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Skill-Structure Shocks, the Share of the High-Tech Sector and Economic Growth Dynamics

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Skill-Structure Shocks, the Share of the High-Tech Sector and Economic Growth Dynamics

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By means of an endogenous growth model of directed technical change with vertical and horizontal R&D, we study a transitional-dynamics mechanism that is consistent with the changes in the share of the high- versus the low-tech sectors found in recent European data. Under the hypothesis of a positive shock in the proportion of high-skilled labour, the technological-knowledge bias channel leads to nonbalanced sectoral growth with a noticeable shift of resources across sectors. A simple calibration exercise suggests that, under prevailing market-scale effects, the model is able to account for up to 50 to 100 percent of the increase in the share of the high-tech sector observed in the data from 1995 to 2007. However, the model predicts that the dynamics of the share of the high-tech sector has no significant impact on the economic growth rate.

Keywords: industry dynamics, high tech, low tech, directed technical change, economic growth

JEL Classification: O41, O31

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1. Introduction

Over more than a decade, European politicians have emphasised the need to increase the share of the high-tech sector as part of the European growth strategy (see, e.g., Johansson, Karlsson, Backman, and Juusola, 2007 and European Commission, 2010, on the "Lisbon Strategy 2000-2010" and "Europe 2020 Strategy"). But while two complementary measures of industry structure are of interest to assess the relative performance of the high-tech sector – the share of the high-tech sector with respect to production and the share with respect to the number of firms –, casual empiricism has mainly focused on the latter and highlighted its slow growth. Notably, available data shows that the performance of the production share has been clearly better, thus implying an increase of average firm size (i.e., production per firm) in the high- vis-à-vis the low-tech sector. Figure 1 depicts the time-series data for *relative* production (production in the high- versus the low-tech manufacturing sectors), over the 1980-2007 period, and the *relative* number of firms (the number of firms in the high- versus the low-tech sectors), over 1995-2007, for 14 European countries.[1] In order to compare more finely the behaviour of relative production and the relative number of firms over time, we considered the longest period with available data for both variables (1995-2007) and computed their cross-country weighted average. We found that the average annual growth rate was positive for both relative production and the relative number of firms, but that the former exceeded the latter by 0.52 percentage points/year (1.22 percent/year versus 0.7 percent/year). In the period 1995-2000, both variables grew at a faster pace (2.8 percent/year versus 1.06 percent/year) and the drift between them was also larger (1.74 percentage points/year).

What are the factors underlying the dynamics of the share of the high-tech sector in Europe in the recent decades? And what can be expected with respect to the impact of that dynamics on economic growth? In our paper, we address these questions from the perspective of transitional dynamics within an extended theoretical model of endogenous growth, summarised below. We conjecture that the disparity between the dynamics of production and of the number of firms is due to the asymmetric role played by the extensive and the intensive margin of industrial growth, where the former pertains to the creation of new products/firms and the latter to the increase of product quality of existing products and, thereby, of production per firm. Therefore, although in line with the general view that industrial growth proceeds both along an intensive and an extensive margin in the long run (e.g., Freeman and Soete, 1997), we expect a rich

[1] The source is the Eurostat on-line database on Science, Technology and Innovation, available at `http://epp.eurostat.ec.europa.eu`, where the OECD classification of high- and low-tech sectors is used (see Hatzichronoglou, 1997). High-tech sectors are aerospace, computers and office machinery, electronics and communications, and pharmaceuticals, while the low-tech sectors are petroleum refining, ferrous metals, paper and printing, textiles and clothing, wood and furniture, and food and beverages. By crossing the data on both variables – production and the number of firms – and considering a minimum time-span of 12 years (which is the maximum time span available for the number of firms), we end up with a sample of 14 European countries, as depicted by Figure 1. To the best of our knowledge, the Eurostat on-line database is the only one with available data for the number of firms in manufacturing broke down according to the referred to OECD classification.

interaction between the two margins for shorter time horizons, namely in response to structural shocks. Having in mind (i) the observed specificity of the high- and low-tech sectors regarding the proportion of high-skilled labour,[2] (ii) the swift change in the skill structure measured by the proportion of high skilled labour found in the data between the 80's and the 90's across a number of developed countries (see, e.g., Acemoglu, 2003 and Barro and Lee, 2010)[3] and (iii) the acceleration of relative production through the 90's (see the upper panel in Figure 1), we emphasise in particular the hypothesis of a shock in the form of an increase in the relative supply of skills (i.e., the ratio of high- to low-skilled workers). This shock is transmitted through a mechanism of directed technical change and has an asymmetric impact on the intensive and the extensive margin, both within and across the high- and the low-tech sectors.[4] As explained further below, the different nature of the intensive and the extensive margin should play a central role here.

Then, we show that by isolating the initial shock to the relative supply of skills as the driver of the change in the industry structure, the model predicts that the economic growth rate will experience, at best, a mild level effect. Indeed, as the economy evolves towards the balanced-growth path (BGP), there is a significant shift of economic activity from the low- to the high-tech sectors (or vice versa), but the aggregate growth rate remains approximately unchanged.[5]

To uncover the analytical mechanism through which the empirical evidence can be

[2] Empirical evidence suggests that high-tech sectors are more intensive in high-skilled labour than the low-tech sectors. For instance, according to the data for the average of the European Union (27 countries, 2007), 30.9% of the employment in the high-tech manufacturing sectors is high skilled ("college graduates"), against 12.1% of the employment in the low-tech sectors. The source is the Eurostat on-line database on Science, Technology and Innovation (`http://epp.eurostat.ec.europa.eu`).

[3] According to Barro and Lee (2010)'s data set, the proportion of the high skilled (measured by the ratio of college to non-college graduates) in the 10 countries with available data for relative production depicted by Figure 1 accelerated from 4.11 to 5.76 percent in 1980-1995 and then slowed down from 3.35 to 0.51 percent in 1995-2007.

[4] The relative supply of skills is usually treated as exogenous in the literature of directed technical change, in order to isolate the impact of the increase of the proportion of the high skilled observed in the data through the technological knowledge bias channel (e.g., Acemoglu and Zilibotti, 2001;Acemoglu, 2003). In principle, causality can run both ways: for instance, an increase in the share of high-skilled labour may imply higher economic growth, but also the latter may increase enrollment rates and thereby the share of the high skilled. However, recent empirical literature has found evidence that supports causality running from human capital to growth (e.g., Hanushek and Kimko, 2000; Sequeira, 2007; Hanushek and Woessmann, 2012), while some authors emphasise the relationship between the share of high-skilled labour and 'exogenous' institutional factors (see, e.g., Jones and Romer, 2010). Particularly strong evidence on causality from human capital to growth relates to the importance of fundamental economic institutions using identification through historical factors (e.g., Acemoglu, Johnson, and Robinson, 2005). In the same line, in Appendix A, we present own evidence supporting (statistical) causality running from the share of the high skilled to the share of production of the high-tech sector.

[5] A sister paper (Gil, Afonso, and Brito, 2012) focuses on the related issue concerning the relationship between high-/low-tech structure, skill structure and economic growth on the BGP. There, it is shown that the share of the high-tech sector matters for growth because this sector employs high-skilled labour, which has an absolute productivity advantage over low-skilled labour, but this effect on growth tends to be dampened by the high entry costs into the high-tech sector.

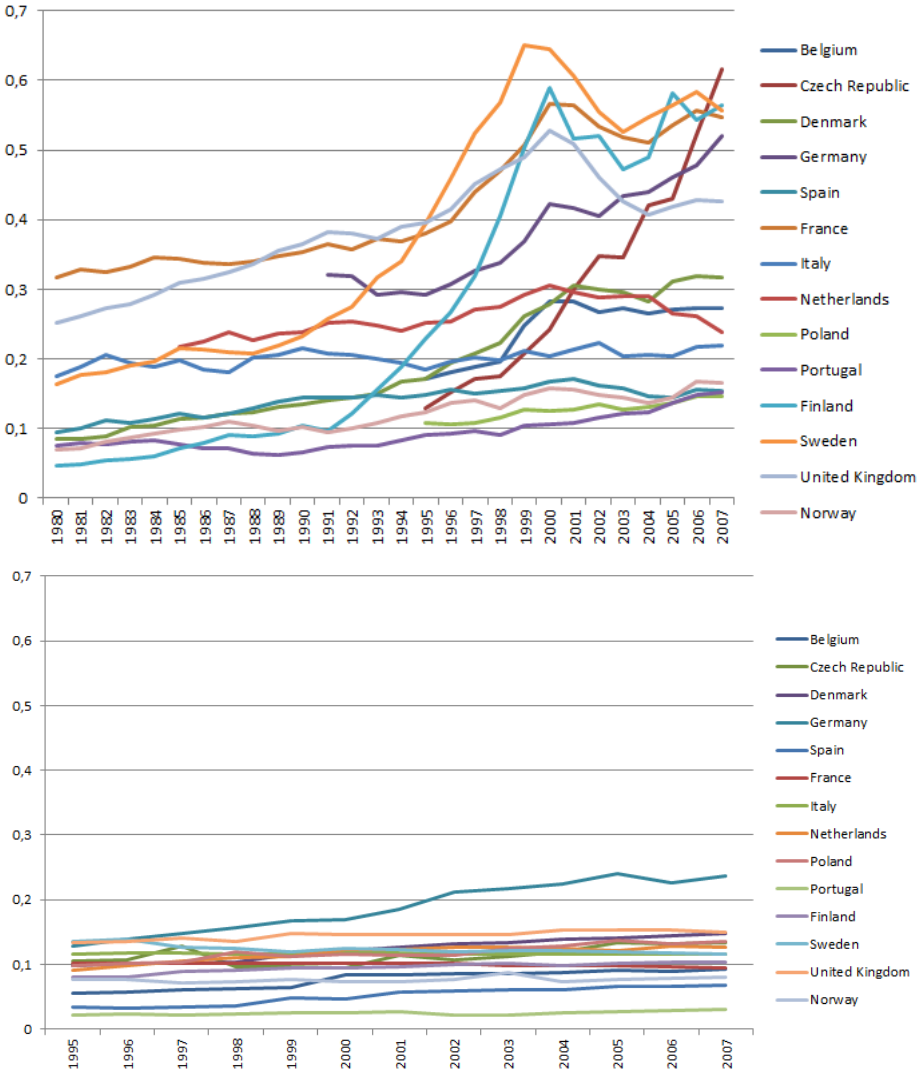

Figure 1: The share of the high-tech sectors through time: relative production (upper panel) and the relative number of firms (lower panel) according to the high-tech low-tech OECD classification in 14 European countries. Source: Eurostat on-line database on Science, Technology and Innovation – table "Economic statistics on high-tech industries and knowledge-intensive services at the national level", available at `http://epp.eurostat.ec.europa.eu` .

accommodated, we develop a general equilibrium growth model that incorporates endogenous directed technical change with vertical R&D (increase of product quality) and horizontal R&D (creation of new products/firms). Final-goods production uses either low- or high-skilled labour with labour-specific intermediate goods, while R&D can be directed to either the low- or the high-skilled labour complementary technology. Thus, "sector" herein represents a group of firms producing the same type of labour-specific intermediate goods. Since the data shows that the high-tech sectors are more intensive in high-skilled labour than the low-tech sectors (see fn. 2), we consider the high- and low-skilled labour-specific intermediate-good sectors in the model as the theoretical counterpart of the high- and low-tech sectors (e.g., Cozzi and Impullitti, 2010).

We consider an R&D specification, as proposed by Gil, Brito, and Afonso (2013), that implies that the choice between vertical and horizontal innovation is related to the splitting of R&D expenditures, which are fully endogenous. Thus, we endogenise the rate of both intensive and extensive growth, and thereby production and the number of firms in each sector.[6] Given the inherently distinct nature of vertical and horizontal innovation (immaterial versus physical) and the consequent asymmetry in terms of R&D complexity and congestion costs, vertical R&D emerges as the ultimate growth engine, while horizontal R&D allows for an explicit link between aggregate and industry-structure variables (the number of firms and production in high- and in low-tech sectors).

Furthermore, we take a flexible view of scale effects on industrial growth. The complete removal of scale effects as sometimes posited in the theoretical growth literature is a knife-edge case, as Peretto and Smulders (2002) have recently stressed. Indeed, the existence of scale effects at the aggregate level is disputed, with the empirical results rejecting it in secular trend but not over transitional dynamics (e.g., Jones, 1995; Jones, 2002; Sedgley and Elmslie, 2010), whereas early empirical studies clearly indicate the existence of scale effects at the industry (manufacturing) level (e.g., Backus, Kehoe, and Kehoe, 1992). Thus, because the literature does not offer a clear cut answer to the issue of the existence of scale effects, we consider a number of scenarios, from no scale effects on growth (only price-channel effects exist) to full scale effects (only market-size-channel effects exist). This will then allow for a flexible relationship between the number of firms and production per firm across the high- and the low-tech sectors.

In our analysis, we focus on global transitional dynamics: global dynamics, as opposed to local dynamics, allows us to carry out a comparative dynamics exercise without restricting the analysis to a sufficiently close neighbourhood of the steady state and, thus, to small shifts in the parameters and the exogenous variables.[7] Since the dynamic system in our model is four dimensional (in appropriately detrended variables), with three prede-

[6] An alternative approach in the literature assumes that the allocation of resources between vertical and horizontal R&D implies a division of labour between the two types of R&D. Since the total labour level is determined exogenously, the rate of growth along the horizontal direction is exogenous, i.e., the BGP flow of new products and industries occurs at the same rate as (or is proportional to) population growth.

[7] Atolia, Chatterjee, and Turnovsky (2010) investigate the reliability of employing linearisation to evaluate the transitional dynamics in neoclassical growth models and conclude that, when transition is slow – as is the case in our model –, linearisation tends to yield misleading predictions.

termined endogenous variables, and is highly non linear, we resort to numerical methods to study global dynamics. In particular, the dynamic system is solved by numerical integration using a finite difference method implementing the three-stage Lobatto IIIa formula provided through the software MatLab.

We analyse transitional dynamics by considering the effects of a unanticipated one-off shock in the relative supply of skills. An interesting asymmetry between the high- and the low-tech sectors then arises working through the technological-knowledge bias channel, because of the difference in profitability between those two sectors induced by the initial rise in the proportion of high-skilled labour: under prevailing market-size-channel effects (price-channel effects), the vertical innovation rate targeting the low-tech sector experiences an immediate decrease (increase) while the rate in the high-tech sector takes an upward (downward) jump; then, given the complementarity between vertical and horizontal R&D, this sets off an asymmetric adjustment over transition of *both* the vertical and the horizontal innovation rate – and hence of growth rates in the intensive and extensive margin – across sectors. As the economy slowly adjusts towards the new BGP, industry dynamics coexists with aggregate stability.

We highlight, in particular, the result that the economic growth rate remains approximately constant over the adjustment. This arises from the fact that the economic growth rate is a weighed average of the two sectoral growth rates, with the weights being a function of the share of the high-tech sector in terms of the technological-knowledge stock, i.e., the measure of the technological-knowledge bias. Thus, the weights also move endogenously in response to the shock in the relative supply of skills, through the technological-knowledge bias channel. The combined effect of the opposing movements of the sectoral growth rates and the shift in the share of the high-tech sector then implies that the economic growth rate is roughly unchanged over transition.

Moreover, our model implies a speed of convergence to the new BGP that is faster at the sectoral than at the aggregate level, in particular if one compares the share of the high-tech sectors in production with the economic growth rate. More generally, transitional dynamics is flexible in the sense that the transition speed is different both across variables and through time, even if the time paths are derived from a linearised version of the dynamic system, which reflects the existence of a multi-dimensional stable manifold. Such a result was firstly explored within an endogenous-growth setup by Eicher and Turnovsky (2001). However, while in the latter a multi-dimensional stable manifold arises from the removal of scale effects in a Jones (1995)-type model, we derive our results under less strict conditions with this respect: given our parametric approach to the modelling of scale effects, the dimension of the dynamic system is independent of the removal of scale effects.[8]

[8]Eicher and Turnovsky (2001) analyse the dynamics of an endogenous growth model with physical capital and horizontal R&D, in which labour is the input, based on Jones (1995), and show that the removal of scale effects in that type of models raises the dimension of the dynamic system such that the latter becomes four-dimensional and the stable manifold two dimensional. In our model of vertical and horizontal R&D and two intermediate-good sectors, where the homogeneous final-good is the input to R&D activities, we are able to derive a four-dimensional dynamic system featuring a

In the case of prevailing market-scale channel effects, the theoretical results are consistent with the time-series data depicted by Figure 1. That is, there is an increase in the share of the high-tech sectors both in terms of production and of the number of firms, paralleled by an increase in production per firm relatively to the low-tech sectors. The former result stems from the positive response of the two measures of industry structure to the shock through the technological-knowledge bias channel (a larger market, measured by employed high-skilled labour, expands profits and, thus, the incentives to allocate resources to both types of R&D in the high-tech sectors), while the latter is explained by the stronger complexity and congestion costs impinging on horizontal R&D, which slow down and dampen the response of the number of firms relatively to that of production. According to a simple calibration exercise, the model is able to account for up to 50 to 100 percent of the increase in the share of the high-tech sectors observed in the European data from 1995 to 2007.

