

Delamination behaviour of composites

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## Introduction

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1 Laminated composites are becoming the preferred material system in a variety  
2 of industrial applications, such as aeronautical and aerospace structures, ship  
3 hulls in naval engineering, automotive structural parts, micro-electro-  
4 mechanical systems as also civil structures for strengthening concrete members.  
5 The increased strength and stiffness for a given weight, increased toughness,  
6 increased mechanical damping, increased chemical and corrosion resistance  
7 in comparison to conventional metallic materials and potential for structural  
8 tailoring are some of the factors that have contributed to the advancement of  
9 laminated composites. Their increased use has underlined the need for  
10 understanding their modes of failure and evolving technologies for the continual  
11 enhancement of their performance.

12 The principal mode of failure of layered composites is the separation  
13 along the interfaces of the layers, *viz.* delamination. This type of failure is  
14 induced by interlaminar tension and shear that develop due to a variety of  
15 factors such as: Free edge effects, structural discontinuities, localized  
16 disturbances during manufacture and in working condition, such as impact  
17 of falling objects, drilling during manufacture, moisture and temperature  
18 variations and internal failure mechanisms such as matrix cracking. Hidden  
19 from superficial visual inspection, delamination lies often buried between  
20 the layers, and can begin to grow in response to an appropriate mode of  
21 loading, drastically reducing the stiffness of the structure and thus the life of  
22 the structure. The delamination growth often occurs in conjunction with  
23 other modes of failure, particularly matrix cracking.

24 A study of composite delamination, as does any technological discipline,  
25 has two complementary aspects: An in depth understanding of the phenomenon  
26 by analysis and experimentation and the development of strategies for  
27 effectively dealing with the problem. These in turn lead to a number of  
28 specific topics that we need to consider in the present context. These comprise  
29 of:

- 30 1. An understanding of the basic principles that govern the initiation of  
31 delamination, its growth and its potential interaction with other modes  
32 of failure of composites. This is the theme of the first chapter, but several  
33 authors return to this theme in their own respective contributions.
- 34 2. The determination of material parameters that govern delamination  
35 initiation and growth by appropriate testing. These must necessarily be  
36 interfacial strength parameters which govern interlaminar fracture initiation  
37 and interlaminar fracture toughness parameters, *viz.* critical strain energy  
38 release rates that must govern interlaminar crack growth. The book contains  
39 several valuable contributions from leading international authorities in  
40 the field of testing of composites.
- 41 3. Development of analytical tools : What are the methodologies one may  
42 employ to assess the possibility of delamination onset and growth under  
43

- typical loading scenarios? This may be approached from the points of view of fracture mechanics, damage mechanics, cohesive modeling approach and approaches which draw from and combine these. In particular, the cohesive modeling approach has proven to be a powerful and versatile tool in that when embedded in a nonlinear finite element analysis, it can trace the two-dimensional delamination growth without user interference, is robust from the point of view of numerical convergence, and can potentially account for a variety of interfacial failure mechanisms. This subject is discussed thoroughly in several authoritative contributions.
4. Detection of delamination: Ability to diagnose the presence of delamination and to be able to capture in graphical terms the extent of delamination damage is a desideratum towards which the composite industry is continuing to make progress. Several nondestructive evaluation tools are available and have been used with varying degrees of success. Acoustic emission, Lamb-wave and Piezo-electric technologies are discussed in the context of delamination detection in the present work.
  5. Prevention of delamination: Several techniques of either inhibiting delamination or altogether suppressing it are available. The book contains a section treating the following techniques of delamination prevention/inhibition: 'Self-healing' composites which internally exude adhesive material as soon as crack advances thus effectively arresting the crack; Z-pin bridging in which fibers are introduced across the interlaminar surfaces, liable to delaminate, artfully tapering off discontinuities which are sources of potential delamination and the use of toughened epoxies.
  6. Delamination driven structural failure: Certain loading scenarios can cause delamination growth if there is some preexisting delamination in the structural component which in turn can lead to structural failure. Typically these are: Impact, cyclic loading (delamination due to fatigue), compressive loading causing localized buckling in the vicinity of delamination and dynamic loading in the presence of in-plane compression. Impact loading and any form of dynamic loading in the presence of significant compressive stress in sandwich structures are known to trigger delamination failure which is abrupt and total. These aspects have been discussed in several contributions.

The book has been divided into several sections to address the issues mentioned in the foregoing. It has been a pleasure to work with a number of authors of international standing and reputation who had spent a great deal of effort in developing their respective chapters. The references cited at the end of each chapter should supplement and corroborate the concepts developed in the chapter. We hope that researchers and engineers who are concerned to apply state of the art technologies to composite structural analysis, design and evaluation of risk of failure will find this book useful and a valuable source of insight.

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## Interaction of matrix cracking and delamination

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### 11.1 Introduction

Laminated composite materials have high strength-to-weight and stiffness-to-weight ratios. They can be considered as laminar systems with weak interfaces. Consequently, they are very susceptible to interlaminar damage designated by delamination. In the presence of delaminations the material stiffness and, consequently, of the associated structure, can be drastically reduced which can lead to its catastrophic failure. Moreover, delamination is an internal damage and is not easily detected, which increases the associated risks.

