U.PORTO
FEP FACULDADE DE ECONOMIA
UNIVERSIDADE DO PORTO

New Economic Geography: perspectives, multiple regions and individual heterogeneity

José Maria Lopes Gaspar

January 2017

Doctoral Thesis in Economics

Supervisors:

Sofia Balbina Santos Dias Castro Gothen
João Oliveira Correia da Silva

Para o meu pai, Alfredo (in memoriam).

Biography

José Maria Lopes Gaspar was born on September 28, 1988. He lived in his home-town in Lisbon until 2006, after which he moved to Oporto to get his Bachelor (BSc) degree in Economics at the School of Economics and Management (FEP) of the University of Porto.

He joined the Master of Science (MSc) in Economics at that same institution in 2010. His MSc dissertation "The Footloose Entrepreneur Model with three regions" was supervised by Professors Sofia B.S.D. Castro and João Correia da Silva. In 2011, he tutored students from the portuguese-speaking african countries (PALOP) in support classes of Macroeconomics I at FEP. Later in that year, he received a merit scholarship from the University of Porto for outstanding academic achievements during the MSc.

In 2012, he was enrolled at the Doctoral Programme at FEP, also under the supervision of Professors Sofia B.S.D. Castro and João Correia da Silva.

Around 2014 he joined Católica Porto Business School as an Assistant Professor, teaching the courses of Information and Technological Systems and Applied Quantitative Methods to the undergraduate programmes in Economics, Management, and the double degree in Law and Management.

Throughout his academic course, he presented his PhD related research in several international conferences in the fields of Regional Science and Economic Geography. In 2013, he received the Timberlake Prize for young research best paper at the 1st International Conference on Algebraic and Symbolic Computation in Lisbon, which was later published in the journal *Economic Modelling.*

Acknowledgements

I am very grateful to Fundação para a Ciência e Tecnologia (FCT) for their financial support through a Ph.D scholarship.

I would like to thank all staff members of the School of Economics and Management at the University of Porto. All of them have directly or indirectly contributed to the final outcome of this Thesis. I am particularly thankful to all the teachers who have tutored me throughout my BSc, MSc and PhD courses.

A special note of gratitude is devoted to the late Professor António Almodovar, whose guidance throughout the course of History of Economic Thought helped me gain a deeper understanding of Spatial Economics. As an outstanding scholar and person, he will truly be missed.

I am also grateful for all the suggestions provided by the several acquaintances in international workshops, conferences and seminars during my PhD course. I am particularly grateful to Pascal Mossay, whose suggestions in meetings both in Porto and Newcastle have allowed us to substantially improve this work.

I would like to acknowledge all the counselling and motivation from all of my friends and family. Their love and patience are very important to me. I am specially thankful to all the students of the PhD in Economics at FEP, who have helped me and guided me throughout this arduous course, mostly, but not only, in the now (in)famous "Room 252". Their friendship and kindness is very well cherished. I am also grateful to my fellow colleagues at Católica Porto Business School, with a special mention to those who have shared the assistants' office at "Edifício Paraíso".

Needless to say, I am (once again) indebted to my Supervisors Sofia Castro Gothen and João Silva for the countless hours spent together reviewing this work with paper and pencil. A large part of who I am today, both academically and personally, is due to their tutoring throughout these several years (more than 5, and still counting!). For that invaluable contribution, I can only be forever thankful.

Last but not least, I would like to thank and dedicate this work to my mother, Maria Teresa, my sister, Catarina, and to Isabel, who are the most important people in my life and without whom the completion of this work would be impossible.

Abstract

This dissertation focuses on the role of regions and heterogeneity as determinants of the spatial distribution of economic activities.

The first essay provides a prospective assessment of some new avenues of research for New Economic Geography. We also identify new possible directions along which the field could develop.

In the second essay, we study the Footloose Entrepreneur (FE) model with a finite number of equidistant regions, focusing on the qualitative properties of the following equilibria: agglomeration, dispersion and partial dispersion. We find that the tendency for agglomeration increases with the number of regions, whereas numerical evidence suggests that partial dispersion is always unstable. Locational hysteresis appears to be a persistent feature with an arbitrary number of regions. Finally, we introduce exogenous regional heterogeneity in the model and conclude that having more (less) farmers on the core (periphery) region improves the likelihood of a core-periphery pattern.

In the third essay, we extend the FE model with quasi-linear log utility to multiple regions. We show that industry cannot disperse evenly among regions if there is at least one other region with no industry. We find that a spatial distribution where industry is evenly distributed among all regions except one requires that one region is considerably more industrialized than the others. As transportation costs decrease: (i) if inter-regional mobility is low, there is catastrophic agglomeration once dispersion becomes unstable; (ii) if it is high, the transition towards agglomeration is smoother. Finally, we find that the welfare of mobile workers is highest at agglomeration, whereas the welfare of the immobile workers is highest at dispersion. For the society as a whole, however, there is a tendency towards over-agglomeration of economic activities.

In the last essay, we study a New Economic Geography model where all consumers are inter-regionally mobile and have Hotelling-type heterogeneous preferences for location. This heterogeneity produces a dispersive *home-sweet-home* effect. We demonstrate that different functional forms of preferences for location induce different spatial distributions in the long-run. We also find that a higher trade integration leads to less spatial inequality.

Keywords: New economic geography, Multiple regions, Heterogeneity
JEL Classification Numbers: R10, R12, R23

Resumo

Esta dissertação foca-se no papel das regiões e da heterogeneidade como determinantes da distribuição espacial das atividades económicas.

O primeiro ensaio fornece uma avaliação prospectiva de algumas novas vias de pesquisa para a Nova Economia Geográfica. Identificamos, também, novas direções possíveis ao longo das quais a área poderá desenvolver-se.

No segundo ensaio, estudamos o modelo Footloose Entrepreneur (FE) com um número finito de regiões equidistantes, com ênfase nas propriedades qualitativas dos equilíbrios de aglomeração, dispersão e dispersão parcial. Constatamos que a tendência para a aglomeração aumenta com o número de regiões, enquanto a evidência numérica sugere que a dispersão parcial é sempre instável. A histerese locacional parece ser uma característica persistente no modelo com um número arbitrário de regiões. Finalmente, introduzimos heterogeneidade regional exógena no modelo e concluímos que a existência de mais (menos) agricultores na região do centro (periferia) aumenta a probabilidade de um padrão Centro-Periferia.

No terceiro capítulo, estendemos o modelo FE com utilidade quase-linear logarítmica para várias regiões. Mostramos que a indústria não se pode dispersar uniformemente entre as regiões se houver pelo menos uma região sem indústria. Uma distribuição geográfica onde a indústria está uniformente distribuída por todas as regiões exceto uma requer que esta última seja consideravelmente mais industrializada do que as outras. À medida que os custos de transporte diminuem, concluimos que: (i) se a mobilidade inter-regional é baixa, existe aglomeração catastrófica quando a dispersão se torna instável; (ii) se a mobilidade for alta, a transição para a aglomeração é suave. Finalmente, verificamos que o bem-estar dos trabalhadores móveis é máximo na aglomeração, enquanto que o bem-estar dos trabalhadores imóveis é máximo na dispersão. Para a sociedade como um todo, no entanto, existe uma tendência para a sobre-aglomeração das atividades económicas.

No último ensaio, estudamos um modelo da Nova Economia Geográfica onde todos os consumidores são inter-regionalmente móveis e têm preferências heterogéneas do tipo Hotelling relativamente à sua residência. Esta heterogeneidade produz um efeito *home-sweet-home* dispersivo. Demontramos que diferentes formas funcionais para estas preferência induzem distribuições espaciais muito diferentes no longo prazo. Constatamos também que uma maior integração comercial reduz as desigualdades regionais.

Palavras-chave: Nova Economia Geográfica, Regiões Múltiplas, Heterogeneidade
Código JEL: R10, R12, R23

Contents

Contents **vi**

List of Figures **ix**

1 Introduction **1**

2 Economic Geography: a prospective review **6**

2.1 Introduction 7
2.2 An overview on NEG 8
2.3 Reassessing New Economic Geography 12
2.3.1 Persistent features in NEG models 12
2.3.2 Costly icebergs 14
2.3.3 Monopolistic competition 15
2.3.4 A lack of geography 15
2.3.5 The role of the computer: empirical work and policy implications 17
2.3.6 Myopic migration and multiple equilibria selection 18
2.4 Breaking through the strait-jacket 20
2.4.1 Individual heterogeneity 20
2.4.2 Beyond CES preferences 22
2.4.3 Towards a multi-regional framework 23
2.4.4 Linking knowledge and culture to economics 26
2.5 Concluding remarks 28

3 The Footloose Entrepreneur Model with an arbitrary number of equidistant regions **30**

3.1 Introduction 31
3.2 Economic environment 34

3.3 Short-run equilibrium 36
3.4 Long-run equilibria and stability 39
3.4.1 Stability of total dispersion 41
3.4.2 Stability of agglomeration 42
3.4.3 Stability of partial dispersion 44
3.5 Impact of the number of regions 46
3.5.1 Numerical evidence of locational hysteresis 48
3.5.1.1 In the 3-region FE model 48
3.5.1.2 In the $\boldsymbol{n}$-region FE model 49
3.6 Agglomeration under exogenous regional heterogeneity 51
3.7 Conclusion 53
3.A - Nominal wage in the 3-region model 55
3.B - Jacobian and total dispersion 56
3.C - Jacobian and partial dispersion 59
3.D - Comparing models 61

4 Agglomeration patterns in a multi-regional economy without income effects 63
4.1 Introduction 64
4.2 The Quasi-linear log model with n regions 69
4.2.1 Demand and indirect utility 69
4.2.2 Supply 70
4.2.3 Short-run equilibrium 72
4.3 Long-run equilibria 73
4.3.1 Agglomeration 74
4.3.2 Symmetric dispersion 75
4.3.3 Boundary dispersion 76
4.3.4 Partial agglomeration 76
4.3.4.1 Existence of partial agglomeration equilibria 76
4.3.4.2 Stability of partial agglomeration 78
4.4 Bifurcations in the n-region model 80
4.4.1 Primary and secondary bifurcations 81
4.4.2 A note on the black hole condition and on the role of inter-regional mobility 89
4.5 Welfare 90

4.5.1 Entrepreneurs 90
4.5.2 Farmers 91
4.5.3 Social welfare 92
4.6 Conclusion 96
4.A - Proofs of section 4.2 99
4.B - Proofs of section 4.3 100
4.C - Proofs of section 4.4 108
4.C - Proofs of section 4.5 111

5 Economic Geography meets Hotelling: a *home-sweet-home* effect 117
5.1 Introduction 118
5.2 The model 122
5.3 Long-run equilibria 127
5.3.1 Agglomeration 130
5.3.2 Interior equilibria 132
5.3.2.1 Symmetric dispersion 133
5.3.2.2 Partial agglomeration 134
5.4 The impact of the home-sweet-home effect 135
5.4.1 Logit home-sweet-home effect 136
5.4.2 A linear home-sweet-home effect 140
5.5 A note on trade integration and spatial inequality 143
5.6 Concluding remarks 144
5.A - Wages and freeness of trade 147
5.B - Utility differentials 151
5.C - Stability of symmetric dispersion 152
5.D - Partial agglomeration 153
5.E - Logit and linear heterogeneity 155

6 Conclusions and discussion 158

Bibliography 162

List of Figures

3.1 Parameter regions in which $\beta \leq 0$ and $\gamma \leq 0$ for $n = 3$. 45
3.2 Stability of agglomeration for 2 and 3 regions. 47
3.3 Agglomeration and total dispersion in parameter space. 49
3.4 Location hysteresis in (ϕ, n) space, with $\sigma = 5$ and $\mu = 0.4$. 50
3.5 Locational hysteresis in (ϕ, n, μ) space for $\sigma = 4$. 50

4.1 Illustration of $\lambda^*(h)$ for $n = 3$. 78
4.2 Regions of stability $\delta_n < 0$ in (h, ϕ) space, for $n = \{3, 5, 10\}$. 80
4.3 Bifurcation diagram for the 3-region QLLog model. 83
4.4 Migration dynamics of entrepreneurs inside the 2-simplex for the QLLog model. 84
4.5 Locational hysteresis in the QLLog model. 86
4.6 Bifurcation diagram for the 3-region QLLog model (2). 87
4.7 A 2-simplex with stable agglomeration and dispersion. 88
4.8 Bifurcation diagram for the QLLog model without dispersion. 90
4.9 Social welfare along the invariant space. 95

5.1 Short-run equilibrium relative wage. 126
5.2 Utility differential $\Delta u = u_L - u_R$ for different levels of θ. 130
5.3 Utility differential when all consumers reside in country in (ϕ, θ) space. 132
5.4 Utility differential and Logit home-sweet-home effect for different degrees of heterogeneity. 138
5.5 Utility differential and Logit home-sweet-home effect for different levels of the freeness of trade. 139
5.6 Bifurcation diagrams for the Logit home-sweet-home effect. 140
5.7 Utility differential and linear home-sweet-home effect for different degrees of heterogeneity. 141

5.8 Utility differential and linear home-sweet-home effect for changing levels of the freeness of trade. 142
5.9 Bifurcation diagrams for the linear home-sweet-home effect. 143

Chapter 1

Introduction

Throughout its history, the field of New Economic Geography (NEG) has continuously sought to uncover new ways to explain the impact of economic integration on the geographical distribution of economic activities and how these contribute to shape regional income disparities. Particularly, its main study object remains the tendency towards spatial agglomeration of industry as a result of decreasing trade barriers.

The purpose of this dissertation is to contribute to NEG by providing new insights and explanations for the aforementioned issues. In a broad sense, our aim is to study the interdependencies among regions and how they impact the spatial structure of the economy. In this regard, our objectives are mainly threefold. First, we provide an overall assessment of NEG, reviewing the literature and also identifying paths along which the field could yet improve. Second, we incorporate multiple regions, equidistant among themselves, into two otherwise well-known distinct Footloose Entrepreneur (FE) models (Forslid and Ottaviano, 2003; Baldwin *et al.*, 2004; Pflüger, 2004), that belong to a class of analytically solvable versions of Krugman's seminal Core-Periphery (CP) model (Krugman, 1991). Lastly, we study the impact of the interaction between differences in how each individual perceives different regions and market-factor driven agglomeration forces. The rest of this introduction is devoted to explaining the structure and contents within each chapter.

Chapter 2 contains a prospective review of the field of NEG and lays down the context for the topics addressed in the subsequent main chapters of the Thesis. We start by providing a general overview of the state of the art of NEG and then describe some of its main limitations and shortcomings. We then proceed to characterize possible research paths along which NEG has improved or could develop further. Among several issues, we identify the two main topics that comprise the following chapters: the role of multiple regions and the impact of heterogeneity in preferences for residential location. Therefore, Chapter 2 also serves as in introductory approach to our main research topics and hopefully motivates the reader to our main points of interest by putting them into a broad perspective. In the subsequent chapters, we narrow down our focus and adopt a more technical and detailed approach that rigorously addresses these two main topics.

The first major topic seeks to overcome the limited dimensionality in Economic Geography. We depart from the usual two-region setup and thus embrace a multi-regional framework. We analyse the spatial distribution of economic activities across an arbitrary number of equidistant regions using two analytically solvable Core-Periphery models: first, we extend the FE model of Forslid and Ottaviano (2003);[1] second, we use the FE model with quasi-linear log utility (QLLog) without income effects developed by Pflüger (2004). This covers up Chapters 3 and 4, respectively, of the Thesis. The second topic, addressed in Chapter 4, tackles the role of consumer heterogeneity in Economic Geography. Particularly, we introduce preferences *à la* Hotelling in order to study how different preferences for residential location affect the resulting spatial patterns.

Chapter 3 of the Thesis, called "The Footloose Entrepreneur Model with an arbitrary number of equidistant regions", extends the FE model (Forslid and Ottaviano, 2003; Baldwin *et al.*, 2004) to an arbitrary number of equidistant regions. The FE setup departs from the original CP model in that each manufacturing firm's scale of production does not depend on the skilled, inter-regionally mobile workforce; rather, each firm uses a fixed amount of entrepreneurs. This renders the model analytically solvable and allows us to obtain the entrepreneur's wage as an explicit function of the spatial distribution of workers. Since the main qualitative properties of the CP model are preserved in the FE model, the latter becomes a good potential candidate for a multi-regional extension. We show that, whatever the global size of unskilled and skilled labour, as the number of regions increases, single-region agglomeration becomes more likely while symmetric dispersion becomes less likely. We also provide numerical evidence that a spatial equilibrium with at least one region absent of industry is always unstable. Moreover, the simultaneity of stability of agglomeration and dispersion (and therefore, the existence of locational hysteresis) in the original CP and FE models seems to persist for the multi-regional case. Finally, we introduce exogenous asymmetries by allowing regional endowments of the immobile (unskilled) labour to differ across regions and study the qualitative properties of agglomeration under changes in these endowments.

Chapter 4, "A multi-regional agglomeration model without income effects", follows up the previous multi-regional extension by considering the slightly different quasi-linear log (QLLog) model by Pflüger (2004). In its original formulation, the QLLog model considers a Footloose Entrepreneur setup (described above) with a quasi-linear upper tier

[1] See also Baldwin *et al.* (2004).

utility function for consumers, instead of the usual Cobb-Douglas functional form. The modification, albeit small, produces two important effects. Firstly, the model becomes even simpler and more tractable than the FE model. Secondly, the consequent removal of income effects on the demand for manufactures leads to the emergence of asymmetric spatial equilibria, which may correspond to any possible spatial distribution, depending on the level of transportation costs.

Unlike the FE model, many of the findings of the two-region QLLog model are not preserved by its multi-regional extension. In other words, the inclusion of additional regions significantly modifies the qualitative structure of the space economy. Most noticeably, we find that along a space with $n-1$ evenly distributed regions, an asymmetric equilibrium requires one region to be significantly more industrialized compared to the others. Moreover, the transition from symmetric dispersion towards agglomeration, along a smooth path where transport costs fall steadily, depends crucially on the global worker mobility. Specifically: if mobility is low, agglomeration becomes catastrophic as in the original CP model; if mobility is high, the transition towards agglomeration is more progressive. Lastly, the greater tractability of the QLLog model allows us to analytically confirm that any spatial distribution whereby at least one region is absent of industry (while all the other regions are evenly distributed) is not possible.

In Chapter 5, “Economic Geography meets Hotelling: a home-sweet-home effect”, we use a modified version of the two-region CP model where all labour is free to migrate between regions, as in Helpman (1998), and incorporate consumer heterogeneity in the preferences for residential location. This is achieved by considering a uniform distribution of consumer preferences along a line segment in the fashion of Hotelling’s (1929) linear city model. Each consumer is identified by his position on this line which corresponds to his preference towards residing in a given country. Therefore, whereas a more industrialized region provides a higher utility from consumption of manufactures for all consumers, it may fail to attract consumers who have a higher idiosyncratic preference for the less industrialized region.

Hotelling-type preferences allow us to formulate different utility penalties as a function of these preferences, i.e., to infer about the impact of different consumer behaviours in the long-run spatial distributions. The widely used logit model used in previous works (Tabuchi and Thisse, 2002; Murata, 2003; Combes *et al.* 2008), which yields an equivalent closed form in our setting, corresponds to a very specific type of consumer heterogeneity regarding location preferences. On the other hand, our Hotelling-type heterogeneity is general enough that we are able to compare the predictions based on the logit model with other types of consumer behaviour. Doing this, we are able to conclude that the long-run geographical distribution of economic activities is not invariant to the type of consumer heterogeneity. However, irrespective of the latter, we find that lower transport costs promote symmetric distributions and discourage agglomeration, a result which is at odds with Krugman's original predictions (and those of models similar to the CP model).

In Chapter 6 we provide a brief discussion on the more general results obtained in the Thesis. We take the opportunity to discuss how different assumptions across the different models are likely to influence different outcomes. By way of conjecture, we try to infer about how some specifications determine different results, and leave some considerations for possible future work.

Chapter 2

Economic Geography: a prospective review

2.1 Introduction

Since the path-breaking contribution of Krugman's (1991) Core-Periphery (CP) model, which marked the surge of the field of New Economic Geography (NEG), many researchers have devoted their attention towards providing theoretical and empirical works that add to the study of the geographical agglomeration of economic activities. Throughout its rather brief history, up to recent years, NEG has remained yet an attractive and vibrant field for many economists. In spite of this, NEG has also been constantly susceptible to a wide range of criticism. Moreover, the lack of departure from some main, albeit restrictive, assumptions has confined the field to a theoretical strait-jacket, thus thwarting efforts to come up with new insights and providing new relevant developments. In this review, we summarize the contents of a number of papers in NEG literature which we consider to be very successful in the attempt to provide an overall assessment of the discipline.

What these works have in common is the fact that they are all aware of the technical limitations of the frameworks generally assumed in the literature, and they all try to analyse the causes for the shortcomings in terms of new developments in the field. Generally speaking, we refer to the stringent assumption of constant elasticity of substitution models of monopolistic competition, the unrealistic assumption of iceberg transportation costs, the lack of calibration and attention devoted to the use of numerical computations, the strict homogeneity assumed across consumers and firms, the homogeneity in location space, and the overuse of migration adjustment processes where agents are assumed to be short-sighted, to mention but a few. The papers reviewed throughout this survey are aware of these limitations and thus try to give a prospective account of new directions that the field of NEG should follow.

At a later stage, we briefly describe some notable contributions in the literature that address some of the issues raised by the recent criticism surveyed in this work. Some of the issues pertain to: the influence of firm or consumer heterogeneity in spatial agglomeration outcomes; new general models of monopolistic competition; reduced dimensionality; the role of the spatial topology in a multi-regional setting; forward-looking expectations in the migration adjustment process; the role of the computer for the sake of numerical simulations; and new analytical approaches that provide valuable insights on bifurcation mechanisms in CP models.

The rest of the paper is organized as follows. In Section 2 we review the literature from the dawn of NEG, starting from Krugman's 1991 seminal Core-Periphery model, up to subsequent models that allowed NEG to span across different (but not unrelated) fields such as urban economics and economic growth. Section 3 overviews the main limitations that have confined the field of NEG in a theoretical strait-jacket. We then proceed to identify possible avenues of search within the field and some of the recent contributions that have provided new path-breaking insights in Section 4. Section 5 is left for some concluding remarks.

2.2 An overview on NEG

In this section we summarize the state of the art in New Economic Geography literature and the recent discussions aiming to provide new insights and future developments in the field. These contributions provide an overview on the economic history of the early developments in NEG up the the more recent theories at the frontier of the field. Furthermore, they underline the main restrictions in NEG and show what needs to change in the field if new path-breaking conclusions are to be drawn from the study of economic geography.

Recently, urban economics has had the merit of delivering important contributions, such as providing micro-foundations of urban agglomeration economies and explaining the impact of neighbourhood effects and spatial externalities on the stratification of cities. Location theory, deeply rooted in Hotelling's "Stability in Competition" (1929) and Lösch's "The Economics of Location" (1954), studies the geographical distribution of industry and geographical variations in mark-ups. Though relevant fields in their own right, neither of them has achieved the interest reached by NEG. One of the reasons might be that, unlike the other two, NEG has a well defined and broad enough objective. As Walter Isard (1956) complained, classical location theory, for instance, was disentangled from mainstream economics because it confined itself to a disintegrated framework of partial equilibrium theory, constant coefficients, linear transport costs, and *ad hoc* demands (Blaug, 1997).

On the other hand, NEG is first field in economics to provide a detailed description of spatial inequalities that emerge as the outcome of a general equilibrium model. Paul Krugman, as a precursor of NEG with his seminal article from 1991, was the first to show how regional imbalances arise. The reason why it took so long to come up with an explanation probably hinged on the technical impossibilities imposed by dominant paradigms of economic theory up to that date. Authors such as Duranton and Puga (2004), Fujita and Thisse (2009) and Puga (2010) discuss what results are consistent with the neoclassical theory in terms of the spatial distribution of economic activities. To sum up their overview, it is useful to state Starret's (1978) "Spatial Impossibility Theorem". The theorem states that in an Arrow-Debreu economy under constant returns to scale, with a finite number of agents and locations, homogeneous space and costly transportation, there is no competitive equilibrium involving inter-locational trade (transportation). By homogeneous space it is meant that both preferences and the set of production technologies are independent of location. In other words, if economic activities are perfectly divisible, there is a competitive equilibrium such that each location is autarkic.

Bearing the previous in mind, it is not difficult to find that, in order to explain spatial inequalities and regional specialization, one must violate at least one of the assumptions stated in the Theorem. This led to the surge of models of comparative advantages based on spatial heterogeneities among regions, models of agglomeration externalities that arise from non-market interactions that yield increasing returns external to the firms, and models of imperfect competition. The first type of models cannot account for the existence of full-fledged agglomerations and very high spatial imbalances. The second type appeared long before NEG, but, as the previous type models, works under the framework of constant returns and perfect competition. As such, models with agglomeration externalities fail to give an explanation for the microeconomic interactions that give rise to those spatial externalities. Moreover, such externalities seem to be relevant in the small but not as essential in the large. With models of imperfect competition, pricing decisions by firms depend on the spatial distribution of both consumers and other firms. Models of monopolistic competition are favoured in particular. One reason is that, since there is no strategic interaction between firms, the common problem of existence of equilibrium which occurs frequently in oligopolistic competition, is not that problematic in this case. Another reason pertains to the higher tractability of monopolistic competition. Since the framework implies increasing returns to scale at the plant level, and transportation is costly, location decisions are not trivial. Hence, both ingredients are essential to any model that aims at explaining the space-economy. A particular case is the Dixit-Stiglitz monopolistic competition model (Dixit and Stiglitz, 1973), where consumers value variety of a horizontally differentiated good and where there exists a continuum of firms operating under internal economies of scale for each good.

Seeing that the space-economy is a result of agglomeration forces operating against dispersion forces, we scrutinize the main principles at work that led to NEG. Price competition is known to be a strong dispersion force (see Tirole, 1988). However, product differentiation alleviates price competition and hence allows firms to locate where they have access to a bigger market and higher demand, and where transport costs are lower. The other principle at work is that of the home market effect (HME), whereby large markets are relatively more attractive to firms. But this view assumes that the market size is exogenous, i.e., that consumers are not allowed to migrate between regions. The path-breaking contribution to tackle this issue is the Core-Periphery (CP) model by Krugman (1991). The starting point of the CP model is that the migration of some workers affects the global welfare and thus changes the relative attractiveness of both origin and destination. These effects can be seen as externalities because workers do not take them into account in their decisions (they are short-sighted). The basic layout of the CP model comprises two regions, and two sectors: one operating under monopolistic competition *à la* Dixit-Stiglitz and the other operating under perfect competition; and two factors of production. One factor is regionally immobile and is used as an input in the agricultural sector. The other is regionally mobile and is used as input in the industrial sector. There is a cumulative process whereby market size (inducing the HME) and cost of living effects work in a way that promotes agglomeration of industry in one region. As this region becomes bigger, so does the market, thus attracting more industry (the HME at work). This circular causation of forward linkages and backward linkages, noted by Krugman (1991), generates a centripetal force. On the other hand, a more concentrated market enhances price competition, thus working as a dispersion force (market crowding effect). This is also called a centrifugal force. All things considered, the key factor for determining the spatial distribution of industry is the level of transportation costs. Hence, contrary to the neoclassical model that predicts only convergence, the CP model accounts for both convergence and divergence. Inasmuch as there is also a need to explain how agglomeration outcomes are possible when factor mobility is reduced, several models have come up with explanations based on input-output linkages. One such contribution is the paper by Krugman and Venables (1995). The main idea is that agglomeration of a sector in a region occurs because there is a vertically linked sector that is already agglomerated in that region. The forces at work in this framework are different. There is also a market expansion effect but in this case it is due to higher income (higher wages since labour supply is inelastic) that leads to higher consumer demand. However, if wages are too high, some firms will want to relocate their production to the periphery, so there is also a dispersion force. The advantage of this framework is that a self-perpetuating agglomeration process may not happen. Instead, economic integration yields a bell-shaped curve of spatial development. This kind of model accounts for the possibility of re-industrialization of the periphery, supporting the hypothesis of a "spatial" Kuznets curve whereby market forces initially increase, and then decrease, economic inequalities. This prediction has also been uncovered in CP models where consumers are heterogeneous concerning their preferences for different locations (Tabuchi and Thisse, 2002; Murata, 2003; Combes *et al.*, 2008). The mechanism that precludes agglomeration in this setting is different. Simply put, heterogeneity in preferences towards residing in one region or the other implies that regional utility differentials differ across individuals, all other things being equal. This implies that some consumers are less willing to migrate than others. A higher dispersion of consumer preferences naturally strengthens its role as an additional dispersion force.

Krugman's contribution from 1991 has also lead to insights on city formation and urban systems. The NEG models that belong to the class of urban and regional systems deal with the spatial distribution of agglomerations without considering their internal spatial structure (Fujita and Mori, 2005). Henderson (1974) developed an approach that allows him to describe the emergence of a hierarchy of cities. Fujita, Krugman and Mori (1999) further extend this approach by considering several differentiated industrial goods in a model similar to an NEG model. Another precursor in this literature refers to the "Racetrack Economy" by Krugman (1993) which extends the original CP model to include 12 regions around an homogeneous circle with the same distance between each adjacent region. The main conclusion reached by Krugman was that a simulation with a nearly uniform initial spatial distribution of industry around the circle would always end up with all manufacturers concentrated equally at two regions located at exact opposite sides of the circle. A more realistic approach is taken by Fujita and Krugman (1995) which sets out from the von Thünen's city centre, where industry is concentrated, surrounded by concentric rings where agriculture takes place. The main prediction of the model is that, as population increases, the borders of the agriculture hinterland locate sufficiently far from the city centre so that firms find it attractive to locate out of the city centre, thus giving rise to a new city. This process is self-perpetuating as more cities arise. A key feature here lies in the role of natural exogenous regional differences between regions in explaining economic geography. These differences determine the initial advantages of a region which may become a new centre; it then grows through a self-perpetuating process until a point from which the initial advantages no longer matter (or are relatively unimportant) for further agglomeration.

The original CP model can also be extended to study regional growth. Baldwin and Martin (2004) and Fujita and Thisse (2003) have attempted to take advantage of the common "tools" shared by both NEG and "new growth" theories. In the latter, the authors show that the growth rate of the global economy depends on the spatial distribution of an innovation sector (that applies mobile skilled workers) across regions. This sopports the evidence that there is a trade-off between growth and spatial equity. Overall, the result is desirable in the sense that, as the economy agglomerates in a region, the innovation rate tends to increase, and all workers benefit from this including those living in the periphery. Hence, the overall outcome is Pareto superior. Of course, workers in the core benefit from a higher welfare than workers with the same set of skills that live in the periphery. Thus, the main implication of these works is that there may be a trade-off between social cohesion and economic growth.

2.3 Reassessing New Economic Geography

The fact that NEG models are intrinsically difficult to work with has led most authors in the field to stick to a narrow framework, almost mimicking Krugman's original setup. Altogether, even accounting for the models departing the common framework, like the class of footloose entrepreneur models (Forslid and Ottaviano, 2003; Baldwin *et al.,* 2004), these models display relatively common equilibrium properties, thus asserting the robustness of the insights provided by the original CP model. Though this may seem attractive, it also means that building models based on frameworks close to the original CP model is very unlikely to bring additional knowledge on NEG. Thus, new important insights can only be obtained by departing from some of the canonical assumptions and functional forms underlying the original CP model.

2.3.1 Persistent features in NEG models

Whatever the agglomeration mechanism, be it labour migration as in the original CP model, input-output linkages as in Venables (1996) or capital accumulation as in Baldwin (1999), and even with marginal changes in functional forms, the key features of

Krugman's original model do not change. Robert-Nicoud (2005), Garretsen and Martin (2010) and Behrens and Robert-Nicoud (2011) argue that, as long as goods are manufactured under increasing returns to scale and markets are segmented in a standard model with 2 regions, what happens in one region impacts the other one. The effects of the economic geography of the two regions is similar across most CP models, as the change in the relevant market sizes for CES-good producers is at the heart of agglomeration and dispersion forces. Following Baldwin *et al.* (2004), we summarize the seven features displayed by most NEG preliminary models:

i. *Home-market effect.* A more than proportional re-location of industry due to CES demand shifts;

ii. *Circular causality between rents and industry size.* The reciprocal relation between higher income and a larger industry;

iii. *Endogenous asymmetries.* They relate to how one region grows at the expense of the other;

iv. *Catastrophic agglomeration.* Small variations in parameter values lead to sudden single-region agglomeration;

v. *Multiple equilibria and self-fulfilling expectations.* History and expectations matter for the selection of the relevant spatial distribution;

vi. *Locational hysteresis.* Persistence of the stability of a core-periphery pattern after a decrease in transport costs below a certain threshold, even if the decrease is temporary.

vii. *Bell-shaped agglomeration rents.* Agglomeration rents exist and are highest for intermediate values of transportation costs;

It turns out that all simple NEG models with Cobb-Douglas-*cum*-CES preferences, iceberg costs and constant returns to labour share the aforementioned features and "are isomorphic in an economically meaningful state space" (Robert-Nicoud, 2005). Ottaviano and Robert-Nicoud (2006) have also pointed this out.

Slight modifications in subsequent NEG models have come to contradict some of these, but not all, persistent features. Of the aforementioned, features (iv) and (vi) are the less likely to hold under slight modifications to the original CP model. For instance, Pflüger (2004) introduces a quasi-linear upper-tier utility function that removes all income effects from manufactured goods and, by giving rise to a continuum of asymmetric distributions between dispersion and agglomeration, excludes both catastrophic agglomeration and locational hysteresis. In Ottaviano *et al.* (2002), where utility is quadratic in the consumption of manufactured goods, catastrophic agglomeration is possible but locational hysteresis is precluded. Another departure from the original CP model is the early footloose capital (FC) model by Martin and Rogers (1995). Noteworthy, in the FC model, the production shift from one region to another is not accompanied by a shift in expenditure. This strips the model of the typical demand and cost linkages, making it completely tractable while still featuring agglomeration.

Each of the following subsections analyses in detail one stringent feature common in most NEG models and suggests ways to overcome it.

2.3.2 Costly icebergs

One limitation commonly present in NEG models is that of assuming iceberg costs (Neary, 2001; Fujita and Mori, 2005; Fujita and Thisse, 2009; Behrens and Robert-Nicoud, 2011). Any variation in mill prices is accompanied by a proportional variation in transport costs. However, transport costs are known to decline steadily as a share of a firm's total cost. Therefore, such an assumption is hardly credible. Ottaviano, Tabuchi and Thisse (2002) assume that trading the industrial good requires a given amount of the numéraire good per unit shipped, which is more in line with empirical evidence as it allows to dwell into the relationship between agglomeration and firms' pricing decisions. In spite of the lack of realism, Fujita and Thisse (2009) argue that the main properties of NEG do not hinge on the specification of transportation costs. This view is perhaps shared by most researchers, as the issue of iceberg transport costs has been continuously under-emphasized in NEG literature up to recently.

On the other side of the spectrum, Fujita and Mori (2005), and Behrens and Robert-Nicoud (2011), call for an endogeneization of transport costs. They argue that NEG has yet to account for the fact that transport costs and trade costs are not symmetric. Furthermore, in the presence of firm heterogeneity, iceberg costs imply that more productive firms face lower per unit trade costs, since the iceberg amounts to a scaling of the firm's marginal cost. As a result, exploring these issues requires explicit modelling of the transport sector.

Fujita and Mori (2005) call for the explicit modelling of the transport sector, something that was attempted by Takahashi (2006), who developed a microfounded formulation of economies of transport density, endogeneizing the transport sector in the 2-region context. However, such a setup precludes any insights on the interdependence between agglomeration patterns and the structure of the transport network.

In a broader sense of geography and transport costs, NEG has also continuously overlooked important location factors such as costs associated with labour, taxes, climate, topography, or the political environment. What is more, these factors are not likely to vary proportionally with distance (Blaug, 1997) or with firms' costs. Therefore, further development of NEG should build on more comprehensive studies on the nature and modelling of transportation activities.

2.3.3 Monopolistic competition

Another resilient feature of most NEG models is the Dixit-Stiglitz framework of monopolistic competition. The main argument supporting the framework is that it is analytically convenient, while still conveying the integration of imperfect competition and increasing returns. However, there are shortcomings. Its framework is lacking in that it contradicts both economic theory and some well known empirically observed facts (Fujita and Thisse, 2009; Zhelobodko *et al.*, 2012), namely: varying markups and prices with firm entry and market size; flexible preferences; and the fact that the size of firms depends on the number of consumers. Fujita and Thisse (2009) claim that this modelling strategy thus reduces both agglomeration and dispersion forces. This reduction is a direct consequence of the lack of pro-competitive effects in firms' markups.

Behrens and Murata (2007) provided a model of monopolistic competition with income and price competition effects. Particularly, under quasi-separable preferences, they showed that profit-maximizing prices may be decreasing in market size and converge to the level of marginal costs (the competitive limit) when the number of firms is arbitrarily large. Zhelobodko *et al.* (2012) attempt to develop a general monopolistic competition model, thus trying to improve on the stringent CES specification. This particular contribution shall be discussed in more detail in Section 4.2 of this paper.

2.3.4 A lack of geography

Other important issues discussed in the literature relate to the spatial origin of costs, the consideration of only two sectors and two regions and the restricted and highly stylized role of the agricultural sector. Out of these three, the dimensionality issue appears to be the most problematic. One reason is the theoretical understanding of realistic interdependencies to guide empirical studies (Fujita *et al.*, 1999). The other stems from the fact that a two-region setup overlooks the variability of market access across regions. Of course, the latter issue clearly departs from models with symmetric transportation costs across regions. Another insight relates to the more complex impacts that may arise from a multi-regional setup compared to a two-region one (Neary, 2001; Fujita and Thisse, 2009).

In the original CP model (Krugman, 1991) with two regions, the transportation cost parameter is the embodiment of geography. In an abstract two-region setting, the distance between hypothetical regions is normalized to unity and "dimensionality" is captured by transport costs, without which there is absolutely no role for geography. Subsequent developments of the original CP model have provided qualitatively different and more realistic results compared to Krugman's CP model. Notwithstanding, these more recent NEG models do not offer a more substantive analysis when it comes to the role of space (Garretsen and Martin, 2010). In reality, the depiction of geographical scale has to take account of the fact that it cannot be represented in terms of just transport costs, and is more complex than the simple geometries used in NEG theory to portray geography. According to Harvey (1985), a region pertains to a bounded geographical unit that is characterized by its own unique economic, social and cultural "structured coherence". It turns out that regional spaces are rarely structurally coherent or contiguous, but are rather characterized by economic, social, cultural and spatial discontinuities. Moreover, regions are produced and modified by both economic and social structures operating over time.

It is true that NEG models internalize location into the economic process. In fact, growth in NEG models affects the distribution of economic activities and vice versa (recall Fujita and Thisse, 2002; Baldwin and Martin, 2004). However, as geographers would argue, regions in NEG models remain spaceless entities: they have no spatial extent and thus no internal spatial structure. While this may be true, at least to some extent, this fact has also already been acknowledged by some NEG theorists. In fact, Behrens and Robert-Nicoud (2011) not only are aware of this but also emphasize the role of urban hierarchy formation and the spatial sorting of heterogeneous individuals across cities as potential paths for the development of NEG. Moreover, some NEG models allow for regions to differ in size. Notwithstanding, these differences are always exogenous (or pre-given), fixed and purely quantitative, thus belonging to an absolute spatial frame.

All things considered, one may argue that NEG treats the economic landscape in the form of an abstract geometric space. Similarly, history and time in NEG are also circumscribed in the sense that focus lies on equilibrium outcomes and local stability analysis, rather than on real time, or history (Martin, 1999; Boschma and Frenken, 2006; Garretsen and Martin, 2010). More emphasis should be placed on novelty, selection, adaptation and self-organization, as these processes stimulate and form the ongoing evolution of the space economy. However, instead of an evolutionary approach, NEG treats innovation and technological change as determinants of the extent of spatial agglomeration while neglecting its impact on the nature of economic activity.

2.3.5 The role of the computer: empirical work and policy implications

The computer has long been an essential tool to study NEG models, due to their inherent intractability or lack of closed form solutions (Neary, 2001). Fujita and Thisse (2009) argue that we do not need to use the computer if the goal is to study the qualitative properties of equilibria. Moreover, they argue that recent developments have come to confirm Krugman's numerical results (Fujita *et al.*, 1999; Robert-Nicoud, 2005; and Mossay, 2006). Thus, they argue, the computer is no longer needed in NEG. Behrens and Robert-Nicoud (2011) think differently on this matter. They point out that serious calibration is rarely done in NEG and conjecture that this may be one explanation for the under-representation of NEG in top journals of economic theory. Moreover, most NEG models lack policy implications, thus causing many researchers in the field to refrain from prescribing policy recommendations. This issue is closely related to the aforementioned lack of serious model calibration, and also lack of useful empirical work.