Finally, we note that while the empirical literature rejects the existence of scale effects in *secular* trend, as cited earlier, our quantitative results suggest scale effects play a role as regards the *medium* term behaviour of the economies – in particular in the light of the relatively short time span of the time-series data that support our calibration exercise. In this sense, our results are complementary to the long-term vision of industrial growth as a non-scale phenomenon.[9]

The remainder of the paper has the following structure. In Section 2, we present the model of directed technological change with vertical and horizontal R&D, derive the dynamic general equilibrium and characterise the BGP. In Section 3, we detail the comparative dynamics results by considering the impact of a shock in the relative supply of skills on the aggregate and the industry-level variables, and carry out an illustrative calibration exercise. Section 4 gives some concluding remarks.

2. The model

The model used herein is drawn from Acemoglu and Zilibotti (2001), augmented with vertical R&D and developed under flexible scale effects. Thus, we study a directed technological change model with vertical and horizontal R&D, built into a dynamic general equilibrium setup of a closed economy where the aggregate competitively-produced final good can be used in consumption, production of intermediate goods and R&D. The economy is populated by a fixed number of infinitely-lived households who inelastically supply one of two types of labour to final-good firms: low-skilled, L, and high-skilled labour, H. The final good is produced by a continuum of firms, indexed by $n \in [0, 1]$, to whom two substitute technologies are available: the "Low" (respectively, "High") technology uses a combination of L (H) and a continuum of L-specific (H-specific) intermediate goods indexed by $\omega_L \in [0, N_L]$ ($\omega_H \in [0, N_H]$).

Potential entrants can devote resources to either horizontal or vertical R&D, and di-

three-dimensional stable manifold irrespective of the degree of scale effects.

[9] In fact, scale effects over transitional dynamics obtain in several theoretical models; see, e.g., Jones (1995), Dinopoulos and Thompson (1998); Jones (2002), and Sedgley and Elmslie (2013).

rected to either the high- or the low-skilled labour-specific technology. Horizontal R&D increases the number of industries, N_m, $m \in \{L, H\}$, in the m-specific intermediate-good sector,[10] while vertical R&D increases the quality level of the good of an existing industry, indexed by $j_m(\omega_m)$. Then, the quality level $j_m(\omega_m)$ translates into productivity of the final producer by using the good produced by industry ω_m, $\lambda^{j_m(\omega_m)}$, where $\lambda > 1$ is a parameter measuring the size of each quality upgrade. By improving on the current best quality index j_m, a successful R&D firm will introduce the leading-edge quality $j_m(\omega_m) + 1$ and hence render inefficient the existing input. Therefore, the successful innovator will become a monopolist in ω_m. However, this monopoly, and the monopolist earnings that come with it, are temporary, because a new successful innovator will eventually substitute the incumbent.

2.1. Production and price decisions

This section briefly describes the familiar components of Acemoglu and Zilibotti's (2001) model, augmented with vertical R&D. Aggregate output at time t is defined as $Y_{tot}(t) = \int_0^1 P(n,t)Y(n,t)dn$, where $P(n,t)$ and $Y(n,t)$ are the relative price and the quantity of the final good produced by firm n. Each final-good firm n has a constant-returns-to-scale technology possibly using low- and high-skilled labour and a continuum of labour-specific intermediate goods with measure $N_m(t)$, such that $N_{tot}(t) = N_L(t) + N_H(t)$ and

$$\begin{aligned} Y(n,t) \;=\; & A\left[\int_0^{N_L(t)} \left(\lambda^{j_L(\omega_L,t)} \cdot X_L(n,\omega_L,t)\right)^{1-\alpha} d\omega_L\right] [(1-n)\cdot l \cdot L(n)]^{\alpha} + \\ & + A\left[\int_0^{N_H(t)} \left(\lambda^{j_H(\omega_H,t)} \cdot X_H(n,\omega_H,t)\right)^{1-\alpha} d\omega_H\right] [n \cdot h \cdot H(n)]^{\alpha} \end{aligned}, \; 0 < \alpha < 1, \tag{1}$$

where $A > 0$ is the total factor productivity, $L(n)$ and $H(n)$ are the labour inputs used by n and α is the labour share in production, and $\lambda^{j_m(\omega_m,t)} \cdot X_m(n,\omega_m,t)$ is the input of m-specific intermediate good ω_m measured in efficiency units at time t.[11] An *absolute*-productivity advantage of H over L is captured by $h > l \geq 1$; a *relative*-productivity advantage of each labour type is determined by terms n and $(1-n)$, implying that H is relatively more productive for larger n, and vice-versa. As explained below, at each t there is a competitive equilibrium threshold $\bar{n}(t)$, endogenously determined, where the switch from one technology to the other becomes advantageous, so that each n produces exclusively with one technology, either L- or H-technology.

Final producers take the price of their final good, $P(n,t)$, wages, $W_m(t)$, and input prices $p_m(\omega_m,t)$ as given. From the usual profit maximisation conditions, we determine the demand of intermediate good ω_m by firm n, at each t[12]

[10] Henceforth, we will also refer to the "m-specific intermediate-good sector" as "m-technology sector".

[11] In equilibrium, only the top quality of each ω_m is produced and used; thus, $X_m(j,\omega_m,t) = X_m(\omega_m,t)$.

[12] The first-order conditions require the equation of the marginal product of each intermediate good to its price. Although, given (1), the profit of final good firms is a function of time, profit maximisation amounts to a static optimisation problem since there are no intertemporal linkages impacting on profits. Thus, the producer of $Y(n)$ selects $X(n,\omega_m)$ at each date to maximise the flow of profits at that date (see, e.g., Barro and Sala-i-Martin, 2004; Acemoglu, 2009).

$$\begin{aligned} X_L(n,\omega_L,t) &= (1-n)\cdot l\cdot L(n)\cdot\left[\frac{A\cdot P(n,t)\cdot(1-\alpha)}{p_L(\omega,t)}\right]^{\frac{1}{\alpha}}\lambda^{j(\omega_L,t)\left(\frac{1-\alpha}{\alpha}\right)} \\ X_H(n,\omega_H,t) &= n\cdot h\cdot H(n)\cdot\left[\frac{A\cdot P(n,t)\cdot(1-\alpha)}{p_H(\omega,t)}\right]^{\frac{1}{\alpha}}\lambda^{j(\omega_H,t)\left(\frac{1-\alpha}{\alpha}\right)} \end{aligned}. \tag{2}$$

There is monopolistic competition if we consider the whole sector: the monopolist in industry $\omega_m \in [0, N_m(t)]$ fixes the price $p_m(\omega_m, t)$ but faces an isoelastic demand curve, $X_L(\omega_L,t) = \int_0^{\bar{n}(t)} X_L(n,\omega_L,t)dn$ or $X_H(\omega_H,t) = \int_{\bar{n}(t)}^1 X_H(n,\omega_H,t)dn$ (see (2)). We assume that intermediate goods are non-durable and entail a unit marginal cost of production, measured in terms of the final good, whose price is taken as given (numeraire). Profit in ω_m is thus $\pi_m(\omega_m,t) = (p_m(\omega_m,t)-1)\cdot X_m(\omega_m,t)$, and the profit maximising price is a constant markup over marginal cost

$$p_m(\omega_m,t) \equiv p = \frac{1}{1-\alpha} > 1,\ m \in \{L,H\}. \tag{3}$$

Given $\bar{n}$ and (3), we can then write the final-good output as

$$Y(n,t) = \begin{cases} A^{\frac{1}{\alpha}}P(n,t)^{\frac{1-\alpha}{\alpha}}\cdot(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}}\cdot(1-n)\cdot l\cdot L(n)\cdot Q_L(t) & ,\ 0\le n\le\bar{n} \\ A^{\frac{1}{\alpha}}P(n,t)^{\frac{1-\alpha}{\alpha}}\cdot(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}}\cdot n\cdot h\cdot H(n)\cdot Q_H(t) & ,\ \bar{n}\le n\le 1 \end{cases}, \tag{4}$$

where the aggregate quality index

$$Q_m(t) = \int_0^{N_m(t)} q_m(\omega_m,t)d\omega,\ q_m(\omega_m,t) \equiv \lambda^{j_m(\omega_m,t)\left(\frac{1-\alpha}{\alpha}\right)},\ m\in\{L,H\}, \tag{5}$$

measures the technological-knowledge level in each m-technology sector. Thus, $Q \equiv Q_H/Q_L$ measures the technological-knowledge bias. The allocation of the low- and high-skilled labour inputs to the L- and the H-technology sector verifies $L = \int_0^{\bar{n}} L(n)dn$ and $H = \int_{\bar{n}}^1 H(n)dn$. With competitive final-good producers, economic viability of either L- or H-technology relies on the relative productivity and price of labour, as well as on the relative productivity and prices of intermediate goods, due to complementarity in production. Labour prices depend on quantities, H and L. In relative terms, the productivity-adjusted quantity of H is $\mathcal{H}/\mathcal{L}$, where $\mathcal{H} \equiv hH$ and $\mathcal{L} \equiv lL$. As for the productivity and prices of intermediate goods, they depend on complementarity with either H or L, on the technological knowledge embodied and on the markup. These determinants are summed up in Q_L and Q_H. The endogenous threshold $\bar{n}$ follows from equilibrium in the inputs markets, and relies on the determinants of economic viability of the two technologies, such that

$$\bar{n}(t) = \left[1+\left(\frac{\mathcal{H}}{\mathcal{L}}\frac{Q_H(t)}{Q_L(t)}\right)^{\frac{1}{2}}\right]^{-1}. \tag{6}$$

$\bar{n}(t)$ implies that L- (H-)specific technology is exclusively used by final-good firms indexed by $n \in [0,\bar{n}(t)]$ ($n \in [\bar{n}(t),1]$), and it can be related to the ratio of price indices of final goods produced with L- and H-technologies:

$$\frac{P_H(t)}{P_L(t)} = \left(\frac{\bar{n}(t)}{1-\bar{n}(t)}\right)^{\alpha}, \text{ where } \begin{cases} P_L(t) = P(n,t)\cdot(1-n)^{\alpha} = \exp(-\alpha)\cdot\bar{n}(t)^{-\alpha} \\ P_H(t) = P(n,t)\cdot n^{\alpha} = \exp(-\alpha)\cdot(1-\bar{n}(t))^{-\alpha} \end{cases} . \quad (7)$$

In (7), we first define the price indices, $P_L(t)$ and $P_H(t)$, by recognising that, in equilibrium, the marginal value product, $\frac{\partial}{\partial m(n)}(P(n,t)Y(n,t))$, must be constant over n, implying that $P(n,t)^{\frac{1}{\alpha}}\cdot(1-n)$ and $P(n,t)^{\frac{1}{\alpha}}\cdot n$ must be constant over $n\in[0,\bar{n}(t)]$ and $n\in[\bar{n}(t),1]$, respectively. Then, considering that at $\bar{n}(t)$ the L- and the H- technology firms must break even, we relate $P_L(t)$ and $P_H(t)$ with $\bar{n}(t)$. Equation (6) shows that if either the technology is highly H-biased or if there is a large relative supply of H, the share of final goods using the H-technology is large and $\bar{n}(t)$ is small. By (7), small $\bar{n}(t)$ implies a low $P_H(t)/P_L(t)$. In this case, the demand for $\omega_H\in[0,N_H(t)]$ is low, which discourages R&D activities directed to H-technology.

From (2), (3) and (7), we find the optimal intermediate-good production, $X_m(\omega_m)$, and thus the optimal profit accrued by the monopolist in ω_m is

$$\pi_m(\omega_m,t) = \pi_{0m}\cdot P_m(t)^{\frac{1}{\alpha}}\cdot q_m(\omega_m,t)\ ,\ m\in\{L,H\}\,, \quad (8)$$

where $\pi_{0L}\equiv\mathcal{L}A^{\frac{1}{\alpha}}\left(\frac{\alpha}{1-\alpha}\right)(1-\alpha)^{\frac{2}{\alpha}}$ and $\pi_{0H}\equiv\mathcal{H}A^{\frac{1}{\alpha}}\left(\frac{\alpha}{1-\alpha}\right)(1-\alpha)^{\frac{2}{\alpha}}$ are positive constants.

Total intermediate-good optimal production, $X_{tot}(t)\equiv X_L(t)+X_H(t)\equiv\int_0^{N_L(t)}X_L(\omega_L)d\omega_L+\int_0^{N_H(t)}X_H(\omega_H)d\omega_H$, and total final-good optimal production, $Y_{tot}(t)\equiv Y_L(t)+Y_H(t)\equiv\int_0^{\bar{n}(t)}P(n,t)Y(n,t)dn+\int_{\bar{n}(t)}^1P(n,t)Y(n,t)dn$, are, respectively,

$$X_{tot}(t) = \chi_X\Gamma(t) \quad (9)$$

and

$$Y_{tot}(t) = \chi_Y\Gamma(t), \quad (10)$$

where $\chi_X\equiv A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{2}{\alpha}}$, $\chi_Y\equiv A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}}$ and $\Gamma(t)\equiv P_L(t)^{\frac{1}{\alpha}}\cdot\mathcal{L}\cdot Q_L(t)+P_H(t)^{\frac{1}{\alpha}}\cdot\mathcal{H}\cdot Q_H(t)$.

Finally, by considering the condition that the real wage, W_m, must equal the marginal productivity of labour in equilibrium in the m-technology sector $m\in\{L,H\}$, we get, from equation (10), the skill premium as a function of the technological-knowledge bias, $Q\equiv Q_H/Q_L$,

$$W(t)\equiv\frac{W_H(t)}{W_L(t)} = \frac{h}{l}\left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{-\frac{1}{2}}(Q(t))^{\frac{1}{2}}\,. \quad (11)$$

2.2. R&D

We consider two R&D sectors, one targeting horizontal innovation and the other endeavoring vertical innovation. We assume that the pools of innovators performing the two

types of R&D are different. Each new design (a new variety or a higher quality good) is granted a patent and thus a successful innovator retains exclusive rights over the use of his/her good. We also take the simplifying assumptions that both vertical and horizontal R&D are performed by (potential) entrants, and that successful R&D leads to the set-up of a new firm in either an existing or in a new industry (e.g., Howitt, 1999; Strulik, 2007; Gil, Brito, and Afonso, 2013). There is perfect competition among entrants and free entry in R&D business.

2.2.1. Vertical R&D

By improving on the current top quality level $j_m(\omega_m, t)$, $m \in \{L, H\}$, a successful R&D firm earns monopoly profits from selling the leading-edge input of $j_m(\omega_m, t) + 1$ quality to final-good firms. A successful innovation will instantaneously increase the quality index in ω_m from $q_m(\omega_m, t) = q_m(j_m)$ to $q_m^+(\omega_m, t) = q_m(j_m + 1) = \lambda^{(1-\alpha)/\alpha} q_m(j_m)$. In equilibrium, lower qualities of ω_m are priced out of business.

