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Quantifying Symmetry in Periodontal Disease: A Novel Measure for Clinical and Epidemiological Applications

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Abstract: Symmetry quantification in periodontal disease is crucial for understanding disease progression and facilitating population-based studies, especially with incomplete data. This study introduces a novel measure for symmetry assessment using a decay-type exponential function. Designed to maximize spatial predictability and align with clinical perceptions of symmetry, this measure demonstrated high efficacy. An evaluation involving periodontists showed a strong correlation (0.96) between clinical assessments and the symmetry scores generated by the measure. Additionally, the measure enhanced predictive models, outperforming simpler models in terms of RMSE, MAE, and R^2 values. Future research should validate this measure across diverse populations and explore its broader applications

keywords: symmetry quantification; periodontal disease; predictive modeling

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1 Introduction

From an epidemiological perspective, symmetry quantification facilitates the estimation of population disease parameters, especially when data is incomplete. This is evident in studies designed with half-mouth evaluations. Clinically, the significance of symmetry is underscored by the notion that asymmetric values of periodontal disease indicators might be influenced by asymmetric factors that affect both the onset and the progression of disease.

In the realm of fuzzy symmetry, when examining periodontal structures, it becomes imperative to quantify the symmetry between distinct entities. For this purpose, consider two values, A and A', each representing specific attributes or measurements of contralateral sites taken on the same scale. This means that both A and A' are quantified using the same units and methodology, allowing for direct comparison or estimation between the two.

2 Proposed Measure

To create a measure that captures similarity information, maximizes spatial predictability, and mimics the clinician's perception of symmetry grade, we introduce a decay-type exponential function with the form:

$$SM(A, A', \alpha, \beta) = e^{-\frac{|A-A'|}{\alpha+\beta\frac{A+A'}{2}}} \quad (1)$$

Where: - α is a scaling parameter. - β is the association parameter between A and A' (its contralateral counterpart).

2.1 Parameter α

Alpha (α) acts as a scaling factor that helps control the steepness of the exponential decay in the function. The α adjusts the sensitivity of the SM function to absolute differences between A and A'; a smaller α makes the function more sensitive. The default values of α are set to 1, 0.1, and 0.01 according to the scale of A, when A and A' are integers, decimals, or centesimals, respectively.

When A and A' are integers, the potential differences between A and A' are larger in absolute terms compared to when A and A' range from 0 to 0.9 on a decimal scale. Consequently, α needs to be adjusted to maintain a function response similar to what was observed with larger integer values. α must be reduced to increase the function's sensitivity to these smaller differences. This feature allows for the comparison of symmetry grades between pairs A and A' measured on different scales.

2.2 Parameter β

Making the parameter β equal to the correlation coefficient between A and A', we are effectively stating that the higher the correlation, the more predictable the behavior of one site based on the other. This fits well with the goal of using symmetry grading for prediction, as highly correlated contralateral sites will have similar characteristics and responses.

In this application of SM, β is always positive, as there is no biological justification for it to be negative. This positivity is crucial because it ensures that the function emphasizes similarity rather than dissimilarity in the grading of periodontal lesions between sites. Positive values of β reduce the denominator in the SM function for pairs of sites with higher correlation, thereby diminishing the impact of absolute differences $|A-A'|$ and highlighting their inherent similarity. The use of β as a correlation coefficient in the context of symmetry grading for periodontal lesions leverages statistical relationships to enhance contralateral spatial predictivity. Additionally, directionality of the difference was incorporated into the function by creating SM_dir, which involves multiplying SM by γ, a parameter that takes values of -1 and $+1$.

2.3 Empirical Results

Evaluation of SM Function in Clinical Setup To evaluate the effectiveness of the SM function in capturing the clinical magnitude of symmetry, a study was conducted involving ten experienced periodontists. These experts were asked to score twenty pairs of pocket probing depth (PPD) values on a scale from zero to ten. The results of this expert assessment were then compared to the symmetry scores generated by the SM function for the same pairs of values. The analysis revealed a high correlation coefficient of 0.96, indicating a strong agreement between the periodontists' evaluations and the SM function's outputs.