The concern of Fujita and Mori (2005) toward the role of the computer in NEG is more closely related to the opinion of Behrens and Robert-Nicoud (2011). They find that the use of the computer for numerical simulations is rare in NEG and that the recent developments in computer technology may and should turn this around. While it is still important to continue providing analytically solvable and tractable models, the limitations in terms of richness and completeness this imposes on NEG modelling means that going beyond a symmetric 2-region and 2-industry framework model of trade and geography requires the use of the computer to attain useful empirical analyses and policy implications.

While abstractions from a realistic geometrical space have been common practice in NEG (as identified in Section 3.4), the recent and increasing surge of sophisticated computational methods, together with the availability of geographical data, has given rise to analyses based on models of agglomeration externalities with explicit geographical space, such as Redding and Sturm (2008), and Allen and Arkolakis (2014).

2.3.6 Myopic migration and multiple equilibria selection

One concern in NEG relates to the excessive use of evolutionary dynamics to model migration in most CP models. Under this specification, mobile workers only care about current utility levels, something that is not consistent with fully rational forward-looking behaviour. Therefore, there is a need for a dynamic process that mimics observed migration behaviour of workers. More importantly, one would like to know if Krugman's results are robust to a more realistic formulation of the dynamic process.

For specifications such as the widely used *replicator dynamics*,[1] the critique is that agents are myopic, i.e., they base their location decisions solely on current utility differentials. Under myopic dynamics and through the analysis of local stability, the standard CP model predicts two possible outcomes: full agglomeration and symmetric dispersion. Moreover, since migration is a continuous process, individual location revisions are implicitly burdened by frictions (Oyama, 2009), and a core-periphery pattern, once achieved, may not be upset. If agglomeration is stable, however, there is no endogenous mechanism able to select among the multiple stable agglomeration configurations, the resulting long-run outcome being determined by historical initial advantages. In reality, notwithstanding, it seems reasonable to claim that agents are also concerned with expected future utility rather than current utility levels. In this case, future location configurations matter and self-fulfilling expectations may drive the economy from one core-periphery pattern to another (Oyama, 2009).

Suppose workers have a common belief about the future of the space-economy. Pro-

[1]See Fujita *et al.* (1999) for a more comprehensive description.

vided that workers care enough about the future, it is possible that workers in a region with a currently larger market will migrate to the currently smaller region, thus reversing the historically inherited advantage of the larger region. For this to happen, we need only consider that the currently larger region actually has a lower endowment of immobile workers compared to the other. Then, because workers value future utility enough, they anticipate that their overall discounted utility will be higher if they all migrate to the region with more immobile workers. These theoretical findings have been confirmed by Oyama (2009), who has incorporated self-fulfilling expectations in a mixed setup of the FC model (Marting and Rogers, 1995) with logarithmic quasi-linear utility and multiple regions (as in Pflüger, 2004). This builds on the literature concerning the selection of equilibria through the consideration of absorbing states and global accessibility. In Oyama's model, each entrepreneur builds expectations about the future path of the distribution of entrepreneurs and chooses to locate in a region that maximizes his expected discounted indirect utility. There is a feasible path along which a revising agent optimizes against the future location pattern of industry. The stationary states of this path are the equilibrium states of a societal game with a continuum of homogeneous players and correspond to the spatial equilibria of the static model. However, there may be also perfect foresight paths that escape from a stationary state when the degree of migration friction is low enough. That state is thus considered as unstable and motivates the following stability concepts, due to Matsui and Matsuyama (1995): a given state is *absorbing* if any perfect foresight path converges to it for an initial state close enough to it; and is *globally accessible* if there exists a perfect foresight path converging to the state, given any initial condition. An additional assumption is that trade costs depend only on the destination country, i.e., they are predominantly destination-specific.

One main conclusion concerns agglomeration configurations. If trade barriers are low enough, and, if a region has a larger market size and/or higher trade barriers, then the state corresponding to full agglomeration in that region is absorbing and globally accessible. A direct implication of this result is that full symmetry deems the choice of forward looking expectations and myopic migration for the dynamics irrelevant. In this case, initial advantages and history necessarily matter. With exogenous asymmetries, however, allowing for perfect foresight dynamics enables one to select a unique equilibrium through the consideration of global stability which is not possible if agents are myopic. This globally stable equilibrium corresponds to full agglomeration in the region which is most relatively protected or has the potentially largest market size, whatever the initial state. The policy implications in terms of trade policy under forward-looking expectations are straightforward. A country looking to attract industry may have incentives to increase its trade barriers, thus inducing relocation of firms toward it. This is not possible with myopic migrants, inasmuch as each agglomeration state is locally stable under myopic dynamics.

Oyama's and subsequent frameworks still lack explicit modelling of the formation of expectations, something that could be relevant for policy issues. Though more realistic than the standard replicator dynamics and other specifications imported from evolutionary game theory, there is still some lack of explicit dynamic modelling based on microfoundations.

2.4 Breaking through the strait-jacket

Having sought to describe the state of the art and the new directions for future developments of NEG, we now present a class of recent contributions that address, to some extent, some of the issues raised in the previous section.

2.4.1 Individual heterogeneity

A new explored path of research in NEG takes account of heterogeneity at the consumer and firm level. One can claim that the typical Dixit-Stiglitz monopolistic competition, which continuously dominates NEG, has been enriched with firm heterogeneity, an attractive feature that allows us to say something about firms, industry dynamics and selection processes (see, e.g., Ottaviano, 2012). Differences in productivity at the firm level have some clear implications for the spatial distribution of industry. There are two empirical facts that have remained unexplained by NEG models: the spatial sorting of heterogeneous individuals across cities and an explanation for an urban hierarchy following the rank-size rule. According to Behrens and Robert-Nicoud (2011), factor heterogeneity and urban ingredients are important for future developments of NEG.

NEG should build on a stronger role of firm and consumer heterogeneity, in complement with imperfect competition and increasing returns at the firm level. The base for this argument can be summed up in the setup used by Ottaviano (2011), with a simple two symmetric regions, two firms model. As usual, under differentiated products, costly transportation and agglomeration economies, endogenous concentration in one region driven by a self-sustaining process occur whenever market size and cost-of-living advantages are enough to offset the competitive stress of co-location, for a low enough level of transportation costs. According to Ottaviano (2011), this setup could be enriched once we account for efficiency differences at the firm level and study their interaction with the differences in production costs and market size. If, for instance, location space is heterogeneous, a less efficient firm would have more incentives to avoid tougher competition and would thus locate in the less advantageous location. As a result, firm heterogeneity acts as an additional dispersion force, the more so the higher the transport costs and the lower the differentiation between both firms' products. The aforementioned example focuses on technological differences, but the argument could be extended to differences in quality of the products offered by the firms. Thus, whereas horizontal differentiation acts as an agglomeration force, vertical differentiation clearly favours dispersion outcomes. The former is usually captured by NEG models by the parameter referring to the elasticity of substitution between varieties. The latter could be accounted for by some sort of degree of firm heterogeneity. Hence, we would have an additional microeconomic parameter contributing to the determination of the geographical distribution of industry.

Heterogeneity in firm efficiency does not always lead to more dispersive outcomes. In fact, Ehrlich and Seidel (2013) argue that higher discrepancies in firms' total factor productivity, through self-selection, raise the number of exporting firms and thus foster the agglomeration of economic activities.

While heterogeneity across agents has been traditionally dealt with in urban economics (see, e.g., Duranton and Puga, 2004), it could further add to the understanding of agglomeration economies. Some attempts to study how micro-heterogeneity influences some key results of most NEG models are those of Baldwin and Okubo (2006, 2009), Nocke (2006), Melitz and Ottaviano (2008) and Okubo *et al.* (2010).

Another important source of individual heterogeneity lies at the consumer level. In papers such as Tabuchi and Thisse (2002), Murata (2003), Mossay (2003), Combes *et al.* (2008) and Redding (2016), heterogeneity is incorporated by considering that preferences for residential location are idiosyncratic to each individual. The premise is that each location has its own specific characteristics, be it climate, culture or provision of public infrastructures, which are perceived differently by each individual (Rodríguez-Pose and Ketterer, 2012). These individual specific amenities are coupled with regional asymmetries in the form of location specific amenities, generating asymmetries in equilibrium real wages across different locations, thus allowing for a broader range of spatial configurations. Region specific amenities translate into first nature advantages, i.e., exogenous asymmetries which make a region more attractive compared to another for all individuals. However, even if regions are otherwise completely symmetric, individual idiosyncrasies, by constituting an actual dispersive force, may help explain why some locations are more industrialized than others. This type of heterogeneity is included in NEG models by considering that migration responds to the realization of a random unobserved variable. In order to keep the analysis tractable, probabilistic migration is usually modelled according to the Logit model by MacFadden (1974), as in Tabuchi and Thisse (2002), Murata (2003), Combes *et al.* (2008), or Akamatsu *et al.* (2012). However, the qualitative structure of spatial distributions remains by and large invariant under the Logit specification. Consequently, by considering variations in the scale parameters that govern the particular Logit models, instead of allowing for the distribution of preferences itself to change, predictions about the influence of heterogeneity on NEG are contained within a limited scope. This should motivate the deepening of the connection between NEG and the study of residential location choice in order to better understand the influence of heterogeneity in preferences for location on the spatial distribution of economic activities.

2.4.2 Beyond CES preferences

Some developments in NEG have sought to overcome the lack of pro-competitive effects and variable elasticity substitution envisaged by the Dixit-Stiglitz setup. Ottaviano *et al.* (2002) address these problems by considering a quasi-linear utility function with quadratic sub-utility and linear additive transport costs. Their model, while retaining the main qualitative properties of Krugman's original model, is able to account for pro-competitive market effects; particularly, profit maximizing prices are decreasing in market size. Moreover, additive transportation costs are useful to study the relationship between agglomeration and pricing decisions (Ottaviano, 2000). Other theoretical contributions have been developed recently with a focus on pro-competitive effects (see, e.g., Behrens, 2005). Another contribution worth mentioning in this regard is the class of Chamberlinian monopolistic competition models analysed by Behrens and Murata (2007). They show that additively quasi-separable preferences yield constant absolute risk aversion (CARA); the latter allows for profit maximizing prices as a decreasing function of market size.

The paper by Zhelobodko *et al.* (2012) deals with a more general model of monopolistic competition with symmetric additively separable preferences. The main aim of the paper is to develop a general model that encompasses the CES as a special case, but that allows for a better description of real world markets and issues brought out by oligopoly theory, while being tractable enough to enable the study of market equilibrium properties. The framework in the paper has the attractive feature of precluding any specific functional form for utility functions. The only requirement is that the utility be concave so as to guarantee that consumers are lovers of variety. Such a specification allows them to deal with different patterns of substitution through the concept of relative love for variety (RLV) which is tantamount to the elasticity of marginal utility. The RLV measures consumers' attitude toward variety loving. It is the inverse of the elasticity of substitution associated with the consumption level of a given good.[2] When preferences display an increasing (decreasing) RLV, consumers care less (more) about variety when their consumption level is lower. The concept of relative love for variety in this framework is so important that identifiable opposite market outcomes depend on the RLV. If it is increasing, the market generates pro-competitive effects, i.e., a larger market size leads to lower equilibrium prices. If it is decreasing, the market generates price-increasing (anti-competitive) effects: a larger market size leads to higher market prices because the elasticity of substitution now decreases. For the CES case (constant RLV), competitive effects are completely washed out. These results seem to hold under a framework with several sectors, heterogeneous firms, and specific utility functional forms such as the quadratic and translog utility. As a result, this general monopolistic competition model provides a new application to NEG models, with the particularity that its results hold under at least some utility specifications. Also, being able to account for heterogeneity among producers, it conveniently allows to tackle some of the problems discussed in the previous subsection.

2.4.3 Towards a multi-regional framework

As argued in section 3, one relevant issue in NEG theory is that most results are mainly restricted to two-region models. Firstly, the 2-region framework is oversimplifying as

[2] It follows that the CES is the special case of a constant RLV.

it overlooks the potential variability of market access across multiple regions (Fujita and Thisse, 2009), along with other complex interdependencies that may arise from more extensive regional networks (Fujita *et al.*, 1999; Fujita and Mori, 2005; Behrens and Robert-Nicoud, 2011; Tabuchi, 2014). Two main issues, raised by Akamatsu *et al.* (2012), pertain to the lack of explanation of different agglomeration patterns than that of full concentration in one region, and the fact that these models are not able to account for spatial interactions in a well defined sense.

There have been a series of extensions of NEG models to a multi-regional set-up so far. Some focus on the heterogeneity in location space, by considering stylized geometries such as the "racetrack economy", where regions are equally spaced around a circumference (Krugman, 1993; Fujita *et al.*, 1999; Picard and Tabuchi, 2010; Castro *et al.*, 2012; Ikeda *et al*, 2012; Mossay, 2013). Ago *et al.* (2006) studied equally spaced regions along a line segment, while Ikeda and Murota (2014) considered hexagonal configurations. Others, such as Barbero and Zofío (2012), have tried to discern about the role of different space topologies to explain the locational advantages of some regions.

Other contributions focus on providing new analytical and numerical approaches that make the inherently cumbersome multi-dimensional models more tractable. For instance, Akamatsu *et al.* (2012) combine spatial discounting matrices and Fourier transformations with discrete choice theory and a racetrack economy. Their aim is to provide a complete gallery of the process of agglomeration patterns in Core-Periphery models as transportation costs fall steadily over time, starting from a very high value at which only the symmetrically dispersed outcome is a stable equilibrium. In a multi-regional setup, the first critical value for the transportation costs under which the agglomeration structure emerges, i.e., the first bifurcation occurs, is obtained analytically. While this bifurcation is the only one in most 2-region CP models, in this n-region model, the spatial configuration of industry further evolves with further decreases in transport costs, giving rise to a second bifurcation. At each bifurcation, the number of regions in which firms locate is reduced by half and the spacing between each pair of adjacent "core" regions doubles after each bifurcation. This gives rise to what is called *spatial period doubling bifurcation*.[3] As a result, the economy may eventually converge

[3] A feature which is also present in Ikeda *et al.* (2012).

to a scenario of full agglomeration in a single region for some level of transportation costs. However, this only happens if consumers have homogeneous preferences toward location patterns. If preferences are heterogeneous, it is shown that a bifurcation may occur for some transport cost level that deems some agglomeration unstable and reverts to the uniform distribution of the fully symmetric outcome. Furthermore, repetitions of agglomeration and full dispersion may be observed, as is shown through some simulations depicted in the paper. This lends support to the idea of a bell-shaped spatial development curve found in previous 2-region CP models.

Fabinger (2015) introduces an analytical method based on finding roots for a two-dimensional function that holds for a fairly general class of agglomeration models. When space is discrete instead of continuous, he is able to account for equilibria exhibiting asymmetries in the distributions of multiple cities with varying extensions and population densities, whose properties can be explained in terms of their sensitive dependence on initial distributions. In other words, discretizing space allows to interpret equilibrium properties in the light of deterministic chaos theory.

Extending beyond the 2-region framework is particularly useful if it is susceptible to empirical validation. In the early decades of NEG, multi-regional models have been seldom subject to empirical testing. However, with the advent of computer sophistication and the increasing availability of disaggregated data, noteworthy contributions have been made in recent years. Some worth mentioning are the works by Davis and Weinstein (2002), Niebuhr (2006), Redding and Sturm (2008), and Allen and Arkolakis (2014). Bosker *et al.* (2010) show that most conclusions from 2-region NEG models hold under more realistic settings. This includes not only the extension to the equidistant multi-regional case (Puga, 1999), but also the consideration of non-equidistant regions. A notable difference, however, is that, in the second case, exogenous asymmetries give rise to the possibility of spatial distributions other than agglomeration or dispersion. Recently, Tabuchi (2014) has used a multi-regional version of Krugman's original CP model to show that it can account for the historical trend of agglomeration in the capital regions over the past few centuries. Behrens *et al.* (2004) test the home market effect in a multi-country economy by extending the model by Krugman (1980). They find evidence that the extended model predicts a home market effect only after accounting for the impact of countries' differential access to world market on actual production and trade, and find that the home market effect is indeed strong in the world trade data.

2.4.4 Linking knowledge and culture to economics

Throughout the last decades there has been a narrow focus of NEG on pecuniary externalities through linkage effects. Other possible sources of agglomeration economies such as knowledge externalities and technological spillovers are left out. Fujita and Mori (2005) argue that this is done out of convenience. Such a narrow focus enables researchers to design a microfounded model based on the firms' perspective using modern tools of economic theory. However, it is true that further development in NEG requires modelling the creation and transfer of knowledge. In particular, the role of K-linkages has become increasingly relevant in NEG literature. Building upon pioneering works such as Berliant and Fujita (2008, 2009 and 2012), one should hope that a new comprehensive NEG theory fully integrates the linkage effects among consumers and producers and K-linkages in space. According to Fujita (2007), geography is an essential feature of knowledge creation and diffusion. For instance, people residing in the same region interact more frequently and thus contribute to develop the same, regional set of cultural ideas. However, while each region tends to develop its unique culture, the economy as a whole evolves according to the synergy which results from the interaction across different regions (i.e., different cultures). That is, according to Duranton and Puga (2001), knowledge creation and location are inter-dependent. Berliant and Fujita (2012) developed a model of spatial knowledge interactions and showed that higher cultural diversity, albeit hindering communication, promotes the productivity of knowledge creation. This corroborates the empirical findings of Ottaviano and Peri (2006; 2008). Ottaviano and Prarolo (2009) show how improvements in the communication between different cultures fosters the creation of multicultural cities in which cultural diversity promotes productivity. This happens, they argue, because better communication allows different communities to interact and benefit from productive externalities without risking losing their cultural identities. Berliant and Fujita (2011) take a first step towards using a micro-founded R&D structure to infer about its effects on economic growth. They find that long-run growth is positively related to the effectiveness of interaction among workers as well as the effectiveness in the transmission of public knowledge.

Therefore, combining the typical pecuniary externalities in NEG models with the spatial diffusion of knowledge spawned from intra-regional and inter-regional interactions alike is important if we want to infer about an eventual circular causality between migration and the circulation of knowledge. In other words, NEG may shed light on the importance of knowledge exchanged between different regions through trade networks compared to "internally" generated knowledge.

Besides the importance of heterogeneity in knowledge, it is also important to discern about the *relatedness* of variety. This relatedness measures the cognitive proximity and distance between sectors that allows for a higher intensity of knowledge spillovers. According to Frenken *et al.* (2007), a higher *related* variety increases the inter-sectoral knowledge spillovers between sectors that are technologically related. This potentially adds a new dimension to the role of heterogeneity and location in the creation and diffusion of knowledge. Tavassoli and Carbonara (2014) have tested the role of knowledge intensity and variety using regional data for Sweden and found evidence that the different types of cognitive proximity have an important weight. This confirms the relevance of disentangling between these different concepts in order to infer about the spatial determinants of innovation and knowledge creation.

The incorporation of knowledge creation and diffusion into NEG could also benefit from the introduction of agglomeration mechanisms in endogenous growth models with innovation. Particularly, innovations that affect quality or a firm's cost efficiency are usually driven by stochastic processes. Typically, the production of knowledge involves some sort of uncertainty. Therefore, we can think of quality as a proxy, or at least a function, of a given firm's stock of knowledge. In the literature following Schumpeterian growth models such as Aghion and Howitt (1992), Young (1998), Howitt (1999), or more recently Dinopoulos and Segerstorm (2010), innovations occur with a probability that depends on factors such as the amount of the firm's research effort, the common pool of public knowledge available to all firms, and the individual firm's quality level. Introducing geography and worker mobility in these models allows the success of innovations to depend also on the magnitude of regional interaction through the exchange of ideas between workers and producers alike among regions. If each region holds its own set of ideas, or culture, then more localized spillovers translate into higher related variety, as innovation benefits more from a regional common pool of ideas. However, the interaction between researchers hailing from different cultures is also important for innovation. This adds a potential new role for transportation costs in NEG. For instance, higher trade integration, as usually captured by lower transport costs, is likely to foster the inter-regional communication between researchers, thus adding relevance to the interaction between researchers from different regions. If cognitive proximity is relatively less important than the interaction between different cultures, then this *unrelated* variety implies that lower transport costs are likely to induce the dispersion of economic activities.

2.5 Concluding remarks

In this survey we have sought to make a description of recent contributions to the field, which may be separated in two different groups. The first group describes the literature which provides an overall assessment of early and recent contributions in the field of NEG. After considering the state of the art, what these works share in common is the fact that they seek to critically and prospectively point out the theoretical limitations in the field and provide new insights for the future research and development of NEG. The second group belongs to a selection of works that we feel provide new path-breaking contributions by, intentionally or not, tackling many of the issues raised by the review articles in the former group. A part of the literature on which this survey is lacking is that concerning recent contributions in NEG which deal with the inter-disciplinarity between NEG and other fields, such as urban economics and proper economic geography. Other issues that have been conveniently left out of this reading are those relating to the exclusion of more sectors other than the typical agricultural non-skilled and manufacturing skilled sectors, and those related with the explicit modelling of transportation costs. Of course, this does not reflect any lack of consideration toward those issues, but rather reflects the broad and extensive area of new avenues of research that will be able to maintain NEG as a yet vibrant and dynamic field.

Another concern that has been raised by researchers, mainly those outside NEG, is the lack of inter-disciplinarity between the economics and geography profession. On account of these critiques, NEG is many times referred to as being too narrowly focused, thus neglecting important issues that are determinant if new important insights are to be achieved.

To sum up all of the recent contributions, while still being able to provide a fair enough description of the contents of those papers, would be unimaginable. Instead of a very general and superficial look, the aim of this survey is to provide a somewhat extensive comprehension on the topics and papers covered by it. Other excellent reviews on the literature on NEG models are provided in works such as Fujita *et al.* (1999), Fujita and Thisse (2002), Baldwin et al. (2003), Henderson and Thisse (2004), Combes et al. (2008) and Brakman et al. (2009). As a final remark, it cannot be overemphasized that there is still room for further developments in NEG; this, however, is only possible if we keep trying to break through the conceptual strait-jacket strapping the field of new economic geography.

Acknowledgements: I thank my supervisors João Correia-da-Silva and Sofia B.S.D. Castro for revising this work. I am grateful for the financial support by Portuguese Public Funds through Fundação para a Ciência e Tecnologia (FCT) in the framework of the Ph.D. scholarship SFRH/BD/90953/2012.

Chapter 3

The Footloose Entrepreneur Model with an arbitrary number of equidistant regions

3.1 Introduction

The secular tendency for spatial agglomeration of economic activity is well known and has always been a matter of profound debate. Recent developments have allowed a more rigorous treatment of such phenomena, with recourse to microeconomic foundations.[1] The benchmark in this literature is the Core-Periphery (CP) model, introduced by Krugman (1991).

Many issues have been raised in recent literature about the shortcomings of New Economic Geography, mainly due to the transversality of some crucial assumptions across many models which do not allow for new meaningful insights and breakthroughs. Not least important of these issues is the dimensionality problem; in particular that of the 2-region framework.

Theoretical insights on a model with three or more regions are interesting for different reasons. One reason is the theoretical understanding of interdependencies among many regions to guide empirical studies. The other stems from the fact that a two-region set-up overlooks the variability of market access across regions (Fujita and Thisse, 2009).[2] Another insight relates to the more complex impacts that may arise from a multi-regional set-up compared to a two-region one (Fujita and Mori, 2005; Fujita and Thisse, 2009; Behrens and Robert-Nicoud, 2011). As pointed out by Fujita *et al.* (1999), the consideration of only two regions stems from the advantage of dealing with more tractable problems, although it seems implausible that the geographical dimension of economic activity can be reduced to a 2-region framework. It is important, therefore, to understand to what extent the main conclusions that were obtained using 2-region models extend to models with more regions.

This motivated a number of different studies that try to overcome the over-simplistic dimensionality of the two-region set-up. Castro *et al.* (2012) study a 3-region version of the CP model by Krugman (1991). Comparing the behaviour of the 3-region model relatively to the 2-region model, their main conclusion was that the additional region favours the agglomeration of economic activity and hinders the dispersion of economic

[1] See Fujita *et al.* (1999), Ottaviano *et al.* (2002), Baldwin *et al.* (2004), Robert-Nicoud (2005) and the references therein.

[2] Of course, the latter issue clearly departs from models with symmetric transportation costs across regions.

activity. The paper by Akamatsu *et al.* (2012) seeks to explain spatial agglomeration in a CP model with 2^n regions. They provide a description of the process of particular agglomeration patterns in the CP model as transportation costs steadily decrease over time, giving rise to the conjecture of *spatial period doubling bifurcation,* which they later prove analytically. According to this result, the number of regions in which firms locate is reduced by half and the spacing between each pair of adjacent "core" regions doubles after each bifurcation. Oyama (2009) has incorporated self-fulfilling expectations in migration decisions in a multi-regional variant of the CP model to allow for global stability in just one core region in the presence of asymmetries or trade barriers. Tabuchi and Thisse (2011) study the rise of a hierarchical system of central places in a multi-location space. Barbero and Zofío (2012) considered different network topologies in order to show how locational advantages due to more heterogeneous configurations enforces the likelihood of agglomeration in regions that are best located. The role of heterogeneous distances between regions has also been addressed in other frameworks such as the racetrack economy (Krugman, 1993; Fujita *et al.*, 1999; Picard and Tabuchi; 2010; Mossay, 2013), equally spaced regions on the line segment (e.g. Ago *et al.*, 2006) or hexagonal distributions such as in the monograph by Ikeda and Murota (2014). Other recent contributions concerning NEG models with more than 2 regions include works such as Behrens *et al.* (2006), Akamatsu and Takayama (2009), Ikeda *et al.* (2012), Forslid and Okubo (2012), Fabinger (2015) and Commendatore *et al.* (2015a).[3]

Some of the inherent technical difficulties that stem from the extension of a 2-region model to a multiregional framework call for a base model that is more tractable than the original CP model (Krugman, 1991). An analytically solvable version of the CP model, dubbed the Footloose Entrepreneur (FE) model, was developed by Forslid and Ottaviano (2003). The only difference with respect to the original CP model is that, in the FE model, the variable input in the mobile sector is immobile labour instead of mobile labour. The role of mobile (footloose) labour becomes limited to the fixed input (entrepreneurship) in the mobile sector. This subtle modification renders the model analytically solvable because the marginal production cost becomes independent of the spatial distribution of economic activity.

[3]For a more comprehensive and insightful overview of some of the main contributions concerning multiregional NEG models, see Commendatore *et al.* (2015b).

This motivates us to consider a n-region version of the analytically solvable FE model as it allows us to obtain a closed-form solution for the regional utility level as a function of the spatial distribution of economic activity.[4] In Tabuchi (2014), the author develops a multi-regional model based on Fujita *et al.* (1999) with exogenous asymmetries both in trade costs and in the distribution of the immobile workers and finds that it is able to predict the historical tendency of agglomeration in the capital regions. The paper focuses on a limit analysis of transportation costs (i.e., autarky and near free trade), whereas our set-up, by considering equidistant regions, allows us to characterize analytically the stability of equilibria.

Our main finding is that, as the number of regions increases, agglomeration becomes more likely while dispersion becomes less likely. More precisely, we conclude that: (i) the set of parameter values for which agglomeration is stable in an economy with n regions is contained in the set of parameters for which it is stable in an economy with $n+1$ regions; and (ii) the set of parameter values for which dispersion is stable in an economy with $n+1$ regions is contained in the set of parameters for which it is stable in an economy with n regions. This is an improvement on the results of Castro et al. (2012). This happens because, when unskilled workers are evenly distributed among regions, more regions implies less immobile consumers in each region and, therefore, lower local demand. As a result, a large region becomes relatively more attractive as the market-size effect becomes stronger relative to the market-crowding effect, which induces agglomeration.

We also present numerical evidence that strongly suggests that partial dispersion (i.e., symmetric dispersion across m regions, with $m < n$) is never stable, whatever the parameter values. Our simulations additionally indicate that locational hysteresis is a persistent feature of the FE model with an arbitrary number of regions, but disappears as n tends to infinity because agglomeration becomes the only possible stable equilibrium. The scope for simultaneity of stability of dispersion and agglomeration, as given by an interval in the level of transportation costs, seems to attain a maximum for some n.

[4]Ottaviano *et al.* (2002) and Pflüger (2004) have also built analytically solvable CP models by considering quasi-linear preferences and are therefore also good potential candidates for a multi-regional extension.

As in Forslid and Ottaviano (2003), we introduce exogenous regional asymmetries in our n-region version of the FE model by considering regional heterogeneity in the endowment of unskilled labour across regions. By studying the stability conditions for agglomeration, we find that an increase (decrease) of unskilled labour in the core region (peripheral regions) leads to the strengthening of agglomeration similar to the increase in the number of regions in the symmetric case. This is achieved through an analysis of skilled labour wages providing further insight into the agglomeration mechanism for a finite number of regions.

We conclude that the impact of considering additional regions in the FE model is analogous to that of considering additional regions in the CP model. In this sense, the FE model behaves similarly to the CP model (as desired by its creators). Even though the FE model is more tractable than the CP model as the number of regions increase, we still feel the need for a base model yet more tractable, more easily extensible to a higher number of regions than the ones that have been used so far. This however is beyond the scope of this article.

The remainder of the paper is structured as follows. In Section 2, we underline the main assumptions of the FE model with n regions. In Section 3, we obtain the general expressions for nominal and real wages as functions of the spatial distribution of the entrepreneurs. In Section 4, we address the dynamics of the model and find the stability conditions for three possible kinds of equilibria: agglomeration, total dispersion and partial dispersion. We also discuss how each of these outcomes becomes more or less likely as the parameters of the model change. In Section 5, we assess the effect of increasing the number of regions on the behaviour of the FE model. In Section 6, we allow for exogenous heterogeneity in the unskilled immobile labour factor in the FE model and determine a more general local stability condition for agglomeration in order to study how the spatial distribution of farmers is likely to influence the agglomerative outcome. In Section 7, we make some concluding remarks.

3.2 Economic environment

The economy is composed by $n \geq 2$ regions that are assumed to be structurally identical and equidistant from each other. The framework is exactly as that of the 2-region FE model by Forslid and Ottaviano (2003), except for the fact that an arbitrary number of regions is considered instead of only two.

The total endowments of entrepreneurs and unskilled labour are, respectively, $H(n)$ and $L(n)$.[5] Entrepreneurs can move freely between regions: $\sum_{i=1}^{n} H_i = H(n)$; while unskilled workers are immobile and assumed be evenly spread across the n regions: $L_i = \frac{L(n)}{n}, \forall i$.

The representative consumer of region i has the usual Cobb-Douglas utility function:

$$U_i = X_i^{\mu} A_i^{1-\mu}, \tag{3.1}$$

where A_i is the consumption of agricultural products in region i and X_i is the consumption of a composite of differentiated varieties of manufactures in region i, defined by:

$$X_i = \left[\int_{s \epsilon N} d_i(s)^{\frac{\sigma-1}{\sigma}} ds \right]^{\frac{\sigma}{\sigma-1}}, \tag{3.2}$$

where $d_i(s)$ is the consumption of variety s of manufactures in region i, N is the mass of existing varieties, and $\sigma > 1$ is the constant elasticity of substitution between different varieties of manufactures. From utility maximization, $\mu \in (0, 1)$ is the share of expenditure in manufactured goods.

Production of a variety of manufactures requires, as inputs, α units of entrepreneurs and β units of unskilled labour for each unit that is produced. Therefore, the production cost of a firm in region i is:

$$C_i(x_i) = w_i \alpha + w_i^L \beta x_i, \tag{3.3}$$

where w_i is the nominal wage of skilled workers in region i and w_i^L is the nominal wage of unskilled workers in region i.

Trade of manufactures between two regions is subject to iceberg costs $\tau \in (1, +\infty)$. Let τ_{ij} denote the number of units that must be shipped at region i for each unit that is delivered at region j. Since the regions are assumed to be equidistant from each other,

[5] The dependence of these endowments on the number of regions increases the generality of the comparison between models with different numbers of regions.

we have the following trade cost structure:

$$\begin{cases} \tau_{ij} = 1, & \text{if } j = i \\ \tau_{ij} = \tau, & \text{if } j \neq i. \end{cases}$$

The agricultural good is produced using one unit of unskilled labour for each unit that is produced (constant returns to scale), and is freely traded across regions.

3.3 Short-run equilibrium

Let $p_{ji}(s)$ and $d_{ji}(s)$ denote the price and demand in region i of a variety, s, that is produced in region j. Utility maximization by consumers in region i yields the following aggregate regional demand:

$$d_{ji}(s) = \frac{p_{ji}(s)^{-\sigma}}{P_i^{1-\sigma}} \mu Y_i, \tag{3.4}$$

where P_i is the regional price index of manufactures, associated with (3.2):

$$P_i = \left[\sum_{j=1}^{n} \int_{s \epsilon N} p_{ji}(s)^{1-\sigma} ds \right]^{\frac{1}{1-\sigma}}, \tag{3.5}$$

and Y_i is the regional income:

$$Y_i = w_i H_i + w_i^L \frac{L(n)}{n}, \tag{3.6}$$

Turning to the supply side and starting with the agricultural sector, absence of transport costs implies that its price is the same everywhere ($p_1^A = ... = p_n^A$). Furthermore, under perfect competition, we have marginal cost pricing: $p_i^A = w_i^L, \forall i$. Consequently, there is unskilled workers' wage equalization among regions: $w_1^L = ... = w_n^L$. Hence, by choosing the agricultural good as numeraire, we can set $p_i^A = w_i^L = 1, \ \forall i$. We assume that the *non-full-specialization* (NFS) condition (Baldwin *et al.*, 2004) holds, guaranteeing that agriculture is active in all regions even if all manufacturing activity takes place in a single region.[6] This is guaranteed if we assume that:

$$\mu < \frac{\sigma}{n(\sigma - 1) + 1}. \tag{3.7}$$

.

In the industrial sector, given the fixed cost in (3.3), the number of varieties manufactured in region i is $v_i = H_i/\alpha$. A manufacturing firm in region i facing the total cost in (3.3) maximizes the following profit function:

$$\pi_i(s) = \sum_{j=1}^{n} p_{ij}(s) d_{ij}(s) - \beta \left[\sum_{j=1}^{n} \tau_{ij} d_{ij}(s) \right] - \alpha w_i. \tag{3.8}$$

Total supply to region $j \neq i$, including the fraction of product that "melts", is equal to $\tau d_{ij}(s)$. The first order condition for maximization of (3.8) yields the same pricing equation as that of the 2-region model by Forslid and Ottaviano:

$$p_{ij}(s) = \tau_{ij} \beta \frac{\sigma}{\sigma - 1}. \tag{3.9}$$

Note that $p_{ij}(s)$ is independent of s, which implies that $d_{ij}(s)$ also is. All varieties produced in region i are sold at the same price and are equally demanded in region j.

Using (3.9), the CES price index (3.5) becomes:

$$P_i = \alpha^{\frac{1}{\sigma - 1}} \beta \frac{\sigma}{\sigma - 1} \left(\sum_{j=1}^{n} \phi_{ji} H_j \right)^{\frac{1}{1-\sigma}}, \tag{3.10}$$

where $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in (0, 1)$ represents the "freeness of trade" between regions i and j.

Absence of entry barriers in the manufacturing industry translates into zero profits in equilibrium. Operating profits must totally compensate fixed costs, which are equal

[6] This condition requires world expenditure on agricultural goods to be greater than the total production of agricultural goods in all regions except one, i.e., $(1 - \mu) \sum_{i=1}^{n} Y_i > \frac{n-1}{n} L(n)$.

to the wages paid to the entrepreneurs:

$$\alpha w_i = \sum_{j=1}^{n} p_{ij} d_{ij} - \beta \sum_{j=1}^{n} \tau_{ij} d_{ij},$$

which becomes, considering the prices in (3.9):

$$w_i = \frac{\beta x_i}{\alpha(\sigma - 1)}, \tag{3.11}$$

where $x_i \equiv \sum_{j=1}^{n} \tau_{ij} d_{ij}$ is the total production by a manufacturing firm in region i.

Using (3.4), (3.9) and (3.10), we obtain an expression for x_i that depends on regional incomes and on the number of firms in each region:

$$x_i = \frac{\mu(\sigma - 1)}{\alpha\beta\sigma} \sum_{j=1}^{n} \frac{\phi_{ij} Y_i}{\sum_{m=1}^{n} \phi_{mj} v_m}. \tag{3.12}$$

Replacing (3.12) in (3.11), and given that $v_i = H_i/\alpha$, we obtain:

$$w_i = \frac{\mu}{\sigma} \sum_{j=1}^{n} \frac{\phi_{ij} Y_j}{\sum_{m=1}^{n} \phi_{mj} H_m}. \tag{3.13}$$

By (3.6), regional income equals:

$$Y_i = \frac{L(n)}{n} + w_i H_i. \tag{3.14}$$

The spatial distribution of entrepreneurs can be described, in relative terms, by the vector $h \equiv (h_1, ..., h_n)$, where $h_i \equiv \frac{H_i}{H(n)}, \forall i$. The fraction of entrepreneurs in region n may be omitted because it is implicit in the other fractions: $h_n = 1 - \sum_{i=1}^{n-1} h_i$.

The price index of manufactures, P_i, becomes, after (3.10):

$$P_i(h) = \frac{\beta\sigma}{1 - \sigma} \left(\frac{\alpha}{H(n)}\right)^{\frac{1}{\sigma-1}} \left(\sum_{j=1}^{n} \phi_{ji} h_j\right)^{\frac{1}{1-\sigma}}. \tag{3.15}$$

Using (3.13) and (3.14), we obtain a system of n equations that determines the nominal wages in each region:

$$w_i = \frac{\mu}{\sigma} \sum_{j=1}^{n} \frac{\phi_{ij} \left[\frac{L(n)}{nH(n)} + w_j h_j\right]}{\sum_{m=1}^{n} \phi_{mj} h_m}, \quad \forall i \in \{1, ..., n\}. \tag{3.16}$$

For illustrative purposes, we present the closed-form solution for the equilibrium nominal wages of the entrepreneurs as a function of their spatial distribution for $n = 3$.

Proposition 3.1. *When $n = 3$, the nominal wages of entrepreneurs in region i are given by:*

$$w_i = \frac{\frac{\mu L}{3\sigma H(n)} \left[\sum_{j=1}^{3} \frac{\phi_{ij}}{r_j} + \frac{\mu}{\sigma}\left(\phi(\phi-1)\frac{\sum_{k\neq i} h_k}{\prod_{k\neq i} r_k} + \frac{\phi^2-1}{r_i}\sum_{k\neq i}\frac{h_k}{r_k}\right) + \frac{\mu^2}{\sigma^2}\left(2\phi^3 - 3\phi^2 + 1\right)\frac{1}{r_i}\prod_{k\neq i}\frac{h_k}{r_k}\right]}{1 - \frac{\mu}{\sigma}\sum_{j=1}^{3}\frac{h_j}{r_j} + \frac{\mu^2}{\sigma^2}\left(1-\phi^2\right)\left(\frac{h_1 h_2}{r_1 r_2} + \frac{h_1 h_3}{r_1 r_3} + \frac{h_2 h_3}{r_2 r_3}\right) - \frac{\mu^3}{\sigma^3}\left(2\phi^3 - 3\phi^2 + 1\right)\prod_{j=1}^{3}\frac{h_j}{r_j}}, \tag{3.17}$$

where $r_j \equiv \sum_{m=1}^{3} \phi_{mj} h_m$.

Proof. See Appendix A. □

Since agents spend a fraction μ of their income on manufactures, whose price index is P_i, and the price of agricultural goods is unity, the real wage or indirect utility level of entrepreneurs is given by:

$$\omega_i = w_i / P_i^{\mu}. \tag{3.18}$$

3.4 Long-run equilibria and stability

Entrepreneurs migrate to the region that offers them the highest real wage or indirect utility (3.18). For concreteness, we consider *replicator dynamics*: the flow of entrepreneurs to a region is proportional to the difference between the region's real wage and the weighted average real wage and to the number of entrepreneurs in the region.

Formally, the dynamics are described by the following system of $n - 1$ ordinary differential equations defined in the simplex $\Delta \equiv \left\{ h \in \mathbb{R}_+^n : \ \sum_{i=1}^n h_i = 1 \right\}$:[7]

$$\begin{cases} \dot{h}_1 = (\omega_1 - \bar{\omega})\, h_1 \\ \ldots \\ \dot{h}_{n-1} = (\omega_{n-1} - \bar{\omega})\, h_{n-1} \end{cases}, \quad h_i \in [0,1]\,, \quad \forall i \in \{1, ..., n-1\}, \tag{3.19}$$

where $\bar{\omega} \equiv \sum_{i=1}^n h_i \omega_i$ is the weighted average real wage. Migration to the omitted region in (3.19) is, consistently, given by $\dot{h}_n = -\sum_{i=1}^{n-1} \dot{h}_i = (\omega_n - \bar{\omega})\, h_n$.

