, but does not cause *per se*, a boom-bust cycle. In fact, fluctuations in both real and financial variables are markedly amplified only when a persistently accommodative monetary policy environment is coupled with perverse incentives in the financial sector.

## 2 Motivation

### 2.1 The subprime crisis

Following the 2007 collapse of the U.S. subprime mortgage market and the resulting global financial and economic crisis, several authors have discussed the causes and consequences of the house price bubble and the boom-bust cycle. These analyses, either coming from the academia (*e.g.*, Borio, 2008 and Blanchard, 2009) or from policy-makers (*e.g.*, Trichet, 2009, Bean et al., 2010 and Bernanke, 2010), have overall concluded that the seeds of the crisis lay in a combination of both micro and macro factors.<sup>2</sup>

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<sup>1</sup> Appendix A presents the complete model, while technical details are described in appendices B, C and D.

<sup>2</sup> See Borio (2008), Brunnermeier (2009) and FED (2009) for a chronology of the events relating to the subprime crises.

Microeconomic factors are mostly related to recent innovations in financial instruments, institutions and markets. A non-exhaustive list of these factors includes: the reduced incentives for lenders to properly screen and monitor borrowers due to pay packages encouraging the pursuit of short-term returns; the under-estimation of the true risk of complex (and often not transparent) structured financial products arising from the replacement of sound risk management practices with mathematical and statistical models of risk; the distorted incentives faced by ratings agencies; the moral hazard behavior of financial institutions considered too big or too important to fail; and, additionally, an inadequate regulation and supervision of individual financial institutions and markets and of the financial system as a whole.

Potential macroeconomic factors include a protracted period of very low (and in some cases negative) real interest rates and plentiful liquidity; large international payments imbalances resulting from a “savings glut” in surplus countries; and the benign macroeconomic environment at the beginning of the 21<sup>st</sup> century as side effect of the Great Moderation.

Evidence for assigning a central role, as cause of the subprime crisis, to excessively loose monetary policy is, nevertheless, mixed. To date, the 2010 Jackson Hole Symposium provides the most recent debate on this issue. On the one hand, Bean et al. (2010) argue that low policy rates played only a modest direct role. As they state, “although monetary policy may have played a role in the credit/house-price boom that preceded the crisis, it is rather more Rosencrantz than Hamlet.” On the other hand, Taylor (2010) disagrees that the role of monetary policy was only a modest one without implications for future policy.

In this paper we do not attempt to explicitly model the subprime crisis. First, it is unlikely that any of the aforementioned factors in isolation could explain the crisis. Second, it would be too complex to comprise all of them in a DSGE model. Nevertheless, we do try to capture some micro factors (related in particular with the behavior of financial intermediaries) relevant to analyzing whether the Fed’s policy to keep interest rates low for a prolonged period may have played a key role for the run-up of the crisis. To explore this hypothesis, we rely on a model in which the financial sector, rather than being passive, plays a central role in driving the boom-bust cycle.

## 2.2 Financial system in DSGE models

The DSGE model, currently the state-of-the-art macroeconomic model, results from a fusion of the Real Business Cycle models of the 1980s with the New Keynesian sticky-price models of the early 1990s. In its primordial version, this model incorporated no role for credit and financial factors at all. Works that followed have continued to assume frictionless financial markets so that financial intermediaries played a passive role, despite of the increasing awareness about their importance in affecting the performance of the economy, including though the transmission of monetary policy. For example, in the DSGE models currently used for monetary policy analysis at the main central banks – *e.g.*, the SIGMA model at the FED (Erceg et al., 2006), the Smets and Wouters model at the ECB (Smets and Wouters, 2003) and the Bank of England’s Quarterly Model (Harrison et al., 2005) – the financial sector hardly plays a prominent role.

A first attempt to introduce a financial sector in a New Keynesian DSGE framework has been made by Bernanke et al. (1999). In their model, the financial sector is limited to a banking sector that amplifies the effects of the shocks *via* the financial accelerator effect. More recently, some authors have enhanced the structure and role of the financial sector in DSGE models. Iacoviello (2005) extends the Bernanke et al. (1999) model by introducing collateral constraints for firms, as in Kiyotaki and Moore (1997). Christiano et al. (2003, 2008, 2010) and Goodfriend and McCallum (2007) consider a perfectly competitive banking sector that offers agents a variety of financial assets with different returns, while Kobayashi (2008) and Gerali et al. (2010) consider imperfect competition in the banking sector so as to model the setting of interest rates by banks. Cúrdia and Woodford (2010) also allow for a time-varying spread between deposits and lending rates. Finally, a number of papers (see, for instance, Van den Heuvel, 2008, Gertler and Karadi, 2009, de Walque et al., 2010 and Meh and Moran, 2010) study the role of bank capital in the transmission of macroeconomic shocks.<sup>3,4</sup>

While most of the literature focuses on financial frictions that arise from the behavior of borrowers, the subprime crisis has highlighted the need to analyze the behavior of financial intermediaries themselves. In this paper we take a step toward determining whether the financial sector plays an active role in the boom-bust cycle. We do so by augmenting the Christiano et al. (2010) model with a shadow banking system. Our microfounded financial system is thus composed of two different financial intermediaries (retail and investment banks) that intermediate funds from households (lenders) to two groups of entrepreneurs (borrowers).

Following the Bernanke et al. (1999) framework, in the retail banking sector there is an “agency / information” problem between borrowers and lenders. Information is asymmetric, in that the entrepreneur’s realized return may be observed at no cost only by the entrepreneur, while it can be observed by the retail bank only after paying a monitoring cost. Thus, the model is of the costly state verification type.

In the shadow banking sector we introduce an “agency / money” problem, in that the investment bank manager may pursue his own private objectives, which need not coincide with those of the stockholders. This problem arises because the manager faces perverse incentives – in the form of side payments – to boost his private revenue at the expense of stockholders’ profits, *i.e.* the bank manager can divert a fraction of stockholders’ profits for his own benefit.

We then use the model to address the following questions:

1. How do perverse incentives in the financial sector affect the transmission of monetary policy shocks through the economy? How different are our findings from those of a workhorse DSGE model?
2. Does a “too low for too long” interest rate policy cause a boom-bust cycle?
3. What are the effects of perverse incentives in the financial sector when coupled with a persistently low interest rate environment?

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<sup>3</sup> Drumond (2009) provides an exhaustive survey on the theoretical literature on the bank capital channel of propagation of exogenous shocks as well as on the regulatory framework of capital requirements under the Basel Accords.

<sup>4</sup> This brief review only focuses on DSGE models and, in view of the growth of this literature, does not aim to be exhaustive.

### 3 An overview of the model

The core of our framework is a simplified version of the *Financial Accelerator Model* described in Christiano et al. (2010), hereafter CMR.<sup>5</sup> It essentially corresponds to the models in Smets and Wouters (2003) and Christiano et al. (2005) enlarged with the financial accelerator mechanism developed by Bernanke et al. (1999). To this we add a shadow banking system that intermediates funds between households and an additional set of entrepreneurs. The model is thus composed of government, households, firms, capital producers, entrepreneurs, and banks. Figure 1 sketches the structure of the model. Agents drawn in black are those already present in the CMR model, while the “new agents” are drawn in blue.

Government expenditures represent a constant fraction of final output and are financed by lump-sum taxes imposed to the households. The central bank sets the nominal interest rate.

Households consume, save and supply labor services monopolistically. Two types of financial instruments, offered by banks, are available to households: time deposits and corporate bonds. To keep this part of the model as simple as possible, we assume that the rate of return is the same for both financial instruments, so households are indifferent between holding deposits or bonds.

On the production side, monopolistically competitive intermediate-good firms use labor (supplied by households) and capital (rented from entrepreneurs) to produce a continuum of differentiated intermediate goods. Perfectly competitive final-good firms buy intermediate goods and produce the final output, which is then converted into consumption, investment and government goods.

Capital producers combine investment goods with undepreciated capital purchased from entrepreneurs to produce new capital, which is then sold back to entrepreneurs.

Capital services are supplied by entrepreneurs, who own the stock of physical capital and choose how intensively to use it. Entrepreneurs purchase capital using their own resources – net worth, or equity, resulting from net proceeds of their activities from one period to the next – as well as external finance. In fact, entrepreneurs’ net worth is not enough to finance the full amount of capital they acquire, so they finance a part of their capital expenditures either by issuing bonds or by means of bank loans.

The setting up of the shadow banking system is paralleled by the division of the entrepreneurial sector into two groups: the riskier entrepreneurs and the safer entrepreneurs, who have access to two different sources of external funding.<sup>6</sup> We assume that the riskier entrepreneurs obtain financing via retail bank loans, while the safer entrepreneurs issue bonds resorting to investment banks.<sup>7</sup> In particular, we

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<sup>5</sup> The simplified version excludes long-run growth, the fixed cost in the production function and distortionary taxes on capital and labor income and on household consumption. While not changing the model’s dynamic responses to monetary policy shocks, these simplifications reduce its complexity.

<sup>6</sup> A number of papers (see, among many others, Diamond, 1991, Chemmanur and Fulghieri, 1994, Holmstrom and Tirole, 1997, Berlin and Loeys, 1988, Bolton and Freixas, 2000, 2006, Repullo and Suarez, 2000, Chakraborty and Ray, 2007, Hale, 2007 and Gerber, 2008) characterize equilibria when bank lending and direct financing through securities issues are both present. Usually, in equilibrium, firms are segmented by risk classes in their choice of funding, with safer firms choosing bond financing and riskier firms preferring bank loans.

<sup>7</sup> Typically, a firm going public hires an investment bank to sell its securities. The investment bank (the underwriter) acts as an intermediary between the issuing firm and the ultimate investors. The most common type of underwriting arrangement is the “firm commitment” underwriting, according to which the underwriter buys the entire stock of bonds from the firm and resells it to investors at a higher price (*i.e.*, at a lower interest rate). This spread represents the investment bank’s profits. See Ellis et al. (2000) for an in-depth analysis of the underwriting process.

consider the entrepreneurs of the CMR model as riskier because they may default (since they face an idiosyncratic productivity shock), while we consider the additional set of entrepreneurs as safer because we assume that they always have enough wealth to repay their debt and thus never default. Accordingly, we calibrate the model so as to guarantee that, in equilibrium, safer entrepreneurs finance themselves at a lower interest rate than riskier entrepreneurs.

Lending to riskier entrepreneurs involves an agency/information problem, because they costlessly observe their idiosyncratic shocks, whereas the retail bank must pay a monitoring cost to observe those shocks. The optimal lending contract is of the costly state verification type. In particular, a standard debt contract is set up specifying a loan amount and an interest rate to pay whenever the entrepreneur is solvent. If the entrepreneur cannot pay the required interest because of an unfavorable realization of his productivity shock, he goes into bankruptcy and turns over his remaining equity to the retail bank, after being monitored. The rate of return paid by solvent entrepreneurs must thus be high enough to cover the cost of funds to the bank, as well as the monitoring costs net of the resources that the bank can recover from bankrupt entrepreneurs (for further details see Bernanke et al., 1999).

The investment banking sector is the core part of our shadow banking system.<sup>8</sup> We assume that the bond market is populated by a continuum of monopolistic competitive investment banks, who set the coupon rate on bonds in order to maximize profits, which are then rebated to the stockholders, *i.e.* to the households.<sup>9</sup> Within each investment bank, the agent that makes the decision is the investment bank manager, whom we call henceforth the underwriter.

Two distinct mechanisms – optimism and perverse incentives – are at work in the investment banking sector. First, we consider that an optimistic underwriter is willing to underwrite bonds at a lower – relatively to its “normal” value – coupon interest rate.<sup>10</sup> We assume that the underwriter turns out to be optimistic when the entrepreneur pledges more collateral and, accordingly, we model underwriter’s optimism as a positive function of the entrepreneur’s net worth. An unexpected increase of the entrepreneur’s net worth – as a result of a monetary easing or a favorable productivity shock – triggers optimism and *may* result in a lower bond coupon interest rate. Second, we introduce perverse incentives by assuming that the safer entrepreneur offers side payments to the underwriter in order to borrow at a more favorable interest rate. In exchange of those side payments, an optimistic underwriter may *de facto* facilitate the extension of credit by setting a “discounted” – relatively to the “normal” – bond coupon rate. Defining how much the coupon rate deviates from the normal rate depends upon the underwriter’s utility function, in which the trade-off between maximizing his private revenue and the investment bank’s profits (hence, the stockholders’ profits) is explicitly modeled. An agency conflict between investment bank managers and stockholders arises because side payments represent a

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<sup>8</sup> The expression “shadow banking system” has been suggested originally by Paul McCulley of PIMCO at the 2007 Jackson Hole conference, where he defined it as “the whole alphabet soup of levered up non-bank investment conduits, vehicles, and structures” (McCulley, 2007, pag. 2). See Pozsar et al. (2010) for a comprehensive and up-to-date description of the shadow banking system.

<sup>9</sup> The empirical evidence on the U.S. market of bond underwriting suggests an oligopolistic market structure. For example, Fang (2005) shows that the largest five investment banks underwrite more than 60% of all deals, and the largest fifteen banks account for roughly 95% of all deals.

<sup>10</sup> There is considerable evidence that economic agents may be too optimistic. See De Bondt and Thaler (1994) for an exhaustive survey on behavioral finance and Puri and Robinson (2007) for an empirical analysis on how optimism affects economic decisions.

compensation for the underwriter to the sacrifice of stockholders' profitability.<sup>11</sup>

Having briefly presented the main features of our model, in the next section we describe the financial system, with particular emphasis on the shadow banking system. The rest of the model is standard in the literature and is set out in appendix A.

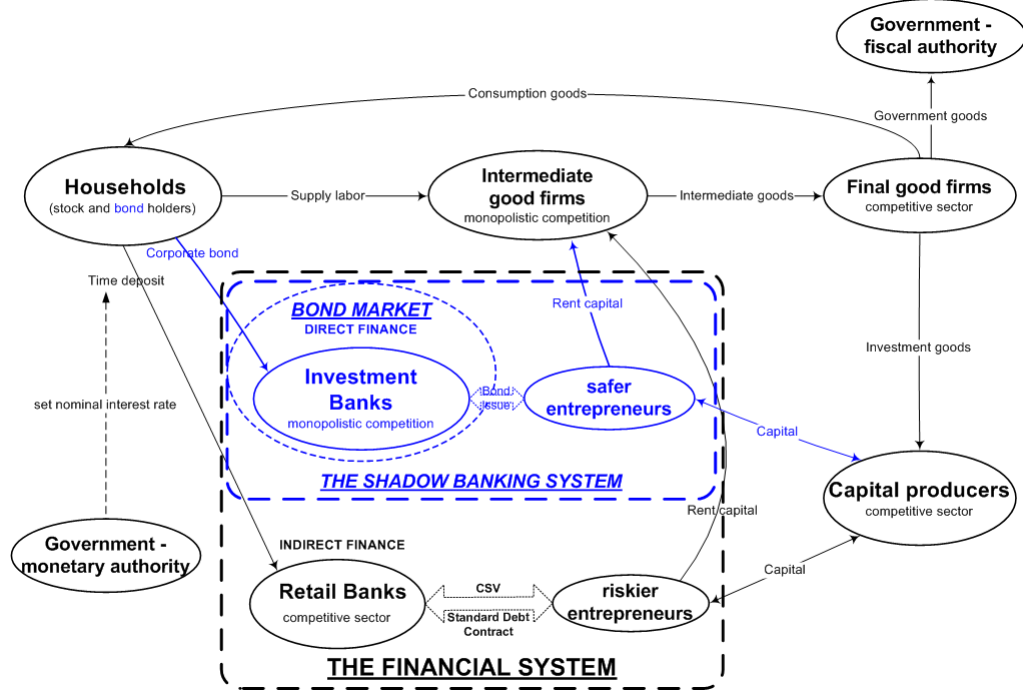


Figure 1: Structure of the model

## 4 The financial system

We assume that riskier entrepreneurs represent a fraction  $\eta$  of the total population of entrepreneurs, while safer entrepreneurs represent the remaining fraction,  $1 - \eta$ . In what follows, the superscripts “ $H$ ” and “ $H, r$ ” (“ $L$ ” and “ $L, l$ ”) refer to variables associated with the riskier (safer) entrepreneurs.

### 4.1 Riskier entrepreneurs and retail banks

Riskier entrepreneurs own a share of the economy's stock of physical capital. Entrepreneurs' net worth is enough to finance only a part of their holdings of physical capital, the rest being financed by loans from a representative retail bank. Entrepreneurial loans are risky because the returns on

<sup>11</sup> The agency conflict between managers and stockholders is a central theme in the corporate-finance literature (see Stein, 2003, for a survey). The manager-stockholder agency conflict arises because the managers may pursue their own private objectives rather than those of outside stockholders. Studies on conflicts of interest in investment banking industry include, among others, Michaely and Womack (1999) and Mehran and Stulz (2007).



their investments are subject to idiosyncratic shocks. In particular, entrepreneurs who suffer a large unfavorable shock and who therefore cannot pay the required interest, go bankrupt. Financial frictions arise because the idiosyncratic shock is observed by the entrepreneurs at no cost, and by the bank only if it incurs in a fixed monitoring cost. To mitigate costs stemming from this source of asymmetric information, entrepreneurs and bank sign a standard debt contract, according to which the entrepreneur commits to pay back the loan principal and a non-default interest rate, unless he declares default. In case of default, the bank conducts a verification of the residual value of the entrepreneur's assets and takes in all of the entrepreneur's net worth, net of monitoring costs.

The debt contracts extended by the bank to entrepreneurs are financed by bank's issuance of time deposits to households. Although individual entrepreneurs are risky, the bank itself is not: by lending to a large number of entrepreneurs, the bank can diversify the idiosyncratic risk and thus can guarantee a safe return on households' deposits. Nevertheless, financial frictions – reflecting the costly state verification problem between entrepreneurs and the bank – imply that bank hedges against credit risk by charging a premium over the rate at which it can borrow from households.

In particular, as shown by Bernanke et al. (1999), the first order conditions of the contracting problem yield the following relationship linking the expected return on capital ( $R_{t+1}^{k,H}$ ) relative to the risk-free interest rate ( $R_{t+1}^e$ ) and the entrepreneur's leverage ratio ( $\frac{Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}}$ ):

$$\frac{E_t \left( 1 + R_{t+1}^{k,H} \right)}{1 + R_{t+1}^e} = \Psi \left( \frac{Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}} \right), \quad (1)$$

where  $Q_{\bar{k}',t}$ ,  $\bar{K}_{t+1}^{H,r}$  and  $N_{t+1}^{H,r}$  denote, respectively, the price of capital, the entrepreneur's stock of capital and the entrepreneur's net worth and the function  $\Psi$  is such that  $\Psi' > 0$  for  $N_{t+1}^{H,r} < Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}$ . The ratio  $\frac{E_t(1+R_{t+1}^{k,H})}{1+R_{t+1}^e}$ , which Bernanke et al. (1999) interpreted as the external finance premium faced by the entrepreneur, depends positively on the entrepreneur's leverage ratio. Intuitively, all else equal, higher leverage means higher exposure, implying a higher probability of default, hence a higher credit risk, which the bank translates into a higher required return on lending.

The cost of borrowing fluctuates endogenously with the cycle due to two general equilibrium mechanisms.

The first one is the Bernanke et al. (1999) “financial accelerator” effect, whereby induced changes in the asset price alter the value of the collateral that the entrepreneur can pledge and, hence, the contractual loan rate. Specifically, a positive shock to the asset price – as a result of a monetary easing or a favorable shock to productivity – increases the entrepreneur's net worth and decreases the external finance premium, which in turn stimulates the demand for investment. The increase in net worth also reduces the expected default probability and allows the entrepreneur to take on more debt and to further expand investment. An accelerator effect arises, since the investment boom raises the asset price, further pushing up the entrepreneur's net worth and investment.

The second mechanism – which CMR refer to as the “Fisher deflation” effect – is absent in Bernanke

et al. (1999) and works through a debt-deflation effect.<sup>12</sup> This effect arises because of the assumption that the return received by households on time deposits is nominally non-state contingent, while loans to entrepreneurs are state-contingent. Therefore, unexpected movements in the price level alter the *ex-post* real burden of entrepreneurial debt and, hence, the entrepreneur's net worth. Namely, following an unexpected increase in inflation, the total real resources transferred from the entrepreneur to households are reduced and, as a consequence, the entrepreneur's net worth increases.

As CMR point out, the “accelerator” and “Fisher” effect mechanisms reinforce each other in the case of shocks that move inflation and output in the same direction (*e.g.*, monetary policy shocks), whereas they dampen the macroeconomic transmission of shocks that move inflation and output in opposite directions (*e.g.*, technology shocks).

