DESIGN AND OPTIMIZATION OF PREFABRICATED TEE BEAMS

Alfredo Soeiro, Fernando Fagundo, Sung Yong Yu

Pro-Rector, University Porto, Portugal
Professor, Department of Civil Engineering, University Florida, USA
Professor, Department Architectural Engineering, DongGuk University, Seoul, Korea

Email: avsoeiro@reit.up.pt, fef@ce.ufl.edu, yu@unitel.co.kr

STRUCTURAL OPTIMIZATION REVIEW

Overview

Optimization is a state of mind that is always implicitly present in structural engineers activity. From experience they learn to recognize good initial dimension ratios so that their preliminary designs demand small changes and the elements are not overdesigned. Motivation behind this attitude is to create a structure that for given purposes is simultaneously useful and economic.

Structural optimization theory tries to rationalize this methodology for several reasons. The main one is to reduce the design time, specially for repetitive projects. It provides a systematized logical design procedure and yields some design improvement. It tries to avoid bias due to engineering intuition and experience. It also increases the possibility of obtaining improved designs and requires a minimal amount of human-machine interaction.

There are however some limitations and disadvantages like the increase in computational time when the number of design variables becomes large. Also the applicability of the analysis program is limited to the specified purpose to which it was developed.

Conceptual errors and incomplete formulations are frequent. Optimization results are often misleading and should always be examined. Most optimization algorithms have difficulty in dealing with nonlinear and discontinuous functions so care must be provided when formulating the design problem. Another factor is that it is rarely guaranteed that the optimization algorithm will lead to the global optimum design, yielding most of the times local optimum points.

For the above reasons the suggestion that the word optimization in structural design should be replaced by design improvement is a better expression to materialize the root of this structural area (1). The recognition that it is a convenient and valuable tool to improve has been increasing in the structural design community. It must however be noted that expectations to obtain the absolute best design will probably lead to a series of unsuccessful attempts.

Methods

Researchers have developed considerably the techniques of optimum design in the last twenty five years. Research and exploration of these methods were mainly developed in the aeronautical and mechanical industries where the repetition of designs, for instance, created the need to search for more economical and efficient final products. More recently, with the availability of increasing computer capabilities civil engineering researchers and designers increased their participation in structural optimization following the lines defined by the other engineering disciplines. Optimization methods are however common to the different areas and are mainly divided in two groups. These are commonly known by the names Optimality Criteria and Mathematical Programming (6). Another area in structural optimization researched by a few scientists is based on duality theory concepts that is an attempt to unify these two other methodologies (7).

Optimality Criteria theory is based on a iterative approach where the conditions for an optimum solution are previously defined. The concept is used as the basis for the selection of a structure with minimum volume. This methodology derives from the extreme principles of structural mechanics and has been limited to simple structural forms and loading conditions. The formulation is the following:

$$\underline{x}^{K+1} = \varphi(\underline{x}^{K}, \underline{u}^{K+1})$$

where $\underline{x}$ is the vector of design variables, $\underline{u}^{K+1}$ is an estimative of lagrangian multipliers and φ is an adequate recurrence relation. Estimation of the lagrangian multipliers is made using the active constraints, those inequality or equality constraints with value close to zero. Recurrence relation φ and lagrangian multipliers represent the necessary conditions for optimality known as Kuhn-Tucker conditions.

On the other hand the Mathematical Programming group establishes an iterative method that updates the search direction. It seeks the maximum or minimum of multivariable function subject to limitations expressed by constraint functions. The iterative procedure may be defined as follows:

$$\underline{x}^{K+1} = \underline{x}^{K} + \alpha^{k} d^{K}$$

where α^{K} is the step size and d^{K} is the search direction. The search direction is obtained through an analysis of the optimization problem and the step size depends on designer experience. Methods of the second group may be divided in two areas. These areas are transformation methods, like penalty functions, barrier functions and method of multipliers, and primal methods, such as sequential linear and quadratic programming, gradient projection method, generalized reduced gradient and method of feasible directions.