Let $I_m^i(j_m)$ denote the Poisson arrival rate of vertical innovations (vertical-innovation rate) by potential entrant i in industry ω_m when the highest quality is j_m. The rate $I_m^i(j_m)$ is independently distributed across firms, across industries and over time, and depends on the flow of resources $R_{vm}^i(j_m)$ committed by entrants at time t. As in, e.g., Barro and Sala-i-Martin (2004, ch. 7), $I_m^i(j_m)$ features constant returns in R&D expenditures, $I_m^i(j_m) = R_{vm}^i(j_m) \cdot \Phi_m(j_m)$, where $\Phi_m(j_m)$ is the R&D productivity factor, which is assumed to be homogeneous across i in ω_m. We assume

$$\Phi_L(j_L) = \frac{1}{\zeta \cdot q_L(j_L + 1) \cdot \mathcal{L}^\epsilon} \text{ and } \Phi_H(j_H) = \frac{1}{\zeta \cdot q_H(j_H + 1) \cdot \mathcal{H}^\epsilon}, \tag{12}$$

where $\zeta \equiv \zeta_L \equiv \zeta_H > 0$ is a constant (flow) fixed vertical-R&D cost, and $\epsilon \geq 0$. Hence, an R&D complexity effect is considered (e.g., Barro and Sala-i-Martin, 2004, ch. 7; Etro, 2008), implying dynamic decreasing returns to vertical R&D: the larger the level of quality, q_m, the costlier it is to introduce a further jump in quality.[13] Equation (12) also implies that an increase in market scale, $\mathcal{L}$ or $\mathcal{H}$, may dilute the effect of R&D outlays on innovation probability (market complexity effect); this captures the idea that the difficulty of introducing new qualities and replacing old ones is proportional to the market size measured by employed labour in efficiency units (e.g., Barro and Sala-i-Martin, 2004), due to coordination, organisational and transportation costs and rental protection actions by incumbents (e.g., Dinopoulos and Thompson, 1999; Sener, 2008). Depending on the effectiveness of those costs and actions, they may partially ($0 < \epsilon < 1$), totally ($\epsilon = 1$) or over ($\epsilon > 1$) counterbalance the scale benefits on profits, which accrue to the R&D successful firm at each t. Thus, we take a parametric approach to the removal of scale effects, defined over a continuous support (in contrast to, e.g., Jones, 1995), such that there may be, respectively, positive, null or negative net scale effects on industrial

[13] The way Φ depends on j implies that the increasing difficulty of creating new product generations over t exactly offsets the increased rewards from marketing higher quality products; see (12) and (8). This allows for constant vertical-innovation rate over t and across ω in BGP (on *asymmetric* equilibrium in quality-ladders models and its growth consequences, see Cozzi, 2007).

growth, as measured by $1-\epsilon$. Aggregating across i in ω_m, we get $R_{vm}(j_m) = \sum_i R^i_{vm}(j_m)$ and $I_m(j_m) = \sum_i I^i_m(j_m)$, and thus

$$I_L(j_L) = R_{vL}(j_L) \cdot \Phi_L(j_L) \text{ and } I_H(j_H) = R_{vH}(j_H) \cdot \Phi_H(j_H). \tag{13}$$

As the terminal date of each monopoly arrives as a Poisson process with frequency $I_m(j_m)$ per (infinitesimal) increment of time, the present value of a monopolist's profits is a random variable. Let $V_m(j_m)$ denote the expected value of an incumbent firm with current quality level $j_m(\omega_m, t)$,[14]

$$V_m(j_m) = \pi_{0m} \cdot q_m(j_m) \int_t^\infty P_m(s)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s (r(v)+I_m(j_m))dv} ds, \; m \in \{L, H\}, \tag{14}$$

where r is the equilibrium market real interest rate, and $\pi_{0m} q_m(j_m) = \pi_m(j_m) P_m^{-\frac{1}{\alpha}}$, given by (8) and (7), is constant in-between innovations. Free-entry prevails in vertical R&D such that the condition $I_m(j_m) \cdot V_m(j_m + 1) = R_{vm}(j_m)$ holds, which implies that

$$V_L(j_L + 1) = \frac{1}{\Phi_L(j_L)} \text{ and } V_H(j_H + 1) = \frac{1}{\Phi_H(j_H)}. \tag{15}$$

Next, we determine $V_m(j_m + 1)$ analogously to (14), then consider (15) and time-differentiate the resulting expression. Thus, if we also consider (8), we get the no-arbitrage condition facing a vertical innovator

$$r(t) + I_L(t) = \frac{\pi_0 \cdot \mathcal{L}^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}}}{\zeta}, \; r(t) + I_H(t) = \frac{\pi_0 \cdot \mathcal{H}^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}}}{\zeta}, \tag{16}$$

where $\pi_0 \equiv \pi_{0L}/\mathcal{L} = \pi_{0H}/\mathcal{H}$.[15] It has two implications: the present value of "basic" profits $\pi_0 \cdot \mathcal{L}^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}}$ and $\pi_0 \cdot \mathcal{H}^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}}$ (i.e., the profit flows that accrue when $j_m = 0$, or $q_m = 1$), using the effective rate of interest $r(t) + I_m(t)$ as a discount factor, should be equal to the fixed cost of entry; and the rates of entry are symmetric across industries $I_m(\omega_m, t) = I_m(t)$.

If we equate the effective rate of return for both R&D sectors by considering (16), the no-arbitrage condition obtains

$$I_H(t) - I_L(t) = \frac{\pi_0}{\zeta} \left(\mathcal{H}^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}} - \mathcal{L}^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}} \right). \tag{17}$$

[14] We assume that entrants are risk-neutral and, thus, only care about the expected value of the firm.

[15] From (8) and (13), we have $\frac{\dot{\pi}_m(\omega_m,t)}{\pi_m(\omega_m,t)} - \frac{1}{\alpha}\frac{\dot{P}_m(t)}{P_m(t)} = I_m(\omega_m, t) \cdot \left[\dot{j}_m(\omega_m, t) \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right]$ and $\frac{\dot{R}_{vm}(\omega_m,t)}{R_{vm}(\omega_m,t)} - \frac{\dot{I}_m(\omega_m,t)}{I_m(\omega_m,t)} = I_m(\omega_m, t) \cdot \left[\dot{j}_m(\omega_m, t) \cdot \left(\frac{\alpha}{1-\alpha} \right) \cdot \ln \lambda \right]$. Thus, if we time-differentiate (15) considering (14) and the equations above, we get $r(t) = \frac{\pi_m(j_m+1) \cdot I_m(j_m)}{R_{vm}(j_m)} - I_m(j_m + 1)$, which can then be re-written as (16).

Solving (13) for $R_{vm}(\omega_m, t) = R_{vm}(j_m)$ and aggregating across industries ω_m, we determine total resources devoted to vertical R&D, $R_{vm}(t)$; e.g., with $m = L$, $R_{vL}(t) = \int_0^{N_L(t)} R_{vL}(\omega_L, t)\, d\omega_L = \int_0^{N_L(t)} \zeta \cdot \mathcal{L}^{\epsilon} \cdot q_L^{+}(\omega_L, t) \cdot I_L(\omega_L, t)\, d\omega_L$. As the innovation rate is industry independent, then

$$R_{vL}(t) = \zeta \cdot \mathcal{L}^{\epsilon} \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot I_L(t) \cdot Q_L(t),\ R_{vH}(t) = \zeta \cdot \mathcal{H}^{\epsilon} \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot I_H(t) \cdot Q_H(t). \tag{18}$$

2.2.2. Horizontal R&D

Variety expansion arises from R&D aimed at creating a new intermediate good. Again, innovation is performed by a potential entrant, which means that, because there is free entry, the new good is produced by new firms. Under perfect competition among R&D firms and constant returns to scale at the firm level, instantaneous entry obtains as $\dot{N}_m^e(t)/N_m^e(t) = R_{hm}^e(t)/\eta_m(t)$, where $\dot{N}_m^e(t)$ is the contribution to the instantaneous flow of new m-specific intermediate goods by R&D firm e at a cost of $\eta_m(t)$ units of the final good (cost of horizontal entry) and $R_{hm}^e(t)$ is the flow of resources devoted to horizontal R&D by innovator e at time t. The cost $\eta_m(t)$ is assumed to be symmetric within the m-technology sector. Then, $R_{hm}(t) = \sum_e R_{hm}^e(t)$ and $\dot{N}_m(t) = \sum_e \dot{N}_m^e(t)$, implying

$$R_{hm}(t) = \eta_m(t) \cdot \dot{N}_m(t)/N_m(t),\ m \in \{L, H\}\,. \tag{19}$$

Concerning the cost of horizontal entry, $\eta_m(t)$, we follow Gil, Brito, and Afonso (2013) and assume that it is increasing in both the number of existing varieties, $N_m(t)$, and the number of new entrants, $\dot{N}_m(t)$,

$$\eta_m(t) = \phi \cdot N_m(t)^{1+\sigma} \cdot \dot{N}_m(t)^{\gamma},\ m \in \{L, H\}\,, \tag{20}$$

where $\phi > 0$ is a fixed (flow) cost, while $\sigma > 0$ and $\gamma > 0$ relate η with N and $\dot{N}$, respectively. Indeed, equation (20) introduces two types of decreasing returns associated to horizontal innovation. Dynamic decreasing returns to scale are modeled by the dependence of η on N and result from complexity (e.g., Evans, Honkapohja, and Romer, 1998; Barro and Sala-i-Martin, 2004, ch. 6), in the sense that the larger the number of existing varieties, the costlier it is to introduce new varieties. It is noteworthy that the elasticity regulating the horizontal-R&D complexity costs is larger than the one in the vertical-R&D case (i.e., $1 + \sigma > 1$), in line with what should be expected bearing in mind the distinct nature of the two types of R&D (physical versus immaterial). Static decreasing returns to scale (at the aggregate level) are modeled by the dependence of η on $\dot{N}$ and mean that one potential entrant exerts an externality on other entrants due to, e.g., congestion effects. The dependence of the entry cost on the number of entrants introduces dynamic second-order effects from entry, implying that new varieties are brought to the market gradually, instead of through a lumpy adjustment. This is

in line with the stylised facts on entry, according to which entry occurs mostly at small scale since adjustment costs penalise large-scale entry (e.g., Geroski, 1995).

Every horizontal innovation results in a new intermediate good whose quality level is drawn randomly from the distribution of existing varieties (e.g., Howitt, 1999). Thus, the expected quality level of the horizontal innovator is

$$\bar{q}_m(t) = \int_0^{N_m(t)} \frac{q_m(\omega_m, t)}{N_m(t)} d\omega_m = \frac{Q_m(t)}{N_m(t)}, \ m \in \{L, H\}. \tag{21}$$

As his/her monopoly power will be also terminated by the arrival of a successful vertical innovator in the future, the benefits from entry are given by

$$V_m(\bar{q}_m) = \pi_{0m} \cdot \bar{q}_m(t) \int_t^{\infty} P_m(t)^{\frac{1}{\alpha}} \cdot e^{-\int_t^s [r(\nu) + I_m(\bar{q}_m)] d\nu} ds, \ m \in \{L, H\}, \tag{22}$$

where $\pi_{0m}\bar{q}_m = \bar{\pi}_m P_m^{-\frac{1}{\alpha}}$. The free-entry condition is now $\dot{N}_m \cdot V(\bar{q}_m) = R_{hm}$, which simplifies to

$$V_m(\bar{q}_m) = \frac{\eta_m(t)}{N_m(t)}, \ m \in \{L, H\}. \tag{23}$$

Substituting (22) into (23) and time-differentiating the resulting expression, yields the no-arbitrage condition facing a horizontal innovator

$$r(t) + I_m(t) = \frac{\bar{\pi}_m(t)}{\eta_m(t)/N_m(t)}, \ m \in \{L, H\}. \tag{24}$$

2.2.3. Intra-sector no-arbitrage conditions

No-arbitrage in the capital market requires that the two types of investment – vertical and horizontal R&D – yield equal rates of return. Thus, by equating the effective rate of return $r + I_m$ for both types of entry, from (16) and (24), we get the intra-sector no-arbitrage conditions

$$\bar{q}_L(t) = \frac{Q_L(t)}{N_L(t)} = \frac{\eta_L(t)}{\zeta \cdot \mathcal{L}^{\epsilon} \cdot N_L(t)}, \ \bar{q}_H(t) = \frac{Q_H(t)}{N_H(t)} = \frac{\eta_H(t)}{\zeta \cdot \mathcal{H}^{\epsilon} \cdot N_H(t)}. \tag{25}$$

These conditions equate the average cost of horizontal R&D, η_L/N_L (respectively, η_H/N_H), to the average cost of vertical R&D, $\bar{q}_L \zeta \mathcal{L}^{\epsilon}$ ($\bar{q}_H \zeta \mathcal{H}^{\epsilon}$).

On the other hand, bearing in mind (20), (25) can be equivalently recast as

$$\dot{N}_m(t) = x_m(Q_m(t), N_m(t)) \cdot N_m(t), \ m \in \{L, H\}, \tag{26}$$

where

$$x_L(Q_L, N_L) \equiv \left(\frac{\zeta}{\phi} \cdot \mathcal{L}^{\epsilon}\right)^{\frac{1}{\gamma}} \cdot Q_L^{\frac{1}{\gamma}} \cdot N_L^{-\frac{\sigma+\gamma+1}{\gamma}}, \tag{27}$$

$$x_H(Q_H, N_H) \equiv \left(\frac{\zeta}{\phi} \cdot \mathcal{H}^{\epsilon}\right)^{\frac{1}{\gamma}} \cdot Q_H^{\frac{1}{\gamma}} \cdot N_H^{-\frac{\sigma+\gamma+1}{\gamma}}. \tag{28}$$

In a small time interval, the growth rate of average quality is equal to the expected arrival rate of a successful innovation multiplied by the quality shift it introduces: $\dot{\bar{q}}_m/\bar{q}_m = I_m \cdot (q_m^+ - q_m)/q_m$, where both the innovation rate and the quality shift are industry-independent. Time-differentiating (5), and using (26) yields

$$\dot{Q}_m(t) = (\Xi \cdot I_m(t) + x_m(Q_m(t), N_m(t))) \cdot Q_m(t),\ m \in \{L, H\}, \tag{29}$$

where the quality shift is denoted by $\Xi \equiv (q_m^+ - q_m)/q_m = \lambda^{\frac{1-\alpha}{\alpha}} - 1$. The vertical innovation rate is endogenous and will be determined as an economy-wide function below. From (5) and (21), we see that the technological-knowledge stock, Q_m, has two components: an expanding-variety or extensive component, N_m, and a quality-ladder or intensive component, $\bar{q}_m$, i.e., $Q_m(t) = \bar{q}_m(t) \cdot N_m(t)$.[16]

Then, the instantaneous growth rate of average quality $\bar{q}_m$ is a linear function of the vertical-innovation rate,

$$\frac{\dot{\bar{q}}_m(t)}{\bar{q}_m(t)} = \frac{\dot{Q}_m(t)}{Q_m(t)} - \frac{\dot{N}_m(t)}{N_m(t)} = \Xi \cdot I_m(t), \tag{30}$$

whereas we can rewrite x_m as

$$x_L(\bar{q}_L, N_L) = \left(\frac{\zeta \cdot \mathcal{L}}{\phi}\right)^{\frac{1}{\gamma}} \cdot \bar{q}_L(t)^{\frac{1}{\gamma}} \cdot N_L(t)^{-\left(\frac{\sigma+\gamma}{\gamma}\right)}, \tag{31}$$

$$x_H(\bar{q}_H, N_H) = \left(\frac{\zeta \cdot \mathcal{H}}{\phi}\right)^{\frac{1}{\gamma}} \cdot \bar{q}_H(t)^{\frac{1}{\gamma}} \cdot N_H(t)^{-\left(\frac{\sigma+\gamma}{\gamma}\right)}. \tag{32}$$

Equations (31)-(32) clarify the adopted mechanism of entry by explicitly incorporating a channel between vertical innovation and firm dynamics. The latter depends positively on the average quality level, $\bar{q}_m$, and negatively on the number of varieties, N_m. The first effect represents complementarity going from vertical innovation to the horizontal-entry rate, and the second results from the complexity and the congestion effects in horizontal entry (see (20)).

2.3. Households

The economy is populated by a fixed number of infinitely-lived households who consume and collect income from investments in financial assets and from labour. Households inelastically supply low-skilled, L, or high-skilled labour, H. Thus, total labour supply, $L + H$, is exogenous and constant. We assume consumers have perfect foresight concerning the technological change over time and choose the path of final-good aggregate consumption $\{C(t), t \geq 0\}$ to maximise discounted lifetime utility

[16] In contrast, in Acemoglu and Zilibotti (2001)'s model, where only horizontal R&D is considered, the technological-knowledge stock is simply represented by $N_m(t)$.

$$U = \int_0^{\infty} \left(\frac{C(t)^{1-\theta} - 1}{1-\theta} \right) e^{-\rho t} dt, \tag{33}$$

where $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution, subject to the flow budget constraint

$$\dot{a}(t) = r(t) \cdot a(t) + W_L(t) \cdot L + W_H(t) \cdot H - C(t), \tag{34}$$

where a denotes households' real financial assets holdings. The initial level of wealth $a(0)$ is given and the non-Ponzi games condition $lim_{t\to\infty} e^{-\int_0^t r(s)ds} a(t) \geq 0$ is also imposed. The Euler equation for consumption and the transversality condition are standard,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} \cdot (r(t) - \rho), \tag{35}$$

$$\lim_{t\to\infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0. \tag{36}$$

2.4. Macroeconomic aggregation and equilibrium innovation rates

The aggregate financial wealth held by all households is $a(t) = \sum_{m=L,H} \int_0^{N_m(t)} V_m(\omega_m, t) d\omega_m$, which, from the arbitrage condition between vertical and horizontal entry (25), yields $a(t) = \sum_{m=L,H} \eta_m(t) \cdot N_m(t)$. Taking time derivatives and comparing with (34), we get an expression for the aggregate flow budget constraint which is equivalent to the product market equilibrium condition

$$Y_{tot}(t) = C(t) + X_{tot}(t) + R_h(t) + R_v(t), \tag{37}$$

where $R_h = \sum_{m=L,H} R_{hm}$ and $R_v = \sum_{m=L,H} R_{vm}$. Substituting the expressions for the aggregate outputs (10) and (9), and for total R&D expenditures (18) and (19), we have

$$\chi_Y \cdot \Gamma(t) = C(t) + \chi_X \cdot \Gamma(t) + \eta_L(t) \cdot \dot{N}_L(t) + \eta_H(t) \cdot \dot{N}_H(t) +$$

$$+ \zeta \cdot \lambda^{\frac{1-\alpha}{\alpha}} \left(\mathcal{L}^{\epsilon} \cdot I_L(t) \cdot Q_L(t) + \mathcal{H}^{\epsilon} \cdot I_H(t) \cdot Q_H(t) \right). \tag{38}$$

Solving for, e.g., I_L, and using (25) and (26), we get the endogenous vertical-innovation rate at equilibrium in the L-technology sector

$$I_L(Q_L, Q_H, N_L, N_H, C, I_H) = \frac{1}{\zeta \cdot \lambda^{\frac{1-\alpha}{\alpha}} \cdot \mathcal{L}^{\epsilon}} \left[\chi \cdot \left([P_H(Q_H, Q_L)]^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot \frac{Q_H}{Q_L} + [P_L(Q_H, Q_L)]^{\frac{1}{\alpha}} \cdot \mathcal{L} \right) - \frac{C}{Q_L} \right] -$$

$$- \left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{\epsilon} \cdot \frac{Q_H}{Q_L} \cdot I_H - \frac{1}{\lambda^{\frac{1-\alpha}{\alpha}}} \cdot \left[\left(\frac{\mathcal{H}}{\mathcal{L}} \right)^{\epsilon} \cdot \frac{Q_H}{Q_L} \cdot x_H(Q_H, N_H) + x_L(Q_L, N_L) \right], \tag{39}$$

where $\chi \equiv \chi_Y - \chi_X = A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{2}{\alpha}} \left[(1-\alpha)^{-2} - 1\right] > 0$. Observe that P_L and P_H are (non-linear) functions of Q_H/Q_L alone (see (6) and (7)). If we further use (17) to eliminate I_H from (39), we get $I_L \equiv I_L(Q_L, Q_H, N_L, N_H, C)$. As functions $I_m(Q_L, Q_H, N_L, N_H, C)$ can be negative, the relevant innovation rates at the macroeconomic level are

$$I_m^+(Q_L, Q_H, N_L, N_H, C) = \max\left\{I_m(Q_L, Q_H, N_L, N_H, C), 0\right\}, \, m \in \{L, H\}. \tag{40}$$

Thus, there is also a complementary effect of horizontal innovation on vertical innovation: if the number of varieties is too *low*, vertical R&D shuts down.[17] From (16), we get the rate of return of capital as $r(Q_L, Q_H, N_L, N_H, C) = r_{0m} - I_m^+(Q_L, Q_H, N_L, N_H, C)$, where $r_{0L} \equiv \pi_0 \mathcal{L}^{1-\epsilon} P_L^{\frac{1}{\alpha}}/\zeta$ and $r_{0H} \equiv \pi_0 \mathcal{H}^{1-\epsilon} P_H^{\frac{1}{\alpha}}/\zeta$.