In this dynamical system, migration to empty regions has to be started exogenously. If a region is empty, $h_i = 0$, then the corresponding differential equation for the dynamics yields $\dot{h}_i = 0$, which means that the region remains empty. Hence, the boundary of the simplex is invariant for the dynamics.

Direct substitution in equations (3.19) shows that the configurations:

$$h \in \left\{ (1, 0, ..., 0)\,, \left(\tfrac{1}{n}, ..., \tfrac{1}{n}\right), \left(\tfrac{1}{m}, ..., \tfrac{1}{m}, 0, ..., 0\right) \right\}$$

and their permutations are equilibria. The equilibria represented by $(1, 0, ..., 0)$ and its permutations correspond to full agglomeration of industry in one of the regions while the others remain empty. This outcome is called *agglomeration* or *concentration*. The second configuration describes an even distribution of industry among the n regions. This outcome is called *total dispersion*. The last configuration represents an even distribution of industry among only m of the n regions, while the remaining regions are deprived of industry. This is what we call *partial dispersion*.

The description of the dynamics relies on the study of the stability of the aforementioned equilibria. The stability of each equilibrium is preserved by permutation so that the same stability conditions hold for agglomeration or partial dispersion in any of the regions. Equilibria are stable if, after some small exogenous migration of skilled workers to any of the regions, the spatial distribution of skilled workers is pulled back

[7]Note that, even though the spatial distribution of entrepreneurs can be described using $n-1$ coordinates, we sometimes use all n coordinates for the sake of clarity.

to the initial one.[8]

3.4.1 Stability of total dispersion

Since total dispersion is an interior configuration, its stability is given by the sign of the real part of the eigenvalues of the Jacobian matrix of the system in (3.19) at $h = \left(\frac{1}{n}, ..., \frac{1}{n}\right)$.

This matrix has a repeated real eigenvalue with multiplicity $n-1$, which is given by (see Appendix B):

$$\alpha \equiv \frac{\partial \omega_i}{\partial h_i}\left(\frac{1}{n}, ..., \frac{1}{n}\right). \tag{3.20}$$

Total dispersion is stable if α is negative. This occurs if and only if :

$$\frac{\partial w_i/\partial h_i}{w_i} \quad < \quad \frac{\partial P_i^{\mu}/\partial h_i}{P_i^{\mu}},$$

which means that the stability of dispersion can be described in terms of semi-elasticities.

Entrepreneurs remain equally dispersed across the n regions if a migration of entrepreneurs to a region induces a percentage change in the nominal wage smaller than the corresponding percentage change in the real prices. In this case, the loss in purchasing power due to an increase of the share of entrepreneurs, h_i, leads to an exodus from that region until the initial share of entrepreneurs is restored, that is, until $h_i = \frac{1}{n}$.

Proposition 3.2. *Total dispersion is a stable configuration if:*[9]

$$\phi < \phi_b \equiv \frac{(\sigma-\mu)(\sigma-1-\mu)}{\mu^2+\sigma(\sigma-1)+\mu(2\sigma-1)(n-1)}. \tag{3.21}$$

Proof. See Appendix B. □

[8]For configurations in the interior of the simplex, stability depends on the sign of the real part of the eigenvalues of the Jacobian matrix for the dynamics. For configurations on the boundary, such as agglomeration and partial dispersion, stability depends on the sign of the difference between the real wage in the populated regions and the empty regions.

[9]For a necessary condition, replace "$<$" with "$\leq$".

Low transportation costs (high ϕ) discourage dispersion. Following Fujita *et al.* (1999), we call *break point* the critical value ϕ_b which is such that $\alpha(\phi_b) = 0$.

Note that if ϕ_b is negative, total dispersion is not stable for any value of ϕ. We rule out this possibility throughout the paper by assuming:

$$\sigma > \mu + 1, \tag{3.22}$$

commonly referred to as the *no black-hole* condition.[10] With $\phi_b > 0$, there always exists a level of transportation costs above which dispersion is stable (given the values of μ and σ). On the other hand, the fact that $\phi_b < 1$ means that there always exists a level of transportation costs below which dispersion is unstable.

Observing that the derivative of ϕ_b with respect to μ is negative, we conclude that a higher fraction of spending on manufacturing discourages total dispersion. In the extreme case in which μ tends to zero, ϕ_b approaches unity, rendering total dispersion stable.

The effect of σ can be understood by noting that when σ tends to infinity, ϕ_b approaches unity. This means that if the preference for variety is sufficiently low (i.e., if σ is sufficiently high), dispersion is stable.

Total dispersion is always a stable outcome when we are either approaching an economy absent of industry or an economy in which consumers give almost no value to variety in consumption of manufactures. This fits well with intuition.

3.4.2 Stability of agglomeration

Agglomeration, $(h_1, h_2, ..., h_n) = (1, 0, ..., 0)$, is a corner solution on a vertex of the simplex. At such an equilibrium, by symmetry, the real wages in the empty regions are equal. Given the absence of entrepreneurs in all regions except one, the weighted average real wage is simply $\bar{\omega} = \omega_1$.

[10] If agents have a very strong preference for variety ($\sigma < 1 + \mu$), total dispersion is never a stable equilibrium, for any magnitude of the transportation costs.

Lemma 3.3. *Agglomeration in region i is a stable configuration if:*[11]

$$\omega_i > \omega_j, \quad \forall j \neq i.$$

Proof. Without loss of generality, let $h_2 = ... = h_n = 0$. Then, $\bar{\omega} = \omega_1$, that is, the weighted average real wage is the same as the real wage in region 1. That $\omega_1 > \omega_j$ is sufficient for agglomeration to be stable follows from the fact that entrepreneurs migrate to regions with higher real wages, together with continuity of real wages with respect to the spatial distribution. Thus, if an empty region, j, has a lower real wage than region 1, a small exogenous migration of entrepreneurs from region 1 to region j will be followed by their return to region 1. □

If the other regions are to remain empty over time, then there can be no incentives for entrepreneurs to migrate. That is, the indirect utility of entrepreneurs must be higher in region 1 than in the other regions:

$$\frac{w_1}{P_1^\mu} > \frac{w_2}{P_2^\mu} \Leftrightarrow \frac{w_1}{w_2} > \left(\frac{P_1}{P_2}\right)^\mu \Leftrightarrow \frac{w_1}{w_2} > \phi^{\frac{\mu}{\sigma-1}}.$$

Since $\phi^{\frac{\mu}{\sigma-1}} < 1$, a sufficient condition for the stability of agglomeration is that the nominal wage is higher in the core than in the periphery.

Proposition 3.4. *Agglomeration is a stable equilibrium if:*[12]

$$SP(\phi) \equiv (\sigma-\mu)(1-\phi)^2 + n\phi\left[\sigma(1-\phi^{-\frac{\mu}{\sigma-1}}) - \mu(1-\phi)\right] < 0. \tag{3.23}$$

Since $SP(\phi)$ is a convex function and we have $SP(0) > 0$, $SP(1) = 0$ and $SP'(1) > 0$, the function $SP(\phi)$ has exactly one zero in $\phi \in (0,1)$. This value, denoted ϕ_s, is the n-region FE model's *sustain point*, i.e., it is the threshold of ϕ above which agglomeration is a stable equilibrium. If the "freeness of trade" parameter is high enough (i.e., if transport costs are low enough), agglomeration is stable. This is because low transport

[11] For a necessary condition, replace "$<$" with "$\leq$".

[12] For a necessary condition, replace "$<$" with "$\leq$".

costs imply that price indices become relatively higher in the regions that are deserted and thus real wages become relatively lower.

One can also verify that $SP(\phi)$ becomes positive as σ approaches infinity or as μ approaches zero.[13] Therefore, $\lim_{\sigma\to\infty}\phi_s = \lim_{\mu\to 0}\phi_s = 1$. For a given $\phi \in (0,1)$, agglomeration becomes unstable for a sufficiently high σ or a sufficiently low μ. One limit case ($\mu \to 0$) refers to a situation of absence of the manufacturing sector, because μ is the fraction of expenditure on manufactures. The other ($\sigma \to +\infty$) corresponds to the manufacturing sector operating under perfect competition, since σ close to infinity means that variety of manufactures is not valued by consumers. It is as if manufactures were a homogenous good.

3.4.3 Stability of partial dispersion

We now address the stability of configurations in which entrepreneurs are equally dispersed across m regions, with $1 < m < n$, while the remaining $n-m$ regions are empty. Such configurations are always equilibria of the dynamical system (3.19). For simplicity, we focus on the possibility of dispersion across two regions, i.e., of the configuration $h = \left(\frac{1}{2}, 0, ..., 0, \frac{1}{2}\right)$ and its permutations.

Lemma 3.5. *Partial dispersion is stable if:*[14]

$$\omega_i\left(h^p\right) > \omega_j\left(h^p\right) \ \text{ and } \frac{\partial \omega_i}{\partial h_i}\left(h^p\right) < 0, \ i = \{1, n\}, \forall j \neq i \tag{3.24}$$

where $h^p = \left(\frac{1}{2}, 0, ..., 0, \frac{1}{2}\right)$.

Proof. See Appendix C. □

Let $\gamma \equiv \omega_i\left(h^p\right) - \omega_j\left(h^p\right)$ and $\beta \equiv \dfrac{\partial \omega_i}{\partial h_i}\left(h^p\right)$. From Lemma 3.5, after some manipulation, we obtain the following result.

[13] Note that $\lim_{\mu\to 0} SP(\phi) = \sigma(1-2\phi+\phi^2) > 0$ and $\lim_{\sigma\to\infty} SP(\phi) = \lim_{\sigma\to\infty}\sigma(1-2\phi+\phi^2) > 0 = +\infty$.

[14] For necessary conditions, replace "$>$" with "$\geq$".

Proposition 3.6. *Partial dispersion is stable if $\beta < 0$ and $\gamma > 0$, where:*[15]

$$\begin{cases} \gamma \;\equiv\; \sigma(1+\phi)\left[1-\phi^{1-\frac{\mu}{\sigma-1}}\left(\frac{1+\phi}{2}\right)^{\frac{\mu}{\sigma-1}}\right]n-\left[2\sigma+\mu(n-2)\right](1-\phi)(2\phi+1). \\ \beta \;\equiv\; \mu(\sigma+3\sigma\phi-2\phi)n-\mu^2(1-\phi)n-\mu(1-\phi)-2\sigma(\sigma-1-\mu)(1-\phi) \end{cases} \tag{3.25}$$

Proof. See Appendix C. □

Partial dispersion appears to be unstable for all parameter values. However, because of the non-linearity of $\gamma(\phi)$, this appears to be impossible to prove analytically. Numerical inspection of both conditions in (3.25) suggests that these are never simultaneously met, as illustrated in Figure 3.1 for the case of three regions. This conclusion is not surprising, since partial dispersion has also been numerically shown to be unstable in the 3-region CP model by Castro *et al.* (2012).

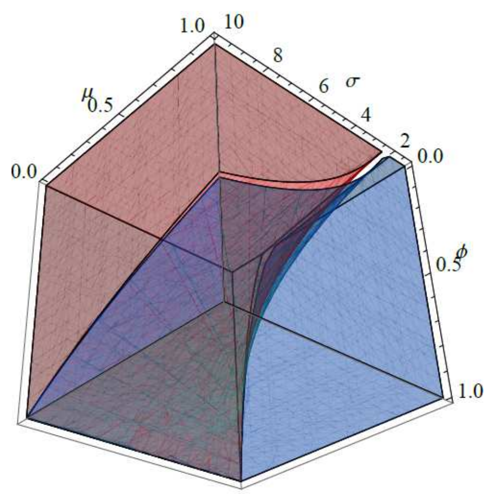

Figure 3.1 – Parameter regions in which $\beta \leq 0$ and $\gamma \leq 0$ for $n = 3$. These do not seem to overlap, indicating that the two stability conditions are never simultaneously satisfied.

The consideration of a greater number of regions seems to reinforce this conclusion. The derivative of β with respect to n is positive and the derivative of γ with respect

[15] For necessary conditions, replace "<" with "≤".

to n also appears, from numerical inspection, to be positive. We thus conjecture that partial dispersion is never stable for any number of regions.

3.5 Impact of the number of regions

A reason to build a n-region model is to be able to understand the impact of the number of regions on the behaviour of the FE model. Castro *et al.* (2012) proved that, in an extension of Krugman's CP model to three regions, more regions favour agglomeration as an outcome. Here, we obtain a stronger result in the same direction.

Proposition 3.7. *The parameter region for which agglomeration is stable in the FE model with $n+1$ regions contains that of the FE model with n regions.*

Proof. The derivative of SP in (3.23) with respect to n is negative. This implies that an increase in the number of regions decreases ϕ_s, for any given values of the remaining parameters. □

In other words, an increase in the number of regions favours stability of agglomeration in the FE model. Figure 3.2 illustrates this effect. The following result provides an explanation for why this happens.

Proposition 3.8. *Comparing agglomeration in the FE model with $n+1$ regions and in the FE model with n regions, we find that: (i) the ratio between price indices in the core and in the periphery are the same; (ii) the ratio between nominal wages in the core and periphery is higher in the model with $n+1$ regions.*

Proof. See Appendix D. □

An entrepreneur who migrates to the periphery will find a cost-of-living that is independent of the number of regions, but the size of the internal market is smaller compared to the core if there are more regions because the fraction of unskilled workers that live there is smaller. All the other entrepreneurs and the remaining unskilled workers would constitute the external market. Given the existence of transportation costs to the other regions, this entrepreneur will face a lower relative global demand in a model with more regions, and, therefore, will earn a lower nominal wage. This leads

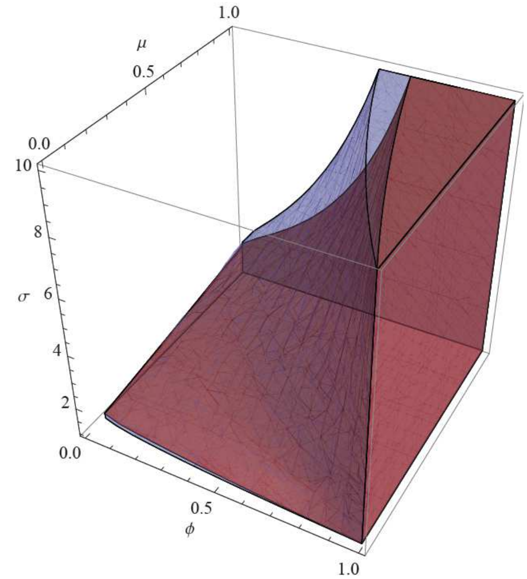

Figure 3.2 – The region in parameter space where $SP_2 < 0$ is painted in red (more opaque), while the region in which $SP_3 < 0$ is painted in blue (less opaque). The blue region contains the red one, illustrating that agglomeration in the 3-region model is more likely than in the 2-region model.

to the fact that agglomeration is more likely in a model with more regions. In the limit, when the number of regions tends to infinity, agglomeration is always stable.

We are also able to compare the stability conditions of total dispersion in the FE model with n regions and the FE model with $n+1$ regions.

Proposition 3.9. *The parameter region for which total dispersion is stable in the FE model with n regions contains that of the FE model with $n+1$ regions.*

Proof. Observe that ϕ_b in (3.21) decreases as the number of regions increases. This means that an increase in the number of regions makes total dispersion less likely to be stable. □

We conclude that an increase in the number of regions hinders stability of total dispersion. After a small deviation from total dispersion, the market-size effect and the market-crowding effect in the larger region becomes stronger (resp. weaker) as the number of regions increases, because each of the smaller regions individually faces a lower local demand given less unskilled workers. Therefore, as the number of immobile workers per region decreases, so diminishes its role as a dispersive force. In the limit, when the number of regions tends to infinity, total dispersion is never stable because the amount of immobile workers in each region becomes negligible.

3.5.1 Numerical evidence of locational hysteresis

One feature of the 2-region CP and FE models is the existence of a subcritical pitchfork bifurcation. This kind of bifurcation implies that there are values of the freeness-of-trade parameter, $\phi \in (\phi_s, \phi_b)$, for which total dispersion and agglomeration are simultaneously stable. Therefore, the long-run distribution of economic activity depends on the initial distribution of economic activity. The model exhibits locational hysteresis.

3.5.1.1 In the 3-region FE model

Regarding the 3-region FE model, numerical inspection of the stability conditions of total dispersion and agglomeration in (3.21) and (3.23) suggests that, with $n = 3$ and any pair (μ, σ), we have $\phi_s < \phi_b$.[16] This implies that there exist values of ϕ for which both total dispersion and agglomeration are stable equilibria, because agglomeration is stable for $\phi > \phi_s$ and total dispersion is stable for $\phi < \phi_b$. This means that the 3-region FE model also exhibits locational hysteresis.

Figure 3.3 illustrates this by plotting the surfaces $SP = 0$ and $\alpha = 0$. However, it also shows that the parameter region for which total dispersion and agglomeration are simultaneously stable is relatively small. The distance between ϕ_s and ϕ_b seems to be bigger for parameter values near the *no black-hole* condition and to decrease as σ increases or μ decreases.

One thing that is possible to establish analytically is the existence of an open subset in parameter space (ϕ, σ, μ) in which both total dispersion and agglomeration are stable outcomes. Consider the point in parameter space $(\phi, \sigma, \mu) = \left(\frac{3}{5}, 5, \frac{2}{5}\right)$. At this point, we have $\alpha < 0$ and $SP < 0$. Therefore, both total dispersion and agglomeration are stable equilibria. Since α and SP are continuous functions of (ϕ, σ, μ), they remain negative in an open neighbourhood of $\left(\frac{3}{5}, 5, \frac{2}{5}\right)$.

[16] If both equilibria are to be simultaneously stable, we must have $SP\left(\phi_b\right) < 0$, $\forall(\mu, \sigma)$. This seems to be the case. However, nonlinearity of $SP(\phi)$ prevents us from demonstrating this analytically.

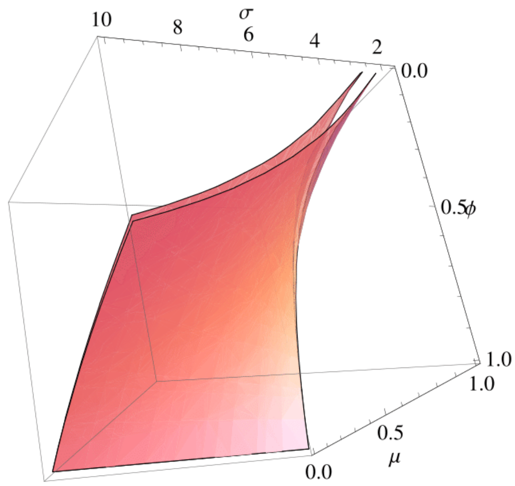

Figure 3.3 – Surfaces $SP = 0$ (top) and $\alpha = 0$ (bottom) in parameter space. Agglomeration and total dispersion are both stable between the two surfaces, where we have $SP < 0$ and $\alpha < 0$.

3.5.1.2 In the n-region FE model

In the n-region FE model, it also seems that, for any given values of n, μ and σ, there always exist values of the freeness-of-trade parameter that render total dispersion and agglomeration simultaneously stable, i.e., we always have $\phi_s < \phi_b$.

Using the analytical expressions for α and SP in (3.21) and (3.23) for a grid of values of μ, σ we have always obtained a region in (n, ϕ) space where we have simultaneity of agglomeration and dispersion.

Our results are summarized in the pictures portrayed in Figure 3.4 and Figure 3.5. They depict α and SP, in (ϕ, n) space and (ϕ, n, μ) space, respectively.[17] In the former, we set both μ and σ. The region between the dashed lines corresponds to both $SP(\phi, n) < 0$ and $\alpha(\phi, n) < 0$, which implies simultaneity of agglomeration and total dispersion. The pictures show strong evidence that this simultaneity always exists whatever the number of regions. Moreover, the scope for simultaneity, captured by the

[17]The choice to fix σ was due to the fact that our simulations evidenced invariance of the results in the choice of values for this parameter.

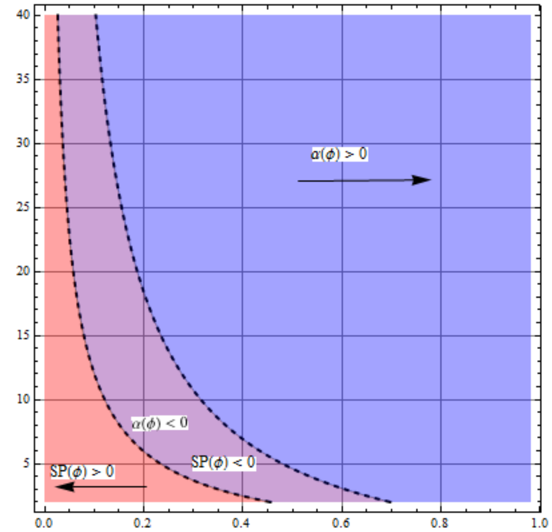

Figure 3.4 – Depiction of $\alpha(\phi, n)$ and $SP(\phi, n)$. Between the dashed lines we have $\alpha(\phi, n) < 0 \cap SP(\phi, n) < 0$, thus $\phi_s < \phi_b$ and there is locational hysteresis. Parameter values: $\sigma = 5, \mu = 0.4$;

difference $\phi_b - \phi_s$, seems to be very low, either when there are very few regions or when there are too many.

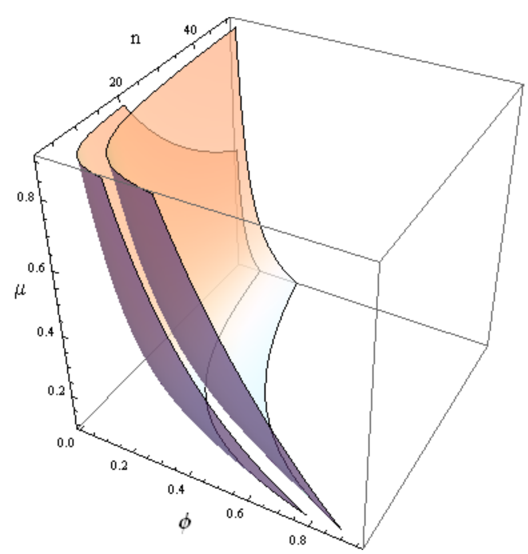

Figure 3.5 – Depiction of the surfaces $\alpha(\phi, n, \mu) = 0$ and $SP(\phi, n, \mu) = 0$ for $\sigma = 4$. Between the surfaces we have $\alpha(\phi, n, \mu) < 0 \cap SP(\phi, n, \mu) < 0$, thus $\phi_s < \phi_b$ and there is locational hysteresis.

The latter picture only fixes σ and contains the surfaces $\alpha(\phi, n, \mu) = 0$ and $SP(\phi, n, \mu) = 0$. In between, α and SP are both negative, so agglomeration and dispersion are both stable. For given μ, σ and n, evidence suggests that $SP(\phi_b) < 0$; thus, there always exist values of ϕ for which total dispersion and agglomeration are simultaneously stable. In this sense, temporary decreases in transport costs below the sustain point may trigger agglomeration in one region permanently.

3.6 Agglomeration under exogenous regional heterogeneity

In this section we let the regions differ in terms of their endowments of farmers. As a result, we are introducing exogenous regional differences such that now each L_i may be different from L_j, $i \neq j$. Take full agglomeration in region i. From (3.15), regional price indices remain unchanged compared to the symmetric case: $P_i = \frac{\beta\sigma}{1-\sigma}\left(\frac{\alpha}{H(n)}\right)^{\frac{1}{\sigma-1}}$; $P_j = \frac{\beta\sigma}{1-\sigma}\left(\frac{\alpha}{H(n)\phi}\right)^{\frac{1}{\sigma-1}}$. Nominal wages are determined by (3.13) and by the adapted version of (3.14):

$$Y_i = L_i + w_i H_i. \tag{3.26}$$

We have the following Lemma.

Lemma 3.10. *Agglomeration in region i is stable if:*

$$\frac{w_i}{\max_{L_{j\in S\setminus\{i\}}}\{w_j\}} > \phi^{\frac{\mu}{\sigma-1}}.$$

Proof. Since regional prices are remain unchanged, Lemma 3.3 asserts that agglomeration is stable if:

$$\frac{w_i}{P_i^\mu} > \max_{L_{j\in S\setminus\{i\}}}\left\{\frac{w_j}{P_j^\mu}\right\} \Leftrightarrow \frac{w_i}{\max_{L_{j\in S\setminus\{i\}}}\{w_j\}} > \left(\frac{P_i}{P_j}\right)^\mu \Leftrightarrow \frac{w_i}{\max_{L_{j\in S\setminus\{i\}}}\{w_j\}} > \phi^{\frac{\mu}{\sigma-1}},$$

concluding the proof. $\square$

Proposition 3.11. *Agglomeration is stable under exogenous asymmetries if:*

$$AA \equiv \sigma\phi - \phi^{\frac{\mu}{\sigma-1}}\left[l_m\left(\mu\left(\phi^2-1\right)+\sigma\right) + \sum_{j\neq i,m}^{n} l_j\phi(\mu(\phi-1)+\sigma) + l_i\sigma\phi^2\right] < 0, \tag{3.27}$$

where $l_m = (\max\{L_j\})/L(n)$; $l_j = L_j/L(n)$, for $j \neq \{i,m\}$; and $l_i = L_i/L(n)$.

Proof. See Appendix E. □

The peripheral region with the highest nominal wage w_m is the region with the highest fraction of farmers (see Appendix E). Inspection of (3.27) shows that agglomeration becomes more likely as L_i increases, and less likely as L_m and L_j decrease.

From Appendix E, we get the entrepreneur's nominal wage at the core region:

$$w_i = \frac{\mu L(n)}{H(n)(\mu-\sigma)} = \frac{\mu\left(l_m + \sum_{j\neq i,m}^{n} l_j + l_i\right)}{H(\mu-\sigma)}. \tag{3.28}$$

The entrepeneur's nominal and real wage at the core is the same as if the regions were symmetric but now L is distributed differently across the regions. An increase in the number of farmers in any of the regions impacts the core's nominal wage with the same positive magnitude, as can be seen from (3.28).

From the proof of Proposition 6.2 (see Appendix E), the highest peripheral nominal wage, $w_m \equiv \max\{w_j\}$ is given by:

$$w_m = \frac{\mu\left(l_m\left(\mu\left(\phi^2-1\right)+\sigma\right) + \sum_{j\neq i,m}^{n} l_j\phi(\mu(\phi-1)+\sigma) + l_i\sigma\phi^2\right)}{H\sigma\phi(\sigma-\mu)}. \tag{3.29}$$

After careful inspection of (3.29), an increase in the number of farmers in any region also increases w_m, but increases in the number of farmers in the peripheral regions has a higher impact.

Let us now take the nominal wage ratio using (3.28) and (3.29):

$$\frac{w_i}{w_m} = \frac{\left(l_m + \sum_{j\neq i,m}^{n} l_j + l_i\right)\sigma\phi}{l_m\left(\mu\left(\phi^2-1\right)+\sigma\right) + \sum_{j\neq i,m}^{n} l_j\phi(\mu(\phi-1)+\sigma) + l_i\sigma\phi^2}.$$

The wage ratio is increasing in l_i and decreasing in l_m and l_j, as expected, which makes agglomeration more (less) likely as l_i (resp. l_m, l_j) increases. The economic intuition does not stray from that given in section 5. In fact, the present case encompasses the symmetric one, as an increase in l_i or a decrease in l_m decreases the entrepreneur's relative nominal (and real) wage in the periphery m, i.e., the periphery with the highest nominal (real) wage. Analogous reasoning in the symmetric case would be to consider an increase in the number of regions and resulting decrease in a periphery's $l_j = 1/n$. Both cases lead to a strengthening of the agglomerative outcome.

3.7 Conclusion

Building on the 2-region FE model by Forslid and Ottaviano (2003), we have obtained both analytical and numerical results from a FE model with an arbitrary number of equidistant regions. We have shown that an increase in the number of regions, other things being equal, favours stability of agglomeration and hinders stability of total dispersion. This happens because the amount of immobile unskilled workforce per region decreases which in turn diminishes its role as a dispersive force. In the limit, when the number of regions tends to infinity, agglomeration becomes the unique stable equilibrium.

Upon introducing exogenous asymmetries in the regional distribution of unskilled labour, we have concluded that market size effects operate through the nominal (hence real) wages in a way such that an increase in unskilled labour in the core region strengthens the stability of agglomeration. The same happens if unskilled labour in the peripheral regions is decreased. This is what actually happens in the symmetric case when the number of regions increases.

We have also provided numerical evidence that strongly suggests that dispersion of entrepreneurs between two regions is never stable in a model with more than two regions, where it corresponds to an outcome of partial dispersion.

Finally, we have found numerical evidence in that, for every triple (μ, σ, n), there exists $\phi \in (0, 1)$ where agglomeration and total dispersion can be simultaneously stable, though this outcome is relatively unlikely. This means that, like the 2-region model, the FE model with more regions exhibits a core-periphery pattern based on a "subcritical pitchfork" bifurcation. The scope for multiplicity of equilibria seems to be lower if the number of regions is either very high or very low.

These results are in the same direction as those obtained by Castro *et al.* (2012) for the CP model with three regions. Through the more tractable framework of the FE model we were able to obtain explicit solutions for the wages of entrepreneurs and show that most key features of Krugman's seminal CP model are likely to hold in a multi-regional set-up.

Acknowledgements: We are thankful to Paulo Guimarães, Pascal Mossay, Vasco Leite and participants in the 2nd Ph.D. Workshop on Industrial and Public Economics at U. Rovira i Virgili for very useful comments and suggestions. We are grateful for the financial support by Portuguese Public Funds through Fundação para a Ciência e

Tecnologia (FCT) in the framework of the Ph.D. scholarship SFRH/BD/90953/2012.

3.A - Nominal wage in the 3-region model

Proof of Proposition 3.1. After (3.13) and (3.14), for $n = 3$, we have the following linear system of equations:

$$\begin{cases} w_1 \left(1 - \frac{\mu}{\sigma}\frac{H_1}{R_1}\right) - w_2 \left(\frac{\mu}{\sigma}\frac{\phi H_2}{R_2}\right) - w_3 \left(\frac{\mu}{\sigma}\frac{\phi H_3}{R_3}\right) & = \frac{\mu}{\sigma}\frac{L}{3}\left(\frac{1}{R_1} + \frac{\phi}{R_2} + \frac{\phi}{R_3}\right) \\ w_1 \left(-\frac{\mu}{\sigma}\frac{\phi H_1}{R_1}\right) + w_2 \left(1 - \frac{\mu}{\sigma}\frac{H_2}{R_2}\right) - w_3 \left(\frac{\mu}{\sigma}\frac{\phi H_3}{R_3}\right) & = \frac{\mu}{\sigma}\frac{L}{3}\left(\frac{\phi}{R_1} + \frac{1}{R_2} + \frac{\phi}{R_3}\right) \\ w_1 \left(-\frac{\mu}{\sigma}\frac{\phi H_1}{R_1}\right) - w_2 \left(\frac{\mu}{\sigma}\frac{\phi H_2}{R_2}\right) + w_3 \left(1 - \frac{\mu}{\sigma}\frac{\phi H_3}{R_3}\right) & = \frac{\mu}{\sigma}\frac{L}{3}\left(\frac{\phi}{R_1} + \frac{\phi}{R_2} + \frac{1}{R_3}\right), \end{cases}$$

where $R_j \equiv \sum_{m=1}^{3} \phi_{mj} H_m$. This may be written in matrix form as $AW = B$, where A stands for the coefficients matrix, W the vector of nominal wages w_i, while B is the column vector of independent terms in the right-hand side of the system of equations above. Applying Cramer's Rule, the solution to this system is of the following form:

$$w_i = \frac{Dw_i}{D},$$

where the denominator D stands for the determinant of matrix A and Dw_i is the determinant of the matrix obtained by replacing the i-th column of A by the column vector B. We have:

$$D = 1 - \frac{\mu}{\sigma}\sum_{j=1}^{3}\frac{H_j}{R_j} + \frac{\mu^2}{\sigma^2}\left(1 - \phi^2\right)\left(\frac{H_1 H_2}{R_1 R_2} + \frac{H_1 H_3}{R_1 R_3} + \frac{H_2 H_3}{R_2 R_3}\right) - \frac{\mu^3}{\sigma^3}\left(2\phi^3 - 3\phi^2 + 1\right)\prod_{j=1}^{3}\frac{H_j}{R_j}.$$

For $i = 1$, Dw_1 is given by:

$$\begin{aligned} Dw_1 & = \frac{\mu}{\sigma}\frac{L}{3}\left\{\left(\sum_{j=1}^{3}\frac{\phi_{1j}}{R_j}\right) + \frac{\mu}{\sigma}\left[\phi\left(\phi - 1\right)\frac{H_2 + H_3}{R_2 R_3} + \frac{\phi^2 - 1}{R_1}\left(\frac{H_2}{R_2} + \frac{H_3}{R_3}\right)\right] + \right. \\ & \left. + \frac{\mu^2}{\sigma^2}\left(2\phi^3 - 3\phi^2 + 1\right)\frac{H_2 H_3}{R_1 R_2 R_3}\right\} \end{aligned}$$

The expression for the nominal wage in region 1 is:

$$w_1 = \frac{\frac{\mu}{\sigma}\frac{L}{3}\left\{\sum_{j=1}^{3}\frac{\phi_{1j}}{R_j} + \frac{\mu}{\sigma}\left[\phi\left(\phi - 1\right)\frac{H_2 + H_3}{R_2 R_3} + \frac{\phi^2 - 1}{R_1}\left(\frac{H_2}{R_2} + \frac{H_3}{R_3}\right)\right] + \frac{\mu^2}{\sigma^2}\left(2\phi^3 - 3\phi^2 + 1\right)\frac{H_2 H_3}{R_1 R_2 R_3}\right\}}{1 - \frac{\mu}{\sigma}\sum_{j=1}^{3}\frac{H_j}{R_j} + \frac{\mu^2}{\sigma^2}\left(1 - \phi^2\right)\left(\frac{H_1 H_2}{R_1 R_2} + \frac{H_1 H_3}{R_1 R_3} + \frac{H_2 H_3}{R_2 R_3}\right) - \frac{\mu^3}{\sigma^3}\left(2\phi^3 - 3\phi^2 + 1\right)\prod_{j=1}^{3}\frac{H_j}{R_j}}.$$

Generically,

$$w_i = \frac{\frac{\mu}{\sigma}\frac{L}{3}\left\{\sum_{j=1}^{3}\frac{\phi_{ij}}{R_j} + \frac{\mu}{\sigma}\left[\phi\left(\phi-1\right)\frac{\sum\limits_{k\neq i}H_k}{\prod\limits_{k\neq i}R_k} + \frac{\phi^2-1}{R_i}\sum_{k\neq i}\frac{H_k}{R_k}\right] + \frac{\mu^2}{\sigma^2}\left(2\phi^3-3\phi^2+1\right)\frac{1}{R_i}\prod_{k\neq i}\frac{H_k}{R_k}\right\}}{1-\frac{\mu}{\sigma}\sum_{j=1}^{3}\frac{H_j}{R_j} + \frac{\mu^2}{\sigma^2}\left(1-\phi^2\right)\left(\frac{H_1H_2}{R_1R_2} + \frac{H_1H_3}{R_1R_3} + \frac{H_2H_3}{R_2R_3}\right) - \frac{\mu^3}{\sigma^3}\left(2\phi^3-3\phi^2+1\right)\prod_{j=1}^{3}\frac{H_j}{R_j}}.$$

□

3.B - Jacobian and total dispersion

As a prerequisite to Proposition 3.2, establishing the stability of total dispersion, we have the following Lemma.

Lemma. B.1 *Concerning total dispersion: (i) The Jacobian matrix of the system in (3.19) has a repeated real eigenvalue with multiplicity* $n-1$ *given by:*

$$\alpha \equiv \frac{\partial \omega_i}{\partial h_i}\left(\tfrac{1}{n}, \tfrac{1}{n}, ..., \tfrac{1}{n}\right),$$

and (ii) the equilibrium is stable if α *is negative.*

Proof. An element of the Jacobian is given by:

$$J_{ii} = \frac{\partial \omega_i}{\partial h_i} - \frac{\partial \bar{\omega}}{\partial h_i}.$$

We will show that $\partial_{h_i}\bar{\omega} = 0, \forall i$ at total dispersion.

Assume, by way of contradiction, that $\partial_{h_1}\bar{\omega} > 0$. Then $\bar{\omega}(\frac{1}{n}+\varepsilon, \frac{1}{n}, ..., \frac{1}{n}) > \bar{\omega}(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}) > \bar{\omega}(\frac{1}{n}-\varepsilon, \frac{1}{n}, ..., \frac{1}{n})$.[18] However, the model's symmetry asserts that $\bar{\omega}$ is invariant in the permutation of any h_i and h_j, $i \neq j$, and thus also h_1 and h_n. Hence, $\bar{\omega}(\frac{1}{n}-\varepsilon, \frac{1}{n}, ..., \frac{1}{n}) > \bar{\omega}(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})$, and we arrive at a contradiction. As a result, we have $J_{11} = \partial_{h_1}\omega_1$. Moreover, symmetry implies that $\partial_{h_1}\omega_1 = \partial_{h_2}\omega_2 = \ldots = \partial_{h_{n-1}}\omega_{n-1}$ at

[18] Changes in any h_i are reflected symmetrically in h_n , which is however only implicitly defined.

total dispersion. This leads to the conclusion that $J_{11} = J_{22} = \ldots = J_{n-1,n-1}$, implying that $J_{ii} = \alpha = \partial_{h_i}\omega_i$.

We will next show that all elements outside the main diagonal are zero at total dispersion. An element outside the main diagonal is given by:

$$J_{ij} = \frac{\partial \omega_i}{\partial h_j} - \frac{\partial \bar{\omega}}{\partial h_j}$$

We have seen that $\partial_{h_i}\bar{\omega} = 0$ at total dispersion. Assume, again by way of contradiction, that $\partial_{h_2}\omega_1 > 0$ at total dispersion. It must follow that $\omega_1(\frac{1}{n}, \frac{1}{n} + \varepsilon, \frac{1}{n}, ..., \frac{1}{n}) > \omega_1(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}) > \omega_1(\frac{1}{n}, \frac{1}{n} - \varepsilon, \frac{1}{n}, ..., \frac{1}{n})$. However, the real wage ω in one region is invariant in the permutation of coordinates in any of the other $n-1$ regions. Therefore we have $\omega_1(\frac{1}{n}, \frac{1}{n} + \varepsilon, \frac{1}{n}, ..., \frac{1}{n}) = \omega_1(\frac{1}{n}, \frac{1}{n} - \varepsilon, \frac{1}{n}, ..., \frac{1}{n})$ and arrive at a contradiction. It must follow that $\partial_{h_2}\omega_1 = 0$. Symmetry established analogous results for every $\partial_{h_j}\omega_i$ at total dispersion. As a result, we have $J_{ij} = 0, \forall i \neq j$. Hence, the Jacobian matrix at total dispersion is a scalar multiple of the identity, αI, where:

$$\alpha \equiv \frac{\partial \omega_i}{\partial h_i}\left(\tfrac{1}{n}, \tfrac{1}{n}, ..., \tfrac{1}{n}\right).$$

Thus, α is the $n-1$ repeated eigenvalue of the matrix. The symmetric equilibrium is stable if it is negative, which concludes the proof. □

Proof of Proposition 3.2. We compute $\partial w_1/\partial h_1$ at total dispersion while avoiding having to solve a system of $n-1$ linear equations using just two wage equations from (3.16), e.g., for w_1 and w_n. The reason is that, after lemma B.1, there are $n-2$ wages equal to w_j for which $\partial w_j/\partial h_1 = 0$. As a result, we still have w_1 and w_n that depend on h_1. We have:

$$\begin{cases} w_1\left(1 - \frac{\mu}{\sigma}\frac{h_1}{r_1}\right) - (n-2)w_j\frac{\mu}{\sigma}\phi\left(\frac{1/n}{r_j}\right) - w_n\left(\frac{\mu}{\sigma}\phi\frac{\frac{2}{n} - h_1}{r_n}\right) & = \frac{1}{H(n)}\frac{\mu}{\sigma}\frac{L(n)}{n}\left(\frac{1}{r_1} + (n-2)\frac{\phi}{r_j} + \frac{\phi}{r_n}\right) \\ -w_1\left(\frac{\mu}{\sigma}\phi\frac{h_1}{r_1}\right) - (n-2)w_j\frac{\mu}{\sigma}\phi\left(\frac{1/n}{r_j}\right) + w_n\left(1 - \frac{\mu}{\sigma}\frac{\frac{2}{n} - h_1}{r_n}\right) & = \frac{1}{H(n)}\frac{\mu}{\sigma}\frac{L(n)}{n}\left(\frac{\phi}{r_1} + (n-2)\frac{\phi}{r_j} + \frac{1}{r_n}\right), \end{cases} \tag{3.30}$$

where $r_1 = h_1 + \phi\frac{n-2}{n} + \phi\left(\frac{2}{n} - h_1\right)$, $r_j = \frac{1}{n} + \frac{(n-1)\phi}{n}$ and $r_n = h_1\phi - h_1 + \frac{(n-2)\phi}{n} + \frac{2}{n}$.