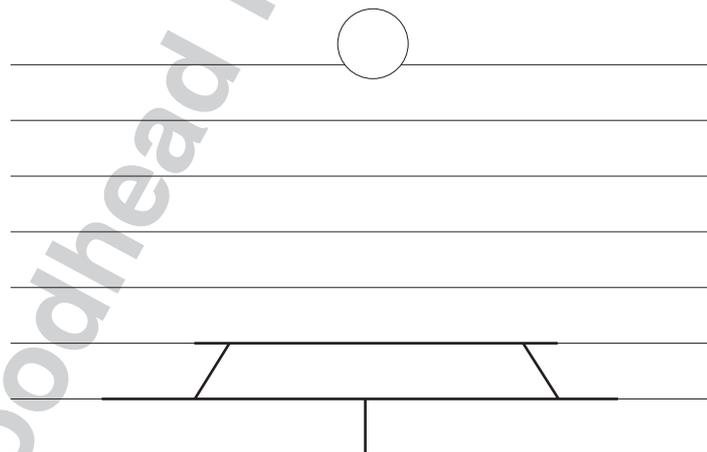
#### 11.1.1 Damage mechanism

In the majority of real applications delamination does not occur alone. It is known that matrix cracking inside layers and delamination are usually associated and constitute a typical damage mechanism of composites (Takeda *et al.*, 1982; Joshi and Sun, 1985), especially when structures are submitted to bending loads. In fact, although the phenomenon can occur under tensile loading it acquires a remarkable importance under bending loads, e.g. low velocity impact (Choi *et al.*, 1991a). It is commonly accepted that there is a strong interaction between matrix cracking inside layers and delamination between layers. This coupling phenomenon is initiated by matrix cracking, i.e., shear and/or bending cracks in the early stages of the loading process. These cracks can originate delamination and significantly affect its propagation. Delamination occurs between layers with differing fibre orientations. In fact, when a bending or shear crack in a layer reaches an interface between two differently oriented layers it is unable to easily penetrate the other layer, thus propagating as a delamination. On the other hand, two adjacent laminae having different fibre angles induce extensional and bending stiffness mismatch which, combined with the low strength of the matrix, make composite materials very sensitive to delamination at those interfaces. In general, the delamination

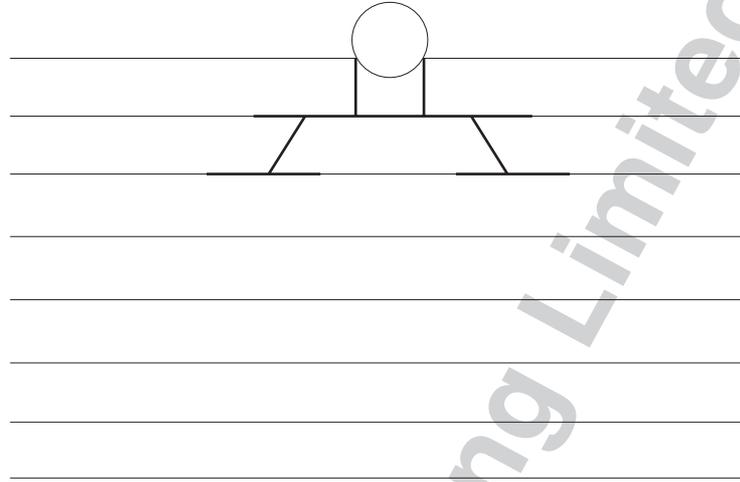
1 resistance of a given interface decreases with the increase of its stiffness  
 2 mismatching degree (Liu and Malvern, 1987). During delamination propagation  
 3 extensive micro-matrix cracks are generated in the adjacent layers.

4 It is worth noting that this complex interaction damage mechanism can  
 5 occur in different forms depending on the bending stiffness of the structure.  
 6 Flexible components respond primarily in a flexural mode, inducing significant  
 7 tensile stresses at farther layers from the loaded surface. Therefore, the cracks  
 8 located in these outermost layers are vertical and caused by bending effects  
 9 (Choi and Chang, 1992) and their initiation and growth occur in an almost  
 10 mode I fracture process. These cracks will generate a delamination along the  
 11 upper adjacent interface, which will interact with other matrix cracks of the  
 12 neighbour upper ply, leading to delamination on the second interface and so  
 13 on (see Fig. 11.1). Delamination patterns in flexible laminates present a  
 14 frustum-conical shape, where delaminations' size increases towards the lower  
 15 face of the laminate (Levin, 1991). A different failure mechanism occurs for  
 16 stiffer laminates. In this case, only small deflections take place and damage  
 17 initiates near to the loaded surface as a result of contact forces. Shear cracks  
 18 develop near to the impact indentation and propagate through the upper ply  
 19 up to the neighbour interface degenerating in a delamination. This delamination  
 20 extends from the loaded region until it also deflects into a lower ply by shear  
 21 and inclined cracks (see Fig. 11.2). Further delamination growth in stiff  
 22 laminates occurs in a barrel shape region by growth of delaminations around  
 23 the midplane (Olsson *et al.*, 2000).

24 Matrix bending cracks are a predominant mode I fracture process governed  
 25 by transverse normal tensile stresses whilst matrix shear cracks are governed  
 26 by interlaminar shear and transverse normal tensile stresses (Choi *et al.*,



42 11.1 Schematic representation of damage initiation mechanism in  
 43 flexible laminates.



11.2 Schematic representation of damage initiation mechanism in stiff laminates.

1991b). Delamination initiation is controlled by mode I although its growth is typically a mixed shear mode (II and III) fracture process (Choi and Chang, 1992), which is explained by the bending stiffness mismatching between adjacent differently oriented layers.

