U.PORTO
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STRATEGIC CHOICE OF SPACIAL PRICE POLICY AND COLLUSION IN MARKETS WITH NETWORK EFFECTS

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Dissertation

Master in Economics

Supervised by

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2024

Acknowledgments

Writing this dissertation was undoubtedly one of the most challenging yet profoundly rewarding experiences of my life. Throughout the many highs and lows of this process, all my limits, dreams and beliefs were deeply tested in ways that I had never anticipated. Even though the path was not always the smoothest, having the right people by my side made this journey significantly easier. I will forever be sincerely thankful to those who helped me navigate and overcome the numerous obstacles that appeared along the way.

First and foremost, I want to express my deepest gratitude to my supervisor, João Correia da Silva. Your patience, availability, expertise, insightful guidance, and unwavering support were indispensable throughout this whole process. From our early discussions that helped shaping the foundations of this dissertation to your meticulous feedback on the final draft, you have consistently inspired me to strive for excellence. Your encouragement during the difficult moments, and your belief in my potential, provided the motivation I needed to persevere through all the complexities of this journey. Your influence has not only shaped the outcome of this dissertation but has also left an incredible mark on my academic and personal life. From the bottom of my heart, thank you for everything!

I would also like to extend my heartfelt gratitude to all the professors who took part in my journey at FEP. Your invaluable lessons had a profound impact on both my intellectual growth and personal development. Each of you has left an indelible mark on my education, and for that, I am truly grateful.

To all my friends, thank you for all the amazing moments we have shared and for the incredible memories we have created.

I also want to thank my family for all their love and support. In particular, I want to thank my godfather, João, who, regardless of the circumstances, always had the kindest words and the most heartwarming smile. Thank you for gracing my life with your sweet presence and for blessing all our family with the honesty, altruism, compassion, and generosity of your soul. I miss you greatly! This dissertation is wholeheartedly dedicated to you. I hope it makes you proud!

Lastly and most importantly, I want to thank my parents for showing me every day the true meaning of unconditional love and support. To my mother, the strongest and most courageous person I know, thank you for teaching me the power that lies in believing and never giving, despite all the adversities and hardships of life. And, to my father, thank you for teaching me that with hard work, dedication and perseverance, all dreams can indeed become a reality. Thank you both for always believing in me and loving me through all the ups and downs. Thank you for being my rock when I had no ground to stand on and for being the light that always shows me the way out of the darkness. You two are my home, my shelter, my safe spot and my happy place. You are the most beautiful and important part of my life, and I love you with all my heart!

Abstract

This dissertation analyses firms' choice of spatial price policy in a Hotelling (1929) duopoly with direct network effects. Specifically, it aims at understanding how firms that operate in these markets decide between following a uniform or a personalized pricing strategy. In their seminal paper, Thisse and Vives (1988) concluded that, in the absence of network effects, firms are trapped in a Prisoner's Dilemma situation, where they choose personalized pricing, even though they could achieve higher profits by collectively committing to the use of uniform pricing. The findings of this dissertation confirm that this Prisoner's Dilemma situation persists even with weak positive or relatively weak negative direct network effects. However, it is also shown that, in the presence of relatively strong negative direct network effects, an asymmetric equilibrium emerges, with one firm adopting personalized pricing while the other prices uniformly. Additionally, this research work demonstrates that an equilibrium in which both firms use uniform pricing can arise if the costs of the sophisticated technology that is required to employ a personalized pricing scheme are sufficiently high.

It is also explored whether firms can avoid the Prisoner's Dilemma situation in an infinitely repeated game of price competition by engaging in a collusive agreement to choose uniform pricing as their pricing policy, while letting the exact price level be competitively determined in the market. It is ascertained that this type of collusive agreements is more likely in the presence of positive direct network effects, if the firm that adheres to the agreement can adapt its uniform price during the period in which a deviation to personalized pricing occurs. On the contrary, if she cannot promptly detect the defection and adjust her unique price in that period, collusion is easier when there are negative network effects. Moreover, it is shown that if the negative network effects are sufficiently strong, collusion is more likely to occur if the other firm does not become aware of the defection in the period it occurs and, thus, cannot adjust its uniform price. This result brings a new insight to the existing economic literature regarding this subject, since it demonstrates that, under very specific circumstances, an early detection and response to a defection can be detrimental to the sustainability of a collusive agreement.

JEL Codes: D43, L13

Keywords: Pricing Policies, Personalized Pricing, Network Effects, Collusion

Resumo

A presente dissertação analisa o modo como as empresas, que operam num duopólio de Hotelling (1929) com efeitos de rede diretos, escolhem a sua política de preços. De um modo particular, procura-se averiguar se as mesmas tenderão a recorrer ao uso de preços personalizados ou a fixar um preço uniforme para todos os consumidores. No seu artigo seminal, Thisse e Vives (1988) concluíram que, na ausência de efeitos de rede, as empresas enfrentam um Dilema do Prisioneiro, uma vez que optam pelo uso de preços personalizados, ainda que os seus lucros fossem superiores se ambas recorressem a preços uniformes. Os resultados desta dissertação demonstram que, quando os efeitos de rede diretos são positivos e fracos ou negativos e relativamente fracos, este Dilema do Prisioneiro persiste. No entanto, na presença de efeitos de rede negativos relativamente fortes, surge um equilíbrio assimétrico, onde uma das empresas pratica preços personalizados e a outra um preço uniforme. Cumulativamente, esta dissertação evidencia que um equilíbrio onde ambas as empresas usam preços uniformes pode ser alcançado caso os custos da tecnologia necessária para implementar uma estratégia de preços personalizados sejam suficientemente elevados.

Este trabalho de investigação procura ainda averiguar se, num jogo de repetição infinita, as empresas conseguem sustentar um acordo colusivo que prescreva a escolha de uma política de preços uniformes, mas que não designe o nível de preços, devendo o mesmo ser determinado competitivamente no mercado. Os resultados deste estudo demonstram que este tipo de conluio é mais provável na presença de efeitos de rede positivos, se a empresa que cumpre o acordo puder ajustar o seu preço único no período em que a rival se desvia e usa preços personalizados. Contrariamente, se a empresa não conseguir detetar prontamente o desvio e ajustar o seu preço uniforme no período em que o mesmo ocorre, o conluio torna-se mais fácil de sustentar na presença de efeitos de rede negativos. Conclui-se, ainda, que se os efeitos de rede negativos forem suficientemente fortes, o conluio é mais fácil de ocorrer se a outra empresa não tiver conhecimento da deserção no período em que a mesma ocorre e, consequentemente, não puder ajustar o seu preço uniforme. Este resultado contribui com uma nova perspetiva para literatura económica relativa a este tópico, uma vez que demonstra que, sob circunstâncias muito específicas, uma deteção rápida e uma resposta precoce a um desvio podem ser prejudiciais para a sustentabilidade de um acordo colusivo.

Códigos JEL: D43, L13

Palavras-Chave: Políticas de Preços, Preços Personalizados, Efeitos de Rede, Conluio

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1. Introduction

From the moment a product is conceived until it is finally acquired by a consumer, firms have to make a wide range of strategic decisions that can ultimately dictate its success or failure. One of the most crucial and challenging dilemmas that they face is choosing a pricing policy, since it not only directly affects the profits but also shapes the perception and the feelings that consumers have towards the product and the firm itself. In this context, firms usually need to decide between setting a uniform price for all consumers or implementing a price discrimination strategy.

The most refined form of price discrimination is commonly referred to in the literature as personalized pricing and occurs when a firm charges each buyer according to his willingness to pay for the product or service (Shapiro & Varian, 1999). For a long time, this pricing policy was thought to be a purely theoretical concept, that could not be successfully implemented due to the fact that it requires a perfect knowledge about consumers and their preferences. However, the growth of the digital economy and the advancements in big data analytics have enabled firms to collect, store, process and analyse a large amount of customers' data, which is seemingly creating the right conditions for the use of personalized pricing schemes (Belleflamme & Peitz, 2015). This paradigm shift has, therefore, intensified the need to thoroughly understand firms' pricing policy decisions, and, particularly, their choice between uniform and personalized pricing.

Nonetheless, this issue is not new in economic literature. In their seminal paper, Thisse and Vives (1988) demonstrated that, in a Hotelling (1929) duopoly, both firms are trapped in a Prisoner's Dilemma situation and end up choosing personalized pricing, even though they would be better off by collectively committing to the use of uniform pricing. In this dissertation, we revisit their work in order to examine if this result holds in markets with direct network effects.

This type of network effects can be classified as either positive or negative depending on whether the value of the good increases or decreases with the number of consumers acquiring it. Some digital goods, such as videogames with a multiplayer environment, social networking applications and computer software, are characterized by the presence of positive network effects, whereas luxury goods are one of the most well-known examples of the existence of negative network effects. Many of these products belong to some of the most profitable and fastest-growing markets at the present moment. Therefore, due to their significant importance, this dissertation aims at better understanding firms' pricing decisions in these markets in order to provide some helpful insights to their managers.

In some cases, these goods have an associated subscription fee that is charged on a periodic basis. This implies that firms "meet" regularly in the marketplace to choose their pricing policy and determine their prices, which provides a scenario that can be modelled by an infinitely repeated game of price competition. When firms have this type of interaction in the marketplace, it is possible for them to reach and sustain a collusive outcome, which would not occur in a single-period game (Motta, 2004). Thus, to help competition authorities better identifying cases in which collusion might be present, this dissertation also analyses whether firms can sustain a situation in which they collude on the choice of the pricing policy but allow prices to be competitively determined in the market.

So, to add a new layer of depth to the existing economic literature regarding firms' choice of pricing policy, network effects and collusion, the present dissertation addresses the following questions: What type of pricing policy will firms choose in a Hotelling duopoly with direct network effects? Does the cost of the technology that is necessary to implement a personalized pricing strategy influences the firms' choice of pricing policy? What are the welfare implications of this choice? Can firms collude on a specific pricing policy in an infinitely repeated game of price competition? How does the presence of direct network effects impacts the sustainability of such a collusive agreement? How is the likelihood of that agreement affected by the timing of the detection of a defection?

The remainder of the dissertation is structured as follows. Chapter 2 provides a review of the related literature that encompasses a description of the historical and theoretical framework that surrounds the research questions and an explanation of the key concepts. Chapter 3 presents the model and Chapter 4 analyses firms' choice of spatial price policy in the presence of direct network effects. In turn, Chapter 5 studies the sustainability of the collusion on pricing policies in an infinitely repeated game of price competition. Lastly, Chapter 6 presents the main conclusions of this dissertation, and Chapter 7 contains the detailed calculations.

2. Literature Review

This chapter presents a review of the existing economic literature regarding firms' choice of pricing policy, network effects and collusion, which are the main phenomena studied in this dissertation. Simultaneously, it provides a brief description of the historical and theoretical framework that surrounds the research questions and an explanation of the key concepts.

2.1 Firms' Choice of Pricing Policy

When placing a product in the market, firms must decide which type of pricing policy to follow. Specifically, they need to choose between following a uniform pricing policy and, thus, set a unique price for all the consumers or employing a price discrimination strategy. According to Stigler (1987), the latter occurs when the ratio between the price of a certain good and its marginal cost differs between consumers.

One of the forms of price discrimination is personalized pricing (Shapiro & Varian, 1999), also known as first-degree price discrimination (Pigou, 1920), which takes place when a firm, with complete information about individual preferences, charges each buyer according to its willingness to pay for the product. This practice is only possible if arbitrage between consumers does not exist, which means that consumers who acquire the good at a lower price must find it too costly, or even impossible, to resell it to the ones with a higher willingness to pay. Moreover, the firm must possess some market power, so that it can set prices above the marginal cost, which implies that this strategy can only be applied in imperfectly competitive markets (Varian, 1989).

Throughout the years, economists have developed a significant number of studies aimed at understanding the firms' choice of pricing policy and, specifically, at identifying what factors could drive companies to use personalized instead of uniform pricing.