TEE BEAM PROBLEM DESCRIPTION

The double tee design for the purpose of this problem is based on the allowable stresses theory. The purpose of this phase of optimization of the design intends to make an approximate determination of section dimensions, prestressing force and sag of prestressing tendons at mid and end sections based on the acting flexural forces. It does not include any verification of shear and torsion. Other details like the reinforcement, verification of shear transfer between the concrete topping and the prefabricated beam and the compliance with ultimate flexural analysis are not performed. The designing rules for allowable stresses are those prescribed in the Building Codes Requirements for Structural Concrete (ACI 318-95). The materials used are concrete strength (f_c') of 7000 psi and prestressing steel of seven wire strand tendons Grade (f_{pu})= 270 ksi. The concrete at time of prestress transfer has a minimum f_{ci}' of 5000 psi. The external loads considered are 50 psf of dead load and 122.8 psf of live load. The weight of double tee beam and of the two to four inches topping are considered as self weight in addition to the referred loads. The span of the simply supported beam is 41 feet. The prestressing tendons profile are linear with constant eccentricity. The percentage of losses in relation to the prestressing force at transfer is estimated as 15% admitting adequate curing of the precast elements.

TEE BEAM OPTIMIZATION PROBLEM

The variables considered for this problem are

X1 - prestressing force
X2- cable eccentricity
X3 - beam height
X4 - flange thickness

The height of the prefabricated beam is variable and between fifteen and twenty six inches. The value of the eccentricity is limited by the values imposed by the ACI. Web and flange dimensions have upper and lower limits. Web width is fixed at 7.75 inches at the flange interface. The total flange width is 94.5 inches.

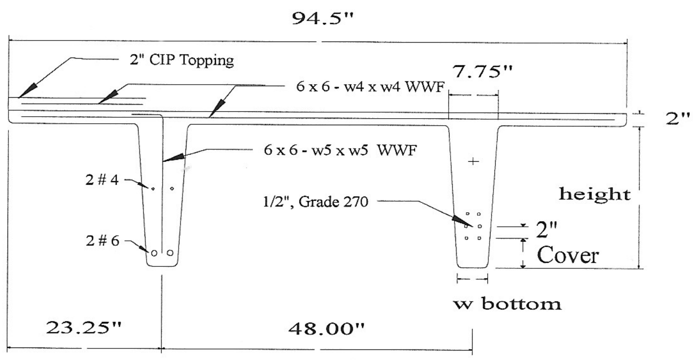

Figure 1. Tee beam section

The objective considered to evaluate the different designs and to find the optimal solution is defined by a function that minimizes the height of the double tee beam, including the prefabricated element and the concrete topping poured in a second phase. Then the optimization problem is to minimize the objective function and respecting the constraints expressing the dimensional constraints and the code regulations.

These limitations imposed six dimensional constraints to the optimization problem. The constraints considered were those imposed by the geometric definition of the section and by the ACI limits of the allowable stresses. The limits dictated by the ACI are verified at the top and bottom of the section for the compressive and tensile stress limits. The verification is made for the end and mid sections at the following stages

a) transfer of prestress for the prefabricated beam;
b) superimposed dead load for the composite beam;
c) superimposed dead plus live load for the composite beam.

Constraints imposed by the limits of the allowable stresses create a set of twelve constraints that are complemented by the other six. The mathematical model is defined by

$$\text{Minimize } F(X)$$

$$\text{subject to}$$

$$g_i(X) \leq \text{limit}$$

Mathematical Model

The objective function is

Minimize F(X) = X3

The first six constraints (dimension limits) are

$g1(X) = X2 \geq 0$ in
$g2(X) = X2 \leq X3 - 4.5$ in
$g3(X) = X3 \geq 15$ in
$g4(X) = X3 \leq 26$ in
$g5(X) = X4 \leq 4$ in
$g6(x) = X4 \geq 2$ in

The twelve constraints stating the limits for compressive and tensile stresses during the three phases at the end section and mid span section are expressed by

A) Precast section - Transfer

End of beam

$g7(X) = (-P/A + (P e)/wt) \leq 424$ psi Top
$g8(X) = (-P/A - (P e)/wb) \geq -3000$ psi Bottom

Mid span

$g9(X) = -Mb/wt + (-P/A + (P e)/wt) \leq 212$ psi Top
$g10(X) = Mb/wb + (-P/A - (P e)/wb) \geq -3000$ psi Bottom

B) Dead load on composite beam

End of beam

$g11(X) = (-RP/A + (RP e)/wt) \leq 502$ psi
$g12(X) = (-RP/A - (RP e)/wb) \geq -3150$ psi

Mid span - approximate

$g13(X) = -Md/wtc + (-RP/A + (RP e)/wt) \geq -3150$ psi
$g14(X) = Md/wbc + (-RP/A - (RP e)/wb) \leq 502$ psi

C). Dead plus live load on composite beam.