### 11.1.2 Classical prediction methodologies

Two main approaches have been used in order to simulate the damage mechanism described above. One is based on strength of materials concepts where materials are assumed to be free of defects. However, in many situations the problem of stress concentrations nearby to a notch or a flaw leads to mesh dependency in numerical approaches. To overcome this problem the stresses obtained analytically or numerically are used in a point stress or average stress criteria (Whitney and Nuismer, 1974) in order to evaluate the occurrence of failure. In the point stress criterion the stresses are evaluated at a characteristic distance whereas in the average stress criterion they are averaged over a distance. An example is given by Choi *et al.* (1991b) where quadratic average stress criteria were used to predict matrix cracking when

$$\left(\frac{{}^n \bar{\sigma}_{22}}{{}^n Y}\right)^2 + \left(\frac{{}^n \bar{\sigma}_{23}}{{}^n S_i}\right)^2 = 1 \tag{11.1}$$

and delamination when

$$D_a \left[ \left(\frac{{}^n \bar{\sigma}_{23}}{{}^n S_i}\right)^2 + \left(\frac{{}^{n+1} \bar{\sigma}_{13}}{{}^{n+1} S_i}\right)^2 + \left(\frac{{}^n \bar{\sigma}_{22}}{{}^n Y}\right)^2 \right] = 1 \tag{11.2}$$

The subscripts 1, 2 and 3 are the orthotropic local coordinates of the  $n$ th or  $(n + 1)$ th layers, which correspond, respectively, to the upper and lower plies of the  $n$ th interface.  $Y$  and  $S_i$  are the *in situ* ply transverse and shear interlaminar strengths, respectively, and  $D_a$  is an empirical constant which was determined from experiments using a fitting procedure. The stress components are averaged within the ply thickness

$${}^n \bar{\sigma}_{ij} = \frac{1}{h_n} \int_{t_{n-1}}^{t_n} \sigma_{ij} dz \quad 11.3$$

where  $t_n$  and  $t_{n-1}$  correspond to the position of the upper and lower interfaces of the  $n$ th ply,  $h$  is the ply thickness and  $z$  the axis normal to the plate. These criteria were applied in two steps:

- the matrix cracking criterion is initially applied in each layer;
- if matrix cracking is predicted in a given layer, the delamination criterion is applied subsequently considering two different circumstances; this criterion is applied to the lower adjacent interface if a shear crack was predicted and to the upper one if a bending crack was found.

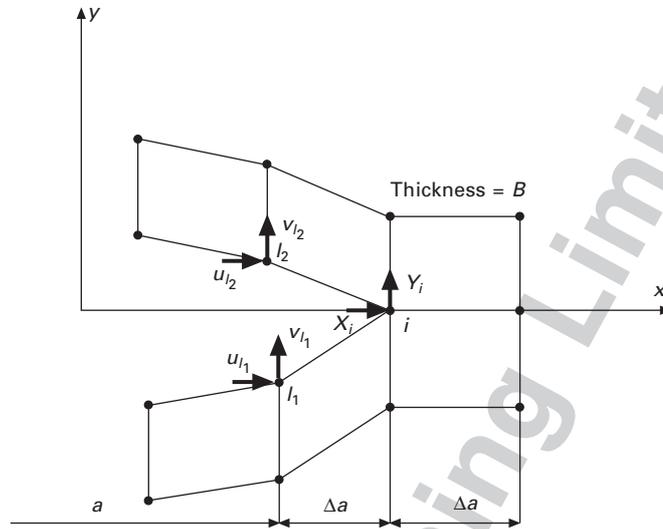
Unlike what happens in strength of materials based approaches, the fracture mechanics approach assumes the presence of an inherent defect in the material. The majority of the proposed works are based on the concepts of strain energy release rate. It is usually assumed that damage propagation occurs when the strain energy at the crack front is equal to the critical strain energy release rate, which is a material property. The strain energy release rates are commonly obtained by using the Virtual Crack Closure Technique (VCCT) (Krueger, 2002). Considering a two dimensional problem (see Fig. 11.3), the strain energies ( $G_I$  and  $G_{II}$ ) can be calculated by the product of the relative displacements at the 'opened point' (nodes  $l_1$  and  $l_2$ ) and the loads at the 'closed point' (node  $i$ )

$$G_I = \frac{1}{2B\Delta a} Y_i \Delta v_i$$

$$G_{II} = \frac{1}{2B\Delta a} X_i \Delta u_i \quad 11.4$$

being  $B\Delta a$  the area of the new surface created by an increment of crack propagation (see Fig. 11.3). It should be assured that self similar propagation occurs and an adequate refined mesh should be used. Liu *et al.* (1993) used the VCCT to obtain the strains energy release rates and a linear mixed-mode criterion

$$\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} = 1 \quad 11.5$$



11.3 Scheme of the VCCT.

to simulate the propagation of a small initial delamination crack introduced after the occurrence of the initial matrix cracking failure.  $G_{Ic}$  and  $G_{IIc}$  represent the interlaminar critical strain energy release rates in modes I and II, respectively. A similar approach was followed by Zou *et al.* (2002) where a mixed-mode delamination growth is considered when

$$\left(\frac{G_I}{G_{Ic}}\right)^\alpha + \left(\frac{G_{II}}{G_{IIc}}\right)^\beta + \left(\frac{G_{III}}{G_{IIIc}}\right)^\gamma = 1 \quad 11.6$$

and  $\alpha$ ,  $\beta$  and  $\gamma$  are mixed-mode fracture parameters determined from material tests.

The stress and fracture mechanics based criteria present some disadvantages. The stress based methods present mesh dependency during numerical analysis due to stress singularities. On the other hand, the point/average stress criteria require the definition of a critical dimension which depends on the material and stacking sequence (Tan, 1989), and do not have a physically powerful theoretical foundation. Fracture mechanics approach relies on the definition of an initial flaw or crack length. However, in many structural applications the locus of damage initiation is not obvious. On the other hand, the stress-based methods behave well at predicting delamination onset, and fracture mechanics has already demonstrated its accuracy in the delamination propagation modelling. In order to overcome the referred drawbacks and exploit the usefulness of the described advantages, cohesive damage models and continuum damage mechanics emerge as suitable options. These methodologies combine aspects of stress based analysis to model damage

1 initiation and fracture mechanics to deal with damage propagation. Thus, it  
 2 is not necessary to take into consideration an initial defect and mesh dependency  
 3 problems are minimized.