2.5. The dynamic general equilibrium

The dynamic general equilibrium is defined by the paths of allocations and price distributions $\left(\{X_m(\omega_m, t), p_m(\omega_m, t)\}, \omega_m \in [0, N_m(t)]\right)_{t\geq 0}$ and of the number of firms, quality indices and vertical-innovation rates $\left(\{ N_m(t), Q_m(t), I_m(t)\} \right)_{t\geq 0}$ for sectors $m \in \{L, H\}$, and by the aggregate paths $(C(t), r(t))_{t\geq 0}$, such that: (i) consumers, final-good firms and intermediate-good firms solve their problems; (ii) free-entry and no-arbitrage conditions are met; and (iii) markets clear. Total supplies of high- and low-skilled labour are exogenous. We focus on the region of the state space where $I_m^+(\cdot) = I_m(\cdot) > 0$, $m \in \{L, H\}$,[18] such that the equilibrium paths can be obtained from the system

$$\dot{C} = \frac{1}{\theta} \cdot (r_{0m} - I_m(Q_L, Q_H, N_L, N_H, C) - \rho) \cdot C, \tag{41}$$

$$\dot{Q}_m = (I_m(Q_L, Q_H, N_L, N_H, C) \cdot \Xi + x_m(Q_m, N_m)) \cdot Q_m, \tag{42}$$

$$\dot{N}_m = x_m(Q_m, N_m) \cdot N_m, \tag{43}$$

given $Q_m(0)$ and $N_m(0)$, and the transversality condition (36), which may be re-written as

$$\lim_{t\to\infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot \zeta \cdot (\mathcal{L}^{\epsilon} \cdot Q_L(t) + \mathcal{H}^{\epsilon} \cdot Q_H(t)) = 0. \tag{44}$$

2.6. The balanced-growth path

As the functions in system (41)-(43) are homogeneous, a BGP exists only if: (i) the asymptotic growth rates of consumption and of the quality indices are constant and equal to the economic growth rate, $g_C = g_{Q_L} = g_{Q_H} = g$; (ii) the asymptotic growth rates of the

[17] This effect is analysed in more detail in Gil, Brito, and Afonso (2013).

[18] As one can see below in Section 3 and illustrated in Appendix B, these conditions are met by our numerical simulations.

number of varieties are constant and equal, $g_{N_L} = g_{N_H}$; (iii) the vertical-innovation rates and the final-good price indices are asymptotically trendless, $g_{I_L} = g_{I_H} = g_{P_L} = g_{P_H} = 0$; and (iv) the asymptotic growth rates of the quality indices and the number of varieties are monotonously related, $g_{Q_L}/g_{N_L} = g_{Q_H}/g_{N_H} = (\sigma + \gamma + 1)$, $g_{N_m} \neq 0$, $m \in \{L, H\}$. Observe, from 26, that $x_m = g_{N_m}$ is always positive if $N_m > 0$.

It will be convenient to recast system (41)-(43), by considering the growth rate of the number of varieties, x_m, as defined by (27)-(28), the consumption rate, $z_L \equiv C/Q_L$, and the technological-knowledge bias, $Q \equiv Q_H/Q_L$, into an equivalent system in detrended variables. We then get, again with $I_m^+ = I_m > 0$,

$$\dot{x}_L = \left[\frac{\Xi}{\gamma} \cdot I_L - \left(\frac{\sigma + \gamma}{\gamma}\right) \cdot x_L\right] \cdot x_L, \tag{45}$$

$$\dot{z}_L = \left[\frac{1}{\theta} \cdot (r_{0L} - \rho) - \left(\frac{1}{\theta} + \Xi\right) \cdot I_L - x_L\right] \cdot z_L, \tag{46}$$

$$\dot{x}_H = \left[\frac{\Xi}{\gamma} \cdot I_L - \left(\frac{\sigma + \gamma}{\gamma}\right) \cdot x_H + \frac{\Xi}{\gamma} \cdot \frac{\pi_0}{\zeta} \cdot \left(\mathcal{H}^{1-\epsilon} \cdot P_H^{\frac{1}{\alpha}} - \mathcal{L}^{1-\epsilon} \cdot P_L^{\frac{1}{\alpha}}\right)\right] \cdot x_H, \tag{47}$$

$$\dot{Q} = \left[\Xi \cdot \frac{\pi_0}{\zeta} \cdot \left(\mathcal{H}^{1-\epsilon} \cdot P_H^{\frac{1}{\alpha}} - \mathcal{L}^{1-\epsilon} \cdot P_L^{\frac{1}{\alpha}}\right) + x_H - x_L\right] \cdot Q, \tag{48}$$

where $I_L \equiv I_L(Q, x_L, x_H, z_L) = I_L(Q_L, Q_H, N_L, N_H, C)$, $I_H \equiv I_H(Q, x_L, x_H, z_L) = I_H(Q_L, Q_H, N_L, N_H, C)$, $P_L \equiv P_L(Q) = P_L(Q_L, Q_H)$, and $P_H \equiv P_H(Q) = P_L(Q_L, Q_H)$. These equations, together with the transversality condition (44) and the initial conditions on $x_L(0)$, $x_H(0)$ and $Q(0)$, describe the transitional dynamics and the BGP, by jointly determining $x_L(t), z_L(t), x_H(t)$ and $Q(t)$. Then, we can determine the level variables $N_m(t)$, $C(t)$ and $Q_L(t)$ (respectively, $Q_H(t)$), for a given $Q_H(t)$ ($Q_L(t)$).

The households transversality condition (44) can also be related to the detrended variables,

$$\lim_{t \to \infty} e^{-\rho t} \cdot z_L(t)^{-\theta} \cdot \zeta \cdot (\mathcal{L}^{\epsilon} + \mathcal{H}^{\epsilon} \cdot Q(t)) \cdot Q_L(t)^{1-\theta} = 0, \tag{49}$$

where z_L and Q are stationary along the BGP, as shown above. Let $Q_L = \hat{q}_L e^{gt}$, where $\hat{q}_L$ denotes detrended Q_L (i.e., stationary along the BGP), and substitute in (49), to see that the transversality condition implies $\rho \geq (1 - \theta)g$. Using the Euler equation, $g = (r - \rho)/\theta$, the latter condition can be written alternatively as $r > g$. This condition also guarantees that attainable utility is bounded, i.e., the integral (33) converges.

Proposition 1. *Let* $\tilde{r}_{0m} - \rho > 0$, *and* $0 < \frac{\frac{\Xi}{\theta}(\tilde{r}_{0m} - \rho)}{\Xi(\sigma+\gamma+1)+(\sigma+\gamma)/\theta} < \overline{x} \equiv \chi \cdot \tilde{\Gamma}/(\tilde{Q}_L \mathcal{Z})$, $m \in \{L, H\}$. *The interior steady state,* $(\tilde{x}_L, \tilde{z}_L, \tilde{x}_H, \tilde{Q})$, *exists and is unique, with:*

$$\tilde{Q} = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{1-2\epsilon}, \tag{50}$$

$$\tilde{x}_L = \tilde{x}_H = \frac{\frac{\Xi}{\theta}(\tilde{r}_{0m} - \rho)}{\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma)}, \tag{51}$$

$$\tilde{z}_L = \chi\left(\tilde{P}_H^{\frac{1}{\alpha}}\mathcal{H}\tilde{Q} + \tilde{P}_L^{\frac{1}{\alpha}}\mathcal{L}\right) - \left(\zeta\lambda^{\frac{1-\alpha}{\alpha}}\tilde{I}_L + \zeta\tilde{x}_L\right)\left(\mathcal{H}^{\epsilon}\tilde{Q} + \mathcal{L}^{\epsilon}\right), \tag{52}$$

where

$$\tilde{I}_L = \tilde{I}_H = \left(\frac{\sigma + \gamma}{\Xi}\right)\tilde{x}_L = \left(\frac{\sigma + \gamma}{\Xi}\right)\tilde{x}_H, \tag{53}$$

and $\tilde{r}_{0L} \equiv \frac{\pi_0}{\zeta}\mathcal{L}^{1-\epsilon}\tilde{P}_L^{\frac{1}{\alpha}} = \tilde{r}_{0H} \equiv \frac{\pi_0}{\zeta}\mathcal{H}^{1-\epsilon}\tilde{P}_H^{\frac{1}{\alpha}}$, $\tilde{P}_L = \exp(-\alpha)\cdot\left[1 + (\mathcal{H}/\mathcal{L})^{1-\epsilon}\right]^{\alpha}$, $\tilde{P}_H = \exp(-\alpha)\cdot\left[1 + (\mathcal{H}/\mathcal{L})^{\epsilon-1}\right]^{\alpha}$, *and* $\mathcal{Z} \equiv \zeta\left[(\sigma + \gamma)/\Xi + \sigma + \gamma + 1\right]\left(\mathcal{H}^{\epsilon}\tilde{Q} + \mathcal{L}^{\epsilon}\right) > 0$.

Proof is given in Appendix B.

Equations (50)-(52) represent a steady-state equilibrium with balanced growth in the usual sense, such that the endogenous growth rates are positive,

$$\tilde{g}_{N_L} = \tilde{g}_{N_H} = \tilde{x}_L = \tilde{x}_H > 0, \tag{54}$$

$$\tilde{g}_{Q_L} = \tilde{g}_{Q_H} = \tilde{g} = \frac{\frac{\Xi}{\theta}(\tilde{r}_{0L} - \rho)(\sigma + \gamma + 1)}{\Xi(\sigma + \gamma + 1) + \frac{1}{\theta}(\sigma + \gamma)} > 0. \tag{55}$$

Thus, our model predicts, under a sufficiently productive technology, a BGP with constant positive growth rates, g and g_{N_m}, where the former exceeds the latter by the growth of intermediate-good quality, $\Xi \cdot I_m$ (see equation (30)). It is clear from (53) that such a BGP only exists if both the growth rate of the number of varieties and the vertical-innovation rate are positive. Then, the economic growth rate is propelled by the growth rate of the number of varieties plus the growth rate of intermediate-good quality, $\tilde{g} = \tilde{g}_{N_m} + \Xi \cdot \tilde{I}_m$ (see (29)), in line with the well-known view that industrial growth proceeds both along an intensive and an extensive margin. However, reflecting the distinct nature of vertical and horizontal innovation (immaterial versus physical) and the consequent asymmetry in terms of R&D complexity costs (see (12) and (20)), variety expansion is ultimately sustained by the endogenous quality upgrade, as the expected growth of intermediate-good quality due to vertical R&D makes it attractive, in terms of intertemporal profits, for potential entrants to always put up an entry cost, in spite of its increase with N_m.[19] Thus, vertical innovation arises as the ultimate growth engine, in the sense that it sustains both variety expansion and aggregate output growth.

The level variables are $\tilde{C}$, $\tilde{N}_m$, and $\tilde{Q}_m$, $m \in \{L, H\}$, where $\tilde{Q}_L$ is undetermined and

$$\tilde{C} = \tilde{z}\tilde{Q}_L, \tag{56}$$

$$\tilde{N}_L = \left(\frac{\zeta}{\phi}\mathcal{L}^{\epsilon}\right)^{\frac{1}{\sigma+\gamma+1}}(\tilde{x}_L)^{\frac{-\gamma}{\sigma+\gamma+1}}\left(\tilde{Q}_L\right)^{\frac{1}{\sigma+\gamma+1}}, \tag{57}$$

[19] Indeed, it is shown in Barro and Sala-i-Martin (2004, ch. 6) that, in a setting with only horizontal R&D, the complexity cost in (20) generates a constant N along the BGP (provided population growth is zero).

$$\tilde{N}_H = \left(\frac{\zeta}{\phi}\mathcal{H}^{\epsilon}\right)^{\frac{1}{\sigma+\gamma+1}} (\tilde{x}_H)^{\frac{-\gamma}{\sigma+\gamma+1}} \left[\left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{1-2\epsilon} \tilde{Q}_L\right]^{\frac{1}{\sigma+\gamma+1}}. \tag{58}$$

From the expressions for X_L and X_H (see (9)) and for N_L and N_H above, combined with (50) and (51), we derive the steady-state expressions for relative production and the relative number of firms (i.e., H- vis-à-vis L-technology sector),

$$\tilde{X} \equiv \left(\frac{\tilde{X_H}}{X_L}\right) = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{1-\epsilon}, \tag{59}$$

$$\tilde{N} \equiv \left(\frac{\tilde{N_H}}{N_L}\right) = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{\frac{1-\epsilon}{\sigma+\gamma+1}}. \tag{60}$$

Finally, by considering equations (11) and (50), we get the steady-state skill premium

$$\tilde{W} = \frac{h}{l}\left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{-\epsilon}. \tag{61}$$

In order to characterise the interior steady state $(\tilde{x}_L, \tilde{z}_L, \tilde{x}_H, \tilde{Q})$ in terms of local stability, we linearise the dynamical system (45)-(48) in a neighbourhood of $(\tilde{x}_L, \tilde{z}_L, \tilde{x}_H, \tilde{Q})$ and obtain the following fourth-order system

$$\begin{pmatrix} \dot{x}_L \\ \dot{z}_L \\ \dot{x}_H \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} a_{11}\tilde{x}_L & a_{12}\tilde{x}_L & a_{13}\tilde{x}_L & \frac{\Xi}{\gamma}\left(\frac{\tilde{\partial I_L}}{\partial Q}\right)\tilde{x}_L \\ a_{21}\tilde{z}_L & a_{22}\tilde{z}_L & a_{23}\tilde{z}_L & a_{24}\tilde{z}_L \\ a_{31}\tilde{x}_H & a_{32}\tilde{x}_H & a_{33}\tilde{x}_H & a_{34}\tilde{x}_H \\ -\tilde{Q} & 0 & \tilde{Q} & -\Xi\mathcal{S}_1\tilde{Q} \end{pmatrix} \begin{pmatrix} x_L - \tilde{x}_L \\ z_L - \tilde{z}_L \\ x_H - \tilde{x}_H \\ Q - \tilde{Q} \end{pmatrix}, \tag{62}$$

given the initial conditions $x_L(0)$, $x_H(0)$ and $Q(0)$ and the transversality condition (49). The Jacobian matrix $J\left(\tilde{x}_L, \tilde{x}_H, \tilde{z}_L, \tilde{Q}\right)$, in (62), is evaluated at the steady state, where we define

$a_{11} \equiv -\frac{1}{\gamma}\left(\frac{\Xi}{\Xi+1}\right)\mathcal{S}_0 - \frac{\sigma+\gamma}{\gamma}$; $a_{12} \equiv -\frac{1}{\zeta\mathcal{L}^{\epsilon}}\frac{1}{\gamma}\left(\frac{\Xi}{\Xi+1}\right)\mathcal{S}_0$; $a_{13} \equiv -\frac{1}{\gamma}\left(\frac{\Xi}{\Xi+1}\right)\mathcal{S}_0\left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{1-\epsilon}$;

$a_{21} \equiv \left(\frac{1}{\theta}+\Xi\right)\frac{1}{\Xi+1}\mathcal{S}_0 - 1$; $a_{22} \equiv \left(\frac{1}{\theta}+\Xi\right)\frac{1}{\Xi+1}\frac{1}{\zeta\mathcal{L}^{\epsilon}}\mathcal{S}_0$; $a_{23} \equiv \left(\frac{1}{\theta}+\Xi\right)\frac{1}{\Xi+1}\left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{1-\epsilon}\mathcal{S}_0$;

$a_{24} \equiv \frac{\pi_0}{\theta\zeta}\frac{1}{2e}\mathcal{L}^{1-\epsilon}\left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{\epsilon} - \left(\frac{1}{\theta}+\Xi\right)\left(\frac{\tilde{\partial I_L}}{\partial Q}\right)$;

$a_{31} \equiv a_{11} + \frac{\sigma+\gamma}{\gamma}$; $a_{32} \equiv a_{12}$; $a_{33} \equiv a_{13} - \frac{\sigma+\gamma}{\gamma}$; $a_{34} \equiv a_{14} - \frac{\Xi}{\gamma}\mathcal{S}_1$;

with

$\mathcal{S}_0 \equiv 1/\left[1 + \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{1-\epsilon}\right]$; $\mathcal{S}_1 \equiv \frac{\pi_0}{\zeta}\frac{1}{2e}\mathcal{L}^{1-\epsilon}\frac{1}{\tilde{Q}\mathcal{S}_0}$;

$\left(\frac{\tilde{\partial I_L}}{\partial Q}\right) = \left[\mathcal{S}_1 - \left(\frac{1}{\Xi+1} + \frac{\sigma+\gamma}{\Xi}\right)\tilde{x}_H\right]\left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{\epsilon}\mathcal{S}_0 + \frac{1}{\zeta}\frac{1}{e}\mathcal{H}^{1-\epsilon}\frac{1}{\Xi+1}\chi\frac{1}{\tilde{Q}}$.