Evaluation of SM Function in Contralateral Prediction To evaluate the importance of $SM\_dir11.Site$ for contralateral prediction, two gradient boosting machine (GBM) models were fitted using the National Health and Nutrition Examination Survey (NHANES) 2011-2012 data and evaluated. The first model, GBM1, included both the values of PPD for the modeled *site*, each one of the six sites per tooth: Disto-Vestiblular (DV), Vestibular (V), Mesio-Vestibular (MV), Disto-Lingual (DL), Lingual (L) and Mesio-Lingual (ML) of the upper right central incisor ($11Site$) and the mean directional SM ($SM\_dir11.Site$) as predictors. The second model, GBM2, included only the PPD of the modeled $Site$ ($11.Site$) as a predictor.

Table 1: Performance Metrics for GBM Models Including Right Side Values ($Site1$) and Directional SM (GBM1) and only Right Side Values ($Site2$) (GBM2)

	GBM models											
Metric	**DV1**	**DV2**	**V1**	**V2**	**MV1**	**MV2**	**DL1**	**DL2**	**L1**	**L2**	**ML1**	**ML2**
RMSE	0.167	0.760	0.230	0.653	0.210	0.682	0.252	0.740	0.220	0.705	0.350	0.685
MAE	0.037	0.484	0.049	0.416	0.037	0.457	0.064	0.535	0.055	0.468	0.086	0.465
MSE	0.028	0.578	0.053	0.427	0.044	0.466	0.063	0.547	0.049	0.497	0.122	0.469
R^2	0.969	0.357	0.930	0.375	0.954	0.502	0.932	0.417	0.943	0.417	0.871	0.506
Adj. R^2	0.967	0.322	0.927	0.340	0.951	0.474	0.928	0.385	0.940	0.385	0.864	0.479
Exp. Var.	0.969	0.356	0.923	0.375	0.951	0.493	0.932	0.415	0.943	0.416	0.871	0.505

Abreviatures: RMSE – Root Mean Squared Error, MAE – Mean Absolute Error, MSE – Mean Squared Error, R^2 – R-squared, Adj. R^2 – Adjusted R^2, Exp. Var. – Explained Variance, DV – Disto-Vestiblular, V – Vestibular, MV – Mesio-Vestibular, DL – Disto-Lingual, L – Lingual, ML – Mesio-Lingual *sites* of the upper right central incisor ($11Site$), $SM\_dir11.Site$ – Mean directional SM

Table 2: Variable Importance in GBM1 Models per Site of Upper Central Incisors

Variable	**DV1**	**V1**	**MV1**	**DL1**	**L1**	**ML1**
SM_dir11.Site	69.554	61.598	54.865	64.871	66.539	47.927
Site	30.446	38.402	45.135	35.130	33.461	52.073

The performance of both models was assessed on a hold-out test set using Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Squared Error (MSE), R-squared (R^2), Adjusted R^2 (Adj. R^2) and Explained Variance (Exp. Var.) as evaluation metrics across multiple sites for the two GBM models by site GBM1 (DV1, V1, MV1, DL1, L1 and ML1) and GBM2 (DV2, V2, MV2, DL21, L2 and ML2).

For site DV, GBM1 (which incorporated both $11DV$ and $SM\_dir11.DV$) achieved an RMSE of 0.167, an MAE of 0.037, an MSE of 0.028, and an R^2 of 0.969, significantly outperforming GBM2, which had an RMSE of 0.760, an MAE of 0.484, an MSE of 0.578, and an R^2 of 0.357. Similar results where found for the other five sites, as can be observed in table 1

The trend observed in the performance metrics across all sites indicates that GBM1, which includes the additional predictor $DM\_dir11.Site$, consistently outperforms GBM2. This is evident from the lower RMSE and MAE values and higher R^2 values for GBM1, suggesting a better fit to the test data.

Table 2 shows the relative importance of the predictors in the GBM1 models for different sites of the central incisors. The mean directional SM ($DM\_dir11.Site$) had a higher importance than the site value alone in most cases, highlighting its significant contribution to the improved performance of GBM1. For example, the importance of $SM\_dir11.DV$ was highest for site DV1 (69.554) compared to the DV PPD *site* value alone (30.446). Similar trends are seen across other sites, emphasizing the value of including directional SM as a predictor.

3 Conclusions and Future Work

The proposed measure for symmetry quantification in periodontal disease offers a robust framework for both clinical and epidemiological applications. By adjusting α and β, this measure can be tailored to different scales and correlation levels, enhancing its utility in various contexts. Future research should focus on validating this measure in larger and more diverse populations and exploring its application in other fields.

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