## 4 5 **11.2 Mixed-mode cohesive damage model**

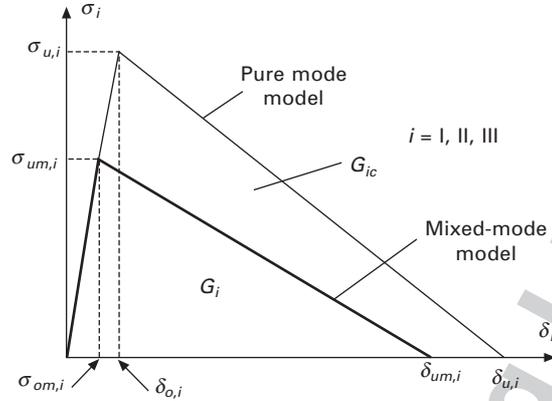
6  
7 Cohesive damage models are frequently used to simulate damage onset and  
 8 growth. They are usually based on a softening relationship between stresses  
 9 and relative displacements between crack faces, thus simulating a gradual  
 10 degradation of material properties. They do not depend on a predefined  
 11 initial flaw unlike conventional fracture mechanics approaches. Typically,  
 12 stress based and energetic fracture mechanics criteria are used to simulate  
 13 damage initiation and growth, respectively. Usually cohesive damage models  
 14 are based on spring (Cui and Wisnom, 1993 and Lammerant and Verpoest,  
 15 1996) or interface finite elements (Mi *et al.*, 1998, Petrossian and Wisnom,  
 16 1998; de Moura *et al.*, 2000) connecting plane or three-dimensional solid  
 17 elements. Those elements are placed at the planes where damage is prone to  
 18 occur which, in several structural applications, can be difficult to identify *a*  
 19 *priori*. However, an important characteristic of delamination is that its  
 20 propagation is restricted to a well defined plane corresponding to the interface  
 21 between two differently oriented layers, thus leading to a typical application  
 22 of cohesive methods. Taking this into consideration, a cohesive mixed-mode  
 23 damage model based on interface finite elements is presented.

24 The formulation is based on the constitutive relationship between stresses  
 25 on the crack plane and the corresponding relative displacements

$$26 \quad \sigma = \mathbf{E} \delta_r \quad 11.7$$

27  
28 where  $\delta_r$  is the vector of relative displacements between homologous points,  
 29 and  $\mathbf{E}$  a diagonal matrix containing the penalty parameter  $e$  introduced by  
 30 the user. Its values must be quite high in order to hold together and prevent  
 31 interpenetration of the element faces. Following a considerable number of  
 32 numerical simulations (Gonçalves *et al.*, 2000), it was found that  $e = 10^7$   
 33 N/mm<sup>3</sup> produced converged results and avoided numerical problems during  
 34 the non-linear procedure.

35 The interface finite element includes a damage model to simulate damage  
 36 onset and growth. Equation 11.7 is only valid before damage initiation. The  
 37 considered damage model combines aspects of strength-based analysis and  
 38 fracture mechanics. It is based on a softening process between stresses and  
 39 interfacial relative displacements and includes a mixed-mode formulation.  
 40 After peak stress the material softens progressively or, in other words, undergoes  
 41 damage (see Fig. 11.4). To avoid the singularity at the crack tip and its  
 42 effects, a gradual rather than a sudden degradation, which would result in  
 43 mesh-dependency, is considered. It is assumed that failure occurs gradually



11.4 Pure and mixed-mode damage model.

as energy is dissipated in a cohesive zone behind the crack tip. This is equivalent to the consideration of a 'Fracture Process Zone', defined as the region in which the material undergoes softening deterioration by different ways, e.g., micro-cracking, fibre bridging and inelastic processes. Numerically, this is implemented by a damage parameter whose values vary from zero (undamaged) to unity (complete loss of stiffness) as the material deteriorates. For pure mode (I, II or III) loading, a linear softening process starts when the interfacial stress reaches the respective strength  $\sigma_{u,i}$  (see Fig. 11.4). The softening relationship can be written as

$$\boldsymbol{\sigma} = (\mathbf{I} - \mathbf{D})\mathbf{E}\boldsymbol{\delta}_r \quad 11.8$$

where  $\mathbf{I}$  is the identity matrix and  $\mathbf{D}$  is a diagonal matrix containing, on the position corresponding to mode  $i$  ( $i = \text{I, II, III}$ ), the damage parameter,

$$d_i = \frac{\delta_{u,i}(\delta_i - \sigma_{o,i})}{\delta_i(\delta_{u,i} - \sigma_{o,i})} \quad 11.9$$

where  $\delta_{o,i}$  and  $\delta_{u,i}$  are, respectively, the onset and ultimate relative displacements of the softening region (see Fig. 11.4), and  $\delta_i$  is the current relative displacement. The maximum relative displacement,  $\delta_{u,i}$ , at which complete failure occurs, is obtained by equating the area under the softening curve to the respective critical strain energy release rate

$$G_{ic} = \frac{1}{2} \sigma_{u,i} \delta_{u,i} \quad 11.10$$

In general, structures are subjected to mixed-mode loadings. Therefore, a formulation for interface elements should include a mixed-mode damage model, which, in this case, is an extension of the pure mode model described

1 above (see Fig. 11.4). Damage initiation is predicted by using a quadratic  
2 stress criterion

$$\begin{aligned} 3 & \\ 4 & \left(\frac{\sigma_I}{\sigma_{u,I}}\right)^2 + \left(\frac{\sigma_{II}}{\sigma_{u,II}}\right)^2 + \left(\frac{\sigma_{III}}{\sigma_{u,III}}\right)^2 = 1 \quad \text{if } \sigma_I \geq 0 \\ 5 & \\ 6 & \left(\frac{\sigma_{II}}{\sigma_{u,II}}\right)^2 + \left(\frac{\sigma_{III}}{\sigma_{u,III}}\right)^2 = 1 \quad \text{if } \sigma_I \leq 0 \\ 7 & \\ 8 & \end{aligned} \quad 11.11$$

