Visualization of electric field lines in an engineering education context

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***Abstract*— The electromagnetic theory presents a unifying explanation of electric and magnetic phenomena underlying our technological society. It is a fundamental physical theory taught in engineering schools at university level. In this theory the electromagnetic field is a vector field permeating space. An important aspect relating to students difficulties and misconceptions is the difficulty in visualizing vector fields. With the goal of enhancing student understanding and studying student engagement we have developed high quality 3D visualizations of electromagnetic situations. These make use of accurate computation of the field lines, together with realistic rendering using the open source software Blender. We present examples of electrostatic situations with both an assessment of the student understanding and an evaluation of the students' perceptions of the importance of the visualizations. Complex interplay between visualization specific issues and the abstract notion of the field is identified in the students' conceptions. It is found that the visualizations are not used as substitutes of other learning resources. They are perceived as allowing a quick access to content and prompting motivation. The adequacy of the visualization to the subject content as well as the capacity to use it as self-assessment is valued by the students.**

Keywords—electric field, 3D visualization, students' beliefs and misconceptions, students' attitudes and perceptions

I. Introduction

A field is a scalar, vectorial or higher dimensional (e.g. tensor) quantity that is defined in every point of space. Fields are central quantities in engineering: the temperature distribution is a scalar field; the velocity of a fluid is a vector field; the stress distribution a tensor field. In this paper we focus on the electric field, in the context of a "Physics II" course. This course (with different designations) takes place in all engineering degrees in our School of Engineering, typically in the 1st semester of the 2nd year. For many students it presents the first systematic contact with the concept of vector field and with the difficulties in understanding this abstract concept and in visualizing it. This difficulty is enhanced by an incomplete understanding of the notion of vector. The visualization dimension is very important in engineering education, where the vast majority of engineering students are visual learners (e.g. [1]).

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The visualization of fields is a research topic in the computing sciences; however we focus on a specific kind of visualization – the field line, and its interaction with student learning. The field line concept is introduced in pre-university physics education and ubiquitous in university physics textbooks. There are more advanced visualization concepts but their introduction often requires costly software packages, the climbing of usability learning curves and adaptation to new visualization concepts.

A. The electrostatic field and field lines

The electrostatic field in free-space is completely described by Maxwell's equations for electrostatics. These prescribe the rotational and divergence of the electrostatic field ($\vec{E}$)

$$\vec{\nabla} \times \vec{E} = \vec{0} \qquad (1)$$

$$\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0 \qquad (2)$$

where ρ is the volume density of electric charge and ε_0 the permittivity of free-space. The situations presented in this paper consist of charge distributions with symmetry. The spatial regions considered for the electrostatic field are void of charge.

One situation is that of a thin homogenous sheet of positive charge coplanar to the xy plane. The planar and axial symmetry of the situation implies that $\vec{E} = E(z)\hat{k}$. Furthermore, outside the sheet no charge exists and the divergence is zero. This implies that the field is constant, more exactly

$$E_{\text{plane}}(z) = \begin{cases} E_0, z > 0 \\ -E_0, z < 0 \end{cases} \qquad (3)$$

where E_0 is a constant.

Another situation is that of an infinite cylinder of charge, with a charge density that is function only of the cylindrical radius r. This situation has cylindrical symmetry and the field is of the form $\vec{E} = E(r)\hat{r}$. Outside the cylinder no charge exists and the field has zero divergence. This implies that in this region

$$E_{\text{cylinder}}(r) = E_0 \frac{r_0}{r} \qquad (4)$$

where E_0 is the electrostatic field at the surface of the cylinder of radius r_0.

The third situation is that of a spherical charge distribution, with a charge density that is function only of spherical radius r. The spherical symmetry of the situation implies that $\vec{E} = E(r)\hat{r}$. In the region where no charge exists the field has zero divergence. This implies that the field has the form

$$E_{\text{sphere}}(r) = E_0 \left(\frac{r_0}{r}\right)^2 \qquad (5)$$

where E_0 is the electrostatic field at the surface of the sphere of radius r_0.