In one of the first works regarding this theme, it was concluded that under a static monopoly setting, where competition is absent, the firm would choose to use personalized pricing instead of uniform pricing, since it would allow her to fully extract consumer surplus and, consequently, maximize the profit (Pigou, 1920).

Later, the pioneering work of Thisse and Vives (1988) analysed firms' strategic choice of spatial pricing policy in a static duopoly landscape based on the Hotelling (1929) model.

Assuming that there is perfect information about all the consumers in the market, the authors concluded that the firms' choice would also be to always use personalized pricing. However, in this case, the fact that personalized pricing allows firms to compete individually for each consumer without having to change the prices set for the others, intensifies the competition for each buyer, and reduces prices, as a result. Therefore, the use of personalized pricing ends up benefiting consumers and hurting the profitability of the sellers, who get trapped in a Prisoner's Dilemma-type situation, as they would be better off by collectively committing to the implementation of a uniform pricing policy.

Later, Aguirre and Martín (2001) showed that these results crucially depend on the fact that Thisse and Vives (1988) assumed that if both firms choose the same pricing policy, they set the prices at the same time, but, if they choose different policies, the firm that uses uniform pricing is the leader, while the one that uses personalized pricing is the follower. In their work, Aguirre and Martín (2001) showed that if these leader-follower roles are reversed, the pricing policies adopted in equilibrium depend on the consumers' reservation value. If the reservation value is low, there are two equilibria in pure strategies, in which either both firms price uniformly or both firms price discriminate. For intermediate levels of the reservation value, the Prisoner's Dilemma situation emerges, with both firms choosing to personalize prices. Lastly, if the reservation value is high enough, the equilibria are asymmetric, with one firm using uniform pricing and the other using personalized pricing. These authors also analysed the case where firms choose their prices simultaneously, regardless of the pricing policy that they have committed to follow. In this situation, the authors demonstrate that there are two equilibria, with either both firms using personalized pricing or both using uniform pricing and note that the equilibrium in which both firms price uniformly Pareto dominates the other. Thereby, these authors managed to show that the strategic choice of spatial price policy under a static duopoly setting critically depends on the rules of price competition that are adopted.

Eber (1997) extended the framework of Thisse and Vives (1988) to encompass endogenous product differentiation and concluded that if firms choose where to locate before deciding the type of pricing policy, the unique equilibrium remains the same, with both firms choosing personalized pricing. Conversely, if firms choose the pricing policy before the location, the unique equilibrium is characterized by both firms committing to the use of uniform pricing.

Some studies developed afterwards also indicated that this Prisoner's Dilemma situation might not arise and, thereby, personalized pricing might have a positive effect on profits if firms are asymmetric in size and consumers have heterogeneous brand loyalty (Shaffer & Zhang, 2002), if companies are asymmetric in terms of costs (Matsumura & Matsushima, 2015), if firms also customize the quality of their products (Ghose & Huang, 2009), if consumers' demand is heterogeneous (Esteves, 2022) or if there are imperfectly informed costumers and advertising is not too expensive (Esteves & Resende, 2019).

However, there has never been a study aimed at understanding firms' choice of pricing policy in markets with direct network effects, which constitutes a significant void in the literature, that this dissertation aims to start fulfilling.

2.2 Network Effects

In their seminal paper, Katz and Shapiro (1985) note that the utility that an individual derives from consuming certain goods depends upon the consumption decisions of other agents. This means that there is an externality associated with consumption, an idea that had been previously presented by Veblen (1899) and Leibenstein (1950).

According to Katz and Shapiro (1985), this consumption externality can be generated by the presence of direct network effects, when the value of the good is a function of the number of other agents consuming it, or indirect network effects, when the value of the good hinges on the quantity of complementary products or services available. More specifically, the direct network effects, which are at the centre of this dissertation, can either be positive, also known in the literature as bandwagon effects, if the value of the commodity increases with the number of other individuals that buy it, or negative, sometimes referred to as snob or congestion effects, if the value of the product decreases with the number of other agents acquiring it.

In a work that has some resemblances with this dissertation, Navon et al. (1995) incorporated linear direct network effects in the Hotelling (1929) model with linear transportation costs and uniform pricing. The authors concluded that when bandwagon effects exist but are not too strong, both stores remain operating, but price competition is fiercer, which leads to lower equilibrium prices. However, when the bandwagon effects are strong enough to dominate the transportation cost, different price equilibria may coexist, with either of the firms serving the entire market. In this case, a monopoly emerges endogenously, and, just like in the theory of contestable markets, the active store sets a price below the monopoly one because of the existence of entry threats. On the contrary, snob effects tend to lessen competition, thereby, increasing the market power of each firm and raising equilibrium prices, which may invite more stores to enter the market. In other words, negative direct network effects may induce the presence of a larger variety of brands, as it is often observed in the industry of luxury goods. These conclusions were, later, reiterated by Grilo et al. (2001), that used both quadratic transportation costs and network effects, and by Di Cintio (2007), who resorted to linear transportation costs and quadratic network effects.

Nonetheless, ever since Rochet and Tirole (2003), Caillaud and Jullien (2003), and Armstrong (2006) the majority of the research on network effects has been developed in the context of multi-sided markets. However, in this dissertation, we return to the study of a standard (single-sided) market with direct network effects, in order to fill a gap that was left unaddressed: the firms' choice of spatial pricing policy.

2.3 Collusion

According to Motta (2004), collusion can be defined as a situation in which firms agree to restrict or soften competition in order to maintain prices above a competitive benchmark. As the author stresses, this benchmark usually corresponds to the equilibrium price of a game where firms only meet once in the marketplace, because, in that case, collusion would not arise.

These collusive agreements can take a wide variety of forms, ranging from committing to the practice of a certain price to allocating market shares, refusing to supply certain clients, fixing the quantities that are produced and coordinating the behaviour regarding other dimensions like investments and advertising.

Moreover, there is a multiplicity of institutional arrangements that firms can follow in order to reach and sustain a collusive outcome, and, based on those, we can classify collusion as either tacit or explicit. Explicit collusion occurs when firms communicate and exchange sensitive information with each other, which allows them to easily coordinate their actions and make joint decisions regarding some strategic areas. Conversely, under tacit collusion, firms do not communicate with each other and use the market to signal their intentions to coordinate on a certain outcome. This makes coordination much harder and costly since firms might have to go through long periods of experimentation in the market, with multiple adjustments, until the desired outcome is reached.

However, even if firms communicate with each other, sustaining a collusion outcome is not easy, since every firm naturally has the incentive to unilaterally deviate from it, by setting a lower pricing or selling higher quantities than the ones agreed upon, since it allows her to increase its own profit. So, to ensure that a collusive agreement is reached and sustained, there are two elements that must necessarily be present. Firstly, its participants must be able to detect in a timely manner that a deviation has occurred. Nonetheless, as Stigler (1964) noted, detecting a deviation can sometimes be significantly hard, because, in many markets, firms' prices and outputs remain private and, thus, are not directly observable. But, identifying the deviation is not enough. In fact, there must also be a credible mechanism to punish eventual deviators, which usually takes the form of a more aggressive market behaviour from the other firms that depresses the future profits of the deviator.

So, a collusive outcome can only arise if a firm knows that a deviation will be quickly identified and subsequently punished. In turn, this implies that firms need to repeatedly meet in the marketplace for collusion to be sustained, since, otherwise, the punishment cannot occur. Thus, collusion will never happen in a single-period game and, therefore, should always be modelled, in the literature, through dynamic (repeated) games (Motta, 2004).

It is also important to note that the punishment will only make firms stick to the agreement if it is designed in a way that ensures that the immediate gain that a firm earns by deviating is lower than the present value of the profits that she will lose once the punishment starts. We can, therefore, conclude that, other things equal, collusion is more likely if the punishment is stronger, if the deviation profit is lower, if the collusive profits are higher and if the discount factor is high, that is, if firms attach more weight to future profits, since these are the ones affected by the punishment (Motta, 2004).

The most strictly unforgiving punishment strategy that firms can implement is usually known in game theory as *grim-trigger* and was initially introduced by Friedman (1971). Under this strategy, firms will cooperate as long as no one deviates. However, if at any given time, one firm defects, they will never cooperate again for the remainder of the iterated game. Nevertheless, there are other punishing strategies in which, for example, firms restart colluding after a phase of punishment. Abreu (1986, 1988) has, in fact, shown that the optimal punishment usually takes the form of a *stick and carrot* strategy, with a very strong punishment in just one period ("stick") and a reversion to collusion immediately afterwards ("carrot").

As previously stated, collusive practices usually allow firms to distort, restrict or even eliminate market competition between firms and are, thereby, prohibited by antitrust laws across the globe. Particularly, in the European Union, this type of agreements between firms is prohibited under Article 101 of the Treaty on the Functioning of the European Union (TFEU). Due to their very sensitive nature and their potential to significantly harm consumers, competition authorities devote a lot of time and effort into identifying and fighting such practices. Therefore, to guarantee that cases of collusion are swiftly detected and punished by these authorities, it is extremely important to have a thorough understanding of the factors that can facilitate or hinder the sustainability of such practices (Motta, 2004).

Economists in the field of industrial organization have been conducting numerous research works aimed at identifying those factors. Succinctly, it has been concluded that collusion is typically easier to occur in markets where firms interact regularly, where price adjustments are frequent, where there are entry barriers and where there is a higher level of transparency regarding prices and selling conditions. Additionally, if firms are present in multiple markets and if they are more symmetric, namely, in terms of market shares, production capacity and costs, collusive agreements are generally more likely to be reached. For a fixed number of market participants, collusion is also easier to sustain in growing markets, where present profits are small compared with the future ones. On the contrary, collusion is typically harder in innovative-driven markets or in markets with a larger number of competitors. Similarly, the existence of business cycles and demand fluctuation also tends to hinder collusion (Ivaldi et al., 2003).

This dissertation, then, aims at expanding the existing research on the factors that affect the sustainability of a collusive practice, by ascertaining how direct network effects influence the likelihood of reaching an agreement to use a specific pricing policy in an infinitely repeated game of price competition.

3. Model

Consider a market setting with two competing firms, A and B, exogenously located at opposite extremes of the Hotelling line. These firms sell a product, whose constant marginal cost of production is normalized to zero. There is a unit mass of consumers uniformly distributed over the interval $[0,1]$ and each consumer, whose relative preference for firm B over firm A is indexed by its location $x \in [0,1]$, has a unit demand.

Assuming that firm A is located at point 0 and firm B at point 1, the utility of a consumer located at x is given by:

$$U(x) = \begin{cases} v - p_A - tx + \alpha n_A & \text{when buying from A} \\ v - p_B - t(1-x) + \alpha n_B & \text{when buying from B} \end{cases}$$

where v stands for the gross, intrinsic utility that an individual derives from consuming the product. This parameter is assumed to be sufficiently large in order to ensure that all consumers prefer acquiring the good rather than staying out of the market. Moreover, p_A and p_B denote the prices charged by firms A and B, respectively, and $t > 0$ represents the transportation cost per unit of distance. The direct network effects are captured in the utility function by the terms αn_A and αn_B, in which α indicates the sign and the strength of the network effect, while n_A and n_B denote the number of consumers buying from firms A and B, respectively, with $n_A + n_B = 1$. To ensure that both firms are always active in the market, it is assumed that $t > 3\alpha$ or, equivalently, that $\frac{\alpha}{t} < \frac{1}{3}$, which encompasses the cases where the network effects are absent $(\alpha = 0)$, negative $(\alpha < 0)$ or weakly positive $\left(0 < \alpha < \frac{t}{3}\right)$.

The timing of the game is similar to the one used by Thisse & Vives (1988) and is the following:

- Stage 1 – Firms simultaneously and independently choose their pricing policy: uniform pricing (U) or personalized pricing (P).
- Stage 2 – Each firm observes the rival's pricing policy and, if the policies are the same, they decide their prices simultaneously and independently. On the contrary, if the pricing policies are different, the firm that chose uniform pricing (U) sets its price first and is followed by the one that chose to use personalized pricing (P)[1].

