NON-LINEAR TRANSPORT MODEL FOR ROD MILL SIMULATION SCALE UP OF BATCH KINETICS

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ABSTRACT

In the first part of the paper, a model aiming at describing the well-known size-dependent transport that occurs inside the charge of continuous flow rod mills is presented. The mechanism of selective transport is simulated as a time-dependent process by a non-linear classification function, in which the classification phenomenon is described as dependent on size composition of mill hold up. The explicit dependence on the time variable defines the model as belonging to the class referred to as *distributed parameters models*, in opposition to the traditional lumped parameter type models, based on residence time distributions (RTDs) only suited for simulation of steady state flow. The model is led by a set of condensed parameters correlated with operational variables.
A methodology for experimental evaluation of those parameters from size analysis of the mill products and starting from a batch comminution kinetics previously calculated, is presented. On account of the non-linear character of the model, the samples for model fitting must be collected along a transient period as well as from the steady state.

It is theoretically shown that the feasibility of the parameter evaluation procedure is only guaranteed by the existence of the dynamic model for transport simulation. It is also proved that the access to data from a transient period is a necessary condition to allow the convergence of the algorithm.

INTRODUCTION

The evolution of models for simulation of the comminution process, from batch to continuous flow, a necessary step for their embedding into the algorithms for circuit simulatoion and control in the industry, has up to now been achieved by means of the concept of Residence Time Distribution (RTD).
The option for this type of solution to the problem of the transport mechanism that occurs inside the mill obliges a previously choice of a model in which the time becomes implicit. In fact, those models are called lumped because they correspond to a broad description of the phenomenon, previously postulated, and do not allow for a time dependent process description. Being so, models based on RTD's are only suited for steady state simulation.

This paper reports some results already achieved by a research project (BAPTISTA, 1994) in which the central idea is to develop phenomenological models for describing rod

mill comminution kinetics in terms of non-linear interferences of some size classes on the kinetic reduction of the others and the transport along the mill as a size dependent process. According to some previous work on this theme, we are interested in the development of a model driven methodology that will be able to identify the information structure that underlies the set of experimental data.
Obviously, this type of non-linear phenomenology we are particularly interested in, is not accessible by means of a steady state approach. In fact, under these conditions, all the competitive and/or cooperative interferences between different size classes will be hidden on account of the postulated invariability of size that characterizes this sate. Thus, *only the data collected during a transient period are able to show the relevant information that supports the phenomenological identification we are looking for.*

In terms of main objectives, this paper aims at presenting a phenomenological model that fits to the well-known size dependent transport mechanism that occurs inside wet rod mills. Starting from such a model, a methodology for evaluation of both mill comminution and transport kinetic parameters, directly from data collected in an industrial plant is proposed.

The methodology for parameter evaluation is based on a back calculation procedure, supported by the powerful non-linear Levenberg-Marquardt optimization algorithm. In the background, models of comminution and transport, both presented as time dependent processes, are at work.
However, the use of phenomenolgical models for comminution and transport, instead of the traditional approach based on RTD's, will rapidly lead to very complex description, with a large number of degrees of freedom and, particularly, to a great instability for the solution.
Our experience has demonstrated that the problem presents several local minima and that there is a negligible chance of access to the region of the principal minimum.
Our point of view about this matter is that we can reach the end point if two principal conditions are guaranteed:

- first of all the models we are going to use should have a good chance to justify the data that will be collected;
- data must include information, not only about the steady state but, necessarily, from a transient period expressive enough to reveal the non-linearities we are interested in.

On the other hand, having in mind the complexity of the problem, the chance to obtain a good solution depends on the possibility of fine tunning optimization procedure itself, using a method of successive approximations in which intermediate solutions are reached starting from simple models; this can be resumed as follows:

- evaluation of comminution kinetic parameters from batch tests as a first approximation to the order of magnitude of the their real values in the continuous mill; with this procedure we expect to obtain values for the breakage function quite close to the real ones;
- to make a good guess for the mill size composition; this could be obtained by:
  i) sampling the charge along the mill (which is a very difficult task in an industrial mill !);
  ii) starting the mill with a known size composition, or with the mill empty of ore;
  iii) to fit in advance a lumped model of RTD (e.g., the standard model of three perfect mixers in series, one big unit followed by two small ones) in order to obtain the previous guess;

- *accepting the batch comminution kinetics, or even the kinetics obtained from* the model of RTD above referred, as a good guess for starting the optimization procedure, the non-linear transport model is fitted to the experimental data collected along the transient period and the steady state as well;
- the stability of the solution increases with the amount of available data from the transient period; that is, the probability of reaching the end point depends on the number of intermediate size compositions of the transient, to which the model is fitted.