Once the electrostatic field is known the field lines are computed as tangents to the field. In the case of a 3D Cartesian electrostatic field they are the solutions of the differential equations

$$\frac{dx}{E_x(x,y,z)} = \frac{dy}{E_y(x,y,z)} = \frac{dz}{E_z(x,y,z)} \qquad (6)$$

For the symmetrical situations presented previously the field points in the direction of one of the coordinates' versors and is only function of that coordinate. Therefore the field lines are straight lines with an intensity given by Eqs. 3-5. These equations were used to implement the field lines.

A further set of rules is required to implement the field lines, the typical rules are: a) the field lines sources are positive charges and the sinks the negative charges; b) the lines must depart/arrive with a uniform angular separation from a point charge; c) the number of field lines departing from a point charge must be proportional to the charge magnitude; d) the density of field lines is proportional to the electric field intensity; e) at large distances of a charge distribution with non-zero total charge the field lines behave as a for a point charge.

B. *Previous research on student understading of 2D electric field lines*

The 2D representation of electric field lines was previously addressed in the literature.

[2] found confusion between the field line and vector concepts, with some students drawing curved vectors. Difficulties in the connection between electric field and electric force were pointed out. The incapacity to imagine possible complex field distributions, in a region where the field was only known in one point, was underlined and explained with the simplistic situations found in textbooks. The importance of making the transition from a vector field to a field line was pointed out and the suggestion made to present both representations. It was found that some students give a physical nature to the field line and don't comprehend it as a visualization of the real physical quantity – the electric field.

[3] referred mistakes and lack of physical realism in the field line drawings of simple charge distributions in university physics textbooks.

The most important critique is the one of [4], who presented the fundamental limitations of 2D diagrams of electrostatic field lines, which are 3D by nature. Except in very simple situations, can the field line density encode field line strength. This issue is related with the 2D nature of the diagrams. Another important effect is related to the anisotropic clumping of field lines around charges. Field lines must depart/arrive isotropically from/to a point charge. In configurations of several charges it is impossible to maintain isotropy if a 2D representation is chosen. Finally, it was found that in configurations of charges with zero total charge, open field lines would be present, suggesting a non-zero total charge. The previous arguments where extended to non-electrostatic situations by [5].

[6] studied students understanding of the electric field in several aspects: definition, source of the field, representation and superposition. They found that students, prior to university instruction, were incapable of representing electric fields, in spite of having pre-university physics instruction. No correlation between grade and electric field mental construct was found. A persistent difficulty in the representation of electric fields was found, with students confusing electric fields with equipotentials and failing the representation in when multiple charges where present.

The current state of the art of field visualization is that of the MIT TEAL programme [7]. Most of the field line visualizations are 2D using the line integral convolution technique. However, some visualizations are 3D (related to the magnetic field and induction). The authors introduced visualizations in an active learning environment and found a gain in overall student achievement in Electromagnetics when compared with a control group.

II. THE IMPLEMENTATION OF 3D ELECTRIC FIELD LINE VISUALIZATIONS

3D visualizations of the electric field are generally implemented in free software allowing interactivity. They follow two broad classes: vector implementation (e.g. [8]) and field line implementation (e.g. [9]). Our approach was to implement the visualization in Blender allowing for high quality rendering as a first step in the development of more complex and interactive solutions.

A. *Blender implementation*

Blender [10] was chosen to make the 3D visualizations of the electric fields since it is free and open-source software for 3D modelling and animation, with a physics based render engine (Cycles).

Having the possibility to write scripts in Python allows the user to parameterize the creation of geometries making it easier to analyse different scenarios. The implementation of the Open Shading Language (OSL) in recent versions, allows the use of mathematical functions to define material properties as a function of other geometrical properties as surface position, orientation, normal vector, among others. Blender also has the possibility to create interactive applications, either by using its internal game or by exporting its 3D models to third part solutions.