[1] This assumption, that is standard in the literature on personalized pricing (Thisse and Vives, 1988; Shaffer and Zhang, 2002; Choe et al., 2018), allows us to find the subgame perfect Nash equilibrium in pure strategies.

- Stage 3 – Each consumer observes the prices and, taking into account the expected network sizes, decides to buy the good from the firm that yields him a higher utility.

As usual in the literature, this sequential game is solved by backward induction and all the equilibria found are subgame perfect Nash equilibria in pure strategies.

4. Strategic Choice of Pricing Policy in Markets with Direct Network Effects

4.1 Market Shares, Prices and Profits

To find the subgame perfect Nash equilibria of the game and understand firms' strategic choice of spatial price policy in the presence of direct network effects, we must start by computing the location of the marginal consumer, the prices and the firms' payoffs in three different cases: (i) when one firm uses personalized pricing and the other uses uniform pricing; (ii) when both firms use personalized pricing; and (iii) when both firms use uniform pricing.

4.1.1 When one firm uses personalized pricing and the other uses uniform pricing

We start by analysing the case in which one firm, A, uses personalized pricing, $p_A(x)$, while the other, B, sets a uniform price, p_B. According to the timing of the game, firm B is, then, the price leader and firm A will react optimally to its price.

Assuming that consumers are rational, it is possible to find a marginal consumer, z, that is indifferent between buying the product from firm A or firm B. All the consumers to the left of the marginal consumer, indexed by $x \in [0, z]$, purchase from firm A and the ones to the right, indexed by $x \in [z, 1]$, purchase from firm B. The marginal consumer must, then, satisfy the following equation:

$$U_A(z) = U_B(z) \Leftrightarrow$$

$$\Leftrightarrow v - p_A(z) - tz + \alpha n_A = v - p_B - t(1 - z) + \alpha n_B$$

Considering the existence of self-fulfilled expectations, in equilibrium, the network sizes expected by each consumer are the actual network sizes, so:

$$n_A = z \text{ and } n_B = 1 - z$$

and, therefore, the marginal consumer must satisfy:

$$v - p_A(z) - tz + \alpha z = v - p_B - t(1 - z) + \alpha(1 - z) \Leftrightarrow$$

$$\Leftrightarrow z = \frac{1}{2} + \frac{p_B - p_A(z)}{2(t-\alpha)} \tag{4.1}$$

When deciding its pricing schedule, firm A, which is the price follower, observes p_B and sets its personalized prices so that all consumers on the interval $[0, z]$ are left indifferent between choosing either firm[2], which means that for these consumers:

$$U_A(x) = U_B(x) \Leftrightarrow p_A^{PU}(x) = p_B + t - 2tx - \alpha + 2\alpha z$$

The remainder of the consumers, who are located between $[z, 1]$, are offered $p_A(x) = p_A(z)$ and choose firm B.

The profit of firm A is, then, given by:

$$\pi_A^{PU} = \int_0^z p_A^{PU}(x)\ dx = \int_0^z (p_B + t - 2tx - \alpha + 2\alpha z)\ dx = \\ = (p_B + t - \alpha)z - (t - 2\alpha)z^2 \tag{4.2}$$

Since z depends on $p_A(z)$, which is the variable of decision of firm A, we can write firm A's optimization problem as:

$$\max_z \pi_A^{PU} = (p_B + t - \alpha)z - (t - 2\alpha)z^2$$

This means that, given the unique price previously set by firm B, firm A will optimally decide its demand by identifying the location of the last consumer she wants to serve.

From the first-order condition[3], we get that the optimal value of z is:

$$z^{PU} = \frac{p_B + t - \alpha}{2(t - 2\alpha)} \tag{4.3}$$

Anticipating firm A's behaviour and the $p_A(z)$ that it will set in order to reach the optimal value of z, firm B sets the uniform price (p_B) that allows the maximization of its own profit:

$$\max_{p_B} \pi_B^{PU} = p_B(1 - z^{PU}) = p_B\left(\frac{t - 3\alpha - p_B}{2(t - 2\alpha)}\right) \tag{4.4}$$

[2] In case of indifference, consumers between [0,z] choose firm A.

[3] The second-order condition is satisfied for the values of t and α considered in the model.

Solving the first-order condition[4] with respect to p_B yields that:

$$p_B^{PU} = \frac{t - 3\alpha}{2} \tag{4.5}$$

By substituting (4.5) into (4.3), we can write the location of the marginal consumer as a function of the parameters of the model:

$$z^{PU} = \frac{3t - 5\alpha}{4(t - 2\alpha)} \tag{4.6}$$

To ensure that both firms are active in the market and face a positive demand, the marginal consumer must be located between 0 and 1. This implies that the parameters t and α must satisfy the restriction $t < 3\alpha \Leftrightarrow \frac{\alpha}{t} < \frac{1}{3}$, which encompasses the cases where the network effects are absent $(\alpha = 0)$, negative $(\alpha < 0)$ or weakly positive $\left(0 < \alpha < \frac{t}{3}\right)$.

As it has been previously stated, the marginal consumer is always the one that is indifferent between buying the product at either firm, so it must also satisfy the relation expressed (4.1). By equalling the value of z in that equation to the one in (4.6) we can find $p_A(z)$, which is the personalized price charged by firm A to the marginal consumer.

$$\frac{p_B - p_A(z) + t - \alpha}{2(t - \alpha)} = \frac{3t - 5\alpha}{4(t - 2\alpha)} \Leftrightarrow$$

$$\Leftrightarrow \frac{\frac{t - 3\alpha}{2} - p_A(z) + t - \alpha}{2(t - \alpha)} = \frac{3t - 5\alpha}{4(t - 2\alpha)} \Leftrightarrow \frac{3t - 5\alpha - 2p_A(z)}{t - \alpha} = \frac{3t - 5\alpha}{t - 2\alpha} \Leftrightarrow$$

$$\Leftrightarrow p_A^{PU}(z) = \frac{5\alpha^2 - 3t\alpha}{2(t - 2\alpha)}$$

By analysing this result, we can conclude that in the absence of direct network effects, the personalized price charged by firm A to the marginal consumer is equal to zero, which coincides with the results of Thisse & Vives (1988). This means that firm A maximizes its profits by serving all the consumers that, given the uniform price set by firm B, are willing to pay a positive price for its product. It is important to note that these prices are set so that consumers are indifferent between acquiring the product at either one of the firms, which implies that the personalized prices linearly decrease as we move along the Hotelling line and away from the location of firm A.

[4] The second-order condition is satisfied for the values of t and α considered in the model.

However, the profit maximization strategy followed by firm A in the presence of direct network effects is different. In fact, if there are weak positive network effects, the price charged to the marginal consumer is negative, which implies that the firm is subsidizing some consumers to buy its product. By doing so, the firm is able to increase its network size and, as a result, charge higher prices to its customer base, which yields gains that more than compensate the loss in the profits propelled by the subsidies conceded. So, in markets like the ones for multiplayer videogames or social media applications, if one firm can employ personalized pricing and the other cannot, due, for example, to the lack of data or the absence of the necessary technology, the first one will end up paying some consumers to use its product.

On the other hand, when negative network effects are present, the personalized price charged to the marginal consumer is positive and, therefore, the firm chooses not to serve some customers that are located to the right of the marginal consumer and that would still be willing to pay a positive price for its product. The reasoning behind this decision lies in the fact that it allows the firm to reduce its network size and, thus, charge higher prices to the consumers who buy its product. This strategy provides an increase in the profit that is higher than the gains that the firm would obtain by selling the product to all the buyers that would be willing to pay a positive price for the good. So, in the markets for luxury goods, for example, the firm that is able to employ personalized pricing will deliberately choose not to sell the product to some consumers, even though they would be willing to pay for it, as we often see in reality.

To sum up, the prices charged by the firms are, then:

$$p_A^{PU}(x) = \begin{cases} t\left(\frac{3}{2} - 2x\right) + \frac{-2t\alpha + 5\alpha^2}{2(t-2\alpha)} & \text{if } x \in \left[0, \frac{3t-5\alpha}{4(t-2\alpha)}\right] \\ \frac{5\alpha^2 - 3t\alpha}{2(t-2\alpha)} & \text{if } x \in \left[\frac{3t-5\alpha}{4(t-2\alpha)}, 1\right] \end{cases}$$

and

$$p_B^{PU} = \frac{t-3\alpha}{2}$$

By replacing the p_B expressed in (4.5) and the z expressed in (4.6) into the equations defined in (4.2) and (4.4), we can obtain the profits of firm A and B:

$$\pi_A^{PU} = \left(\frac{t-3\alpha}{2} + t - \alpha\right)\left(\frac{3t-5\alpha}{4(t-2\alpha)}\right) - (t-2\alpha)\left(\frac{3t-5\alpha}{4(t-2\alpha)}\right)^2 = \frac{(3t-5\alpha)^2}{16(t-2\alpha)}$$

$$\pi_B^{PU} = \left(\frac{t-3\alpha}{2}\right)\left(\frac{t-3\alpha-\frac{t-3\alpha}{2}}{2(t-2\alpha)}\right) = \frac{(t-3\alpha)^2}{8(t-2\alpha)}$$

The firm that uses personalized pricing and is the price-follower can achieve higher profits than the leader, who uses uniform pricing, regardless of the magnitude or the sign of the direct network effects. There is, therefore, a second-mover advantage that stems from the fact that the follower can optimally react to the unique price set by the leader by defining personalized prices that allow the capture of the maximum possible surplus from each of the customers it chooses to serve.

4.1.2 When both firms use personalized pricing

We now analyse the case in which both firms use personalized prices, $p_A(x)$ and $p_B(x)$, and choose them simultaneously.

Once again, the marginal consumer is the one that is indifferent between purchasing the product at either firm, so it must satisfy:

$$U_A(z) = U_B(z) \Leftrightarrow z = \frac{1}{2} + \frac{p_B(z) - p_A(z)}{2(t-\alpha)} \tag{4.7}$$

All the consumers located in the interval $[0, z]$ acquire the product at firm A. For these consumers, firm B offers a price that is equal to the one charged to the marginal consumer, that is, $p_B(x) = p_B(z)$ and firm A sets its personalized prices in a way that ensures that they are indifferent between choosing either firm[5]:

$$U_A(x) = U_B(x) \Leftrightarrow p_A(x) = p_B(x) + t - 2tx - \alpha + 2\alpha z \Leftrightarrow$$

$$\Leftrightarrow p_A(x) = p_B(z) + t - 2tx - \alpha + 2\alpha z$$

Conversely, all the consumers located at $[z, 1]$ buy the product from firm B. For those, firm A sets a price that is equal to $p_A(z)$, while firm B chooses its personalized prices so that they are indifferent between purchasing at either one of the sellers[6]:

$$U_A(x) = U_B(x) \Leftrightarrow p_B(x) = p_A(x) - t + 2tx + \alpha - 2\alpha z \Leftrightarrow$$

[5] In case of indifference, consumers between [0,z] choose firm A.