## 4.2 The shadow banking system

### 4.2.1 Safer entrepreneurs

#### Profit maximization

At the beginning of period  $t$ , the representative  $l$ -th entrepreneur provides capital services to intermediate-good firms. Capital services,  $K_t^{L,l}$ , are related to the entrepreneur's stock of physical capital,  $\bar{K}_t^{L,l}$ , by  $K_t^{L,l} = u_t^{L,l} \bar{K}_t^{L,l}$ , where  $u_t^{L,l}$  denotes the level of capital utilization. In choosing the capital utilization rate, the entrepreneur takes into account the increasing and convex utilization cost function  $a(u_t^{L,l})$ , that denotes the cost, in units of final goods, of setting the utilization rate to  $u_t^{L,l}$ .<sup>13</sup>

Then, at the end of period  $t$ , the entrepreneur sells the undepreciated capital to capital producers at price  $Q_{\bar{K}',t}$ , pays the nominal coupon rate ( $R_t^{coupon}$ ) on bonds issued and purchases new capital from capital producers at price  $Q_{\bar{K}',t}$ . The capital acquisition is financed partly by his net worth,  $N_{t+1}^{L,l}$ , and partly by issuing new bonds. The amount of bonds issued,  $BI_{t+1}^{L,l}$ , is given by:

$$BI_{t+1}^{L,l} = Q_{\bar{K}',t} \bar{K}_{t+1}^{L,l} - N_{t+1}^{L,l} . \quad (2)$$

The entrepreneur's time- $t$  profits,  $\Pi_t^{L,l}$ , are given by:

$$\begin{aligned} \Pi_t^{L,l} = & \left[ u_t^{L,l} r_t^{k,L} - a(u_t^{L,l}) \right] \bar{K}_t^{L,l} P_t + (1 - \delta) Q_{\bar{K}',t} \bar{K}_t^{L,l} \\ & - Q_{\bar{K}',t} \bar{K}_{t+1}^{L,l} - R_t^{coupon} \left( Q_{\bar{K}',t-1} \bar{K}_t^{L,l} - N_t^{L,l} \right) , \end{aligned}$$

where  $r_t^{k,L}$  denotes the real rental rate,  $P_t$  the price of the final good and  $\delta$  the depreciation rate.

<sup>12</sup> Fisher (1933) emphasizes the “debt deflation” effect that arises when debt contracts are set in nominal terms. Other papers that analyze the debt-deflation effect include Iacoviello (2005) and Gerali et al. (2010).

<sup>13</sup> The functional form that we use is  $a(u_t^{L,l}) = \frac{r^{k,L}}{\sigma_a^L} \left[ \exp^{\sigma_a^L (u_t^{L,l}-1)} - 1 \right]$ , where  $r^{k,L}$  is the steady state value of the rental rate of capital,  $a(1) = 0$ ,  $a''(1) > 0$  and  $\sigma_a^L = a''(1)/a'(1)$  is a parameter that controls the degree of convexity of costs.

In period  $t$  the entrepreneur chooses the capital utilization rate and the desired capital to use in period  $t + 1$  so as to maximize  $\Pi_t^{L,l}$ , taking as given the coupon rate to be paid on the bonds issued. The first order conditions with respect to  $u_t^{L,l}$  and  $\bar{K}_{t+1}^{L,l}$  are, respectively:

$$r_t^{k,L} = a' \left( u_t^{L,l} \right) \quad (3)$$

$$Q_{\bar{k}',t} = \beta E_t \left\{ \left[ u_{t+1}^{L,l} r_{t+1}^{k,L} - a \left( u_{t+1}^{L,l} \right) \right] P_{t+1} + (1 - \delta) Q_{\bar{k}',t+1} - R_{t+1}^{coupon} Q_{\bar{k}',t} \right\} . \quad (4)$$

Equation (3) states that the rental rate on capital services equals the marginal cost of providing those services. As the rental rate increases it becomes more profitable to use capital more intensively up to the point where the extra profits match the extra utilization costs. The capital Euler equation (4) equates the value of a unit of installed capital at time  $t$  to the expected discounted return of that extra unit of capital in period  $t + 1$ .

The entrepreneur's equity at the end of period  $t$ ,  $V_t^{L,l}$ , is given by

$$V_t^{L,l} = \left\{ \left[ u_t^{L,l} r_t^{k,L} - a \left( u_t^{L,l} \right) \right] P_t + (1 - \delta) Q_{\bar{k}',t} \right\} \bar{K}_t^{L,l} - (1 + R_t^{coupon}) \left( Q_{\bar{k}',t-1} \bar{K}_t^{L,l} - N_t^{L,l} \right) .$$

The first term represents the rental income of capital, net of utilization costs, and the proceeds from selling undepreciated capital to capital producers. The second term represents the payment (coupon and principal) of the bonds issued in period  $t - 1$ .

To avoid a situation in which the entrepreneur accumulates enough net worth to become self-financed, we assume that there is a constant probability of death. Namely, in each period the entrepreneur exits the economy with probability  $1 - \gamma^L$ . In that case, the entrepreneur rebates his equity to households in a lump-sum way:

$$transfer \text{ to households} = (1 - \gamma^L) V_t^{L,l} .$$

To keep the entrepreneurs' population constant, a new entrepreneur is born with probability  $1 - \gamma^L$ .

The total entrepreneur's net worth  $N_{t+1}^{L,l}$  combines total equity and a transfer,  $W_t^{e,L,l}$ , received from households, which corresponds to the initial net worth necessary for the entrepreneur's activity to start. The law of motion for the entrepreneur's net worth is:

$$N_{t+1}^{L,l} = \gamma^L V_t^{L,l} + W_t^{e,L,l} .$$

### Financing cost minimization problem and funds demand curve

We assume that each investment bank  $z$  has some market power in conducting its intermediation services. An entrepreneur seeking an amount of borrowing for period  $t + 1$  equal to  $BI_{t+1}^{L,l}$ , defined by (2), would therefore allocate his borrowing among different investment banks,  $BI_{t+1}^{L,l}(z)$ , so as to minimize the total repayment due. At the end of period  $t$ , the entrepreneur chooses how much to borrow from bank  $z$  by solving the following problem:

$$\min_{BI_{t+1}^{L,l}(z)} \int_0^1 [1 + R_{t+1}^{coupon}(z)] BI_{t+1}^{L,l}(z) dz$$

subject to the Dixit-Stiglitz aggregator

$$BI_{t+1}^{L,l} = \left\{ \int_0^1 \left[ BI_{t+1}^{L,l}(z) \right]^{\frac{\varepsilon_{t+1}^{coupon} - 1}{\varepsilon_{t+1}^{coupon}}} dz \right\}^{\frac{\varepsilon_{t+1}^{coupon}}{\varepsilon_{t+1}^{coupon} - 1}},$$

where  $R_{t+1}^{coupon}(z)$  is the interest rate charged by the  $z$ -th bank and  $\varepsilon_{t+1}^{coupon} > 1$  is the time-varying interest rate elasticity of the demand for funds. The first order condition yields the following entrepreneur's demand for funds:

$$BI_{t+1}^{L,l}(z) = \left( \frac{1 + R_{t+1}^{coupon}(z)}{1 + R_{t+1}^{coupon}} \right)^{-\varepsilon_{t+1}^{coupon}} BI_{t+1}^{L,l},$$

where  $R_{t+1}^{coupon}$  is the nominal average coupon rate prevailing in the market at time  $t + 1$ , defined as:

$$1 + R_{t+1}^{coupon} = \left\{ \int_0^1 \left[ 1 + R_{t+1}^{coupon}(z) \right]^{1 - \varepsilon_{t+1}^{coupon}} dz \right\}^{\frac{1}{1 - \varepsilon_{t+1}^{coupon}}}.$$

As expected, the funds demand curve has a negative slope: when the interest rate that the  $z$ -th bank sets increases relatively to the average rate, the entrepreneur decides to borrow less funds from that bank.

#### 4.2.2 Investment banks

The investment banking sector comprises a continuum of monopolistic competitive investment banks, indexed by  $z \in [0, 1]$ , owned by households. To keep the analysis as simple as possible, we follow the recent DSGE banking literature and assume perfect competition in the market for households' deposits in these banks.<sup>14</sup> We also rule out the entry and exit of investment banks. The investment bank therefore maximizes its profits, taking as given the return to pay to the households. The appendix A shows that the required return on bonds by households is equal to the risk-free rate, *i.e.* the central bank nominal interest rate  $R_t^e$  (see equations A.10 and A.11).

At the end of period  $t$ , the  $z$ -th investment bank thus solves the following profit maximization problem:

$$\max_{R_{t+1}^{coupon}(z)} \Pi_{t+1}^{IB}(z) = \left\{ \left[ 1 + R_{t+1}^{coupon}(z) \right] BI_{t+1}^{L,l}(z) - \left[ 1 + R_{t+1}^e \right] BI_{t+1}^{L,l}(z) \right\} \quad (5)$$

$$\text{subject to} \quad BI_{t+1}^{L,l}(z) = \left( \frac{1 + R_{t+1}^{coupon}(z)}{1 + R_{t+1}^{coupon}} \right)^{-\varepsilon_{t+1}^{coupon}} BI_{t+1}^{L,l}.$$

The first order condition is

$$\left( \frac{1 + R_{t+1}^{coupon}(z)}{1 + R_{t+1}^{coupon}} \right)^{-\varepsilon_{t+1}^{coupon}} - \varepsilon_{t+1}^{coupon} \frac{1 + R_{t+1}^{coupon}(z) - (1 + R_{t+1}^e)}{1 + R_{t+1}^{coupon}} \left( \frac{1 + R_{t+1}^{coupon}(z)}{1 + R_{t+1}^{coupon}} \right)^{-\varepsilon_{t+1}^{coupon} - 1} = 0.$$

<sup>14</sup> See, for instance, Aliaga-Diaz and Olivero (2007), Andr s and Arce (2009), Kobayashi (2008) and Teranishi (2008).

Imposing a symmetric equilibrium and rearranging yields

$$1 + R_{t+1}^{coupon} = \frac{\varepsilon_{t+1}^{coupon}}{\varepsilon_{t+1}^{coupon} - 1} (1 + R_{t+1}^e) , \quad (6)$$

that is, the coupon rate is set as a markup,  $\frac{\varepsilon_{t+1}^{coupon}}{\varepsilon_{t+1}^{coupon} - 1}$ , over the policy interest rate. The profits of the investment banking sector in period  $t + 1$  are given by

$$\Pi_{t+1}^{IB} = (R_{t+1}^{coupon} - R_{t+1}^e) (1 - \eta) BI_{t+1}^{L,l} , \quad (7)$$

and are rebated to households.

Assuming that the interest rate elasticity of the demand for funds is constant,  $\varepsilon_{t+1}^{coupon} = \varepsilon^{coupon,a}$ , the coupon rate becomes a constant markup applied to the required return by households:

$$1 + R_{t+1}^{coupon,a} = \frac{\varepsilon^{coupon,a}}{\varepsilon^{coupon,a} - 1} (1 + R_{t+1}^e) . \quad (8)$$

In what follows, we consider  $R_{t+1}^{coupon,a}$  as the “normal” interest rate on bonds.

#### 4.2.3 Optimism and perverse incentives in the shadow banking sector

In this subsection we extend the model presented in the previous subsection in two respects. First, we introduce optimism among underwriters in the investment banking sector by considering that an optimistic underwriter is willing to underwrite bonds at a lower – than the normal – coupon interest rate. Second, we introduce perverse incentives by assuming that the representative safer entrepreneur offers side payments to the underwriter in order to borrow at a more favorable interest rate. In exchange of those side payments, an optimistic underwriter may *de facto* facilitate the extension of credit by setting a “discounted” – relatively to its normal value – bond coupon rate.

We do so by endogenizing the choice of the interest rate elasticity underlying the demand for funds,  $\varepsilon_{t+1}^{coupon}$ . Note, from (6), that an increase in the interest rate elasticity leads to, *ceteris paribus*, a lower coupon rate. This relation between the elasticity and the coupon rate allows us to separate and solve the investment bank’s profit maximization problem in two steps. First, the underwriter chooses the interest rate elasticity according to his preferences. Second, he solves the maximization problem (5), which leads to equation (6). In practice, after determining  $\varepsilon_{t+1}^{coupon}$ , the underwriter has implicitly determined the coupon rate that solves the investment bank’s profit maximization problem (5).

#### Optimism

First, we assume that the underwriter becomes optimistic if the entrepreneur pledges a higher value as collateral. We thus model underwriter’s optimism,  $\chi_t$ , as a positive function of the entrepreneur’s net worth. To take into account the fact that human beliefs are highly correlated and persistent (Carlson, 2007), we furthermore model optimism as an  $AR(1)$  process with high persistence. Accordingly, the

law of motion for optimism is given by

$$\chi_t = \rho_\chi \chi_{t-1} + (1 - \rho_\chi) \left[ \bar{\chi} + \alpha_3 \left( N_{t+1}^{L,l} - N^{L,l} \right) \right] , \quad (9)$$

where  $\bar{\chi}$ ,  $\bar{\chi} = 0$ , is the steady state level of optimism,  $\rho_\chi$  captures the degree of persistence in optimism and  $\alpha_3 > 0$  the sensitivity of optimism with respect to the deviation of the entrepreneur's net worth from its steady state value ( $N^{L,l}$ ).

Second, we assume that the interest rate elasticity of the demand for funds is computed as follows:

$$\varepsilon_{t+1}^{coupon,biasd} = \varepsilon^{coupon,a} (1 + \chi_t) , \quad (10)$$

which means that positive deviations of optimism from its steady state level increase the interest rate elasticity of the demand for funds, relatively to its normal value of  $\varepsilon^{coupon,a}$ . The biased elasticity results in a lower coupon rate, which may be seen substituting (10) into (6), yielding the following expression

$$1 + R_{t+1}^{coupon,biasd} = \frac{\varepsilon_{t+1}^{coupon,biasd}}{\varepsilon_{t+1}^{coupon,biasd} - 1} (1 + R_{t+1}^e) , \quad (11)$$

where  $R_{t+1}^{coupon,biasd}$  is the biased coupon rate that an optimistic underwriter would set on the bonds issued. Comparing (11) and (8), it is clear that the optimistic underwriter would underwrite bonds at a lower than the normal interest rate.

The coupon rate set by the underwriter ( $R_{t+1}^{coupon}$  in 6) thus varies from a maximum of  $R_{t+1}^{coupon,a}$  (corresponding to  $\varepsilon_{t+1}^{coupon,a}$ ) to a minimum of  $R_{t+1}^{coupon,biasd}$  (corresponding to  $\varepsilon_{t+1}^{coupon,biasd}$ ). In between these extremes, the value of the coupon rate chosen corresponds to a specific value of  $\varepsilon_{t+1}^{coupon}$ . In the next subsection we thus describe how such interest rate elasticity and, as a consequence, the bond coupon rate, are determined.

### Perverse incentives and the optimal choice of the coupon rate

Suppose that the entrepreneur offers side payments to the underwriter in order to borrow at a more favorable coupon rate, *i.e.* at an interest rate lower than the normal rate  $R_{t+1}^{coupon,a}$  defined by (8). Suppose also that households are not aware of this possibility. We assume that the amount of side payments paid to the underwriter at the end of period  $t + 1$  is given by

$$side\ payments_{t+1} = \Omega \left( R_{t+1}^{coupon,a} - R_{t+1}^{coupon} \right) V_{t+1}^{L,l} , \quad (12)$$

that is, side payments represent a fixed share,  $\Omega$ , of the entrepreneurial equity and are proportional to the difference between  $R_{t+1}^{coupon,a}$  and  $R_{t+1}^{coupon}$ . In principle, the underwriter should ignore these side payments and protect stockholders' interests, that is, the underwriter should maximize the bank's profits setting  $R_{t+1}^{coupon} = R_{t+1}^{coupon,a}$ . In that case, as equation (12) shows, he would not receive any side payments. However, the underwriter may alternatively choose to underwrite a bond at a lower rate ( $R_{t+1}^{coupon} < R_{t+1}^{coupon,a}$ ) benefitting from those side payments. Clearly, side payments lead to an agency conflict within the investment bank, between its stockholders (*i.e.*, households) and its staff (*i.e.*, the

underwriters). Note that the lower the  $R_{t+1}^{coupon}$  (compared to  $R_{t+1}^{coupon,a}$ ), the higher will be the side payments that the underwriter receives and the lower will be the stockholders' return for a given level of  $BI_{t+1}^{L,l}$  (as given by equation 7).<sup>15</sup>

The top part of figure 2 sketches the trade-off faced by the underwriter – maximization of his own benefit versus maximization of stockholders' profits. To endogenize the choice of  $\varepsilon_{t+1}^{coupon}$ , we model this trade-off considering the following quadratic utility function for the underwriter:

$$u(\varepsilon_{t+1}^{coupon}) = -r_2 \left( \varepsilon_{t+1}^{coupon,bias} - \varepsilon_{t+1}^{coupon} \right)^2 - (1 - r_2) \left( \varepsilon_{t+1}^{coupon} - \varepsilon_{t+1}^{coupon,a} \right)^2, \quad 0 \leq r_2 \leq 1. \quad (13)$$

The first part mirrors the underwriter's private objective to maximize the amount of side payments received. Recall that, from equation (12), side payments are maximized when  $R_{t+1}^{coupon}$  is as low as possible, that is, when  $R_{t+1}^{coupon} = R_{t+1}^{coupon,bias}$ . This happens when  $\varepsilon_{t+1}^{coupon} = \varepsilon_{t+1}^{coupon,bias}$ . Parameter  $r_2$  represents the importance the underwriter attaches to his private objective. The second part displays the underwriter's objective to maximize stockholders' profits by setting an  $\varepsilon_{t+1}^{coupon}$  that is as close as possible to  $\varepsilon_{t+1}^{coupon,a}$ . This objective enters the underwriter's utility function with a weight of  $(1 - r_2)$ .

The underwriter chooses  $\varepsilon_{t+1}^{coupon}$  so as to maximize (13). The first order condition is:

$$2r_2 \left( \varepsilon_{t+1}^{coupon,bias} - \varepsilon_{t+1}^{coupon} \right) - 2(1 - r_2) \left( \varepsilon_{t+1}^{coupon} - \varepsilon_{t+1}^{coupon,a} \right) = 0.$$

Using (10) and rearranging, the first order condition then becomes

$$\varepsilon_{t+1}^{coupon} = \varepsilon_{t+1}^{coupon,a} (1 + r_2 \chi_t). \quad (14)$$

Substituting (14) into (6) yields the following expression for the coupon interest rate

$$1 + R_{t+1}^{coupon} = \frac{\varepsilon_{t+1}^{coupon,a} (1 + r_2 \chi_t)}{\varepsilon_{t+1}^{coupon,a} (1 + r_2 \chi_t) - 1} (1 + R_{t+1}^e). \quad (15)$$

The coupon rate is therefore a time-varying markup,  $\frac{\varepsilon_{t+1}^{coupon,a} (1 + r_2 \chi_t)}{\varepsilon_{t+1}^{coupon,a} (1 + r_2 \chi_t) - 1}$ , over the policy rate and is influenced both by the level of optimism and by the weight that the underwriter attaches to his private benefit. As a result, the optimistic underwriter may *de facto* set a coupon rate for the issued bonds that is lower than the rate that maximizes the bank's profits in the context of no optimism and no side payments. Note, in particular, that it is the combination of underwriter's optimism and his willingness to receive side payments ( $\chi_t > 0$  and  $r_2 > 0$ ) that leads to a discounted coupon rate.

<sup>15</sup> If there are side payments, before knowing whether exiting the economy, the entrepreneur transfers a share  $\Omega(R_t^{coupon,a} - R_t^{coupon})$  of his equity to the underwriter as side payments. After that, with probability  $1 - \gamma^L$  the entrepreneur exits the economy and rebates his equity to households in a lump-sum way:

$$transfer\ to\ households = (1 - \gamma^L) [1 - \Omega(R_t^{coupon,a} - R_t^{coupon})] V_t^{L,l}.$$

Entrepreneurial net worth is thus given by

$$N_{t+1}^{L,l} = \gamma^L [1 - \Omega(R_t^{coupon,a} - R_t^{coupon})] V_t^{L,l} + W_t^{e,L,l}.$$

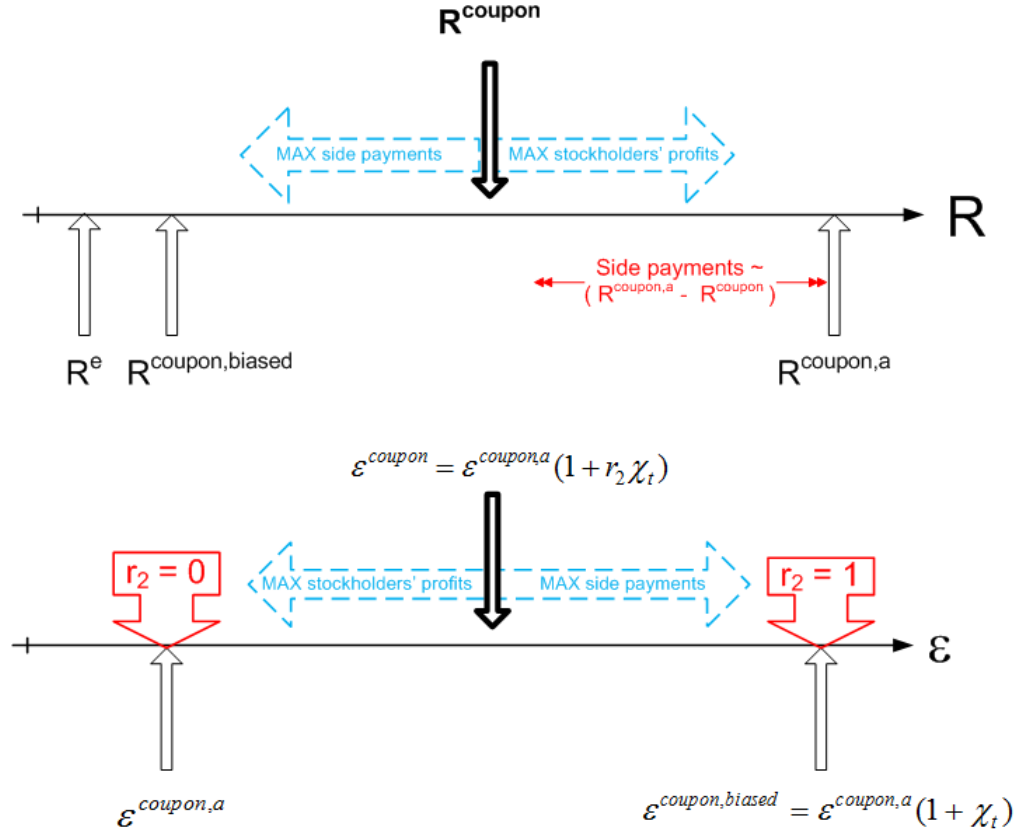


Figure 2: The underwriter's trade-off

The bottom part of figure 2 shows that the exact value of the coupon rate that is chosen – corresponding to a unique value of  $\varepsilon^{coupon}$  – depends upon the value of  $r_2$  and the degree of optimism.