ROD MILL NON-LINEAR TRANSPORT MODEL

In rod milling, the effect of size classification that occurs inside the mill on account of the resistance to the transport operated by the charge is well known and profusely referred by the literature.
This resistance is a non-linear phenomenon because it is size dependent and can not be expressed as a single delay process. Models based on cascades of units of known RTD's, although they could offer in certain cases a sensitive prediction of the final results, can not be seen as final solutions. They are not effective solutions, but only simple black-box approaches. The grinding kinetics obtained, being dependent on the postulated transport, is as unreal as the transport model itself is.
The only way to ensure a good chance to calculate a real grinding kinetics is to develop a dynamic model for the transport mechanism, that includes, particularly, its selectivity on particle size.

To create such a phenomenological model, the mill is considered to be divided into several elemental volumes Δv. Particles held inside this volume work as an obstruction to the flow of the particles that are held in the anterior volume Δv. This obstruction, dependent on the particle size and pulp viscosity, can be expressed by a function we call ***PASSAGE FUNCTION***.
As the comminution is going on and the size composition of each elemental volume is becoming smaller, the Passage Function varies along the mill towards the end side.

For describing the Passage Function we associate each mill segment to a classifier driven by a partition curve. However, the common occurrence of a non-negligible amount of large particles in the discharge mill product suggests that a clear effect of indiscriminate dragging in the upper size range must be included into the partition curve.

We will obtain the form of the partition curve as a function of the mill charge viscosity, this depending on the pulp solids density, while the mill throughput will be decisive for determining the partition curve's cut size of each elemental volume.
The above referred partition curve represents the probability of a particle leaving the elemental volume in the next time interval, and is associated with each size class present in the mill holdup. Accordingly, we call this partition curve the ***filter function***.
We propose for the analytical form of this filter, $F_{j,i}$ a conventional classification function (Lynch, Rao, Plitt) having in the mind the similitude of physical significance of both processes:

$$F_{j,i} = 1 - \left(\frac{\exp\left(PF_i \times \frac{s_j}{PV_i^{50}}\right) - 1}{\exp\left(PF_i \times \frac{s_j}{PV_i^{50}}\right) + \exp(PF_i) - 2} \right) \qquad [1]$$

$F_{j,i}$ - filter function of elemental volume m_i;
s_j - particle size class;
PF_i - parameter that describes the shape of the filter;
PV_i^{50} - cut size (d_{50} of the elemental filter partition function.

Parameter PV_i^{50} can be read as an indicator of the average particle size that comes to the next elemental volume m_{i+1}, while PF_i is associated with the pulp viscosity.

Meanwhile, the direct application of the mass conservation principle obliges that in each time interval each elemental volume discharges the same mass of material that came into it from the previous segment in the anterior time interval. However it can happen that there is not yet enough material at the convenient size to be discharged, because the filter of the present volume is more restrictive, although this mass of particles has undergone a comminution event inside the present elemental volume. In order to overcome this situation, an indiscriminate dragging effect in the upper size range has been added to the *filter function*. That allows for an additional discharge of particles to satisfy the mass conservation principle. This effect, as a first approximation, is considered to affect proportionally all size classes. This mechanism can be seen as a specific non-linear size dependent process because is only demanded by the model in certain physical conditions.
This dragging effect is called a ***passage stabilizer***, Φ_i, and has a clear physical significance: it represents the volume percentage of the elemental volume m_i that is needed to submit to a classification process in order to obtain the total material of size composition required by the filter function that will pass to the next mill segment. When this stabilizer assumes a value greater than unity, the non-linearity comes into effect and an instantaneous discharge of non classified material occurs.

When we put together the filter function and the stabilizer, the ***passage function*** for the elemental volume m_i is computed as follows:

$$Q_j \cdot \Delta t = \sum_j \left[\left(F_{ji} \cdot \Phi_i\right) \cdot \left(L_{ji} \cdot V_i\right) \right] \qquad [2]$$

where,
V_i - is the mass content of the elemental volume i.
Q_j - is the mass flow of class **j** through the elemental volume m_i;
L_{ji} - is the size composition of the elemental volume m_i;

The mill size composition of the elemental volume, L_{ji}, computed by a non-linear comminution kinetic model proposed by LEITE (1990), is the linkage element of the comminution model with the model of transport.