[6] In case of indifference, consumers between [z,1] choose firm B.

$$\Leftrightarrow p_B(x) = p_A(z) - t + 2tx + \alpha - 2\alpha z$$

To sum up, the price schedules are given by:

$$p_A(x) = \begin{cases} p_B(z) + t - 2tx - \alpha + 2\alpha z & \text{if } x \in [0, z] \\ p_A(z) & \text{if } x \in [z, 1] \end{cases} \tag{4.8}$$

and

$$p_B(x) = \begin{cases} p_B(z) & \text{if } x \in [0, z] \\ p_A(z) - t + 2tx + \alpha - 2\alpha z & \text{if } x \in [z, 1] \end{cases} \tag{4.9}$$

The profit of A can, then, be written as:

$$\pi_A^{PP} = \int_0^z p_A(x)\, dx = \int_0^z (p_B(z) + t - 2tx - \alpha + 2\alpha z)\, dx =$$

$$= p_B(z)\, z + tz - tz^2 - \alpha z + 2\alpha z^2$$

If $p_B(z)$ is replaced by the expression in (4.9), we get:

$$\pi_A^{PP} = [p_A(z) - t + 2tz + \alpha - 2\alpha z]\, z + tz - tz^2 - \alpha z + 2\alpha z^2 =$$

$$= p_A(z) z + tz^2$$

and, by substituting z by the equality in (4.7), we lastly have that the profit of firm A is:

$$\pi_A^{PP} = p_A(z) \left[\frac{p_B(z) - p_A(z) + t - \alpha}{2(t - \alpha)}\right] + t \left[\frac{p_B(z) - p_A(z) + t - \alpha}{2(t - \alpha)}\right]^2$$

Firm A chooses $p_A(z)$ in order to maximize its profits, which yields the following first-order condition[7]:

$$\frac{d\pi_A^{PP}}{dp_A(z)} = 0 \Leftrightarrow$$

$$\Leftrightarrow \left[\frac{p_B(z) - p_A(z) + t - \alpha}{2(t - \alpha)}\right] - \frac{p_A(z)}{2(t - \alpha)} + 2t \left[\frac{p_B(z) - p_A(z) + t - \alpha}{2(t - \alpha)}\right] \left[\frac{-1}{2(t - \alpha)}\right] = 0$$

Solving the equation with respect to $p_A(z)$ allows us to find the best response function of firm A:

$$p_A(z) = \frac{[p_B(z) + t - \alpha]\, \alpha}{2\alpha - t}$$

[7] The second-order condition is satisfied for the values of t and α considered in the model.

Similarly, the profit of firm B is given by:

$$\pi_B^{PP} = \int_z^1 p_B(x)\, dx = \int_z^1 (p_A(z) - t + 2tx + \alpha - 2\alpha z)\, dx =$$

$$= p_A(z)(1 - z) + \alpha - 3\alpha z + tz - tz^2 + 2\alpha z^2$$

If $p_A(z)$ is substituted by the corresponding value in (4.8), it can be written as:

$$\pi_B^{PP} = [p_B(z) + t - 2tz - \alpha + 2\alpha z](1 - z) + \alpha - 3\alpha z + tz - tz^2 + 2\alpha z^2 =$$

$$= p_B(z)(1 - z) + t - 2tz + tz^2$$

and, by replacing z by the expression in (4.7), we finally have the profit of firm B:

$$\pi_B^{PP} = p_B(z)\left[\frac{p_A(z) - p_B(z) + t - \alpha}{2(t-\alpha)}\right] + t - 2t\left[\frac{p_B(z) - p_A(z) + t - \alpha}{2(t-\alpha)}\right] + t\left[\frac{p_B(z) - p_A(z) + t - \alpha}{2(t-\alpha)}\right]^2$$

Given the latter profit function, firm B chooses the $p_B(z)$ that allows its maximization, which yields the following first-order condition[8]

$$\frac{d\pi_B^{PP}}{dp_B(z)} = 0 \Leftrightarrow$$

$$\Leftrightarrow \left[\frac{p_A(z) - p_B(z) + t - \alpha}{2(t-\alpha)}\right] - \frac{p_B + 2t}{2(t-\alpha)} + 2t\left[\frac{p_B(z) - p_A(z) + t - \alpha}{2(t-\alpha)}\right]\left[\frac{1}{2(t-\alpha)}\right] = 0$$

Solving this equation with respect to $p_B(z)$ allows us to find the best response function of firm B:

$$p_B(z) = \frac{[p_A(z) + t - \alpha]\,\alpha}{2\alpha - t}$$

Firms simultaneously set their prices and, in equilibrium, both of them play their best response:

$$\begin{cases} p_A(z) = \dfrac{[p_B(z) + t - \alpha]\,\alpha}{2\alpha - t} \\ p_B(z) = \dfrac{[p_A(z) + t - \alpha]\,\alpha}{2\alpha - t} \end{cases} \Leftrightarrow \begin{cases} p_A(z) = -\alpha \\ p_B(z) = -\alpha \end{cases}$$

So, in equilibrium, $p_A(z) = p_B(z) = -\alpha$. Replacing this into (4.7), we get that the marginal consumer is located at $z^{PP} = \frac{1}{2}$.

[8] The second-order condition is satisfied for the values of t and α considered in the model.

The price schedules expressed in (4.8) and (4.9) can, then, be rewritten as:

$$p_A^{PP}(x) = \begin{cases} t - 2tx - \alpha & \text{if } x \in \left[0, \frac{1}{2}\right] \\ -\alpha & \text{if } x \in \left[\frac{1}{2}, 1\right] \end{cases}$$

and

$$p_B^{PP}(x) = \begin{cases} -\alpha & \text{if } x \in \left[0, \frac{1}{2}\right] \\ 2tx - t - \alpha & \text{if } x \in \left[\frac{1}{2}, 1\right] \end{cases}$$

Recovering the expressions of the profits and substituting z, $p_A(z)$ and $p_B(z)$ by the equilibrium values, we get that:

$$\pi_A^{PP} = p_A(z)\,z + tz^2 = \frac{t - 2\alpha}{4}$$

and

$$\pi_B^{PP} = p_B(z)(1 - z) + t - 2tz + tz^2 = \frac{t - 2\alpha}{4}$$

4.1.3 When both firms use uniform pricing

Lastly, we study the case in which both firms use uniform prices, p_A and p_B, and choose them simultaneously.

Similarly to the other two cases the location of the marginal consumer must satisfy the following restriction to ensure that he is indifferent between buying the product at either one of the firms:

$$U_A(z) = U_B(z) \Leftrightarrow z = \frac{1}{2} + \frac{p_B - p_A}{2(t - \alpha)} \tag{4.10}$$

The profit of firm A if given by:

$$\pi_A^{UU} = p_A\,z = p_A\left(\frac{1}{2} + \frac{p_B - p_A}{2(t - \alpha)}\right)$$

and the firm chooses the price (p_A) that allows its maximization, which results in the following first-order condition[9]:

$$\frac{d\pi_A^{UU}}{dp_A} = 0 \Leftrightarrow \frac{1}{2} + \frac{p_B - p_A}{2(t-\alpha)} - \frac{p_A}{2(t-\alpha)} = 0$$

By solving the equation with respect to p_A, we are able to find the function that prescribes the best response of firm A to the price chosen by firm B:

$$p_A = \frac{p_B + t - \alpha}{2}$$

Meanwhile, the profit of firm B can be expressed as:

$$\pi_B^{UU} = p_B(1-z) = p_B\left[\frac{1}{2} + \frac{p_A - p_B}{2(t-\alpha)}\right]$$

As a rational economic agent, firm B also chooses the price (p_B) that maximizes its profit, which yields the next first-order condition[8]:

$$\frac{d\pi_B^{UU}}{dp_B} = 0 \Leftrightarrow \frac{1}{2} + \frac{p_A - p_B}{2(t-\alpha)} - \frac{p_B}{2(t-\alpha)} = 0$$

and, by isolating p_B, we get the best response function of this firm:

$$p_B = \frac{p_A + t - \alpha}{2}$$

According to the timing of the game, firms simultaneously set the price that maximizes their profits, which means that, in equilibrium, both firms play their best response to the action of the rival:

$$\begin{cases} p_A = \dfrac{p_B + t - \alpha}{2} \\ p_B = \dfrac{p_A + t - \alpha}{2} \end{cases} \Leftrightarrow \begin{cases} p_A = t - \alpha \\ p_B = t - \alpha \end{cases}$$

So, in equilibrium, the uniform prices are $p_A^{UU} = p_B^{UU} = t - \alpha$ and, by replacing them into equation (4.10) we get that the marginal consumer is located at $z^{UU} = \frac{1}{2}$.

The equilibrium profits are, then, given by:

$$\pi_A^{UU} = p_A^{UU} z^{UU} = \frac{t-\alpha}{2} \quad \text{and} \quad \pi_B^{UU} = p_B^{UU}(1 - z^{UU}) = \frac{t-\alpha}{2}$$

[9] The second-order condition is satisfied for the values of t and α considered in the model.

4.2 Equilibria in The Absence of Fixed Costs from Personalized Pricing

In the first stage of the game, firms simultaneously and independently commit to a pricing policy based on the payoffs that are expected to occur in the last stage of the game. These payoffs were computed in Section 4.1 and are summarized in Table 1.

		Firm B	
		U	**P**
Firm A	**U**	$\left(\frac{t-\alpha}{2}, \frac{t-\alpha}{2}\right)$	$\left(\frac{(t-3\alpha)^2}{8(t-2\alpha)}, \frac{(3t-5\alpha)^2}{16(t-2\alpha)}\right)$
	P	$\left(\frac{(3t-5\alpha)^2}{16(t-2\alpha)}, \frac{(t-3\alpha)^2}{8(t-2\alpha)}\right)$	$\left(\frac{t-2\alpha}{4}, \frac{t-2\alpha}{4}\right)$

Payoffs: (Profit of Firm A, Profit of Firm B)

Table 1 - Summary of The Firms' Payoffs

Therefore, by analysing the profits, we are able to understand firms' decisions and find the subgame perfect Nash equilibria of the game, that is, the strategy profiles that ensure that, in every subgame, both firms are playing their best response to their rival's actions.

Comparing the different payoffs, we can conclude that when one firm uses uniform pricing, the best response of the other is always to choose personalized pricing, since it yields a higher profit, for the all the values of the parameters t and α considered in the model. However, when one firm employs a personalized pricing strategy, the best response of the other is to also use personalized pricing if $\frac{-1}{\sqrt{2}-1} \leq \frac{\alpha}{t} < \frac{1}{3}$, and to choose uniform pricing if $\frac{\alpha}{t} < \frac{-1}{\sqrt{2}-1}$.

Given these results, it is possible to conclude that the subgame perfect Nash equilibria depend on the sign and on the relative strength of the network effects:

- If $\frac{-1}{\sqrt{2}-1} \leq \frac{\alpha}{t} < \frac{1}{3}$, that is, if there are weak positive network effects or relatively weak negative network effects, we have a unique equilibrium, (P,P), in which both firms commit to the use of personalized pricing. This corresponds to the typical Prisoner's Dilemma situation that Thisse and Vives (1988) found in the absence of network effects, in which the use of personalized pricing is a dominant strategy, but firms would make higher profits if both used uniform pricing.

- If $\frac{\alpha}{t} < \frac{-1}{\sqrt{2-1}}$, that is, if the negative network effects are sufficiently strong, we have the asymmetric equilibria (P,U) and (U,P), which means that one of the firms commits to the use of uniform pricing while the other chooses personalized pricing.

These conclusions bring a new insight into the literature on personalized pricing and show that the equilibrium found in the seminal paper of Thisse and Vives (1988), in which the two firms choose to use personalized pricing, might not always exist, depending on the characteristics of the markets. Specifically, we demonstrate that in markets with relatively strong negative network effects, like the markets for luxury goods, one of the firms will choose to use uniform pricing while the other uses personalized pricing, even though both of them have the means and the possibility to employ a personalized pricing scheme.

4.3 Equilibria in The Presence of Fixed Costs from Personalized Pricing

So far, we have assumed that firms can costlessly implement a personalized pricing strategy. However, that is not often the case in reality. In fact, firms usually need to acquire or develop advanced technologies that allow them to collect, store, process and analyse a vast amount of information about consumers and their preferences that is necessary to the practice of personalized prices. We now consider that these technologies have an associated fixed cost of F, and rewrite the firms' profits in Table 2.