## 5 The response to monetary policy shocks

Having presented the model, we now analyze its dynamics. We first describe the calibration of the model and then present the impulse responses to two types of monetary policy shocks.

The central bank sets the short-term nominal interest rate,  $R_t^e$ , following a Taylor-type interest rate rule. Specifically, the monetary policy rule allows for interest rate smoothing and interest rate responses to deviations of expected inflation ( $E_t \pi_{t+1}$ ) and current output ( $Y_t$ ) from their steady states:

$$R_t^e = (R_{t-1}^e)^{\tilde{\rho}} \left[ R^e \left( \frac{E_t \pi_{t+1}}{\bar{\pi}} \right)^{\alpha_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\alpha_y} \right]^{(1-\tilde{\rho})} \varepsilon_t^{MP}, \quad (16)$$

where  $R^e$ ,  $\bar{\pi}$  and  $\bar{Y}$  are the steady state values of  $R_t^e$ ,  $\pi_t$  and  $Y_t$ , respectively,  $\alpha_\pi$  and  $\alpha_y$  are the weights assigned to expected inflation and output,  $\tilde{\rho}$  captures interest rate smoothing and  $\varepsilon_t^{MP}$  is a white noise monetary policy shock.



We first solve numerically the model, for the steady state, using the computational procedure described in appendix C. We then compute the first-order Taylor series approximation to the equilibrium conditions in the neighborhood of the steady state.<sup>16</sup>

We compare the responses to monetary impulses under three different variants of the above-described model:

- variant 1: the simplified version of the *Financial Accelerator Model* of CMR, which corresponds to setting the share of riskier entrepreneurs  $\eta$  equal to 1;
- variant 2: our model including the shadow banking system but excluding optimism and side payments, which is obtained setting  $r_2 = 0$ ;
- variant 3: our model including optimism and side payments in the shadow banking system, assuming  $r_2 = 1$ , *i.e.* the version in which the underwriter only cares about his own benefit and maximizes the amount of side payments received.

Recall from subsection 4.1 that the transmission mechanism in variant 1 is affected by two general equilibrium mechanisms. The first one is the Bernanke et al. (1999) “financial accelerator” effect, whereby induced changes in asset prices alter the value of the collateral that the entrepreneur can pledge and, hence, the contractual loan rate. The second mechanism is a CMR-type “Fisher deflation” effect, whereby unexpected movements in the price level alter the *ex-post* real burden of entrepreneurial debt and, hence, the entrepreneur’s net worth.

To these two channels, variant 2 adds a new set of monopolistic investment banks. The monopolistic power in setting bond interest rates affects the credit supply conditions of a set of entrepreneurs through the introduction of a constant interest rate spread. Hence, this variant allows us to analyze whether the interest-rate-setting by banks interacts with the aforementioned channels and to what extent it does modify the monetary transmission mechanism.<sup>17</sup>

Finally, variant 3 adds optimism and perverse incentives to the “monopolistic banking competition” effect. In this variant, the underwriter diverts a fraction of stockholders’ profits for his own benefit and extends credit at a lower interest rate. In this framework, the behavior of the underwriter influences the credit supply conditions of a set of borrowers, which in turn influences the real economy through a countercyclical time-varying interest rate spread. This variant allows us to study the importance of the bank manager’s behavior in shaping the monetary transmission mechanism, which has been virtually ignored in the literature so far.

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<sup>16</sup> All simulations in this paper have been conducted with Dynare. Codes are available from the corresponding author upon request.

<sup>17</sup> We should note that, as far as we know, at most two out of these three effects – “financial accelerator”, “Fisher deflation” and “monopolistic banking competition” – have been analyzed within a single model. As regards studies that address only one of the effects, when compared to standard models with frictionless financial markets, CMR show that the “financial accelerator” effect, as well as the “Fisher deflation” effect, amplify and propagate the transmission of monetary policy shocks, while Andrés and Arce (2009) find that monopolistic competition in the banking sector dampens the macroeconomic transmission of policy shocks. As regards studies that combine two of these effects, Iacoviello (2005) and CMR show that the “accelerator” and the “Fisher” effect reinforce each other in what concerns the response of the economy to monetary policy shocks, whereas Mandelman (2010) finds that the assumption of an imperfectly competitive banking system in the Bernanke et al. (1999) framework magnifies the propagation and amplification of policy shocks to the economy.

We conduct two policy experiments. The first experiment is a one-period expansionary monetary policy shock. It allows us to assess whether (and how) the transmission mechanism of monetary policy is affected by the presence of a shadow banking system using as a benchmark the impulse responses of a workhorse DSGE model (variant 1).

In the second experiment, we create a “persistently low interest rate” scenario by forcing the nominal interest rate to be 25 basis points lower than its steady state value during 8 quarters. This experiment allows for a) determining if an extended period of loose monetary policy generates *per se* a boom-bust cycle and b) analyzing whether the interaction between long periods of accommodative monetary policy and perverse incentives in the financial sector causes and/or amplifies fluctuations in real and financial activity.

The next subsection briefly describes the calibration, before turning to the analysis of the impulse response functions in subsections 5.2 and 5.3.

## 5.1 Calibration

The model is calibrated to the U.S. economy, assuming that a period is a quarter. The values are chosen so that the model’s steady state reproduces some key features in the U.S. data. In this subsection we only describe the calibration of the parameters related with the shadow banking system. The values of the remaining parameters are calibrated within the range usually considered in the New Keynesian literature.<sup>18</sup> Table 1 reports the values of the calibrated parameters, and tables 2 and 3 report the steady state implications of the model and their empirical counterparts.

To match the return on time deposits (which is also equal to the steady state central bank nominal interest rate), we set the discount factor  $\beta$  to 0.9875. Equation (6) shows that the steady state spread between the coupon rate and the risk-free rate (the yield spread) depends on the interest rate elasticity  $\varepsilon^{coupon}$ . Chen et al. (2007) report an average annual yield spread of AAA bonds of 84 basis points. Accordingly, we set  $\varepsilon^{coupon}$  to 510 so that the annual yield spread is around 80 basis points. As a result, the coupon rate paid by safer entrepreneurs is 5.9%/year.

To match the observed average leverage ratio, we set the survival probability of safer entrepreneurs  $\gamma^L$  to 0.96. In the law of motion for optimism (9), we set the persistence parameter  $\rho_\chi$  to 0.9 and the sensitivity to entrepreneur’s net worth  $\alpha_3$  to 40.<sup>19</sup>

The parameter  $\Omega$  (the fraction of equity that the entrepreneur is willing to pay as side payments) is chosen so as to guarantee that the entrepreneur is always better off when he pays side payments. In principle, the safer entrepreneur may choose between two options. He can either pay the coupon rate  $R_{t+1}^{coupon,a}$  given by equation 8 or he can offer side payments and obtain a lower coupon rate ( $R_{t+1}^{coupon}$

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<sup>18</sup> The values of the parameters related with the riskier entrepreneurial sector are primarily chosen to match the cost of external finance, *i.e.*, the contractual, no-default interest rate on entrepreneurial debt ( $Z_t$  resulting from A.7). Setting the fraction of realized payoffs lost in bankruptcy,  $\mu$ , to 0.15 and the standard deviation of the entrepreneur idiosyncratic productivity shock,  $\sigma$ , to 0.55 yields  $Z = 6.8\%/year$ , which is close to observed data. This in turn also guarantees that, in equilibrium, bond financing is cheaper than bank financing (safer entrepreneurs finance themselves at a more favorable interest rate). To match the observed leverage ratio, we set the survival rate  $\gamma^H$  to 0.97.

<sup>19</sup> This calibration guarantees that  $\varepsilon_{t+1}^{coupon} = \varepsilon^{coupon,a} (1 + r_2 \chi_t) > 1 \forall t$ .

given by equation 15). This choice depends on the value of  $\Omega$ . Given our baseline calibration, in appendix D we show that, in the steady state, the entrepreneur is better off whenever  $\Omega$  is smaller than a threshold level  $\bar{\Omega} = 0.25$ . Accordingly, we set  $\Omega$  to 0.1, thus assuming that the entrepreneur gives away 10 % of his equity as side payments to obtain a lower coupon rate.

Finally, we calibrate the parameter  $\eta$  (the share of riskier entrepreneurs in the economy) by replicating the ratio of bond finance to bank finance in the U.S. economy which, according to De Fiore and Uhlig (2005), is equal to 1.34. We closely match this ratio by setting  $\eta$  to 0.3.

As tables 2 and 3 show, the model is successful in reproducing most of the salient features of the U.S. economy: key macroeconomic and leverage ratios, interest rates and, importantly, its financial market structure.

## 5.2 The economy's response to an unanticipated one-period expansionary monetary policy shock

In this subsection we study the transmission of a monetary policy shock by analyzing the impulse responses to a one-period innovation in the short-term nominal interest rate ( $\varepsilon_t^{MP}$  in 16), corresponding to a 25 basis points reduction of the annualized nominal interest rate. Figures 3-5 illustrate the impulse responses of the key variables under the three variants of the model (variant 1: blue solid line; variant 2: red crossed line; and variant 3: black circled line).

In all figures presented, variables are expressed in percent deviation from their steady state values, except for inflation, that is expressed as annualized percent deviation from its steady state, and interest rates, that are expressed in percentage points at annual rate. The horizontal axis represents time on a quarterly scale.

The responses of aggregate variables in variant 1 are qualitatively standard. After the initial drop, the nominal interest rate gradually returns to its steady state value. Aggregate quantities – output, consumption and investment – as well as inflation display a hump-shaped response and peak after about three to six quarters. The price of capital shows maximum upward reactions at impact before returning to its steady state. The effects on aggregate variables are long-lived despite the fact that the effects on the nominal interest rate only last for roughly two years.

Overall, the response of lending activity is weak at the aggregate level: although entrepreneurs accumulate more capital (*stock of capital*  $\uparrow$ ), the sharp increase in the aggregate net worth ( $N$   $\uparrow$ ) leads to a decrease of total credit ( $q\bar{K} - N$ ) below its steady state level.

While in most cases the responses in variant 2 are pretty similar to those in variant 1, it is notable that the impact of the monetary policy shock is somewhat dampened under this variant. We find, in line with other studies, that the introduction of market power in banking results in smoother effects. A striking difference is evident, however, when we compare the responses in variant 3 with those in the other two variants of the model. First, the business cycle is amplified – in particular, the peak in investment is two times greater than under variant 2. Second, at its height, the response of investment is roughly twice as big, in percent terms, as the response of output (while it is nearly the same in

the other variants). Finally, and in contrast with the other two variants of the model, total credit increases: the rise in aggregate entrepreneurial capital purchases more than compensates the more pronounced (compared to the other variants) increase in the aggregate net worth, so that the net effect is an increase of total credit above its steady state value. This can be explained by analyzing each type of entrepreneur separately.

Therefore, turning to the variables specific to the entrepreneurial sector, we conclude that under the three variants of the model the riskier entrepreneur's net worth increases in response to the shock because of both the "accelerator" and the "Fisher" effects. The rise in the price of capital leads to a boost in the value of the assets of the entrepreneur, which in turn reduces the probability of bankruptcy ( $\bar{\omega} \downarrow$ ). Moreover, because of the drop in entrepreneur's leverage, retail bank charges a lower interest rate on loans ( $Z \downarrow$ ). This reflects the fact that the cost of external financing depends on the borrower's leverage: as predicted by equation (1), all else equal, the lower the leverage, the lower the external finance premium, hence the lower the interest rate on loans. This "accelerator" effect is then reinforced by the "Fisher" effect: the *ex-post* value of existing entrepreneurial debt decreases as inflation rises. As a consequence, the entrepreneur's net worth increases further.

In both variants 2 and 3, the monetary policy shock leads to a lower coupon rate paid by the safer entrepreneur. Under variant 2 the coupon rate is set as a constant markup over the policy rate (recall equation 8), therefore  $R^{coupon}$  follows the nominal interest rate path. In variant 3, however, the coupon rate also depends on the underwriter's behavior (equation 15). In particular, the rise in the price of capital increases the value of the collateral held by the entrepreneur (*net worth*  $\uparrow$ ), which in turn triggers optimism (equation 9). The underwriter's optimism, combined with his willingness to receive side payments ( $r_2 = 1$ ), leads to a drop in the bond coupon rate larger than that in variant 2. As figure 5 shows,  $R^{coupon}|_{variant\ 3}$  is smaller than  $R^{coupon}|_{variant\ 2}$  for about 20 quarters, that is, the underwriter persistently extends credit at a lower interest rate in exchange of side payments.

Thus, in both variants 2 and 3, the monetary policy shock leads to a lower borrowing cost for both types of entrepreneurs – both  $Z$  and  $R^{coupon}$  decrease. However, note that the underwriter's behavior and preferences influence the spread between the cost of financing for the riskier entrepreneur and the coupon rate on bonds for the safer entrepreneur ( $Z - R^{coupon}$ , in figure 5). This spread in turn strongly influences the allocation of funds between safer and riskier entrepreneurs and, consequently, total capital and total credit dynamics.

In fact, in variant 2 the drop in  $Z$  is larger than the drop in  $R^{coupon}$  ( $Z - R^{coupon} \downarrow$ ). That is, financing in the loan market becomes relatively cheaper than funding in the bond market. As a result, there is an increase in the amount of borrowing from retail banks (*loans*  $\uparrow$ ) and a reduction in the flow of funds to the safer entrepreneur (*bond amount*  $\downarrow$ ). Differently, in variant 3,  $Z - R^{coupon}$  increases, *i.e.* given the marked reduction in the coupon rate due to optimistic behavior, bond financing becomes relatively cheaper than bank financing, leading safer entrepreneurs to invest more (*bond amount*  $\uparrow$ ), while the amount of borrowing from retail banks drops (*loans*  $\downarrow$ ). Riskier entrepreneur thus prefer to use capital more intensively (when compared with variant 2). Overall, the increase in capital stock is higher than under variant 2 since the increase in bonds issued more than offsets the decrease of loan amount.

These findings suggest that financial market frictions alone – in the form of monopolistic competition in banking system – do not change significantly the model’s dynamics, whereas the behavior of the financial intermediary – driven by optimism and perverse incentives – does play a role in the transmission of the monetary policy shock: the effects of monetary policy on real and financial activity are in fact amplified in the variant of the model in which the financial intermediary plays a more active role.

### 5.3 The economy’s response in a “persistently low interest rate” scenario

At the macroeconomic level, it has been recognized that accommodative monetary policies have historically been a key factor in driving boom-bust cycles of all types.<sup>20</sup> Although the low level of the federal funds rate in the early 2000s is generally considered to have helped fuel the housing bubble that burst in 2007, it is still an open debate whether lax monetary policies played a key role in generating the boom-bust cycle.<sup>21</sup>

In addition to and interacting with the low interest rate environment prevailing at the beginning of the 2000s, microeconomic factors related to recent innovations in the financial market structure and products may have also contributed to the subprime crisis. Even though the interaction between microeconomic distortions in the financial sector and a persistently loose monetary policy environment seems to have been relevant in generating and/or amplifying the boom-bust cycle, the relative importance of each of these factors is still open to debate.

Our model is well-suited to analyze the interaction between long periods of accommodative monetary policy and financial market distortions, as well as to disentangle their relative importance. To do so, in this subsection we create a “persistently low interest rate” scenario and analyze the model’s dynamics. We reproduce such a scenario by combining the right sequence of monetary policy innovations ( $\varepsilon_t^{MP}$  in equation 16) in order to hold the nominal interest rate 25 basis points lower than its steady state value during 8 quarters. There are thus eight consecutive monetary policy shocks, each coming as a surprise to the agents. The overall impulse responses are then obtained by summing up the responses to each of the successive monetary policy shocks.

Figures 6-8 display the impulse responses of several variables under the three variants of the model. By construction, the nominal interest rate deviates from its steady-state value by 25 basis points during 8 quarters. Then, from period 9 onwards, its dynamics is governed by the Taylor rule with response to deviations of expected inflation and current output from their respective steady states. In period 8, inflation and output are well above their steady state values. Hence, starting from period 9, the nominal interest rate rises and gradually reverts to its steady state value.

The dynamic responses of aggregate variables are qualitatively similar across the three variants of the model. Output, investment, consumption, inflation and the price of capital rise until period 8. The subsequent monetary tightening leads to a contraction of output, consumption and investment and a rapid decline in the price of capital.

<sup>20</sup> See, for instance, Bordo (2008) and Calomiris (2008).

<sup>21</sup> The Fed funds rate was gradually reduced from around 6.50% in November 2000 to around 1.75% in December 2001 and was kept at that level until December 2002. Then, after two policy interventions (November 2002 and June 2003), it was reduced and kept to 1% until June 2004.

Nevertheless, variant 3 exhibits responses that are quantitatively different. The effects of monetary policy shocks on the real economy are considerably amplified – the peak in output is about 35% higher and the response in investment is about twice as large as that in the other variants of the model. The effects on financial variables are magnified as well. The percentage increase in the price of capital, at its peak, is roughly the double of the increase that occurs in the other variants. Moreover, after the initial jump, the price of capital rises 60% during the boom phase, which is more than four (eleven) times the increase in variant 1 (2) during the same period. Note also that the pattern of the price of capital mimics the typical shape of an asset price bubble: the large and rapid asset price increase is followed by a burst and then a collapse.

The most striking difference which, of course, underlies the dynamics of the other aggregate variables, is that, whereas lending activity is weak at the aggregate level under variants 1 and 2, the persistent monetary easing leads to a lending boom in variant 3 that lasts well after the roughly 4 years it takes for the effects on the nominal interest rate to die away. Looking at the entrepreneurial variables, it is evident that the boom in total credit is driven by the safer entrepreneur’s demand for funds (*bond amount*  $\uparrow$ ).

Figure 8 allows us to trace the monetary transmission mechanism in variant 3. The rise in the price of capital leads to a boost in the safer entrepreneur’s net worth, which in turn triggers an optimistic sentiment by the underwriter. This optimism, when combined with the underwriter’s willingness to receive side payments – as well as with the increase in the amount of these payments induced by the increase in the entrepreneur’s net worth (equation 12) – leads the underwriter to set a significantly lower coupon rate on the bonds issued. In particular, the discount relative to the “normal” rate occurs on impact and continues further during the period of persistently low interest rate, as  $R^{coupon}$  is well below the normal coupon rate  $R^{coupon,a}$  and further declines as time goes on. The protracted opportunity for the safer entrepreneur to have access to an abnormally cheap source of funds – both in absolute terms ( $R^{coupon}$   $\downarrow$ ) and relatively to the cost of borrowing in the loan market ( $Z - R^{coupon}$   $\uparrow$ ) – leads him to accumulate capital aggressively. As a result, the safer entrepreneur’s demand for capital rises, pushing up aggregate demand and causing a boom in the price of capital. The rise in safer entrepreneur’s capital purchases more than compensates the increase in his net worth, so that the net effect is an increase of bond issued much above its steady state value. Finally, in general equilibrium, relatively higher borrowing cost for the riskier entrepreneur (when compared to bond coupon rate) induces him to cut capital expenditures and to use his capital more intensively.

Overall, these findings indicate that a persistently loose monetary policy does not cause *per se* a boom-bust cycle. In fact, neither in the CMR model, nor in its augmented version with monopolistic banking competition, a “too low for too long” interest rate policy generates a boom-bust cycle. However, monetary policy does create the preconditions for a boom-bust: optimism and perverse incentives in the financial sector, when coupled with a persistently low interest rate environment, result in greatly amplified fluctuations in both real and financial variables.<sup>22</sup>

<sup>22</sup> We have checked the sensitivity of our results to changes in the values of parameters  $\eta$  and  $\Omega$ . We have considered three different values of  $\eta$ , namely 0.368, 0.45 and 0.81, which imply a bond to bank finance ratio of, respectively, 1, 0.7 and 0.13 (the latter value reproduces the financial market structure of the Euro Area). Although the qualitative responses are quite similar to those of the baseline calibration, quantitatively the effects on both real and financial variables become dampened as bank financing becomes more important. Finally, changing the value of parameter  $\Omega$

## 6 Conclusion

This paper analyzes whether long periods of loose monetary policy play a key role in generating a boom-bust cycle, as well as the role of perverse incentives in the financial sector in causing and/or amplifying fluctuations in real and financial activity during periods of accommodative monetary policy.

Starting from a model that nests most contemporary DSGE monetary models, we have introduced a microfounded bond market comprised of a monopolistically competitive investment banking sector. The underwriter within the investment bank, who sets the coupon rate on the bonds issued either as a constant markup over the nominal interest rate, or at a discounted rate due to the likelihood of receiving side payments, is the pivotal agent in our model.

We have first analyzed the responses to a one-period expansionary monetary policy shock. The results show that financial market frictions alone – in the form of monopolistic competition in the banking sector – do not change significantly the model’s dynamics (when compared with a workhorse DSGE model). Yet, the effects of monetary policy on economic activity are amplified in the model in which the underwriter facilitates the extension of credit when optimism and perverse incentives are taken into account.