Using any of the equations in (3.30) we can easily find the nominal wage at total dispersion:

$$w_i(\tfrac{1}{n}, \tfrac{1}{n}, ..., \tfrac{1}{n}) = \frac{\mu L(n)}{H(n)(\sigma - \mu)}.$$

In order to compute the derivative $\partial_{h_1} w_1$, we should first solve the second equation in (3.30) for w_n and replace it in the first one to obtain:

$$w_1 = -\frac{\mu\left(h_1(\phi-1)-\phi\right)\left((n-2)\phi w_j\left(-h_1 n(\phi-1)(\mu-\sigma)+2\mu(\phi-1)+\sigma((n-2)\phi+2)\right)+\frac{L(n)}{H(n)n(h_1(\phi-1)-\phi)}\Phi(h_1,\phi,n)\right)}{((n-1)\phi+1)\left(h_1(\phi-1)\left(h_1 n-2\right)(\mu-\sigma)(\phi(\mu+\sigma)+\mu-\sigma)+\sigma\phi(\sigma((n-2)\phi+2)-2\mu)\right)},$$

where:

$$\begin{aligned}\Phi(h_1,\phi,n) \quad \equiv \quad & h_1 n(\phi-1)\left(-h_1(n-2)n(\phi-1)\phi(\mu-\sigma)+\mu+\mu\left(n^2+n-5\right)\phi^2-\mu(n-4)\phi-\sigma(\phi-1)((n-3)\phi-1)\right)- \\ & -2\mu(\phi-1)(\phi(((n-1)n-1)\phi+n)+1)+\sigma(\phi(n(-\phi(((n-3)n+3)\phi+3(n-2))-3)-2\phi+4)-2.\end{aligned}$$

This, together with the knowledge that $\partial w_j/\partial h_1 = 0$ at total dispersion enables us to find $\partial w_1/\partial h_1$ at total dispersion. After some manipulation, we get:

$$\frac{\partial w_1}{\partial h_1}\left(\tfrac{1}{n}, \tfrac{1}{n}, ..., \tfrac{1}{n}\right) = \frac{\mu L(n)n(\phi-1)(\mu+\mu(n-1)\phi+\sigma(\phi-1))}{H(n)(\mu-\sigma)((n-1)\phi+1)(\mu(\phi-1)+(n-1)\sigma\phi+\sigma)}.$$

By (3.15), we have:

$$\frac{\partial P_1^{\mu}/\partial h_1}{P_1^{\mu}} = \frac{\mu n(\phi-1)}{(\sigma-1)((n-1)\phi+1)}.$$

We can then compute the derivative of the real wage in region 1 with respect to h_1 :

$$\frac{\partial \omega_1}{\partial h_1}(\tfrac{1}{n}, \tfrac{1}{n}, ..., \tfrac{1}{n}) = \frac{\mu L(n)n(\phi-1)\left(\mu^2(\phi-1)+\mu(2\sigma-1)((n-1)\phi+1)+(\sigma-1)\sigma(\phi-1)\right)}{H(n)(\sigma-1)(\mu-\sigma)((n-1)\phi+1)((\mu+(n-1)\sigma\phi+\sigma-\mu))}.$$

Note that the denominator is negative as is the factor $(\phi - 1)$ in the numerator. Hence, the sign of the derivative is equal to the sign of:

$$BP(\phi) \equiv \mu^2(\phi-1) + \mu(2\sigma-1)((n-1)\phi+1) + (\sigma-1)\sigma(\phi-1)$$

which is a linear function of ϕ with positive coefficient. Equating $BP(\phi)$ to zero and solving for ϕ gives us:

$$\phi_b \equiv \frac{(\mu-\sigma)(\mu-\sigma+1)}{\mu^2+\mu(n-1)(2\sigma-1)+(\sigma-1)\sigma},$$

and total dispersion is stable if ϕ is below this threshold, thus concluding the proof. □

3.C - Jacobian and partial dispersion

The following result concerns partial dispersion (but can also be applied to total dispersion).

Lemma. C.1 *Configurations of the form* $h_j = \frac{1-h_i}{2}$*, with* $0 \le h_i \le 1$*, satisfy* $\frac{\partial \bar{\omega}}{\partial h_j} = 0$.

Proof. We may look at configurations of the form $(b, ..., b, a, a)$, where $a = (1-b)/2$ and $b \in [0, 1]$. Note that if $b = 1/n$ we have full dispersion. Assume, without loss of generality, that $i = 2$. Suppose $\frac{\partial \bar{\omega}}{\partial h_2}(b, ..., b, a, a) \neq 0$. Assume it is positive. Then

$$\bar{\omega}(b, ..., b, a+\varepsilon, a-\varepsilon) > \bar{\omega}(b, a, a).$$

But, $\bar{\omega}$ is invariant by the permutation that interchanges identically populated regions and therefore

$$\bar{\omega}(b, ..., b, a-\varepsilon, a+\varepsilon) > \bar{\omega}(b, a, a)$$

indicating that $\frac{\partial \bar{\omega}}{\partial h_2} < 0$, which contradicts the assumption and finishes the proof. □

Proof of Lemma 3.5. First, since $h_j = 0$ for $j = \{2, ..., n-1\}$, by similar arguments as those in the proof of Lemma 3.3, we have the following necessary condition for stability of partial dispersion:

$$\omega_1 > \omega_j.$$

Second, we need to ensure not only that h_j will remain zero but also that both h_1 and h_n will remain at $\frac{1}{2}$. If any of the skilled workers migrates, e.g. to region 2, we need them to want to return to region 1 (symmetry implies the same in the opposite direction). This is achieved when an increase in h_1 leads to a decrease in the difference between the real wage ω_1 and the real wage average $\bar{\omega}$:

$$\frac{\partial \omega_1}{\partial h_1}\left(\tfrac{1}{2}, 0\right) - \frac{\partial \bar{\omega}}{\partial h_1}\left(\tfrac{1}{2}, 0\right) < 0.$$

However, since we have $h_1 = \frac{1-h_2}{2}$ at $\left(\frac{1}{2}, 0\right)$, Lemma 3.7 asserts that $\frac{\partial \bar{\omega}}{\partial h_1}\left(\frac{1}{2}, 0\right) = 0$. As a result, the second condition reduces to

$$\frac{\partial \omega_1}{\partial h_1}(h^p) < 0.$$

If the first condition fails to hold, partial dispersion is unstable. □

Proof of Proposition 3.6. First, we determine the nominal wages in regions 1 and n at partial dispersion. For regions 1 and n we use equation (3.16) to get:

$$w_1\left(h^p\right) = w_n\left(h^p\right) = \frac{\mu L(n)}{H(n)(\sigma-\mu)}.$$

As for the other j regions we have:

$$w_j\left(h^p\right) = \frac{\mu L(n)\left(n\left(\frac{2\mu\phi^2}{\sigma-\mu}+\phi+1\right)+2(\phi-1)(2\phi+1)\right)}{H(n)n\sigma\phi(\phi+1)}.$$

Then, the first condition for stability of partial dispersion requires:

$$\begin{aligned}
\frac{w_1}{w_j} &> \left(\frac{P_1}{P_j}\right)^{\mu} \iff \\
\frac{n\sigma\phi(\phi+1)}{\mu(n-2)(\phi-1)(2\phi+1)+\sigma\left((n-2)\phi+n+4\phi^2-2\right)} &> \left(\frac{2\phi}{1+\phi}\right)^{\frac{\mu}{\sigma-1}} \iff \\
\mu(n-2)(\phi-1)(2\phi+1)+\sigma\left[2(\phi-1)(2\phi+1)-n(\phi+1)\left(\phi 2^{\frac{\mu}{1-\sigma}}\left(\frac{\phi}{\phi+1}\right)^{\frac{\mu}{1-\sigma}}-1\right)\right] &< 0.
\end{aligned}$$

Rewriting it, we obtain:

$$\gamma \equiv \sigma(1+\phi)\left[1-\phi^{1-\frac{\mu}{\sigma-1}}\left(\frac{1+\phi}{2}\right)^{\frac{\mu}{\sigma-1}}\right]n-\left[2\sigma+\mu(n-2)\right](1-\phi)(2\phi+1)<0,$$

thus concluding the first part of the proof.

In order to determine the second condition, we follow an approach analogous to that of the proof of Proposition 3.2 in Appendix B, selecting two wage equations from (3.16) and using lemma B.1 to state that only w_1 and w_n depend on h_1:

$$\begin{cases}
w_1\left(1-\dfrac{\mu}{\sigma}\dfrac{h_1}{h_1+\phi(1-h_1)}\right)-w_n\left(\dfrac{\mu}{\sigma}\dfrac{\phi(1-h_1)}{\phi h_1+1-h_1}\right) = \dfrac{\mu L(n)}{\sigma n H(n)}\left(\dfrac{1}{h_1+\phi(1-h_1)}+n-2+\dfrac{\phi}{\phi h_1+1-h_1}\right)\\
w_n\left(1-\dfrac{\mu}{\sigma}\dfrac{1-h_1}{\phi h_1+1-h_1}\right)-w_1\left(\dfrac{\mu}{\sigma}\dfrac{\phi h_1}{h_1+\phi(1-h_1)}\right) = \dfrac{\mu L(n)}{\sigma n H(n)}\left(\dfrac{\phi}{h_1+\phi(1-h_1)}+n-2+\dfrac{1}{\phi h_1+1-h_1}\right)
\end{cases}$$

Solving the second equation for w_n and replacing in the first we obtain w_1 as a function of h_1 :

$$w_1 = \frac{\mu L\left(h_1(\phi-1)\left(h_1(n-2)(\phi-1)(\mu-\sigma)+\mu(-2n\phi+n+3\phi-3)+(n-3)\sigma(\phi-1)\right)+\mu(\phi-1)((n-1)\phi+1)+\sigma\phi(n+\phi-2)+\sigma\right)}{Hn\left(\left(h_1-1\right)h_1(\phi-1)(\mu-\sigma)(\phi(\mu+\sigma)+\mu-\sigma)+\sigma\phi(\sigma-\mu)\right)}.$$

Taking the derivative with respect to h_1 we get:

$$\frac{\partial w_1}{\partial h_1} = \frac{\mu L(\phi-1)\left(-2h_1\left(\phi^2(\mu+\sigma)-\mu+\sigma\right)+h_1^2(\phi-1)(\phi(\mu+\sigma)+\mu-\sigma)+\phi^2(\mu+\sigma)-\mu+\sigma\phi+\sigma\right)(\mu+\mu(n-1)\phi+\sigma(\phi-1))}{H(n)n(\mu-\sigma)\left(\sigma\phi-(h_1-1)\,h_1(\phi-1)(\phi(\mu+\sigma)+\mu-\sigma)\right)^2},$$

which, evaluated at $h_1 = 1/2$, yields:

$$\frac{\partial w_1}{\partial h_1}(h^p) = \frac{4\mu L(n)(\phi-1)(\mu+\mu(n-1)\phi+\sigma(\phi-1))}{H(n)n(\phi+1)(\mu-\sigma)(\phi(\mu+\sigma)-\mu+\sigma)}.$$

Differentiating the price index, by (3.15):

$$\frac{\partial P_1^\mu/\partial h_1}{P_1^\mu}(h^p) = \frac{2\mu(\phi-1)}{(\sigma-1)(\phi+1)}.$$

Finally, we can compute the derivative of the real wage in region 1 with respect to h_1 at partial dispersion:

$$\frac{\partial \omega_1}{\partial h_1}(h^p) = \frac{2(\phi-1)\left(-\frac{\mu}{\sigma-1}-\frac{2(\mu+\mu(n-1)\phi+\sigma(\phi-1))}{n(\phi(\mu+\sigma)-\mu+\sigma)}\right)}{\phi+1},$$

which is negative if and only if:

$$\beta \equiv \mu^2 n(\phi-1)+\mu(\sigma(3n\phi+n-2\phi+2)-2n\phi+2\phi-2)+2(\sigma-1)\sigma(\phi-1) < 0,$$

which gives us the second condition, concluding the proof. □

3.D - Comparing models

Proof of Proposition 3.8. In the n-region model, the core-periphery price index ratio equals:

$$\left(\frac{P_i}{P_j}\right)^\mu = \phi^{\frac{\mu}{\sigma-1}},$$

which does not depend on the number of regions and proves part (i) of the proposition. The nominal wage in the core region i is given by:

$$w_i = \frac{\mu L(n)}{(\sigma-\mu)H(n)}.$$

The peripheral nominal wage is equal to:

$$w_j = \frac{\mu L(n)(\mu(\phi-1)((n-1)\phi+1)+\sigma\phi(n+\phi-2)+\sigma)}{H(n)n\sigma\phi(\sigma-\mu)}$$

The wage ratio is given by:

$$\frac{w_i}{w_j} = \frac{n\sigma}{\mu(\phi-1)((n-1)\phi+1)+\sigma\phi(n+\phi-2)+\sigma},$$

which one can check to be increasing in n, as:

$$\frac{\partial\left(w_i/w_j\right)}{\partial n} = \frac{\sigma(\phi-1)^2(\sigma-\mu)}{(\mu(\phi-1)((n-1)\phi+1)+\sigma\phi(n+\phi-2)+\sigma)^2},$$

thus concluding the final part of the proof.

Appendix E - Exogenous asymmetries

Proof of Proposition 3.11. At concentration in region i, we have, after (3.13):

$$w_i = \frac{\mu L(n)}{H(n)(\sigma-\mu)},$$

which means that the core's nominal wage is still the same.

As for the peripheral regions, let $L_m = \arg\max_{L_{j\in S\setminus\{i\}}}\{w_j\}$. Then:

$$w_m \equiv \max_{L_{j\in S\setminus\{i\}}}\{w_j\} = \frac{\mu\left(L_m\left(\mu\left(\phi^2-1\right)+\sigma\right)+\sum_{j\neq i,m}^{n} L_j\phi(\mu(\phi-1)+\sigma)+L_i\sigma\phi^2\right)}{H(n)\sigma\phi(\sigma-\mu)}.$$

Clearly, it follows that $L_m = \max_{j\in S\setminus\{i\}}\{L_j\}$. The nominal wage ratio is given by:

$$\frac{w_i}{w_m} = \frac{L(n)\sigma\phi}{L_m\left(\mu\left(\phi^2-1\right)+\sigma\right)+\sum_{j\neq i,m}^{n} L_j\phi(\mu(\phi-1)+\sigma)+L_i\sigma\phi^2}.$$

Given the ratio between price indices, agglomeration is stable if:

$$AA \equiv \mu\left\{L(n)\sigma\phi - \phi^{\frac{\mu}{\sigma-1}}\left[L_m\left(\mu\left(\phi^2-1\right)+\sigma\right)+\sum_{j\neq i,m}^{n} L_j\phi(\mu(\phi-1)+\sigma)+L_i\sigma\phi^2\right]\right\} < 0,$$

thus concluding the proof. $\square$

Chapter 4

Agglomeration patterns in a multi-regional economy without income effects

4.1 Introduction

New Economic Geography (NEG) has been on the forefront in recent decades as an economics subject that seeks to explain the spatial distribution of economic activity. Many theoretical models have been built along the lines of the seminal Core-Periphery (CP) model (Krugman, 1991), in which skilled labour mobility combined with a general equilibrium framework under increasing returns, monopolistic competition and transport costs contribute to explain how demand linkages and supply linkages interplay to determine the geographical distribution of industry. Other NEG models have built on the original CP model trying to improve on the former's tractability and attempting to reach new insights (Fujita *et al.,* 1999; Ottaviano *et al.*, 2002; Forslid and Ottaviano, 2003; Pflüger, 2004).

Among many of the common features and results, one that we find empirically hard to justify is the prediction of catastrophic agglomeration, from an evenly dispersed distribution, into a single region as transport costs fall below a certain level. One exception is Pflüger's Quasi-Linear Log (QLLog) model (Pflüger, 2004), where the absence of income effects on regional demand for manufactures, due to the assumption of a quasi-linear upper-tier utility function, significantly simplifies stability analysis. This model reverses the predictions of the seminal CP model by Krugman (1991) that industry is doomed to stay dispersed among two symmetric regions or fully agglomerated in one region. What is more, catastrophic agglomeration is ruled out as transport costs fall below some threshold level, and thus agglomeration is rather a smooth and gradual process. This means that there is some scope for stable asymmetric distribution of industry in the 2-region QLLog model, contrarily to the predictions of many NEG models.[1]

Although insightful, the lack of a multi-regional framework in the QLLog model potentially overlooks complex interdependencies among different regions, which do not arise in the 2-region set-up (Fujita *et al.*, 1999; Fujita and Mori, 2005; Tabuchi *et al.,* 2005; Behrens and Thisse, 2007; Fujita and Thisse, 2009; Behrens and Robert-Nicoud,

[1]The importance of preferences and how these operate to influence long-run dynamics of industry in the space economy are widely discussed in the paper by Pflüger and Südekum (2008a), where the authors discuss qualitative migration pattern changes across different CP models.

2011; Tabuchi, 2014). Moreover, the often assumed one-dimensional geography in NEG hinders any serious empirical work on the field, since the latter typically faces a more heterogeneous and multi-regional setting in the real world (Bosker *et al.*, 2010). A number of different extensions of NEG models to a multi-regional set-up have been made so far, each with its own specificities. Some attempts focus on the role of heterogeneous distances between regions, such as the "Racetrack Economy" where regions are equally spaced around a circumference (Krugman, 1993; Fujita *et al.,* 1999; Picard and Tabuchi, 2010; Mossay (2013). Heterogeneity in location space has also been tackled in other papers, such as the role of different network topologies to explain locational advantages of some regions (Barbero and Zofío, 2012), equally spaced regions along a line segment (Ago *et al.*, 2006), or hexagonal configurations (Ikeda and Murota, 2014). Along different lines, Oyama (2009) considered an equidistant multi-regional CP model with self-fulfilling expectations in migration that lead to global stability of a single core region in the presence of exogenous asymmetries. Other works, such as Tabuchi and Thisse (2011), have built an NEG model that accounts for the rise of a hierarchical system of central places in a multi-regional set-up. Castro *et al.* (2012) studied a version of Krugman's CP model with 3 and more regions and concluded that additional regions favour single-region agglomeration and discourage the dispersion of economic activities. Akamatsu *et al.* (2012) and Ikeda *et al.* (2012) used a 2^n-region CP model to show that decreasing transport costs leads to *spatial period doubling agglomeration*, whereby the number of regions (cities) in which firms locate is reduced by half and the spacing between each evenly agglomerated regions doubles after each bifurcation. Among these contributions, several other works considering multi-regional NEG models are worth mentioning (e.g., Behrens *et al.*, 2006; Forslid and Okubo, 2012; Tabuchi, 2014; Fabinger, 2015; Commendatore *et al.*, 2015a).[2]

Many of the original main conclusions and implications of most 2-region NEG models have been seldom challenged. As an example, Bosker *et al.* (2010) show that most conclusions from 2-region NEG models hold under more realistic settings. This includes not only the extension to the equidistant multi-regional case, as they argue using the paper by Puga (1999) as benchmark, but also the consideration of non-equidistant

[2]While it is not our purpose to provide an extensive overview of the literature concerning multi-regional models, we refer the reader to the review of Commendatore *et al.* (2015b).

regions. A notable difference, however, is that, in the second case, exogenous asymmetries gives rise to the possibility of spatial distributions other than agglomeration or dispersion. Recently, Tabuchi (2014) has used a multi-regional version of Krugman's (1991) CP model to show that it can account for the historical trend of agglomeration in the capital regions over the past few centuries.

This motivates us to extend the QLLog model by Pflüger (2004) to an arbitrary number of equidistant regions. Its simplicity allows us to obtain analytical expressions for the indirect utilities of the inter-regionally mobile skilled workers in each region. One of the most important features of our paper is that the assumption of equidistance does not conflict with the existence of asymmetric spatial distributions. In fact, we uncover novel spatial distributions that do not emerge in the 2-region setting. Therefore, while exogenous asymmetries may help explain observed spatial imbalances, they should not be seen as the only factors causing asymmetries in the distribution of economic activity. Second, the assumption of equidistant regions coupled with the removal of income effects for the demand of manufactured good allows us to obtain explicit expressions for the indirect utilities of the inter-regionally mobile workers and, thus, to fully characterize the stability of several kinds of spatial equilibria under general parameter values.[3] Moreover, our model is rich enough that we are able to study how long-run transitions from dispersion to agglomeration depend not only on the change in transportation costs, but also on the global size of the inter-regionally immobile (unskilled) workforce relative to mobile (skilled) labour.[4] Tabuchi *et al.* (2005) also developed a multi-region model with equidistant regions and quasi-linear utility, but considered quadratic sub-utility, as in Ottaviano *et al.* (2002), instead of CES, as in Pflüger (2004). They also consider urban congestion costs (housing and commuting), which act as an additional dispersion force. Studying the impact of falling transport costs on the size and number of cities (non-empty regions), they find that cities initially grow in size, and then shrink at a later stage, a situation which corresponds to agglomeration followed by re-dispersion of industry. Their results are driven by the interplay between inter-regional transport

[3] Noteworthy, this is an important departure from Tabuchi's (2014) multi-regional framework, who focuses on stability analysis in limit cases for transportation costs.

[4] This is captured by a parameter that also exists in Pflüger's model, but whose influence in the spatial distributions is enhanced by the addition of an arbitrary number of regions.

costs and intra-regional urban congestion costs. In contrast, our results do not hinge on the existence of urban congestion costs.

In Castro *et al.* (2012) and Gaspar *et al.* (2013), the authors provided numerical evidence that in CP models (resp. 3-region CP model and 3-region Footloose Entrepreneur model) a configuration bearing a region without industry and two evenly populated regions could not be a stable outcome. Using the QLLog model, we provide an analytical confirmation of this result, extending it to an arbitrary number of regions; that is, at least one empty region paired with a set of evenly distributed regions cannot be a stable outcome.

Another feature of the n-region QLLog model is that interior asymmetric distributions of industry may arise. For simplicity, we focus on the particular one-dimensional subspace of $n-1$ evenly populated regions and one region with any industry size.[5] For these distributions, entrepreneurs face two decisions: that of migrating between any of the evenly populated regions; and that of migrating between the smaller (bigger) region and any of the other evenly distributed regions. This is true for $n \geq 3$ because the dimension of this particular sub-space is invariant in the number of regions. [6]

We show that an asymmetric equilibrium where one region has comparatively less industry than the other regions cannot be stable. This happens because an entrepreneur who migrates between any of the evenly distributed regions will see his utility rise, leading to further migration to the receiving region which, having now become the largest, will attract more and more entrepreneurs up unto the point where that region will eventually become an industrialized core.

We show that the QLLog model with $n \geq 3$ exhibits a primary transcritical bifurcation at the symmetric equilibrium and a secondary saddle-node bifurcation that branches from an interior asymmetric equilibrium, a feature which suggests that structural changes in the migration dynamics of entrepreneurs as transport costs decrease is more complex than previously thought. This has important implications in the tran-

[5] In an equidistant n-region model, there are many other invariant spaces. For instance, a more general invariant subspace is one whereby k regions share the same industry size a/k and the other $n-k$ regions each have a share equal to $(1-a)/(n-k)$. We focus on the particular case corresponding to $k=1$.

[6] In a 2-region model, there is only one single decision because "moving away from one region automatically implies that workers and firms necessarily go to the other" (Tabuchi *et al.*, 2005).

sition towards more agglomerated equilibria as transport costs steadily decrease. The existence of a saddle-node bifurcation implies that industry will stay fully dispersed even after transport costs have fallen below some threshold level that would deem a partial agglomerative equilibrium stable. However, if the industry is initially at some partial agglomeration equilibrium, a temporary rise in transportation costs will force the industry to permanently fully disperse across regions.

We find that, along a smooth parameter path where transport costs decrease, the transition from symmetric dispersion to agglomeration depends on the global size of the inter-regionally immobile (unskilled) labour relative to mobile (skilled) labour. If this ratio is very high (low worker mobility), there is immediate catastrophic agglomeration of skilled workers in one single region once dispersion loses stability. If the ratio is low (high worker mobility), there is a discontinuous jump from dispersion to a partially agglomerated equilibrium, and a smooth transition towards agglomeration thereafter. Finally, for sufficiently low values of the global unskilled to skilled worker ratio, dispersion is not possible, and the only possible structural change as transport costs steadily fall is that of a smooth transition from partial agglomeration towards full agglomeration.

We find that the symmetric equilibrium yields the worst possible welfare to entrepreneurs. Still, even if migrating increases their average utility across all regions, the utility of the migrant decreases when dispersive forces outweigh agglomerative forces. For the immobile workers it is quite the opposite: farmers attain their highest welfare at symmetric dispersion. For the population as a whole, we show that agglomeration (partial or full), even when stable, may be socially inferior to more symmetric spatial distributions. We thus conclude that the multi-regional QLLog model exhibits a tendency towards over-agglomeration for intermediate levels of transport costs.

The rest of the paper is organized as follows. Section 2 presents the n-region FE model with quasi-linear log utility and discusses the short-run equilibrium of the model. Section 3 is the first part of the study of long-run equilibria, where the focus lies on local stability analysis of some particular configurations. One goal is to study full agglomeration and total dispersion to find how stability changes with the number of regions. Second, we want to study whether partial agglomeration patterns are now stable (contrary to the results of Fujita *et al.*, 1999; Castro *et al.,* 2012; and Gaspar *et al.*, 2013). We try to fully determine the number of interior equilibria along some invariant spaces in order to study their local stability. Bifurcation patterns are accounted for in section 4. In section 5 we study social welfare in the QLLog model at different possible distributions, first disentangling between mobile and immobile workers, and then studying the economy as a whole. Section 6 is left for some concluding remarks.

4.2 The Quasi-linear log model with n regions

Many derivations in this model are a combination of those present in Forslid and Ottaviano (2003), Pflüger (2004) and Gaspar *et al.* (2013). As a result, for the sake of presentation, we omit calculations whenever reasonable. The set of regions is $N = \{1, ..., n\}$. The masses of entrepreneurs and farmers are, respectively, H and L. Entrepreneurs can move freely among regions ($H = H_1 + H_2 + ... + H_n$), while farmers are immobile and evenly distributed among the regions ($L_i = L/n, \forall i \in N$).

4.2.1 Demand and indirect utility

As in Pflüger (2004), the representative consumer of region $i \in N$ faces a translog quasi-linear upper-tier utility function:

$$U_i = \mu \ln M_i + A_i, \quad 0 < \mu < 1 \tag{4.1}$$

where A_i is the consumption of agricultural products in region i and M_i is the consumption of a CES composite of differentiated varieties of manufactures in region i, defined by:

$$M_i = \left[\int_{s \epsilon S} d_i(s)^{\frac{\sigma-1}{\sigma}} ds \right]^{\frac{\sigma}{\sigma-1}}, \tag{4.2}$$

where $d_i(s)$ is consumption of variety s of manufactures in region i, S is the mass of existing varieties, and $\sigma > 1$ is the constant elasticity of substitution between manufactured varieties. The regional price index P_i associated with (4.2) equals:

$$P_i = \left[\sum_{j=1}^{n} \int_{s \epsilon S} p_{ji}(s)^{1-\sigma} ds \right]^{\frac{1}{1-\sigma}}. \tag{4.3}$$

Let $p_{ji}(s)$ and $d_{ji}(s)$ denote the price and individual demand in region i of a variety, s, that is produced in region j. Consumers in each region i maximize utility subject to the budget constraint:

$$Y_i = P_i M_i + A_i,$$

where P_i is given by (4.3) and the price of the agricultural good is, as usual, normalized to unity. This yields the following demand functions:

$$d_{ij}(s) = \mu \frac{p_{ij}(s)^{-\sigma}}{P_i^{1-\sigma}}, \quad M_i = \mu P_i^{-1}, \quad A_i = Y_i - \mu. \tag{4.4}$$

Individual consumption of the agricultural good is positive if and only if $\mu < 1$ (for farmers), which is true, and $w_i > \mu$ (for entrepreneurs).[7]

Using the upper-tier utility in (4.1) and the optimal consumptions of M_i and A_i in (4.4), we reach the consumer's indirect utility:

$$V_i = Y_i - \mu \ln P_i + \mu(\ln \mu - 1). \tag{4.5}$$

4.2.2 Supply

The agricultural good is produced using one unit of unskilled labour (farmers) for each unit that is produced (constant returns to scale), and is freely traded across the n regions. Absence of transport costs implies that its price is the same everywhere ($p_1^A = p_2^A = ... = p_n^A$). Furthermore, under marginal cost pricing: $p_i^A = w_i^L$. Consequently, there is farmer's wage equalization among regions, $w_1^L = w_2^L = ... = w_n^L$. By choosing the agricultural good as numeraire, we can set $p_i^A = w_i^L = 1, \forall i$. We are assuming that the agricultural good is produced in all regions. This holds if global expenditure on agricultural goods exceeds the total production of A_i in $n-1$ regions. Given regional demand in (4.4), global expenditure on agricultural goods is given by $\bar{w}H + L - (H+L)\,\mu$, where:

$$\bar{w} = \sum_{j=1}^{n} h_j w_j, \tag{4.6}$$

[7] For the particular distributions $H_i = H, H_j = 0 \forall j \neq i$, and $H_i = H/n, \forall i \in N$, the nominal wage is $w_i = \frac{\mu}{\sigma}(1 + \frac{L}{H})$ (see equation (4.13) in section 2.3). For these distributions, a necessary and sufficient condition is $\frac{L}{H} > \sigma - 1$, which we assume to hold. For the minimum wage at any spatial distribution, a sufficient condition is given by $\lambda > n(\sigma - 1)$ (the proof is extensive and we omit it for the sake of space). We do not assume the latter because it is too restrictive and mostly not necessary for our results.

stands for the weighted average nominal wage paid to entrepreneurs. Total production of agricultural goods in $n-1$ regions is given by $L(n-1)/n$. The non-full-specialization (NFS) condition (Baldwin *et al.*, 2004) is then given by:

$$\mu < \frac{\bar{w} + \lambda/n}{1+\lambda}, \tag{4.7}$$

where $\lambda = L/H$ where $\lambda = L/H$ is the global unskilled (immobile) to skilled (mobile) labour ratio. As such, a higher λ implies a lower worker mobility. We will later show that $\bar{w} = \frac{\mu}{\sigma}(1+\lambda)$. The NFS condition (4.7) then becomes:

$$\mu < \frac{\lambda\sigma}{n(\lambda+1)(\sigma-1)}.$$

which we assume to hold henceforth.

Production of a variety of manufactures requires α units of skilled labour and β units of unskilled labour for each unit that is produced (Forslid and Ottaviano, 2003). Therefore, the production cost of a firm in region i is:

$$C_i(x_i) = w_i\alpha + \beta x_i, \tag{4.8}$$

where w_i is the nominal wage of skilled workers in region i. Let the iceberg cost parameterτ_{ij} denote the number of units that must be produced in region i for each unit that is delivered at region $j \in N$. We assume that transportation costs are equal between any two (distinct) regions. If $i = j$, then $\tau_{ij} = 1$. If $i \neq j$, then $\tau_{ij} = \tau \in (1, +\infty)$.

In the industrial sector, the number of varieties manufactured in region i is $S_i = H_i/\alpha$. A manufacturing firm in region i facing the total cost in (4.8) maximizes the following profit function:

$$\pi_i(s) = \sum_{j=1}^{n} d_{ij}(s)\left(H_j + \frac{L}{n}\right)(p_{ij}(s) - \beta) - \alpha w_i, \tag{4.9}$$

The first order condition for maximization of (4.9) yields the following pricing equation:

$$p_{ii}(s) = \beta\frac{\sigma}{\sigma-1} \quad \text{and} \quad p_{ij}(s) = \tau\beta\frac{\sigma}{\sigma-1}, \; j \neq i \tag{4.10}$$

Using (4.10), the CES price index (4.3) becomes:

$$P_i = \beta \frac{\sigma}{\sigma - 1} \left(\sum_{j=1}^{n} \phi_{ij} S_j \right)^{\frac{1}{1-\sigma}}, \tag{4.11}$$

where $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in (0, 1]$ represents the "freeness of trade" between regions i and j. Note that when $i = j$ then $\phi_{ij} = 1$.

4.2.3 Short-run equilibrium

Free entry in the manufacturing industry implies zero profit in equilibrium. As such, operating profits must totally compensate fixed costs, which must equal the wage bill paid to entrepreneurs:

$$\alpha w_i = \sum_{j=1}^{n} d_{ij} \left(H_j + \frac{L}{n} \right) (p_{ij} - \beta),$$

which becomes, considering the prices in (4.10):

$$w_i = \frac{\beta \sum_{j=1}^{n} d_{ij}(s) \left(H_j + L/n \right)}{\alpha(\sigma - 1)}, \tag{4.12}$$

Using (4.4), (4.10) and (4.11), and replacing in (4.12) we end up with:

$$w_i = \frac{\mu}{\sigma} \sum_{j=1}^{n} \frac{\phi_{ij} \left(L/n + H_j \right)}{\sum_{m=1}^{n} \phi_{mj} H_m}. \tag{4.13}$$

In the original Footloose Entrepreneur model (Forslid and Ottaviano, 2003) with income effects, the term inside the first-tier summation would read $\phi_{ij} Y_j$, where $Y_j = L/n + w_j H_j$ would correspond to the regional income that depends on the entrepreneurs nominal wage. The simplification due to Pflüger (2004) imposed on U_i removes all income effects on the demand for manufactures, implying that the entrepreneur's compensation does not depend on the nominal wage of entrepreneurs residing in different regions.

We describe the spatial distribution of industry in the economy by working with the share of entrepreneurs residing in each region i, i.e., $h_i = H_i/H$. The set of possible spatial distributions is the $(n-1)$-dimensional simplex defined by $\triangle = \left\{ \mathbf{h} \in \mathbb{R}_+^n : \sum_{i=1}^{n} h_i = 1 \right\}$.

In terms of h_i, the nominal wage in region i can be expressed as:

$$w_i = \frac{\mu}{\sigma} \sum_{j=1}^{n} \frac{\phi_{ij} \left(\lambda/n + h_j\right)}{\sum_{m=1}^{n} \phi_{mj} h_m}. \tag{4.14}$$

The price index may be reformulated as:

$$P_i = \beta \frac{\sigma}{1-\sigma} \left(\frac{H}{\alpha} \sum_{j=1}^{n} \phi_{ij} h_j \right)^{\frac{1}{1-\sigma}}. \tag{4.15}$$

The indirect utility V_i becomes, after replacing (4.15) and (4.14) in (4.5):[8]

$$V_i = \frac{\mu}{\sigma} \sum_{j=1}^{n} \frac{\phi_{ij} \left(\lambda/n + h_j\right)}{\sum_{m=1}^{n} \phi_{mj} h_m} + \frac{\mu}{\sigma - 1} \ln \left(\sum_{j=1}^{n} \phi_{ij} h_j \right) + \eta, \tag{4.16}$$

where $\eta = \mu \left(\ln \mu - 1\right) - \mu(1-\sigma)^{-1} \ln \left[\beta(\sigma-1)^{-1} H/\alpha\right]$ is a constant.

Proposition 4.1. *The weighted average nominal wage paid to entrepreneurs at any spatial distribution is given by:*

$$\bar{w} = \frac{\mu}{\sigma}(1+\lambda). \tag{4.17}$$

Proof. See Appendix A. □

4.3 Long-run equilibria

NEG frequently borrows the evolutionary dynamics from evolutionary game theory used to study the evolution of states in population games. The popular *replicator dynamics* are generally well suited to describe the migration of entrepreneurs when they are short-sighted (Baldwin *et al.*, 2004). The rate of change of the share of entrepreneurs in a region i is assumed to be proportional to the difference between region i's indirect utility, V_i, and the weighted average utility across all regions, $\bar{V}(h) = \sum_{j=1}^{n} h_j V_j(h)$. The dynamical system is thus given by:

$$\dot{h}_i = h_i \left[V_i(h) - \bar{V}(h) \right], \tag{4.18}$$

[8] For an entrepreneur, we have $Y_i = w_i$.

where $i \in N \setminus \{n\}$.[9]

A spatial distribution of entrepreneurs h is said to be an equilibrium if it satisfies $\dot{h}_i = 0, \forall i \in N$. The boundaries of $\triangle$ are invariant for the dynamics. As a result, if a region is initially empty of entrepreneurs, it will remain so unless some exogenous migration to that region occurs.[10] Second, every spatial distribution h such that $h_i = 1/k$ for $k \in N$ regions is an equilibrium.

Our first goal is to study the local stability of equilibria. First, we study full *agglomeration* in a single region i ($h_i = 1$). We then study *symmetric dispersion*, where the entrepreneurs are evenly distributed among the n regions ($h_i = 1/n, \forall i$). We also study equilibria whereby entrepreneurs are evenly dispersed among fewer regions. Such configurations are given by $h_i = 0$ for at least one region i and $h_j = 1/k$ for $k \in \{2, ..., n-1\}$. We refer to each one of these as *boundary dispersion*, since $h_i = 0$ for at least one i implies that the equilibrium is placed at the boundary of the simplex $\triangle$.

The last class of equilibria we study throughout this paper are particular asymmetric interior configurations whereby one region has a share of entrepreneurs $h \in (0, 1)$ and the other $n - 1$ regions are evenly populated with shares $(1 - h)/(n - 1)$. We call this type of equilibrium *partial agglomeration*.

4.3.1 Agglomeration

At agglomeration, since the core region i has all the industry, the weighted average utility equals $\bar{V} = V_i$. Agglomeration is thus stable if $V_j < \bar{V}, \forall j \in N \setminus \{i\}$. The following proposition gives the condition for local stability in parameter space.

Proposition 4.2. *Agglomeration is stable if:*

$$\frac{(1-\phi)\left[\lambda(1-\phi) - n\phi\right]}{n\sigma\phi} + \frac{\ln\phi}{\sigma - 1} < 0. \tag{4.19}$$

Proof. See Appendix B. □

[9] The dynamics of the n^{th} region are residually given by $\dot{h}_n = -\left(\sum_{j=1}^{n-1} \dot{h}_j\right)$.

[10] This might seem unreasonable if an "empty" region has a positive utility differential. However, the *replicator dynamics* are generally used to capture the effect of migration driven by imitation, so one could simply assume that any entrepreneur would be reluctant to be the first to migrate.

Let us the call *sustain point* ϕ_s to the level of ϕ above which agglomeration is stable. Rewriting (4.19) in terms of ϕ_s, we get:

$$\ln(\phi_s) = -\frac{(\sigma-1)(1-\phi)\left[\lambda(1-\phi)-n\phi\right]}{n\sigma\phi}, \tag{4.20}$$

and agglomeration is stable if $\phi > \phi_s$. The RHS of (4.20) must be negative, otherwise agglomeration is always stable and condition (4.19) becomes trivial. To rule this out, we need only consider $\lambda > n\phi/(1-\phi)$. It shall be useful for Section 4 to have a stability condition in terms of λ . Rewriting (4.19), we get:

$$\lambda < \lambda_s \equiv \frac{n\phi\left[(\sigma-1)(1-\phi)-\sigma\ln\phi\right]}{(\sigma-1)(1-\phi)^2}. \tag{4.21}$$

That is, agglomeration is stable if the worker mobility is high enough.

4.3.2 Symmetric dispersion

The following proposition establishes a sufficient condition for local stability of dispersion of entrepreneurs.

Proposition 4.3. *Symmetric dispersion is stable if:*

$$\phi < \phi_b \equiv \frac{\sigma(1-\lambda)+\lambda}{\lambda+n-\sigma\left(\lambda+2n-1\right)}. \tag{4.22}$$

Proof. See Appendix B. □

The RHS of (4.22) is called the *break point,* ϕ_b. Substituting $n = 2$ we recover the corresponding break point of Pflüger's (2004) original model.[11] If $\phi_b < 0$, the symmetric equilibrium can never be stable. Therefore, a common assumption in NEG literature is that $\phi_b > 0$ so that stability of symmetric dispersion cannot be precluded. This sufficient condition is called *no black hole* and is equivalent to:

$$\lambda > \frac{\sigma}{\sigma-1}. \tag{4.23}$$

[11] Which is $\phi_b = \frac{-2\rho\sigma+2\rho+\sigma}{-2\rho\sigma+2\rho-3\sigma+2}$, with $\rho = \frac{\lambda}{2}$.