Since there are three predetermined variables, x_L, x_H and Q, and one jump variable, z_L, saddle-path stability of the interior equilibrium $\left(\tilde{x}_L, \tilde{x}_H, \tilde{z}_L, \tilde{Q}\right)$ requires that

$J\left(\tilde{x}_L, \tilde{x}_H, \tilde{z}_L, \tilde{Q}\right)$ has three eigenvalues with a negative real part and one with a positive real part, hence implying $\det(J(\tilde{x}_L, \tilde{x}_H, \tilde{z}_L, \tilde{Q})) < 0$. However, as the latter condition is compatible with both one and three eigenvalues with negative real part, further conditions must be satisfied so that saddle-path stability applies. These conditions are particularly hard to check analytically, considering that $J\left(\tilde{x}_L, \tilde{x}_H, \tilde{z}_L, \tilde{Q}\right)$ is a 4×4 matrix with just one zero element.[20] In this context, we perform a numerical exercise to check the existence of three eigenvalues with negative real part and one with a positive real part (see Appendix C) and conclude that:

Remark 1. *The interior steady state is locally saddle-path stable for the typical baseline parameter values, but also over a wide range of parameter sets.*

Finally, it is noteworthy that, since the dimension of the stable manifold is larger than unity (it is three-dimensional), there are multiple independent sources of stability in the dynamic system, but which interact between themselves. Thus, non-monotonic trajectories can emerge in the predetermined variables along transition even in the case of a linearised dynamic system (see, e.g., Eicher and Turnovsky, 2001, whose endogenous growth model features a two-dimensional stable manifold).

3. Industry and aggregate dynamics

3.1. Comparative dynamics

This section focuses on the change of the industry structure (high- versus low-tech sectors) over time and on its relationship with the dynamics of the aggregate variables, namely the economic growth rate and the real interest rate. To that end, we explore the transitional dynamics results of the model triggered by an unanticipated one-off shock in the proportion of high-skilled labour.[21] Global dynamics, as opposed to local dynamics, allows us to carry out a comparative dynamics exercise without restricting the analysis to a sufficiently close neighbourhood of the steady state and, thus, to small shifts in the parameters and the exogenous variables. As shown in the previous section, the dynamic system in detrended variables is four dimensional, with three predetermined endogenous variables, and is highly non linear. Therefore, we resort to numerical methods to study its global dynamics.

[20] Since the characteristic polynomial for the linearised system (62) is of the form $P_o(\beta) = \beta^4 + b_3\beta^3 + b_2\beta^2 + b_1\beta + b_0$, where β denote the characteristic roots of matrix $J\left(\tilde{x}_L, \tilde{x}_H, \tilde{z}_L, \tilde{Q}\right)$, and the coefficients b_{4-k}, $k = 1, .., 4$, equal the sum of the kth-order principal minors (in particular, $b_0 = det(J)$ and $b_3 = -tr(J)$), those conditions rely on the solution for a quartic equation (see, e.g., Barnett, 1971; King, 1996; Brito, 2004). Considering partitions in the space of b_{4-k} for the number of pairs of complex eigenvalues, it can be shown that the necessary and sufficient conditions for the existence of three eigenvalues with negative real part and one with a positive real part are:
(i) for zero complex eigenvalues, $b_0 < 0$ and $(b_1 < 0, b_2 < 0$ or $b_2 > 0, b_3 > 0)$;
(ii) for one pair of complex conjugate eigenvalues, $b_0 < 0$ and $(b_3 > 0, h_1 < 0$ or $b_1 < 0, b_3 > 0, h_1 = 0$ or $b_1 < 0, h_1 > 0)$, where $h_1 = b_0 b_3^2 + b_1^2 - b_1 b_2 b_3$.

[21] In Appendix A, we present evidence supporting (statistical) causality running from the share of the high skilled to the share of production of the high-tech sector found in European data.

We start by considering that the economy is in the (pre-shock) steady state; then, we posit an unanticipated one-off shock that shifts the steady state (the post-shock steady state). Together with the transversality condition (equation (49)) and the initial conditions on the predetermined variables, $x_L(0)$, $x_H(0)$ and $Q(0)$ (which are the respective pre-shock steady-state values), the dynamic system (45)-(48) describes the transitional dynamics after the shock, towards the new (post-shock) steady state. Since these boundary conditions apply at different points in time, this amounts to a boundary-value problem: we are given initial conditions on the predetermined variables, which apply at $t = 0$ (immediately after the shock occurs), and a terminal condition, the transversality condition, which applies asymptotically at the new steady state. The job of the numerical algorithm is to express this latter condition in terms of the a priori unknown initial value of the jump variable, $z_L(0)$, and the ensuing time path (of z_L and, thereby, of x_L, x_H and Q) towards the new steady state, in case a stable manifold exists. The dynamic system is solved by numerical integration using a finite difference method implementing the three-stage Lobatto IIIa formula with the software MatLab (version R2014a).[22,23]

Bearing in mind the dynamic system (45)-(48), the time-path solutions of the three predetermined variables, x_L, x_H and Q, and the jump variable, z_L, allow us to assess the industry dynamics, measured by time-path of the relative number of firms (the ratio of the number of firms in the H- to the L-technology sector),

$$N(t) = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{\frac{\epsilon}{\sigma+\gamma+1}} \left(\frac{x_H(t)}{x_L(t)}\right)^{-\left(\frac{\gamma}{\sigma+\gamma+1}\right)} Q(t)^{\frac{1}{\sigma+\gamma+1}}, \tag{63}$$

relative production (the ratio of production in the H- to the L-technology sector),

$$X(t) = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{\frac{1}{2}} Q(t)^{\frac{1}{2}}, \tag{64}$$

the sectoral growth rates in the H- and in the L-technology sectors,

$$g_{Q_H}(t) = I_L(t) \cdot \Xi + x_L(t), \tag{65}$$

$$g_{Q_L}(t) = I_H(t) \cdot \Xi + x_H(t), \tag{66}$$

and the skill premium,

$$W(t) = \frac{h}{l}\left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{-\frac{1}{2}} Q(t)^{\frac{1}{2}}. \tag{67}$$

[22] The code, which is provided through the MatLab bvp4c function, performs a mesh selection and error control based on the residual of the continuous solution (further information can be found in the MatLab help-documentation).

[23] As an alternative numerical procedure, we also used the "Forward Shoot 1D" algorithm by Atolia and Buffie (2009), which is a Mathematica software implementation. In the case of our dynamic system, which has three pre-determined endogenous variables, this numerical method yielded similar results to the MatLab built-in algorithm but with a prohibitive computational time, especially when several executions were to be made.

At the aggregate level, the dynamics are analysed by computing the time-path of the economic growth rate,

$$g(t) = \frac{\mathcal{L}^{\frac{1}{2}} \cdot g_{Q_L}(t) + (Q(t) \cdot \mathcal{H})^{\frac{1}{2}} \cdot g_{Q_H}(t)}{\mathcal{L}^{\frac{1}{2}} + (Q(t) \cdot \mathcal{H})^{\frac{1}{2}}}, \tag{68}$$

and the real interest rate,

$$r(t) = \frac{\pi_0}{\zeta} \cdot \mathcal{L}^{1-\epsilon} \cdot P_L(t)^{\frac{1}{\alpha}} - I_L(t) = \frac{\pi_0}{\zeta} \cdot \mathcal{H}^{1-\epsilon} \cdot P_H(t)^{\frac{1}{\alpha}} - I_H(t). \tag{69}$$

The effects of a shock in the relative supply of skills, H/L, on the variables of interest are then studied under three different scenarios for the market complexity cost parameter, ϵ (and thus the degree of scale effects on industrial growth, $1-\epsilon$). The three scenarios feature, relatively to the baseline case, a rise in H/L by considering a jump in high-skilled labour, H, from 0.1 to 0.19, while the low-skilled labour, L, is normalised to unity.[24] This then implies that the initial and the new steady state are characterised by, respectively, $H/L = 0.1$ and $H/L = 0.19$. These correspond to the average value of the proportion of the high skilled (measured by the ratio of college to non-college graduates) in the 14 countries presented in Figure 1, as found in Barro and Lee (2010)'s data set for 1980 and 1995, respectively.[25]

In Scenario 1, we focus on $\epsilon = 0$ or values of ϵ near zero, that is, following, e.g., Acemoglu (1998) and Acemoglu and Zilibotti (2001), market-scale effects prevail. Scenario 2 is characterised by $\epsilon = 1$ or values of ϵ near unity, in which case the market-scale effects are (totally or almost totally) removed and the price-channel effects prevail, in line with Jones (1995) and others. Finally, in Scenario 3, we let $\epsilon = 0.5$, meaning that market-size-channel and price-channel effects offset each other exactly, such that the technological-knowledge bias, Q, is independent of the relative supply of skills, H/L, on the BGP.

As for the remaining parameters of the model, we define the following set of baseline values:$\rho = 0.02$; $\theta = 1.5$; $A = 1$; $\phi = 1$; $\alpha = 0.6$; $\lambda = 2.5$; $\sigma = 1.2$; $\gamma = 1.2$; $l = 1.0$; $h = 1.3$.[26] Given that, along the BGP, we have $g_{Q_m} - g_{N_m} = (\sigma+\gamma)g_{N_m}$, we let $\sigma+\gamma = 2.4$ to match the ratio between the growth rate of the average firm size and the growth rate of the number of firms found in cross-section data for European countries in the period 1995-2007, while the values for l and h are in line with Afonso and Thompson (2011), also drawn from European data. Since it has no impact on the growth rates, ϕ was normalised to unity, while the values for θ, ρ, λ and α were set in line with the standard literature

[24] Available data suggests that increases in H have been clearly larger than those in L over time. For instance, the annual average variation of college (the usual proxy for high-skilled labour) and non-college graduates (the proxy for the low-skilled) was, respectively, 5.04 and 0.15 percent, computed as the average of the 14 European countries presented in Figure 1 for the 1980-1995 period. The data is from the Barro and Lee (2010)'s data set.

[25] The first year (1980) is determined by data availability for production, whereas the final year (1995) was chosen by observing that by that time there is a significant acceleration of the share of the high skilled and of the share of production of the high-tech sector (see fn. (3) and Figure 1).

[26] The value of the discount rate, ρ, implies that each period in our model represents a year.

	ζ	$\tilde{z}$	$\tilde{Q}$	$\tilde{x}_m$	$\tilde{g}$	$\tilde{I}_m$	$\tilde{r}$
Scenario 1 ($\epsilon = 0$)	0.42	0.1204	0.2600	0.0073	0.0248	0.0208	0.0572
Scenario 2 ($\epsilon = 1$)	0.66	0.3032	3.8461	0.0074	0.0252	0.0211	0.0577
Scenario 3 ($\epsilon = 0.5$)	0.50	0.1729	1.0000	0.0074	0.0251	0.0210	0.0577

Table 1: Calibration of the vertical-R&D flow fixed cost, ζ, under three scenarios for ϵ, in order to match the cross-country average of the per capita GDP growth rate over the period 1995-2007, for a sample of 14 European countries in the Eurostat on-line database (available at `http://epp.eurostat.ec.europa.eu`), when $H/L = 0.19$.

	ζ	$\tilde{z}$	$\tilde{Q}$	$\tilde{x}_m$	$\tilde{g}$	$\tilde{I}_m$	$\tilde{r}$
Scenario 1 ($\epsilon = 0$)	0.42	0.0976	0.1300	0.0063	0.0214	0.0179	0.0521
Scenario 2 ($\epsilon = 1$)	0.66	0.3032	7.6923	0.0074	0.0252	0.0211	0.0577
Scenario 3 ($\epsilon = 0.5$)	0.50	0.1414	1.0000	0.0064	0.0217	0.0182	0.0526

Table 2: Steady state values when $H/L = 0.1$, considering the calibrated values of ζ presented in Table 1.

(see, e.g., Barro and Sala-i-Martin, 2004). The values of the remaining parameters, A and ζ, were chosen in order to calibrate the after-shock BGP economic growth rate, g, around 2.5 percent/year (see Table 1), matching the average of the per capita GDP growth rate across European countries over the period 1995-2007.[27] Then, the implied value for the Poisson rate, I, is around 2.1 percent/year; this means that the model predicts an average lifetime of a design of 47.6 years, which is within the range of values considered in the empirical literature (e.g., Caballero and Jaffe, 1993). Moreover, the implied value for the real interest rate, r, is about 5.8 percent, broadly in line with the empirical value for the long-run average real return on the stock market, and which should be taken as the equilibrium rate of return to R&D (e.g., Mehra and Prescott, 1985). Nonetheless, extensive sensitivity analysis has shown that the results presented hereafter are robust, in qualitative terms, to changes in the underlying parameters.

[Table 1 goes about here]

[Table 2 goes about here]

In what follows, we are interested in analysing both the long-run effects (shift in the BGP values) and its decomposition into short-run and transitional-dynamics effects of

[27] The source of the referred to cross-country data is the Eurostat on-line database (link at `http://epp.eurostat.ec.europa.eu`). The sample of 14 European countries used to compute the cross-section average is the same as the one used in Figure1, Section 1. See also fn.1 .

a unanticipated one-off increase in the relative supply of skills, H/L. In particular, we consider an increase in the amount of high-skilled labour, H, with the low-skilled labour, L, remaining constant through time. As we will see, the degree of scale effects, $1-\epsilon$, is a key, albeit indirect, determinant of the characteristics of transitional dynamics, by influencing simultaneously the short- and the long-run response to the shock.

Scenario 1 - "Market-size-channel effect prevails" (small ϵ, Figure 2)

Industry dynamics: short-run effect The increase in H generates an increase in resources in terms of the final good (see (10)) available for R&D. However, the allocation of resources is nonbalanced between sectors. The direct strong positive impact on the profitability of the production of intermediate goods in the H-technology sector (see (8)) more than compensates for the decrease in the price index, P_H, due to the fall in the marginal productivity of labour of that sector; then, an increase in the vertical-innovation rate I_H occurs due to the predominance of the market-size channel. Moreover, given that L is constant, profits in the H-technology sector increase more than in the L-technology sector. The diversion of resources from the latter to the former sector induces a fall in I_L, although only slightly because of the countervailing effect of the upward jump in the price index, P_L . As a result, the sectoral growth rate in the H-technology sector, g_{Q_H} jumps upwards, while the growth rate in the L-technology sector, g_{Q_L}, experiences a small shift downwards.[28]

Industry dynamics: transitional-dynamics effect. After the initial jump, g_{Q_H} takes a downward path, while g_{Q_L}follows an upward path; the former reflects the behaviour of the *intensive* margin (the vertical innovation rate, I_H, falls over transition) which more than compensates for the extensive margin (the growth rate of the number of varieties, x_H, increases); in contrast, the increase in g_{Q_L} reflects the behaviour of both the *intensive* and the *extensive* margin (I_L and x_L increase). After the initial level effect, we have $I_H > I_L$, whereas the time-paths of I_H and I_L respond to a feedback effect: I_H and I_L are commanded by the dynamics of the price indices – P_H decreases and P_L increases towards the new steady state –, which, in turn, reflects the increase in the technological-knowledge bias, Q; the bias rises, at a decreasing rate, due to the difference in profitability between the H- and the L-technology sector, and hence between I_H and I_L, induced by the initial jump in H. In turn, x_H and x_L rise due to the increase in the sectoral technological-knowledge, Q_H and Q_L (given $I_H > 0$ and $I_L > 0$), reflecting the complementarity between the horizontal-entry rate and the technological-knowledge stock (see (26)); however, the fact that $I_H > I_L$ means that the costs pertaining to horizontal entry are only slightly compensated for in the L-technology sector at the beginning of the transition path (see (45) and (47)), while the opposite occurs in the other sector, therefore explaining the different shape of the time-paths of x_H and x_L (concave and convex, respectively). Since $x_H > x_L$ throughout transition, the relative number of firms, N, increases. However, the H-technology sector experiences an acceleration

[28] Notice that, since x_L, x_H and Q are pre-determined variables in the dynamic system (45)-(48), they do not experience any short-term response to the exogenous shock.

in terms of the extensive margin that exceeds the one in the L-technology sector, as explained earlier; as a result, the congestion effects in horizontal R&D reduce the velocity of convergence of N (see (63)). In contrast, the absence of congestion effects in vertical R&D determines a faster increase in relative production, X, commanded by Q (see (64)),[29] and thus also a rise in the relative firm size, X/N.[30]

Industry dynamics: long-run effect. Both g_{Q_H} and g_{Q_L} settle down at a level that is higher than the pre-shock BGP level, reflecting the net positive scale effect (market-size effect) associated to the exogenous shock. Overall, the model predicts that the short-run positive scale effect in the economic growth rate overshoots the long-run positive scale effect in the H-technology sector, while, in the L-technology sector, the negative short-run scale effect is more than compensated by the long-run positive scale effect. The relative number of firms, relative production and relative firm size all increase relatively to the pre-shock BGP level.