		Firm B	
		U	**P**
Firm A	**U**	$\left(\frac{t-\alpha}{2}, \frac{t-\alpha}{2}\right)$	$\left(\frac{(t-3\alpha)^2}{8(t-2\alpha)}, \frac{(3t-5\alpha)^2}{16(t-2\alpha)} - F\right)$
	P	$\left(\frac{(3t-5\alpha)^2}{16(t-2\alpha)} - F, \frac{(t-3\alpha)^2}{8(t-2\alpha)}\right)$	$\left(\frac{t-2\alpha}{4} - F, \frac{t-2\alpha}{4} - F\right)$

Payoffs: (Profit of Firm A, Profit of Firm B)

Table 2 - Summary of The Firms' Profits in The Presence of a Fixed Cost (F) Associated with Personalized Pricing

By comparing the payoffs presented in Table 2, we can find the different subgame perfect Nash equilibria that might arise, depending on the type and on the relative strength of the direct network effects and on the cost of personalized pricing. These equilibria are thoroughly computed in the Appendix (Section 7.1), and the equilibrium pricing policies followed by the firms can be summed up as follows:

- If $\frac{\alpha}{t} < \frac{-1}{2\sqrt{3}+1}$, that is, if the negative network effects are sufficiently strong and if:
  - $F \leq \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)}$, both firms choose personalized pricing, and are in a Prisoner's Dilemma situation since they would achieve higher profits by pricing uniformly;
  - $\frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)} < F \leq \frac{(t-3\alpha)^2}{16(t-2\alpha)}$, we have the asymmetric equilibria (U,P) and (P,U), in which one firm uses personalized pricing and the other uses uniform pricing;
  - $F > \frac{(t-3\alpha)^2}{16(t-2\alpha)}$, both firms follow a uniform pricing policy.

- If $\frac{-1}{2\sqrt{3}+1} \leq \frac{\alpha}{t} < \frac{1}{3}$, that is, if there are weak positive network effects or relatively weak negative network effects and if:
    - $F \leq \frac{(t-3\alpha)^2}{16(t-2\alpha)}$, both firms implement a personalized pricing scheme, although they would be better off by using uniform pricing;
    - $\frac{(t-3\alpha)^2}{16(t-2\alpha)} < F \leq \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)}$, (U,U) and (P,P) are the equilibria, but it is credible that firms converge into the equilibrium in which both use uniform pricing, since it is the one that is Pareto efficient;
    - $F > \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)}$, both firms choose to price uniformly.

It is, however, important to note that the complete definition of the subgame perfect Nash equilibria, for each case, evolves not only the aforementioned pricing policies, but also the corresponding equilibrium market prices, that were calculated in Section 4.1.

To provide a better understanding of the way that the type of network effects and the cost of personalized pricing affects the firms' choice of pricing policy, we depict the different equilibria in Figure 1.

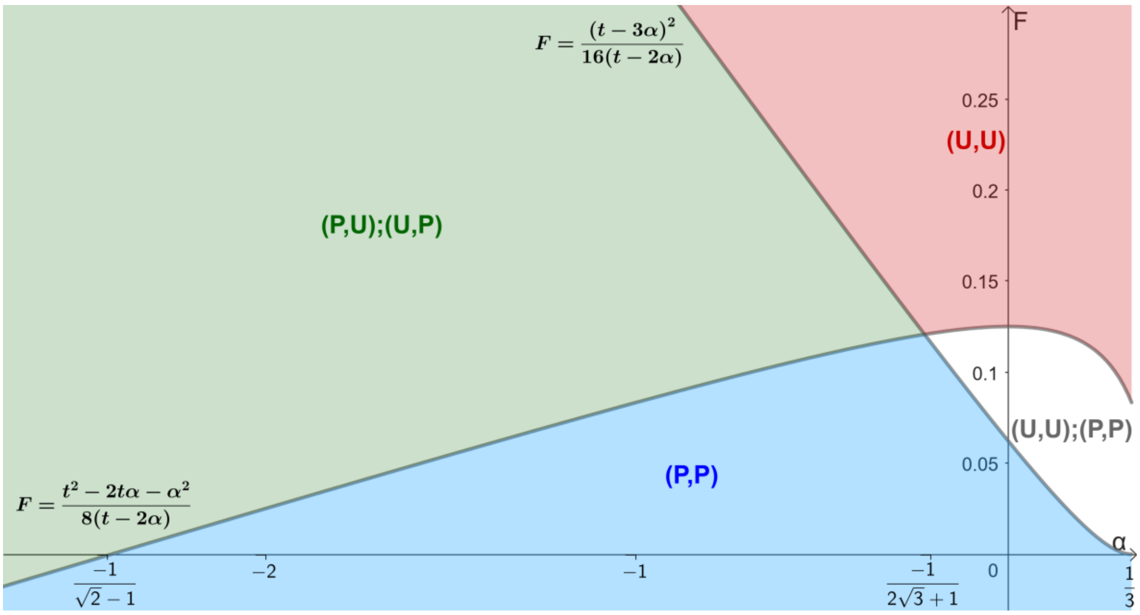


Figure 1 - Equilibria in the Presence of Fixed Costs Associated with Personalized Pricing (for $t = 1$)

In order to guarantee that the conclusions of this study are more aligned with reality, we now assume that $F \geq 0$, which means that the technology needed to implement a personalized pricing strategy either is freely available and, thus, has a cost of zero or has an associated positive fixed cost.

In Figure 1, we can observe that when the value of α is significantly negative $\left(\frac{\alpha}{t} \leq \frac{-1}{\sqrt{2}-1}\right)$, that is, when there are sufficiently strong negative network effects, we either have the asymmetric equilibria (U,P) and (P,U) or the equilibrium in which both firms use uniform pricing, with the latter only happening for higher values of F. In this case, the strategy profile in which both firms choose a personalized pricing policy never constitutes an equilibrium, regardless of the cost of employing that type of pricing scheme.

On the other hand, for intermediate levels of the negative direct network effect $\left(\frac{-1}{\sqrt{2}-1} < \frac{\alpha}{t} < \frac{-1}{2\sqrt{3}+1}\right)$, we can have three different types of equilibria depending on the cost associated with the use of personalized pricing. For small values of F, we have the typical Prisoner's Dilemma situation in which both firms use personalized pricing, even though they would be better off by pricing uniformly. In this case, the Prisoner's Dilemma situation is even further exacerbated by the extra loss in profits that comes from the investment in the necessary technology to implement personalized pricing. As the cost of this technology increases, we move into the asymmetric equilibria in which one firm still uses that pricing strategy while the other prices uniformly. Ultimately, when F reaches high enough values, the equilibrium evolves both firms choosing to use uniform pricing.

Lastly, in the absence of network effects and in the presence of relatively weak negative or weak positive network effects $\left(\frac{-1}{2\sqrt{3}+1} \leq \frac{\alpha}{t} < \frac{1}{3}\right)$, we either have the two firms using personalized pricing, if the cost of the associated technology is low, or we have both of them pricing uniformly if that cost is sufficiently high. As it is possible to see in Figure 1, for intermediate values of F, there are two subgame perfect Nash equilibria: (U,U) and (P,P). However, it is credible that, in this case, firms converge into the equilibrium in which they use uniform pricing since it yields higher profits.

It is, therefore, important to underline that if the cost of the technologies necessary to practice personalized pricing is sufficiently high or if there is a significantly strong negative network effect, the equilibrium in which both firms use personalized pricing does not exist. Moreover, we can also conclude that the higher the cost associated with personalized pricing, the lower is the likelihood that firms employ such a strategy, as it would be expected. Thus, we can state that firms generally prefer a situation where the cost of the technology that is necessary to implement a personalized pricing policy is higher, since it allows them to reach an equilibrium in which they price uniformly and, therefore, avoid the famous Prisoner's Dilemma situation identified by Thisse and Vives (1988).

4.4 Welfare Analysis

In addition to finding the different equilibria and analysing the firms' choice of spatial price policy, it is also important to understand the welfare implications of the various policies that firms might adopt. To do so, we must compute the consumer, the producer, and the total surpluses in the three different strategy profiles that can arise in equilibrium. These results are summarized in Table 3 and the detailed calculations are presented in the Appendix (Section 7.2).

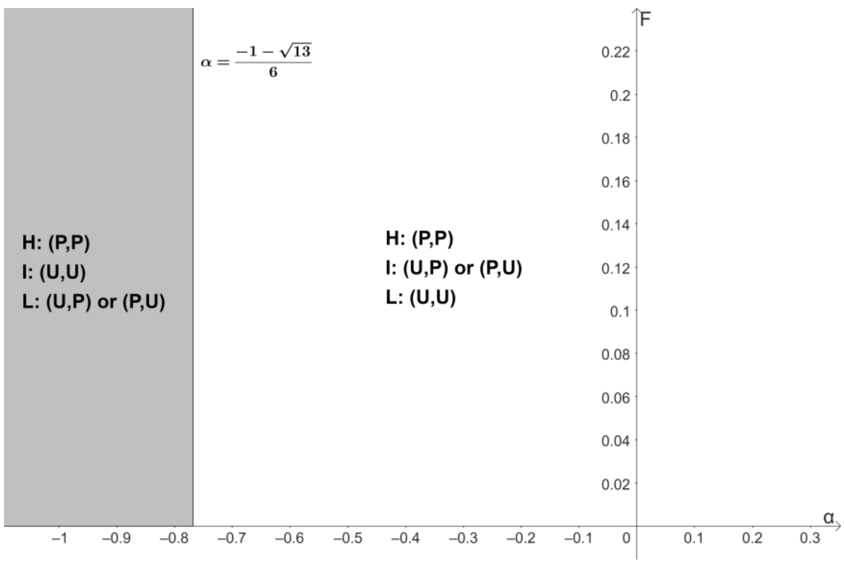
	(U,P) or (P,U)	**(P,P)**	**(U,U)**
Consumer Surplus (CS)	$v+\frac{-4t^2+15\alpha t-15\alpha^2}{4(t-2\alpha)}$	$v-\frac{3t}{4}+\frac{3\alpha}{2}$	$v-\frac{5t}{4}+\frac{3\alpha}{2}$
Producer Surplus (PS)	$\frac{11t^2+43\alpha^2-42\alpha t}{16(t-2\alpha)}-F$	$\frac{t}{2}-\alpha-2F$	$t-\alpha$
Total Surplus (CS+PS)	$v+\frac{-5t^2-17\alpha^2+18\alpha t}{16(t-2\alpha)}-F$	$v-\frac{t}{4}+\frac{\alpha}{2}-2F$	$v-\frac{t}{4}+\frac{\alpha}{2}$

Table 3 - Consumer, Producer and Total Surpluses

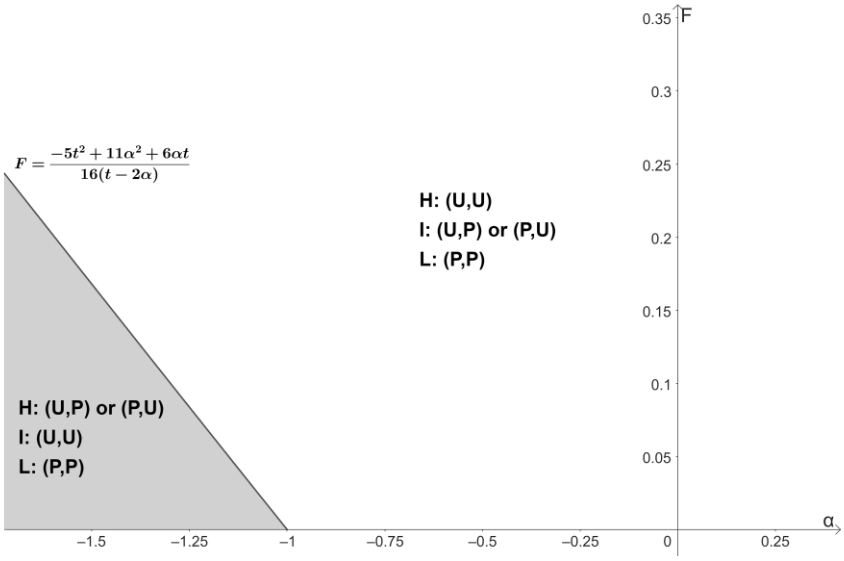
Firstly, it is important to note that, regardless of the pricing policy followed by the firms, a higher value of α leads to a lower profit for each firm, a reduced producer surplus and a higher consumer surplus, which is consistent with the existing literature (Di Cintio, 2007; Grilo et al., 2001; Navon et al., 1995). The rationale is that, in the presence of positive direct network effects, firms seek to expand the number of consumers in their network, which intensifies price competition and decreases equilibrium prices. Conversely, when snob effects are present, competition is softened, allowing firms to maintain higher prices.