End of beam

$g15(X) = (-RP/A + (RP e)/wt) \leq 502$ psi
$g16(X) = (-RP/A - (RP e)/wb) \geq -4200$ psi

Mid span - approximate

$g17(X) = -Mdl/wtc + (-RP/A + (RP e)/wt) \geq -4200$ psi
$g18(X) = Mdl/wbc + (-RP/A - (RP e)/wb) \leq 502$ psi

where

Mdl - moment at mid span due to beam, concrete topping, dead load plus live load (function of X3 and X4);
Md - moment at mid span due to beam, concrete topping plus dead load (function of X3 and X4);
Mb - moment at mid span due to beam weight (function of X3 and X4);
wt - section modulus of the top section (function of X3 and X4);
wtc - section modulus of the top composite section (function of X3 and X4);
wb - section modulus of the bottom section (function of X3 and X4);
wbc - section modulus of the bottom composite section (function of X3 and X4);
P - prestressing force (X1);
R - loss of prestress 15%;
A - area of concrete section (function of X3 and X4);
e - eccentricity of group of tendons;
f_c' - concrete compressive strength = 7000 psi;
f_{ci}' - concrete compressive strength at transfer = 4000 psi;

STRATEGY ADOPTED AND SOLUTION

The problem is nonlinear for the objective function and for the eighteen constraints. The strategy adopted to solve the problem is based on using the Sequential Quadratic Programming (SQP) method (8). The aim is to transform the problem into an easier sub-problem which can be solved and used as the basis of an iterative process.

This method allows that at each iteration an approximation is made for the Hessian of the Lagrangian function using a quasi-Newton method (9). For that reason the function is approximated in the neighborhood of the design point by a quadratic function. An overview is found on M. J. D. Powell, "Variable Metric Methods for Constrained Optimization", Springer Verlag, 1983. The software program used is the Optimization Toolbox of MATLAB produced by Mathworks, Inc., 1993.

The results of the mathematical model solution are

X1 = 310 kips at transfer
X2 = 13.91 in (between center of gravity of composite beam and center of prestress force)
X3 = 23.8 in (height of precast beam)
X4 = 2 in

The stress constraints are

g7 = 428 psi
g8 = - 3056 psi
g9 = 37 psi
g10 = -2209 psi
g11 = 354 psi
g12 = - 2428 psi
g13 = - 221 psi
g14 = - 887 psi
g15 = 354 psi
g16 = - 2428 psi
g17 = - 724 psi
g18 = 446 psi

CONCLUSION

All constraints verified limits within a range of less than 1%. As a consequence of these case studies the definition of section types for the prefabricated elements the software available could be used without modifications and using constraints that reflect the designing steps taken in a procedure without the optimization phase. Time spent to make these improvements was small when compared to the complete design operation. The apparent difficulty for a prestress designer seems to be the lack of understanding about the optimization methods available and of the respective performances. It is necessary then to promote continuing professional development courses in optimization so that a wider use of optimization tools is a characteristic of current design. This is mostly important for elements produced in a large scale or for important designs. Engineering needs development and for that purpose certainly optimization plays an important role.

REFERENCES

(1) - U. Kirsch, Optimum Structural Design, McGraw-Hill, New York, 1981.

(2) - Brandt, A. M., Criteria and Methods of Structural Optimization, Warszawa/Martinus Nijhoff Publishers, The Hague, 1984.

(3) - Spillers, W. R., Iterative Structural Design, North-Holland Publishing Company, Amsterdam, 1975, Appendix A.

(4) - Magnel, G., Prestressed Concrete, McGraw-Hill, New York, 1954.

(5) - Schmit, L. A., Structural Design by Systematic Approach, Proceedings of the Second National Conference on Electronic Computation, Structural Division of ASCE, Pittsburgh, 1960.

(6) - Morris, A. J., Foundations of Structural Optimization:a Unified Approach, John Wiley and Sons, New York, 1982.

(7) - Fleury, C., and Geradin, M., Optimality Criteria and Mathematical Programming in Structural Weight Design, Computers and Structures, Vol. 8, no. 1, 1978, pg. 7-18.

(8) - Haftka, R. T., and Kamat, M. P., Elements of Structural Optimization, Martinus Nijhoff, Amsterdam, 1985.

(9) - Vanderplaats, G. N., Numerical OptimizationTechniques for Engineering Design, McGraw-Hill, New York, 1984.