We have then simulated a “persistently low interest rate” scenario by keeping the central bank nominal interest rate 25 basis points below its steady state value for 8 quarters. Our main result is that a “too low for too long” interest rate policy does create the preconditions for, but does not cause *per se*, a boom-bust cycle. In fact, fluctuations in both real and financial variables are markedly amplified only when optimism and perverse incentives in the financial sector are coupled with such a persistently accommodative monetary policy environment. These findings suggest that, to reduce the odds of future booms, busts and asset price bubbles, policymakers should focus on tuning the financial architecture and reinforcing the financial supervision to restrain optimistic behaviors and perverse incentives. In doing so, policymakers will protect the financial system and the economy as a whole from the negative and often disruptive effects associated with economic booms and busts.

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does not affect the overall dynamics of the model, as long as  $\Omega < \bar{\Omega}$ .

## Appendix A - The complete model

### Final-good firms

Perfectly competitive firms produce the final good that is converted into household consumption goods, investment goods, government goods, goods used up in capital utilization and in bank monitoring as well as for underwriter consumption goods.

The representative firm produces the final good  $Y_t$ , using the intermediate goods  $Y_{i,t}$ , and the production technology

$$Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f},$$

where  $\lambda_f, \infty > \lambda_f \geq 1$ , is the markup for the intermediate-good firms. The representative firm chooses  $Y_{i,t}$  to maximize its profits, taking the output price,  $P_t$ , and the input prices,  $P_{i,t}$ , as given. The maximization problem of the representative firm is thus given by:

$$\begin{aligned} \max_{Y_{i,t}} \quad & P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \\ \text{subject to} \quad & Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}. \end{aligned}$$

Solving the profit maximization problem yields the following demand function for the intermediate good  $i$

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_f}{1-\lambda_f}} Y_t.$$

Perfect competition in the final goods market implies that the price of the final good can be written as:

$$P_t = \left[ \int_0^1 P_{i,t}^{\frac{1}{1-\lambda_f}} di \right]^{1-\lambda_f}. \quad (\text{A.1})$$

### Intermediate-good firms

Monopolistic competitive firms, indexed by  $i \in [0, 1]$ , produce differentiated intermediate goods using the following production function:

$$Y_{i,t} = (K_{i,t})^\alpha (L_{i,t})^{1-\alpha}, \quad (\text{A.2})$$

where  $0 < \alpha < 1$  and  $K_{i,t}$  and  $L_{i,t}$  denote, respectively, the capital and labor input for the production of good  $i$ .

The capital input is assumed to be a composite of two entrepreneur-specific bundles of capital services,  $K_{i,t}^H$  and  $K_{i,t}^L$  which in turn combine the capital services of the individual members of the two



entrepreneur sectors,  $K_{i,t}^{H,r}$  and  $K_{i,t}^{L,l}$ . Formally,

$$K_{i,t} = \left[ \eta^{1-\rho} (K_{i,t}^H)^\rho + (1-\eta)^{1-\rho} (K_{i,t}^L)^\rho \right]^{\frac{1}{\rho}}, \quad (\text{A.3})$$

where  $\rho$  denotes the degree of substitutability between the two entrepreneur-specific bundles of capital services and, since all entrepreneurs are identical within each group,  $K_{i,t}^H = \eta K_{i,t}^{H,r}$  and  $K_{i,t}^L = (1-\eta) K_{i,t}^{L,l}$ .

The  $i$ -th firm hires labor and rents capital in competitive markets by minimizing its production costs, taking as given the nominal wage rate,  $W_t$ , and the real rental rates of capital,  $r_t^{k,H}$  and  $r_t^{k,L}$ . The firm  $i$ 's optimal demand for capital and labor services must thus solve the following cost minimization problem:

$$\min_{\{L_{i,t}, K_{i,t}^H, K_{i,t}^L\}} C(\cdot) = \frac{W_t L_{i,t}}{P_t} + K_{i,t}^H r_t^{k,H} + K_{i,t}^L r_t^{k,L} \quad (\text{A.4})$$

subject to (A.2) and (A.3).

Since all firms  $i$  face the same input prices and since they all have access to the same production technology, real marginal costs  $s_t$  are identical across firms and are given by

$$s_t = \left[ \frac{\tilde{w}_t}{1-\alpha} \right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left[ \frac{\alpha}{r_t^{k,H}} (K_t^{H,r})^{\rho-1} \right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}} (Y_t)^{\frac{\alpha(\rho-1)}{\rho+\alpha(1-\rho)}} \frac{\rho}{\rho+\alpha(1-\rho)},$$

where  $\tilde{w}_t$  denotes the real wage.

## Price setting

Prices are determined through a variant of the Calvo's (1983) mechanism. In particular, every firm faces a constant probability,  $1 - \xi_p$ , of reoptimizing its price in any given period, whereas the non-reoptimizing firms set their prices according to the indexation rule

$$P_{i,t} = P_{i,t-1} (\bar{\pi})^{\iota_1} (\pi_{t-1})^{1-\iota_1},$$

where  $\bar{\pi}$  represents the steady state inflation rate,  $\pi_{t-1} = P_{t-1}/P_{t-2}$  is the inflation rate from  $t-2$  to  $t-1$  and the parameter  $\iota_1$ ,  $0 \leq \iota_1 \leq 1$ , represents the degree of price indexation to steady state inflation. The  $i$ -th firm that optimizes its price at time  $t$  chooses  $P_{i,t} = \tilde{P}_{i,t}$  that maximizes the present value of future expected nominal profits:

$$\max_{P_{i,t}} \Pi_t^{IGF} = E_t \sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \lambda_{t+\tau} [(P_{i,t+\tau} - S_{t+\tau}) Y_{i,t+\tau}]$$

$$\text{subject to} \quad Y_{i,t+\tau} = \left( \frac{P_{i,t+\tau}}{P_{t+\tau}} \right)^{\frac{\lambda_f}{1-\lambda_f}} Y_{t+\tau},$$

where  $E_t$  denotes the mathematical expectations operator conditional on information available at time  $t$ ,  $\lambda_{t+\tau}$  the multiplier in the households' budget constraint,  $S_{t+\tau}$  the firm's nominal marginal cost and  $\beta \in (0, 1)$  the discount factor. At the end of each time period, profits are rebated to households.

We consider only the symmetric equilibrium at which all firms choose the same  $\tilde{P}_t = \tilde{P}_{i,t}$ . Thus, from (A.1), the law of motion for the aggregate price index is

$$P_t = \left\{ (1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left[ P_{t-1} (\bar{\pi})^{\iota_1} (\pi_{t-1})^{1-\iota_1} \right]^{\frac{1}{1-\lambda_f}} \right\}^{1-\lambda_f} .$$

## Capital producers

A continuum of competitive capital producers produce the aggregate stock of capital  $\bar{K}_t$ . New capital produced in period  $t$  can be used in productive activities in period  $t+1$ . At the end of period  $t$ , capital producers purchase existing capital,  $x_{K,t}$ , from entrepreneurs and investment goods in the final good market,  $I_t$ , and combine them to produce new capital,  $x'_{K,t}$ , using the following technology:

$$x'_{K,t} = x_{K,t} + A(I_t, I_{t-1}) .$$

Old capital can be converted one-to-one into new capital, while the transformation of the investment good is subject to quadratic adjustment costs. The function  $A(\cdot)$  summarizes the technology that transforms current and past investment into installed capital.

Investment goods are purchased in the final good market at price  $P_t$ . Let  $Q_{\bar{K}',t}$  be the nominal price of new capital. Since the marginal rate of transformation between new and old capital is unity, the price of old capital is also  $Q_{\bar{K}',t}$ . The representative capital producer's period- $t$  profit maximization problem is thus given by

$$\begin{aligned} \max_{\{I_{t+\tau}, x_{K,t+\tau}\}} \quad & E_t \sum_{\tau=0}^{\infty} \beta^\tau \lambda_{t+\tau} \left\{ Q_{\bar{K}',t+\tau} [x_{K,t+\tau} + A(I_{t+\tau}, I_{t+\tau-1})] \right. \\ & \left. - Q_{\bar{K}',t+\tau} x_{K,t+\tau} - P_{t+\tau} I_{t+\tau} \right\} . \end{aligned} \quad (\text{A.5})$$

Let  $\delta$  denote the depreciation rate and note that, from (A.5), any value of  $x_{K,t+\tau}$  is profit maximizing. Thus considering  $x_{K,t+\tau} = (1 - \delta) \bar{K}_{t+\tau}$  is consistent with both profit maximization and market clearing.

The first order condition with respect to  $I_t$  is:

$$E_t \left[ \lambda_t (Q_{\bar{K}',t} A_{1,t} - P_t) + \beta \lambda_{t+1} Q_{\bar{K}',t+1} A_{2,t+1} \right] = 0 ,$$

where

$$A_{1,t} = \frac{\partial A(I_t, I_{t-1})}{\partial I_t} ; \quad A_{2,t+1} = \frac{\partial A(I_{t+1}, I_t)}{\partial I_t} .$$

This is the standard Tobin's  $Q$  equation that relates the price of capital to the marginal costs of

producing investment goods.<sup>23</sup>

The aggregate capital stock evolves according to

$$\bar{K}_{t+1} = \eta \bar{K}_{t+1}^{H,r} + (1 - \eta) \bar{K}_{t+1}^{L,l} = (1 - \delta) \left[ \eta \bar{K}_t^{H,r} + (1 - \eta) \bar{K}_t^{L,l} \right] + A(I_t, I_{t-1}) \quad .$$

## Riskier entrepreneurs and retail banks

The role of the representative retail bank in the model is to collect time deposits from households in order to finance riskier entrepreneur's investment project. The bank hedges against credit risk by charging a premium over the risk-free rate at which it can borrow from households. The risk-free rate that the bank views as its opportunity cost to lending is a contractual nominal interest rate that is determined at the time the bank liability to households is issued. Unlike in Bernanke et al. (1999), this rate is not contingent on the shocks that intervene before the entrepreneurial loan matures.

At each point in time there is a continuum of heterogeneous entrepreneurs of total measure  $\eta$ , indexed by  $(H, r)$ . At the end of time  $t$ , each entrepreneur is characterized by his net worth,  $N_{t+1}^{H,r}$ , which is used, in combination with a bank loan, to purchase the time- $(t+1)$  stock of capital,  $\bar{K}_{t+1}^{H,r}$ . After the purchase, the entrepreneur experiences an idiosyncratic productivity shock,  $\omega_{t+1}^{H,r}$ , which transforms the purchased capital  $\bar{K}_{t+1}^{H,r}$  into  $\omega_{t+1}^{H,r} \bar{K}_{t+1}^{H,r}$ . By assumption,  $\omega^{H,r}$  is independently and identically distributed over time and across entrepreneurs and follows a log-normal distribution,

$$\ln(\omega^{H,r}) \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right) \quad ,$$

where  $\sigma$  is the standard deviation of  $\ln(\omega^{H,r})$ .

## Capital utilization decision

At the beginning of period  $t$ , the representative entrepreneur provides capital services to intermediate-good firms. Capital services,  $K_t^{H,r}$ , are related to the entrepreneur's stock of physical capital,  $\bar{K}_t^{H,r}$ , by  $K_t^{H,r} = u_t^{H,r} \bar{K}_t^{H,r}$ , where  $u_t^{H,r}$  denotes the level of capital utilization. In choosing the capital utilization rate, the entrepreneur takes into account the increasing and convex utilization cost function

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<sup>23</sup> We adopt the following investment adjustment costs function:

$$A(I_t, I_{t-1}) = \left[ 1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \quad , \quad S\left(\frac{I_t}{I_{t-1}}\right) = \frac{S''}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2$$

so that  $S(1) = S'(1) = 0$  and  $S''(1) = S'' > 0$  in steady state. Therefore

$$A_{1,t} = \frac{\partial A(I_t, I_{t-1})}{\partial I_t} = 1 - \frac{S''}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - S'' \frac{I_t}{I_{t-1}} \left( \frac{I_t}{I_{t-1}} - 1 \right)$$

and

$$A_{2,t+1} = \frac{\partial A(I_{t+1}, I_t)}{\partial I_t} = S'' \left( \frac{I_{t+1}}{I_t} \right)^2 \left( \frac{I_{t+1}}{I_t} - 1 \right) \quad .$$

$a(u_t^{H,r})$ , that denotes the cost, in units of final goods, of setting the utilization rate to  $u_t^{H,r}$ .<sup>24</sup>

The entrepreneur chooses  $u_t^{H,r}$  solving the following maximization problem:

$$\max_{u_t^{H,r}} \left[ u_t^{H,r} r_t^{k,H} - a(u_t^{H,r}) \right] \omega^{H,r} \bar{K}_t^{H,r} P_t .$$

After determining the utilization rate of capital and earning rent on it, the entrepreneur sells the undepreciated part to capital producers at price  $Q_{\bar{k}',t}$ . The entrepreneur's nominal gross rate of return on capital purchased at time  $t-1$ ,  $1 + R_t^{k,H,r}$ , is given by

$$1 + R_t^{k,H,r} = \frac{\left[ u_t^{H,r} r_t^{k,H} - a(u_t^{H,r}) \right] P_t + (1 - \delta) Q_{\bar{k}',t} \omega^{H,r}}{Q_{\bar{k}',t-1}} .$$

Because the mean of  $\omega^{H,r}$  across entrepreneurs is unity, we may define the average nominal gross rate of return on capital across all entrepreneurs as follows

$$1 + R_t^{k,H} = \frac{\left[ u_t^{H,r} r_t^{k,H} - a(u_t^{H,r}) \right] P_t + (1 - \delta) Q_{\bar{k}',t}}{Q_{\bar{k}',t-1}} . \quad (\text{A.6})$$

### Loan decision and the standard debt contract

At the end of period  $t$ , the entrepreneur has available net worth,  $N_{t+1}^{H,r}$ , which he uses to finance his capital expenditures,  $Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}$ . To finance the difference between expenditures and net worth, he borrows an amount  $B_{t+1}^{H,r} = Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r} - N_{t+1}^{H,r}$  from the retail bank.

After the purchase, the entrepreneur experiences an idiosyncratic productivity shock,  $\omega_{t+1}^{H,r}$ , which transforms the purchased capital  $\bar{K}_{t+1}^{H,r}$  into  $\omega_{t+1}^{H,r} \bar{K}_{t+1}^{H,r}$ . Financial frictions arise from asymmetric information between entrepreneur and bank. In particular, the entrepreneur costlessly observes his idiosyncratic shock, whereas the bank must pay a monitoring cost – which represent a fraction  $\mu$ ,  $0 < \mu < 1$ , of the entrepreneur's gross return – to observe it. The optimal financing mechanism is a standard debt contract which gives the lender the right to all liquidation proceeds in the event of an entrepreneur's default.

At the end of time  $t$ , the bank offers a debt contract to the entrepreneur, which specifies the loan amount,  $B_{t+1}^{H,r}$ , and the gross interest rate on the loan,  $Z_{t+1}^{H,r}$ . At time  $t+1$ , the entrepreneur declares bankruptcy if  $\omega_{t+1}^{H,r}$  is smaller than the default threshold level,  $\bar{\omega}_{t+1}^{H,r}$ , defined by

$$\bar{\omega}_{t+1}^{H,r} \left( 1 + R_{t+1}^{k,H,r} \right) Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r} = Z_{t+1}^{H,r} B_{t+1}^{H,r} . \quad (\text{A.7})$$

Therefore, if  $\omega_{t+1}^{H,r} > \bar{\omega}_{t+1}^{H,r}$ , the entrepreneur pays the lender the amount  $Z_{t+1}^{H,r} B_{t+1}^{H,r}$  and keeps the remaining  $\left( \omega_{t+1}^{H,r} - \bar{\omega}_{t+1}^{H,r} \right) \left( 1 + R_{t+1}^{k,H,r} \right) Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}$ . On the other hand, if  $\omega_{t+1}^{H,r} < \bar{\omega}_{t+1}^{H,r}$ , the entrepreneur

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<sup>24</sup> The functional form that we use is  $a(u_t^{H,r}) = \frac{r^{k,H}}{\sigma_a^H} \left[ \exp^{\sigma_a^H (u_t^{H,r} - 1)} - 1 \right]$ , where  $r^{k,H}$  is the steady state value of the rental rate of capital,  $a(1) = 0$ ,  $a''(1) > 0$  and  $\sigma_a^H = a''(1)/a'(1)$  is a parameter that controls the degree of convexity of costs.

defaults and receives nothing, while the bank monitors the entrepreneur at cost  $\mu \left(1 + R_{t+1}^{k,H,r}\right) \omega_{t+1}^{H,r} Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}$  and receives all of the residual net worth  $(1 - \mu) \left(1 + R_{t+1}^{k,H,r}\right) \omega_{t+1}^{H,r} Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}$ .

The bank raises the funds that are necessary to finance the entrepreneurs activities issuing time deposits to households, and pays them a nominal rate of return  $R_{t+1}^e$ . Perfect competition in the banking sector implies that the following bank's zero profit condition holds in each period:

$$\underbrace{\left[1 - F_t \left(\bar{\omega}_{t+1}^{H,r}\right)\right] Z_{t+1}^{H,r} B_{t+1}^{H,r}}_{\text{revenue from non-bankrupt entrepreneurs}} + \underbrace{(1 - \mu) \int_0^{\bar{\omega}_{t+1}^{H,r}} \omega_{t+1}^{H,r} dF \left(\omega_{t+1}^{H,r}\right) \left(1 + R_{t+1}^{k,H,r}\right) Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}}_{\text{revenue, after monitoring cost, from bankrupt entrepreneurs}} = \underbrace{(1 + R_{t+1}^e) B_{t+1}^{H,r}}_{\text{payment to households}}, \quad (\text{A.8})$$

where  $F_t \left(\omega_{t+1}^{H,r}\right)$  is the cumulative distribution function of  $\omega_{t+1}^{H,r}$ .

Let  $k_{t+1}^{H,r} = \frac{Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}}$  denote the ratio of capital expenditures to net worth. Combining (A.7) with (A.8) and using the definition of  $k_{t+1}$  yields

$$\left[\Gamma_t \left(\bar{\omega}_{t+1}^{H,r}\right) - \mu G_t \left(\bar{\omega}_{t+1}^{H,r}\right)\right] k_{t+1}^{H,r} \frac{1 + R_{t+1}^{k,H}}{1 + R_{t+1}^e} = k_{t+1}^{H,r} - 1,$$

where  $G_t \left(\bar{\omega}_{t+1}^{H,r}\right) = \int_0^{\bar{\omega}_{t+1}^{H,r}} \omega_{t+1}^{H,r} dF \left(\omega_{t+1}^{H,r}\right)$  and  $\Gamma_t \left(\bar{\omega}_{t+1}^{H,r}\right) = \bar{\omega}_{t+1}^{H,r} \left[1 - F_t \left(\bar{\omega}_{t+1}^{H,r}\right)\right] + G_t \left(\bar{\omega}_{t+1}^{H,r}\right)$ . The term  $\Gamma_t \left(\bar{\omega}_{t+1}^{H,r}\right)$  represents the share of entrepreneurial earnings received by the bank and  $\mu G_t \left(\bar{\omega}_{t+1}^{H,r}\right)$  the expected monitoring costs. Therefore  $1 - \Gamma_t \left(\bar{\omega}_{t+1}^{H,r}\right)$  is the share of profits going to the entrepreneur.

The contract determines the division of the expected profits between borrower and lender. In particular, the optimal contract maximizes the entrepreneur's expected return at time  $t + 1$  subject to the zero profit condition on banks. The optimal contracting problem may be written in the following way:

$$\begin{aligned} & \max_{\{k_{t+1}^{H,r}, \bar{\omega}_{t+1}^{H,r}\}} E_t \left\{ \left[1 - \Gamma_t \left(\bar{\omega}_{t+1}^{H,r}\right)\right] \frac{1 + R_{t+1}^{k,H}}{1 + R_{t+1}^e} k_{t+1}^{H,r} \right\} \\ & \text{subject to} \quad \left[\Gamma_t \left(\bar{\omega}_{t+1}^{H,r}\right) - \mu G_t \left(\bar{\omega}_{t+1}^{H,r}\right)\right] k_{t+1}^{H,r} \frac{1 + R_{t+1}^{k,H}}{1 + R_{t+1}^e} = k_{t+1}^{H,r} - 1. \end{aligned}$$

The first order conditions of the contracting problem yield the following relationship between the leverage ratio,  $\frac{Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}}$ , and the expected discounted return to capital (see Bernanke et al., 1999 for details):

$$\frac{E_t \left(1 + R_{t+1}^{k,H}\right)}{1 + R_{t+1}^e} = \Psi \left( \frac{Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}} \right),$$

where  $\Psi' > 0$  for  $N_{t+1}^{H,r} < Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}$ . The ratio  $\frac{E_t(1 + R_{t+1}^{k,H})}{1 + R_{t+1}^e}$ , which Bernanke et al. (1999) interpreted as the external finance premium faced by entrepreneur, depends positively on the entrepreneur's leverage ratio. Intuitively, all else equal, lower leverage means lower exposure, implying a lower probability of default, hence a lower credit risk, which the bank translates into a lower required return on lending.