B. *The three electric field situations*

Three different scenarios were analysed, representing the electric field created by three positive charge geometries: a sphere, an infinite height cylinder and an infinite plane. As shown in Section I.A, the symmetry of the scenarios translates in straight electric field lines. The field lines are distributed uniformly on the surfaces.

For the sphere, the uniform distribution of the lines along the surface is a non-trivial problem, as already referred by [4]. A solution common to the Thompson problem was used to compute uniformly distributed points and their respective coordinates on the sphere [11]. From those points, the field lines were then extruded perpendicular to the surface and converted to thin cylinders in order to have some thickness. To complete the electric field lines representation, the direction must also be represented. In this case we used a cone in each line, placed at a fixed distance from the surface and with its vertex pointing along the electric field direction. In order to represent the electric field intensity, an emission shader was used in the lines material. The emission intensity being proportional to the electric field intensity of the sphere E_{sphere} (Eq. 5). An image of the electric field of the sphere is represented in Fig. 1. An animation was also created for better 3D perception of the field and is presented at [12].

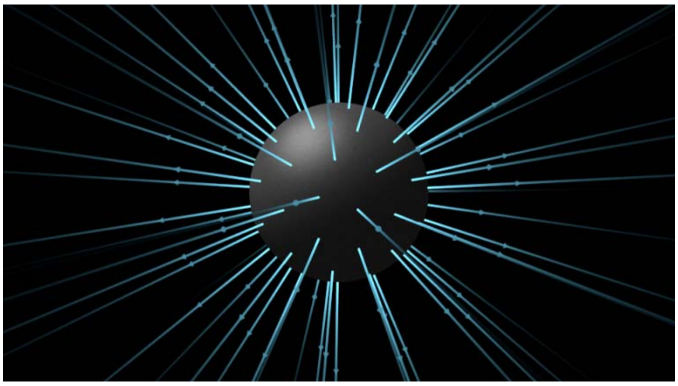


Fig. 1 Representation of the electric field of a positively charged sphere, the full animation is available at [12]

For the cylinder scenario, the same approach as for the sphere was followed, but using instead E_{cylinder} (Eq. 4). An image of the cylinder electric field is presented in Fig. 2, with the respective animation available at [13].

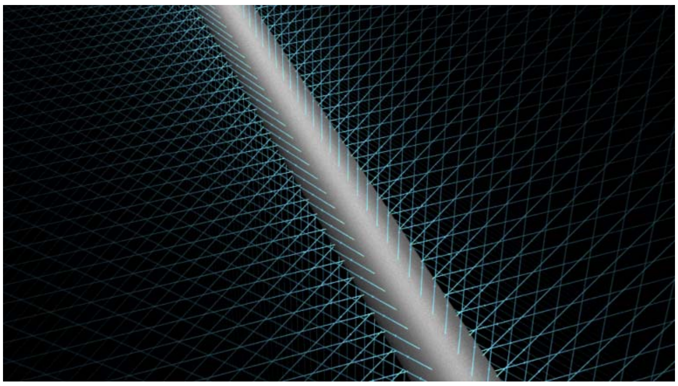


Fig. 2 Representation of the electric field from a positively charged infinite cylinder, the full animation is available at [13].

The plane is physically the simplest case, with constant electric field intensity (cf. Eq. 3) and all the lines parallel to each other. However it represents a challenge to be nicely displayed due to the absence of emission fading and to visual effects created by the distribution of the lines. A sample image is presented in Fig. 3 and the animation can be obtained at [14].

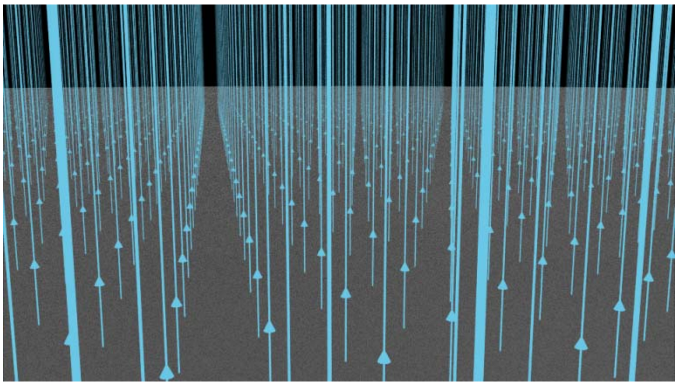


Fig. 3 Representation of the electric field from a positively charged infinite plane, the full animation is available at [14].