Additionally, we can conclude that, in the three cases, total welfare, measured by total consumer surplus, increases with α, which indicates that the gain in consumer surplus that results from the intensified competition always outweighs the loss in profits.

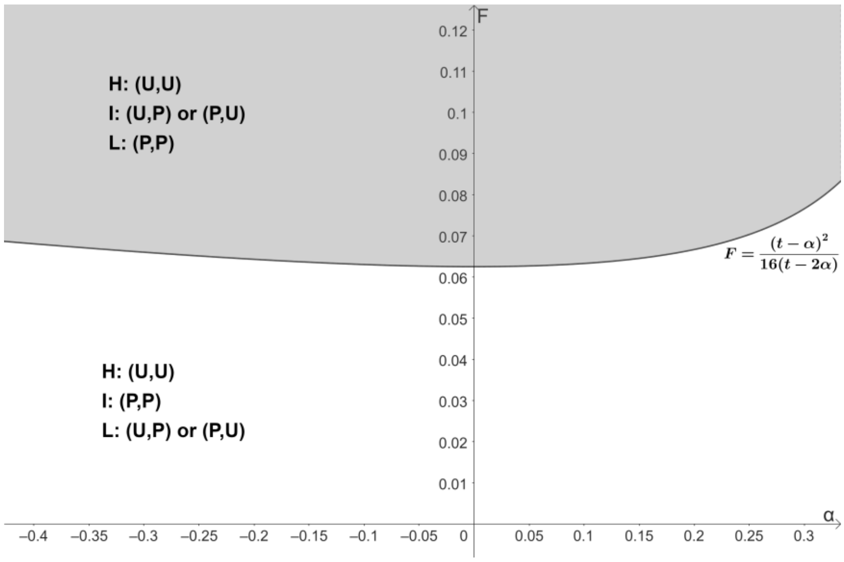
It is also very important to directly compare the value of each type of surplus in the different equilibria. The surpluses presented in each entry of Table 3 can be expressed as a function of the parameters $\frac{\alpha}{t}, \frac{F}{t}$ and $\frac{v}{t}$. However, in order to make the aforementioned comparison, we can ignore $\frac{v}{t}$. Therefore, we can plot the regions of the space $\left(\frac{\alpha}{t}, \frac{F}{t}\right)$ where a certain ordering of surplus holds. In Figure 2, we provide these plots, assuming that $t = 1$, i.e., that the transportation cost has been normalized to 1.



(i) Consumer Surplus



(ii) Producer Surplus



(iii) Total Surplus[10]

Figure 2 - Comparison of the Consumer, Producer and Total Surpluses Between the Different Equilibria (for $t = 1$)

(H: Highest Surplus; I: Intermediate Level Surplus; L: Lowest Surplus)

In Figure 2, we can clearly observe that, when both firms use personalized pricing, consumer surplus is the highest and producer surplus is the lowest. This occurs because the use of personalized pricing allows both firms to compete individually for each consumer without altering the prices charged to the others, which ends up intensifying the competition for each buyer and reducing prices.

In Thisse and Vives (1988) seminal paper, it is possible to observe that, in the absence of network effects, consumer surplus is the lowest when both firms use uniform pricing. We conclude that this result still holds if there are weak positive or relatively weak negative direct network effects. However, if the negative network effects are sufficiently strong, the strategy profile in which only one firm uses uniform pricing is the one that leads to the lowest consumer surplus.

Another crucial insight lies in the fact, that, if $\frac{\alpha}{t} \geq -1$, producer surplus is always the highest when both firms use uniform pricing. However, if the negative network effects are significantly strong $\left(\frac{\alpha}{t} < -1\right)$ and the cost associated with personalized pricing is sufficiently small $\left(F \leq \frac{-5t^2+11\alpha^2+6\alpha t}{16(t-2\alpha)}\right)$, the asymmetric equilibrium is the one that leads to the highest

[10] When $F = 0$, total welfare is equal in (U,U) and (P,P).

producer surplus.

In what concerns to total surplus, we conclude that in the absence of costs associated with personalized pricing, total surplus is always smaller in the asymmetric equilibria when compared to the cases where both firms use either uniform or personalized pricing. In the latter two cases, total welfare is equal, and the type of pricing policy only affects its distribution between firms and consumers.

However, we show that when personalized pricing has an associated fixed cost ($F > 0$), the scenario where both firms use uniform pricing is the only one that leads to the highest total surplus. We also show that, regardless of the type of direct network effects, total surplus is the lowest when both firms use personalized pricing, if the cost of implementing such strategy is significantly high $\left(F > \frac{(t-\alpha)^2}{16(t-2\alpha)}\right)$. Conversely, if that cost is not too high, total surplus reaches its lowest value in the presence of the asymmetric equilibria.

5. Sustainability of Collusion on Pricing Policies in an Infinitely Repeated Game

In the previous chapters, we have considered a single-period game of price competition. However, that type of game is not always the best representation of the way that firms interact in the marketplace. In reality, there are some goods whose price takes the form of a subscription fee that is charged on a recurring basis, which implies that firms have to "meet" regularly in the marketplace to choose their pricing policy and determine their prices. This scenario is, thereby, better modelled by an infinitely repeated game of price competition. As Motta (2004) noted, when firms engage in this kind of iterated interaction in the marketplace, they can potentially reach and sustain a collusive outcome, which would not occur in a single-period game. This is because the ongoing nature of their interaction allows them to employ punishment strategies in response to any defection from the original agreement, thereby, discouraging deviations and promoting long-term cooperation.

In Section 4.2, we concluded that, in a single-period game of price competition, both firms choose to use personalized pricing if $\frac{-1}{\sqrt{2}-1} \leq \frac{\alpha}{t} < \frac{1}{3}$, even though they would be better off by collectively committing to the use of uniform pricing. However, they are unable to achieve the later outcome because each firm has an incentive to unilaterally deviate in order to increase its own profits. In contrast, in an infinitely repeated game, there is a possibility that firms can reach and sustain an agreement in which both commit to the use of uniform pricing, since they can introduce a punishing mechanism that dissuades any deviation, thus, maintaining cooperation over time.

In this chapter, we analyse whether firms can sustain an agreement to collude on the choice of a uniform pricing policy while allowing the price level to be competitively determined in the market. There are several reasons that can lead firms to only collude on the type of pricing policy but not on the price level itself. First, we can assume that selecting a pricing policy is a higher-level managerial decision since it is a strategic choice that needs to be aligned with the company's overall goals and market positioning, whereas the decision of the exact price level within the chosen pricing policy framework is typically a lower-level (operational) decision that is based on the market conditions and on the cost structure. Therefore, it is possible that the collusive agreement only occurs between the top-level managers that are responsible for deciding the pricing policy, which implies that the specific prices are left to be competitively determined in the market. Second, by deliberately maintaining a certain level of price competition in the market, firms can better disguise the existence of a collusive agreement, thereby, delaying or even avoiding its detection by competition authorities. Lastly, if they are caught, they can argue that the choice of a uniform pricing policy was merely a coincidence, perhaps motivated by fairness concerns raised by consumers, rather than an agreement. They can further claim that such action did not significantly restrict or distort competition as prices were still set at a competitive level.

To perform this analysis, we assume that the single-period game with the following stages is infinitely repeated:

- Stage 1 – Firms' top-level managers simultaneously choose the pricing policy: uniform pricing (U) or personalized pricing (P).
- Stage 2 – Firms' lower-level management personnel simultaneously and independently set the prices within the chosen pricing policy framework.[11]
- Stage 3 – Each consumer observes the prices and, taking into account the expected network sizes, decides to buy the good from the firm that yields a higher utility.[12]

Furthermore, we consider that firms follow a *grim-trigger strategy* (Friedman, 1971), which means that they collude and, thus, choose to use uniform pricing until one of them defects from the agreement. If a deviation occurs, a punitive mechanism is triggered, causing the firms to revert to the use of personalized pricing – the equilibrium of the single-period game – in all the subsequent periods.

Colluding on the choice of uniform pricing in the first stage and setting the competitive price $p_A^{UU} = p_B^{UU} = t - \alpha$ in the second stage is, therefore, a subgame perfect Nash equilibrium in the infinitely repeated game if neither firm has an incentive to deviate. This condition only holds if the present value of the profits that a firm gets from always colluding (V_C) is equal to or greater than the sum of profit of deviating (π_D) with the present value of the profits post-deviation (V_P). We can, therefore, write the following incentive

[11] The choice of pricing policy that was made by top-level managers in stage 1 cannot be changed by the lower-level management personnel in stage 2.

[12] We assume that there are no switching costs, which implies that consumers can freely switch firms from one period to the next.

compatibility constraint (ICC), which must be satisfied in order to ensure that the collusive agreement is sustained:

$$V_C \geq \pi_D + V_P$$

Moreover, we consider that δ (with $0 < \delta < 1$) is the discount factor of future profits, which represents the weight that firms attach to future profits compared to the present ones. Therefore, a discount factor close to 1 implies that future profits are almost as valuable as the ones in the present, whereas a discount factor close to 0 means that firms place very little value on future earnings.

As previously emphasized, when deciding whether to adhere to a collusive agreement or to deviate, firms must compare the immediate gains generated by the deviation to the losses that they will have in the future due to the rivals' retaliation. Thus, collusion is only sustainable if firms place sufficient weight on future profits, i.e., if their discount factor is significantly high.

If firms collude in all the periods of the game, the present value of the profits (V_c) is given by:

$$V_C = \sum_{t=0}^{+\infty} \delta^t \pi^{UU} = \frac{\pi^{UU}}{1-\delta} = \frac{\frac{t-\alpha}{2}}{(1-\delta)} = \frac{t-\alpha}{2(1-\delta)}$$

However, if one firm deviates, they revert to using personalized pricing and, specifically, the price schemes computed in Section 4.1.2, in all the subsequent periods. Accordingly, the present value of the profits in the post-deviation periods (V_P), when the punishment is enforced, is given by:

$$V_P = \sum_{t=1}^{+\infty} \delta^t \pi^{PP} = \frac{\delta\, \pi^{PP}}{1-\delta} = \frac{\delta\left(\frac{t-2\alpha}{4}\right)}{1-\delta} = \frac{(t-2\alpha)\delta}{4(1-\delta)}$$

We now proceed to compute the profit that stems from a deviation from the agreed-upon pricing policy. We consider two different scenarios based on the moment that the deviation is detected.

In the first scenario, the firms' choice of pricing policy becomes publicly observable after the first stage of the game. This means that if a deviation occurs, i.e., if a firm opts for personalized pricing in the first stage, the other firm will be aware of it at the end of that stage. Consequently, in the second stage of that period, she will be able to set her unique price according to the rival's chosen pricing policy. Nonetheless, as specified in the timing of the game, in that stage, she cannot change her chosen type of pricing policy from uniform to personalized.

So, in the second stage, the prices of both firms are competitively determined, with the deviating firm setting her personalized prices and the other choosing her uniform price. The equilibrium pricing schedules for this scenario are detailed in Section 4.1.1, where it is also shown that the profit that arises from deviating (π_{D_1}) is given by:

$$\pi_{D_1} = \frac{(3t - 5\alpha)^2}{16(t - 2\alpha)}$$

The ICC, in this scenario, is, therefore, satisfied if:

$$V_C \geq \pi_{D_1} + V_P \Leftrightarrow \frac{t - \alpha}{2(1 - \delta)} \geq \frac{(3t - 5\alpha)^2}{16(t - 2\alpha)} + \frac{(t - 2\alpha)\delta}{4(1 - \delta)} \Leftrightarrow$$

$$\Leftrightarrow \delta \geq \frac{9\left(\frac{\alpha}{t}\right)^2 - 6\frac{\alpha}{t} + 1}{9\left(\frac{\alpha}{t}\right)^2 - 14\frac{\alpha}{t} + 5} = \delta_1^*$$

where δ_1^* represents the critical discount factor in this scenario, i.e., the value of the discount factor above which collusion is sustainable.