Since we assume that consumption of the agricultural good is positive at total dispersion, it then follows that $\lambda > \max\left\{\sigma - 1, \frac{\sigma}{\sigma-1}\right\}$.[12]

Rewriting the stability condition in (4.22) in terms of λ, we get:

$$\lambda > \lambda_b \equiv \frac{\sigma(2n\phi + 1 - \phi) - n\phi}{(\sigma - 1)(1 - \phi)}. \tag{4.24}$$

Therefore, stability of dispersion requires a sufficiently high worker mobility.

4.3.3 Boundary dispersion

At boundary dispersion, entrepreneurs are evenly distributed among $k \in \{2, ..., n-1\}$ regions. Without loss of generality, suppose that $h_j = 0$ for $j = 1, ..., n-k$ and $h_j = \frac{1}{k}$ for $j = n-k+1, ..., n$. Note that these configurations are particular of models with more than 2 regions. The next result shows that entrepreneurs cannot disperse evenly along regions while other regions remain empty.

Theorem 4.4. *Boundary dispersion is always unstable.*

Proof. See Appendix B. □

Theorem 4.4 provides analytical confirmation of the numerical evidence found in Fujita *et al.* (1999), Castro *et al.* (2012), Gaspar *et al.* (2013). In their works, the authors used a Core-Periphery model with 3 (and more) regions to provide numerical evidence in that border dispersion (resp. partial dispersion) in two regions is always unstable.[13] In the present paper, we further add to the conjecture that industry cannot disperse evenly among regions if there is at least another region with no industry.

4.3.4 Partial agglomeration

4.3.4.1 Existence of partial agglomeration equilibria

In an n-region model, there potentially exist several different kinds of asymmetric equilibria. For simplicity, we shall restrict our analysis to a one-dimensional subspace of $\triangle$,

[12] The no black hole condition is satisfied by $\lambda > \sigma - 1$ if$\sigma > \frac{1}{2}(3+\sqrt{5})$ and thus becomes redundant.

[13] The first two used the original CP model by Krugman (1991), whereas the latter two used the FE model by Forslid and Ottaviano (2003).

defined by $\triangle_{inv} = \left\{\mathbf{h} \in \triangle : h_j = \frac{1-h_1}{n-1}, \forall j \neq 1\right\}$, which is invariant for the dynamics.[14] Note that this is a particular case of a more general one-dimensional subspace whereby k regions have a share of entrepreneurs h_i/k and the other $n-k$ regions have a share $(1-h)/(n-k)$. We are focusing on the case where $k=1$ (or, equivalently by symmetry, $k=n-1$), i.e., only one region differs from the others in its industry size. Note that agglomeration and dispersion are patterns in $\triangle_{inv}$. The next proposition determines the maximum number of equilibria in $\triangle_{inv}$.

Proposition 4.5. *There can be at most two interior asymmetric equilibria for* $h \in (0,1)$*. However, at most one interior equilibrium exists for* $h \in (0,1/n)$*, whereas at most two can exist for* $h \in (1/n,1)$*. These equilibria exist if and only if:*

$$\lambda = \lambda^*(h) \equiv \frac{n(\sigma-1)(1-\phi)\phi(hn-1) - n\sigma\left[h(1-\phi)-\phi\right]\left[\phi(h+n-2)-h+1\right]\ln\left\{\frac{\phi(h+n-2)-h+1}{(n-1)[h(1-\phi)+\phi]}\right\}}{(\sigma-1)(1-\phi)^2(hn-1)}. \tag{4.25}$$

Proof. See Appendix B. □

Equation (4.25) expresses the value of λ for which a spatial distribution $h \in (0,1) \setminus \left\{\frac{1}{n}\right\}$ is an interior equilibrium. Note that $h \in (1/n,1)$ means that region 1 has more industry than the remaining regions and we thus refer to it as a *partial core*.

We illustrate in Figure 4.1 the multiplicity of partial agglomeration, drawing $\lambda^*(h)$ for $n=3$ and $\sigma=4$, for two different cases.

[14] All 1-dimensional spaces of the form $h_j = (1-h_i)/(n-1)$, $i \neq j$, for $i \in N$, are invariant.

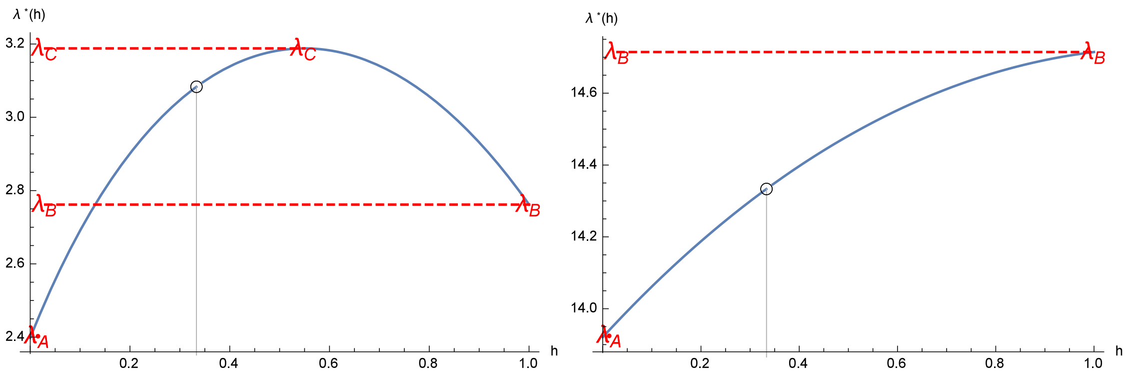

Figure 4.1 – Illustration of $\lambda^*(h)$ for $n = 3$. On the vertical axis we present values of λ such that $h \in (0, 1) \setminus \{1/3\}$ is an interior equilibrium. To the left, we have $\phi = 0.2$ and to the right we have $\phi = 0.65$. The equilibria occur at the intersection of a horizontal line (λ constant) with the lines depicted in each figure.

In the first case (picture to the left), ϕ is low. For $\lambda_A < \lambda < \lambda_B$, there is only one equilibrium: $h < 1/3$. For $\lambda_B < \lambda < \lambda_C$, there are two interior asymmetric equilibria. In this region, at least one of these equilibria has a partial core. If λ is relatively high, both equilibria are characterized by a partial core. If $\lambda > \lambda_C$, there are no equilibria. In the second case (picture to the right), there is at most one interior asymmetric equilibrium. The higher is λ, the more likely it is that the interior equilibrium corresponds a partial core.

4.3.4.2 Stability of partial agglomeration

At a partial agglomeration equilibrium, two types of migrations can take place. One migration concerns movements along the invariant space $\triangle_{inv}$, i.e., from region 1 to the evenly populated regions. If $n - 1$ entrepreneurs leave region 1, each of the other regions will get 1 of those entrepreneurs. Since, along the invariant space, regions $\{2, ..., n\}$ share the same size, the decisions of these $n - 1$ entrepreneurs are the same. The other migration concerns that of an entrepreneur who chooses to move exogenously between two regions other than region 1 (transversally to $\triangle_{inv}$). We have the following result.

Theorem 4.6. *Partial agglomeration is: (i) unstable for $h \in (0, 1/n)$; and (ii) stable if $h \in (1/n, 1)$ and:*

$$\delta \equiv (1 - \phi)(hn - 1)\left[(n - 1)\phi + 1\right] + \Phi\nu < 0, \tag{4.26}$$

where $\Phi = h^2 n(1-\phi)^2 - 2h(1-\phi)^2 + \phi\left\{n\left[(n-3)\phi+2\right]+3\phi-4\right\}+1$ *and* ν *is the log term in (4.25).*

Proof. See Appendix B. □

Partial agglomeration can only be stable if one region is a partial core. Its stability depends solely on the effect of migration between a partial core and any of the other regions on utilities, i.e., δ.

It is important to detail further the economic intuition that leads to part (i) of Theorem 4.6. First, a small exogenous migration from region n to region $j \neq 1$ decreases the nominal wage in that region:

$$\frac{\partial w_j}{\partial h_j}\left(h, \frac{1-h}{n-1}\right)\bigg|_{\lambda=\lambda^*(h)} = \frac{(n-1)^2\sigma(h(1-\phi)+\phi)\nu}{(\sigma-1)(hn-1)\left[\phi(h+n-2)-h+1\right]},$$

which is negative for $h \in (0, 1/n)$. This decrease happens because there is a shift in the demand for manufactures produced in region j towards manufactures produced in region n. On the other hand, there is a positive cost-of-living effect in region j due to immigration, which increases the utility of entrepreneurs residing in region j. This is captured by differentiating the second term of V_j using (4.16) and evaluating at partial agglomeration, which gives:

$$\frac{\mu(n-1)(1-\phi)}{(\sigma-1)\left[\phi(h+n-2)-h+1\right]} > 0. \tag{4.27}$$

When $h < 1/n$, the agglomerative cost-of-living effect due to an exogenous migration in the receiving region always outweighs the dispersive decrease in nominal wage. The receiving region, having become the largest after migration, will then further develop into an industrialized core. This explains the fact that it is not possible for industry to spread evenly among $n-1$ regions when the other region is less industrialized.
Inspection of δ shows that partial agglomeration is unstable in an open interval $h \in (\frac{1}{n}, \frac{1}{n}+\varepsilon)$ for anyϕ.[15] That is, stability of partial agglomeration requires a significantly industrialized partial core. Moreover, any partial agglomeration h is unstable for $\phi \in$

[15]This stems from the fact that $\delta(h=1/n)=0$, $\frac{\partial\delta}{\partial h}(h=1/n)=0$ and $\frac{\partial^2\delta}{\partial h^2}(h=1/n)>0$.

$(1-\varepsilon, 1)$.[16] In order to convey a better picture, we illustrate in Figure 4.2 the region $\delta < 0$ in (h, n) space, for $n = \{3, 5, 10\}$.

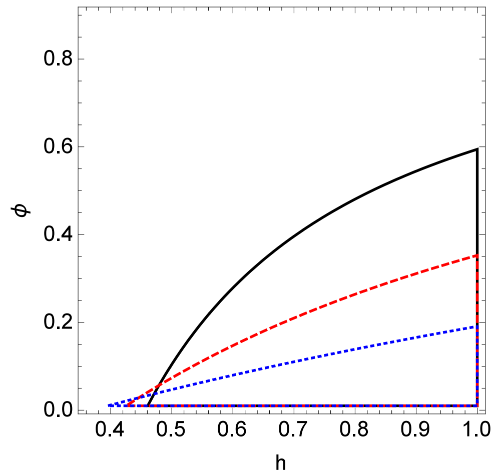

Figure 4.2 – Regions of stability $\delta_n < 0$ in (h, ϕ) space, for $n = \{3, 5, 10\}$. The region below the solid line corresponds to $\delta_3 < 0$; below the dashed line we have $\delta_5 < 0$; the dotted line contain $\delta_{10} < 0$.

The numerical evidence suggests that, with more regions, partial agglomeration requires higher barriers to trade (lower ϕ). With more regions, some partial agglomeration equilibria may arise with less industry in the partial core because total dispersion implies a lower share of entrepreneurs in each region ($h = \frac{1}{n}$ is decreasing in n). Finally, Figure 4.2 shows that partial agglomeration $h \in (1/n, 1)$ is stable if the freeness of trade is low enough. The more industrialized is the partial core, the higher is the range of freeness of trade for which partial agglomeration is a stable equilibrium.

4.4 Bifurcations in the n-region model

Most 2-region NEG models under exogenous symmetry undergo pitchfork bifurcations at the symmetric dispersion when the parameter path concerns smooth changes only in transportation costs. Pflüger and Südekum (2008a) have shown that changes in the functional form of the utility function produce modifications in the qualitative

[16]This is a consequence of $\delta(\phi = 1) = 0$, $\frac{\partial \delta}{\partial \phi}(\phi) = 0$ and $\frac{\partial^2 \delta}{\partial \phi^2}(\phi = 1) > 0$.

structure of a class of footloose-entrepreneur models. However, these changes affect the type of pitchfork bifurcation rather than the type of bifurcation itself.[17] For instance, the 2-region QLLog (Pflüger, 2004) undergoes a supercritical pitchfork bifurcation at symmetric dispersion. We next show that when more than 2 regions come to interplay, the QLLog model exhibits instead a primary transcritical bifurcation at the symmetric equilibrium and a secondary saddle-node bifurcation that branches from an interior asymmetric equilibrium. A distinctive feature of these bifurcations is that the qualitative change in spatial distributions as transport costs decrease here is not reflected symmetrically across all regions, even though the model is completely symmetric in all respects. This suggests that the role of transport costs in an equidistant multi-regional model may be more complex than what is envisaged in most agglomeration models.[18]

The remainder of the section is dedicated to explaining the details of these bifurcations and their consequences on the qualitative transition towards more agglomerated distributions as trade integration increases.

4.4.1 Primary and secondary bifurcations

We have the following results.

Proposition 4.7. *In the n-region ($n \geq 3$) QLLog model symmetric dispersion undergoes a transcritical bifurcation at the break point.*

Proof. See Appendix C. □

From this result, symmetric dispersion looses stability as ϕ rises above ϕ_b. A primary branch of partial agglomeration equilibria crosses symmetric dispersion at the bifurcation point $\phi = \phi_b$. For $\phi < \phi_b$, this branch lies in the region $h \in (1/n, 1)$. For $\phi > \phi_b$, the branch lies in the region $h \in (0, 1/n)$. Locally, both before and after the bifurcation occurs, the partial agglomeration equilibria along the primary branch are

[17] In the model of Ottaviano *et al.* (2002), the resulting bifurcation is a borderline case between a supercritical and subcritical pitchfork.

[18] Bifurcation in core-periphery models has been addressed by Berliant and Kung (2009) in a different context. The variety of bifurcations is obtained through the addition of parameters to the original model.

unstable in a neighbourhood of the bifurcation point.[19]

In order to understand the behaviour of partial agglomeration equilibria on the entire invariant space Δ_{inv} for $h \in (1/n, 1)$, we take a further step by verifying the conditions for a secondary bifurcation along the primary branch that occurs at a *fold point,* ϕ_f. Because we are looking at the invariant subspace $(h_1, h_j(h_i)) = (h, (1-h)/(1-n))$, the study of bifurcations at the symmetric dispersion equilibrium in this restriction of the n-region model is reduced to a one dimensional case.

Proposition 4.8. *Along the primary branch for $h \in (1/n, 1)$, the n-region QLLog model undergoes a saddle-node bifurcation as ϕ decreases from ϕ_b.*

Proof. See Appendix C. □

The existence of a secondary saddle-node bifurcation for $h \in (1/n, 1)$, together with the direction of the transcritical bifurcation and stability of its branches, ensures that $\phi_f < \phi_b$ and $\phi_f < \phi_s$. As ϕ increases above ϕ_f, two partial agglomeration equilibria appear. The one with the more industrialized partial core is stable, whereas the other one is unstable. In other words, the saddle-node bifurcation is characterized by a curve of partial agglomeration equilibria along a primary branch for $h \in (1/n, 1)$ that is tangent to the line $\phi = \phi_f$ and lies to its right.[20]

One important question concerns whether $\phi_b < \phi_s$ or $\phi_b > \phi_s$. The relative position of these thresholds determines the smoothness of the progressive industrialization process as the freeness of trade increases. On the other hand, the qualitative structure of the model ensures that there always exists a stable spatial distribution with $h \in [1/n, 1]$, for every freeness of trade value $\phi \in (0, 1)$.

We first focus on the case $\phi_b < \phi_s$, which requires a relatively low λ. Figure 4.3 depicts the QLLog model's bifurcation diagram in the invariant space $\triangle_{inv}$ for $n = 3$. For the illustrations we set $\mu = 0.3$, $\sigma = 4$ and $\lambda = 2.5$.[21] The interpretation strays from that of the typical bifurcation diagrams of 2-region models in NEG literature,

[19] The branch for $h \in (0, 1/n)$ is stable (only) along the invariant space. From Theorem 4.6, however, it is unstable.

[20] The details that support these claims about the bifurcations are provided by the derivative in (T3), (T4), and (SN3) in Appendix C.

[21] The parameter values chosen for the simulations ensure that $w_i, w_j > \mu$ at every partial agglomeration equilibrium, so that entrepreneurs consume both goods at every possible distribution.

because we are restraining ourselves to the invariant subspace $\triangle_{inv} \subset \triangle$. Just like the simplex in the 2-region model, the subspace is also one dimensional here and migration movements that affect region 1 will affect all other regions. A migration from (to) region 1 would result in $2x$ entrepreneurs leaving (entering) region 1 for x entrepreneurs that enter (leave) each of the other 2 regions.[22]

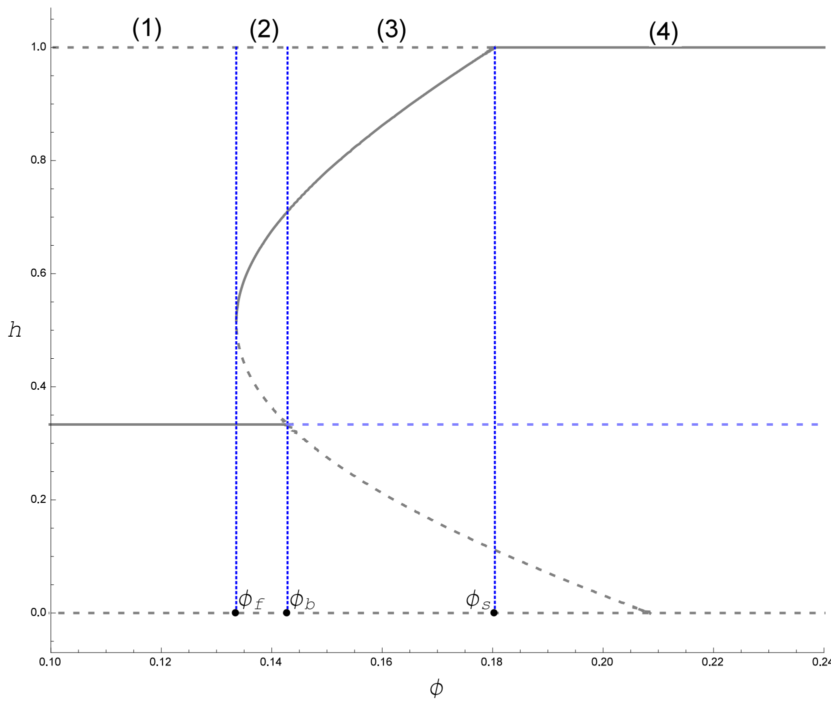

Figure 4.3 – Bifurcation diagram for the 3-region model. Solid and dashed lines represent stable and unstable equilibria, respectively. Vertical dashed lines delimit areas (1) to (4) as follows: (1) stability of symmetric dispersion; (2) stability of both dispersion and partial agglomeration; (3) stability of partial agglomeration; (4) stability of full agglomeration.

It thus comes as no surprise to see that $h = 0$ corresponds to a qualitatively different spatial distribution when $n = 2$ and when $n \geq 3$. When $n = 2$, it corresponds to agglomeration. When $n \geq 3$, it corresponds to border dispersion, which is never stable.

In Figure 4.3 we can see the primary transcritical bifurcation branching from the break point ϕ_b and the secondary saddle-node bifurcation occurring along the primary

[22] In a 3-region model, along the invariant space, region 1 has $h_1 = h$ entrepreneurs and regions 2 and 3 have $h_2 = h_3 = (1 - h)/2$.

branch, whose fold point, ϕ_f, is located in the upper part of the invariant subspace. Notice that an initially partially agglomerated industry will be forced to fully disperse across regions if there is a decrease in the freeness of trade below ϕ_f. If this decrease is temporary, the industry will remain fully dispersed, so there are permanent effects, which means that the n-region QLLog model exhibits locational hysteresis. Formally, this happens because $\phi_f < \phi_b$.

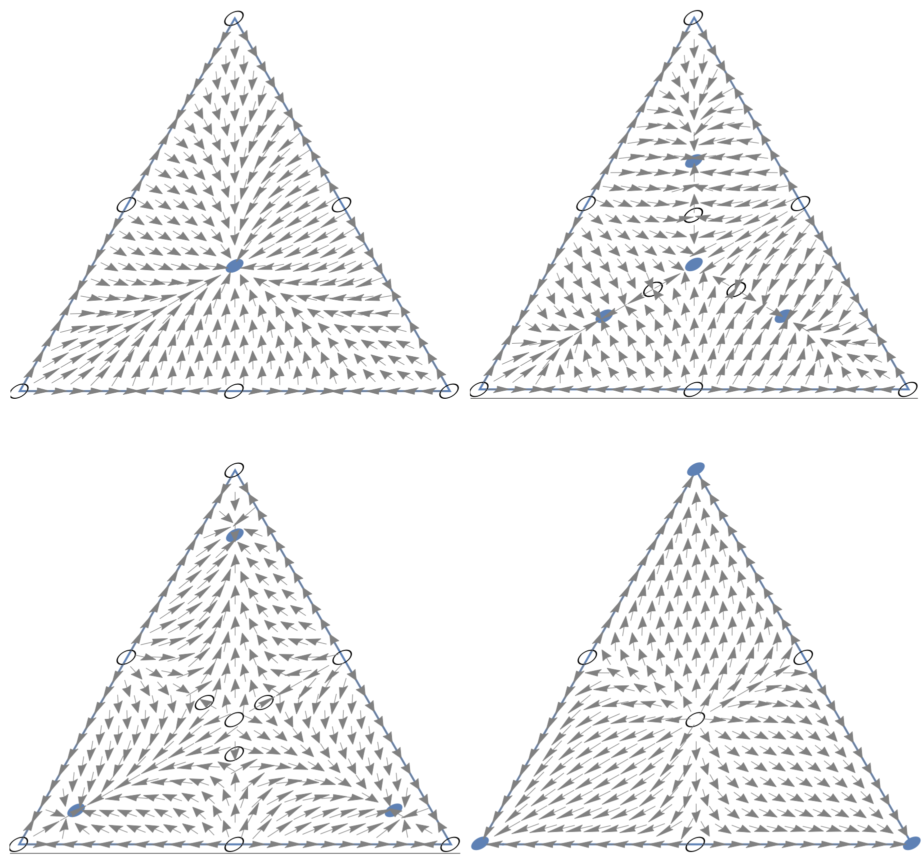

Figure 4.4 – Migration dynamics of entrepreneurs inside the 2-simplex. From left to right to bottom ϕ, each picture corresponds to a region in Figure 4.3 indexed from (1) to (4). In the first picture, only symmetric dispersion is stable. In the second, both partial agglomeration and symmetric dispersion are stable. In the third picture only partial agglomeration is stable. In the last picture, agglomeration is the only stable equilibrium.

Figure 4.4 contains the different dynamics of the 3-region model inside the 2-dimensional simplex, with correspondence to the regions (1)-(4) of Figure 4.3. We start with low levels of ϕ. When the freeness of trade is very low, we have $\phi < \phi_f < \phi_b < \phi_s$, so only dispersion is stable (upper left picture). However, once $\phi_f < \phi < \phi_b < \phi_s$, two partial agglomeration equilibria immediately arise on the upper part (for $h \in (1/3, 1)$) of the invariant space $\triangle_{inv}$ of the simplex (upper left picture). The one that lies closer to agglomeration is the only stable equilibrium. In the bottom left picture, ϕ has risen just above the break point but lies just below the sustain point, i.e., $\phi_f < \phi_b < \phi < \phi_s$. At this point there are still two partial agglomeration equilibria; however, one of them lies in the inferior part of the invariant space $\triangle_{inv}$ and is unstable whereas the other is stable. As ϕ approaches the sustain point, the stable partial agglomeration equilibrium approaches agglomeration until it disappears once $\phi > \phi_s$, after which agglomeration becomes the only stable spatial distribution.

Intuitively, the absence of income effects mitigates agglomeration forces as transport costs decrease, which justifies the existence of partial agglomeration just as in the 2-region model. On the other hand, higher market access variability due to the presence of more regions implies that lower transport costs enhance the relative strength between centripetal and centrifugal forces, resulting in a spatial distribution where one region is considerably more industrialized. Further decreases in transport costs then lead to a progressive and smooth transition towards full agglomeration.

We now focus on the case whereby $\phi_b > \phi_s$. Note that, in Pflüger's 2-region QLLog model, the existence of a supercritical bifurcation at the break point ϕ_b precludes the converse case, i.e., $\phi_b \geq \phi_s$. For $n \geq 3$, however, we have the following result.

Proposition 4.9. *There exists a $\lambda \in (\lambda_b, \lambda_s)$ such that agglomeration and dispersion are both stable if:*

$$n > n_T \equiv -\frac{(1-\phi)^2}{\phi(1-\phi+\ln\phi)}, \tag{4.28}$$

Proof. See Appendix C. $\square$

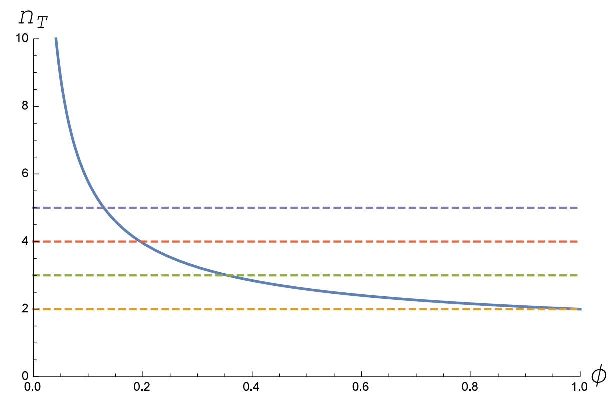

Figure 4.5 – Threshold n_T in (4.28). Agglomeration and dispersion are simultaneously stable in the graph above the thick line. For $n \geq 3$, simultaneity requires higher transport costs for a higher number of regions.

Figure 4.5 illustrates n_T from (4.28) in ϕ space. It shows that simultaneity of stability of agglomeration and dispersion is not possible in the 2-region model. It also shows that an $(n+1)$-region model favours simultaneity of agglomeration and dispersion for a wider range of transport cost values compared to an n-region model.

Corollary 4.10. *For a sufficiently high λ, there exists a range of transport cost values for which agglomeration and dispersion are both stable.*

Proof. If condition (4.28) holds, then, for some λ, agglomeration and dispersion are simultaneously stable. Since stability of both equilibria requires $\phi > \phi_s$ and $\phi < \phi_b$, then it follows that, for some λ, we must forcibly have $\phi_b \geq \phi_s$. Moreover, we have $d(\phi_b - \phi_s)/d\lambda > 0$, which means that $\phi_b - \phi_s > 0$ occurs for a sufficiently high λ. $\square$

We now proceed to illustrate the qualitative structure of spatial distributions when $\phi_s < \phi_b$. We increase the ratio of immobile to mobile workers compared to the previous simulations, by setting $\lambda = 6$. The resulting bifurcation diagram is now presented in Figure 4.6. Clearly, the main qualitative difference compared to Figure 4.3 pertains to the region indexed by (3), where we now have $\phi \in (\phi_s, \phi_b)$.

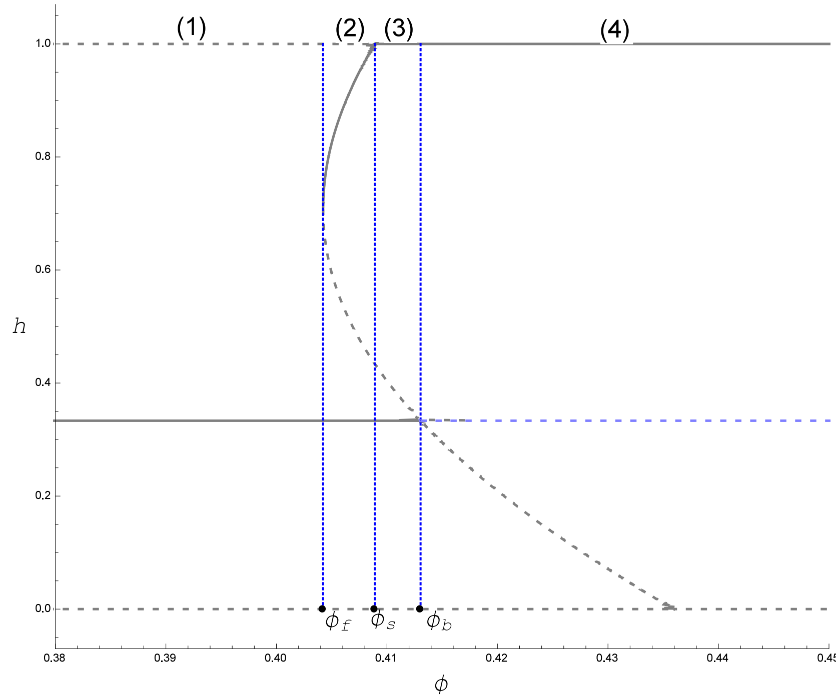

Figure 4.6 – Bifurcation diagram for the 3-region model. Solid and dashed lines represent stable and unstable equilibria, respectively. Vertical dashed lines delimit areas (1) to (4) as follows: (1) stability of symmetric dispersion; (2) stability of both dispersion and partial agglomeration; (3) stability of both dispersion and full agglomeration; (4) stability of full agglomeration. Parameter values are $\sigma = 5$ and $\lambda = 6$.

Figure 4.7 portrays the 2-dimensional simplex for $\phi_b > \phi_s$ once ϕ rises above the sustain point (corresponding to region (3) in Figure 4.6).

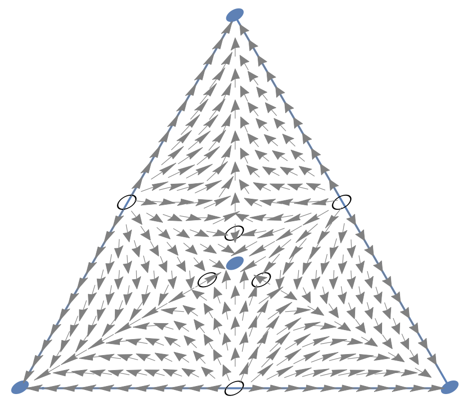

Figure 4.7 – A 2-simplex portraying for $\phi_s < \phi < \phi_b$. We can see that agglomeration and symmetric dispersion are both stable. A single unstable partial agglomeration equilibrium exists in the upper part of $\triangle_{inv}$.

We can observe that for $\phi_s < \phi < \phi_b$, agglomeration and total dispersion are both simultaneously stable, while a single unstable partial agglomeration equilibrium for $h \in (1/3, 1)$ exists in $\triangle_{inv}$ between them.

We can now sum up the implications of the relative position between the break point and sustain point are as follows. If $\phi_b < \phi_s$, once symmetric dispersion looses stability, a significant migration will occur to a partially agglomerated equilibrium. This migration will be followed by a smooth transition towards agglomeration if ϕ rises further. This scenario is illustrated in figures 4.3 and 4.4. Conversely, if $\phi_b > \phi_s$, entrepreneurs will immediately agglomerate in one single region once dispersion becomes unstable. Moreover, if the increase in ϕ above ϕ_b is due to some temporary policy, agglomeration is permanent. This is depicted in Figure 4.6. Smoother transitions require a lower immobile to mobile labour ratio λ, whereas catastrophic agglomeration is more likely under higher values of λ.[23]

In economic terms, when overall inter-regional mobility is low, dispersion forces are higher because firms have larger incentives to relocate to less industrialized regions in order to capture local demand and avoid fiercer competition in more crowded markets. However, dispersion forces due to existence of immobile workers in other markets are

[23]Our results can be shown to extend to a fairly general range of values for λ.

naturally stronger at more symmetric distributions. The implication is that a lower global inter-regional mobility encourages symmetric dispersion more than it discourages full agglomeration. When it is too low, if agglomeration forces exceed dispersion forces at symmetric dispersion, the spatial distribution then immediately shifts towards full agglomeration.

4.4.2 A note on the black hole condition and on the role of inter-regional mobility

We know that there is a condition on which stability of total dispersion hinges crucially; the no black hole condition. In early NEG literature, the no black hole condition may have been a requirement assumed out of necessity or convenience; after all, in a fully symmetric two region, two industry setting, precluding dispersion would doom geography to the unlikely prediction of full agglomeration in one region. In our context, the no black hole condition would seem *ad hoc* to say the least.

Figure 4.8 shows a bifurcation diagram where the ratio of inter-regionally immobile workforce relative to mobile (skilled) labour is lower than unity and, as such, total dispersion is precluded. The results concerning bifurcations in the previous section do not extend to this case since we know that $\phi_f < \phi_b$. In fact, as the freeness of trade increases, the spatial distribution approaches agglomeration monotonically from an interior asymmetric equilibrium along the invariant space $\triangle_{inv}$.

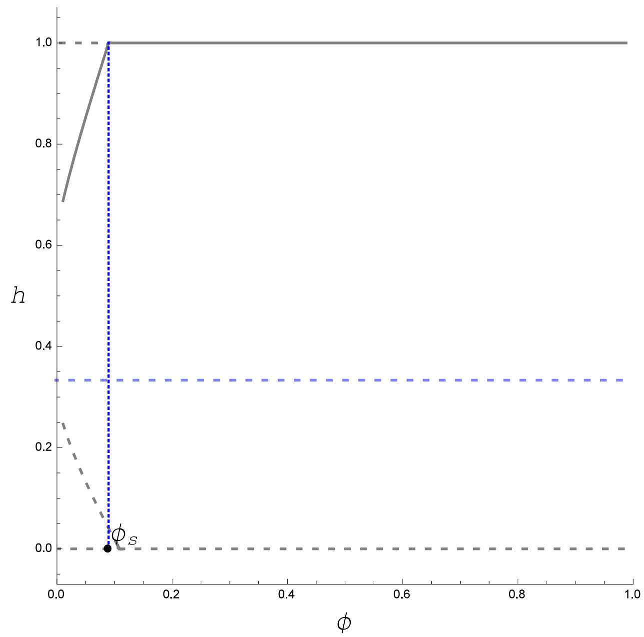

Figure 4.8 – If the unskilled labour force is relatively low, dispersion is not a stable outcome, but partial agglomeration is still possible. We have $\sigma = 2.5$ and $\lambda = 1.6$.

On account of these findings, we conclude that a higher global inter-regional worker mobility (lower λ) leads to smoother transitions towards agglomeration as transport costs steadily decrease.

4.5 Welfare

So far we have analysed the different possible spatial distributions in the the QLLog model with an arbitrary number of equidistant regions. We now analyse whether the possible distributions are desirable from a social perspective. For normative purposes, we adopt a utilitarian criterion similar to that of Pflüger and Südekum (2008b), thus looking at the average indirect utility of entrepreneurs, farmers, and then at the whole economy.

4.5.1 Entrepreneurs

We now discuss how distribution patterns relate to the entrepreneurs' overall average welfare.

Theorem 4.11. *The average utility of entrepreneurs is convex in the spatial distribution of entrepreneurs h, attaining a global minimum at symmetric dispersion.*

Proof. See Appendix D. □

Theorem 4.11 shows that total dispersion is the spatial distribution that yields the worst outcome for the entrepreneurs as measured by average utility. To understand why entrepreneurs may be driven to a situation that reduces their overall welfare, it is useful to perform a thought experiment by assuming an economy with 3 regions whose distribution is initially fully dispersed. Suppose that total dispersion is the only stable equilibrium. Consider now a marginal exogenous exodus occurs from region 1 to region 3, without loss of generality. By assumption that dispersion is stable, utility in region 1 must increase, as does the average utility in all regions. However, the entrepreneurs in region 3 will now observe that $V_1 > \bar{V}$. Since they are short-sighted and seek the region with the highest utility, they will return to region 1, thus restoring the symmetric dispersion distribution. This situation is similar to a Prisoner's Dilemma. As noticed by Pflüger and Südekum (2008b), the continuum of stable equilibria with partial agglomeration that the absence of income effects allows for, as opposed to other NEG models, may contribute to this result. Adding dispersion forces such as a housing sector (Pflüger and Südekum, 2008b) or commuting costs such as in Ottaviano *et al.* (2002) and Tabuchi *et al.* (2005) would most likely improve the welfare of entrepreneurs at less agglomerated outcomes.

4.5.2 Farmers

The average nominal wage paid to farmers is equal to 1. Therefore, average indirect utility for farmers is given by:

$$\bar{V}^L = 1 + \frac{\mu}{(\sigma-1)n} \sum_{j=1}^{n} \ln\left[\phi + (1-\phi)h_j\right] + \eta. \tag{4.29}$$

Average nominal wages are invariant to changes in h. Therefore, all changes in the welfare of both farmers are completely caused by changes in average price indices.[24]

[24] Recalling Proposition 1, the same can be stated about the welfare of entrepreneurs.

Theorem 4.12. *The farmers' average indirect utility is concave in the spatial distribution of entrepreneurs h, attaining a global maximum at symmetric dispersion.*

Proof. See Appendix D. □

Therefore, farmers attain the highest welfare when all entrepreneurs are evenly dispersed among the n regions. A consequence of the concavity of $\bar{V}^L$ is that, for the farmers, the regional indirect utility is strictly increasing in the number of entrepreneurs that reside there. This is explained by the fact that, due to equidistance, the regional price index P_i in (4.15) is a strictly decreasing function of h_i.[25] Therefore, the cost of living in a region decreases steadily as it becomes more industrialized. Industrialization hence leads to a progressive improvement of the farmers' residing in that region. On the other hand, this implies higher spatial inequality while farmers as a whole become relatively poorer in a core-periphery pattern.

4.5.3 Social welfare

The results shown so far evidence a clear trade-off between the welfare enjoyed by entrepreneurs and the welfare enjoyed by farmers at a given spatial distribution. For a given stable equilibrium, the welfare of the economy as a whole thus depends on the global ratio of farmers to entrepreneurs, λ, and the number of regions n. Let us define social welfare as an average of both average indirect utilities that depends on the spatial distribution of entrepreneurs:

$$\Omega(h) = \frac{1}{\lambda + 1} \left[\bar{V}(h) + \lambda \bar{V}^L(h) \right]. \tag{4.30}$$

Rewriting (4.30) using (4.29) and $\bar{V}$ we get:

$$\Omega(h) = \frac{1}{\lambda + 1} \left\{ \varepsilon + \frac{\mu}{(\sigma - 1)} \left[\sum_{j=1}^{n} \ln \left[\phi + (1 - \phi) h_j \right] \left(\frac{\lambda}{n} + h_j \right) \right] \right\}, \tag{4.31}$$

[25] One can rewrite P_i in a way that it depends only on h_i. See proof of Theorem 4.11 for more details.

where $\varepsilon = \lambda(1+\eta) + \bar{w} + \eta$ is a constant. We compare social welfare at the different possible distributions: agglomeration, symmetric dispersion, and partial agglomeration. The next results provide valuable information concerning the local extrema of $\Omega(h)$ and allow us to greatly simplify our analysis.

Proposition 4.13. *Social welfare extrema are located on one-dimensional invariant spaces whereby k regions have a share of entrepreneurs $h_i = h/k$ and the other $n-k$ regions have a share $h_j = (1-h)/(n-k)$.*

Proof. See Appendix D. □

As we have seen before, one particular invariant space of this kind is the one we have studied so far, Δ_{inv}, which corresponds to the particular cases $k = 1$ or $k = n-1$. From Proposition 4.15, a Corollary follows for the 3-region case.

Corollary 4.14. *For $n = 3$, all social welfare extrema are located on the invariant space Δ_{inv}.*

Proof. For the 3-region model, the only invariant spaces that correspond to the ones identified in Proposition 4.15 are the three invariant spaces whereby one region has $h_i = h$ entrepreneurs and the other two regions have $h_j = h/2$. This exactly matches the invariant space Δ_{inv}, for 3 regions, which concludes the proof. □

From Proposition 4.15, the study of social welfare extrema can be reduced to the simpler one-dimensional invariant space, without loss of generality, as it contains all the local extrema of $\Omega(h)$. Additionally, Corollary 4.14 ensures that in order to study welfare in a 3-region model we need only look at Δ_{inv}.[26] Using (4.31) and $n = 3$, we reach:

$$\Omega\left(h \in \Delta_{inv}\right) = \frac{1}{3(\lambda+1)}\left[3\varepsilon + \frac{\mu}{(\sigma-1)}\zeta\right], \tag{4.32}$$

where:

$$\zeta = (3h+\lambda)\ln\left[h(1-\phi)+\phi\right] + \left[2h + 3(1-h)\right]\ln\left[\frac{1-h}{2}(1-\phi)+\phi\right].$$

[26]For $n \geq 4$, there may exist other potential extrema on other invariant spaces (as well as equilibria).