C. Lessons learned from 3D field implementation

During the implementation several issues had to be overcome that were not trivial at the start of the work. One had to do with how to represent the electric field intensity along the line. This is the major limitation of a previous monochromatic display of field lines where the field intensity is encoded in line density. Initially a colour map from red to blue was implemented but soon abandoned due to concerns with confusion with electric polarity. Other multi-colour codes of the field intensity were not implemented as some sort of colour bar was required to pass the information of field intensity. A single colour solution where a brightness shader is proportional to the electric field intensity was selected as the most simple and easy to understand solution by the students. But with that single brightness shader, as the electric field intensity decreases, the lines would tend to be a dark solid. Consequently in the 3D display, the lines in front of the camera would block the visualization of other lines. To overcome this problem the material also used a transparency shader which is inversely proportional to the electric field intensity. As the electric field decreases, the transparency increases, creating a fading effect as the lines get further away from the surface. This solution was applied to the sphere and cylinder but not to the infinite plane scenario, where the field is constant. Another point is related to the thickness and number displayed field lines. A balance between avoiding cluttering the visualization and presenting a large number of lines had to be found, requiring many trial and error implementations. A further aspect was to avoid the formation of patterns due to field line cluttering – such a limitation is still present in the plane visualization.

Another issue had to do with the charges surfaces. Initially colour versions were implemented but they distracted the viewer from the field lines. Specific colours could also confuse the student with regards polarity or conducting nature of the material. A dark grey colour was chosen as it emphasized the bright colour field lines and provided a soft transition from the black background. With regards to the charge surface material, as it could not reflect the light emitted by the lines, the charged surfaces and the surface illumination were rendered in a separate layer and then combined with the lines in post processing. The illumination of the charged surfaces is important to perceive the surfaces curvature. Furthermore the fixed light sources illuminating the charges are critical to create the perception that the charges are in a fixed position and it is the camera that is moving, since no other visual information is given by an uniform dark background.

The camera movement was another important aspect. An elliptical movement, approaching and distancing from the charge distribution was chosen. This motion is simple and allows looping. It probes different spatial scales. The camera path ellipse was tilted with respect to the cylinder and plane to enhance perspective. The camera speed along the elliptical path was chosen to be small when near the charge distribution and large far from it. This variable speed ensured a balanced visualization time, allowing for understanding, near the charged object, at intermediate distances and far from it.

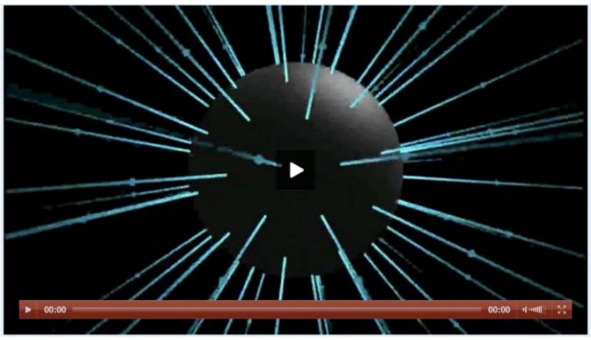
Finally, the perspective given by a 3D view is fundamental to perceive the 3D space and should be close to the human eyes field of view – avoiding on one hand the extreme distortion that can be achieved by a wide field of view angle and on the other, the undistorted practically orthographic view with very small field of view angles.

III. ANALYSIS OF STUDENT INTERACTION WITH THE 3D FIELD LINE VISUALIZATION

In this section an analysis of the student interaction with the visualization is presented. The measurement of the student interaction consisted in a concept question illustrated by the visualization followed by a questionnaire to probe the students' perceptions. The measurements were made in a "Physics II" course four months after completion of the course. The questionnaire was opened by 40 students of the course, but only 24 students submitted the concept question answers and 23 the perception questionnaire answers. The implementation was in the Moodle server of the course, each student answering the questions after login.