In the second scenario, however, the firms' choice of pricing policy is not observable after the first stage. As a result, in the second stage, the firm that adheres to the agreement sets its uniform price under the assumption that the other firm has also not deviated. In fact, she only becomes aware of the defection after the second stage, when the prices are presented to consumers. This means that the firm does not have the possibility to adjust her strategy in order to counter the deviation within the period in which it takes place.

Assuming that firm A is the one that deviates, she will employ a personalized pricing scheme, while firm B sticks to the use of uniform pricing. However, at the stage that the exact prices are set, firm B is not yet aware of the deviation. So, she will set the uniform price $p_B^{UU} = t - \alpha$, since it is the one that would prevail if both firms had complied with the agreement. Knowing this information allows firm A to set the personalized prices that ensure that the consumers on $[0, z]$ are left indifferent between firm A and B[13]:

$$p_A(x) = p_B^{UU} + t - 2tx - \alpha + 2\alpha z = 2t - 2\alpha - 2tx + 2\alpha z$$

[13] In case of indifference, consumers between [0,z] choose firm A.

The profit of the deviating firm (π_{D_2}) is, thus, given by:

$$\pi_{D_2} = \int_0^z (2t - 2\alpha - 2tx + 2\alpha z)\, dx = (2t - 2\alpha)z + (2\alpha - t)z^2$$

The firm, then, decides to serve, with the corresponding personalized prices, all the consumers until the marginal consumer, whose location is chosen in order to maximize the profit. The optimization problem of firm A can, thereby, be written as:

$$\max_z \pi_D = (2t - 2\alpha)z + (2\alpha - t)z^2$$

which yields the following first-order condition[14]:

$$\frac{d\pi_D}{dz} = 2t - 2\alpha + 2(2\alpha - t)z = 0 \Leftrightarrow z = \frac{\alpha - t}{2\alpha - t}$$

Given that the marginal consumer can only be located within the interval $[0,1]$, it must satisfy the restriction:

$$z = \begin{cases} \dfrac{\alpha - t}{2\alpha - t} & \text{if } \dfrac{1}{1-\sqrt{2}} \leq \dfrac{\alpha}{t} \leq 0 \\ 1 & \text{if } 0 < \dfrac{\alpha}{t} < \dfrac{1}{3} \end{cases}$$

This implies that the profit from deviating is:

$$\pi_{D_2} = \begin{cases} \dfrac{-t^2 + 2t\alpha - \alpha^2}{2\alpha - t} & \text{if } \dfrac{1}{1-\sqrt{2}} \leq \dfrac{\alpha}{t} \leq 0 \\ t & \text{if } 0 < \dfrac{\alpha}{t} < \dfrac{1}{3} \end{cases}$$

Therefore, in this scenario, the ICC is satisfied when:

$$V_C \geq \pi_{D_2} + V_P \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \dfrac{t - \alpha}{2(1-\delta)} \geq \dfrac{-t^2 + 2t\alpha - \alpha^2}{2\alpha - t} + \dfrac{(t - 2\alpha)\delta}{4(1-\delta)} \Leftrightarrow \delta \geq \dfrac{2 - 2\frac{\alpha}{t}}{3 - 4\frac{\alpha}{t}} & \text{if } \dfrac{-1}{\sqrt{2}-1} \leq \dfrac{\alpha}{t} \leq 0 \\ \dfrac{t - \alpha}{2(1-\delta)} \geq t + \dfrac{(t - 2\alpha)\delta}{4(1-\delta)} \Leftrightarrow \delta \geq \dfrac{2 + 2\frac{\alpha}{t}}{3 + 2\frac{\alpha}{t}} & \text{if } 0 < \dfrac{\alpha}{t} < \dfrac{1}{3} \end{cases}$$

[14] The second-order condition is satisfied for the values of t and α considered in the model.

which yields the following critical discount factor:

$$\delta_2^* = \begin{cases} \dfrac{2-2\frac{\alpha}{t}}{3-4\frac{\alpha}{t}} & \text{if } \dfrac{-1}{\sqrt{2}-1} \leq \dfrac{\alpha}{t} \leq 0 \\[2ex] \dfrac{2+2\frac{\alpha}{t}}{3+2\frac{\alpha}{t}} & \text{if } 0 < \dfrac{\alpha}{t} < \dfrac{1}{3} \end{cases}$$

In both scenarios, the value of the critical discount factor depends on $\frac{\alpha}{t}$, i.e., it hinges on the nature and on the relative strength of the direct network effects, as it is depicted in Figure 3.

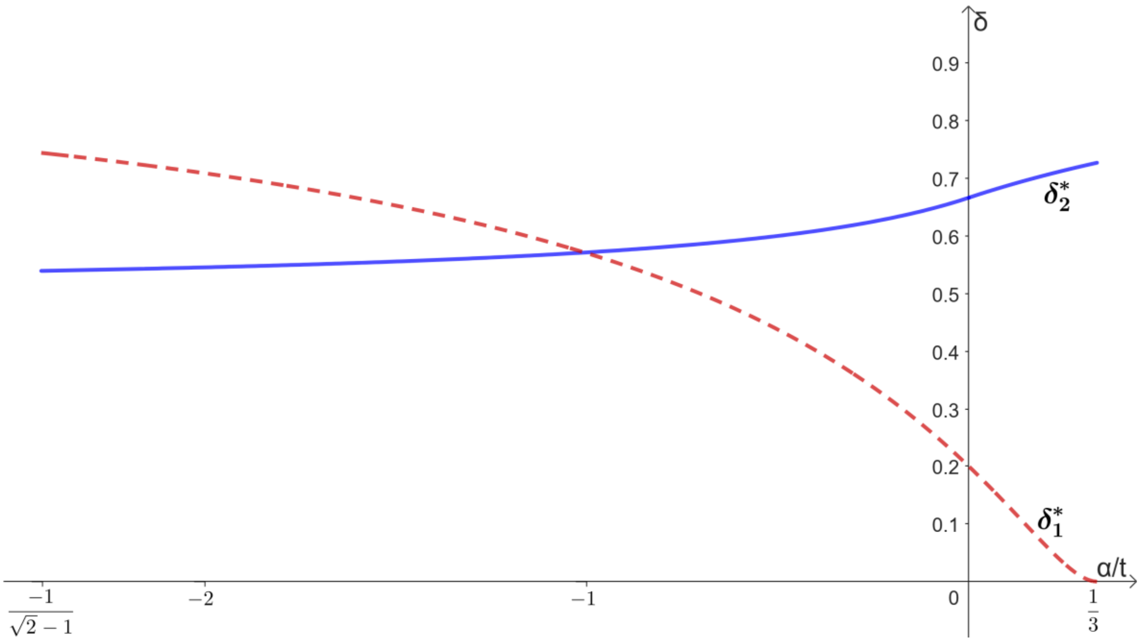


Figure 3 - Impact of the Direct Network Effects on the Critical Discount Factor

Since the critical discount factor represents the threshold of the firms' discount factor above which collusion is sustainable, it can naturally be used as a measure of the likelihood of reaching and sustaining a collusive agreement. The lower the value of δ^*, the easier it is to sustain collusion, since even "impatient" firms, that highly value the immediate gains from deviation, would lack sufficient incentives to deviate. Conversely, if the value of δ^* is higher, collusion becomes more difficult to sustain, because even firms with a high discount factor, who place a substantial weight on the future losses stemming from punishment, might still have incentives to defect from the agreement.

In Figure 3, we can observe that the impact of the direct network effects on the critical discount factor varies depending on the scenario that is considered. Therefore, we can conclude that their impact on the likelihood of collusion critically hinges on the moment in which a deviation is detected.

In the first scenario, where defection is noticeable after the first stage and the other firm can still adjust her unique price accordingly in that period, collusion becomes easier to sustain as the value of $\frac{\alpha}{t}$ increases. Although the presence of network effects does not affect the losses that stem from the punishment in the post-deviation period, the immediate gains from deviation decrease as α increases, which explains why firms have fewer incentives to deviate when α is higher. In fact, if the network effects are positive, collusion on the choice of uniform pricing as the pricing policy is very likely to occur.

Conversely, in the second scenario, where defection is only detected after the prices are presented to consumers, collusion is less likely to be sustained as value of $\frac{\alpha}{t}$ increases, since firms end up deviating across a wider range of discount factors. Once again, the presence of network effects does not influence the losses propelled by the rivals' retaliation. However, here, the immediate gains from deviation increase as α augments, which justifies why firms have greater incentives to deviate when α is higher. Therefore, in this scenario, these collusive agreements are more likely to occur in markets with negative network effects, such as those for luxury goods.

In Figure 3, we can also perceive that the influence of the different scenarios on the likelihood of sustaining a collusive agreement varies according to the relative strength of the direct network effects.

If $-1 \leq \frac{\alpha}{t} < \frac{1}{3}$, collusion is easier to sustain in the first scenario, which implies that a deviation is less likely to occur if it is detected immediately after the first stage of the game. The reasoning behind this result lies in the fact that, in the second stage, the firm that adheres to the agreement can partially counter the deviation by adjusting her uniform price level, thereby, reducing the profits that stem from deviating and, consequently, minimizing the incentives to do so.

On the contrary, if $\frac{-1}{\sqrt{2}-1} \leq \frac{\alpha}{t} < -1$, collusion is more likely to occur in the second scenario. This implies that firms have fewer incentives to deviate from the agreed-upon pricing policy when such action is only detected later in that period, after the exact prices are presented to consumers. This conclusion, then, contradicts the typical notion that collusion is easier to sustain if firms can detect and react to a defection earlier. In fact, for these values of the parameters, when the other firm becomes aware of the defection after the first stage and adjusts her uniform price accordingly, price competition ends up being softened. This, in turn, increases the profits from deviation, making it more appealing. So, when the negative network effects satisfy the aforementioned condition, collusion is easier to sustain if the other firm does not react to the defection in the period it occurs.

6. Conclusion

This dissertation analysed firms' choice of spatial price policy in a Hotelling duopoly with direct network effects. Specifically, it expanded the work of Thisse and Vives (1988) in order to understand if firms that operate in these markets would choose to follow a uniform or a personalized pricing policy.

In their seminal paper, Thisse and Vives (1988) established that, in the absence of direct network effects, both firms adopt a personalized pricing strategy, even though they would be able to achieve higher payoffs by collectively committing to the use of uniform pricing. The firms are, therefore, trapped in a Prisoner's Dilemma-type situation, from which they wish they could escape.

This dissertation concluded that this Prisoner's Dilemma situation also occurs when there are weak positive or relatively weak negative direct network effects. However, it was found that, in the presence of strong negative direct network effects, the equilibrium involves one firm using personalized pricing while the other uses uniform pricing.

It was also analysed how the cost of the sophisticated technologies that are required for implementing a personalized pricing strategy affects firms' pricing decisions. As expected, it was asserted that the higher this cost, the lower is the likelihood that firms employ a personalized pricing strategy. Depending on the cost associated with personalized pricing and on the network effects, the equilibrium might involve two firms, one firm or none using that pricing strategy. When this cost is sufficiently high or when there are significantly strong negative network effects, the equilibrium in which both firms use personalized pricing does not exist. In turn, the asymmetric equilibria, in which only one firm uses personalized pricing, can only occur in the presence of relatively strong or intermediate negative network effects. Finally, the equilibrium where both firms price uniformly, thereby, avoiding the Prisoner's Dilemma situation, is reached when the cost of personalized pricing is sufficiently high.

The latter part of the dissertation was devoted to the study of the sustainability of a collusive agreement on the choice of a pricing policy, but not of a price level, in an infinitely repeated game. In particular, it was assessed whether firms could sustain an agreement to choose uniform pricing and, thereby, escape the Prisoner's Dilemma situation that occurs in the single-period game.