We discuss the analytical details of the shape of $\Omega(h \in \Delta_{inv})$ in Appendix D. Figure 4.9 plots the different possible shapes of social welfare for increasing levels of ϕ, for $\lambda = 3$ and $\sigma = 5$. When ϕ is low (upper left picture), symmetric dispersion is the only stable and is a global welfare maximum. For a slightly higher ϕ (upper right picture), both dispersion and a highly industrialized partial agglomeration dispersion are stable but the former is still a global welfare maximum. Increasing ϕ further (medium left picture) makes dispersion unstable, whereas partial agglomeration is stable but is socially inferior compared to any less agglomerated spatial distribution. [27] As ϕ rises even further (medium right picture) agglomeration is now stable but is dominated by another less asymmetric distribution. However, continuous increases in ϕ steadily improve the welfare at agglomeration until it eventually becomes a global welfare maximum (lower pictures). The evidence here shows that the 3-region model exhibits a tendency towards over-agglomeration for intermediate transport cost levels.

[27] It turns out that no partial agglomeration corresponds to a local (hence global) welfare maximum. See Appendix D for more details on this matter.

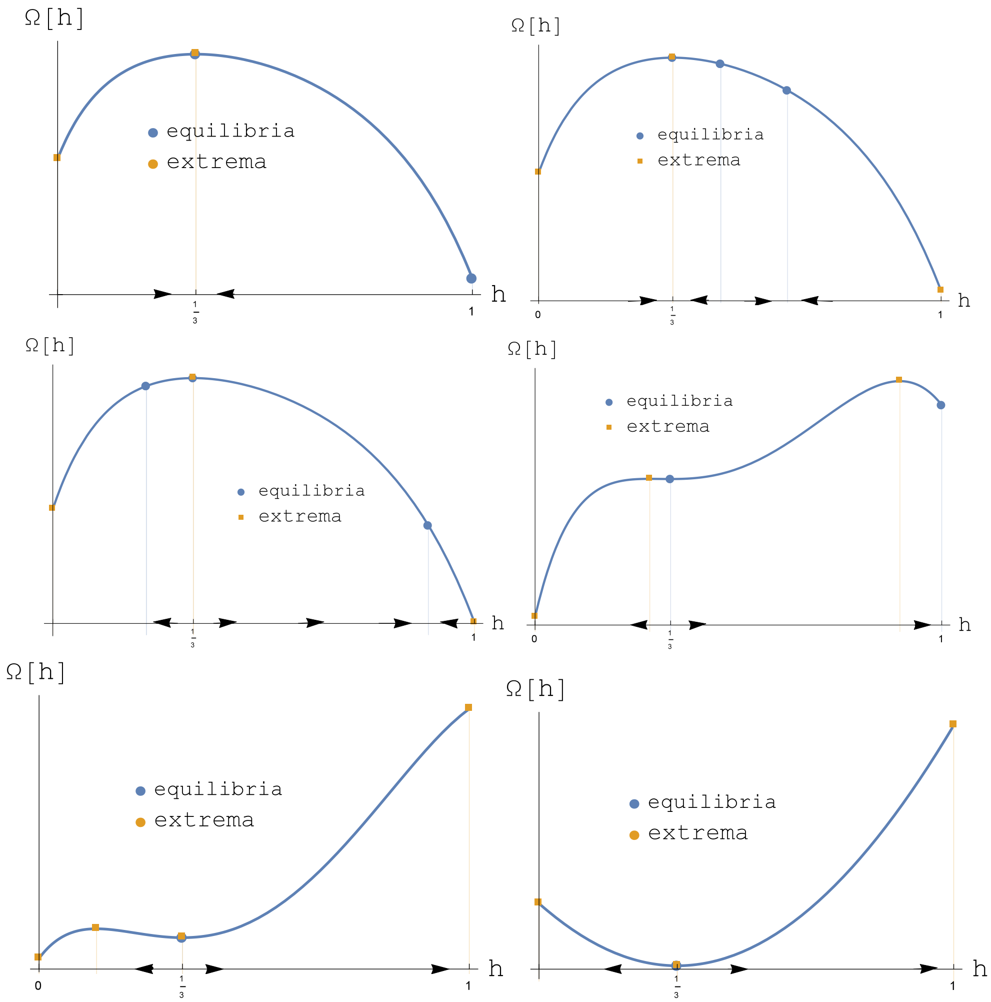

Figure 4.9 – Welfare along the invariant space Δ_{inv} for the 3-region model. From left to right to bottom, ϕ is increasing.

In order to check if this tendency carries over to the general n-region model, we present the following Theorem.

Theorem 4.15. *From a social point of view: (i) if dispersion is stable, it is a local maximum and always superior to agglomeration; (ii) there exists $\phi_z \in (\phi_s, 1)$ such that agglomeration is stable for$\phi \in (\phi_s, \phi_z)$, but is socially inferior to other less asymmetric distributions.*

Proof. See Appendix D. □

From Proposition 4.15 we learn that there is a range for the freeness of trade just above the sustain point for which agglomeration is a stable outcome, but also whereby social welfare is lower compared to other less industrialized distributions. Therefore, there is a tendency towards over-agglomeration. This is particularly notorious if $\phi_b > \phi_s$, because a temporary increase in the freeness of trade just above ϕ_b permanently shifts the spatial distribution to a socially inferior spatial distribution.[28]

4.6 Conclusion

The QLLog model presents itself as a good candidate for extending the study of NEG to a higher number of regions. By considering a quasi-linear upper tier utility function, the absence of income effects on consumers' demand for manufactures enables one to obtain simpler analytical expressions for the entrepreneurs' real wages in each region. In most other NEG models with different utility functional forms, regional wages feed back on the entrepreneurs' incomes in all other regions, which in turn depend on the wages they get, making it progressively harder to obtain tractable expressions as more regions are considered.

As we have seen, the QLLog model allows to study NEG with an arbitrary number of equidistant regions under exogenous symmetry. Moreover, it accommodates for the possibility of partial agglomeration equilibria, a feature which is ruled out in many CP models under exogenous symmetry. This enforces the idea that exogenous asymmetries are not the only source of asymmetric spatial distributions.

We look at equilibria where at least one region is absent of industry and the remaining regions are evenly indutrialized, and find they are always unstable. To the best of our knowledge, this is the first analytical confirmation that an evenly distributed industry among less than the total number of regions is not possible.

We look at partial agglomeration distributions along invariant spaces where all but one region share the same level of industry. Contrary to the 2-region QLLog model (Pflüger, 2004), where the only invariant space is the entire set of spatial distributions itself and any distribution may correspond to a stable equilibrium, with three and more

[28] On the other hand, it can be shown that a higher worker mobility (lower λ) and a higher trade freneess increases the likelihood that global welfare will be greater at agglomeration compared to dispersion. This statement is true if $\lambda > n\phi/(1-\phi)$, which we assume; otherwise dispersion is always stable.

regions, along the aforementioned invariant space, a partial agglomeration equilibrium can only be stable if a single region has a relatively larger industry compared to all other regions. This happens because, when a single region is comparatively smaller, an entrepreneur who migrates between any two of the evenly distributed regions (which are larger) will see his utility rise. Thus, if exogenous migration occurs to any such region, it will attract more and more entrepreneurs until it becomes an industrialized core.

A consequence of the stability analysis of partial agglomeration is that the QL-Log model distribution patterns with three regions and more cannot be explained by the 2-region model's pitchfork bifurcation (Pflüger, 2004). Instead, it undergoes a primary transcritical bifurcation along the invariant space at the symmetric dispersion equilibrium and a secondary saddle-node bifurcation that branches from partial agglomeration. The existence of a saddle-node implies that entrepreneurs, who are initially partially agglomerated, will become permanently dispersed across all regions if transport costs increase temporarily. Moreover, it is possible that agglomeration becomes stable before dispersion becomes unstable, depending on the level of worker mobility. Thus, from a smooth path where transport costs decrease, once symmetric dispersion looses stability, there are two possibilities: (i) if the worker mobility is high, industry converges immediately to partial agglomeration and then smoothly transits towards a single region agglomeration; (ii) if mobility is low, industry immediately agglomerates in a single region.

We have shown that the average utility of the entrepreneurs declines from agglomeration until dispersion, where their average utility is at its lowest. The converse happens to the welfare of farmers. Their average utility is minimal at agglomeration and is highest at dispersion. This evidences a clear trade-off in spatial distributions between entrepreneurs and farmers in terms of social desirability. When we look at the society as a whole, the model exhibits a tendency towards over-agglomeration when transportation costs lie at intermediate levels. The social desirability of more agglomerated distributions is higher when the proportion of farmers is lower, and can be improved for all workers by decreasing the cost of living through lower transportation costs.

Acknowledgements: We are thankful to Pascal Mossay, two anonymous referees, and participants in the 3rd Industrial Organization and Spatial Economics conference at Saint Petersburg for very useful comments and suggestions. Financial support by

Portuguese Public Funds through Fundação para a Ciência e Tecnologia (FCT) in the framework of the Ph.D. scholarship SFRH/BD/90953/2012 is dully acknowledged.

4.A - Proofs of section 4.2

This appendix contains the formal proofs pertaining to section 2 of the paper.

Proof of Proposition 4.1. Consider the nominal wage w_i in (4.13):

$$w_i = \frac{\mu}{\sigma}\sum_{j=1}^{n}\frac{\phi_{ij}\left(\frac{\lambda}{n}+h_j\right)}{\sum_{m=1}^{n}\phi_{mj}h_m}.$$

For w_1 we have:

$$w_1 = \frac{\mu}{\sigma}\left(\frac{\lambda/n+h_1}{h_1+\phi h_2+\ldots+\phi h_n}+\phi\frac{\lambda/n+h_2}{\phi h_1+h_2+\phi h_3\ldots+\phi h_n}+\ldots+\phi\frac{\lambda/n+h_n}{\phi h_1+\phi h_2+\ldots+\phi h_{n-1}+h_n}\right).$$

Notice that $h_i = \phi h_i + (1-\phi)h_i$ and that $\sum_{j=1}^{n} h_j = 1$. Thus, we can simplify w_1 to:

$$w_1 = \frac{\mu}{\sigma}\left[\frac{\lambda/n+h_1}{\phi+(1-\phi)h_1}+\phi\frac{\lambda/n+h_2}{\phi+(1-\phi)h_2}+\ldots+\phi\frac{\lambda/n+h_n}{\phi+(1-\phi)h_n}\right].$$

Analogously, we can compute w_n :

$$w_n = \frac{\mu}{\sigma}\left[\phi\frac{\lambda/n+h_1}{\phi+(1-\phi)h_1}+\phi\frac{\lambda/n+h_2}{\phi+(1-\phi)h_2}+\ldots+\frac{\lambda/n+h_n}{\phi+(1-\phi)h_n}\right]$$

The average nominal wage $\bar{w}$, defined in (4.17), is the weighted sum of the w_i's of the previous form:

$$\bar{w} = \frac{\mu}{\sigma}\sum_{j=1}^{n}\frac{\sum_{m=1}^{n}h_m\phi_{jm}\left(\lambda/n+h_j\right)}{\phi+(1-\phi)h_j}.$$

For each j, let us decompose the summation in the numerator as follows:

$$\bar{w} = \frac{\mu}{\sigma}\sum_{j=1}^{n}\frac{h_1\phi(\lambda/n+h_j)+...+h_j(\lambda/n+h_j)+...+h_n\phi(\lambda/n+h_j)}{\phi+(1-\phi)h_j}.$$

By using $h_j = \phi h_j + (1-\phi)h_j$ and $\sum_{j=1}^{n} h_j = 1$, average wage then becomes:

$$\begin{aligned}
\bar{w} &= \frac{\mu}{\sigma}\sum_{j=1}^{n}\frac{[\phi+(1-\phi)h_j]\,(\lambda/n+h_j)}{\phi+(1-\phi)h_j} \iff \\
\bar{w} &= \frac{\mu}{\sigma}\left(\lambda+\sum_{j=1}^{n}h_j\right) \iff \\
\bar{w} &= \frac{\mu}{\sigma}\left(\lambda+1\right),
\end{aligned}$$

concluding the proof. □

4.B - Proofs of section 4.3

This appendix contains all the proofs concerning both existence and local stability of equilibria (section 3).

Proof of Proposition 4.2. The indirect utility in the core region with $h_i = 1$ is given by:

$$V_i = \frac{\mu}{\sigma}(\lambda+1),$$

which does not depend on transportation costs. For a peripheral region j we have:

$$V_j = \frac{\mu\left[\lambda+\phi^2(\lambda+n)+\lambda(n-2)\phi\right]}{n\sigma\phi}+\mu\frac{\ln\phi}{\sigma-1}.$$

Agglomeration in the core region i is thus stable if:

$$\begin{aligned}
V_i &> V_j \iff \\
\frac{(1-\phi)\left[\lambda(1-\phi)-n\phi\right]}{n\sigma\phi}+\frac{\ln\phi}{\sigma-1} &< 0,
\end{aligned}$$

concluding the proof. □

Proof of Proposition 4.3. Local stability of interior equilibria in $\triangle$ is given by the sign of the real part of the eigenvalues of the Jacobian matrix of the system in (4.18) at $(h_1, h_2, ..., h_{n-1}) = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)$. At symmetric dispersion, the average utility $\bar{V}$ is invariant in the permutation of any two coordinates, due to symmetry. If we interchange the distributions between region 1 and region n we then have $\bar{V}\left(\frac{1}{n}+\varepsilon, \frac{1}{n}, ..., \frac{1}{n}\right) = \bar{V}\left(\frac{1}{n}-\varepsilon, \frac{1}{n}, ..., \frac{1}{n}\right)$. But this implies that $\partial_{h_i}\bar{V}\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right) = 0$.[29] The argument of invariance extends to the indirect utility V_i in the permutation of any two coordinates $j \neq i$, which implies that $\partial_{h_j}V_i\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right) = 0, \forall j \neq i$. Finally, symmetry among regions establishes that we must have $\partial_{h_i}V_i\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right) = \partial_{h_j}V_j\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)$. The Jacobian matrix of (4.18) at the symmetric equilibrium is thus given by:

$$J = \begin{bmatrix} \frac{\partial V_i}{\partial h_i} & 0 & \dots & 0 \\ 0 & \frac{\partial V_i}{\partial h_i} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{\partial V_i}{\partial h_i} \end{bmatrix},$$

which has a repeated real eigenvalue with multiplicity $n-1$ given by:

$$\frac{\partial V_i}{\partial h_i}\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right),$$

and total dispersion is stable if $\partial_{h_i}V_i\left(\frac{1}{n}, ..., \frac{1}{n}\right) < 0$. Computing the derivative $\partial_{h_i}V_i\left(\frac{1}{n}, ..., \frac{1}{n}\right)$, total dispersion is stable if:

$$\partial_{h_i}V_i\left(\frac{1}{n}, ..., \frac{1}{n}\right) \equiv \frac{\mu n(1-\phi)\left[(2n-1)\sigma\phi - \lambda(\sigma-1)(1-\phi) - n\phi + \sigma\right]}{(\sigma-1)\sigma\left[(n-1)\phi+1\right]^2} < 0 \iff$$
$$(2n-1)\sigma\phi - n\phi + \sigma - \lambda(\sigma-1)(1-\phi) < 0.$$

Solving for ϕ, we get the equivalent stability condition:

$$\phi < \frac{\sigma(1-\lambda)+\lambda}{\lambda+n-\sigma\left(\lambda+2n-1\right)},$$

concluding the proof. □

[29] Notice that partial derivatives in this context imply that changes in h_i are reflected symmetrically in h_n.

Proof of Theorem 4.4. The analysis for the stability of partial dispersion is bipartite. An "attracting boundary" condition first requires that regions absent of entrepreneurs remain empty, i.e., that $h_i = 0$. All these regions have the same indirect utility V_i. The other k regions have the same indirect utility V_j. Since there are $n-k$ regions with no entrepreneurs and k regions with the same distribution $1/k$, the average utility equals $\bar{V} = V_j$. Continuity of indirect utility in the spatial distribution ensures that, after an exogenous migration into a region i, h_i will return to zero if $V_i < \bar{V} \equiv V_j$, that is,

$$V_i|_{BD} - V_j|_{BD} < 0$$

Second, one needs to ensure that the spatial distribution does not change "along the boundary". Given symmetry among the k regions, the second condition then requires that an increase in h_j leads to a decrease in the difference between V_j and $\bar{V}$, in order to restore $h_j = 1/k$. This condition thus becomes $\partial_{h_j} V_j - \partial_{h_j} \bar{V} < 0$. If we allow for the implicitly determined region n to take $1/k$ of the spatial distribution, then without loss of generality we can assume that $h_j = 1/k$ and $h_n = 1/k$. As a result, invariance of $\bar{V}$ in the permutation of h_j and h_n (see proof of Proposition 4.3) asserts that $\partial_{h_j} \bar{V} = 0$ at boundary dispersion (BD). Thus, the second condition for stability of boundary dispersion boils down to:

$$\left.\frac{\partial V_j}{\partial h_j}\right|_{BD} < 0.$$

Computing the indirect utilities, V_i and V_j, and the first order derivative $\partial_{h_j} V_j$, we get the first condition and the second condition, respectively, for stability of boundary dispersion:[30]

$$n\sigma\phi\left[(k-1)\phi+1\right]\ln\left[\frac{k\phi}{(k-1)\phi+1}\right] + (\sigma-1)(1-\phi)\left[\lambda(1-\phi)-n\phi\right] < 0$$

$$k\left[\lambda(\sigma-1)(1-\phi) - n(2\sigma-1)\phi\right] + n\sigma(1-\phi) < 0.$$

[30]Standard inspection of the Jacobian matrix at the equilibrium can be shown to boil down to the same conditions.

Rewriting the stability conditions in terms of λ we get:

$$\frac{n\left[k(2\sigma-1)\phi+\sigma(1-\phi)\right]}{k(\sigma-1)(1-\phi)}<\lambda<\frac{n\phi\left\{\sigma\left[(k-1)\phi+1\right]\ln\left[\frac{(k-1)\phi+1}{k\phi}\right]+\sigma(1-\phi)+\phi-1\right\}}{(\sigma-1)(\phi-1)^2}.$$

Therefore, no λ can satisfy the stability condition if:

$$\begin{aligned}\frac{n\phi\left\{\sigma\left[(k-1)\phi+1\right]\ln\left[\frac{(k-1)\phi+1}{k\phi}\right]+\sigma(1-\phi)+\phi-1\right\}}{(\sigma-1)(\phi-1)^2}-\frac{n\left[k(2\sigma-1)\phi+\sigma(1-\phi)\right]}{k(\sigma-1)(1-\phi)}<0\\ \iff -\frac{n\sigma\left[(k-1)\phi+1\right]\times\Omega}{k(\sigma-1)(\phi-1)^2}<0,\end{aligned} \quad (4.33)$$

where $\Omega=k\phi\left\{\ln\left[\frac{k\phi}{(k-1)\phi+1}\right]-1\right\}+1$. The inequality in (4.33) holds if $\Omega(k,\phi)>0$, i.e., if:

$$\ln\left[\frac{k\phi}{(k-1)\phi+1}\right]>-\frac{1+k}{\phi k}.$$

Notice that a sufficient condition for $\Omega>0$ is that:

$$\begin{aligned}\ln\left[\frac{k\phi}{(k-1)\phi+1}\right]>-\frac{1}{\phi k}\iff\ln\left[\frac{(k-1)\phi+1}{k\phi}\right]<\frac{1}{\phi k}\iff\\ \iff\ln\left(\frac{k-1}{k}+\frac{1}{k\phi}\right)<\frac{1}{k\phi}\iff\frac{1}{k\phi}-\ln\left(\frac{k-1}{k}+\frac{1}{k\phi}\right)>0.\end{aligned}$$

We know that $f(x)=x-\ln(x+a)$ has a minimum in $x=1-a$ and $f(1-a)=1-a>0$ if $a<1$. Here, we have:

$$f\left(x(k,\phi)\right)=x(k,\phi)-\ln\left[x(k,\phi)+a(k)\right]\equiv\frac{1}{k\phi}-\ln\left[\frac{1}{k\phi}+\frac{k-1}{k}\right].$$

Therefore, Ω is positive if $(k-1)/k<1$, which is true, and thus the inequality in (4.33) holds, concluding the proof. □

Proof of Proposition 4.5. Interior equilibria along Δ_{inv} satisfy $\left(\dot{h}_1,\dot{h}_j\right)=(0,0)$ in (4.18) if and only if $V_1=V_j,\forall j$ and $1\neq j$. The configuration $(h_1,h_j)=\left(h,\frac{1-h}{n-1}\right)$ is an interior asymmetric equilibrium if and only if:

$$\lambda=\lambda^*(h)\equiv\frac{n(\sigma-1)(1-\phi)\phi(hn-1)-n\sigma\left[h(1-\phi)-\phi\right]\left[\phi(h+n-2)-h+1\right]\nu}{(\sigma-1)(1-\phi)^2(hn-1)}, \quad (4.34)$$

where $\nu = \ln\left\{\frac{\phi(h+n-2)-h+1}{(n-1)[h(1-\phi)+\phi]}\right\}$ and $h \in (0,1) \setminus \left\{\frac{1}{n}\right\}$.

Observe that $\lambda^*(h)$ is always positive. It is given by $(\alpha + \beta\nu)/\gamma$, where $\alpha/\gamma > 0, \beta < 0$ and $\nu/\gamma < 0$.[31] Thus, for each h there is a positive λ that guarantees that the corresponding spatial distribution is an interior asymmetric equilibrium.

The first and second derivatives of $\lambda^*(h)$ in (4.34) with respect to h are given, respectively, by:

$$\frac{\partial\lambda^*(h)}{\partial h} = \frac{\alpha_1 + \beta_1\nu}{(\sigma-1)(1-\phi)^2(hn-1)^2}.$$
$$\frac{\partial^2\lambda^*(h)}{\partial h^2} = \frac{n\sigma\left[(n-1)\phi+1\right]^2(\alpha_2+\beta_2\nu)}{(\sigma-1)(1-\phi)^2(hn-1)^3\left[h(1-\phi)+\phi\right](\phi(h+n-2)-h+1)},$$

where:

$$\begin{aligned}
\alpha_1 &= -(1-\phi)(1-hn)\left[(n-1)\phi+1\right] \\
\beta_1 &= h^2n(1-\phi)^2 - 2h(1-\phi)^2 + \phi\left\{n\left[(n-3)\phi+2\right]+3\phi-4\right\}+1 \\
\alpha_2 &= (1-\phi)^2(hn-1)\left[h(n-2)+(3-2n)\phi-1\right] \\
\beta_2 &= -2(n-1)\left[h(1-\phi)+\phi\right]\left[\phi(h+n-2)-h+1\right].
\end{aligned}$$

For $h \in (0, 1/n)$ we have $\alpha_2 < 0, \beta_2 < 0$ and $\nu > 0$. Moreover, the denominator of $\partial^2\lambda^*(h)/\partial h$ is positive. Thus, the second derivative is negative, which implies that $\lambda^*(h)$ is strictly concave in $(0, 1/n)$.

When $h \in (1/n, 1)$, we have $\alpha_2 > 0, \beta_2 < 0$ and $\nu < 0$. Since the denominator is now negative, $\partial^2\lambda^*(h)/\partial h$ is still negative, which implies that $\lambda^*(h)$ is also strictly concave in $(1/n, 1)$.

By computing the limits of $\lambda^*(h)$ and its derivatives as h approaches total dispersion

[31] The terms of $\lambda^*(h)$ are given by $\alpha = n(\sigma-1)(1-\phi)\phi(hn-1)$, $\beta = -n\sigma\left[h(1-\phi)-\phi\right]\left[\phi(h+n-2)-h+1\right]$, and $\gamma = (\sigma-1)(1-\phi)^2(hn-1)$.

we get:

$$\lim_{h\to\frac{1}{n}\pm}\lambda^*(h) = \frac{\sigma(2n\phi+1)-n\phi}{(\sigma-1)(1-\phi)} > 0.$$

$$\lim_{h\to\frac{1}{n}\pm}\frac{\partial\lambda^*(h)}{\partial h} = \frac{(n-2)n\sigma}{2(n-1)(\sigma-1)} > 0.$$

$$\lim_{h\to\frac{1}{n}\pm}\frac{\partial^2\lambda^*(h)}{\partial h^2} = -\frac{n^4\sigma(1-\phi)}{3(n-1)^2(\sigma-1)\left[(n-1)\phi+1\right]} < 0.$$

The first limit ensures that $\lambda^*(h)$ is continuous at $h = 1/n$. The second limit guarantees that $\lambda^*(h)$ is differentiable at $h = 1/n$, given continuity. Finally, the third limit establishes strict concavity at $h = 1/n$. Thus, the invariant space $\triangle_{inv}$ contains at most two interior asymmetric equilibria in $(0, 1)$.

More specifically, the positive limit concerning the first derivative additionally ensures that $\lambda^*(h)$ increases monotonically in $(0, 1/n)$; therefore, there can be at most one equilibrium in that interval. For $h \in (0, 1/n)$, we know that $\lambda^*(h)$ initially increases and may or may not decrease after some point. As a result, we have at most two equilibria for $h \in (0, 1/n)$. This concludes the proof. □

Proof of Theorem 4.6. On the invariant space $\triangle_{inv}$, region 1 has a distribution $h_1 = h$ and there are $n-1$ regions with distributions $h_j = (1-h)/(n-1)$. The dynamics $\dot{h}_1 = f_1(h)$ on $\triangle_{inv}$ are invariant to the permutation of any of the $j \neq i$ coordinates. As a result, we have $\partial_{h_j} f_1 = 0$ at partial agglomeration. Given symmetry, this implies $\partial_{h_1} f_j = 0$, for $j \neq 1$. Applying the same argument of invariance to all the other $n-2$ regions we have $\partial_{h_{j^-}} f_j = 0$, where $j^- \neq j$. This reduces the Jacobian of the dynamical system at partial agglomeration to:

$$J = \begin{bmatrix} \dfrac{\partial f_1}{\partial h_1} & 0 & \dots & 0 \\ 0 & \dfrac{\partial f_j}{\partial h_j} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \dfrac{\partial f_j}{\partial h_j} \end{bmatrix},$$

or any permutation of its elements at the main diagonal, which gives us the eigenvalues. One of the eigenvalues is given by

$$\begin{aligned} \frac{\partial f_1}{\partial h_1} &= \underbrace{V_1 - \bar{V}}_{0} + h_1 \left(\frac{\partial V_1}{\partial h_1} - \frac{\partial \bar{V}}{\partial h_1} \right) \\ \frac{\partial f_1}{\partial h_1} &= h \left(\frac{\partial V_1}{\partial h_1} - \frac{\partial \bar{V}}{\partial h_1} \right), \end{aligned}$$

where the latter equality is obtained after substituting h_1 and h_j for the partial agglomeration equilibrium. The first term equals zero because it is the condition for an equilibrium. The condition for stability reduces to:

$$\frac{\partial V_1}{\partial h_1} - \frac{\partial \bar{V}}{\partial h_1} < 0.$$

The other eigenvalue δ has a multiplicity of $n-2$ and is given by:

$$\begin{aligned} \frac{\partial f_j}{\partial h_j} &= \underbrace{V_j - \bar{V}}_{0} + h_j \left(\frac{\partial V_j}{\partial h_j} - \underbrace{\frac{\partial \bar{V}}{\partial h_j}}_{0} \right) \\ \iff \frac{\partial f_j}{\partial h_j} &= \frac{1-h}{1-n} \frac{\partial V_j}{\partial h_j}. \end{aligned}$$

The fact that the derivative concerning the average utility is zero stems from its invariance in the permutation of coordinates j and n, since $h_j = h_n$ at the interior equilibrium. The previous condition reduces to:

$$\frac{\partial V_j}{\partial h_j} < 0.$$

As a result, partial agglomeration is stable if:

$$\begin{cases} \left. \dfrac{\partial V_1}{\partial h_1} - \dfrac{\partial \bar{V}}{\partial h_1} \right|_{PA} & < 0 \\ \left. \dfrac{\partial V_j}{\partial h_j} \right|_{PA} & < 0 \end{cases}.$$

Computing the partial derivatives at $(h_1, h_j) = (h, (1-h)/(1-n))$ and evaluating them at equilibrium using $\lambda^*(h)$ in (4.34), the stability conditions for partial agglomeration are given by:

$$\begin{cases} \gamma \equiv \frac{(1-\phi)(1-hn)-(n-1)[h(1-\phi)+\phi]\times\nu}{(1-hn)} & < 0 \\ \delta \equiv \frac{(1-\phi)(hn-1)[(n-1)\phi+1]+\Phi\times\nu}{(hn-1)} & < 0. \end{cases}$$

The rest of the proof is divided in two parts, supporting the claims in (i) and (ii).

(i). We will show that the first stability condition in (4.26) fails to hold for $h \in (0, 1/n)$, that is, $\gamma > 0$.
The log term is always positive, as is the denominator of γ. However, we have $(1-\phi)(1-hn) > 0$ and $-(n-1)\left[h(1-\phi)+\phi\right] < 0$. Hence, we have $\gamma > 0$ if and only if:

$$D \equiv (1-\phi)(1-hn) - (n-1)\left[h(1-\phi)+\phi\right]\nu > 0.$$

As h approaches $1/n$ we have:

$$\begin{aligned} &\lim_{h\to 1/n} D = \lim_{h\to 1/n}\left[(1-\phi)(1-hn)\right] - \lim_{h\to 1/n}(n-1)\left[h(1-\phi)+\phi\right] \times \lim_{h\to 1/n} \nu \iff \\ \iff &\lim_{h\to 1/n} D = 0 - \frac{(n-1)\left[(n-1)\phi+1\right]}{n} \times \lim_{h\to 1/n}\left\{\ln\left[\frac{\frac{(n-1)\phi+1}{n}}{\frac{(n-1)\phi+1}{n}}\right]\right\} \iff \\ \iff &\lim_{h\to 1/n} D = 0. \end{aligned}$$

Together with the knowledge that:

$$\begin{aligned} \frac{\partial D}{\partial h} &= -\frac{(n-1)^2\left[h(1-\phi)+\phi\right]^2\left\{\frac{\phi-1}{(n-1)[h(1-\phi)+\phi]} - \frac{(1-\phi)[\phi(h+n-2)-h+1]}{(n-1)[h(1-\phi)+\phi]^2}\right\}}{\phi(h+n-2)-h+1} - (n-1)(1-\phi)\nu - n(1-\phi) \\ &= -(1-\phi)\left[\frac{(1-\phi)(1-hn)}{\phi(h+n-2)-h+1} + (n-1)\nu\right] < 0, \end{aligned}$$

we find that D is strictly decreasing and thus positive in $(0, 1/n)$. As a result, $\gamma > 0$ for $h \in (0, 1/n)$ implying that partial agglomeration is unstable in that interval.

(ii). We will now show that, provided $h \in (1/n, 1)$, $\gamma < 0, \forall(\phi, n)$.
Observe that ν is negative for $h \in (1/n, 1)$, as is the denominator of γ. Moreover, we have $-(n-1)\left[h(1-\phi)+\phi\right] < 0$ and $(1-\phi)(1-hn) < 0$. This altogether implies that $\gamma < 0$ if and only if:

$$D \equiv (1-\phi)(1-hn) - (n-1)\left[h(1-\phi)+\phi\right]\nu < 0.$$

From proof of Theorem 4.6, we know that the limit of D as h tends to $1/n$ is zero. This, together with the knowledge that $\partial D/\partial h$ is positive for $h \in (1/n, 1)$ ensures that D is also positive in that interval. Thus, γ is always negative for $h \in (1/n, 1)$, which concludes the proof. $\square$

4.C - Proofs of section 4.4

This Appendix contains the formal proofs concerning section 4. For simplicity, we consider $\dot{h}_i = f_i(h)$, where $f_i(h)$ is defined in (4.18).

Proof of Proposition 4.7. The conditions required for a transcritical bifurcation (Guckenheimer and Holmes 2002; pp. 149 and 150) are as follows:

(T1.) For all values of the bifurcation parameter ϕ, we must have $f_i(\frac{1}{n}, ..., \frac{1}{n}; \phi) = 0$.

This condition is satisfied since total dispersion is always an equilibrium.

(T2.) The Jacobian of $f_i(h)$ has a zero eigenvalue at total dispersion.

Condition (T2) requires:

$$\frac{\partial f_i}{\partial h_i}(\tfrac{1}{n}, ..., \tfrac{1}{n}; \phi_b) = 0,$$

which gives us the *break point* ϕ_b in the n-region model.

After (4.22), it is given by:

$$\phi_b \equiv \frac{\sigma(1-\lambda) + \lambda 0}{\lambda + n - \sigma\left[\lambda + 2n - 1\right]}.$$

(T3.) At total dispersion and at the break point we must have $\frac{\partial^2 f_i}{\partial h_i^2}(\frac{1}{n}, ..., \frac{1}{n}; \phi_b) \neq 0$.

The second derivative of f_i with respect to h_i at the symmetric equilibrium is given by:

$$\frac{\partial^2 f_i}{\partial h_i^2}(\tfrac{1}{n}, ..., \tfrac{1}{n}) = \frac{\mu(n-2)(1-\phi)\left\{\phi^2\left[n\sigma(2\lambda + 4n - 3) - 2n(\lambda + n) + \sigma\right] + \sigma\phi\left[(3-2\lambda)n - 2\right] + 2\lambda n\phi + \sigma\right\}}{(\sigma - 1)\sigma\left[(n-1)\phi + 1\right]^3}.$$

When evaluated at the break point, we end up with:

$$\frac{\partial^2 f_i}{\partial h_i^2}(\tfrac{1}{n}, ..., \tfrac{1}{n}; \phi_b) = \frac{\mu(n-2)(1-2\sigma)^2}{(\lambda+1)^2(\sigma-1)^3},$$

which is zero for $n = 2$ and positive for $n \geq 3$.

(T4.) At total dispersion and at the break point we must have $\frac{\partial^2 f_i}{\partial h_i \partial \phi}(\frac{1}{n}, ..., \frac{1}{n}; \phi_b) \neq 0$. Computing $\frac{\partial^2 f_i}{\partial h_i \partial \phi}$ at the symmetric equilibrium yields:

$$\frac{\partial^2 f_i}{\partial h_i \partial \phi}(\tfrac{1}{n}, ..., \tfrac{1}{n}) = \frac{\mu n \left\{2\left[\lambda(\sigma-1)(1-\phi) - n\sigma\phi\right] + n\phi + \phi - 1\right\}}{(\sigma-1)\sigma\left[(n-1)\phi+1\right]^3}.$$

Evaluating at the break point we then get:

$$\frac{\partial^2 f_i}{\partial h_i \partial \phi}(\tfrac{1}{n}, ..., \tfrac{1}{n}; \phi_b) = \frac{\mu(2\sigma-1)\left[\lambda - \sigma(\lambda+2n) + n + \sigma\right]^2}{(\lambda+1)^2 n(\sigma-1)^3\sigma} > 0.$$

Since all conditions are verified, we conclude that the model undergoes a transcritical bifurcation at the break point ϕ_b. □

Proof of Proposition 4.8. A necessary and sufficient condition for an interior distribution with $h \in (1/n, 1)$ to be a partial agglomeration equilibrium is given by $\lambda^*(h)$ in equation (4.34). We use the conditions for a saddle-node bifurcation given by Guckenheimer and Holmes (2002, Theorem 3.4.1). Applied to the QLLog model, they are as follows:

(SN1.) At partial agglomeration we must have $\dfrac{df}{dh}(h; \lambda^*(h); \phi_f) = 0$.

In this instance, $f(h_i)$ is the RHS of (4.18) and the proof of Theorem 78 gives:

$$\frac{df}{dh}(h; \lambda^*(h); \phi_f) = 0 \iff \delta = 0,$$

where δ is as in (4.26). We rewrite $\delta = 0$ as:

$$\frac{(1-\phi_f)(1-hn)\left[(n-1)\phi_f+1\right]}{\Phi(h;\phi_f)} = \nu(h;\phi_f), \tag{4.35}$$

where

$$\Phi(h,\phi_f) = h^2 n(1-\phi_f)^2 - 2h(1-\phi_f)^2 + \phi_f \left\{n\left[(n-3)\phi_f+2\right]+3\phi_f-4\right\}+1,$$
$$\nu(h,\phi_f) = \ln\left\{\frac{\phi_f(h+n-2)-h+1}{(n-1)\left[h(1-\phi_f)+\phi_f\right]}\right\},$$

and ϕ_f is the level of freeness of trade at which the interior equilibrium changes stability.

(SN2.) It is required that $\dfrac{d^2 f}{dh^2}(h;\lambda^*(h);\phi_f) \neq 0$.
From (4.25) and (4.35), we have:

$$\frac{d^2 f}{dh^2}(h;\lambda^*(h);\phi_f) = \frac{(h-1)h\mu(1-\phi)^2\left[(n-1)\phi+1\right]^2\Gamma}{(\sigma-1)\left[h(1-\phi)+\phi\right]^2\left[\phi(h+n-2)-h+1\right]^2\Phi},$$

where $\Gamma(h,\phi) = h^2(n-2)(1-\phi)^2 + 2h(1-\phi)\left[(2n-3)\phi+1\right] - \phi\left\{n\left[(n-5)\phi+2\right]+5\phi-4\right\} - 1$. The term Φ is positive. Therefore, the derivative is zero if $\Gamma(h;\lambda^*(h);\phi_f)=0$. We will show that this leads to a contradiction. The term $\Gamma(h,\phi)$ has only one (meaningful) zero given by:

$$h = h^* \equiv -\frac{\phi(2n(1-\phi)+3\phi-4)-\sqrt{n-1}(1-\phi)\left[(n-1)\phi+1\right]+1}{(n-2)(1-\phi)^2}.$$

It suffices to show that (SN1) does not hold for this value of h. By replacing $h=h^*$ in (4.26) we obtain:

$$\frac{df}{dh}(h^*;\lambda^*(h);\phi_f) = \delta(h^*) \equiv -\frac{\left[(n-1)\phi+1\right]^2}{(n-2)^2}\Xi,$$

where:

$$\Xi = (n-2)\left[\left(\sqrt{n-1}+2\right)n-2\right] + (n-1)\left(n+2\sqrt{n-1}\right)\log(n-1).$$

Since $\Xi>0$, it follows that $\delta(h^*)<0$. Hence, (SN1) does not hold and we arrive at a contradiction. It follows that $\Gamma(h;\lambda^*(h);\phi_f)\neq 0$ and thus $\frac{d^2 f}{dh^2}(h;\lambda^*(h);\phi_f)\neq 0$.

(SN3.) It is required that $\dfrac{df}{d\phi}(h;\lambda^*(h);\phi_f) \neq 0$.

From (4.25) and (4.35), we have:

$$\frac{df}{d\phi}\left(h;\lambda^*(h);\phi_f\right)=\frac{(1-h)h\mu(hn-1)\times\Theta}{(\sigma-1)\sigma\left[h(\phi-1)-\phi\right]\left[\phi(h+n-2)-h+1\right]\times\Phi},$$

where $\Theta=h^2n(1-\phi)^2-h(1-\phi)\left\{(n-2)\sigma\left[(n-1)\phi+1\right]-2\phi+2\right\}-\sigma\left[(n-1)\phi+1\right](2n\phi-3\phi+1)+\phi\left\{n\left[(n-3)\phi+2\right]+3\phi-4\right\}+1<0$. Since the term $hn-1>0$ for $h\in(1/n,1)$, we can conclude that $df/d\phi>0$ when evaluated at partial agglomeration and at ϕ_f, ensuring that (SN3) is satisfied. This concludes the proof. □

Proof of Proposition 4.9. We know that total dispersion is stable if $\lambda>\lambda_b$, whereas agglomeration is stable if $\lambda<\lambda_s$. As a result, both equilibria are simultaneously stable if there is a $\lambda\in(\lambda_b,\lambda_s)$. Using (4.21) and (4.24), simultaneity of stability of stability then requires $\lambda_s-\lambda_b>0$:

$$\begin{aligned}\frac{n\phi\left[(\sigma-1)(1-\phi)-\sigma\ln\phi\right]}{(\sigma-1)(\phi-1)^2}-\frac{\sigma(2n\phi+1-\phi)-n\phi}{(\sigma-1)(1-\phi)}.>0 &\iff\\ -\frac{(1-\phi)\left[(n-1)\phi+1\right]+n\phi\log(\phi)}{(\sigma-1)(1-\phi)^2}>0 &\iff\\ -\frac{(1-\phi)^2}{\phi(1-\phi+\ln\phi)}<n,&\end{aligned}$$

which concludes the proof. □

4.D - Proofs of section 4.5

This Appendix contains the formal proofs concerning section 5.