A. Concept question

The visualization of the electric field from a spherical charge distribution was presented to the students as a video illustrating a multiple-choice concept question (cf. Fig. 4). The multiple choice concept questions require the students to identify the wrong sentence from five different possible choices.



The video above illustrates the electric field created by electric charge distributed on sphere. The sphere is fixed in space and a camera moves very slowly around it. Which of the following sentences is false?

a) In the points crossed by the blue lines, the electric field vector is parallel to the lines.

b) The charge of the sphere is positive.

c) The charge is distributed homogeneously in the sphere surface.

d) The electric field intensity is zero, at a distance starting at a few tens of sphere radii, from the sphere surface.

e) The electric field intensity decreases with the distance to the sphere surface.

Fig. 4 Concept question and frame from the ilustrating visualization of the 3D electric field from a sphere. The full video is available at [12].

B. Concept question results and discussion

The presented statistics are only for the students who submitted an answer to the question.

The (a) choice probes the identification of the electric field direction as parallel to the field line, it is a true sentence. However, 13% of the students identified it as false. This can be explained by an incorrect understanding of the field line concept or by interference from other electromagnetics concepts: a) the magnetic force is perpendicular to the magnetic field line; b) the magnetic field is perpendicular to the lines of current. The (b) and (c) choices, which are true, have the lowest identifications as false – 8%. With regards the (b) choice, the arrow in the field lines identifies the field direction. In our visualization the arrows point outwards and field is of a positive charge. These answers could trace an incorrect understanding of the field line concept. On the other hand visualization specific issues are not to be excluded, namely: a) the incapacity to identify the cone direction in the line visualization; b) association of a negative charge to the dark sphere; c) association of a negative charge to a blue colour of the field line. The answers that assumed that this choice is true do not mean that the students understood its meaning – a "blind" answer assuming a "default" positive charge situation is not excluded. With regards the (c) choice, the answers that assumed it as false could point to the assumption that the field lines are emanating from specific charged points at the sphere. On the other had a "blind" true assumption could be related to the fact that all the problems worked out in class assumed the spheres to have homogeneous charge distributions. The answer could also be visualization related – the sphere has a homogeneous pattern. The (d) choice is the false statement in the concept question. It was correctly identified by 58% of the answers. In contrast with the other choices it refers a more abstract concept, namely that the field decreases to small values far from the charge – but not to zero. In the video visualization, far from the charge, the field lines are not visible anymore due to a combination of transparency and intensity. The students that answered correctly had to recall this abstract notion and identify the limitations of the visualization. The frequency of selection of the choice (d) correlated with the final grade in the course. Finally choice (e), which is true, was identified as false by 13% of the students. This choice simply states a decrease of the field with distance but was selected by students with a high final grade. We speculate that they might have been diverted by the video of the visualization.

C. Perception questionaire

Following some literature (e.g. [15]) on the evaluation of educational resources the following three dimensions were identified to evaluate the video visualizations: D1) learning readiness and motivation; D2) adequacy to content and learning objectives; D3) self-assessment and promotion of learning awareness. These dimensions are present in the eleven questions of the perception questionnaire. The possible answers are in an integer scale of 1 to 7, with 1 meaning no relevance/importance and 7 meaning very relevant/important. The questionnaire was designed to be answered in 3 min. in order not to discourage its filling. The questionnaire is presented in Table I

TABLE I. PERCEPTION QUESTIONNAIRE

N.[a]	Question
Q1-D1	Watched this video because it seemed a quicker way to access course content.
Q2-D1	Watched this video to save time from studying from other sources.
Q3-D1	With this video I realized very clearly the subject it dealt with, which had not happened before.
Q4-D1	Watched this video because I believe that its content will be somehow evaluated.
Q5-D1	The video presented the subject in a challenging way, prompting me to know more about the subject.
Q6-D2	The video was adapted to the learning objectives of the subject.
Q7-D2	The video was adapted to my level of knowledge.
Q8-D2	The video was a clear example of the concepts that it was supposed to demonstrate.
Q9-D3	The video addressed the essential concepts that were supposed to be known in this subject.
Q10-D3	Videos like this allow us to think critically and reflectively about applying these concepts, in other contexts or other courses.
Q11-D3	This video allowed me to assess the knowledge I have on the subject.