## Entrepreneurial net worth

After the loan contract received in  $t - 1$  is settled, the entrepreneurial equity,  $V_t^{H,r}$ , is given by

$$V_t^{H,r} = \left(1 + R_t^{k,H}\right) Q_{\bar{k}',t-1} \bar{K}_t^{H,r} - \left[ 1 + R_t^e + \frac{\mu \int_0^{\bar{\omega}_t^{H,r}} \omega^{H,r} dF_{t-1}(\omega^{H,r}) \left(1 + R_t^{k,H}\right) Q_{\bar{k}',t-1} \bar{K}_t^{H,r}}{Q_{\bar{k}',t-1} \bar{K}_t^{H,r} - N_t^{H,r}} \right] \left( Q_{\bar{k}',t-1} \bar{K}_t^{H,r} - N_t^{H,r} \right) .$$

Equity depends on the profits accumulated from the entrepreneur's activities. The first term represents the proceeds from selling undepreciated capital to capital producers, plus the rental income of capital, net of the costs of utilization (see equation A.6). The term in squared brackets represents the gross rate of return paid by entrepreneur on time- $(t - 1)$  loans.

At this point, to ensure that entrepreneur does not accumulate enough net worth to be fully self-financed, CMR assume that there is a constant probability of death. Namely, in each period entrepreneur exits the economy with probability  $1 - \gamma^H$ . In this case, entrepreneur rebates his equity to households in a lump-sum way:

$$transfer\ to\ households = (1 - \gamma^H) V_t^{H,r} .$$

To keep the population constant,  $1 - \gamma^H$  entrepreneurs are born each period.

Entrepreneurial net worth,  $N_{t+1}^{H,r}$ , combines the total equity and a transfer received from households,  $W_t^{e,H,r}$ , and is given by

$$N_{t+1}^{H,r} = \gamma^H V_t^{H,r} + W_t^{e,H,r} .$$

A feature of the debt contract is that entrepreneurs with no net worth receive no loans. Thus, if newborn entrepreneurs receive no transfers, they would have zero net worth and would therefore not be able to purchase any capital. The same happens with the fraction of entrepreneurs who are bankrupt due to a low realization of  $\omega$ . To avoid this situation, the  $1 - \gamma^H$  entrepreneurs who are born and the  $\gamma^H$  who survive receive the subsidy  $W_t^{e,H,r}$  from households.

## Households

There is a continuum of infinitely lived risk averse households, indexed by  $j \in [0, 1]$ . Each household consumes, supplies a differentiated labor input and allocates his savings between riskless time deposit and corporate bonds. As households differ in hours worked and in income, one would expect that they would also differ in consumption and asset allocations. However, each household  $j$  is assumed to hold state-contingent securities that provide insurance against household-specific wage-income risk. As a result, households are homogeneous with respect to consumption and asset holdings in equilibrium. Therefore, in what follows, consumption and saving decisions are not indexed by  $j$ .<sup>25</sup>

<sup>25</sup> See Erceg et al. (2000) for a discussion about the existence of state-contingent securities.

## Consumption and saving decisions

The instantaneous utility function of a given household is separable in consumption and hours worked and given by:

$$u(\cdot) = \log(C_{t+\tau} - bC_{t+\tau-1}) - \psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L}, \quad (\text{A.9})$$

where  $C_t$  denotes the household consumption at time  $t$  and  $h_{j,t}$  denotes its hours worked in period  $t$ . The parameter  $b > 0$  measures the degree of external habit formation in consumption,  $\sigma_L > 0$  is the inverse of the Frisch elasticity of labor supply and  $\psi_L > 0$  is a preference parameter that affects the disutility of supplying labor.

At the end of period  $t$ , household allocates his savings into time deposits,  $T_t$ , and corporate bonds,  $CB_t$ . At the end of period  $t+1$ , time deposits pay a riskless rate of return equal to  $R_{t+1}^e$ , while the rate of return on corporate bonds is  $R_{t+1}^F$ . We assume that both rates are known when household makes his saving decision and are not contingent on the realization of period- $(t+1)$  monetary policy shock.

The household budget constraint at time  $t$ , written in nominal terms, is given by

$$\begin{aligned} & (1 + R_t^e) T_{t-1} + (1 + R_t^F) CB_{t-1} + W_{j,t} h_{j,t} \\ & + (1 - \gamma^L) [1 - \Omega(R_t^{\text{coupon},a} - R_t^{\text{coupon}})] (1 - \eta) V_t^{L,l} + (1 - \gamma^H) \eta V_t^{H,r} \\ & + \Pi_t^{IGF} + \Pi_t^{IB} + NCS_t - CB_t - T_t - P_t C_t - W_t^e - Lump_t \geq 0 \end{aligned},$$

where  $W_{j,t}$  is the wage earned by the household  $j$ ,  $NCS_{j,t}$  represents the net payoff of the state contingent securities that the  $j^{th}$  household purchases to insulate itself from the uncertainty associated with the ability to re-optimize its wage,  $\Pi_t^{IGF}$  and  $\Pi_t^{IB}$  are the profits received from, respectively, intermediate-good firms and investment banks,  $(1 - \gamma^H) \eta V_t^{H,r}$  are the lump-sum transfers received from riskier entrepreneurs who exit the economy,  $(1 - \gamma^L) [1 - \Omega(R_t^{\text{coupon},a} - R_t^{\text{coupon}})] (1 - \eta) V_t^{L,l}$  are the lump-sum transfers (net of side payments) received from safer entrepreneurs who exit the economy,  $W_t^e$  is the total transfer payment to entrepreneurs and  $Lump_t$  are lump-sum taxes paid to finance government expenditures.

The representative household takes its consumption and saving decisions so as to maximize the expected lifetime utility subject to its intertemporal budget constraint. The optimization problem is given by

$$\begin{aligned} & \max_{\{C_{t+\tau}, T_{t+\tau}, CB_{t+\tau}\}} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[ \log(C_{t+\tau} - bC_{t+\tau-1}) - \psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L} \right] \\ & \text{subject to} \quad (1 + R_{t+\tau}^e) T_{t-1+\tau} + (1 + R_{t+\tau}^F) CB_{t-1+\tau} + W_{j,t+\tau} h_{j,t+\tau} \\ & \quad + (1 - \gamma^L) [1 - \Omega(R_{t+\tau}^{\text{coupon},a} - R_{t+\tau}^{\text{coupon}})] V_{t+\tau}^L + (1 - \gamma^H) V_{t+\tau}^H \\ & \quad + \Pi_{t+\tau}^{IGF} + \Pi_{t+\tau}^{IB} + NCS_{t+\tau} - CB_{t+\tau} - T_{t+\tau} \\ & \quad - P_{t+\tau} C_{t+\tau} - W_{t+\tau}^e - Lump_{t+\tau} \geq 0. \end{aligned}$$

The first order conditions with respect to  $T_t$ ,  $CB_t$  and  $C_t$  are, respectively,

$$\lambda_t = \beta (1 + R_{t+1}^e) E_t (\lambda_{t+1}) \quad (\text{A.10})$$

$$\lambda_t = \beta (1 + R_{t+1}^F) E_t (\lambda_{t+1}) \quad (\text{A.11})$$

$$P_t \lambda_t = \frac{1}{(C_t - bC_{t-1})} - \beta b \frac{1}{(C_{t+1} - bC_t)} \quad , \quad (\text{A.12})$$

where  $\lambda_t$  is the Lagrange multiplier associated to the households' budget constraint. Equation (A.10) represents the standard Euler equation. The right hand side of (A.12) is the marginal utility of consumption, taking into account habit persistence. Comparing (A.10) and (A.11), it must hold that  $R_{t+1}^F = R_{t+1}^e \forall t$ , *i.e.* the return on corporate bonds equals the return on time deposits, which in turn is equal to the central bank nominal interest rate. This result is due to the assumption that both interest rates are known when household makes his optimal decision and are not contingent on the realization of period- $(t+1)$  monetary policy shock.

### Labor supply and wage setting

Each household is a monopolistic supplier of a differentiated labor service,  $h_{j,t}$ , to the production sector. Labor services are bundled together using the aggregator function

$$L_{i,t} = \left[ \int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w} \quad , \quad (\text{A.13})$$

where  $\lambda_w$ ,  $\infty > \lambda_w \geq 1$ , represents the wage markup. The demand curve for the  $j^{th}$  household specialized labor services is

$$h_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t} \quad ,$$

and the aggregate nominal wage,  $W_t$ , is given by

$$W_t = \left[ \int_0^1 (W_{j,t})^{\frac{1}{1-\lambda_w}} dj \right]^{1-\lambda_w} \quad . \quad (\text{A.14})$$

In each period, a fraction  $\xi_w$  of households cannot reoptimize their wages and, therefore, set their wages according to the indexation rule

$$W_{j,t} = W_{j,t-1} (\bar{\pi})^{\iota_{w1}} (\pi_{t-1})^{1-\iota_{w1}} \quad ,$$



where  $i_{w1}$ ,  $0 \leq i_{w1} \leq 1$ , represents the degree of wage indexation to steady state inflation rate. The fraction  $1 - \xi_w$  of reoptimizing households set their wages by maximizing

$$\begin{aligned} \max_{W_{j,t}} \quad & E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau \left[ -\psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L} + \lambda_{t+\tau} W_{j,t+\tau} h_{j,t+\tau} \right] \\ \text{subject to} \quad & h_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t} . \end{aligned}$$

We only consider the symmetric equilibrium in which all households choose the same  $\tilde{W}_t = W_{j,t}$ . Thus, given (A.14), the law of motion of the aggregate wage index is given by

$$W_t = \left\{ (1 - \xi_w) \tilde{W}_t^{\frac{1}{1-\lambda_w}} + \xi_w \left[ W_{t-1} (\bar{\pi})^{i_{w1}} (\pi_{t-1})^{1-i_{w1}} \right]^{\frac{1}{1-\lambda_w}} \right\}^{1-\lambda_w} .$$

## Resource constraint

The aggregate resource constraint is

$$\begin{aligned} C_t + I_t + GC_t + \eta \mu \int_0^{\bar{\omega}_t} \omega dF(\omega) \left( 1 + R_t^{k,H} \right) \frac{Q_{\bar{k}',t-1} \bar{K}_t^{H,r}}{P_t} \\ + UC_t + \eta a \left( u_t^{H,r} \right) \bar{K}_t^{H,r} + (1 - \eta) a \left( u_t^{L,l} \right) \bar{K}_t^{L,l} = Y_t . \end{aligned}$$

Government expenditures,  $GC_t$ , are determined exogenously as a constant fraction,  $\eta_g$ , of final output:  $GC_t = \eta_g Y_t$  and are financed by lump-sum taxes to the households. The fourth term represents final output used by banks in monitoring riskier entrepreneurs, and  $UC_t = \Omega (R_t^{coupon,a} - R_t^{coupon}) (1 - \eta) V_t^{L,l}$  represents the underwriter's consumption in period  $t$ . Finally, the last two terms on the left hand side capture capital utilization costs.

## Aggregate variables and market clearing conditions

Aggregate net worth ( $N_{t+1}^{TOT}$ ) and aggregate leverage ( $lev_{t+1}^{TOT}$ ) are defined, respectively, as

$$\begin{aligned} N_{t+1}^{TOT} &= \eta N_{t+1}^{H,r} + (1 - \eta) N_{t+1}^{L,l} \\ lev_{t+1}^{TOT} &= \eta lev_{t+1}^{H,r} + (1 - \eta) lev_{t+1}^{L,l} = \eta \frac{Q_{\bar{k}',t} \bar{K}_{t+1}^{H,r}}{N_{t+1}^{H,r}} + (1 - \eta) \frac{Q_{\bar{k}',t} \bar{K}_{t+1}^{L,l}}{N_{t+1}^{L,l}} . \end{aligned}$$

Total credit ( $B_{t+1}^{TOT}$ ) is defined as the sum of bank loans and bonds issued and is given by:

$$B_{t+1}^{TOT} = \eta B_{t+1}^{H,r} + (1 - \eta) B I_{t+1}^{L,l} .$$

The capital rental market clearing conditions are:

$$\int_0^1 K_{i,t}^H di = K_t^H = \eta K_t^{H,r} = \eta u_t^{H,r} \bar{K}_t^{H,r}$$

and

$$\int_0^1 K_{i,t}^L di = K_t^L = (1 - \eta) K_t^{L,l} = (1 - \eta) u_t^{L,l} \bar{K}_t^{L,l} .$$

Loan and bond market clearing conditions are, respectively,  $T_t = \eta B_{t+1}^{H,r}$  and  $CB_t = (1 - \eta) BI_{t+1}^{L,l}$ .

The market clearing condition in the labor market is:  $L_t = \int_0^1 \left\{ \left[ \int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w} \right\} di$ .

Finally, the total transfer from households ( $W_t^e$ ) to entrepreneurs must satisfy

$$W_t^e = \eta W_t^{e,H,r} + (1 - \eta) W_t^{e,L,l} .$$

## Appendix B - Technical details

### Final-good firms

The maximization problem solved by the representative final-good firm is the following:

$$\begin{aligned} \max_{Y_{i,t}} \quad & \Pi_t^{FGF} = P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \\ \text{subject to} \quad & Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f} . \end{aligned}$$

The first order condition is:

$$\frac{\partial \Pi_t^{FGF}}{\partial Y_{i,t}} = 0 \Leftrightarrow P_t \lambda_f \left[ \int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f - 1} \frac{1}{\lambda_f} Y_{i,t}^{\frac{1}{\lambda_f} - 1} - P_{i,t} = 0 .$$

A simple algebra shows that the demand function for the intermediate good  $i$  is given by

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_f}{1-\lambda_f}} Y_t . \quad (\text{B.1})$$

Substituting (B.1) into the expression for  $\Pi_t^{FGF}$  yields

$$\Pi_t^{FGF} = P_t Y_t - \int_0^1 P_{i,t} \left( \frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_f}{1-\lambda_f}} Y_t di . \quad (\text{B.2})$$

Perfect competition in the final-good market implies that  $\Pi_t^{FGF} = 0$ . Imposing this condition in (B.2) gives the following expression for the price of the final good:

$$P_t = \left[ \int_0^1 P_{i,t}^{\frac{1}{1-\lambda_f}} di \right]^{1-\lambda_f} . \quad (\text{B.3})$$

### Intermediate-good firms

#### Cost minimization problem

The  $i$ -th firm's cost minimization problem, in real terms, is given by

$$\min_{\{L_{i,t}, K_{i,t}^H, K_{i,t}^L\}} C(\cdot) = \frac{W_t L_{i,t}}{P_t} + K_{i,t}^H r_t^{k,H} + K_{i,t}^L r_t^{k,L} \quad (\text{B.4})$$

$$\text{subject to } Y_{i,t} = (K_{i,t})^\alpha (L_{i,t})^{1-\alpha} \quad (\text{B.5})$$

$$K_{i,t} = \left[ \eta^{1-\rho} (K_{i,t}^H)^\rho + (1-\eta)^{1-\rho} (K_{i,t}^L)^\rho \right]^{\frac{1}{\rho}} . \quad (\text{B.6})$$

Solving (B.5) for  $L_t$  and using (B.6), the minimization problem may be rewritten as

$$\min_{\{K_{i,t}^H, K_{i,t}^L\}} C(\cdot) = \frac{W_t}{P_t} (Y_{i,t})^{\frac{1}{1-\alpha}} \left[ \eta^{1-\rho} (K_{i,t}^H)^\rho + (1-\eta)^{1-\rho} (K_{i,t}^L)^\rho \right]^{-\frac{\alpha}{\rho(1-\alpha)}} + K_{i,t}^H r_t^{k,H} + K_{i,t}^L r_t^{k,L} .$$

The first order conditions with respect to  $K_{i,t}^H$  and  $K_{i,t}^L$  are, respectively,

$$r_t^{k,H} = \frac{W_t}{P_t} (Y_{i,t})^{\frac{1}{1-\alpha}} \frac{\alpha}{1-\alpha} \eta^{1-\rho} \left[ \eta^{1-\rho} (K_{i,t}^H)^\rho + (1-\eta)^{1-\rho} (K_{i,t}^L)^\rho \right]^{-\frac{\alpha}{\rho(1-\alpha)}-1} (K_{i,t}^H)^{\rho-1} \quad (\text{B.7})$$

$$r_t^{k,L} = \frac{W_t}{P_t} (Y_{i,t})^{\frac{1}{1-\alpha}} \frac{\alpha}{1-\alpha} (1-\eta)^{1-\rho} \left[ \eta^{1-\rho} (K_{i,t}^H)^\rho + (1-\eta)^{1-\rho} (K_{i,t}^L)^\rho \right]^{-\frac{\alpha}{\rho(1-\alpha)}-1} (K_{i,t}^L)^{\rho-1} . \quad (\text{B.8})$$

Taking the ratio of (B.7) and (B.8), the following arbitrage condition for the choice of capital services may be derived:

$$\frac{r_t^{k,H}}{r_t^{k,L}} = \left( \frac{\eta}{1-\eta} \right)^{1-\rho} \left( \frac{K_{i,t}^H}{K_{i,t}^L} \right)^{\rho-1} . \quad (\text{B.9})$$

Since

$$K_{i,t}^H = \eta K_{i,t}^{H,r} = \eta \left( u_t^{H,r} \bar{K}_{i,t}^{H,r} \right) , \quad (\text{B.10})$$

then the arbitrage condition may be rewritten in terms of entrepreneur-specific capital services as

$$\frac{r_t^{k,H}}{r_t^{k,L}} = \left( \frac{u_t^{H,r} \bar{K}_{i,t}^{H,r}}{u_t^{L,l} \bar{K}_{i,t}^{L,l}} \right)^{\rho-1} .$$

From (B.7) we can derive the following expression for  $K_{i,t}$ :

$$K_{i,t} = \left[ \frac{W_t}{P_t} \frac{1}{r_t^{k,H}} \frac{\alpha}{1-\alpha} \eta^{1-\rho} (K_{i,t}^H)^{\rho-1} \right]^{\frac{1-\alpha}{\rho+\alpha(1-\rho)}} (Y_{i,t})^{\frac{1}{\rho+\alpha(1-\rho)}} . \quad (\text{B.11})$$

Now compute  $r_t^{k,L}$  from (B.9) and  $K_{i,t}^L$  from (B.6). Using these results and equation (B.11) to substitute in (B.4), it takes a few steps to obtain the following expression for the cost function  $C(\cdot)$ :

$$C(\cdot) = \left[ \frac{W_t}{P_t} \frac{1}{1-\alpha} \right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left[ \frac{\alpha \eta^{1-\rho}}{r_t^{k,H}} (K_{i,t}^H)^{\rho-1} \right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}} (Y_{i,t})^{\frac{\rho}{\rho+\alpha(1-\rho)}} .$$

Real marginal costs are thus given by

$$s_{i,t} = \frac{\partial C(\cdot)}{\partial Y_{i,t}} = \left[ \frac{W_t}{P_t} \frac{1}{1-\alpha} \right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left[ \frac{\alpha \eta^{1-\rho}}{r_t^{k,H}} (K_{i,t}^H)^{\rho-1} \right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}} (Y_{i,t})^{\frac{\alpha(\rho-1)}{\rho+\alpha(1-\rho)}} \frac{\rho}{\rho+\alpha(1-\rho)} .$$

Efficient input choice by firm  $i$  also implies that real marginal costs must be equal to the cost of renting one unit of capital divided by the marginal product of capital ( $\partial Y/\partial K$ ). Since

$$\frac{\partial Y_{i,t}}{\partial K_{i,t}^H} = \alpha \left( \frac{L_{i,t}}{K_{i,t}} \right)^{1-\alpha} \eta^{1-\rho} (K_{i,t}^H)^{\rho-1} \left[ \eta^{1-\rho} (K_{i,t}^H)^\rho + (1-\eta)^{1-\rho} (K_{i,t}^L)^\rho \right]^{\frac{1}{\rho}-1}$$

and

$$\frac{\partial Y_{i,t}}{\partial K_{i,t}^L} = \alpha \left( \frac{L_{i,t}}{K_{i,t}} \right)^{1-\alpha} (1-\eta)^{1-\rho} (K_{i,t}^L)^{\rho-1} \left[ \eta^{1-\rho} (K_{i,t}^H)^\rho + (1-\eta)^{1-\rho} (K_{i,t}^L)^\rho \right]^{\frac{1}{\rho}-1} ,$$

then

$$s_{i,t} = \frac{r_t^{k,H}}{\frac{\partial Y_{i,t}}{\partial K_{i,t}^H}} = \frac{r_t^{k,L}}{\frac{\partial Y_{i,t}}{\partial K_{i,t}^L}} .$$

Since all firms  $i$  face the same input prices and since they all have access to the same production technology, real marginal costs  $s_{i,t}$  are identical across firms, *i.e.*,  $s_{i,t} = s_t$  with

$$s_t = \left[ \frac{W_t}{P_t} \frac{1}{1-\alpha} \right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left[ \frac{\alpha \eta^{1-\rho}}{r_t^{k,H}} (K_t^H)^{\rho-1} \right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}} (Y_t)^{\frac{\alpha(\rho-1)}{\rho+\alpha(1-\rho)}} \frac{\rho}{\rho+\alpha(1-\rho)} ,$$

or, using equation (B.10),

$$s_t = \left[ \frac{W_t}{P_t} \frac{1}{1-\alpha} \right]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}} \left[ \frac{\alpha}{r_t^{k,H}} (K_t^{H,r})^{\rho-1} \right]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}} (Y_t)^{\frac{\alpha(\rho-1)}{\rho+\alpha(1-\rho)}} \frac{\rho}{\rho+\alpha(1-\rho)} .$$