Proof of Theorem 4.11. From (4.17), we know that the average nominal wage paid to entrepreneurs is given by $\bar{w}=\frac{\mu}{\sigma}(1+\lambda)$. Therefore, the entrepreneur's weighted average utility is given by:

$$\bar{V}=\frac{\mu}{\sigma}(1+\lambda)+\frac{\mu}{\sigma-1}\sum_{i=1}^{n}h_i\ln\left(\sum_{j=1}^{n}\phi_{ij}h_j\right)+\eta.$$

By using $h_j = \phi h_j + (1-\phi)h_j$ and $\sum_{j=1}^{n} h_j = 1$, $\bar{V}$ simplifies to:

$$\bar{V} = \frac{\mu}{\sigma - 1} \sum_{j=1}^{n} h_j \ln\left[\phi + (1-\phi)h_j\right] + \frac{\mu}{\sigma}(\lambda + 1) + \eta.$$

Let us now define the summation term in $\bar{V}$ as $F(h) = f(h_1) + f(h_2) + ... + f(h_{n-1}) + g(h_n)$, where $f : \mathbb{R} \mapsto \mathbb{R}$ is continuous and twice differentiable, $h_n : (h_1, h_2, ..., h_{n-1}) \mapsto 1 - h_1 - ... - h_{n-1}$, and $g(h_n) = f \circ h_n : \mathbb{R}^{n-1} \mapsto \mathbb{R}$. For each $i \neq n$, we have:

$$f''(h_i) = \frac{2\left(1-\phi\right)\left[\phi + \left(1-\phi\right)h_i\right] - h_i\left(1-\phi\right)}{\left[\phi + \left(1-\phi\right)h_i\right]^2}.$$

The derivative is positive if $2\left[\phi + \left(1-\phi\right)h_i\right] > h_i\left(1-\phi\right) \iff h_i > -2\phi/\left(1-\phi\right)$, which is always true. Hence, each $f(h_i)$ is a strictly convex function for all $i \neq n$. Next, we will show that the last term of $F(h)$, $g(h_n)$, is also convex. First:

$$\begin{aligned} g\left[t(h_1^0, ..., h_{n-1}^0) + (1-t)(h_1^1, ..., h_{n-1}^1)\right] &= g\left[th_1^0 + (1-t)h_1^1, ..., th_{n-1}^0 + (1-t)h_{n-1}^1\right] = \\ &= f\left[1 - th_1^0 - (1-t)h_1^1 - ...th_{n-1}^0 - (1-t)h_{n-1}^1\right] = \\ &= f\left[1 - t(h_1^0 + ... + h_{n-1}^0) - (1-t)(h_1^1 + ... + h_{n-1}^1)\right]. \end{aligned}$$

Second, we have:

$$tg(h_1^0, ..., h_{n-1}^0) + (1-t)g(h_1^1, ..., h_{n-1}^1) = tf(1 - h_1^0 - ... - h_{n-1}^0) + (1-t)f(1 - h_1^1 - ... - h_{n-1}^1).$$

Since f is strictly convex, it follows that:

$$\begin{aligned} tf(1 - h_1^0 - ... - h_{n-1}^0) + (1-t)f(1 - h_1^1 - ... - h_{n-1}^1) &\geq f\left[t(1 - h_1^0 - ... - h_{n-1}^0) + (1-t)(1 - h_1^1 - ... - h_{n-1}^1)\right] = \\ &= f\left[1 - t(h_1^0 + ... + h_{n-1}^0) - (1-t)\right](h_1^1 + ... + h_{n-1}^1). \end{aligned}$$

We then conclude that:

$$g\left[t(h_1^0, ..., h_{n-1}^0) + (1-t)(h_1^1, ..., h_{n-1}^1)\right] \leq tg(h_1^0, ..., h_{n-1}^0) + (1-t)g(h_1^1, ..., h_{n-1}^1),$$

which implies that $g(h_n)$ is convex. Since $F(h)$ is a sum of strictly convex functions, it is strictly convex in h. Therefore, $\bar{V}$, a constant plus $F(h)$, is also strictly convex.
Since $\partial_{h_i}\bar{V}\left(\frac{1}{n}, ..., \frac{1}{n}\right) = 0$ (see proof of proposition 4.3), the first order condition for a minimum is satisfied. Given that $\bar{V}$ is convex in h, it attains a global minimum at dispersion, concluding the proof. ☐

Proof of Theorem 4.12. Define the summation in (4.29) as $F(h) = f(h_1) + f(h_2) + ... + f(h_{n-1}) + g(h_n)$, where $h_n : (h_1, h_2, ..., h_{n-1}) \mapsto 1 - h_1 - ... - h_{n-1}$. The second derivative of each $f(h_i)$ is given by:

$$f''(h_i) = -\frac{(1-\phi)^2}{[h(1-\phi)+\phi]^{\,2}} < 0.$$

Then, $f(h_i)$ is strictly concave. Analogous reasoning to that of Proof of Theorem 4.11 permits to show that $g(h_n) = f \circ h_n : \mathbb{R}^{n-1} \mapsto \mathbb{R}$ is also strictly concave. Since $F(h)$ is a sum of strictly concave functions, it is itself strictly concave in h. Therefore, $\bar{V}^L$, a constant term plus $F(h)$, is also strictly concave.
Each price index P_i in (4.15) is invariant to the permutation of any two region's coordinates. Therefore, we can assert that $\partial_{h_i} \bar{V}^L \left(\frac{1}{n}, ..., \frac{1}{n}\right) = 0$, $\forall i \in N$.[32] Given strict concavity, $\bar{V}^L$ attains a maximum at $h = (1/n, ..., 1/n)$, which concludes the proof. $\square$

Proof of Proposition 4.13. Rewrite the social welfare function $\Omega(h)$ in (4.31) as:

$$\Omega(h) = a + g(h_1) + g(h_2) + ... + g(h_n). \tag{4.36}$$

The optimization plan for $\Omega(h)$ consists on maximizing (minimizing) (4.36) subject to the constraint $\sum_{j=1}^{n} h_i = 1$. We can write the Lagrangian function as $\mathcal{L}(\mathbf{h}, \gamma) = g(h_1) + g(h_2) + ... + g(h_n) + \gamma(1 - h_1 - ... - h_n)$, where γ is the Lagrange multiplier. The FOC's for the optimization problem are, consistently, given by:

$$\begin{cases} g'(h_1) & = \gamma \\ g'(h_2) & = \gamma \\ \vdots & \vdots \\ g'(h_n) & = \gamma. \end{cases}$$

[32] For a more formal reasoning, see Proof of Proposition 4.3 in Appendix B.

Since the RHS is the same for every FOC, we must have $g'(h_1) = g'(h_2) = ... = g'(h_n)$. Each $g'(h_i)$ is given by:

$$g'(h_i) = \frac{(1-\phi)(h_i n + \lambda)}{h_i n(1-\phi) + n\phi} + \ln\left[h_i(1-\phi) + \phi\right].$$

The second derivative $g''(h_i)$ is given by:

$$g''(h_i) = -\frac{(1-\phi)\left[\lambda - h_i n(1-\phi) + -\phi(\lambda + 2n)\right]}{n\left[h_i(1-\phi)+\phi\right)]^2},$$

which has either one zero for $h \in [0,1]$ or none. Therefore, $g'(h_i)$ is either concave or convex. This implies that, at most, two different values of $h_i \in [0,1]$ may correspond to $g'(h_i) = \gamma$. The consequence of this is that all potential maximizers (or minimizers) of $\Omega(h)$ are characterized by a vector $\boldsymbol{h} = (h_1, h_2, ..., h_n)$ such that k of its elements correspond to a share of entrepreneurs equal to h/k and the remaining $n-k$ elements have a share equal to $(1-h)/(n-k)$.[33] This concludes the proof. $\square$

Social welfare along the invariant space $\Omega(h \in \Delta_{inv})$**.** Social welfare $\Omega(h)$ attains a critical value at symmetric dispersion, because the derivatives of the farmers' and entrepreneurs' average utilities with respect to each h_i are both equal to zero. The other candidates for possible extrema are computed as follows. Equating $\Omega'(h)$ in (4.32) to zero, we get:

$$\lambda^*_{\max}(h) \equiv \lambda = \frac{3(3h-1)(1-\phi)\phi - 3\left[h(1-\phi)-\phi\right]\left[h(1-\phi)-\phi-1\right]\log\left[\dfrac{2(h(1-\phi)+\phi)}{h(\phi-1)+\phi+1}\right]}{(3h-1)(1-\phi)^2}.$$

It is possible to show that $\lambda^*_{\max}$ has at most two solutions for $h \in [0,1] \setminus \{1/3\}$.[34] This implies that $\Omega(h)$ has at most three critical points (including $h = 1/n$). Moreover, $\lambda^*_{\max}(h) < \lambda^*(h)$ defined in (4.25) for $n = 3$ and $h > 1/3$, which means that $\Omega'(h \in \Delta_{inv}) \neq 0$ when evaluated at a partial agglomeration equilibrium. Therefore,

[33] This includes both dispersion if $h = 1/n$ and agglomeration in any region $h = 1$.

[34] The analysis is similar to that of section 3.4.1. In fact, there exists at most one zero in $(0, 1/n)$, and at most two in $(1/n, 1)$.

no (market) stable partial agglomeration equilibrium corresponds to a socially optimal partial agglomeration. Differentiating $\Omega(h)$ twice yields a second order polynomial in h, which means that $\Omega''(h)$ has at most two zeros and thus $\Omega(h)$ changes concavity at most twice. This implies that $\Omega(h)$ can be in principle "W"-shaped or "M"-shaped. The former can be ruled out numerically which means that $\Omega(h)$ can have, at most, two maxima (including $h = 0$ and $h = 1$).[35]

Proof of Theorem 4.15.

(i). Define the summation term of $\Omega(h)$ in (4.31) as $F(h) = f(h_1)+f(h_2)+...+f(h_{n-1})+g(h_n)$, where $h_n : (h_1, h_2, ..., h_{n-1}) \mapsto 1 - h_1 - ... - h_{n-1}$. The second derivative of each $f(h_i)$ evaluated at $h_i = 1/n$ is given by:

$$\left.\frac{\partial^2 f(h_i)}{\partial^2 h_i}\right|_{h_i=\frac{1}{n}} = \frac{n(1-\phi)\left[\phi(\lambda+2n-1)-\lambda+1\right]}{\left[(n-1)\phi+1\right]^2}.$$

The derivative is negative if and only if:

$$\phi < \phi_w \equiv \frac{\lambda - 1}{\lambda + 2n - 1}.$$

Using (4.22), it is easily verified that $\phi_b < \phi_w$. If $\phi < \phi_b < \phi_w$, symmetric dispersion is stable and $f(h_i)$ is concave. Given that $f : \mathbb{R} \mapsto \mathbb{R}$ is strictly concave, replicating the reasoning from the proof of Theorem 4.11 allows us to conclude that $g(h_n) = f \circ h_n : \mathbb{R}^{n-1} \mapsto \mathbb{R}$ is also strictly concave. Therefore, $F(h)$ is strictly concave for $\mathbf{h} = (1/n, ..., 1/n)$ and $\Omega(h)$, a constant term plus $F(h)$, is also strictly concave at symmetric dispersion when the latter is stable. Since $\Omega(h)$ attains a critical value at symmetric dispersion, we conclude that the latter always attains a local maximum when it is stable.

Evaluating welfare at symmetric dispersion gives us:

$$\Omega\left(\frac{1}{n}, ..., \frac{1}{n}\right) = \frac{1}{\lambda+1}\left[\varepsilon + \frac{\mu(\lambda+1)}{(\sigma-1)}\ln\left(\phi + \frac{1-\phi}{n}\right)\right].$$

[35] And also, at most, three minima.

At agglomeration, welfare is given by:

$$\Omega(h_i = 1) = \frac{1}{\lambda + 1}\left[\varepsilon + \frac{\mu\lambda(n-1)}{(\sigma-1)n}\ln\phi\right].$$

This implies that agglomeration yields a higher welfare than dispersion dispersion if and only if:

$$\Delta\Omega \equiv \frac{\lambda(n-1)}{n}\ln\phi - (\lambda+1)\ln\left(\phi + \frac{1-\phi}{n}\right) > 0.$$

The difference $\Delta\Omega$ is concave in ϕ, has a zero for $\phi \in (0,1)$ and another at $\phi = 1$, and is negative at $\phi = \phi_b$. Symmetric dispersion is thus strictly better than agglomeration from a social point of view when the former is stable.

(ii). It can be shown that $\Omega'(1)$ is concave in ϕ with only one root $\phi_z \in (0,1)$ and another at $\phi = 1$. Moreover, it is negative when evaluated at the sustain point ϕ_s, which implies that $\phi_s < \phi_z$. Therefore, there exists a $\phi \in (\phi_s, \phi_z)$ where agglomeration is stable and $\Omega'(1) < 0$, meaning that welfare is higher at another less asymmetric distribution. This concludes the proof.

□

Chapter 5

Economic Geography meets Hotelling: a *home-sweet-home* effect

5.1 Introduction

New Economic Geography (NEG) seeks to explain the observed uneven spatial distribution of economic activities. In the prosecution of this objective, it hinges heavily on pecuniary factors such as inter-regional real wage differentials. As such, it frequently overlooks the fact that potential migrants are heterogeneous, not only regarding their tastes for manufactured goods, or their skill levels, but also regarding their idiosyncratic preferences for residential location. Countries, cities, or generically regions, typically differ in characteristics, each having its own tangible and intangible amenities. Some regions may be perceived as better for most people because they have a considerable advantage in these amenities compared to others (for instance, better provision of facilities such as communal areas, parks, lower crime rates, or more pleasant views). Other regional characteristics are perceived differently by different individuals. This is the case for cultural and historic amenities, whose advantages are subjective as they are typically perceived differently by different individuals and therefore each individual's response or attachment is more idiosyncratic (Rodríguez-Pose and Ketterer, 2012). In this sense, some places may have clear advantages over others for everyone, while others are only advantageous to certain people (Storper and Manville, 2006; Albouy *et al.*, 2015).

Most literature on residential location choice typically focuses on the first case, where all consumers respond in the same way to interregional differences. However, when consumers are differentiated with respect to their (cultural or historical) attachment to different regions, then regions which are more industrialized and developed may fail to attract some potential migrants from less developed regions. The fact that some people think that "there is no place like home" implies that agents may effectively be less willing to leave their region of origin. These individual idiosyncrasies in preferences towards different locations constitutes an actual dispersive force, a fact which should be considered in NEG (Fujita and Mori, 2005; Combes *et al.*, 2008), but is not envisaged by most models. Some notable exceptions in NEG are the works of Murata (2003), Tabuchi and Thisse (2002), Combes et al. (2008), Akamatsu *et al.* (2012) and Redding (2016), who have considered the role of heterogeneous location preferences borrowing from the literature of discrete choice theory (see McFadden, 1974). Another important contribution is that of Mossay (2003), where geography is treated in a continuous circular space, and individual preferences for residential location are assumed to follow a random walk process.

In discrete choice theory models of random utility, agents draw their utility from a deterministic observable component (e.g., consumption from manufactured goods) and another random unobservable component which represents their idiosyncratic tastes (like preferences for residential location that stem from intangible amenities). When each of the unobservable random components of utility are assumed to be independently, identically, extreme value Type I distributed, the choice probabilities for each alternative are given by the well known Logit model (McFadden, 1974). Since it has a very simple closed form, it is generally one of the first choices used for qualitative behavioural choice (Train, 2009). Under the logit, inter-regional migration thus responds to the realization of a random variable (Tabuchi and Thisse, 2002; Anderson *et al.*, 1992).

In this paper, we propose a Core-Periphery (CP) model where consumers are heterogeneous with respect to their preferences toward living in one country or the other. While the countries are dimensionless points in physical space, from a consumer's perspective, they are located at the opposite extremes of a virtual (or psychological) line segment, which we assume to be the unit interval. Each consumer's residential preference is then revealed by his location on the interval. The two consumers who are located at 0 and 1 are those who have an extremely high preference for just one region, and a lower preference for the other. The consumer at 1/2 has a similar preference for either countries. This is an adaptation of Hotelling's (1929) linear city to residential location choice, where two differentiated regions play the role of two horizontally differentiated goods. We thus attempt to reconcile Krugman's (1991) framework, grounded on market factors as determinants of spatial agglomeration, with a Hotelling framework that focuses on heterogeneity concerning non-market factors. This heterogeneity, by attaching consumers to their most preferred region, acts as a dispersive force on the distribution of economic activities.

The short-run equilibrium is identical to Murata (2003) and Tabuchi *et al.* (2014; 2016), with one single good manufactured under Dixit-Stiglitz monopolistic competition and a homogeneous workforce (regarding skill levels) that is interregionally mobile.[1] All consumers who live in a given country draw the same utility from consumption goods.

[1]To the best of our knowledge, the first attempt towards an NEG model where all labour is interregionally mobile was put forward by Helpman (1998), where the dispersive force stems from a nontradable housing sector.

However, each consumer bears an utility decrement which is a decreasing function of his preference toward residing in their country. Therefore, those who have a lower (higher) preference for a country (i.e., who are located farther away from one extreme of the virtual line segment) will get a lower (higher) overall utility if they reside in that country. In the long-run, consumers choose to live in the country that offers them the highest overall utility. The long-run spatial distribution of consumers can then be seen as the result of two counteracting forces. On the one hand, gains from agglomeration due to increasing returns generate a higher utility differential from consumption goods in the more populated country. This promotes the concentration of economic activities. On the other hand, the less a consumer wishes to live in a country (as revealed by his position on the unit interval), the higher the utility penalty differential he bears if he resides there. We call this utility penalty the *home-sweet-home* effect, which contributes to the dispersion of consumers across the countries. In the long-run, this effect is analogous to congestion costs that are increasing in the population size of a country.

We show that the probabilistic migration setting according to the Logit formulation (Tabuchi and Thisse, 2002; Murata, 2003; Akamatsu *et al.*, 2012) yields a specific closed form expression for the decrement in the utility function due to heterogeneous preferences in our model.[2] However, our framework is broader in the sense that we are able to take account of qualitatively different forms of heterogeneity. Borrowing from Hotelling's linear city model, we adapt the linear transportation costs function to our utility penalty.

The Logit model corresponds to a situation where the home-sweet-home effect (utility penalty differential) is exponentially increasing in the distribution of consumer preferences for residential location. For the consumer who has the highest preference toward a country, his utility penalty from residing in the other country tends to infinity. Intuitively, we can say that full agglomeration is precluded, because there is always a fraction of consumers (no matter how small) who will want to live in the less industrialized country due to their high personal attachment to it.

With a linear utility penalty, the home-sweet-home effect is linearly increasing in

[2] In this sense, the Hotelling model can be reinterpreted as a discrete choice random utility model (Anderson *et al.*, 1992).

the distribution of consumer preferences. Under this setting, we show that both the symmetric equilibrium and agglomeration may be simultaneously stable. If this occurs, selection between the two possible spatial outcomes depends on the initial spatial distribution, that is, "history matters" (Krugman, 1991; Matsuyama, 1991; Redding *et al.*, 2011). If spatial imbalances are initially very high, the relative utility in the more industrialized country is so high that all consumers will decide to agglomerate in one single country. Intuitively, even the consumers with stronger preferences will want to avoid exceedingly low standards of living. Conversely, if regional disparities are initially low, consumers will disperse evenly among the countries. The regional utility differential is not enough to offset even the consumers who make almost no distinction between the countries.

We show that, whatever the scale and distribution of consumer preferences for residential location, stability of the symmetric dispersion equilibrium (resp. agglomeration equilibrium) becomes more (less) likely and partial agglomeration becomes more symmetric as transportation costs fall. This happens because regional real wage differentials shrink as trade barriers fall, which reduces the incentives for workers to relocate from the less populated country to the more populated one. This is a reversion of the usual prediction that lower transport costs lead to agglomeration, and contrasts the findings that consumer heterogeneity leads to a bell-shaped relationship between decreasing transport costs and spatial inequality (Murata, 2003; Tabuchi and Thisse, 2002). Hence, when all labour is inter-regionally mobile, spatial imbalances decrease as regions become more integrated. One economic intuition is that the absence of an immobile workforce decreases the incentive for firms to relocate to smaller markets in order to capture a higher share of local demand and avoid competition when transport costs are higher. Krugman and Elizondo (1996), Helpman (1998) and Murata and Thisse (2005), have also obtained similar results concerning the relationship between transportation costs and spatial inequality. The first includes a congestion cost in the core, the second considers a non-tradable housing sector and the treatment is only numerical, while the latter's prediction cannot be disassociated from the interplay between interregional transportation costs and intra-regional commuting costs. Recently, Tabuchi *et al.* (2016) have reached similar conclusions to ours arguing that falling transport costs increase the incentives for firms in peripheral regions to increase production since they have a better access to the core, thus contributing to the dispersion of economic activities. Empirically consistent with our findings is the work of Allen and Arkolakis

(2014) who have estimated that the removal of the US Interstate Highway System, which, by limiting accesses, can be interpreted as an increase in transportation costs, would cause a redistribution of the population from more economically remote regions to less remote regions in the US.

It is the goal of this paper to dwell further in the role of consumer heterogeneity, showing that not only does it matters, but it has the potential to produce very different predictions about the spatial distribution of economic activities. As such, while the structure of spatial distributions remains invariant under changes in particular types of heterogeneity (like the Logit model), it changes when we introduce preferences that are qualitatively different. For instance, under linear or concave preferences, some of the predictions of the original CP model (Krugman, 1991), such as catastrophic agglomeration and locational hysteresis, are recovered.

Nevertheless, while heterogeneity is a contributing factor to reduced labour mobility, it alone bears no implications on the relation between decreasing trade barriers and the level of spatial inequality.

The rest of the paper is organized as follows. In section 2 we describe the model and the short-run equilibrium. In section 3 we study the qualitative properties of the long-run equilibria for a generic utility penalty. In section 4 we integrate discrete choice theory based migration in our set-up and compare its results with different specifications of the home-sweet-home effect. In section 5, we briefly discuss the impact of trade integration on spatial imbalances. Finally, section 6 is left for some concluding remarks.

5.2 The model

There are two countries L and R with a total population of mass 1. Consumers are assumed to have heterogeneous preferences with respect to the country in which they reside. For simplicity, we assume that these preferences are described by a parameter x, uniformly distributed along the interval $[0, 1]$. The consumer with preference described by $x = 0$ (resp. $x = 1$) has the highest preference for residing in country L (resp. country R). An agent whose preference corresponds to $x = 1/2$ is indifferent between either country. We can thus refer to each consumer with respect to his preference towards living in country L as $x \in [0, 1]$. The two countries L and R can be thought of as two points on opposite sides of the line segment $[0, 1]$. This interpretation is analogous to that of the linear city model by Hotelling (1929), which can be conveniently adapted to problems dealing with heterogeneity and diversity of agents across a wide array of economic domains (Rosen, 2002).

For a consumer with preference x, the utilities from living in country L and R are given, respectively, by:

$$\begin{aligned} U_L(x) =& U\left(C_L, t(x)\right) \\ U_R(x) =& U\left(C_R, t(1-x)\right), \end{aligned} \tag{5.1}$$

where C_i denotes the level of consumption of a consumer living in country $i \in \{L, R\}$, and $t(x)$ is the utility penalty associated with living in L, while $t(1-x)$ is the utility penalty a consumer faces from living in country R. Countries are symmetric in all respects. We assume that $t'(x) > 0$, $\partial U/\partial C > 0$, and $\partial U/\partial t < 0$.

The derivations and expressions for the short-run equilibrium are similar to those made by Murata (2003) and Tabuchi *et al.* (2014, 2016). This stems from the fact that we are considering only a monopolistically competitive sector that produces a tradeable good under transport costs. The consumption aggregate is a CES composite given by:

$$C = \left[\int_{s \in S} c(s)^{\frac{\sigma-1}{\sigma}} ds\right]^{\frac{\sigma}{\sigma-1}},$$

where s stands for the variety produced by each monopolistically competitive firm and σ is the constant elasticity of substitution between any two varieties. The consumer is subject to the following budget constraint:

$$P_i C_i = w_i,$$

where P_i is the price index and w_i is the nominal wage in region i. Utility maximization by a consumer in region i yields the following demand for each manufactured variety produced in j and consumed in i:

$$c_{ij} = \frac{p_{ij}^{-\sigma}}{P_i^{1-\sigma}} w_i, \tag{5.2}$$

where:

$$P_i = \left[\int_{s \in S} p_i(s)^{1-\sigma} d(s)\right]^{\frac{1}{1-\sigma}}, \tag{5.3}$$

is the manufacturing price index for region i. A manufacturing firm faces the following cost function:

$$TC(q) = w\left(\beta q + \alpha\right),$$

where q corresponds to a firm's total production of manufacturing goods, β is the input requirement (per unit of output) and α is the fixed input requirement. The manufacturing good is subject to trade barriers in the form of iceberg costs, $\tau \in (1, +\infty)$: a firm ships τ units of a good to a foreign country for each unit that arrives there. Assuming free entry in the manufacturing sector, at equilibrium firms will earn zero profits, which translates into the following condition:

$$\pi \equiv \left(p - \beta w\right) q - \alpha w = 0,$$

which gives the firm's total equilibrium output, symmetric across countries:

$$q_L = q_R = \frac{\alpha(\sigma - 1)}{\beta}, \tag{5.4}$$

and the following profit maximizing prices:

$$p_{LL} = \frac{\beta\sigma}{\sigma - 1} w_L \text{ and } p_{RL} = \frac{\beta\sigma\tau}{\sigma - 1} w_L. \tag{5.5}$$

The number of varieties in each region is, therefore, proportional to the number of agents:

$$\frac{n_L}{n_R} = \frac{h_L}{h_R} = \frac{h}{1-h},$$

where h is the fraction of agents residing in country L. Labour-market clearing implies that the number of agents in region L equals labour employed by a firm times the number of varieties produced in a region:

$$h = n_L\left(\alpha + \beta q_L\right),$$

from where, using (5.4), we get:

$$n_L = \frac{h}{\sigma\alpha} \text{ and } n_R = \frac{1-h}{\sigma\alpha}. \tag{5.6}$$

Choosing labour in region R as the numeraire, we can normalize w_R to 1. The price index P_L in (5.3) then becomes, after (5.6):

$$P_L = \left[\frac{h}{\sigma\alpha} \left(\frac{\beta\sigma}{\sigma - 1} w \right)^{1-\sigma} + \frac{1-h}{\sigma\alpha} \left(\frac{\beta\sigma\tau}{\sigma - 1} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{5.7}$$

whereas the price index in R is given by:

$$P_R = \left[\frac{1-h}{\sigma\alpha} \left(\frac{\beta\sigma}{\sigma - 1} \right)^{1-\sigma} + \frac{h}{\sigma\alpha} \left(\frac{\beta\sigma\tau}{\sigma - 1} w \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{5.8}$$

Rewriting a firm's profit in region L as:

$$\pi_L = (p_{LL} - \beta w) \left[hc_{LL} + (1-h)\tau c_{RL} \right] - \alpha w,$$

the zero profit condition yields, after using (5.2), (5.5) and (5.7), the following wage equation:

$$1 = \frac{hw^{1-\sigma}}{hw^{1-\sigma} + \phi(1-h)} + \frac{\phi(1-h)w^{-\sigma}}{1 - h + h\phi w^{1-\sigma}}, \tag{5.9}$$

where $\phi \equiv \tau^{1-\sigma} \in (0,1)$ is the *freeness of trade*. Since h is given in the short-run, we are interested in finding the nominal wage as a function of the spatial distribution, which is impossible to obtain explicitly, except for very specific values of σ. However, manipulating the expression in (5.9), we can write:

$$h = \frac{w^{\sigma} - \phi}{w^{\sigma} - \phi + w(w^{-\sigma} - \phi)}. \tag{5.10}$$

The nominal wage w can be implicitly defined as a function of the spatial distribution in country L, $h \in [0,1]$.[3] We have $w(0) = \phi^{1/\sigma} < 1$, $w(1/2) = 1$ and $w(1) = \phi^{-1/\sigma} > 1$. We say that L is larger than R when $h > 1/2$, and smaller otherwise. The nominal wage is strictly increasing in the spatial distribution (Proposition 6 in Appendix A).[4]

[3] The conditions for application of the Implicit Function Theorem are shown to be satisfied in Appendix A.

[4] This also guarantees that $h < 1$ in (5.10). This holds if if $w^{-\sigma} - \phi > 0$. Since $w^{-\sigma}(h) \in (\phi, \frac{1}{\phi})$, the statement is true.

Moreover, if $\sigma \geq 2$,[5] the nominal wage is convex when country L is larger than country R (Proposition 7 in Appendix A). The first result corresponds to the home-market effect identified by Krugman (1991). The second result states that the home-market effect is magnified as the industrial size difference between countries becomes more pronounced. Figure 5.1 illustrates $w(h)$ for $h \in [0, 1]$ with parameter values $\sigma = 2$ and $\phi = 0.5$.[6]

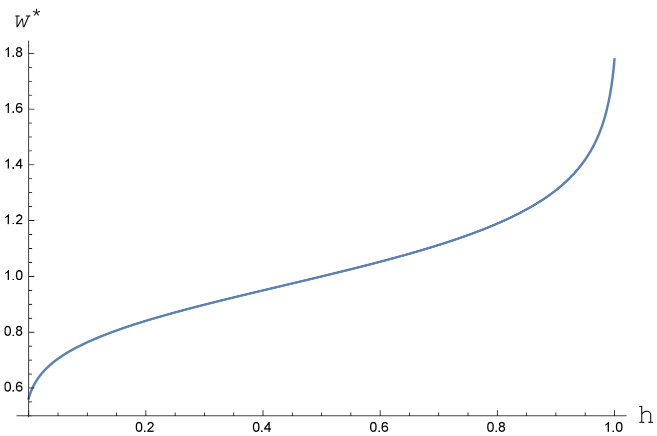

Figure 5.1 – Short-run equilibrium relative wage as a function of the consumers in country L. We set $\sigma = 2$ and $\phi = 0.5$.

Remark 5.1. When $h > 1/2$, the relative wage w increases when trade barriers are higher (see Proposition 8 in Appendix A). Conversely, if $h < 1/2$, higher trade barriers implies lower wages in L. Therefore, whatever the initial spatial distribution of consumers, higher trade barriers increase the wage divergence between the countries.

The intuition is as follows. When all workers are mobile, they can move to the region that offers them a relatively better access to varieties. This advantage of the larger country becomes higher as transport costs increase because markets become more focused on local demand (i. e., more localized). At the same time, the absence of a fixed number of immobile consumers in the smaller region implies that there is a reduced incentive to relocate production in order to capture a larger share of the market while avoiding costly transportation. All this increases expenditures in the

[5] In section 3.2 we discuss that this assumption is innocuous on empirical grounds.

[6] Our choice of parameters is made for graphical convenience and extends to the whole parameter range, unless stated otherwise. Therefore, the conclusions under our choice are consistent with more realistic empirical estimates.

larger region relative to the smaller, which in turn pushes the nominal relative wage upwards. Krugman (1991) identifies this as the *home-market* effect.

5.3 Long-run equilibria

We consider a general isoelastic sub-utility for consumption goods:

$$\begin{cases} u_i = \dfrac{C_i^{1-\theta} - 1}{1-\theta}, & \text{if } \theta \in [0,1) \cup (1,+\infty) \\ u_i = \ln C_i, & \text{if } \theta = 1, \end{cases},$$

where for $\theta = 1$ we take the limit value of the upper expression of u_i. The parameter $\theta = -C_i u_i''(C_i)/u_i'(C_i)$ is the rate at which marginal utility in country i decreases as consumption increases. It also influences how a change in the regional consumption differential, $C_L - C_R$, impacts the regional utility differential, $u_L - u_R$. If $\theta = 0$, changes in $u_L - u_R$ are proportional to the corresponding change in relative consumption. From an economic perspective, we can say that θ defines how the relative attractiveness of a country changes in response to relative regional consumption levels. It thus influences the strength of the self-reinforcing agglomeration mechanism when one country is more populated than the other.

In a long-run spatial equilibrium, h consumers live in country L and $1-h$ consumers live in country R. Knowing from utility maximization that $C = W/P$ and assuming that the utility is linear in $t(x)$, we rewrite the indirect utilities as:

$$\begin{aligned} V_L(h) &= \frac{C_L^{1-\theta} - 1}{1-\theta} - t(h) \\ V_R(h) &= \frac{C_R^{1-\theta} - 1}{1-\theta} - t(1-h), \end{aligned} \tag{5.11}$$

where h satisfies the short-run equilibrium in (5.10). Notice that $t(h)$ in (5.11) is derived from the way preferences for residential location affect each individual differently. However, its impact can be seen as analogous to that of congestion costs that do not affect the demand for consumption goods directly.[7] Take the case of pollution. In that

[7] One commonly source of congestion cost stems from increasing housing prices. However, these

instance, we could look at $t(h)$ as a cost of pollution that is increasing in the population (or industry) size in a country. In other words, our model has a correspondence with others where congestion costs depend solely on population size.

In the short run, the spatial distribution of consumers is given. In the long-run, we say that all consumers in the interval $(0, h)$ prefer to live in country L, whereas the consumers in $(h, 1)$ prefer to live in country R. The indifferent consumer h, identified by his position on the line, is thus obtained from equalization of indirect utilities $V_L(h)$ and $V_R(1-h)$. Using the price indices in (5.7), the indirect utilities in (5.11) become:

$$
\begin{aligned}
V_L(h) &= \eta \frac{w^{1-\theta}\left[(1-h)\phi + hw^{1-\sigma}\right]^{\frac{1-\theta}{\sigma-1}} - 1}{1-\theta} - t(h) \\
V_R(h) &= \eta \frac{\left[1-h+h\phi w^{1-\sigma}\right]^{\frac{1-\theta}{\sigma-1}} - 1}{1-\theta} - t(1-h),
\end{aligned}
\tag{5.12}
$$

where $\eta = \{(\sigma-1)/\left[\sigma\beta\right]\}(\sigma\alpha)^{1/(\sigma-1)}$.

Workers are free to migrate to the region that offers them the highest indirect utility. We focus on the difference between the regional utility in country L and the regional utility in country R. A consumer is indifferent between both countries if the utility differential from consumption goods, $\Delta u \equiv u_L - u_R$, is equal to the utility penalty differential $t(h) - t(1-h)$, with:

$$
\Delta u = \begin{cases} \dfrac{\eta}{1-\theta}\left\{ w^{1-\theta}\left[(1-h)\phi + hw^{1-\sigma}\right]^{\frac{1-\theta}{\sigma-1}} - \left[1-h+h\phi w^{1-\sigma}\right]^{\frac{1-\theta}{\sigma-1}} \right\}, & \text{if } \theta \neq 1 \\ \ln w + \dfrac{1}{\sigma-1}\ln\left[\frac{hw^{1-\sigma}+(1-h)\phi}{h\phi w^{1-\sigma}+(1-h)}\right], & \text{if } \theta = 1. \end{cases}
\tag{5.13}
$$

The utility penalty differential $t(h)-t(1-h)$ is what we call the *home-sweet-home effect*, i.e., the relative cost supported by the indifferent consumer h who lives in country L. The consumers who have an attachment towards country R (at $x \in (1/2, 1]$) face a positive cost when migrating to country L. Therefore, the home-sweet-home effect constitutes a dispersive force driving consumers from country L back to country R.

Henceforth, we adopt the normalizations by Fujita *et al.* (1999), i.e., $\alpha\sigma = 1$ so

---

typically also affect the consumer's budget constrain and thus could not correspond to our setting.

that the number of consumers in a country equals its number of firms. Moreover, we assume that $(\sigma - 1)/(\sigma\beta) = 1$ so that the price of each manufactured variety in a country equals its workers' nominal wage. These imply that $\eta = 1$. For $\theta = 1$, we may discard any of the preceding normalizations as taking differences from utilities cancels out $\ln[\eta/(1-\theta)]$. This means that, if utility in consumption is logarithmic, labour input requirements do not affect differences in regional utilities.

Proposition 5.2. *The utility differential* Δu *is increasing in* h.

Proof. See Appendix C. □

The nominal wage gap w increases as country L becomes more populated. The same happens to the price differential $P_R - P_L$ (see equations 5.7 and 5.8)[8], which constitutes an agglomerative cost-of-living effect. These results explain the fact that the relative utility from consumption goods in country L, as measured by the relative real wage, becomes larger and larger as the population in L increases. To counter this process of self-reinforcing tendency towards full agglomeration, we have the dispersive home-sweet-home effect; it is the interplay between these forces that ultimately determines the spatial configuration of the economy.

Figure 5.2 depicts Δu in (5.13) for different levels of θ and with parameter values $\phi = 0.5$ and $\sigma = 2$.

[8]Refer also to Tabuchi *et al.* (2016) for a more detailed analysis.

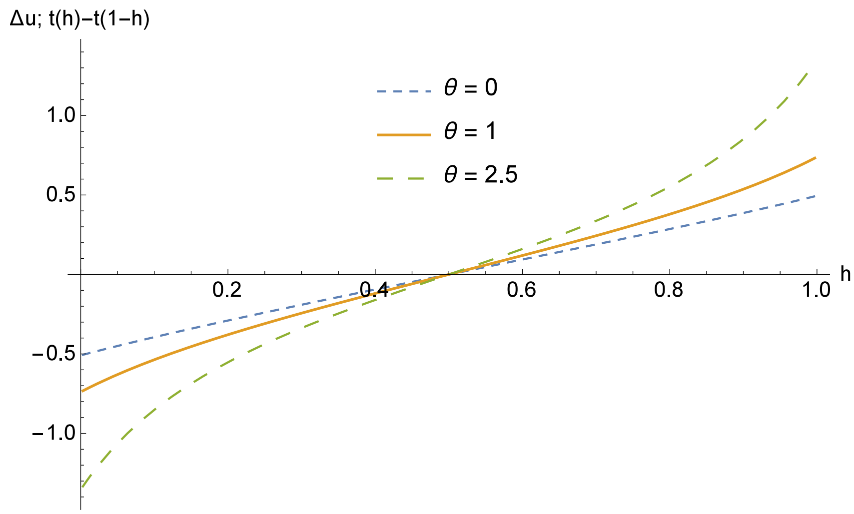

Figure 5.2 – Utility differential $\Delta u = u_L - u_R$ for different levels of θ. Parameter values are $\sigma = 2$ and $\phi = 0.5$.

We observe that a higher θ increases the utility differential Δu for any spatial distribution $h > 1/2$. Therefore, if L is relatively more industrialized, a higher θ increases the attractiveness of L relative to R for consumers. Thus, it strengthens the agglomeration forces towards country L.

5.3.1 Agglomeration

Since the model is symmetric in all respects, we focus on agglomeration in country L, without loss of generality. We say that agglomeration with all population residing in country L is an equilibrium if the individual with propensity $x = 1$ desires to stay at country L, which implies that all other individuals will also want to reside in L. This requires that the utility in the core, L, must be higher than the utility in the periphery, R. If these conditions are met, agglomeration is stable.[9] In formal terms, this implies that $u(1) - t(1) > u(0) - t(0)$. Alternatively, the utility differential $\Delta u(1)$ must exceed the home-sweet-home effect, $t(1) - t(0)$, when country L is the core. We have the following result.

Proposition 5.3. *Stability of agglomeration is as follows:*

[9]Therefore, one can say that agglomeration is stable if it is an equilibrium.

(i). When $\theta \neq 1$, it is stable if

$$\frac{1-\phi^{\frac{(2\sigma-1)(1-\theta)}{\sigma(\sigma-1)}}}{1-\theta} > t(1) - t(0); \tag{5.14}$$

(ii). When $\theta = 1$, it is stable if:

$$-\left[\frac{2\sigma-1}{\sigma(\sigma-1)}\right] \ln \phi > t(1) - t(0); \tag{5.15}$$

Proof. Agglomeration in L is stable if $\Delta u(1) > t(1) - t(0)$. Using the first expression in (5.13) at $h = 1$, for $\theta \neq 1$, we have:

$$\frac{1-\phi^{\frac{(2\sigma-1)(1-\theta)}{\sigma(\sigma-1)}}}{1-\theta} > t(1) - t(0), \text{ for } \theta \neq 1.$$

For $\theta = 1$, Δu at $h = 1$ is given by the limit of the expression above as θ tends to unity:

$$-\left[\frac{2\sigma-1}{\sigma(\sigma-1)}\right] \ln \phi > t(1) - t(0).$$

This concludes the proof. □

Agglomeration in country L is stable when the consumer who likes country L the least still prefers to live in L, rather then to stay alone in country R. This happens if the home-sweet-home effect is not too strong. The utility differential when all workers reside in country L, $\Delta u(1)$, is given by the LHS's of expressions (5.14) and (5.15). Analytical inspection shows that $\Delta u(1)$ is decreasing in ϕ (for any value of θ). Therefore, the utility gain from residing in the core instead of in the periphery is decreasing in the freeness of trade. This means that, as countries become more integrated, agglomeration becomes harder to sustain. As ϕ approaches unity, $\Delta u(1)$ approaches zero, and agglomeration is always unstable: when all consumers are mobile, the incentive to avoid fiercer competition in the core region (market crowding effect) is lower because no local demand exists in the periphery. In the absence of a fixed immobile local demand, the burden of costly transportation can be avoided altogether if everyone agglomerates in a single region.[10] Therefore, the tendency to agglomerate will be stronger when transport costs are higher.