Aggregate dynamics The economic growth rate, g, and the real interest rate, r, experience only a very slight increase along the transition path; thus, the long-run effect of an increase in H results almost entirely from the short-run response to the exogenous shock. The stability of the aggregate variables over transition reflects the opposing movements of the sectoral growth rates, g_{Q_H} and g_{Q_L}, in case of g,[31] and the parallel movements of the vertical innovation rate, I_m, and the price index, P_m, within each m-technology sector, in case of r. As explained above, the common cause is the technological-knowledge bias effect arising from the increase in H.

[Figure 2 goes about here]

Scenario 2 - "Price-channel effect prevails" (large ϵ, Figure 3)

Industry dynamics: short-run effect By removing the scale effects, the chain of effects is induced by the price channel, by which there are stronger incentives to improve technologies when the goods that they produce command higher prices. Hence, the direct positive impact of the increase in H on the profitability of the production of intermediate goods in the H-technology sector is now more than compensated by the decrease in the price index, P_H; then, a decrease in the vertical-innovation rate I_H occurs due to the

[29] Observe that Q has also a direct effect on N (see (63)), but it is dampened by the complexity and congestion effects associated to horizontal R&D and which are regulated by parameters σ and γ.

[30] Eventually, X/N will take a slight fall as the economy gets closer to the new BGP because, since the speed of convergence of X is larger than that of N (see Figure 5, below), the former will stop increasing before the latter.

[31] In fact, since g is a weighed average of the two sectoral growth rates, with the weight being a function of the technological-knowledge bias, Q (see (68)), i.e., the share of the H-technology sector in terms of the technological-knowledge stock, then Q also plays a direct role in the dynamics of g. More specifically, the effect of the relatively intense fall in g_{Q_H} is dampened by the increase in Q over transition.

predominance of the price channel. Consequently, a diversion of resources arises from the H- to the L-technology sector, inducing an increase in I_L. As a result, the sectoral growth rate in the H-technology sector, g_{Q_H} jumps downwards, while the growth rate in the L-technology sector, g_{Q_L}, experiences a shift upwards.

Industry dynamics: transitional-dynamics effect After the initial jump, g_{Q_H} takes an upward path, while g_{Q_L} follows a downward path. In order to decompose this behaviour in terms of intensive and extensive margin, it is convenient to consider two separate cases, one for $\epsilon \in (0.5; \bar{\epsilon})$ and the other for $\epsilon \in (\bar{\epsilon}; 1]$, where $\bar{\epsilon} \in (0.5; 1)$ depends on the values of the other parameters.

(a) With ϵ up to $\bar{\epsilon}$, the reduction of the sectoral growth rate in the L-technology sector reflects the behaviour of the *intensive* margin (i.e., the fall in vertical innovation rate, I_L), which more than compensates the extensive margin (the growth rate of the number of varieties, x_L, increases over most part of the transition path); in contrast, the acceleration of activity in the H-technology sector reflects the behaviour of both the *intensive* and the *extensive* margin (I_H increases monotonically along the transition path, while x_H increases over most part of the transition path). After the initial level effect, we have $I_H < I_L$, with I_H and I_L are commanded by, respectively, the increase in P_H and the decrease in P_L towards the new steady state, which, in turn, reflect the decrease in the technological-knowledge bias, Q; the bias falls, at a decreasing rate, due to the difference in profitability between the H- and the L-technology sector, and hence between I_H and I_L, induced by the initial jump in H. In turn, x_H and x_L rise due to the increase in the sectoral technological-knowledge, Q_H and Q_L, given $I_H > 0$ and $I_L > 0$; however, the fact that $I_H < I_L$ means that the costs pertaining to horizontal entry are only slightly compensated for in the H-technology sector at the beginning of the transition path, while the opposite occurs in the other sector, which explains the distinct shape of the time-paths of x_H and x_L (the shapes are symmetric to the ones in Scenario 1). Since $x_H < x_L$ all over transition, the relative number of firms, N, decreases. However, the L-technology sector experiences an acceleration in terms of the extensive margin that exceeds the one in the H-technology sector, as already explained; hence, the congestion effect pertaining to horizontal R&D reduces the velocity at which N is falling. Benefiting from the absence of congestion effects in vertical R&D, relative production, X, takes a faster fall commanded by Q, and thus inducing a decrease in the relative firm size, X/N.[32]

(b) When $\epsilon > \bar{\epsilon}$, x_H and x_L display marked non-monotonic time paths, the former being convex and the latter being concave. As already explained, after the initial level effect, we have $I_H < I_L$. However, as the price channel gets stronger (i.e., ϵ increases towards unity), the downward jump in I_H becomes larger, such that eventually the vertical-innovation rate is not able to compensate for the costs pertaining to

[32] Eventually, X/N will increase slightly as the economy approaches the new BGP because X converges at a higher speed than N (see Figure 5, below).

horizontal entry at the beginning of the transition path. Under this scenario, the horizontal entry rate x_H will start the transitional dynamics by following a downward path, but since I_H increases monotonically over transition, the latter will eventually become large enough to overturn the costs effect; from that point on, x_H will take an upward path towards the new steady state.[33] In the L-technology sector, an opposite behaviour will occur. Thus, in both sectors, the transition process begins propelled by the intensive margin, although partially countervailed by the extensive margin, but eventually the convergence to the long-run equilibrium is carried out at the expense of both margins. The relative number of firms, relative production and relative firm size are characterised by a behaviour that is similar to the one in (a).

Industry dynamics: long-run effect The effect on the industrial growth rates, relative production and the relative number of firms is very small (if ϵ is near unity) or non-existent (if $\epsilon = 1$).

Aggregate dynamics The growth rate and the real interest rate remain approximately constant in response to the shock in H, exhibiting time-paths that are (slightly) non-monotonic (in the case of the real interest rate) and very flat over transition, since scale effects are totally (or almost totally) removed from the model.

[Figure 3 goes about here]

Scenario 3 - "Balanced market-size-channel and price-channel effects" ($\epsilon = 0.5$, Figure 4)

Industry dynamics: short-run effect For intermediate values of ϵ, the market-size and the price channel are in action with similar strength, which implies that the incentives for vertical R&D arising from the shock in H tend to be shared roughly equally between the L- and the H-technology sector. Overall, this means that more resources become available for a simultaneous, but relatively small, increase in the vertical-innovation rates, I_L and I_H, and hence in the sectoral growth rates, g_{Q_L} and g_{Q_H}.

Industry dynamics: transitional-dynamics effect The endogenous variables experience only a slight (or no) change along the transition path in both sectors, reflecting the balance between the market-size and the price channel; in particular, this balance determines that the technological-knowledge bias, Q, is unresponsive to changes in the proportion of high-skilled labour. Both g_{Q_L} and g_{Q_H} then follow upward paths along the transition to the new steady state, with the acceleration of economic activity now

[33] Notice that when the market-size channel prevails, as in Scenario 1, the fall in I_L is only slight because of the countervailing effect of the upward jump in the price index, P_L. Thus, a non-monotonic behaviour of x_L does not occur or is very mild.

being commanded by the extensive margin in both sectors, since x_H and x_L increase over transition. This more than compensates the intensive margin, as I_H and I_L experience a slight fall: given the unresponsiveness of Q to the exogenous shock, the decrease in the vertical-innovation rates reflects essentially the shift of resources towards the extensive margin over transition. The independence of Q relatively to the relative supply of skills implies that the relative number of firms, relative production and the relative firm size are unchanged along the transition path, too.

Industry dynamics: long-run effect Eventually, both g_{Q_L} and g_{Q_H} will settle down at a level that is higher than the pre-shock steady state level, with the short-run effect of the exogenous shock translating almost one-to-one into the long-run effect. In the case of the relative number of firms, relative production and the relative firm size, the long-run effect results strictly from the short-run response to the exogenous shock.

Aggregate dynamics The growth rate and the real interest rate experience only a very slight increase along the transition path; thus, the long-run effect of an increase in H results almost entirely from the short-run response to the exogenous shock, but which is smaller than in Scenario 1, since scale effects are partially removed from the model.

[Figure 4 goes about here]

3.2. Discussion and a simple calibration

It is noteworthy that, except for the knife-edge case in which market-size-channel and price-channel effects offset each other exactly (Scenario 3), as the economy evolves towards the new BGP, there is a noticeable shift of economic activity between sectors, specially in terms of production but also of the number of firms. For the baseline values of the parameters considered in Section 3.1 and with $\epsilon = 0$ ($\epsilon = 1$), relative production, X, and the relative number of firms, N, increase a total of, respectively, 13.3 and 10.7 percentage points (decrease 42.5 and 19.9 points) over 120 years, while the economic growth rate, g, and the real interest rate, r, increase a total of, respectively, 0.34 and 0.51 percentage points (no accumulated variation).[34] Thus, whatever the scenario considered, the aggregate variables remain roughly unchanged over the 120 years, which implies, in particular, that the share of the high-tech sector has roughly a null correlation with economic growth over the adjustment.

[34] In the model, the shock in H/L implies an immediate jump (the "short-run effect" analysed in Section 3.1) in some of the variables of interest. However, we are obviously conducting an artificial experiment by considering a one-off jump in H/L; in reality, the relative supply of skills should be expected to have followed a continuous time-path, even if at an accelerated rate, between the 80's and the 90's. Thus, more realistically, and in particular in Scenario 1, the short-run impact on those variables should be imagined as being spread out over a certain period of time, instead of as a discontinuous jump. Bearing this in mind, under that scenario, we assess the change in the variables of interest by considering both the discrete short-run adjustment and the ensuing transition path.

We would also like to emphasise that, as depicted by Figure 5, the speed of adjustment to a positive shock in the relative supply of skills may be quite different across variables, whether we compare them at the aggregate or the industry level. The speeds of convergence are also time-varying for each variable. The one with the slowest speed is clearly the relative number of firms, in contrast to relative production, a result that is mainly explained by the asymmetric impact of the complexity and congestion costs on vertical and horizontal R&D, which then implies different speeds of convergence of the two industry-structure variables towards the new BGP. On the other hand, when the market-size channel dominates, the interest rate converges at a higher speed than the economic growth rate, but both are slower than relative production. When the price-channel dominates, the fact that the interest rate follows a non-monotonic time-path that overshoots the new BGP (although only very slightly) after approximately 40 years implies that its speed of convergence will, at that time, become infinite;[35] after passing through that point, the interest rate will eventually converge at a finite rate that is higher than that of the economic growth rate but smaller than that of relative production.

The importance of these features of transitional dynamics has been emphasised within the endogenous growth literature by Eicher and Turnovsky (2001). However, unlike the latter, we obtain flexible transitional dynamics without having to restrict our analysis to a non-scale version of our model, while the dimension of the dynamic system in detrended variables is the same in the two models.

[Figure 5 goes about here]

Bearing in mind the available data at the sectoral level, we assess the adequacy of the theoretical results to the empirical side. According to the time series data for the 14 European countries depicted by Figure 1, both measures of industry structure are growing over time, but with the former outpacing the latter. That is, the shift of economic activity occurs from the low- to the high-tech sectors and with a stronger impact on production than on the number of firms. This evidence suggests that Scenario 1 (Figure 2) is the only one that is qualitatively consistent with the empirical facts on industry dynamics. As explained earlier, this scenario features the technological-knowledge bias working mainly through the market-size channel. Additionally, we observe that Scenario 1 is the only one that is characterised by a rising technological-knowledge bias and thus an increasing skill premium over transition (see (67)), a prediction that also seems to be corroborated by the available data for the same set of European countries.[36]

A simple calibration exercise gives an illustration of the ability of the transitional-dynamics mechanism of the model to quantitatively address the distinct performance of relative production and the relative number of firms observed in the data. We consider

[35] This accounts for the singularity observed in the lower panel of Figure 5 with respect to β_r.

[36] In the 14 European countries considered in Figure 1, the skill premium – measured as the mean annual earnings of the college graduates employed in manufacturing vis-à-vis the mean annual earnings of the non-college graduates – increased from an average of 1.678, in 2002, to 1.742, in 2006, implying a growth rate of 0.94 percent/year. The source is the Eurostat on-line database on Science, Technology and Innovation – table "Annual data on employment in technology and knowledge-intensive sectors at the national level, by level of education" (available at `http://epp.eurostat.ec.europa.eu`).

the shock on H/L as a change from 1980 to 1995, as in Section 3.1, with a one-year lag for the impact on the technological structure (in light of the results presented in Appendix A). However, as a robustness check, we also include a scenario with a 5-year lag for the impact on the technology structure and, thus, in which the shock on H/L is measured considering the time span from 1980 to 1990 (in this case, we let H/L increase from 0.1 to 0.15, according to the data on the selected 14 countries in Barro and Lee, 2010).

This exercise is run for Scenario 1, with $\epsilon = 0$, and by setting ρ, θ, α, λ, ζ, and $\sigma + \gamma$ to their baseline values, as defined in Section 3.1. However, it is also important to note that γ, the parameter that regulates the horizontal R&D congestion cost, is crucial to determine the speed of convergence of N and X and hence their growth rates per period over transition. Given the lack of empirical guidance regarding this parameter, we consider, as a sensitiveness analysis, different values for γ (and thus for σ) in the interval $(0; 2.4)$ – the upper boundary of the interval reflects the value for $\sigma + \gamma$ established in Section 3.1, which is taken as given herein.[37] Figure 6 shows a monotonic relationship between γ and the predicted values for the transitional growth rates of N and X. As expected, lower values of γ yield higher growth rates of both N and X, but with a stronger effect on the former as γ approaches the lower boundary, since shifts in that parameter impact directly on the horizontal entry cost and only indirectly on the vertical entry cost.

Table 3 summarises the results of the calibration exercise, by considering the baseline, the upper and the lower values for γ. The results show that, under Scenario 1 and under the hypothesis of an initial increase in relative supply of skills from 0.1 to 0.19 and 1-year lag impact (from 0.1 to 0.15 and 5-year lag impact), the model accounts for, respectively, 82 to 100 percent and 6 to 87 percent (41 to 50 percent and 5 to 53 percent) of the average annual growth rate of relative production and of the relative number of firms observed in the data. As a final robustness check to these results, we also look into the ability of the model to replicate the dynamic behaviour of the skill premium, and find that the model accounts for 86 to over 100 percent (50 to 60 percent) of the annual growth rate of the skill premium observed in the data.

[Figure 6 goes about here]

[Table 3 goes about here]

A prevailing market-size channel implies scale effects on industrial growth. This is in apparent contrast with the well-known endogenous-growth debate over the counterfactual character of scale effects. However, the existing literature rejects the existence of scale

[37] To be more specific, we consider $\gamma \in [0.1; 2.3]$ in our numerical simulation, since values of γ very close to 0 or to 2.4 render the numerical computation of the time-paths of the endogenous variables unstable (note that the limiting case of $\gamma = 0$ [equivalently, $\sigma = 2.4$] implies that the model displays no transitional dynamics, while $\sigma = 0$ [equivalently, $\gamma = 2.4$] implies that no interior BGP exists).

	Relative production (annual growth rate, %)	Relative number of firms (annual growth rate,%)	Skill premium (annual growth rate,%)
	H/L increases from 0.1 to 0.19 (1-year lag)		
$\gamma = 2.3$	1.00	0.04	0.81
$\gamma = 1.2$ (baseline)	1.02	0.08	0.83
$\gamma = 0.1$	1.23	0.61	1.02
	H/L increases from 0.1 to 0.15 (5-year lag)		
$\gamma = 2.3$	0.50	0.04	0.47
$\gamma = 1.2$ (baseline)	0.51	0.07	0.48
$\gamma = 0.1$	0.62	0.37	0.57
observed	1.22	0.70	0.94

Table 3: Calibration exercise for the annual growth rates of relative production, the relative number of firms and the skill premium. Observed and predicted values for the growth rates of relative production and the relative number of firms are computed as described in Figure 6. The observed values for the growth rates of the skill premium cover the period 2002-2006 (see data description in fn. 36), while the predicted values are those implicit in the numerically-computed time path for W (based on (67)). In the upper (lower) panel, the values were obtained by considering the time path from $t = 8$ to $t = 12$ ($t = 12$ to $t = 16$), where $t = 0$ corresponds to the year of 1990 (1995). Parameter values are the same as in Figure 6.

effects in *secular* trend, while acknowledging their role over transitional dynamics (e.g., Jones, 1995; Jones, 2002; Sedgley and Elmslie, 2010, 2013). Similarly, our quantitative results underline the role of scale effects in the *medium* term, in particular given the relatively short time span of the time-series data that we used in our calibration.