9 assuming that normal compressive stresses do not promote damage.  
10 Considering Equation 11.7, the first equation 1.11 can be rewritten in function  
11 of relative displacements

$$12 \left(\frac{\delta_{om,I}}{\delta_{o,I}}\right)^2 + \left(\frac{\delta_{om,II}}{\delta_{o,II}}\right)^2 + \left(\frac{\delta_{om,III}}{\delta_{o,III}}\right)^2 = 1 \quad 11.12$$

13  $\delta_{om,i}$  ( $i = I, II, III$ ) being the relative displacements corresponding to damage  
14 initiation. Defining an equivalent mixed-mode displacement

$$15 \delta_m = \sqrt{\delta_I^2 + \delta_{II}^2 + \delta_{III}^2} \quad 11.13$$

16 and mixed-mode ratios

$$17 \beta_i = \frac{\delta_i}{\delta_I} \quad 11.14$$

18 the mixed-mode relative displacement at the onset of the softening process  
19 ( $\delta_{om}$ ) can be obtained combining Equations 11.12–11.14.

$$20 \delta_{om} = \delta_{o,I} \delta_{o,II} \delta_{o,III} \sqrt{\frac{1 + \beta_{II}^2 + \beta_{III}^2}{(\delta_{o,II} \delta_{o,III})^2 + (\beta_{II} \delta_{o,I} \delta_{o,III})^2 + (\beta_{III} \delta_{o,I} \delta_{o,II})^2}} \quad 11.15$$

21 The corresponding relative displacement for each mode,  $\delta_{om,i}$ , can be obtained  
22 from Equations 11.13–11.15.

$$23 \delta_{om,i} = \frac{\beta_i \delta_{om}}{\sqrt{1 + \beta_{II}^2 + \beta_{III}^2}} \quad 11.16$$

24 Once a crack has initiated the above stress-based criterion cannot be used in  
25 the vicinity of the crack tip due to stress singularity. Consequently, the  
26 mixed-mode damage propagation is simulated using the linear fracture energetic  
27 criterion

$$28 \frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} + \frac{G_{III}}{G_{IIIc}} = 1 \quad 11.17$$

The released energy in each mode at complete failure can be obtained from the area of the minor triangle of Fig. 11.4.

$$G_i = \frac{1}{2} \sigma_{um,i} \delta_{um,i} \quad 11.18$$

Considering Equations 11.7, 11.13 and 11.14, the energies (Equations [11.10 and 11.18]) can be written in function of relative displacements. Substituting into Equation 11.17, it can be obtained

$$\delta_{um} = \frac{2(1 + \beta_{II}^2 + \beta_{III}^2)}{e \delta_{om}} \left[ \frac{1}{G_{Ic}} + \frac{\beta_{II}^2}{G_{IIc}} + \frac{\beta_{III}^2}{G_{IIIc}} \right] \quad 11.19$$

which corresponds to the mixed-mode displacement at failure. The ultimate relative displacements in each mode,  $\delta_{um,i}$  can be obtained from Equations 11.13, 11.14 and 11.19

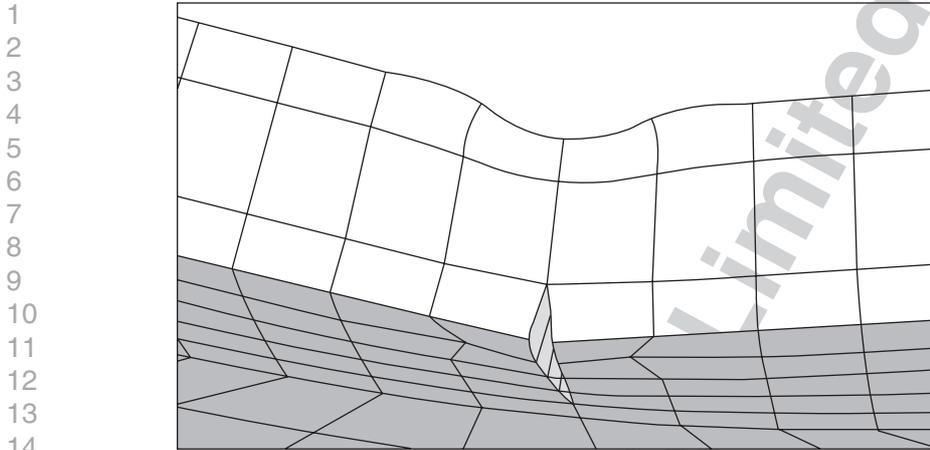
$$\delta_{um,i} = \frac{\beta_i \delta_{um}}{\sqrt{1 + \beta_{II}^2 + \beta_{III}^2}} \quad 11.20$$

The damage parameter for each mode can be obtained substituting  $\delta_{om,i}$  and  $\delta_{um,i}$  in Equation 11.19.

