Optimality Conditions for Impulsive Control*

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This article surveys several results on first and second order necessary conditions of optimality for impulsive control problems recently obtained in [4, 2, 3, 6, 7]. The most complete problem considered is the following:

$$\begin{aligned} \text{Min. } & k_0(t_0, x_0, t_1, x_1) \\ \text{s.t. } & k_1(t_0, x_0, t_1, x_1) = 0, \quad k_2(t_0, x_0, t_1, x_1) \leq 0 \\ & dx(t) = f(t, x(t), u(t))dt + G(t, x(t))d\mu(t) \ \forall t \\ & k_3(t, x(t)) \leq 0 \ \ \forall t \\ & u(t) \in \Omega(t), \mu \in \mathcal{K} \end{aligned}$$

Here, the time endpoints t_i, $i = 0, 1$ are free, $k_i : \Re^{2n+2} \to \Re^{d}(k_i)$ with $d(k_0) = 1$, $f : \Re \times \Re^n \times \Re^m \to \Re^n$, $G : \Re \times \Re^n \to \Re^{n \times q}$, and $k_3 : \Re \times \Re^n \to \Re^{d(k_3)}$ are given functions satisfying appropriate assumptions, $\Omega(t) \subset \Re^m$, and $\mathcal{K}$ is a cone of measures whose values range in a pointed convex cone $K \subset \Re^q$.

We start by presenting the solution concept along the lines of [5] and, then discuss issues when state constraints are present, [3]. In particular we detail the set of assumptions required to ensure that the necessary conditions of optimality of first-order do not degenerate, [1].

Then, we present the first-order conditions for the free-time impulsive control with state constraints that were derived in [3]. These conditions look rather intricate. However, it can be easily seen that, for fixed time problems in the absence of state constraints, they easily reduce to those of [4] in a smooth context. The proof of these conditions require the data of the problem to be smoother than the one considered in [4] and use a combination of penalization techniques of [1] with the methods of [4, 6, 7].

For a simpler problem, i.e., fixed time and without state constraints, we present second-order necessary conditions of optimality that do not degenerate even for abnormal control processes. The basic idea consists in using the second order information in order to select a subset of nondegenerate first order multipliers. This result is proved without any a priori normality assumptions on the data of the problem and it can be regarded as an extension for the impulsive context of a result in [1].

A discussion, illustrated with examples, of these results in the context of the state-of-the-art providing a historical perspective and insight for future developments is also included.

*The research reported here was developed with Prof. Aram Arutyunov of PFRU and MSU, Moscow, Russia, Prof. Geraldo Silva of IBILCE, UNESP, SJRP, Brasil, Prof. Vladimir Dykhta of BSUEL, Irkutsk, Russia, Dr. Vladimir Karamzin, MSU, Moscow, Russia

†The author gratefully acknowledges the support of INVOTAN and of Fundacao para a Ciencia e Tecnologia

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