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Assessing the Symmetry of Contralateral Periodontal Lesions Using Statistical Methods

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Abstract: Periodontal disease is a prevalent condition that can lead to significant oral health issues if left untreated. One aspect of periodontal health that has garnered interest is the symmetry of periodontal lesions between contralateral sites. Symmetry in this context can provide insights into the uniformity of disease progression and provide information for contralateral inference. Previous studies have often assumed symmetry in periodontal lesions, but few have rigorously tested this assumption using robust statistical methods. This study aims to assess the symmetry of contralateral periodontal lesions using data from the National Health and Nutrition Examination Survey (NHANES) 2011-2012. By comparing the probabilistic distributions of periodontal variables from contralateral sites, this research seeks to provide a detailed understanding of periodontal lesions symmetry.

keywords: periodontal disease; symmetry; statistical analysis.

MSC2020: 49-XX; 34-XX; 92-XX.

1 Introduction

Periodontal disease is a prevalent condition that can lead to significant oral health issues if left untreated. One aspect of periodontal health that has garnered interest is the symmetry of periodontal lesions between contralateral sites in the mouth. Symmetry in this context can provide insights into the uniformity of disease progression. Previous studies have often assumed symmetry in periodontal lesions, but few have rigorously tested this assumption using robust statistical methods. This study aims to assess the symmetry of contralateral periodontal lesions using data from the National Health and Nutrition Examination Survey (NHANES) 2011-2012. By comparing the probabilistic distributions of periodontal variables from contralateral sites, this research seeks to provide a detailed understanding of the extent to which periodontal lesions are symmetric.

2 Materials and Methods

The data for this study were obtained from the National Health and Nutrition Examination Survey (NHANES) 2011-2012. NHANES is a program of studies designed to assess the health and nutritional status of adults and children in the United States. It combines interviews and physical examinations, providing a comprehensive dataset for various health-related research.

The study population includes individuals aged 30 and above who participated in the NHANES 2011-2012 cycle and had complete periodontal examinations. Exclusion criteria included individuals with missing data for key periodontal variables .

Periodontal examinations were conducted by trained dental professionals and included measurements of pocket probing depth (PPD) at six sites per tooth (mesio vestibular (MV), vestibular (V), distovestibular(DV) , mesiolingual (ML), lingual (L), distolingual (DL)). For this study, data from upper right (11) and left (21) central incisors were analyzed.

2.1 Statistical Analysis

Comparison of Probability Density Functions (PDFs) Probability density functions for PPD were estimated for each contralateral site using kernel density estimation. The kernel density estimator $\hat{f}(x)$ for a dataset $X = \{x_1, x_2, \ldots, x_n\}$ is given by:

$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

where K is the kernel function (e.g., Gaussian kernel) and h is the bandwidth parameter. The estimated PDFs were plotted for visual inspection of symmetry. Divergence measures, such as Kullback-Leibler divergence, were calculated to quantify differences between the PDFs.

Kolmogorov-Smirnov (K-S) Test The K-S test was used to compare the empirical distribution functions of periodontal variables from contralateral sites. The K-S statistic D is defined as:

$$D = \sup_x |F_n(x) - G_n(x)|$$

where $F_n(x)$ and $G_n(x)$ are the empirical distribution functions of the two samples. The test provides a p-value indicating whether the distributions are significantly different.

Generalized Additive Models for Location, Scale, and Shape (GAMLSS) GAMLSS models were fitted to the data, with periodontal variables as the response and the side (left or right) as a factor. The GAMLSS framework allows modeling not only the mean (μ) but also other parameters such as the variance (σ), skewness (ν), and kurtosis (τ) of the distribution. The general form of a GAMLSS model is:

$$Y_i \sim \text{Distribution}(\mu_i, \sigma_i, \nu_i, \tau_i)$$

$$g(\mu_i) = \eta_{\mu,i} = \mathbf{X}_{\mu,i}\boldsymbol{\beta}_\mu$$

where $g(\cdot)$ is a link function, $\mathbf{X}_{\mu,i}$ is the design matrix for the mean, and $\boldsymbol{\beta}_\mu$ are the regression coefficients. The null model (excluding the side factor) was compared with the side model using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). A likelihood ratio test was conducted to assess the significance of the side effect. The test statistic Λ is given by:

$$\Lambda = -2(\log L_{\text{null}} - \log L_{\text{side}})$$

where L_{null} and L_{side} are the likelihoods of the null and side models, respectively. This statistic follows a chi-squared distribution with degrees of freedom equal to the difference in the number of parameters between the models. K-fold cross-validation was performed to evaluate the predictive performance of both models, ensuring that the inclusion of the side factor improves the model's robustness. The data was split into k subsets, and the model was trained on $k-1$ subsets while the remaining subset was used for validation. This process was repeated k times.