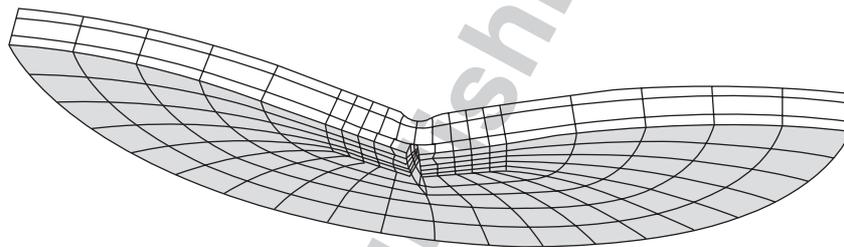
The interaction between matrix cracking and delamination in  $(0_4, 90_4)_s$  carbon-epoxy laminates under low velocity impact was simulated using a cohesive damage model (de Moura and Gonçalves, 2004). Circular clamped plates of 50 mm diameter were tested and damage, identified by X-ray method, was constituted by:

- a longitudinal long crack parallel to fibres direction in the outermost group of equally oriented layers and caused by bending loading;
- an extensive delamination located at the distal interface between differently oriented layers relatively to the loaded surface; the delamination has a characteristic two-lobed shape with its major axis oriented on the direction of the lower adjacent ply.

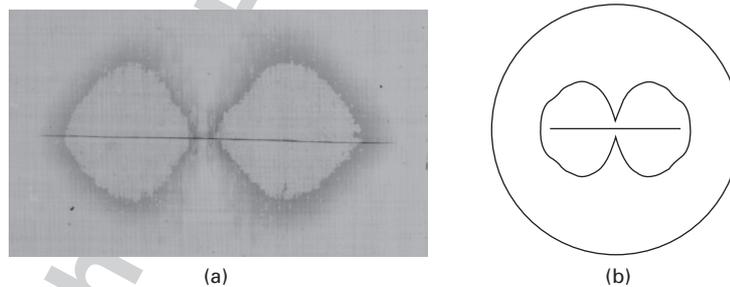
In the numerical model only half plate was considered due to geometrical and material symmetrical conditions. With the aim to numerically simulate this damage mechanism, interface finite elements were placed at the critical interface in order to simulate the observed delamination, and at the vertical symmetry plane of the used mesh. The objective of these vertical elements was to model the onset and growth of the longitudinal bending crack in the outermost group of equally oriented layers which was observed to be the initial damage (see Fig. 11.5). This crack induces delamination in the adjacent interface as it can be seen in Fig. 11.6. This damage mechanism occurs in a progressive way, i.e., the growth of the vertical crack is associated with an increasing delamination. Numerical results agreed with the experiments.



11.5 Detail of the initial bending crack.

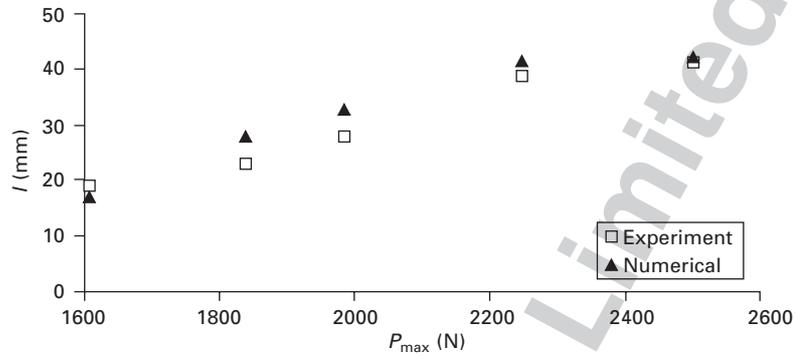


11.6 Delamination induced by the vertical crack.

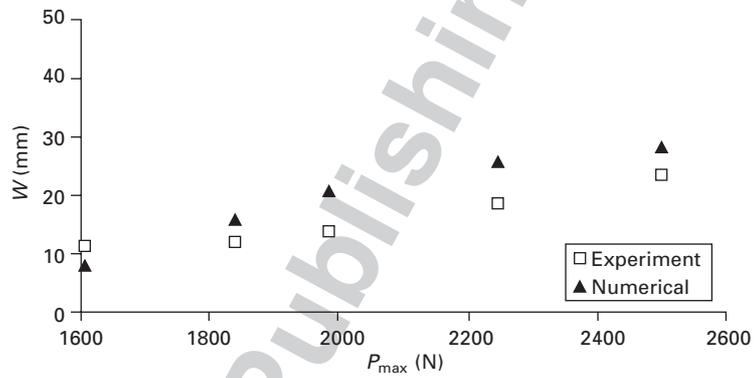


11.7 Experimental and numerical delamination and crack length.

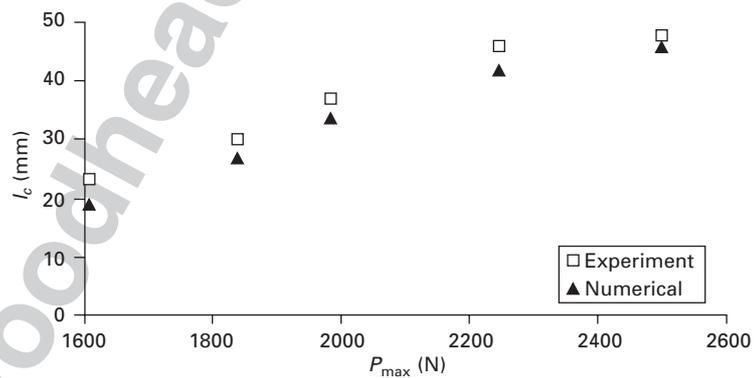
36  
37  
38  
39 The shape of the delamination was accurately predicted (see Fig. 11.7). The  
40 delamination and crack lengths in function of the maximum load were also  
41 in agreement (see Figs 11.8 and 11.10), although some non-negligible  
42 differences were noted in the delaminations width (see Fig. 11.9). The global  
43 trend was captured for all cases.