It was first ascertained that collusion is more likely in the presence of positive direct network effects if the firm that adheres to the agreement can adapt her uniform price during the period in which a deviation to personalized pricing occurs. On the contrary, if she cannot promptly detect the defection and adjust her unique price in that period, collusion is more likely when there are negative network effects.

Additionally, it was concluded that, if $-1 \leq \frac{\alpha}{t} < \frac{1}{3}$, collusion is easier to sustain if the firm that sticks to the agreement can adapt her unique price in the period that the deviation occurs. Conversely, if $\frac{-1}{\sqrt{2}-1} \leq \frac{\alpha}{t} < -1$, it is easier to reach and sustain a collusive agreement, if the firm does not become aware of the defection in the period it occurs and, thus, cannot adjust her uniform price. This latter case is particularly relevant, since it demonstrates that, under certain circumstances, an early detection and response to a defection can be detrimental to the sustainability of a collusive agreement.

7. Appendix

7.1 Equilibria in The Presence of Fixed Costs from Personalized Pricing

By comparing the payoffs presented in Table 2, we are able to find the different subgame perfect Nash equilibria that might arise depending on the parameters of the model. Specifically, we can conclude that:

(A) (P,P) is the equilibrium if:

(A1) When one firm chooses U, the best response of the other is to choose P:

$$\pi_{P}\Big|_{U} \geq \pi_{U}\Big|_{U} \Leftrightarrow \frac{(3t-5\alpha)^2}{16(t-2\alpha)} - F \geq \frac{t-\alpha}{2} \Leftrightarrow$$

$$\Leftrightarrow F \leq \frac{(t-3\alpha)^2}{16(t-2\alpha)}$$

and

(A2) When one firm chooses P, the best response of the other is also to choose P:

$$\pi_{P}\Big|_{P} \geq \pi_{U}\Big|_{P} \Leftrightarrow \frac{t-2\alpha}{4} - F \geq \frac{(t-3\alpha)^2}{8(t-2\alpha)} \Leftrightarrow$$

$$\Leftrightarrow F \leq \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)}$$

So, given the restrictions on the parameters α and t, the equilibrium in which both firms use personalized pricing occurs if:

$$F \leq \frac{(t-3\alpha)^2}{16(t-2\alpha)} \wedge F \leq \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} F \leq \dfrac{(t-3\alpha)^2}{16(t-2\alpha)} & \text{when } \dfrac{-1}{2\sqrt{3}+1} \leq \dfrac{\alpha}{t} < \dfrac{1}{3} \\ F \leq \dfrac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)} & \text{when } \dfrac{\alpha}{t} < \dfrac{-1}{2\sqrt{3}+1} \end{cases}$$

(B) (U,U) is the equilibrium if:

(B1) When one firm chooses U, the best response of the other is to choose U:

$$\pi_{P}\big|_{U} < \pi_{U}\big|_{U} \Leftrightarrow F > \frac{(t-3\alpha)^2}{16(t-2\alpha)}$$

and

(B2) When one firm chooses P, the best response of the other is to choose U:

$$\pi_{P}\big|_{P} < \pi_{U}\big|_{P} \Leftrightarrow F > \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)}$$

So, given the restrictions on the parameters α and t, the equilibrium in which both firms commit to and use uniform pricing occurs if:

$$F > \frac{(t-3\alpha)^2}{16(t-2\alpha)} \wedge F > \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} F > \dfrac{(t-3\alpha)^2}{16(t-2\alpha)} & \text{when } \dfrac{\alpha}{t} < \dfrac{-1}{2\sqrt{3}+1} \\ F > \dfrac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)} & \text{when } \dfrac{-1}{2\sqrt{3}+1} \leq \dfrac{\alpha}{t} < \dfrac{1}{3} \end{cases}$$

(C) (U,P) and (P,U) are the equilibria if:

(C1) When one firm chooses U, the best response of the other is to choose P:

$$\pi_{P}\big|_{U} \geq \pi_{U}\big|_{U} \Leftrightarrow F \leq \frac{(t-3\alpha)^2}{16(t-2\alpha)}$$

and

(C2) When one firm chooses P, the best response of the other is to choose U:

$$\pi_{P}\big|_{P} < \pi_{U}\big|_{P} \Leftrightarrow F > \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)}$$

So, given the restrictions on the parameters α and t, the equilibria in which one firm uses personalized pricing while the other prices uniformly occurs if:

$$\frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)} < F \leq \frac{(t-3\alpha)^2}{16(t-2\alpha)} \text{ when } \frac{\alpha}{t} < \frac{-1}{2\sqrt{3}+1}$$

(D) (U,U) and (P,P) are the equilibria if:

(D1) When one firm chooses U, the best response of the other is to choose U:

$$\pi_{P}\Big|_{U} < \pi_{U}\Big|_{U} \Leftrightarrow F > \frac{(t-3\alpha)^2}{16(t-2\alpha)}$$

and

(D2) When one firm chooses P, the best response of the other is to choose P:

$$\pi_{P}\Big|_{P} \geq \pi_{U}\Big|_{P} \Leftrightarrow F \leq \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)}$$

So, given the restrictions on the parameters α and t, the equilibria in which both firms either employ personalized pricing or uniform pricing occurs if:

$$\frac{(t-3\alpha)^2}{16(t-2\alpha)} < F \leq \frac{t^2-2t\alpha-\alpha^2}{8(t-2\alpha)} \text{ when } \frac{-1}{2\sqrt{3}+1} \leq \frac{\alpha}{t} < \frac{1}{3}$$

However, it is important to note that, for the values of the parameters considered, (U,U) is the Pareto efficient equilibrium when compared to (P,P), since it yields higher profits to both firms. Thus, it is credible that the firms converge into the equilibrium in which they simultaneously choose to price uniformly.

7.2 Consumer, Producer and Total Surpluses

Case 1 – When one firm uses personalized pricing and the other uses uniform pricing - (P,U) or (U,P)

(A) Consumer Surplus

Assuming that firm A is the one that implements personalized pricing, we can conclude that the consumer surplus of the costumers that acquire the product at firm A is:

$$CS_A^{PU} = \int_0^{z^{PU}} U_A(x)\,dx = \int_0^{z^{PU}} (v - p_A^{PU}(x) - tx + \alpha n_A)\,dx =$$

$$= \int_0^{z^{PU}} \left(v - \frac{3t}{2} + \frac{5\alpha}{2} + tx - \alpha z^{PU}\right) dx$$

$$= \left(v - \frac{3t}{2} + \frac{5\alpha}{2}\right) z^{PU} + \left(\frac{t}{2} - \alpha\right)(z^{PU})^2$$

On the other hand, the consumers that buy the product at firm B, which is assumed to be the one that uses uniform pricing, have the following surplus:

$$CS_B^{PU} = \int_{z^{PU}}^{1} U_B(x)\,dx = \int_{z^{PU}}^{1} (v - p_B^{PU} - t(1-x) + \alpha n_B)\,dx =$$

$$= \int_{z^{PU}}^{1} \left(v - \frac{3t}{2} + \frac{5\alpha}{2} + tx - \alpha z^{PU}\right) dx$$

$$= v - t + \frac{5\alpha}{2} + \left(-v + \frac{3t}{2} - \frac{7\alpha}{2}\right) z^{PU} + \left(-\frac{t}{2} + \alpha\right) (z^{PU})^2$$

Given $z^{PU} = \frac{3t-5\alpha}{4(t-2\alpha)}$, the total consumer surplus in this market is equal to:

$$CS^{PU} = CS_A^{PU} + CS_B^{PU} = v - t + \frac{5\alpha}{2} - \alpha z^{PU} =$$

$$= v + \frac{-4t^2 + 15\alpha t - 15\alpha^2}{4(t-2\alpha)}$$

(B) Producer Surplus

The total producer surplus of the market corresponds to the sum of the profits of both firms:

$$PS^{PU} = \pi_A^{PU} + \pi_B^{PU} = \frac{(3t-5\alpha)^2}{16(t-2\alpha)} - F + \frac{(t-3\alpha)^2}{8(t-2\alpha)} =$$

$$= \frac{11t^2 + 43\alpha^2 - 42\alpha t}{16(t-2\alpha)} - F$$

(C) Total Surplus

The total surplus of the market is given by the sum of the consumer and the producer surpluses, and is equal to:

$$TS^{PU} = CS^{PU} + PS^{PU} = v + \frac{-4t^2 + 15\alpha t - 15\alpha^2}{4(t-2\alpha)} + \frac{11t^2 + 43\alpha^2 - 42\alpha t}{16(t-2\alpha)} - F =$$

$$= v + \frac{-5t^2 - 17\alpha^2 + 18\alpha t}{16(t-2\alpha)} - F$$

Case 2 – When both firms use personalized pricing – (P,P)

(A) Consumer Surplus

Since the firms are symmetric and employ the same pricing strategy, the surplus of the consumers that buy the product at firm A and at firm B is the same:

$$CS_A^{PP} = CS_B^{PP} = \int_0^{z^{PP}} U_A(x)\,dx =$$

$$= \int_0^{\frac{1}{2}} (v - p_A^{PP}(x) - tx + \alpha n_A)\,dx = \int_0^{\frac{1}{2}} \left(v - t + tx + \frac{3\alpha}{2}\right) dx =$$

$$= \frac{v}{2} - \frac{3t}{8} + \frac{3\alpha}{4}$$

The total consumer surplus in this market is, thus, equal to:

$$CS^{PP} = CS_A^{PP} + CS_B^{PP} = 2\left(\frac{v}{2} - \frac{3t}{8} + \frac{3\alpha}{4}\right) =$$

$$= v - \frac{3t}{4} + \frac{3\alpha}{2}$$

(B) Producer Surplus

The total producer surplus of the market corresponds to the sum of the profits of both firms:

$$PS^{PP} = \pi_A^{PP} + \pi_B^{PP} = 2\left(\frac{t - 2\alpha}{4} - F\right) = \frac{t}{2} - \alpha - 2F$$

(C) Total Surplus

The total surplus of the market is given by the sum of the consumer and the producer surpluses, and is equal to:

$$TS^{PP} = CS^{PP} + PS^{PP} = v - \frac{3t}{4} + \frac{3\alpha}{2} + \frac{t}{2} - \alpha - 2F =$$

$$= v - \frac{t}{4} + \frac{\alpha}{2} - 2F$$

Case 3 – When both firms use uniform pricing – (U,U)

(A) Consumer Surplus

Since the firms are symmetric and employ the same pricing strategy, the surplus of the consumers that buy the product at firm A and at firm B is the same:

$$CS_A^{UU} = CS_B^{UU} = \int_0^{z^{UU}} U_A(x)\,dx =$$

$$= \int_0^{\frac{1}{2}} (v - p_A^{PP} - tx + \alpha n_A)\,dx = \int_0^{\frac{1}{2}} \left(v - t - tx + \frac{3\alpha}{2}\right) dx =$$

$$= \frac{v}{2} - \frac{5t}{8} + \frac{3\alpha}{4}$$

The total consumer surplus in this market is, thus, equal to:

$$CS^{UU} = CS_A^{UU} + CS_B^{UU} = 2\left(\frac{v}{2} - \frac{5t}{8} + \frac{3\alpha}{4}\right) =$$

$$= v - \frac{5t}{4} + \frac{3\alpha}{2}$$

(B) Producer Surplus

The total producer surplus of the market corresponds to the sum of the profits of both firms:

$$PS^{UU} = \pi_A^{UU} + \pi_B^{UU} = 2\left(\frac{t-\alpha}{2}\right) = t - \alpha$$

(C) Total Surplus

The total surplus of the market is given by the sum of the consumer and the producer surplus, and is equal to:

$$TS^{UU} = CS^{UU} + PS^{UU} = v - \frac{5t}{4} + \frac{3\alpha}{2} + t - \alpha =$$

$$= v - \frac{t}{4} + \frac{\alpha}{2}$$

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