[a] Question number (e.g. Q1) and dimension sampled (e.g. D1).

D. Perception questionaire results and discussion

The perception questionnaire data is presented in Table II. The variable homogeneity was tested with the Kaiser-Meyer-Olkin adequacy test which yielded 0.503 and the Bartletts' test of sphericity with a significance of 0.006. A factorial analysis with varimax rotation yielded that the three dimensions explain 63.9% of the variance: D1 with 26.6%, D2 with 23.6% and D3with 13.6%. The Cronbach's α are: 0.756 for D1, 0.801 for D2 and 0.493 for D3, suggesting internal consistency in the first two dimensions.

TABLE II. PERCEPTION QUESTIONNAIRE DATA

N.	Minimum	Maximum	Mean	Std. deviation
Q1-D1	1	7	4.68	1.56
Q2-D1	1	6	2.68	1.86
Q3-D1	1	7	3.65	1.64
Q4-D1	1	7	3.91	1.88
Q5-D1	2	6	4.17	1.27
Q6-D2	3	7	5.22	1.09
Q7-D2	4	7	5.43	0.99
Q8-D2	2	7	5.22	1.45
Q9-D3	2	7	4.73	1.58
Q10-D3	1	7	4.26	1.63
Q11-D3	3	7	5.48	1.12

The two questions in the dimension D1 (learning readiness and motivation) with highest students' perception of relevance are Q1 and Q5 dealing with speed of access to content and desire to know more. These items also have the lowest standard deviation. With regards to dimension D2 (adequacy to content and learning objectives) all questions have a high students' perception of relevance score and low standard deviation. With regards to dimension D3 the scores are lower than for D2 and higher than D1. Question Q11 had the highest score.

The students valued the video visualization in function of its adequacy to the subject content. Furthermore a stimulation of the students' self-assessment capacity is apparent. The characteristics valued by the students in video visualizations are related to the following four aspects/questions: a) adequacy to the learning objectives (Q6); b) adequacy to the level of knowledge (Q7); c) clear example of the concepts that it was supposed to demonstrate (Q8); d) self-assessment (Q11).

The best course students are a good evaluation panel of the video visualization.

IV. CONCLUSIONS AND FURTHER PROSPECTS

In the paper high quality visualizations of the electrostatic field in simple situations were presented. These visualizations are a first step towards more complex and interactive settings. Implementing visualizations that not to divert and confuse the students is difficult and severe issues were identified. Results of the student interaction with a specific visualization are presented. It is found a complex interplay between students' misconceptions and the visualization. The students value the visualization as a mean to gain fast access to content and as increasing the motivation. The visualizations should be adapted to their knowledge. The importance of the visualization for self-assessment is underlined but it could arise as being presented in a concept question. Interestingly videos are not used as substitutes of other learning resources or other duties of the learner.

Developments include the creation of matrix concept questions probing in more detail the students' physical understanding of the visualization. As the concept question was answered after the student completed the course we cannot probe the importance of visualization in student understanding. The comparison with a control group will be done in the next year.

With regards to the visualizations themselves more complex situations will be implemented and subject to evaluation. An interesting application, although requiring complex computations, would be the visualization of the electric fields in printed boards or electromagnetic compatibility. A limitation of the visualizations is related to the Cycles engine used in Blender which currently does not allow for embedding and spatial manipulation of the scene as e.g. in Sketchfab [16]. To overcome it we had to create an animation. Allowing the user to physically change the scene (by changing the charge values, or adding other charges) would be ideal but is currently impossible given the rendering times.

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