Parameter Condensation

In order to complete de development of the transport model there is still a problem to be solved relating to the passage filter. In expression [2] the parameters required to calculate the model must be condensed to make the model more consistent and independent of the number of elemental volumes of the mill. To do so we propose the same shape factor, $PF_i=PF$, ***for*** the filter function of the different mill segments. For the successive cut sizes, PV_i^{50}, of each filter we use a linear relationship to correlate them all: the slope of this correlation is a parameter to fit upon the data; the known ordinate can be experimentally obtained from the average of the final mill product size composition.

When both models of comminution and transport are put together, the global model obtained is driven by:

- 4 parameters for the grinding selection and breakage functions, plus 1 more parameter if a non-linearity of the "umbrella type" is needed;
- 3 parameters to condense the transport passage function.

The total number of 7 parameters, that corresponds to the same number of degrees of freedom, although not too high for the powerful Levenberg-Marquardt optimization algorithm, makes for some numerical instability of the end point on account of the complexity of the phenomena we are trying to investigate. To overcome this behavior we propose the methodology above referred, which, as we will see in the final part of this paper, seems to be a way to make the scale up from batch comminution kinetics to the industrial scale.

EXPERIMENTAL SETUP

The laboratorial facility used includes a batch wet rod mill, 0.36 m diameter x 0.36 m long and a continuous wet rod mill 0.36 m diameter x 0.72 m long.
The material tested was crystalline quartz rock. The initial mill feed size composition was obtained from the discharge of a roll mill, of size between 20 mesh and 9 millimeters.

First of all, the material is tested for evaluation of batch comminution kinetics. The results can be observed in fig. 1.

The significance of each parameter is as follows:

- PA represents a global measurement of the velocity of grinding; it is the effective element of the selection matrix of the largest size class -- the very low value obtained by the back calculation procedure (0.0059 T^{-1}) means a slow kinetics (as can be expected for a quartz hard rock !);
- PK is a scaling parameter used to determine the other elements of the selection matrix, starting from the PA value -- the algorithm adjusted this parameter to the maximum possible, which means a tendency of the model to increase the velocity of grinding of the smaller size classes; this situation, clearly related with the very low value adjusted for PA value, suggests the existence of an insufficient nip angle for the largest particles present in the mill;
- PW and PG are shape parameters of an Harris function used to describe the breakage function -- the values adjusted make for a tendency of producing a big amount of daughter particles for the classes close to the parent particles from which they were issued by comminution; the shape of the breakage function can be seen in the fig. 2.

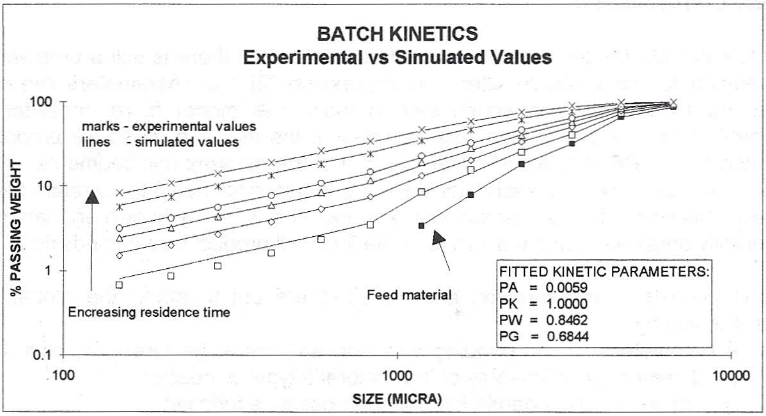

Fig. 1

In order to obtain data from a continuous flow transient state, we use the start up transient with the mill full with an hold up size composition similar to the feed material composition. In other circumstances an impulse-response technique of the pedestal type (rectangle function) acting on a specific size class, has also been used.

In this way, the standard experimental routine consists of a continuous flow grinding operation at constant solids feed rate and starts with a mill charge of known size composition. Water feed rate is also maintained constant.

When steady state is reached, a rectangle impulse for a small time interval (5 minutes) is generated: during this time interval the feed is totally or partially substituted by material all within the same size class.