4. Concluding remarks

This paper builds an endogenous growth model of directed technical change with simultaneous vertical and horizontal R&D and flexible scale effects to study the shifts in the share of the high- vis-à-vis the low-tech sectors within manufacturing in the context of slow, but flexible, transitional dynamics. We show that, under the hypothesis of a positive shock in the proportion of high-skilled labour, the technological-knowledge bias channel leads to nonbalanced sectoral growth, while the aggregate variables are roughly unchanged.

It is worth noting the asymmetric role played by the intensive and the extensive margin in explaining the time-path of the industry-level variables under scale and no-scale effects on growth. Our theoretical results show that a rich interaction between the two margins should be expected when one takes into account the short and transitional-dynamics responses to structural shocks. The fact that the shock in the relative supply of skills occur due to a rise in high-skilled labour paralleled by a stabilisation (or only a slight decrease) in low-skilled labour (which is in accordance to the empirical evidence) further enhances the asymmetry between the different scenarios for the degree of scale effects.

Under prevailing market-scale effects, the theoretical results are qualitative consistent with the increase in the share of the high-tech sectors found in time-series data, computed as a weighed average across 14 European countries. We also presented a simple calibration exercise, which showed that the implied magnitudes for the shift in the share of the high-tech sectors over transition are of up to 50 to 100 percent of the change observed in the data from 1995 to 2007. However, importantly, the model predicts that the dynamics of the share of the high-tech sector has no significant impact on the economic growth rate. Therefore, in as much as the change in the industry structure is mainly driven by a shift in the proportion of high skilled workers, our results suggest that raising the share of the high-tech sector may be largely ineffective in stimulating economic growth.

We leave for future research a full investigation of whether the analytical mechanism proposed in this paper plays a first-order role in nonbalanced high-/low-tech sectoral growth at the empirical level. This could be conducted by implementing a finer calibration of the model in light of the cross-section data for the European countries. On the other hand, it would be interesting to extend our model to a setting in which Total Factor Productivity (TFP) growth rates are not homogeneous across the high- and the low-tech sectors along the BGP, and analyse the implications of the cross-sector differences in TFP growth rates for the dynamics of sectoral input reallocation and of the economic growth rate. However, it should be noted that in this case, and in line with the recent literature on structural change (e.g., Ngai and Pissarides, 2007; Blankenau and Cassou, 2009), the nonbalanced sectoral growth may induce an ever increasing (decreasing) share

of the sector with higher (lower) TFP growth. In contrast, in the model developed in this paper, when the BGP is (asymptotically) reached, balanced growth at both the aggregate and the sectoral level is established, and thus no sector ever vanishes, as seems to be the case empirically.

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Dependent variable: Dif log Relative Supply of Skills			
Excluded	Chi-sq	df	Prob.
Dif log Relative Production	0.917690	1	0.3381
Dependent variable: Dif log Relative Production			
Excluded	Chi-sq	df	Prob.
Dif log Relative Supply of Skills	4.448970	1	0.0349

Table 4: VAR Granger Causality/Block Exogeneity Wald Tests. Sample-period: 1980-1995.

Appendix

A. Granger causality test for the relative supply of skills and relative production

We run a VAR model in order to test Granger causality between the proportion of high-skilled labour (the relative supply of skills) and the share of production of the high-tech sector (relative production), for the period 1980-1995. The starting year (1980) is determined by data availability for production, whereas the final year (1995) was chosen by observing by that time there is a significant acceleration of the share of the high skilled and of the share of production of the high-tech sector (see fn. (3) and Figure 1, in the text). We estimate the VAR model for the cross-country average of the relative supply of skills and of relative production, considering the 10 countries,[38] among the 14 countries presented in Figure 1, which have available data for high- and low-tech production as of 1980.

By considering the condition of stability of the VAR (no roots outside the unit circle) and the VAR lag order selection criteria, we estimate a bivariate VAR with 1 lag on the first log difference of the relative supply of skills and of relative production. We find that the relative supply of skills Granger causes relative production (the null hypothesis that the former does not Granger cause the latter is rejected with a probability of 0.0349), but not the other way around (the null hypothesis that relative production does not Granger cause the relative supply of skills is rejected with a probability of 0.3381).[39]

[Table 4 goes about here]

[38] These countries are Denmark, Finland, France, Italy, Germany, Norway, Portugal, Spain, Sweden, and the UK.

[39] We also run a VAR for the full sample period, 1980-2007, and again found that the relative supply of skills Granger causes relative production, but not the other way around.

B. Proof of Proposition 1

Proof. By equating $\dot{x}_m = \dot{z}_L = \dot{Q} = 0$, $m \in \{L, H\}$, we show that there is just one interior steady state, i.e., $\tilde{x}_L > 0 \ \wedge \ \tilde{z}_L > 0 \ \wedge \ \tilde{x}_H > 0 \ \wedge \ \tilde{Q} > 0$. Given (26) and the BGP condition that $\tilde{g}_{N_L} = \tilde{g}_{N_H}$, then $\tilde{x}_L = \tilde{x}_H$. Together with (48), we find that $\dot{Q} = 0$ implies

$$\tilde{P} \equiv \left(\frac{\tilde{P_H}}{P_L}\right) = \left(\frac{\mathcal{L}}{\mathcal{H}}\right)^{\alpha(1-\epsilon)}, \tag{70}$$

which also guarantees $\tilde{I}_L = \tilde{I}_H$ (see (17)). Next, substitute (70) in (7) and solve in order to Q to get

$$\tilde{Q} \equiv \left(\frac{\tilde{Q_H}}{Q_L}\right) = \left(\frac{\mathcal{H}}{\mathcal{L}}\right)^{1-2\epsilon} > 0. \tag{71}$$

From here, together with (6) and (7), we find that $\tilde{P}_L = e^{-\alpha}\tilde{\tilde{n}}^{-\alpha}$, $\tilde{P}_H = e^{-\alpha}\left(1-\tilde{\tilde{n}}\right)^{-\alpha}$, $\tilde{\tilde{n}} = \left[1 + (\mathcal{H}/\mathcal{L})^{1-\epsilon}\right]^{-1}$ and, thus, $\tilde{P}_L = \exp(-\alpha) \cdot \left[1 + (\mathcal{H}/\mathcal{L})^{1-\epsilon}\right]^{\alpha}$ and $\tilde{P}_H = \exp(-\alpha) \cdot \left[1 + (\mathcal{H}/\mathcal{L})^{\epsilon-1}\right]^{\alpha}$. Now, we turn to the solution of $\dot{x}_L = 0$ and $\dot{z}_L = 0$. By replacing (70) and (71) in (39), we get the linear function $I_L \equiv I_L(x_L, x_H, z_L) = I_0 + I_1 x_H + I_2 x_L + I_3 z_L$, where $I_0 \equiv \Theta\zeta^{-1}\lambda^{-\frac{1-\alpha}{\alpha}}\mathcal{L}^{-\epsilon}\chi\left[\tilde{P}_H^{\frac{1}{\alpha}}\mathcal{H}\tilde{Q} + \tilde{P}_L^{\frac{1}{\alpha}}\mathcal{L}\right]$, $I_1 \equiv -\Theta\lambda^{-\frac{1-\alpha}{\alpha}}(\mathcal{H}/\mathcal{L})^{1-\epsilon}$, $I_2 \equiv -\Theta\lambda^{-\frac{1-\alpha}{\alpha}}$, $I_3 \equiv -\Theta\zeta^{-1}\lambda^{-\frac{1-\alpha}{\alpha}}\mathcal{L}^{-\epsilon}$, and $\Theta \equiv 1/\left[1 + (\mathcal{H}/\mathcal{L})^{1-\epsilon}\right]$. Substituting in (45) and (46), equating $\dot{x}_L = 0$ and $\dot{z}_L = 0$ and solving for z_L and x_L, yields

$$\tilde{z}_L = \left(-I_0 - I_1\tilde{x}_H - I_2\tilde{x}_L + \frac{\sigma+\gamma}{\Xi}\tilde{x}_L\right)\frac{1}{I_3}, \tag{72}$$

$$\tilde{x}_L = \frac{\frac{\Xi}{\theta}\left[\frac{\pi_0}{\zeta}\mathcal{L}^{1-\epsilon}\tilde{P}_L^{\frac{1}{\alpha}} - \rho\right]}{\Xi(\sigma+\gamma+1) + \frac{1}{\theta}(\sigma+\gamma)}. \tag{73}$$

Given that $\tilde{x}_L = \tilde{x}_H$ from the BGP conditions, we can write

$$\tilde{x}_H = \frac{\frac{\Xi}{\theta}\left[\frac{\pi_0}{\zeta}\mathcal{L}^{1-\epsilon}\tilde{P}_L^{\frac{1}{\alpha}} - \rho\right]}{\Xi(\sigma+\gamma+1) + \frac{1}{\theta}(\sigma+\gamma)} = \frac{\frac{\Xi}{\theta}\left[\frac{\pi_0}{\zeta}\mathcal{H}^{1-\epsilon}\tilde{P}_H^{\frac{1}{\alpha}} - \rho\right]}{\Xi(\sigma+\gamma+1) + \frac{1}{\theta}(\sigma+\gamma)}. \tag{74}$$

Then, letting $\tilde{r}_{0L} \equiv \frac{\pi_0}{\zeta}\mathcal{L}^{1-\epsilon}\tilde{P}_L^{\frac{1}{\alpha}}$ and $\tilde{r}_{0H} \equiv \frac{\pi_0}{\zeta}\mathcal{H}^{1-\epsilon}\tilde{P}_H^{\frac{1}{\alpha}}$, where $\tilde{r}_{0L} = \tilde{r}_{0H}$ by construction, we see that $\tilde{x}_m > 0$ iff $\tilde{r}_{0m} > \rho$ (the production technology is sufficiently productive). Finally, using (72) and the definition of I_0, I_1, I_2 and I_3, we get equations (52) and (53), in the text. By rewriting (52) as $\tilde{z}_L = \mathcal{Z} \cdot (\bar{x} - \tilde{x}_L)$, where $\mathcal{Z} \equiv \zeta\left[(\sigma+\gamma)/\Xi + \sigma + \gamma + 1\right]\left(\mathcal{H}^{\epsilon}\tilde{Q} + \mathcal{L}^{\epsilon}\right) > 0$ and $\bar{x} \equiv \chi \cdot \tilde{\Gamma}/(\tilde{Q}_L\mathcal{Z}) > 0$, with $\chi \equiv A^{\frac{1}{\alpha}} \cdot$

$(1-\alpha)^{\frac{2}{\alpha}} \cdot \left[(1-\alpha)^{-2}-1\right] > 0$ and $\tilde{\Gamma} \equiv \tilde{P}_L^{\frac{1}{\alpha}} \cdot \mathcal{L} \cdot \tilde{Q}_L + \tilde{P}_H^{\frac{1}{\alpha}} \cdot \mathcal{H} \cdot \tilde{Q}_H$, we see that $\tilde{z}_L > 0$ iff $\tilde{x}_m < \bar{x}$ (the growth rate of the number of varieties is bounded from above).[40] □

C. Numerical verification of local saddle-path stability

In this appendix, we perform numerical verification of saddle-path stability in the neibourhood of the interior steady state, $(\tilde{x}_L, \tilde{z}_L, \tilde{x}_H, \tilde{Q})$. The analysis consists of: (i) considering a sensible interval of variation for each parameter value; and (ii) re-running the computation of the eigenvalues of matrix J, in (62), by letting a given parameter take the values in that interval, while the other parameters are set to their baseline values. We consider the following typical baseline parameter values (see Barro and Sala-i-Martin, 2004): $\rho = 0.02$; $\theta = 1.5$; $\alpha = 0.6$; $\lambda = 2.5$; $\epsilon = 0$.[41] As for the remaining parameters, we let: $\sigma = 1.2$; $\gamma = 1.2$; $l = 1$; $h = 1.3$, as explained in Section 3.

Table 5 presents the extreme values for each parameter of interest considered in the numerical exercise. A set of practical criteria has commanded the selection of the extreme values for the parameters. The extreme values for α and h were chosen broadly in line with the range of values cited by the empirical literature (e.g., Acemoglu, 2009), whereas the extreme values for ϵ imply that either the market-scale effects or the price-channel effects exist. As regards the other parameters, given the lack of well-established empirical guidance, we have chosen: the lower values in order to be close to the lower bound of the theoretical support; the upper values of ρ, σ, γ and ζ such that the implied BGP economic growth rate is not negative; the upper values for θ and λ by observing that they defined a threshold above which an increase of those parameters has a negligible impact on the BGP economic growth rate.

[Table 5 goes about here]

The experimentation with numerical values shows that there are three eigenvalues with negative real part and one with a positive real part for the considered broad range of parameter values (see Figure **7**), thus satisfying the conditions for local saddle-path stability stated in fn. 20.

[Figure 7 goes about here]

[40] Observe that, given (53), this is equivalent to the condition that the vertical innovation rate is bounded from above, $\tilde{I}_m < \bar{I}$, with $\bar{I} > 0$ properly defined.

[41] We also let $A = 1$ and $\phi = 1$. In particular, parameter ϕ is not considered in this numerical exercise since it has no impact on the steady state values $(\tilde{x}_L, \tilde{z}_L, \tilde{x}_H, \tilde{Q})$ or on the eigenvalues of the system in (62).

	σ	γ	λ	ζ	h
Theoretical support	$(0;\infty)$	$(0;\infty)$	$(1;\infty)$	$(0;\infty)$	$(1;\infty)$
Extreme values considered	$\{0.1;5\}$	$\{0.1;5\}$	$\{1.1;5\}$	$\{0.1;1.5\}$	$\{1.1;2\}$

	θ	ρ	α	ϵ
Theoretical support	$(0;\infty)$	$(0;1)$	$(0;1)$	$[0;\infty)$
Extreme values considered	$\{0.1;5\}$	$\{0.005;0.05\}$	$\{0.5;0.7\}$	$\{0;1\}$

Table 5: Extreme values for the parameters of interest in the numerical exercise.

D. Two-dimensional transition paths in (x_L, x_H) space

Figure 8 depicts a two-dimensional projection in (x_L, x_H) space, plotted under three scenarios for ϵ ($\epsilon = 0$, $\epsilon = 1$, $\epsilon = 0.5$). The lower panels show the trajectories (transition paths) of x_L and x_H from $t = 1$ to $t = 120$ (the same number of periods as in Figures 2-5) after a rise in H/L from 0.1 to 0.19 at $t = 0$, while the upper panels depict the switching curves $I_L(x_L, x_H) = 0$ and $I_H(x_L, x_H) = 0$. Since these locus move as $Q(t)$ and $z(t)$ converge towards the new BGP, we considered $Q(t)$ and $z(t)$ valued at $t = 1$ and $t = 120$ to make the planar representation of the switching curves tractable. As a consequence, two pairs of switching curves appear in each scenario. Those curves divide the state space into three zones: in the northeast area, where $I_m(x_L, x_H) < 0$, $m \in \{L, H\}$, the dynamics will be given by the dynamic system (45)-(48) by replacing I_m with $I_m^+ = 0$ according to equation (40); in the southwest area, where $I_m(x_L, x_H) > 0$, $m \in \{L, H\}$, the dynamics is given by keeping $I_m^+ = I_m$ in (45)-(48); in the area between the two switching curves (which exists iff $\epsilon \neq 0.5$), we either have $I_H(x_L, x_H) > 0$ and $I_L(x_L, x_H) < 0$, and thus $I_H^+ = I_H$ and $I_L^+ = 0$, or $I_H(x_L, x_H) < 0$ and $I_L(x_L, x_H) > 0$, and thus $I_H^+ = 0$ and $I_L^+ = I_L$. Figure 8 shows that, for our numerical simulations and given the considered shock in the relative supply of skills, the saddle-path trajectories for x_L and x_H never cross the locus $I_L(x_L, x_H) = 0$ and $I_H(x_L, x_H) = 0$ and thus never leave the southwest area of the (x_L, x_H) space.