11.8 Delamination length ( $l$ ) in function of maximum load.



11.9 Delamination width ( $W$ ) in function of maximum load.



11.10 Longitudinal crack length ( $l_c$ ) in function of maximum load.

1 Although promising results were obtained with this approach it should be  
 2 emphasized that it cannot be considered an adequate prediction model for  
 3 more general applications. In fact, in laminates with several different oriented  
 4 layers it is not suitable to include numerous vertical interface elements in all  
 5 layers to predict matrix cracking in any layer. The alternative is to consider  
 6 that solid elements can also include a damage model in order to simulate  
 7 damage inside layers. This can be done using continuum damage mechanics  
 8 which is discussed in the next section.

### 11.3 Continuum damage mechanics

12 The classical approaches based on strength of materials usually assume that  
 13 once matrix cracking arises, a sudden loss of material properties occur,  
 14 which is generally denominated by ply discount models (Hwang and Sun,  
 15 1989). However, it is known that in presence of matrix cracking, the composite  
 16 does not loose its load carrying capacity immediately. In fact, damage can be  
 17 considered as the progressive weakening mechanism which occurs in materials  
 18 prior to failure. It can be constituted by micro-cracking, voids nucleation and  
 19 growth, and several inelastic processes that deteriorate the material. The  
 20 analysis of cumulative damage is fundamental in life prediction of components  
 21 and structures under loading. Tan (1991) proposed a progressive damage  
 22 model relating the material elastic properties with internal state variables  
 23  $D_i^T$  and  $D_i^C$  ( $i = 1, 2, 6$ ), ranging between 0 and 1, that are function of the  
 24 type of damage. When a given failure criterion is satisfied, the material  
 25 properties are abruptly reduced according to the respective residual strength  
 26 experimentally observed. Each damage mode is predicted by the subsequent  
 27 expressions:

28 Fibre tensile fracture

$$E_{11}^d = D_1^T E_{11}; v_{12}^d = v_{13}^d = 0 \quad 11.21$$

32 Fibre compressive fracture

$$E_{11}^d = D_1^C E_{11}; v_{12}^d = v_{13}^d = 0 \quad 11.22$$

36 Matrix tensile failure

$$E_{22}^d = D_2^T E_{22}; E_{33}^d = D_2^T E_{33}; v_{12}^d = v_{23}^d = 0 \quad 11.23$$

$$G_{12}^d = D_6^T G_{12}; G_{13}^d = D_6^T G_{13}; G_{23}^d = D_6^T G_{23}$$

40 Matrix compressive failure or shear cracking

$$E_{22}^d = D_2^C E_{22}; E_{33}^d = D_2^C E_{33}; v_{12}^d = v_{23}^d = 0 \quad 11.24$$

$$G_{12}^d = D_6^C G_{12}; G_{13}^d = D_6^C G_{13}; G_{23}^d = D_6^C G_{23}$$

The author (Tan, 1991 and Tan and Perez, 1993) obtained good agreement with experimental results considering  $D_1^T = 0.07$ ,  $D_2^T = D_6^T = 0.2$ ,  $D_1^C = 0.14$  and  $D_2^C = D_6^C = 0.4$ . Although this type of models consider a residual strength in accordance with the physical reality they are mesh dependent during numerical analysis.

To avoid the sudden loss of material properties and mesh dependency, the continuum damage models combining strength of materials and fracture mechanics concepts are an appealing alternative. Material damage is simulated by introducing damage variables into the constitutive equations (Lemaitre and Chaboche, 1985). After the matrix cracking initiation is predicted, a gradual softening post-failure analysis is also performed by appropriately reducing the material properties within the elements where matrix cracking onset occurred.

Ladevèze and Le Dantec (1992) developed a continuum damage mechanics formulation for orthotropic materials to account for ply degradation. Considering a damaged layer in a state of plane stress the strain-stress relationship take the general form

$$\boldsymbol{\epsilon} = \mathbf{S}\boldsymbol{\sigma} \quad 11.25$$

where  $\boldsymbol{\epsilon}$  and  $\boldsymbol{\sigma}$  are vectors of elastic strain and stress, respectively. In a local system associated with orthotropy axes it can be written

$$\boldsymbol{\sigma} = (\sigma_{11}, \sigma_{22}, \sigma_{12})^T; \boldsymbol{\epsilon} = (\epsilon_{11}, \epsilon_{22}, 2\epsilon_{12})^T \quad 11.26$$

and  $\mathbf{S}$  the compliance matrix

$$\mathbf{S} = \begin{bmatrix} \frac{1}{E_1(1-d_1)} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2(1-d_2)} & 0 \\ 0 & 0 & \frac{1}{G_{12}(1-d_{12})} \end{bmatrix} \quad 11.27$$

The damage parameters ( $d_1$ ,  $d_2$  and  $d_{12}$ ) define the damage state for the three types of stress loading and varies between 0 (undamaged state) and 1 (complete loss of stiffness). No additional parameter is used to simulate degradation in Poisson's ratios as they are intrinsically affected during damage progression;  $\nu_{12}$  is reduced by the factor  $(1-d_1)$ , since for a uniaxial stress  $\sigma_{11}$  it can be shown from Equations 11.25, 11.26 and 11.27 that  $-\epsilon_{22}/\epsilon_{11} = \nu_{12}(1-d_1)$ ; similarly it can be easily demonstrated that  $\nu_{21}$  is affected by  $(1-d_2)$ . In order to establish the evolution of damage parameters in function of the damage growth the concept of strain energy density  $\phi$  is used

$$\phi = \frac{1}{2} \boldsymbol{\sigma}^T \mathbf{S} \boldsymbol{\sigma} \quad 11.28$$