## Price setting

Every firm faces a constant probability,  $1-\xi_p$ , of reoptimizing its price in any given period, whereas the non-reoptimizing firms set their prices according to the indexation rule  $P_{i,t} = P_{i,t-1} (\bar{\pi})^{\iota_1} (\pi_{t-1})^{1-\iota_1}$ . The  $i$ -th firm that optimizes its price at time  $t$  chooses  $P_{i,t} = \tilde{P}_{i,t}$  that maximizes the present value of future expected nominal profits. The maximization problem is given by:

$$\max_{P_{i,t}} \Pi_t^{IGF} = E_t \sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \lambda_{t+\tau} [(P_{i,t+\tau} - S_{t+\tau}) Y_{i,t+\tau}]$$

$$\text{subject to} \quad Y_{i,t+\tau} = \left( \frac{P_{i,t+\tau}}{P_{t+\tau}} \right)^{\frac{\lambda_f}{1-\lambda_f}} Y_{t+\tau} .$$

Substituting the demand function and rearranging yields

$$\max_{P_{i,t}} \Pi_t^{IGF} = E_t \sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \lambda_{t+\tau} Y_{t+\tau} P_{t+\tau} \left[ \left( \frac{P_{i,t+\tau}}{P_{t+\tau}} \right)^{1+\frac{\lambda_f}{1-\lambda_f}} - s_{t+\tau} \left( \frac{P_{i,t+\tau}}{P_{t+\tau}} \right)^{\frac{\lambda_f}{1-\lambda_f}} \right] . \quad (\text{B.12})$$

We make use of the following definitions:

$$\tilde{p}_{t+\tau} = \frac{\tilde{P}_{t+\tau}}{P_{t+\tau}}, \quad p_{i,t+\tau} = \frac{P_{i,t+\tau}}{P_{t+\tau}}, \quad \lambda_{n,t+\tau} = \lambda_{t+\tau} P_{t+\tau} \quad .$$

Then

$$\frac{P_{i,t+\tau}}{P_{t+\tau}} = \frac{\tilde{\pi}_{t+\tau} \dots \tilde{\pi}_{t+1} \tilde{P}_t}{\pi_{t+\tau} \dots \pi_{t+1} P_t} = X_{t,\tau} \tilde{p}_t \quad , \quad (\text{B.13})$$

where

$$X_{t,\tau} = \begin{cases} \frac{\tilde{\pi}_{t+\tau} \dots \tilde{\pi}_{t+1}}{\pi_{t+\tau} \dots \pi_{t+1}} & \tau > 0 \\ 1 & \tau = 0 \end{cases}$$

and  $\tilde{\pi}_{t+\tau} = (\bar{\pi})^{\iota_1} (\pi_{t+\tau-1})^{1-\iota_1}$ . Using (B.13) to substitute out  $\frac{P_{i,t+\tau}}{P_{t+\tau}}$  in (B.12), then the profit maximization problem may be rewritten as

$$\max_{\tilde{p}_t} \Pi_t^{IGF} = E_t \sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau J_{t+\tau} \left[ X_{t,\tau} (\tilde{p}_t)^{1+\frac{\lambda_f}{1-\lambda_f}} - s_{t+\tau} (\tilde{p}_t)^{\frac{\lambda_f}{1-\lambda_f}} \right] \quad ,$$

where  $J_{t+\tau} = \lambda_{n,t+\tau} Y_{t+\tau} (X_{t,\tau})^{\frac{\lambda_f}{1-\lambda_f}}$  is exogenous from the point of view of the firm. The first order condition is

$$\frac{\partial \Pi_t^{IGF}}{\partial \tilde{p}_t} = 0 \Leftrightarrow E_t \sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \frac{J_{t+\tau}}{1-\lambda_f} (\tilde{p}_t)^{\frac{\lambda_f}{1-\lambda_f}-1} [X_{t,\tau} \tilde{p}_t - \lambda_f s_{t+\tau}] = 0 \quad .$$

After rearranging, the first order condition becomes

$$\tilde{p}_t = \frac{E_t \sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau J_{t+\tau} \lambda_f s_{t+\tau}}{E_t \sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau J_{t+\tau} X_{t,\tau}} = \frac{K_{p,t}}{F_{p,t}} \quad . \quad (\text{B.14})$$

For computational tractability, it is crucial to write the infinite sums,  $K_{p,t}$  and  $F_{p,t}$ , in a recursive representations. After some manipulations, one can show that

$$K_{p,t} = \lambda_{n,t} Y_t \lambda_f s_t + \beta \xi_p \left( \frac{\pi_t^{1-\iota_1}}{\pi_{t+1}} \right)^{-\frac{\lambda_f}{\lambda_f-1}} K_{p,t+1}$$

and

$$F_{p,t} = \lambda_{n,t} Y_t + \beta \xi_p \left( \frac{\pi_t^{1-\iota_1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} \quad .$$

Note that, when prices are fully flexible ( $\xi_p = 0$ ), then  $K_{p,t} = F_{p,t}$  and  $s_t = 1/\lambda_f$ , that is, the real marginal cost is the reciprocal of the markup.

We have derived the optimum price from the firm's first order condition. We now identify a consistency condition that must hold across all firm prices, which allows us to express  $\tilde{p}_t$  in terms of aggregate

variables only. Expanding (B.3) yields

$$P_t = \left[ \int_0^1 P_{i,t}^{\frac{1}{1-\lambda_f}} di \right]^{1-\lambda_f} = \left[ \int_{1-\xi_p} (P_{i,t})^{\frac{1}{1-\lambda_f}} + \int_{\xi_p} (P_{i,t})^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f}.$$

Regarding the limits of integration,  $1 - \xi_p$  refers to the firms that reoptimize prices in period  $t$ , while  $\xi_p$  refers to the firms that do not. Making use of the fact that whether firms are selected to reoptimize or not is determined randomly, we can rewrite the previous expression as follows:

$$P_t = \left\{ (1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p [P_{t-1} \tilde{\pi}_t]^{\frac{1}{1-\lambda_f}} \right\}^{1-\lambda_f}.$$

Dividing both sides by  $P_t$ , it takes a few step to obtain

$$\tilde{p}_t = \left[ \frac{1 - \xi_p \left( \frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f}. \quad (\text{B.15})$$

Finally, combining (B.15) with (B.14) we obtain

$$\frac{K_{p,t}}{F_{p,t}} = \left[ \frac{1 - \xi_p \left( \frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f}.$$

This expression relates the inflation rate to aggregate variables only.

## Households

### The wage decision

Each household  $j$  supplies a differentiated labor input to the production sector. Following Erceg et al. (2000), we assume that there is a representative employment agency that combines households' specialized labor,  $h_{j,t}$ , into homogeneous labor employed by firm  $i$ ,  $L_{i,t}$ , using the following constant returns to scale technology:

$$L_{i,t} = \left[ \int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w},$$

where  $\infty > \lambda_w \geq 1$  represents the wage markup. The representative employment agency hires  $h_{j,t}$  in order to maximize its time- $t$  profits:

$$\begin{aligned} \max_{h_{j,t}} \quad & W_t L_{i,t} - \int_0^1 W_{j,t} h_{j,t} dj \\ \text{subject to} \quad & L_{i,t} = \left[ \int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}. \end{aligned}$$

The first order condition leads to the following demand curve for the  $j^{th}$  household specialized labor services:

$$h_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t} .$$

Zero profit condition for the perfectly competitive employment agencies gives the following relation between the aggregate nominal wage and the wage earned by the household  $j$ :

$$W_t = \left[ \int_0^1 (W_{j,t})^{\frac{1}{1-\lambda_w}} dj \right]^{1-\lambda_w} . \quad (\text{B.16})$$

In each period, a fraction  $\xi_w$  of households cannot reoptimize their wages and, by assumption, set their wages according to the indexation rule  $W_{j,t} = W_{j,t-1} (\bar{\pi})^{\iota_{w1}} (\pi_{t-1})^{1-\iota_{w1}}$ . The fraction  $1 - \xi_w$  of reoptimizing households set their wages solving the following problem

$$\begin{aligned} \max_{W_{j,t}} \quad & E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau \left[ -\psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L} + \lambda_{t+\tau} W_{j,t+\tau} h_{j,t+\tau} \right] \\ \text{subject to} \quad & h_{j,t+\tau} = \left( \frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} . \end{aligned}$$

Substituting out for  $h_{j,t}$  using the labor demand curve yields:

$$\max_{W_{j,t}} E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau \left\{ -\frac{\psi_L}{1+\sigma_L} \left[ \left( \frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{1+\sigma_L} + \lambda_{t+\tau} W_{j,t+\tau} \left( \frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right\} .$$

This equation can be rewritten as:

$$\begin{aligned} \max_{W_{j,t}} \quad & E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau \left\{ -\frac{\psi_L}{1+\sigma_L} \left[ \left( \frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{1+\sigma_L} \right. \\ & \left. + \lambda_{t+\tau} \frac{P_{t+\tau}}{P_{t+\tau}} W_{j,t+\tau} \frac{W_{t+\tau}}{W_{j,t+\tau}} \left( \frac{W_{j,t+\tau}}{W_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}+1} L_{i,t+\tau} \right\} . \end{aligned} \quad (\text{B.17})$$

We adopt the following definitions:

$$W_{j,t+\tau} = \tilde{W}_{t+\tau} , \quad \tilde{w}_{t+\tau} = \frac{W_{t+\tau}}{P_{t+\tau}} , \quad w_{t+\tau} = \frac{\tilde{W}_{t+\tau}}{W_{t+\tau}} ,$$

$$\lambda_{n,t+\tau} = \lambda_{t+\tau} P_{t+\tau} , \quad \tilde{w}_{t+\tau} w_{t+\tau} = \frac{\tilde{W}_{t+\tau}}{P_{t+\tau}} .$$

Then

$$\frac{W_{j,t+\tau}}{W_{t+\tau}} = X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} , \quad (\text{B.18})$$



where

$$X_{t,\tau} = \begin{cases} \frac{\tilde{\pi}_{w,t+\tau} \dots \tilde{\pi}_{w,t+1}}{\pi_{t+\tau} \dots \pi_{t+1}} & \tau > 0 \\ 1 & \tau = 0 \end{cases}$$

and  $\tilde{\pi}_{w,t+\tau} = (\tilde{\pi})^{\iota_{w1}} (\pi_{t+\tau-1})^{1-\iota_{w1}}$ . Substituting (B.18) in (B.17), we obtain

$$\begin{aligned} \max_{W_{j,t}} \quad & E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau \left\{ -\frac{\psi_L}{1+\sigma_L} \left[ \left( X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{1+\sigma_L} \right. \\ & \left. + \lambda_{n,t+\tau} \tilde{w}_{t+\tau} L_{i,t+\tau} \left( X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}+1} \right\} . \end{aligned} \quad (\text{B.19})$$

Maximizing (B.19) with respect to  $w_t$  yields<sup>26</sup>

$$\begin{aligned} E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau \left\{ -\psi_L \left[ \left( X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{\sigma_L} \frac{\lambda_w}{1-\lambda_w} L_{i,t+\tau} \left( X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}-1} X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right. \\ \left. + \lambda_{n,t+\tau} \tilde{w}_{t+\tau} L_{i,t+\tau} \frac{1}{1-\lambda_w} \left( X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right\} = 0 . \end{aligned}$$

or, after rearranging,

$$\begin{aligned} E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau L_{i,t+\tau} \left( X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\lambda_w}{1-\lambda_w} w_t^{\frac{\lambda_w}{1-\lambda_w}-1} \left\{ -\psi_L \left[ \left( X_{t,\tau} \frac{\tilde{w}_t w_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{i,t+\tau} \right]^{\sigma_L} \right. \\ \left. + \lambda_{n,t+\tau} \frac{\tilde{w}_t w_t}{\lambda_w} X_{t,\tau} \right\} = 0 . \end{aligned}$$

Multiplying this expression by  $w_t^{-\frac{\lambda_w \sigma_L}{1-\lambda_w}}$  we obtain, after some manipulations,

$$\begin{aligned} E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau L_{i,t+\tau} \left( X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \lambda_{n,t+\tau} \frac{\tilde{w}_t}{\lambda_w} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} X_{t,\tau} &= \\ E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau L_{i,t+\tau}^{1+\sigma_L} \left( X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \psi_L . \end{aligned} \quad (\text{B.20})$$

Equation (B.20) can be rewritten as

$$F_{w,t} \tilde{w}_t w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} = \psi_L K_{w,t} ,$$

where

$$K_{w,t} = E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau L_{i,t+\tau}^{1+\sigma_L} \left( X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}}$$

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<sup>26</sup> Whether the household chooses  $w_t$  or  $\tilde{W}_t = W_{j,t}$  makes no difference, since  $w_t$  is  $\tilde{W}_t$  scaled by a variable over which the household has no control.

and

$$F_{w,t} = E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau L_{i,t+\tau} \left( X_{t,\tau} \frac{\tilde{w}_t}{\tilde{w}_{t+\tau}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \lambda_{n,t+\tau} \frac{X_{t,\tau}}{\lambda_w} .$$

Therefore the optimal wage rate results

$$w_t = \left[ \frac{\psi_L K_{w,t}}{\tilde{w}_t F_{w,t}} \right]^{\frac{\lambda_w - 1}{\lambda_w (1 + \sigma_L) - 1}} . \quad (\text{B.21})$$

We have derived the wage rate from the household's first order condition. We now derive an expression for the aggregate real wage,  $\tilde{w}_t$ , just in terms of aggregate variables.

Expanding equation (B.16) yields

$$W_t = \left[ \int_o^1 (W_{j,t})^{\frac{1}{1-\lambda_w}} dj \right]^{1-\lambda_w} = \left[ \int_{1-\xi_w} (W_{j,t})^{\frac{1}{1-\lambda_w}} + \int_{\xi_w} (W_{j,t})^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} .$$

Regarding the limits of integration,  $1 - \xi_w$  refers to the households that reoptimize in period  $t$ , while  $\xi_w$  refers to the households that do not. Making use of the fact that whether households are selected to optimize or not is determined randomly, we can rewrite the previous expression as follows:

$$W_t = \left[ (1 - \xi_w) \left( \tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w (W_{t-1} \tilde{\pi}_{w,t})^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} .$$

After dividing both sides by  $W_t$ , it takes few steps to obtain

$$w_t = \left[ \frac{1 - \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w} , \quad (\text{B.22})$$

where  $\pi_{w,t} = W_t/W_{t-1} = \pi_t \tilde{w}_t/\tilde{w}_{t-1}$ . Equating expressions (B.21) and (B.22) yields

$$K_{w,t} = \frac{F_{w,t} \tilde{w}_t}{\psi_L} \left[ \frac{1 - \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{\lambda_w (1 + \sigma_L) - 1} .$$

This expression relates the real wage to aggregate variables only. Note that, when wages are fully flexible ( $\xi_w = 0$ ), the last expression becomes

$$\tilde{w}_t = \lambda_w \frac{\psi_L L_t^{\sigma_L}}{\lambda_{n,t}} ,$$

that is, the real wage in units of the consumption good,  $\tilde{w}_t$ , is a markup,  $\lambda_w$ , over the household's marginal cost of leisure,  $\psi_L L_t^{\sigma_L}/\lambda_{n,t}$ , also expressed in terms of the consumption good.

For computational tractability, it is crucial to write the infinite sums,  $K_{w,t}$  and  $F_{w,t}$ , in a recursive

representations. After some manipulations, one can show that

$$K_{w,t} = h_t^{1+\sigma_L} + \beta \xi_w \left[ \frac{\pi_t^{1-\iota_{w1}}}{\pi_{t+1} \frac{\tilde{w}_{t+1}}{\tilde{w}_t}} \right]^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} K_{w,t+1}$$

and

$$F_{w,t} = h_t \frac{\lambda_{n,t}}{\lambda_w} + \beta \xi_w \left( \frac{1}{\pi_{t+1} \frac{\tilde{w}_{t+1}}{\tilde{w}_t}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{(\pi_t^{1-\iota_{w1}})^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} \quad .$$

## Appendix C - Model solution

This appendix reports the details on how we solved the model. The solution strategy involves linearization around the model's nonstochastic steady state. We first solve numerically the model, for the steady state, using the computational procedure described later in this appendix. We then employ the Dynare software package to compute the first-order Taylor series approximation of the equilibrium conditions in the neighborhood of the steady state.

In what follows, we adopt the following scaling notation:

$$q_t = \frac{Q_{\bar{k}'}}{P_t} \ , \ \lambda_{n,t} = \lambda_t P_t \ , \ w_t^{e,L,l} = \frac{W_t^{e,L,l}}{P_t} \ , \ w_t^{e,H,r} = \frac{W_t^{e,H,r}}{P_t} \ ,$$

$$n_{t+1}^{H,r} = \frac{N_{t+1}^{H,r}}{P_t} \ , \ n_{t+1}^{L,l} = \frac{N_{t+1}^{L,l}}{P_t} \ .$$

The equations that characterize the model's equilibrium, expressed in scaled form, are listed below.

- Investment bank

- coupon rate (constant markup over the nominal interest rate)

$$1 + R_{t+1}^{coupon,a} = \frac{\varepsilon_{coupon,a}}{\varepsilon_{coupon,a} - 1} (1 + R_{t+1}^e) \quad (C.1)$$

- law of motion for optimism

$$\chi_t = \rho_\chi \chi_{t-1} + (1 - \rho_\chi) \alpha_3 \left( n_{t+1}^{L,l} - n^{L,l} \right) \quad (C.2)$$

- coupon interest rate elasticity (with optimism)

$$\varepsilon_{t+1}^{coupon,biasd} = \varepsilon^{coupon,a} (1 + \chi_t) \quad (C.3)$$

- coupon rate (with optimism)

$$1 + R_{t+1}^{coupon,biasd} = \frac{\varepsilon_{t+1}^{coupon,biasd}}{\varepsilon_{t+1}^{coupon,biasd} - 1} (1 + R_{t+1}^e) \quad (C.4)$$

- coupon interest rate elasticity (with optimism and side payments)

$$\varepsilon_{t+1}^{coupon} = \varepsilon^{coupon,a} (1 + r_2 \chi_t) \quad (C.5)$$

- coupon rate (with optimism and side payments)

$$1 + R_{t+1}^{coupon} = \frac{\varepsilon_{t+1}^{coupon}}{\varepsilon_{t+1}^{coupon} - 1} (1 + R_{t+1}^e) \quad (C.6)$$

- Intermediate-good firms

- arbitrage condition for the choice of capital services

$$\frac{r_t^{k,H}}{r_t^{k,L}} = \left( \frac{u_t^{H,r} \bar{K}_t^{H,r}}{u_t^{L,l} \bar{K}_t^{L,l}} \right)^{\rho-1} \quad (\text{C.7})$$

- two measure of marginal costs

$$s_t = \frac{\rho}{\rho + \alpha(1 - \rho)} \left[ \frac{\tilde{w}_t}{1 - \alpha} \right]^{1 - \frac{\alpha}{\rho + \alpha(1 - \rho)}} \left[ \frac{\alpha}{r_t^{k,H}} \left( u_t^{H,r} \bar{K}_t^{H,r} \right)^{\rho-1} \right]^{-\frac{\alpha}{\rho + \alpha(1 - \rho)}} (Y_t)^{\frac{\alpha(\rho-1)}{\rho + \alpha(1 - \rho)}} \quad (\text{C.8})$$

$$s_t = \frac{r_t^{k,H}}{\alpha \left( \frac{h_t}{\bar{K}_t} \right)^{1-\alpha} \left( u_t^{H,r} \bar{K}_t^{H,r} \right)^{\rho-1} \left[ \eta \left( u_t^{H,r} \bar{K}_t^{H,r} \right)^\rho + (1 - \eta) \left( u_t^{L,l} \bar{K}_t^{L,l} \right)^\rho \right]^{\frac{1}{\rho} - 1}} \quad (\text{C.9})$$

where

$$K_t = \left[ \eta \left( u_t^{H,r} \bar{K}_t^{H,r} \right)^\rho + (1 - \eta) \left( u_t^{L,l} \bar{K}_t^{L,l} \right)^\rho \right]^{\frac{1}{\rho}} \quad (\text{C.10})$$

- Capital producers

- first order condition with respect to investment

$$\lambda_{n,t} q_t \left[ 1 - \frac{S''}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - S'' \frac{I_t}{I_{t-1}} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] - \lambda_{n,t} + \beta \lambda_{n,t+1} q_{t+1} S'' \left( \frac{I_{t+1}}{I_t} \right)^2 \left( \frac{I_{t+1}}{I_t} - 1 \right) = 0 \quad (\text{C.11})$$

- law of motion for aggregate stock of physical capital

$$\eta \bar{K}_{t+1}^{H,r} + (1 - \eta) \bar{K}_{t+1}^{L,l} = (1 - \delta) \left[ \eta \bar{K}_t^{H,r} + (1 - \eta) \bar{K}_t^{L,l} \right] + \left[ 1 - \frac{S''}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t \quad (\text{C.12})$$

- Riskier entrepreneur and retail bank

- first order condition with respect to capital utilization

$$r_t^{k,H} = a' \left( u_t^{H,r} \right) \quad (\text{C.13})$$

- definition of rate of return on capital

$$1 + R_t^{k,H} = \frac{\pi_t}{q_{t-1}} \left\{ \left[ u_t^{H,r} r_t^{k,H} - a \left( u_t^{H,r} \right) \right] + (1 - \delta) q_t \right\} \quad (\text{C.14})$$

– standard debt contract

$$E_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] \frac{1 + R_{t+1}^{k,H}}{1 + R_{t+1}^e} + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})} \left[ [\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})] \frac{1 + R_{t+1}^{k,H}}{1 + R_{t+1}^e} - 1 \right] \right\} = 0 \quad (C.15)$$