[Figure 8 goes about here]

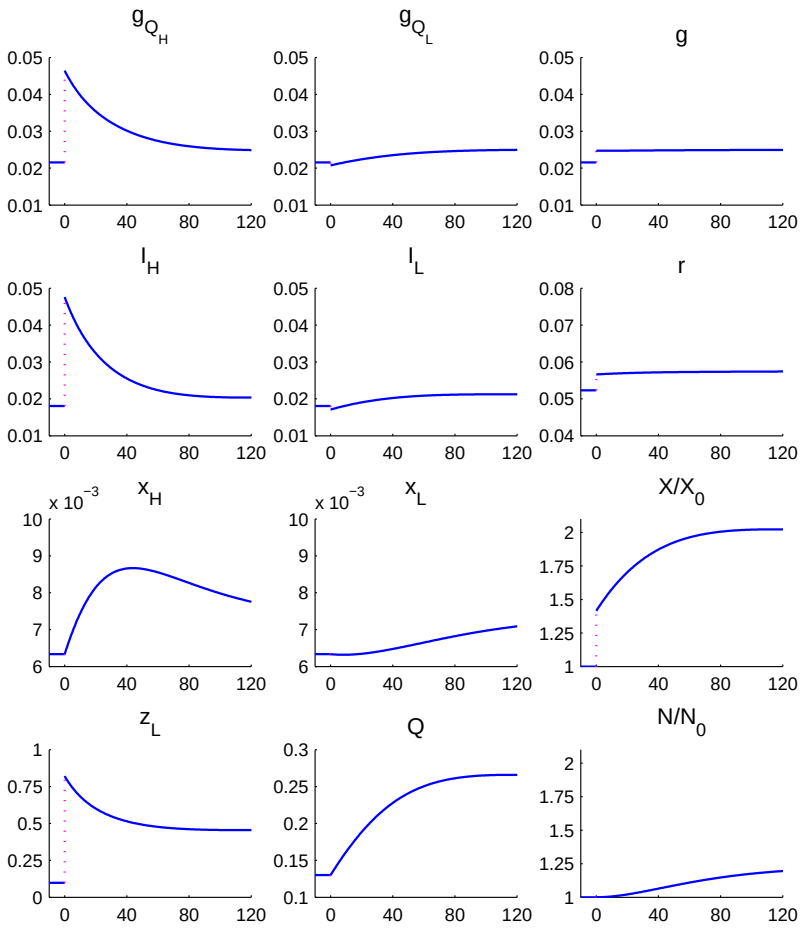

Figure 2: Scenario 1: short-run effects and transitional dynamics of the aggregate and industry-level variables when H increases from 0.1 to 0.19. $\epsilon = 0$. Values for X and N are adjusted by their pre-shock initial values, $X(0) = X_0$ and $N(0) = N_0$, to facilitate the comparison between the time variation of those two variables.

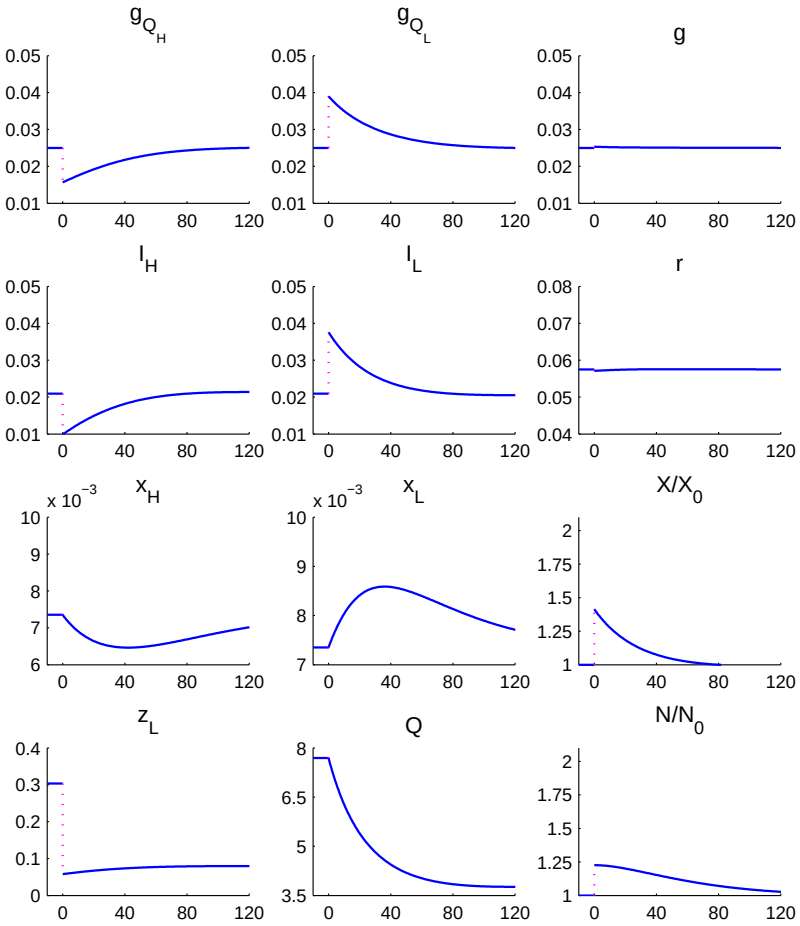

Figure 3: Scenario 2: short-run effects and transitional dynamics of the aggregate and industry-level variables when H increases from 0.1 to 0.19. $\epsilon = 1$.

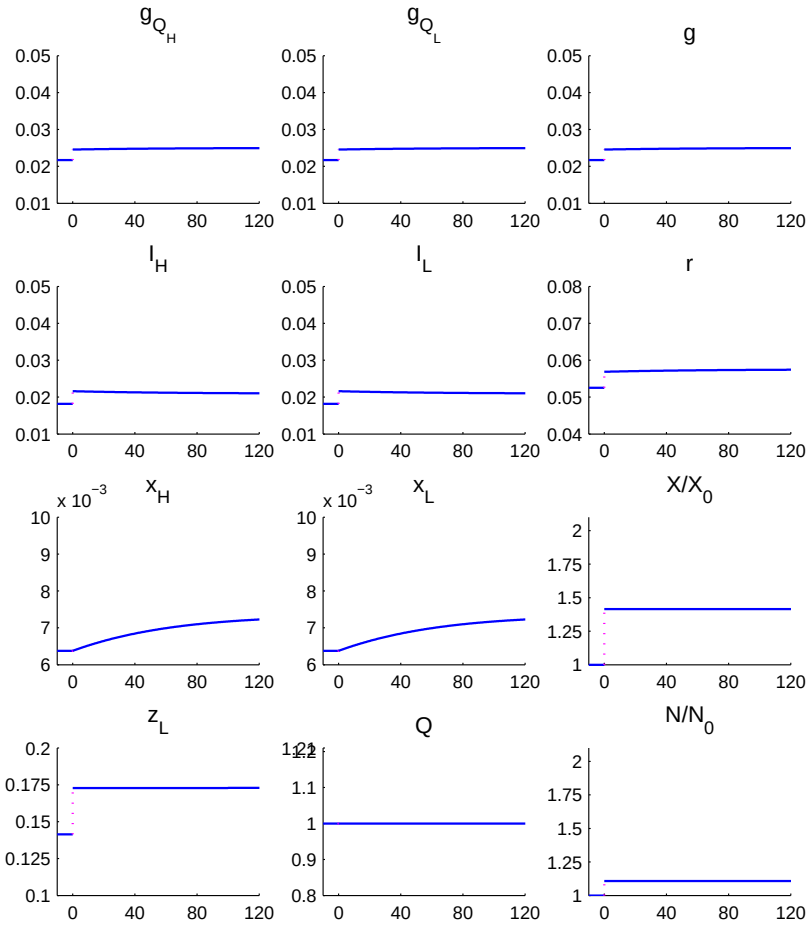

Figure 4: Scenario 3: short-run effects and transitional dynamics of the aggregate and industry-level variables when H increases from 0.1 to 0.19. $\epsilon = 0.5$.

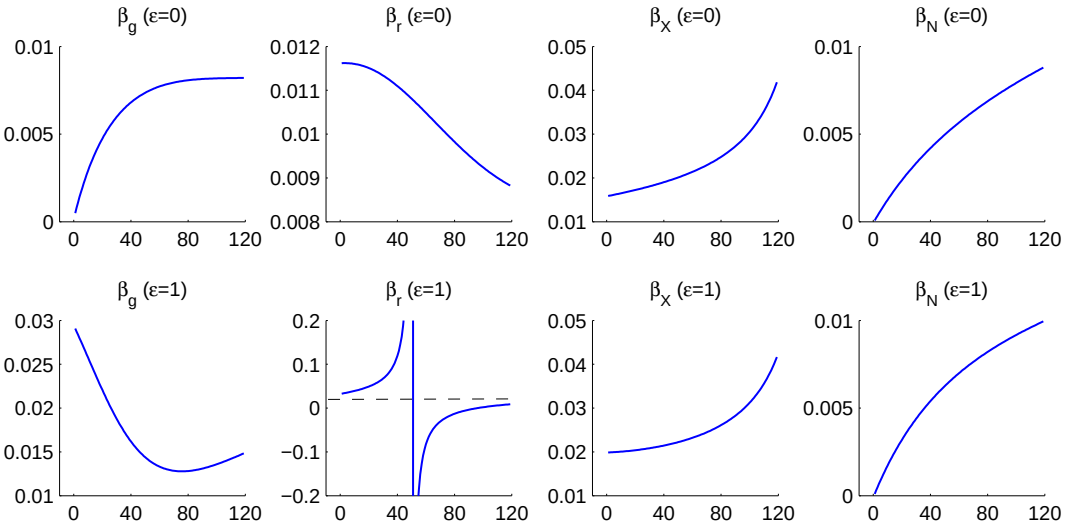

Figure 5: Time-profile of the speed of convergence for the variables of interest (aggregate and industry-level), measured as $\beta_y(t) = -\dot{y}(t)/\left(y(t) - \tilde{y}\right)$ (see Eicher and Turnovsky, 2001), where $\tilde{y}$ is the BGP value of a given variable y and $y \in \{g, r, X, N\}$.

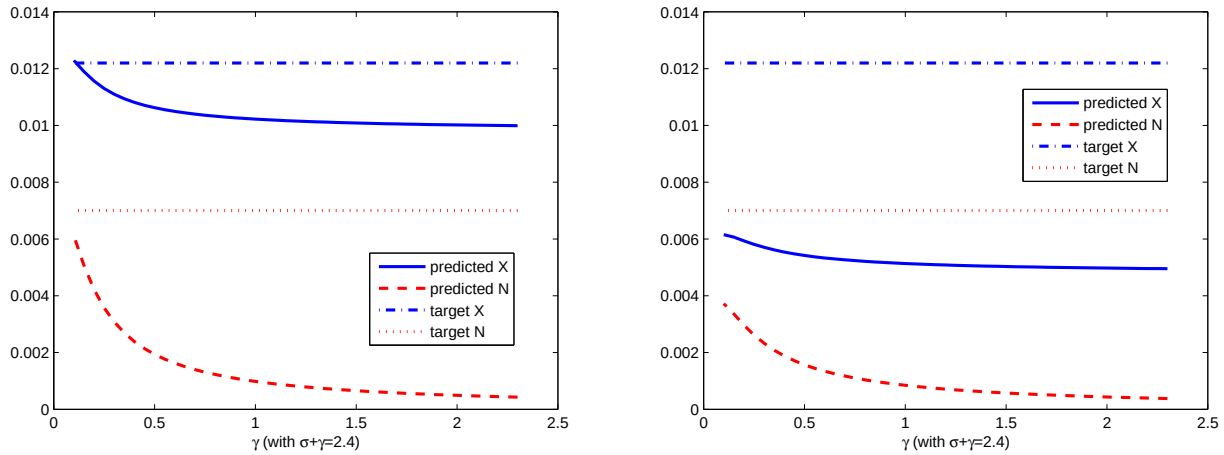

Figure 6: Calibration exercise for the annual growth rates of relative production and the relative number of firms. "Target X" and "Target N" are the observed values for the growth rates, computed as the log differences over the period 1995-2007 of the weighed average of each variable across 14 European countries (see country data and source description in Figure 1). "Predicted X" and "Predicted N" are the predicted values for the transitional growth rates. In the left (right) panel, they were obtained by considering the numerically-computed time paths for N and X (based on (63) and (64)) from $t = 1$ to $t = 13$ ($t = 5$ to $t = 17$), where $t = 0$ corresponds to the year of 1995 (1990). Parameter values are $\epsilon = 0$, $\rho = 0.02$, $\theta = 1.5$, $\alpha = 0.6$, $\lambda = 2.5$, $\zeta = 0.42$, and $\sigma + \gamma = 2.4$, as in Figure 2, while γ takes values in the interval $[0.1; 2.3]$.

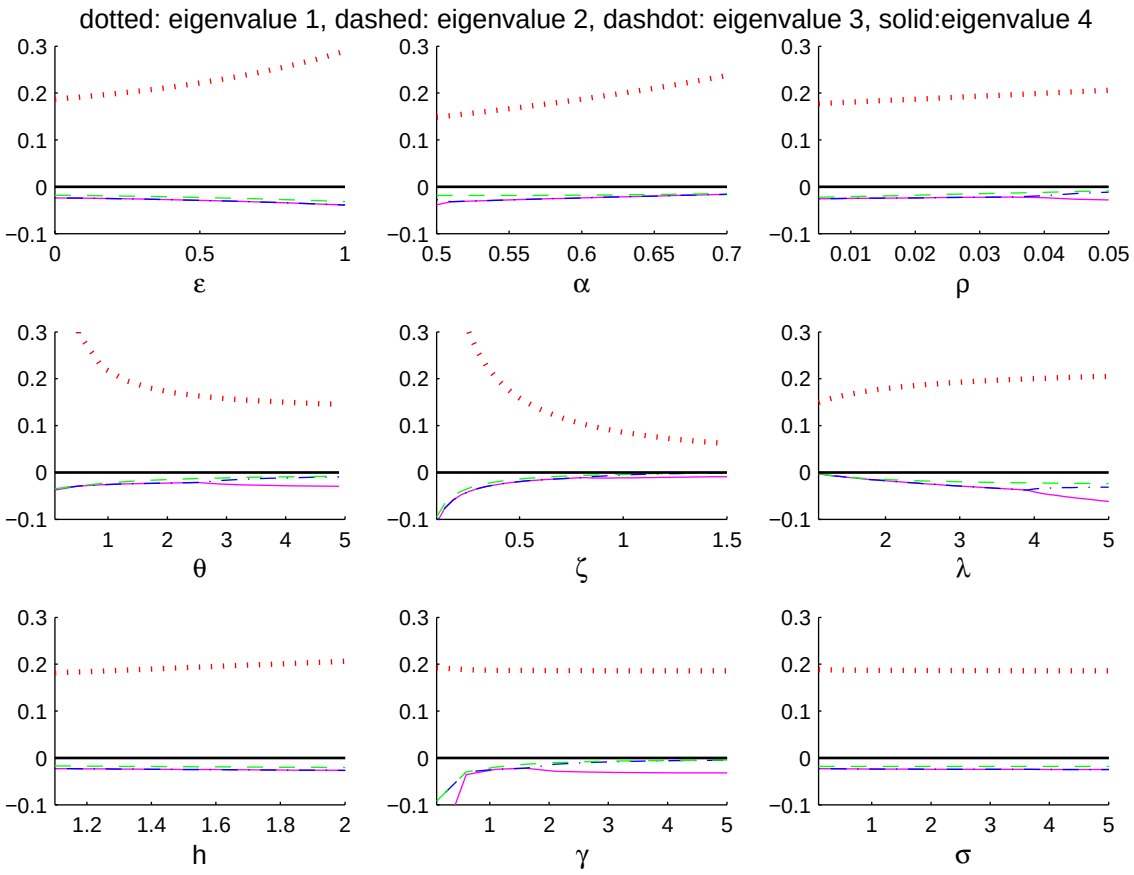

Figure 7: Eigenvalues of matrix J, in (62), considering the set of parameter values depicted by Table 5.

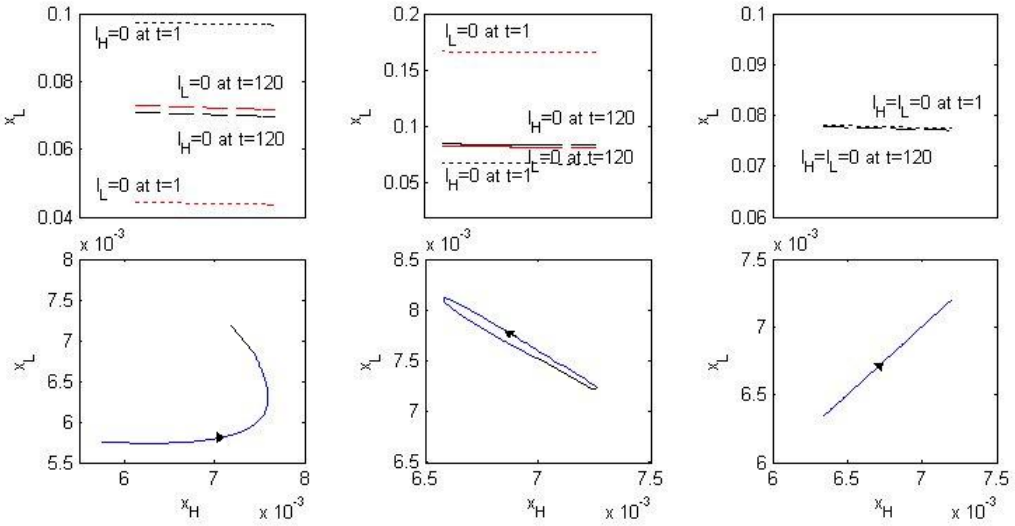

Figure 8: The switching curves $I_L(x_L, x_H) = 0$ and $I_H(x_L, x_H) = 0$ (at $t = 1$ and $t = 120$), and the trajectories of x_L and x_H (from $t = 1$ to $t = 120$) in (x_L, x_H) space under three scenarios for ϵ (respectively, $\epsilon = 0$, $\epsilon = 1$, $\epsilon = 0.5$) after a rise in H/L from 0.1 to 0.19 at $t = 0$. The two panels in each column depict two parts of the same (x_L, x_H) space; we resorted to separate panels to accomodate the very different scale along the vertical axis that corresponds to the $I_m(x_L, x_H) = 0$, $m \in \{L, H\}$, loci and to the trajectories of x_L and x_H.

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