From the beginning, at a constant time interval previously defined, samples of the final product are collected.

All the samples, including the feed material, are analyzed for determination of size composition.

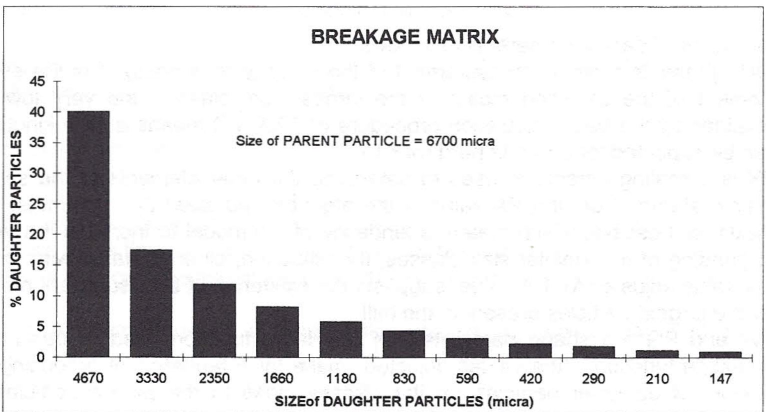

Fig. 2

The experimental results of the continuous flow grinding, since the beginning of the test until the final steady state after the initial start up transient period, can be seen in the fig.3.

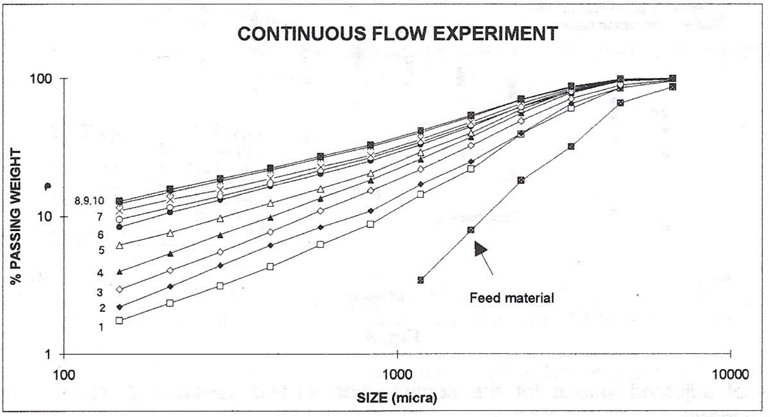

Fig. 3

As can easily be seen, samples numbered from 1 to 7 represent the size evolution during the transient, while samples 8, 9 and 10 are already quite close to the steady state flow. In the explanation we are going to present in the next paragraph, only data from these transient are used.

FITTING MODEL TO DATA
SCALE UP OF COMMINUTION KINETICS AT INDUSTRIAL SCALE

The global model of continuous flow presented in the first part of this paper, lead by a set of 7 parameters (4 for comminution kinetics + 7 for transport model), corresponding to 7 freedom degrees, is fitted to the cumulative size composition of the 10 samples collected. According to the developed methodology, we chose as starting guess for the optimization procedure the kinetic parameters fitted for the batch experiment. However, different guesses were used to test the stability of the solution. Generally speaking, the chance to reach the end point increases with the number of samples collected during the transient. Sometimes, guesses for the transport model too far between lead to different solutions; but in these cases, if we return to the guess of the kinetic batch comminution the solution is reached again.

In fig. 4, results of the fitting procedure are shown. The poor goodness-of-fit that is clear in the fine size range, although enhanced by the logarithmic scale, is related with some sampling difficulties at the beginning of the experiment and also with the initial large average size of the mill contents.

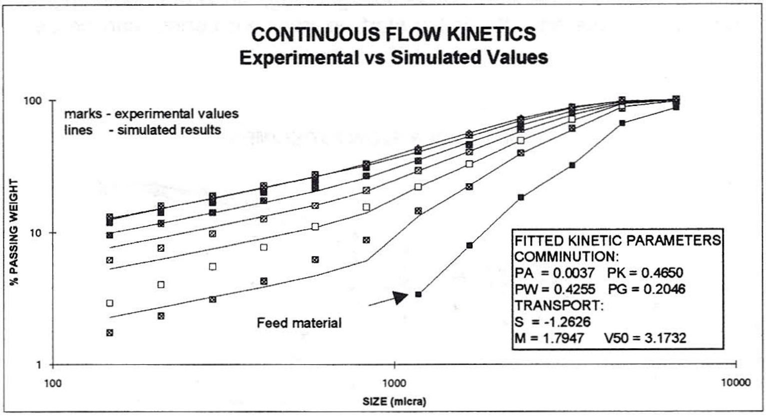

Fig. 4

In terms of adjusted values for the comminution kinetic parameters, some comments should be made:

- PA is 63% of the value calculated for the batch kinetics -- this decrease of the velocity of grinding is compatible with the permanent intake of feed material of the largest size class that is responsible for the lack of nip angle;
- PK fits to a small value -- this means that there is an evident decease of the velocity of grinding towards the fine side range. This effect, absent in the batch test, comes into view because the above referred permanent presence of a significant amount of material of the largest size class makes for the development of a continuous effect of protection of fines of the "umbrella" type (LEITE - 1990);
- PW and PG don't show any important modification when compared with the batch kinetics -- this result is corroborated by some authors (HERBST-1982);

The values obtained for the transport model have the following physical significance:

- V_{50} = 3.1732 mm is the cut size of the partition curve of the mill segment close to the discharge -- this value means that grinding is very slight, which is compatible with the work of a typical rod mill;
- S = -1.2626 is the slope of the straight line that matches the cut sizes of the partition curves of the other mill segments -- the 2 initial segments have cut sizes greater than 3.17 mm, which stresses that grinding affects mainly the first 3 or 4 large size classes;
- M = 1.7947 is the shape factor of the partition curve.

CONCLUSIONS

As main conclusions, is important to underline:

- the transport model, as a phenomenological descriptor, attached itself conveniently to the transient data; experiments with the optimization procedure with less data from the transient lead to instability of the end point;

- the available data portrays a grinding event of very low residence time for the material tested; new experiments should be made with a smaller solids feed rate;
- it was shown batch kinetics is a good initial guess for starting the optimization procedure; if this result is validated when applying the methodology to data from an industrial mill, a real possibility of scaling up the kinetics may be demonstrated;
- when improving the methodology for industrial application, the batch test for comminution kinetics evaluation can be replaced by the kinetics evaluated in the basis of a steady state RTD model.

REFERENCES

Austin, L.G., Klimpel, R.R. et al, 1982. Simulation of Grinding Circuits for Design. In Design and Installations of Comminution Circuits, SME-AIME, New York, pp 301-324.

Baptista, J.M.S., 1994. PhD Thesis to be submitted to the University of Porto, Portugal, during 1994.

Herbst, J.A. and Rajamani, R.K., 1982. Developing a Simulator for Ball Mill Scale-up - a Case Study. In Design and Installations of Comminution Circuits, SME-AIME, New York, pp 325-342.

Lees, M.R. and Lynch, A.J., 1972. Dynamic Behavior of a High Capacity Multi-Stage Grinding Circuit. Transactions of Institution of Mining and Metallurgy, C 81, C227-C235.

Leite, M.R.M., 1990. Kinetic Models for the Simulation of Crushing Circuits. Minerals Engineering, Vol 3, nº 1/2, pp 165-180.

Leite, M.R.M., 1991. A Non-Linear Kinetic Model for Full Autogenuous Grinding Simulation. XVII International Mineral Processing Congress, Dresden, Vol 1, pp 357-370.

Lynch, A.J., 1977. Mineral Crushing and Grinding Circuits. Elsevier, Amsterdam

Madureira, C.N. and Leite, M.R.M., 1991. Modelagem de Operações Unitárias em Tratamento de Matérias Primas. Short Course, AFAIRS/COMETT II UETP, Mining Department, University of Porto.

Madureira, C.N. et al, 1988. Size, Grade and Liberation: a Stochastic Approach to the Fundamental Problem of Mineral Processing. XVI International Mineral Processing Congress, Stockholm, Elsevier, Amsterdam.

Plitt, L.R. et al, 1987. Cyclone Modelling: a Review of Present Technology. C.I.M. Bulletin, Vol 80, nº 905, September 1987, pp 39-50.

Rajamani, R.K. and Herbst, J.A., 1991. Optimal Control of a Ball Mill Grinding Circuit. Chemical Engineering Science, Vol 46, nº 3, pp 861-879, Pergamon Press.