– zero profit condition for bank

$$[\Gamma_t(\bar{\omega}_t) - \mu G_t(\bar{\omega}_t)] \frac{q_{t-1} \bar{K}_t^{H,r}}{n_t^{H,r}} \frac{1 + R_t^{k,H}}{1 + R_t^e} = \frac{q_{t-1} \bar{K}_t^{H,r}}{n_t^{H,r}} - 1 \quad (C.16)$$

– law of motion for net worth

$$n_{t+1}^{H,r} = \gamma^H \frac{q_{t-1}}{\pi_t} \bar{K}_t^{H,r} \left[ R_t^{k,H} - R_t^e - \mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) (1 + R_t^{k,H}) \right] + \gamma^H \frac{n_t^{H,r}}{\pi_t} (1 + R_t^e) + w_t^{e,H,r} \quad (C.17)$$

- Safer entrepreneur

– first order condition with respect to capital utilization

$$r_t^{k,L} = a' \left( u_t^{L,l} \right) \quad (C.18)$$

– definition of rate of return on capital

$$1 + R_t^{k,L} = \frac{\pi_t}{q_{t-1}} \left\{ \left[ u_t^{L,l} r_t^{k,L} - a \left( u_t^{L,l} \right) \right] + (1 - \delta) q_t \right\} \quad (C.19)$$

– first order condition with respect to capital (using the definition of rate of return on capital)

$$R_{t+1}^{coupon} - R_{t+1}^{k,L} - 1 + \frac{1}{\beta} = 0 \quad (C.20)$$

– law of motion for net worth

$$n_{t+1}^{L,l} = \gamma^L [1 - \Omega(R_t^{coupon,a} - R_t^{coupon})] \frac{q_{t-1}}{\pi_t} \bar{K}_t^{L,l} (R_t^{k,L} - R_t^{coupon}) + [1 - \Omega(R_t^{coupon,a} - R_t^{coupon})] \frac{\gamma^L}{\pi_t} (1 + R_t^{coupon}) n_t^{L,l} + w_t^{e,L,l} \quad (C.21)$$

- Households

– first order condition with respect to time deposits

$$\lambda_{n,t} = \frac{\beta}{\pi_{t+1}} (1 + R_{t+1}^e) \lambda_{n,t+1} \quad (C.22)$$

– first order condition with respect to consumption

$$\lambda_{n,t} = \frac{1}{(C_t - bC_{t-1})} - \beta b \frac{1}{(C_{t+1} - bC_t)} \quad (\text{C.23})$$

- Aggregate resource constraint and production function

$$\begin{aligned} C_t + I_t + \eta \left[ \mu \int_0^{\bar{\omega}_t} \omega dF(\omega) \left( 1 + R_t^{k,H} \right) \frac{q_{t-1} \bar{K}_t^{H,r}}{\pi_t} \right] + UC_t \\ + \eta a \left( u_t^{H,r} \right) \bar{K}_t^{H,r} + (1 - \eta) a \left( u_t^{L,l} \right) \bar{K}_t^{L,l} = (1 - \eta_g) Y_t \end{aligned} \quad (\text{C.24})$$

$$Y_t = K_t^\alpha h_t^{1-\alpha} \quad (\text{C.25})$$

- Conditions associated with Calvo sticky prices and wages

$$\lambda_{n,t} Y_t + \beta \xi_p \left( \frac{\pi_t^{1-\iota_1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} - F_{p,t} = 0 \quad (\text{C.26})$$

$$\lambda_{n,t} Y_t \lambda_f s_t + \beta \xi_p \left( \frac{\pi_t^{1-\iota_1}}{\pi_{t+1}} \right)^{-\frac{\lambda_f}{\lambda_f-1}} K_{p,t+1} - K_{p,t} = 0 \quad (\text{C.27})$$

$$h_t \frac{\lambda_{n,t}}{\lambda_w} + \beta \xi_w \left( \frac{1}{\pi_{t+1} \frac{w_{t+1}}{w_t}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{(\pi_t^{1-\iota_{w1}})^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} - F_{w,t} = 0 \quad (\text{C.28})$$

$$h_t^{1+\sigma_L} + \beta \xi_w \left[ \frac{\pi_t^{1-\iota_{w1}}}{\pi_{t+1} \frac{w_{t+1}}{w_t}} \right]^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} K_{w,t+1} - K_{w,t} = 0 \quad (\text{C.29})$$

$$K_{p,t} = F_{p,t} \left[ \frac{1 - \xi_p \left( \frac{\pi_t^{1-\iota_1}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} \quad (\text{C.30})$$

$$K_{w,t} = F_{w,t} \frac{\tilde{w}_t}{\psi_L} \left\{ \frac{1 - \xi_w \left[ \frac{\pi_t^{1-\iota_{w1}}}{\pi_t \frac{w_t}{w_{t-1}}} \right]^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right\}^{1-\lambda_w(1+\sigma_L)} \quad (\text{C.31})$$

- Other variables

– External finance premium

$$P_t^e = \bar{\omega}_{t+1} \left( 1 + R_{t+1}^{k,H} \right) \frac{q_t \bar{K}_{t+1}^{H,r}}{q_t \bar{K}_{t+1}^{H,r} - n_{t+1}^{H,r}} - (1 + R_{t+1}^e) \quad (\text{C.32})$$

- Contractual, no-default interest rate on entrepreneurial debt

$$Z_t = \bar{\omega}_{t+1} \left( 1 + R_{t+1}^{k,H} \right) \frac{q_t \bar{K}_{t+1}^{H,r}}{q_t \bar{K}_{t+1}^{H,r} - n_{t+1}^{H,r}} \quad (\text{C.33})$$

- Aggregate net worth

$$n_{t+1}^{TOT} = \eta n_{t+1}^{H,r} + (1 - \eta) n_{t+1}^{L,l} \quad (\text{C.34})$$

- Bond amount

$$BI_{t+1}^{L,l} = q_t \bar{K}_{t+1}^{L,l} - n_{t+1}^{L,l} \quad (\text{C.35})$$

- Bank loans

$$B_{t+1}^{H,r} = q_t \bar{K}_{t+1}^{H,r} - n_{t+1}^{H,r} \quad (\text{C.36})$$

- Safer entrepreneur's leverage

$$lev_{t+1}^{L,l} = \frac{q_t \bar{K}_{t+1}^{L,l}}{n_{t+1}^{L,l}} \quad (\text{C.37})$$

- Riskier entrepreneur's leverage

$$lev_{t+1}^{H,r} = \frac{q_t \bar{K}_{t+1}^{H,r}}{n_{t+1}^{H,r}} \quad (\text{C.38})$$

- Aggregate leverage

$$lev_{t+1}^{TOT} = \eta lev_{t+1}^{H,r} + (1 - \eta) lev_{t+1}^{L,l} \quad (\text{C.39})$$

- Total credit (bank loans + bonds)

$$B_{t+1}^{TOT} = \eta B_{t+1}^{H,r} + (1 - \eta) BI_{t+1}^{L,l} \quad (\text{C.40})$$

- Monetary policy rule

$$R_t^e = (R_{t-1}^e)^{\tilde{\rho}} \left[ R^e \left( \frac{E_t \pi_{t+1}}{\bar{\pi}} \right)^{\alpha_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\alpha_y} \right]^{(1-\tilde{\rho})} \varepsilon_t^{MP} \quad (\text{C.41})$$

## Steady state

The strategy used for computing the steady state in this model follows the approach used by Christiano et al. (2003). We set one of the endogenous variables of the model to a value that seems reasonable based on empirical evidence, making this variable exogenous in the steady state calculation. We then move a model's exogenous variable into the list of variables that are endogenous in the steady state calculation. This approach allows us to simplify the problem of computing the steady state.

We set the steady state rental rate of capital of the riskier entrepreneur,  $r^{k,H}$ , to 0.0504, in line with the value used by CMR, and we choose the parameter  $\psi_L$  in (A.9) as endogenous variable. The set of



endogenous variables is:

$$\begin{aligned} \pi_t, s_t, I_t, \bar{\omega}_t, R_t^{k,H}, R_t^{k,L}, \bar{K}_t^{H,r}, \bar{K}_t^{L,l}, K_t, n_t^{H,r}, n_t^{L,l}, q_t, \lambda_{n,t}, C_t, \tilde{w}_t, h_t, \\ r_t^{k,L}, R_t^e, F_{p,t}, F_{w,t}, K_{p,t}, K_{w,t}, Y_t, \psi_L, u_t^{H,r}, u_t^{L,l}, \\ \varepsilon_t^{coupon}, \varepsilon_t^{coupon,baised}, R_t^{coupon}, R_t^{coupon,a}, R_t^{coupon,baised}, \chi_t, \\ P_t^{ext}, Z_t, B_t^{H,r}, BI_t^{L,l}, B_t^{TOT}, lev_t^{H,r}, lev_t^{L,l}, lev_t^{TOT}, n_t^{TOT}, \end{aligned}$$

and the equations available for computing the steady state value for these variables are (C.1)-(C.41).

As in Woodford (2003), steady state inflation is set to zero, that is,  $\bar{\pi} = 1$ . By assumption,  $u^{H,r} = u^{L,l} = 1$  and  $\chi = 0$ .

Solve for  $R^e$  and  $q$  using (C.22) and (C.11). Use (C.5) and (C.3) to compute  $\varepsilon^{coupon}$  and  $\varepsilon^{coupon,baised}$ . Solve for the steady state interest rates  $R^{coupon}$ ,  $R^{coupon,a}$  and  $R^{coupon,baised}$  using, respectively, (C.6), (C.1) and (C.4). Take the ratio of (C.26) and (C.27) to obtain the value for  $s$ . Equations (C.20) and (C.19) can be used to obtain  $R^{k,L}$  and  $r^{k,L}$ .

Now we set  $r^{k,H} = 0.0504$  and solve for  $R^{k,H}$  using (C.14). Then solve the non-linear system composed by equations (C.15)-(C.17) to obtain the values for  $n^{H,r}$ ,  $\bar{\omega}$  and  $\bar{K}^{H,r}$ . From (C.7) we get the value for  $\bar{K}^{L,l}$ . Solve for  $n^{L,l}$ ,  $K$  and  $I$  using (C.21), (C.10) and (C.12), respectively. Solve (C.9) for  $h$  and then (C.25) for  $Y$ . Then use (C.8) and (C.24) to solve for  $\tilde{w}$  and  $C$ . Get  $\lambda_n$  using (C.23). Equations (C.26), (C.28) and (C.29) can be used to obtain  $F_p$ ,  $F_w$  and  $K_w$ . It then follows from (C.30) that  $K_p = F_p$ . Finally, solve for  $\psi_L$  using (C.31). The remaining variables are trivial functions of the structural parameters and other steady state values and are computed using equations (C.32)-(C.40).

In these calculations, all variables must be positive, and

$$\bar{K}^{H,r} > n^{H,r} > 0 ; \bar{K}^{L,l} > n^{L,l} > 0 ; Z > R^{coupon} .$$

## Appendix D - Calibration: threshold level for side payments ( $\bar{\Omega}$ )

In this appendix we define the threshold for  $\Omega$  below which the entrepreneur would always be better off when offering side payments.

The entrepreneur has two options. He can:

1. issue bonds at the “normal” coupon rate  $R_{t+1}^{coupon,a}$  (equation 8);
2. offer side payments and obtain a lower coupon rate ( $R_{t+1}^{coupon,bias}$ , in equation 11).

In the first case,  $R_t^{coupon} = R_t^{coupon,a}$ , so entrepreneur’s equity and net worth are given by, respectively,

$$V_t^{L,l,a} = revenues - (1 + R_t^{coupon,a}) BI_t^{L,l}$$

$$N_{t+1}^{L,l,a} = \gamma^L V_t^{L,l,a} + W_t^{e,L,l} ,$$

where  $revenues = \left\{ \left[ u_t^{L,l} r_t^{k,L} - a \left( u_t^{L,l} \right) \right] P_t + (1 - \delta) Q_{\bar{k}',t} \right\} \bar{K}_t^{L,l}$  and  $BI_t^{L,l} = Q_{\bar{k}',t-1} \bar{K}_t^{L,l} - N_t^{L,l}$ .

In the second case,  $R_t^{coupon} = R_t^{coupon,bias}$ , so entrepreneur’s equity and net worth are now given by, respectively,

$$V_t^{L,l,b} = revenues - \left( 1 + R_t^{coupon,bias} \right) BI_t^{L,l}$$

$$N_{t+1}^{L,l,b} = \gamma^L \left[ 1 - \Omega \left( R_t^{coupon,a} - R_t^{coupon,bias} \right) \right] V_t^{L,l,b} + W_t^{e,L,l} .$$

The entrepreneur is therefore better off offering side payments whenever

$$N_{t+1}^{L,l,b} \geq N_{t+1}^{L,l,a}$$

$$\Leftrightarrow \left[ 1 - \Omega \left( R_t^{coupon,a} - R_t^{coupon,bias} \right) \right] V_t^{L,l,b} \geq V_t^{L,l,a}$$

$$\Leftrightarrow V_t^{L,l,b} - V_t^{L,l,a} \geq \Omega \left( R_t^{coupon,a} - R_t^{coupon,bias} \right) V_t^{L,l,b}$$

$$\Leftrightarrow \left( R_t^{coupon,a} - R_t^{coupon,bias} \right) BI_t^{L,l} \geq \Omega \left( R_t^{coupon,a} - R_t^{coupon,bias} \right) V_t^{L,l,b}$$

$$\Leftrightarrow BI_t^{L,l} \geq \Omega V_t^{L,l,b}$$

$$\Leftrightarrow \Omega \leq \frac{BI_t^{L,l}}{V_t^{L,l,b}} .$$

Given the calibration in table 1, in the steady state it results that

$$\Omega \leq \frac{BI_t^{L,l}}{V_t^{L,l,b}} = \frac{\bar{K}^{L,l} - n^{L,l}}{(r^{k,L} - \delta - R^{coupon,bias}) \bar{K}^{L,l} + (1 + R^{coupon,bias}) n^{L,l}} = 0.25 = \bar{\Omega} .$$

Table 1: Model parameters (time unit of model: quarterly)

Households	Value	Source	Description
$\beta$	0.9875	our calibration	discount factor
$\psi_L$	(36)	(endogenous)	weight on disutility of labor
$\sigma_L$	1	CMR	curvature of disutility of labor
$b$	0.63	CMR	habit persistence in consumption
$\xi_w$	0.75	Erceg et al. (2000)	fraction of households that cannot reoptimize wage
$\lambda_w$	1.05	CMR	markup, workers
$\iota_{w1}$	0.29	CMR	weight of wage indexation to steady state inflation
<b>Firms</b>			
$\alpha$	0.36	Levin et al. (2005)	capital share in the production function
$\xi_p$	0.75	Erceg et al. (2000)	fraction of firms that cannot reoptimize price
$\iota_1$	0.16	CMR	weight of price indexation to steady state inflation
$\lambda_f$	1.2	CMR	markup, intermediate good firms
$S''$	29.3	CMR	curvature of investment adjustment cost function
$\delta$	0.03	CMR	depreciation rate on capital
$\rho$	0.6	our calibration	degree of substitutability between capital services
<b>Entrepreneurs</b>			
$\sigma_a^H, \sigma_a^L$	18.9	CMR	curvature of capital utilization cost functions
$\mu$	0.15	our calibration	fraction of realized profits lost in bankruptcy
$\sigma^H$	$\sqrt{0.3}$	our calibration	standard deviation of productivity shock
$\omega^{e,H,r}, \omega^{e,L,l}$	0.02	CMR	transfer from households
$\gamma^L$	0.96	our calibration	survival probability of safer entrepreneurs
$\gamma^H$	0.97	our calibration	survival probability of riskier entrepreneurs
$\eta$	0.3	our calibration	share of riskier entrepreneurs
$\Omega$	0.1	our calibration	percentage of equity paid as side payments
<b>Bond Market</b>			
$\varepsilon^{coupon,a}$	510	Chen et al. (2007)	interest rate elasticity of the demand for funds
$\rho_x$	0.9	our calibration	degree of persistence in optimism
$\alpha_3$	40	our calibration	sensitivity of optimism to entrepreneur's net worth
$\bar{\chi}$	0	our calibration	steady state level of optimism
<b>Policy</b>			
$\tilde{\rho}$	0.88	CMR	interest rate smoothing
$\alpha_\pi$	1.82	CMR	weight of expected inflation in Taylor rule
$\alpha_y$	0.11	CMR	weight of output gap in Taylor rule
$\eta_g$	0.2	CMR	share of government consumption

Table 2: Steady State Properties, Model versus U.S. Data

<i>Variable</i>	<i>Model</i>	<i>U.S. data</i>
$K/Y$	5.48	10.7
$C/Y$	0.63	0.56
$I/Y$	0.17	0.25
$G/Y$	0.2	0.2
leverage ratio = $Q\bar{K}/N$ <sup>1</sup>	<i>safer</i> 1.26	[1.21 ; 1.77]
	<i>riskier</i> 1.35	
<b>bond to bank finance ratio</b> <sup>2</sup>	<b>1.36</b>	<b>1.34</b>

When not specified, the source for U.S. data is CMR and the sample period is 1998Q4-2003Q4. <sup>1</sup> CMR compute the leverage as  $N/(Q\bar{K} - N)$ . We compute the leverage as in Bernanke et al. (1999). <sup>2</sup> Source: De Fiore and Uhlig (2005).

Table 3: Interest Rates, Model versus U.S. Data

<i>Variable</i>	<i>Model</i>	<i>U.S. data</i>
Rate of return on capital, $R^k$	<i>safer</i> 11.38 %	10.32 %
	<i>riskier</i> 8.40 %	
Cost of external finance, $Z$	6.81 %	[7.1 ; 8.1] %
Time deposit, $R^e$	5.16 %	5.12 %
Cost of bond finance, $R^{coupon}$	5.99 %	5.96 % <sup>1</sup>

When not specified, the source for U.S. data is CMR and the sample period is 1987Q1-2003Q4. <sup>1</sup> Chen et al. (2007) find an average yield spread of AAA bonds over the period 1995-2003 of 84 basis points. Adding this spread to the risk-free rate ( $R^e$ ) gives the value displayed in the table.

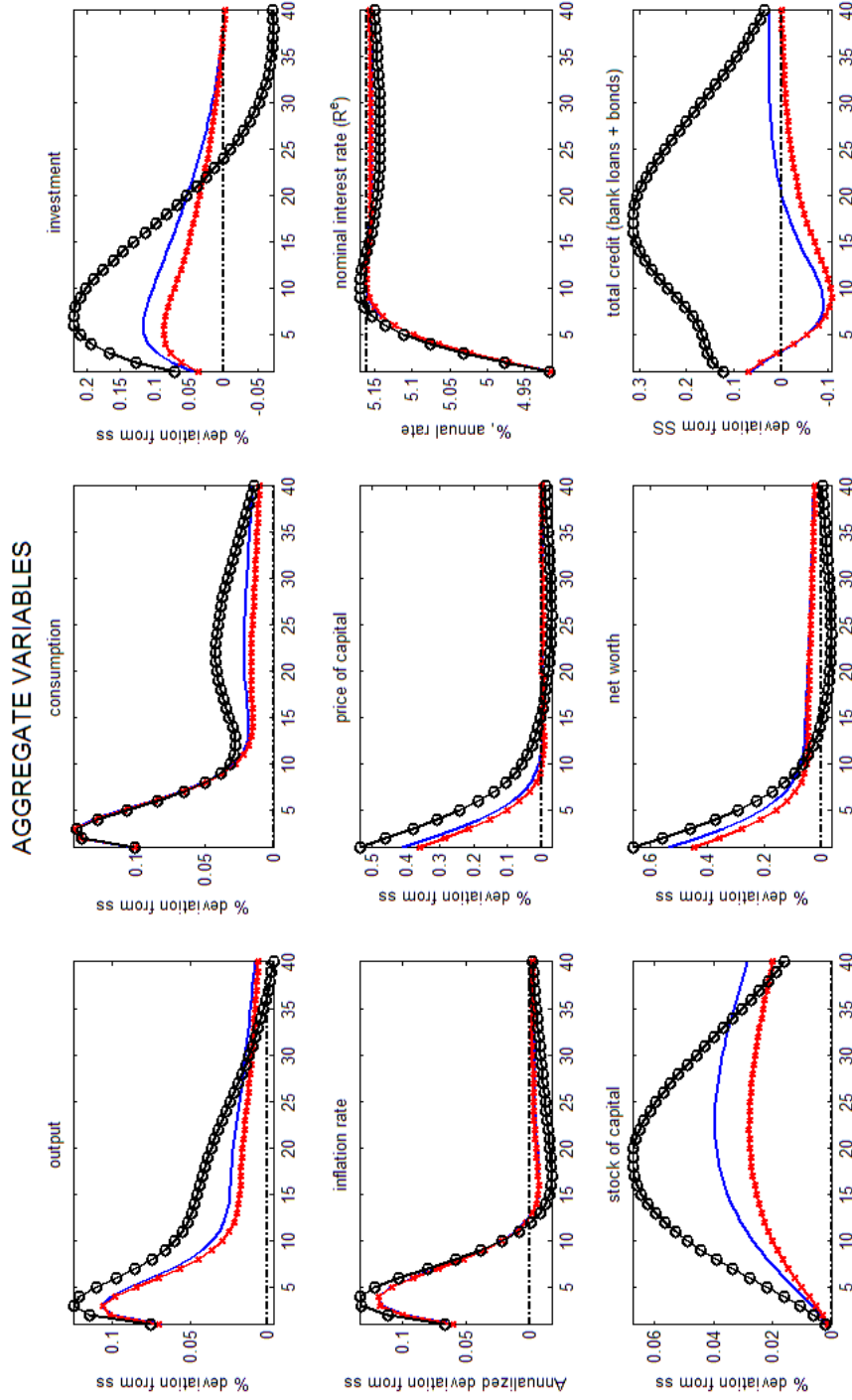


Figure 3: Impulse responses of aggregate variables to a 25 basis point decrease to the nominal interest rate

Note. Values expressed as percentage deviation from steady state values. Inflation is expressed as annualized percent deviation from its steady state and the interest rates are expressed as annual percentage points. Variant 1 (CMR, *Financial Accelerator Model*): blue solid line. Variant 2 (model with  $r_2 = 0$ ): red crossed line. Variant 3 (model with  $r_2 = 1$ ): black circled-dotted line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.