3 Results and discussion

Site	Stats	Teeth		Test Results		K-S	SM	Bhat.	Corr.
		11	21	Stats	p			Coef.	Coef.
DV	Mean	1.40	1.42	t = -0.738	0.461	D = 0.007	0.85	1.000	0.58
	Median	1	1	W = 3589510	0.638		1		
	Variance	0.685	0.761	F = 0.266	0.606	p = 1	0.042		
V	Mean	0.868	0.928	t = -2.936	**3.34e-03**	D = 0.028	0.86	0.999	0.64
	Median	1	1	W = 3474941	**3.02e-03**		1		
	Variance	0.538	0.578	F = 0.065	0.799	p = 0.247	0.047		
MV	Mean	1.31	1.41	t = -4.6212	**3.91e-06**	D = 0.079	0.89	0.997	0.66
	Median	1	1	W = 3354774	**5.33e-09**		1		
	Variance	0.639	0.702	F = 18.218	**2.00e-05**	p = **9.96e-08**	0.036		
DL	Mean	1.48	1.56	t = -3.513	**4.47e-04**	D = 0.050	0.85	0.998	0.62
	Median	1	1	W = 3258241	**2.35e-04**		1		
	Variance	0.729	0.807	F = 11.600	**6.65e-04**	p = **3.18e-03**	0.041		
L	Mean	1.14	1.16	t = -0.925	0.355	D = 0.013	0.86	0.999	0.65
	Median	1	1	W = 3591491	0.267		1		
	Variance	0.731	0.726	F = 0.230	0.647	p = 0.9772	0.046		
ML	Mean	1.73	1.81	t = -3.242	**1.20e-03**	D = 0.055	0.89	0.998	0.73
	Median	2	2	W = 3405239	**5.38e-05**		1		
	Variance	0.826	0.779	F = 7.425	**6.45e-03**	p = **5.21e-04**	0.032		

Abreviatures: 11 – Upper right central incisor; 21 – Upper left central incisor; Stats – Statistics; p – p-value; Statts – Tests statistics; K-S – Kolmogorov-Smirnov test; SM – Symmetry measure; Bhat. Coef.– Bhattacharyya coefficient; Corr. Coef. – Pearson Correlation coefficient

Table 1: Summary of statistical tests comparing central incisors 11 and 21 PPD means, medians and variances across six dental sites; distances between distributions.

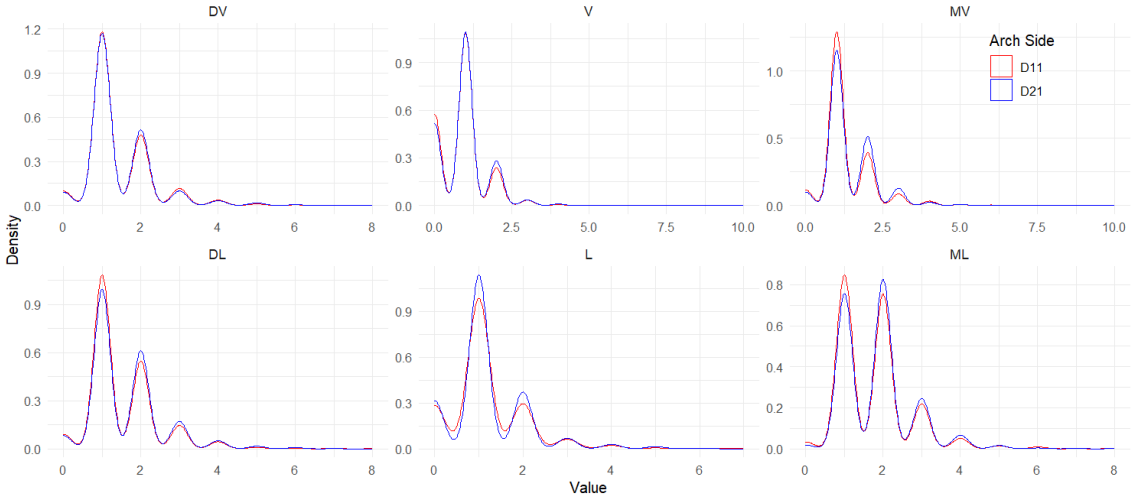

Figure 1: Kernel density plots to compare probability densities of 11 and 21 PPD by site

From the analysis of PPD at DV, V, MV, DL, L, and ML of 11 and 21) (Table 1) significant findings were observed in the DV sites, the comparison of means showed no significant difference (t = -0.738, p = 0.461), and the K-S test, together with the kernel density plots confirmed this with a D-value of 0.007 (p = 1.00) and overlaping curves. Conversely, significant differences

were noted in the V sites, where both the mean difference (t = -2.936, p = 3.34e-03) and the K-S test (D = 0.028, p = 0.247) and discordant kernel density plots, suggested asymmetry. The GAMLSS model for the V sites further indicated a significant side effect, with a notable reduction in AIC and an LRT p-value of 0.03.

Further, the MV sites exhibited significant asymmetry, with the mean PPD significantly different between 11 and 21 (t = -4.621, p = 3.91e-06). The K-S test also highlighted a significant difference (D = 0.079, p = 9.96e-08). The GAMLSS analysis for these sites supported the presence of asymmetry, as indicated by a significant LRT with a p-value of 9.59e-06. Similar patterns were observed in the DL sites, where significant differences were found in mean PPD (t = -3.513, p = 4.47e-04) and the K-S test (D = 0.050, p = 3.18e-03). In contrast, the L sites showed no significant differences (t = -0.925, p = 0.355; D = 0.013, p = 0.977), indicating symmetry at these sites. Cross-validation metrics, including RMSE and MAE, consistently supported these findings, with significant differences noted in the V, MV, and D-L sites but not in the L sites.

Conclusion

The analysis of PPD across six periodontal sites of the upper central incisors (11 and 21) reveals asymmetry in specific sites. Significant differences were observed in the V, MV, and DL sites, indicating that periodontal signals varies between these contralateral sites. However, no significant differences were found in the L sites, suggesting symmetry at these points. Further research should explore the new approachs of symmetry evaluation. The presented symmetry assessment relies on hypothesis testing, which assumes that any observed symmetry falls within a defined confidence interval, allowing for a margin of error. This approach ensures that minor deviations from perfect symmetry are accounted for and are not mistakenly interpreted as significant asymmetry.

4 Conclusions and Future Work

The analyses provide strong evidence of asymmetry in contralateral periodontal lesions but do not quantify the degree of symmetry. Future work should focus on quantifying symmetry to extract valuable information for contralateral inference and improve clinical assessments.

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