The theory also includes the concept of ‘thermodynamic forces’,  $Y_1, Y_2, Y_{12}$ , associated with the internal damage variables  $d_1, d_2, d_{12}$ . Those parameters are also considered as driving forces for damage development and are defined by

$$\mathbf{Y} = \frac{\partial \phi}{\partial \mathbf{d}} \quad 11.29$$

where  $\mathbf{Y} = (Y_1, Y_2, Y_{12})^T$  and  $\mathbf{d} = (d_1, d_2, d_{12})^T$ . Combining Equations 11.28 and 11.29 it follows

$$Y_1 = \frac{\sigma_{11}^2}{2E_1(1-d_1)^2}; Y_2 = \frac{\sigma_{22}^2}{2E_2(1-d_2)^2}; Y_{12} = \frac{\sigma_{12}^2}{2G_{12}(1-d_{12})^2} \quad 11.30$$

In the absence of fibre breakage  $d_1$  is zero throughout the load history and the longitudinal modulus does not degrade; therefore only  $Y_2$  and  $Y_{12}$  driving forces should be considered to model matrix cracking. A linear combination

$$\hat{Y} = Y_{12} + bY_2 \quad 11.31$$

where  $b$  is a material constant, can be used to account for coupling between transverse tension and shear effects. To avoid healing phenomena the maximum value of  $\hat{Y}$  up to the current time  $t$  is defined as

$$\hat{Y}(t) = \max_{\tau \leq t} (Y_{12} + bY_2) \quad 11.32$$

Experimental results for carbon-epoxy laminates showed that damage parameters can be written as

$$d_{12} = \frac{\sqrt{\hat{Y}} - \sqrt{Y_0}}{\sqrt{Y_C}}; d_2 = \frac{\sqrt{\hat{Y}} - \sqrt{Y'_0}}{\sqrt{Y'_C}} \quad 11.33$$

where  $Y_0, Y_C, Y'_0$  and  $Y'_C$  are damage evolution parameters. They are determined experimentally performing tension tests on  $[\pm 45]_s$  and  $[\pm 67.5]_s$  laminates (Ladevèze and Le Dantec, 1992). The model was tested on  $[\pm 45]_{2s}$ ,  $[67.5, 22.5]_{2s}$  and  $[-12, 78]_{2s}$  laminates under tensile loading and excellent agreement was obtained with the respective experimental  $\sigma$ - $\varepsilon$  curves.

An approach similar to the one used in the cohesive damage model described in Section 11.2 is proposed by other authors (Crisfield *et al.*, 1997; Pinho *et al.*, 2006; de Moura and Chousal, 2006). In this case there is a softening relationship between stresses and strains instead of between stresses and relative displacements considered in the cohesive model. Consequently, in this case a characteristic length  $l_c$  must be introduced to transform the relative displacement into an equivalent strain. (see Fig. 11.4)

$$G_{ic} = \frac{1}{2} \sigma_{u,i} \varepsilon_{u,i} l_c \quad 11.34$$

This parameter was considered to be equal to the length of influence of a Gauss point in the given direction and physically can be regarded as the dimension at which the material acts homogeneously. The stress-strain relation can be written considering an equation similar to Equation 11.8

$$\boldsymbol{\sigma} = (\mathbf{I} - \mathbf{D})\mathbf{C}\boldsymbol{\epsilon} \quad 11.35$$

being  $\mathbf{C}$  the stiffness matrix of the undamaged material in the orthotropic directions. Assuming that matrix cracking occur in mixed-mode I+II the damage model described in Section 11.2 can be adopted. The damage parameter is calculated by an expression similar to Equation 11.9 but considering strains instead of relative displacements. A linear softening law is also used. The properties are smoothly reduced due to the energy released at the FPZ. The material properties at a given Gauss point are degraded according to the assumed criterion. This leads to load redistribution for the neighbouring points, thus simulating a gradual propagation process.

In summary, it can be affirmed that these methods allow simulating damage inside solid finite elements used to model composite layers and can be used to simulate matrix cracking phenomenon. A gradual degradation of properties instead of a sudden one avoids the singularity effects and minimizes the consequent mesh sensitivity.

## 11.4 Conclusions

Matrix crack and delamination are intrinsically associated in composite materials, namely under bending loads. The interaction between these two modes of damage constitutes a complex damage mechanism that has not been addressed in a realistic level. Such interaction is fundamental to be considered in a failure model prediction because one mode may initiate the other and they may intensify each other. Two different models come out to deal with the referred damage modes. Mixed-mode cohesive damage models join the positive arguments of stress based and fracture mechanics criteria overcoming their inherent difficulties. These models have being used with success to simulate delamination initiation and growth. They are usually based on interface finite elements including a softening relationship between stresses and relative displacements. The continuum damage mechanics is being applied on the simulation of matrix cracking. These models are based on the introduction of damage parameters into the constitutive equations in order to simulate material damage. These damage parameters increase smoothly with growing damage, leading to a slow degradation of material properties instead of an abrupt one which is not realistic and originate mesh dependencies.

1 In summary, cohesive and continuum damage models are actually the  
2 most prominent numerical tools in order to simulate matrix cracking inducing  
3 delamination damage mechanism of composites. However, it should be  
4 recognized that an accurate methodology addressing all the realistic issues  
5 of this complex damage mechanism is still lacking. The solution points to  
6 the development of a numerical tool incorporating the two kinds of models.  
7 The two methods (cohesive elements and damage mechanics) can coexist. In  
8 fact, both models can be implemented via user subroutines in commercial  
9 software. When the selected damage criterion in a solid element is satisfied  
10 the element fails simulating matrix cracking. This induces important relative  
11 displacements at the adjacent interfaces leading to delamination initiation  
12 and propagation according to the damage criterion of the cohesive elements.  
13 Some aspects should require special attention like the influence of stress  
14 concentration at the intersection of a critical matrix crack with a given interface.  
15 It is not clear up to now if the coexistence of the two methods will be able  
16 to accurately model such particularity. Some efforts should also be dedicated  
17 to the development stress and fracture criteria adequate to the specificities of  
18 this damage mechanism.

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