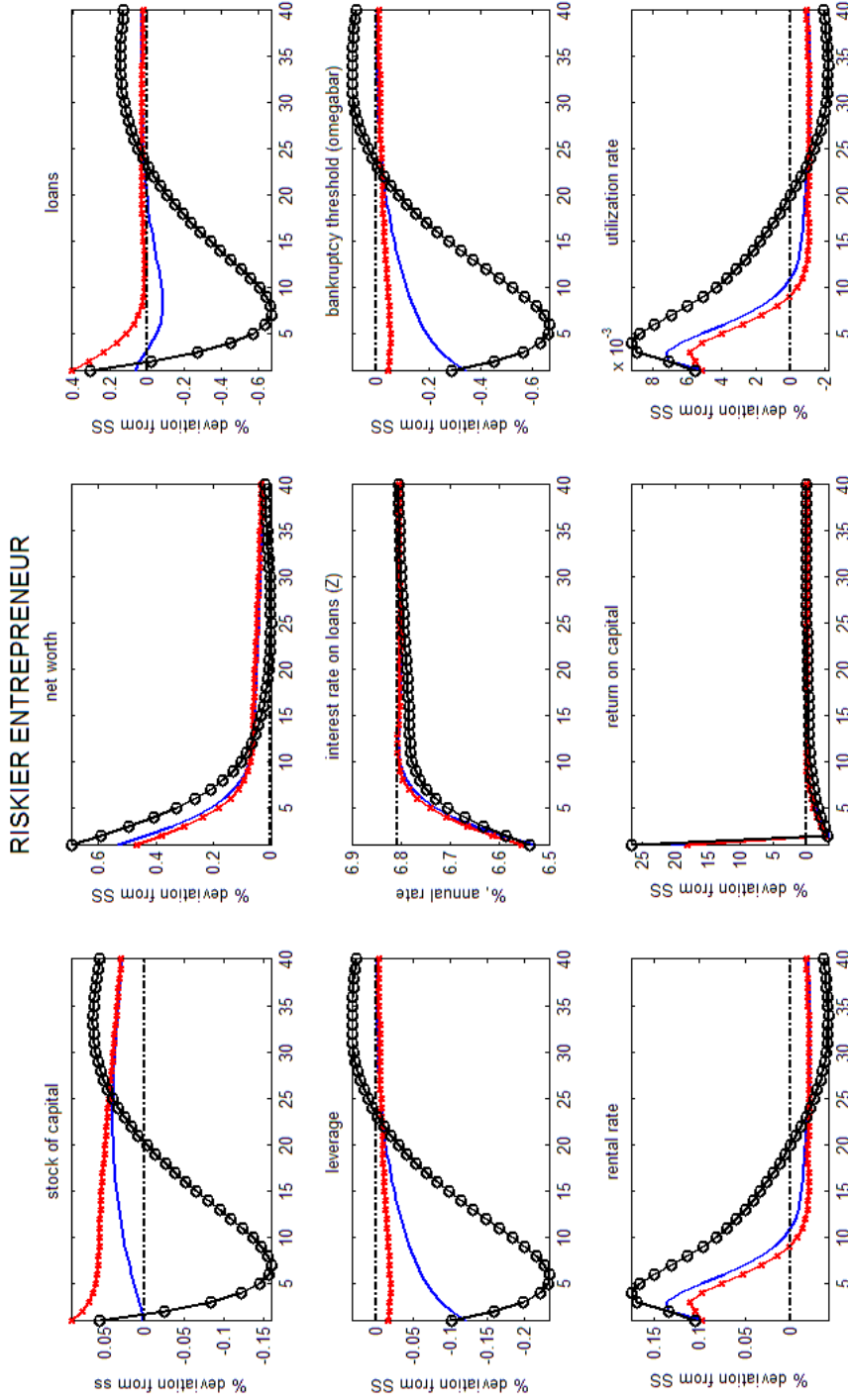


Figure 4: Impulse responses of riskier entrepreneur variables to a 25 basis point decrease to the nominal interest rate

Note. Values expressed as percentage deviation from steady state values, except for interest rates which are expressed as annual percentage points. Variant 1 (CMR, *Financial Accelerator Model*): blue solid line. Variant 2 (model with  $r_2 = 0$ ): red solid line. Variant 3 (model with  $r_2 = 1$ ): black solid line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.

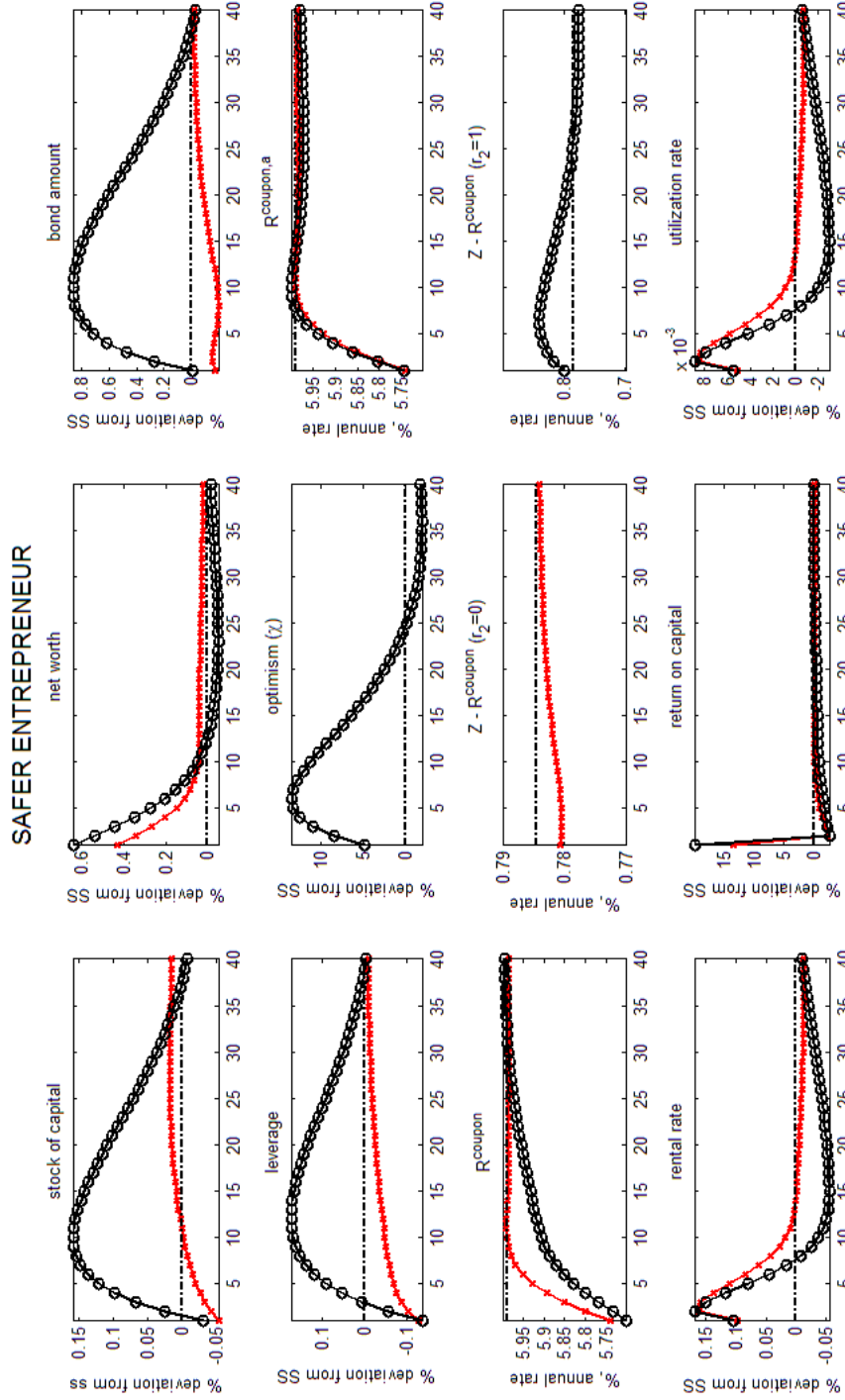


Figure 5: Impulse responses of safer entrepreneur variables to a 25 basis point decrease to the nominal interest rate. Note. Values expressed as percentage deviation from steady state values, except for interest rates which are expressed as annual percentage points. Variant 2 (model with  $r_2 = 0$ ): red crossed line. Variant 3 (model with  $r_2 = 1$ ): black dashed-dotted line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.

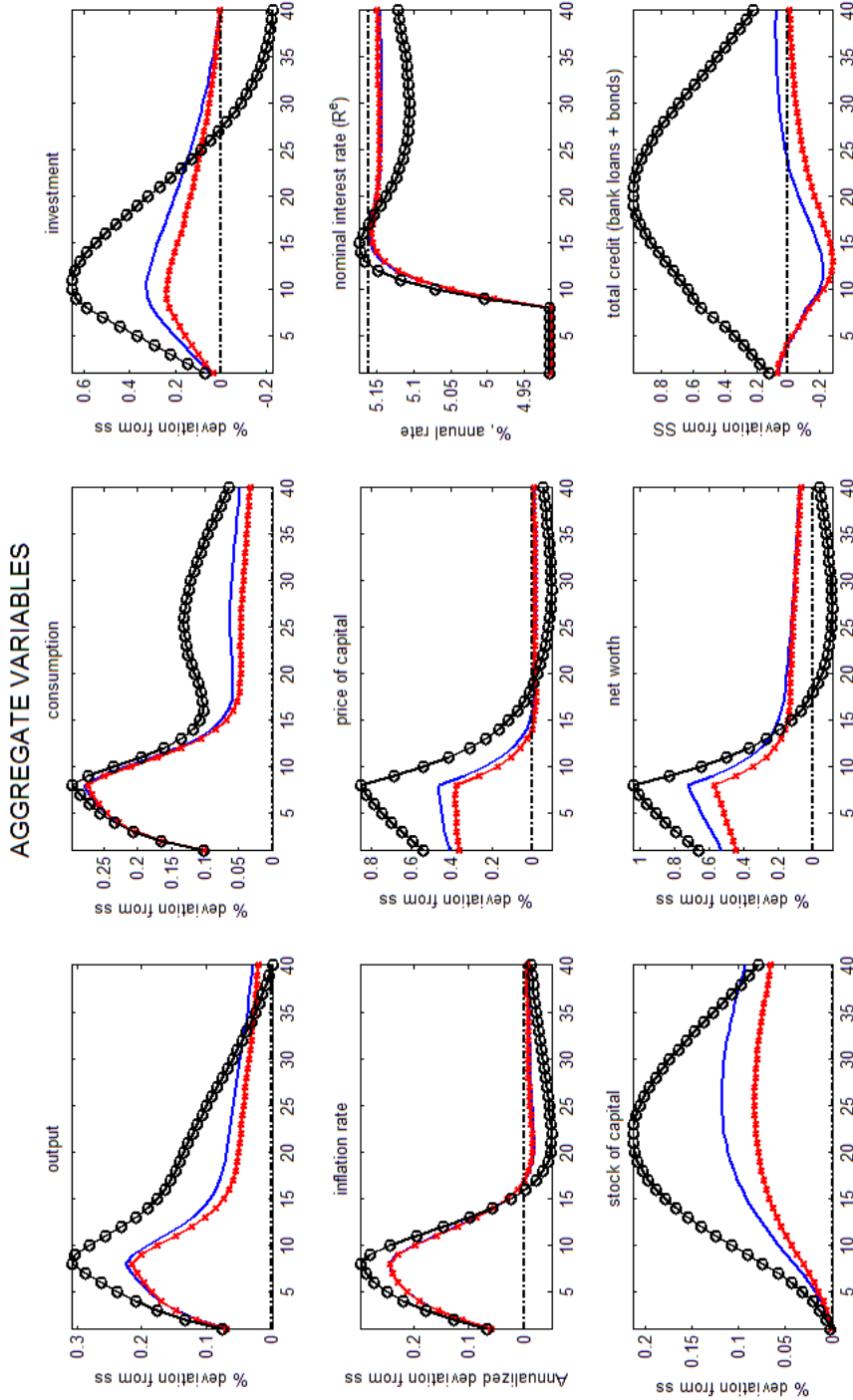


Figure 6: Impulse responses of aggregate variables to a persistent decrease in the nominal interest rate

Note. Values expressed as percentage deviation from steady state values. Inflation is expressed as annualized percent deviation from its steady state and the interest rates are expressed as annual percentage points. Variant 1 (CMR, *Financial Accelerator Model*): blue solid line. Variant 2 (model with  $r_2 = 0$ ): red crossed line. Variant 3 (model with  $r_2 = 1$ ): black dashed-dotted line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.



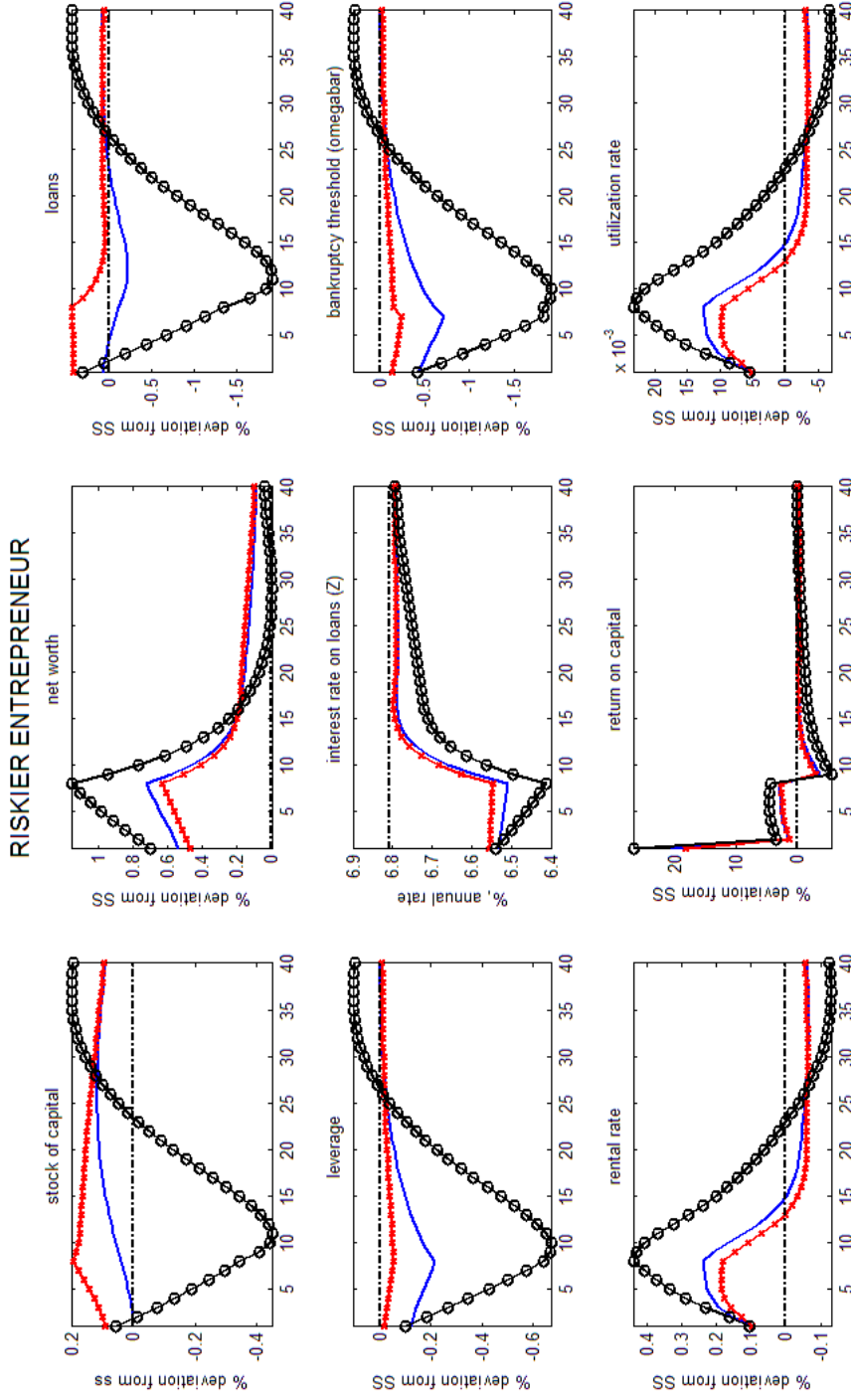


Figure 7: Impulse responses of riskier entrepreneur variables to a persistent decrease in the nominal interest rate

Note. Values expressed as percentage deviation from steady state values, except for interest rates which are expressed as annual percentage points. Variant 1 (CMR, *Financial Accelerator Model*): blue solid line. Variant 2 (model with  $r_2 = 0$ ): red solid line. Variant 3 (model with  $r_2 = 1$ ): black solid line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.

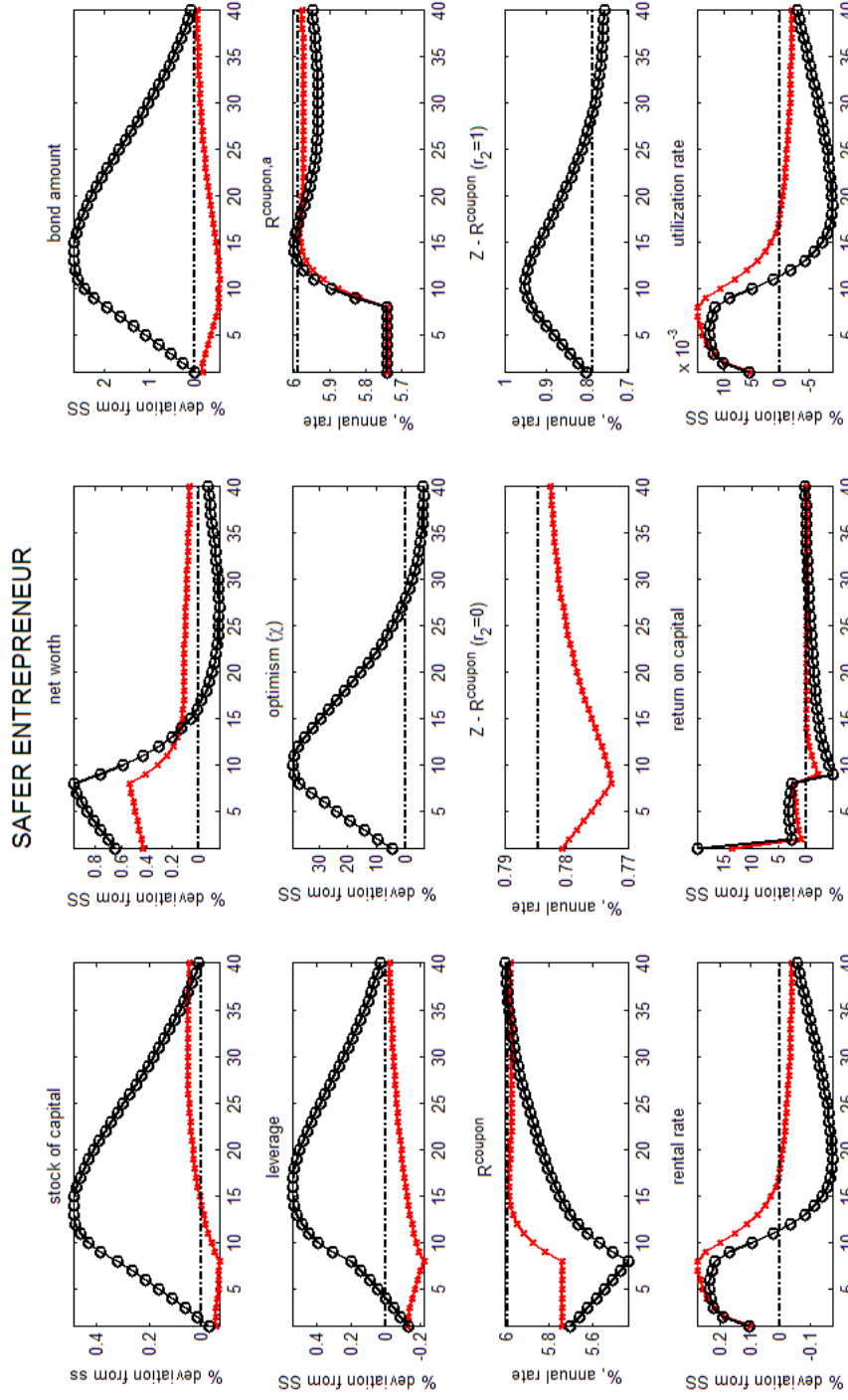


Figure 8: Impulse responses of safer entrepreneur variables to a persistent decrease in the nominal interest rate

Note. Values expressed as percentage deviation from steady state values, except for interest rates which are expressed as annual percentage points. Variant 2 (model with  $r_2 = 0$ ): red crossed line. Variant 3 (model with  $r_2 = 1$ ): black dashed-dotted line. Steady state: black dashed-dotted line. Baseline parameters: see table 1.

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