This reverses the conclusion of many CP models, whereby decreasing transport costs foster agglomeration. One should note, however, that most CP models consider that part of the workforce is inter-regionally immobile, which means that local expenditure (even in smaller regions) is still significant such that firms in larger regions have an incentive to relocate their production in order to avoid fiercer local competition.[11]

From a numerical perspective, the utility differential $\Delta u(1)$ is increasing in θ.

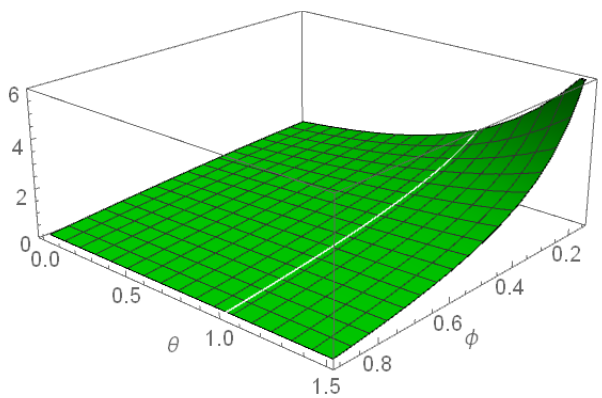

Figure 5.3 – Utility differential when all consumers reside in country L, given by the LHS of (5.14), in (ϕ, θ) space. The utility differential is increasing in θ for any $\phi \in (0, 1)$. We set $\sigma = 2$.

This is clear from Figure 5.3 where we depict $\Delta u(1)$ in (ϕ, θ) space. It shows that a higher θ increases the relative attractiveness of the core region and thus fosters agglomeration.

5.3.2 Interior equilibria

Again, due to symmetry, we focus only on the case where L is larger than R, i.e., $h \geq 1/2$. We define an interior equilibrium $h^* \in [1/2, 1)$ as a spatial distribution h

[10] This fact was noted by Baldwin *et al.* (2004).

[11] In most of these models (e.g., Krugman, 1999; Fujita *et al.*, 1999; Ottaviano *et al.*, 2002; Forslid and Ottaviano, 2003; Pflüger, 2004), the dispersive force is higher when transport costs are very high because markets are more localized. Therefore, as transport costs rise, centrifugal forces increase more than centripetal forces, promoting agglomeration.

that satisfies both (5.10) and $V_L = V_R$ (i.e., an interior equilibrium is given by the indifferent consumer defined at the beginning of section 3). Such an equilibrium is said to be stable if, after a small exogenous migration, the consumer with propensity to locate in country L just slightly lower than the indifferent consumer (i.e., the consumer located at $x + \varepsilon$), will still rather live in L. Formally, if $d\left(V_L - V_R\right)/dh < 0$.

5.3.2.1 Symmetric dispersion

The symmetric dispersion (equilibrium) corresponds to $h^* = 1/2$, which is always a solution to $V_L = V_R$ irrespective of the home-sweet-home effect. We establish the following result.

Proposition 5.4. *Symmetric dispersion is stable if:*

$$\frac{d\Delta u}{dh}\left(\tfrac{1}{2}\right) \equiv \frac{2(2\sigma-1)(1-\phi)\left(\frac{1+\phi}{2}\right)^{\frac{1-\theta}{\sigma-1}}}{(\sigma-1)(2\sigma+\phi-1)} < t'\left(\tfrac{1}{2}\right). \tag{5.16}$$

Proof. See Appendix C. □

Remark 5.5. The condition in (5.16) holds for $\theta \in [0, +\infty)$, as substituting for $\theta = 1$ yields:

$$\frac{2(2\sigma-1)(1-\phi)}{(\sigma-1)(2\sigma+\phi-1)} < t'\left(\tfrac{1}{2}\right),$$

which gives us exactly the same as computing stability using the second expression in (5.13) (see Appendix C).

A careful inspection of (5.16) allows us to conclude that the LHS is decreasing in ϕ if $\theta \geq 1$. For $\theta < 1$, it is decreasing in ϕ if $\sigma \geq 1 + \sqrt{2}/2 \approx 1.71$. Symmetric dispersion then becomes easier to sustain under lower transportation costs if $\sigma > 1 + \sqrt{2}/2$, which, according to recent empirical estimations for σ, is more than reasonable.[12] Since $t(h)$ does not depend on ϕ, this result does not depend on the home-sweet-home effect. As ϕ approaches unity, the LHS of (5.16) goes to zero and dispersion is always stable. Since, at symmetric dispersion, consumers in each country have access to the same amount of

[12]Estimations evidence that σ should be significantly larger than unity (Crozet, 2004; Head and Mayer, 2004; Niebuhr, 2006; Bosker *et al.,* 2010). Anderson and Wincoop (2004), for instance, find that it is likely to range from 5 to 10.

manufactures, an exogenous migration will induce a lower (higher) benefit from local consumption goods in the larger market if transport costs are lower (higher). This is captured by the fact that the relative decrease in prices and increase in wages (at the symmetric equilibrium) is more pronounced when transport costs are higher.

The derivative of the LHS of (5.16) with respect to θ is given by:

$$\left.\frac{d\,(d\Delta u/dh)}{d\theta}\right|_{h=\frac{1}{2}} = -\frac{2(2\sigma-1)(1-\phi)\left(\frac{1+\phi}{2}\right)^{\frac{1-\theta}{\sigma-1}}\ln\left(\frac{1+\phi}{2}\right)}{(\sigma-1)^2(2\sigma+\phi-1)} > 0,$$

implying that dispersion is less likely for higher values of θ. Therefore, if $\theta > 0$, the increase in consumption differential due to small increase in the population size of L leads to a more than proportional increase in the attractiveness of country L relative to R.

5.3.2.2 Partial agglomeration

When the home-sweet-home effect is neither too strong or too weak, agents may distribute themselves asymmetrically between country L and country R at an equilibrium called partial agglomeration. Our first result concerns stability of partial agglomeration in country L, i.e., $h^* \in (1/2, 1)$.

Proposition 5.6. *Partial agglomeration is stable if:*

$$\zeta\left\{\varphi\left[\frac{w^\sigma + w^{1-\sigma} - (w+1)\phi}{w\left(1-\phi^2\right)}\right]^{\frac{\theta-1}{\sigma-1}} - \psi\left[\frac{w^{2\sigma} - (w+1)\phi w^\sigma + w}{w\left(1-\phi^2\right)}\right]^{\frac{\theta+\sigma-2}{\sigma-1}}\right\} < t'\left(h^*\right) + t'\left(1-h^*\right), \tag{5.17}$$

where h^* satisfies the short-run equilibrium condition in (5.10), and:

$$\begin{aligned}
\zeta &= \frac{w^{-(\theta+\sigma+1)}}{(\sigma-1)\left[(\sigma-1)\phi + (\sigma-1)\phi w^{2\sigma} + \left(-2\sigma+\phi^2+1\right)w^\sigma\right]};\\
\varphi &= w\left\{\phi\left[\sigma+(\sigma-1)w\right]w^\sigma - 2\sigma w + w\right\}\left[w^{2\sigma} - (w+1)\phi w^\sigma + w\right];\\
\psi &= \left(1-\phi^2\right)w^{\theta+\sigma+1}\left[(2\sigma-1)w^\sigma - \sigma(w+1)\phi + \phi\right].
\end{aligned}$$

Proof. See Appendix D. □

It is extremely difficult to extract both numerical and analytical information from (5.17) for a general θ. However, we saw through Figure 5.2 that Δu is qualitatively invariant for a fairly low range of values for θ.[13] Therefore, we consider the log-utility case which corresponds to $\theta = 1$, yielding:[14]

$$-\frac{(2\sigma-1)w^{-(\sigma+1)}\left[w^{2\sigma}-(w+1)\phi w^{\sigma}+w\right]^2}{(\sigma-1)\left[(\sigma-1)\phi+(\sigma-1)\phi w^{2\sigma}+\left(-2\sigma+\phi^2+1\right)w^{\sigma}\right]} < t'\left(h^*\right)+t'\left(1-h^*\right), \quad (5.18)$$

Since Δu is increasing in h, the LHS of (5.18) cannot be negative. This requires $(\sigma-1)\phi+(\sigma-1)\phi w^{2\sigma}+\left(-2\sigma+\phi^2+1\right)w^{\sigma} < 0$, defining the following interval for the nominal wage:

$$w^{\sigma} \in \left(1, \frac{1}{2}\left\{\frac{2\sigma-(\phi^2+1)}{\phi(\sigma-1)}+\sqrt{\frac{(1-\phi^2)\left[(1-2\sigma)^2-\phi^2\right]}{(\sigma-1)^2\phi^2}}\right\}\right),$$

which we assume to hold henceforth.

Proposition 5.7. *For $\theta = 1$, a stable partial agglomeration becomes more symmetric as trade barriers decrease.*

Proof. See Appendix D. □

This proposition states that if most consumers reside in country L, increasing the freeness of trade will lead to a smooth exodus from L to country R, irrespective of the home-sweet-home effect. More integration thus increases the incentives for consumers to distribute more equally among the two countries.

5.4 The impact of the home-sweet-home effect

Our discussion so far has left out functional forms for $t(x)$. In this section we analyse the impacts of different functional forms for the utility penalty $t(h)$ on the spatial distributions. We will show that qualitative changes in preferences produce very different spatial patterns.

Assumption 5.8. *Let $\theta = 1$.*

[13] A higher θ increases Δu and its convexity for $h > 1/2$.

[14] The other natural candidate $\theta = 0$ yields linear utility but still produces significantly cumbersome expressions.

Under Assumption 1, we restrict the sub-utility to $u(C) = \ln C$. The parameter θ scales up the strength of agglomeration forces but has no impact on the home-sweet-home effect. Since our focus here is the role of consumer of heterogeneity, Assumption 1 allows us to uncover analytically tractable expressions and to convey our results in a clear way without loss of generality.[15]

5.4.1 Logit home-sweet-home effect

One of the most widely used models in discrete choice theory for choosing between different alternatives is the Multinomial Logit. For the two region case, and following Tabuchi and Thisse (2002), Murata (2003), Combes *et al.* (2008) and Akamatsu *et al.* (2012), the probability that a consumer will choose to reside in country L is given by the binary logit model written as:

$$P_L(h) = \frac{e^{U_L(h)/\mu}}{e^{U_L(h)/\mu} + e^{U_R(h)/\mu}}, \tag{5.19}$$

where $\mu \geq 0$ is a scale parameter which measures the dispersion of consumer preferences. If $\mu = 0$, consumers do not care about their location preferences but rather solely about relative wages. From section 3, the number of agents h is the same as the consumer x who is indifferent between living in country L or in country R. Therefore, the probability $P_L(h)$ is tantamount to the indifferent consumer h in our model. Thus, using (5.19), we can write:

$$h = \frac{e^{U_L(h)/\mu}}{e^{U_L(h)/\mu} + e^{U_R(h)/\mu}}. \tag{5.20}$$

Manipulating (5.20) yields:

$$U_L - \mu \ln h = U_R - \mu \ln(1-h). \tag{5.21}$$

Notice from the (5.21) that $t(h) = \mu \ln h < 0$ for $h \in [0, 1]$, so that the overall utility here is interpreted as the utility from consumption plus a benefit from living in the most preferred country. The overall utility of a consumer in region i thus lies on the interval $[U_i, +\infty]$. It is readily observable that, for a strictly positive μ, the consumer

[15] The qualitative behaviour of the model is indeed similar for a high range of values of θ.

who likes country R the most (at $x = 1$) will never want to live in L, because, at $h = 1$, the overall utility gain from moving from region L to region R is infinite. This means that the Logit penalizes (benefits) the consumers who are less (more) willing to leave a country very strongly. This result is reasonable in contexts where some people have an unbreakable attachment towards a given location. If the consumer who is more willing to leave a given country faces no costs in doing so, he will always migrate to the more preferred country. Therefore, agglomeration in any country is not possible. At the same time, the consumer who is just less willing to live in a country than the indifferent consumer will never want to live there, because the resulting gain from consumption goods is always outweighed by the utility penalty he faces from having to live in a less preferred country.

This discussion allows us to formalize the following result, which summarizes the possible spatial outcomes under Logit type preferences, depending on the degree of heterogeneity μ.

Proposition 5.9. *Under Logit type preferences, the spatial distribution depends on the level of consumer heterogeneity as follows:*

- *Symmetric dispersion is the only stable equilibrium if:*

$$\mu > \mu_d \equiv \frac{(2\sigma - 1)(1 - \phi)}{(\sigma - 1)(2\sigma + \phi - 1)}; \tag{5.22}$$

- *Partial agglomeration* $h^* \in \left(\frac{1}{2}, 1\right)$ *is the only stable equilibrium if* $\mu < \mu_d$.

Proof. See Appendix E. □

Figure 5.4 illustrates how changes in the scale parameter μ affect the resulting spatial distributions.

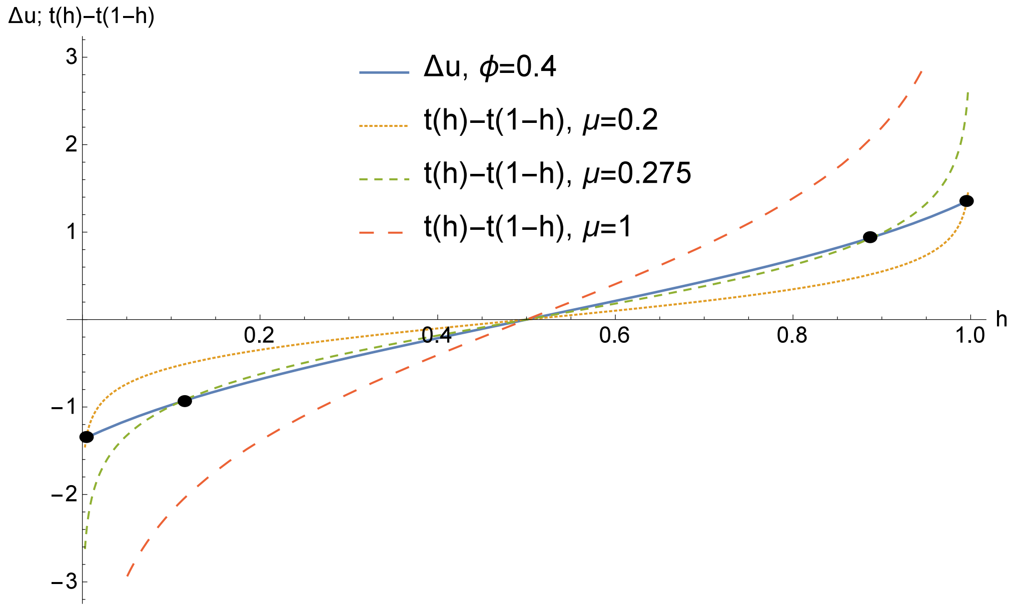

Figure 5.4 – Utility differential Δu (thick line) and penalty differentials $t(h) - t(1-h)$ (dashed lines) as μ changes. For $\mu = 0.2$ (lower dashed line), dispersion is unstable and partial agglomeration is stable. For $\mu = 0.275$ (medium dashed line) dispersion is unstable, partial agglomeration is stable and is less asymmetric. For $\mu = 1$ (upper dashed line) dispersion is stable and is the only equilibrium. Parameters are $\sigma = 2$ and $\phi = 0.4$.

Using $\sigma = 2$ and $\phi = 0.4$, Figure 5.5 shows that when consumer heterogeneity is low there is a single stable partial agglomeration equilibrium that is very asymmetric (close to agglomeration). As consumer heterogeneity increases, partial agglomeration corresponds to a more even distribution. Finally, if consumer heterogeneity is high enough ($\mu > \mu_d$ in (5.22)), the home-sweet-home effect is very high and so consumers disperse symmetrically across the two countries.

In Figure 5.5 we show how the spatial distribution of industry changes as countries become more integrated. We set $\sigma = 2.5$ and $\mu = 0.2$. Even with this value for μ (low overall heterogeneity), the home-sweet-home effect is very high for the more differentiated consumers near $x = 1$. Therefore, at partial agglomeration, there is a fraction $1 - h^*$ consumers for whom the home-sweet-home effect offsets the utility from consumption goods.

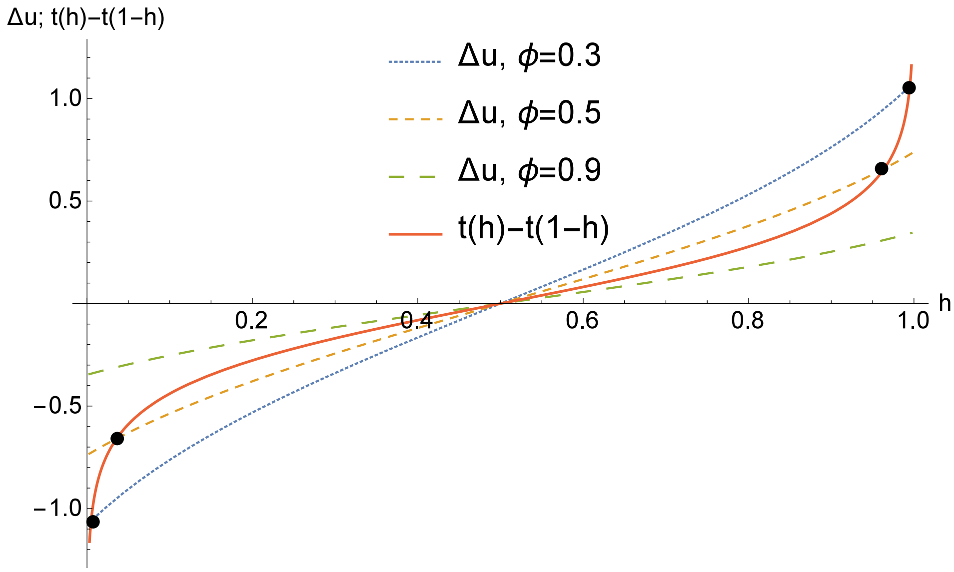

Figure 5.5 – Utility differential Δu (dashed lines) and penalty differential $t(h) - t(1-h)$ (thick line) for increasing levels of ϕ. For $\phi = 0.3$ (upper dashed line), dispersion is unstable and partial agglomeration is stable. For$\phi = 0.5$ (medium dashed line) dispersion is unstable, partial agglomeration is stable and is less asymmetric. For $\phi = 0.9$ (lower dashed line) dispersion is stable and is the only equilibrium. The values for the parameters are $\sigma = 2.5$ and $\mu = 0.2$.

For low levels of the freeness of trade, the home-market effect is stronger at symmetric dispersion, making it unstable. In this case, a unique stable partial agglomeration equilibrium exists where most consumers reside in country L. This means that there is just a small fraction of consumers in R that are not willing to forego their preferred region because the gain in consumption goods from doing so is not high enough. As countries become more integrated, the home-market effect becomes weaker implying that less consumers in R are willing to migrate to country L. This results in a unique partial agglomeration equilibrium which is more evenly populated between both countries. Finally, for a high level of inter-regional integration, agglomeration forces are so weak that no consumer is willing to leave his most preferred region. Therefore, only symmetric dispersion is stable.

The preceding analysis is summarized in the two bifurcation diagrams in Figure 5.6, along a smooth parameter path where μ and ϕ increase. We numerically uncover two pitchfork bifurcations in μ and ϕ, which accurately fit the situations described for Figures5.4 and 5.5. The picture to the right in Figure 5.6 (increasing ϕ) differs from other supercritical pitchforks found in NEG literature (e.g., Pflüger, 2004) crucially in the sense that the direction of change in stability as ϕ increases is reversed, i.e., lower trade barriers leads to more symmetric spatial distributions.

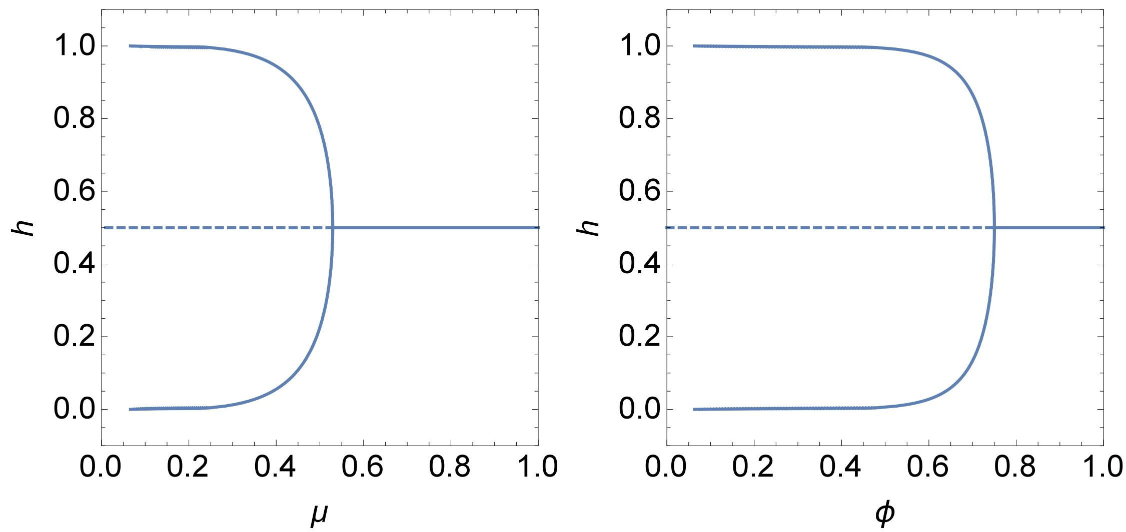

Figure 5.6 – Bifurcation diagrams. To the left, the bifurcation parameter is $\mu \in [0, 1]$ with $\phi = 0.4$. To the right, we have $\phi \in (0, 1)$ as the bifurcation parameter and set $\mu = 0.2$. For both scenarios we use $\sigma = 2$.

5.4.2 A linear home-sweet-home effect

We now consider different functional form for the home-sweet-home effect. We choose a utility penalty $t(h)$ that allows us to relax the "extreme convexity" implicit in the Logit specification, by considering a linear function for the utility penalty, $t(x) = \mu x$, where μ is again a scale parameter.[16] The home-sweet-home effect $t(h) - t(1 - h)$ is also linear in h. One important implication is that the consumer's overall utility V_i is bounded for the consumers at the ends of the distribution, $x = \{0, 1\}$. The next result summarizes the different spatial outcomes under the linear home-sweet-home effect.

Proposition 5.10. *When the home-sweet-home effect is linear in h, the possible spatial patterns are: (i) agglomeration for a low degree of heterogeneity; (ii) either agglomeration or dispersion for an intermediate degree of heterogeneity; and (iii) symmetric dispersion for a high degree of heterogeneity.*

[16]From an analytical point of view, most discrete choice models do not allow for closed form expressions. Therefore, we choose a simple enough alternative that allows us to illustrate the impact of different settings. Other more empirically founded settings could and should be considered.

Proof. See Appendix E. □

We learn from Proposition 5.10 that dispersion and agglomeration are both stable for an intermediate degree of heterogeneity. Hotelling-type consumer heterogeneity allows us to interpret simultaneously stable equilibria in light of the relation between regional income disparities and location preferences. When income inequalities are very low, the differential is not high enough to trigger migration because region specific amenities are relatively more important to potential migrants. In this case, even the consumer who is slightly less indifferent than the indifferent consumer $x = 1/2$ will rather live in his most preferred region. However, as discrepancies increase, potential migrants start to value the gain in real wages more than they value their personal attachment towards a given region. At the opposite extreme of regional disparity, even the consumer who likes the periphery the most would rather live in the core than have to bear a very low (relative) standard of living.

Following the graphical analysis of section 4.1, we first depict the change in the spatial distributions as the scale parameter μ increases.

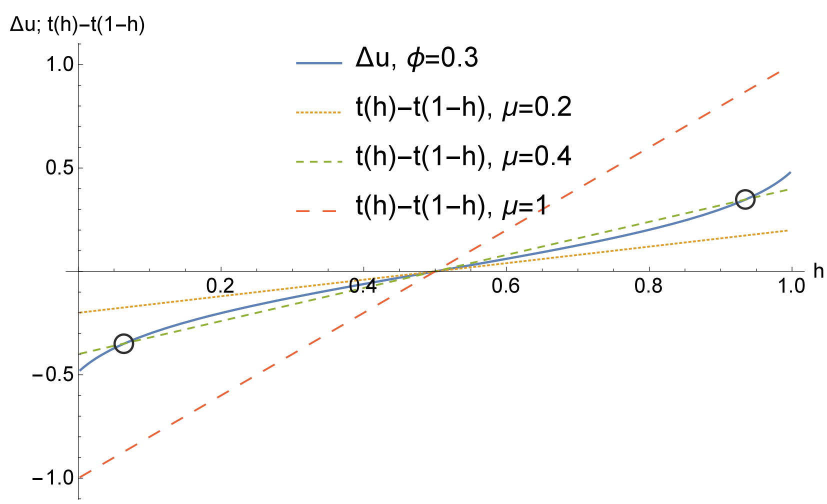

Figure 5.7 – Utility differential Δu (thick line) and penalty differentials $t(h)-t(1-h)$ (dashed lines) as μ changes. For $\mu = 0.2$ (lower dashed line), only agglomeration is stable. For$\mu = 0.4$ (medium dashed line) both agglomeration and dispersion are simultaneously stable. For $\mu = 1$ (upper dashed line) dispersion is the only (stable) equilibrium. Parameters are $\sigma = 5$ and $\phi = 0.3$.

We set $\sigma = 5$ and $\phi = 0.3$. In Figure 5.7, we now observe that when consumer heterogeneity is sufficiently low, all workers will agglomerate in the largest country. If consumer heterogeneity is at an intermediate level: consumers will agglomerate in the largest country if initial disparities are very pronounced; otherwise, consumers will disperse evenly between the countries. Thus, history matters. Finally, if consumer heterogeneity is very high, the only possible outcome is symmetric dispersion. Figure 5.8 illustrates the possible stable equilibria for an increasing level of integration, a fixed level for the degree of heterogeneity $\mu = 0.2$, and $\sigma = 5$.

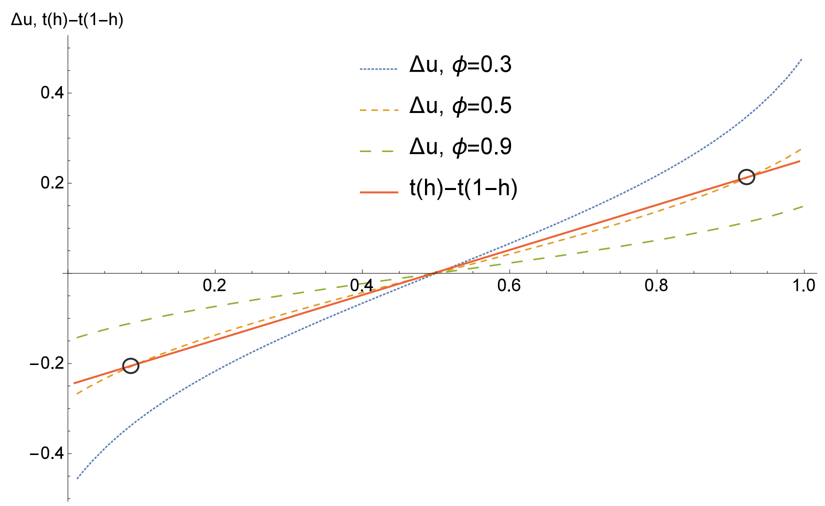

Figure 5.8 – Utility differentials Δu (dashed lines) and penalty differential $t(h) - t(1-h)$ for increasing levels of ϕ. For $\phi = 0.3$ (upper dashed line), only agglomeration is stable. For$\phi = 0.5$ (medium dashed line), both agglomeration and dispersion are simultaneously stable . For $\phi = 0.9$ (lower dashed line) dispersion is stable and is the only equilibrium. We set $\sigma = 5$ and $\mu = 0.2$.

With low values for the freeness of trade there can only be full agglomeration. For intermediate values, both agglomeration and dispersion are stable, and the historical relative sizes of the country matter for the selection of the spatial equilibrium. Finally, when the freeness of trade is very high, the home-sweet-home effect outweighs the agglomerative forces and symmetric dispersion is the only (stable) equilibrium.

In Figure 5.9 we present bifurcation diagrams for variations in μ and ϕ. These diagrams provide a comprehensive view of Figures 5.7 and 5.8. Under the linear home-sweet-home effect, we recover some of the classical predictions of the original CP model (or its "identical twins" (Robert-Nicoud, 2005)). Namely, the model exhibits locational hysteresis and there is the possibility for catastrophic agglomeration (Fujita *et al.,* 1999; Baldwin *et al.,* 2004). The main difference is that the direction in the change of stability as ϕ increases is reversed compared to other subcritical pitchforks identified in NEG literature. As noted before, this happens because higher integration encourages dispersion and discourages agglomeration. It is a temporary increase in transport costs, rather than a decrease, above some threshold level, that may trigger permanent agglomeration in one single country.

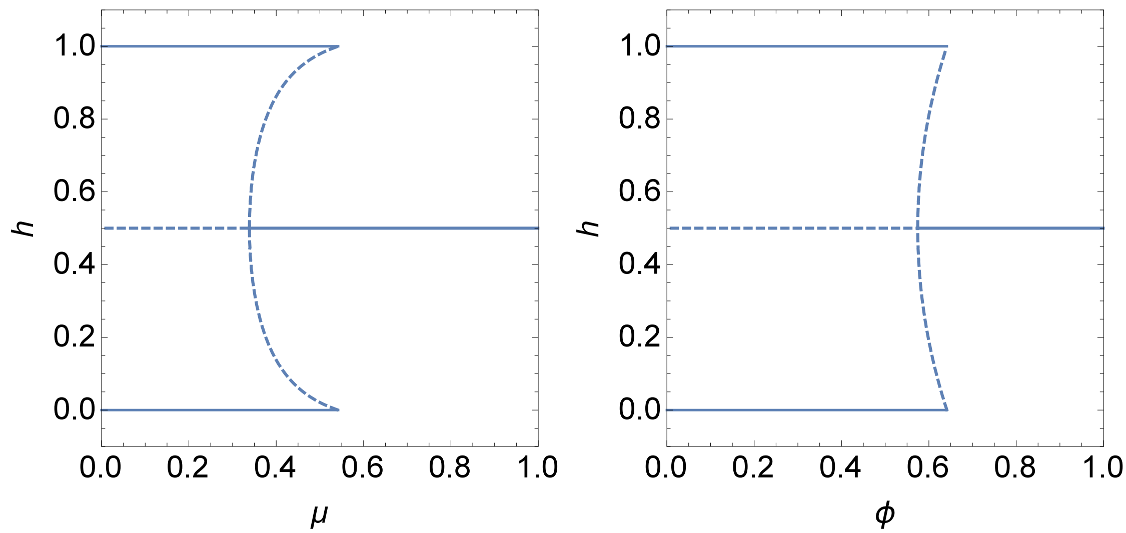

Figure 5.9 – Bifurcation diagrams. To the left, the bifurcation parameter path is $\mu \in [0,1]$ and we set $\phi = 0.3$. To the right, we have $\phi \in (0,1)$ as the bifurcation parameter and set $\mu = 0.2$. For both scenarios we use $\sigma = 5$.

Remark 5.11. Any functional form for which the home-sweet-home effect $t(h)-t(1-h)$ is concave rather than (strictly) convex for $h > 1/2$ provides spatial configurations that are qualitatively similar to those of the linear case.

5.5 A note on trade integration and spatial inequality

Although qualitative differences in the home-sweet-home effect produce significant impacts on structure of the space economy, heterogeneity alone bears no impact on the relationship between trade integration and spatial inequality, which is a monotonic decreasing one. This contrasts the findings in other works with heterogeneity in consumer preferences, such as Tabuchi and Thisse (2002) and Murata (2003), who show evidence of a bell-shaped relationship between trade integration and spatial inequality. The former's setting differs from ours because the authors consider an inter-regionally immobile workforce whose role as a dispersive force is enhanced by higher transportation costs.

However, it is particularly worthwhile to discuss the results of Murata (2003), because his setting is a particular case of our model. Formally, Murata's (2003) model is obtained by setting $\theta = 0$ (linear utility) and $t(h) = \mu \ln h$ (Logit type heterogeneity). Murata (2003) found that the relationship between trade integration and spatial inequality need not be monotonic and depends on the degree of consumer heterogeneity, which is at odds with our findings. For instance, for an intermediate degree of consumer heterogeneity, the author finds that increasing trade integration initially fosters agglomeration and later leads to re-dispersion of industry. However, these conclusions can be shown to stem from the author's particular choice of the value for the elasticity of substitution, $\sigma = 1.25$. As we have argued in Section 3.2.1, such a low value is empirically implausible. For exceedingly low values ($\sigma < 1.71$), increasing returns at the firm level are too strong. Strong enough that the utility gain at dispersion becomes increasing in ϕ, instead of decreasing. This would justify an initial concentration of industry as a result of an increase in ϕ. However, we have seen that for a plausible range of σ the utility gain at dispersion always decreases with ϕ. Moreover, if $\theta > 0$, this holds even for lower values of σ.[17] Therefore, a higher ϕ always promotes more equitable distributions as opposed to asymmetric ones. These findings are shown to hold in a generalized setting.

Hence, when workers are completely mobile, more trade integration ubiquitously reduces the spatial inequalities between the two countries, irrespective of the degree of heterogeneity in location preferences. Therefore, a *de facto* lower inter-regional labour mobility induced by consumer heterogeneity alone cannot account for the predictions that a higher inter-regional integration will lead to more unequal spatial development or an otherwise bell-shaped relationship between the two.

5.6 Concluding remarks

It is widely accepted that individual idiosyncrasies governing preferences over specific locations with a different set of cultural or historical amenities constitute an effective deterrent of inter-regional migration. This helps explaining why some people refuse to

[17]If $\theta \geq 1$, the result holds for $\sigma > 1$.

move to regions where they could otherwise improve their standard of living (as measured exclusively by pecuniary factors). Therefore, heterogeneity concerning preferences for residential location can be seen as a contributing factor for the reduced inter-regional mobility observed in some spatial contexts.

We have built a New Economic Geography model that allows us to arbitrarily specify how the utility from residing in a region changes across different consumers. Modelling the individual utility penalty of migrating to a given location is important because it impacts regional utility differentials with consequent implications on the spatial distribution of economic activities. Consumer heterogeneity toward residential location is usually modelled through probabilistic migration according to the discrete choice Logit model. This imposes an assumption on the distribution of consumer preferences which implies that some consumers would bear a psychological cost, if they migrated to a less preferred region, that is just too great. Therefore, no matter how large the gains from agglomeration due to increasing returns and transportation costs, some people will always choose to live in a relatively poor region. We acknowledge that, in some geographical contexts, some people are in fact too attached to a given location, which would help sustain the claim that full agglomeration in one single region is unlikely. This is even more so when regions have their very own and distinct sets of cultural and historical amenities. However, the importance of these amenities is likely to vary both quantitatively and qualitatively according to the geographical scale. For instance, cultural and historical differences are generally more important at a transnational scale than at the national scale. This would make individuals more reluctant to move to another country than to move to another region within his country.

While changes in the Logit model allow to account for different heterogeneity scales (Scarpa et al., 2008; Train, 2009; Hess and Rose, 2012), they do not capture the fact that consumer preferences may vary qualitatively. For instance, with the Logit, consumers with the highest personal attachment towards a region are always heavily penalized if they migrate to another region.

We illustrate our point of view, by using a very simple framework, where we allow the utility penalty to be a linear (or otherwise strictly concave) function of the distribution of consumer preferences. In the long-run, this means that the more personally attached consumers face a lower utility penalty when they migrate to a less preferred region. This increases the willingness to migrate as a response to regional differences in consumption. Given the pecuniary gains from agglomeration, this allows us to obtain a relationship between agents' reaction to regional non-market factors and regional income inequalities that is potentially empirically relevant. Specifically, we find that when regional size differences are small, the gains in consumption from relocating to the slightly larger region are not enough to offset the decrement in utility of even the consumers who have a just marginally higher preference for the relatively smaller region. However, if initial spatial disparities are very high, then so is the prospective gain in consumption goods of those who consider relocating from the smaller to the larger region. This gain is large enough that it offsets the personal attachment of any consumer toward the less populated region. In this case, the initial spatial distribution will determine if there is a tendency towards spatial convergence or divergence. In other words, history matters.

The variety of possible spatial outcomes conveyed by just two different specifications for consumer preferences, while overlooking other well-known potential determinants of spatial inequality, highlights the importance of considering qualitatively different distributions of individual sensitivities.

By considering that there are no other impediments to migration, i.e., all consumers are allowed to migrate if they so desire, we have shown that a higher inter-regional trade integration always leads to less spatial inequality. This result is independent of the level and impact of consumer heterogeneity.

Acknowledgements: We are thankful to Pascal Mossay and participants in the 2016 RSA Student and Early Career Conference in Newcastle for very useful comments and suggestions. Financial support by Portuguese Public Funds through Fundação para a Ciência e Tecnologia (FCT) in the framework of the Ph.D. scholarship SFRH/BD/90953/2012 is dully acknowledged.

5.A - Wages and freeness of trade

Consider $h = f(w)$ in (5.10). Let us now define $F(h, w) = f(w) - h = 0$. Differentiation of $F(h, w)$ yields:

$$\frac{\partial F(h,w)}{\partial w} = \frac{w^\sigma G(w^\sigma)}{\left[w^{2\sigma} - (w+1)\phi w^\sigma + w\right]^2}, \tag{5.23}$$

where:

$$G(w^\sigma) = -\left[\phi(\sigma - 1) + (\sigma - 1)\phi w^{2\sigma} - \left(2\sigma - \phi^2 - 1\right) w^\sigma\right].$$

The derivative in (5.23) is zero if $G(w^\sigma) = 0$. One can observe that $G(w^\sigma)$ has either two (real) zeros given by some $\{w^-, w^+\}$, or none. Moreover, $G(w^\sigma)$ is concave in w^σ. Since $G(w^\sigma = \phi) = \sigma\left[\phi(1 - \phi^2)\right] > 0$ and $G\left(w^\sigma = \phi^{-1}\right) = -\left[\sigma\phi\left(1 - \phi^{-2}\right)\right] > 0$, it must be that $G(w^\sigma) > 0$ for $w^\sigma \in [\phi, \phi^{-1}] \subset (w^-, w^+)$.

We now proceed to show that $w \in \left(0, \phi^{1/\sigma}\right)$ and $w \in \left(\phi^{-1/\sigma}, +\infty\right)$ are not defined in $h \in [0, 1]$. Using (9), we have the following:

$$\lim_{w \to 0} h(w) = 0; \quad \lim_{w \to +\infty} h(w) = 1; \quad h(\phi^{1/\sigma}) = 0; \quad h(\phi^{-1/\sigma}) = 1;$$

Since $dh/dw = \partial F/\partial w$, we have that $h(w)$ is increasing for $w^\sigma \in (w^-, w^+)$. The limits above, together with the knowledge that the zeros of dh/dw lie to the left and right of $[\phi, \phi^{-1}]$, ensure that $h(w) < 0$ for $w \in \left(0, \phi^{1/\sigma}\right)$ and $h(w) > 1$ for $w \in \left(\phi^{-1/\sigma}, +\infty\right)$.

Knowing that $\partial F/\partial h = -1$ and $\partial F/\partial w$ are continuous, by the IFT we can write $w : h \in [0, 1] \subset \mathbb{R} \mapsto \mathbb{R}$ such that dw/dh exists and $F(h, w(h)) = 0$.

Proposition 5.12. *The nominal wage $w(h)$ is increasing in the spatial distribution.*

Proof. Using implicit differentiation on (5.10), we get:

$$\frac{dw}{dh} = \frac{\left[w^{2\sigma} - (w+1)\phi w^\sigma + w\right]^2}{w^\sigma G(w^\sigma)}. \tag{5.24}$$

This derivative is positive for $w \in [\phi^{1/\sigma}, \phi^{-1/\sigma}]$, which implies that $w(h)$ is increasing in $[0, 1]$ and concludes the proof. □

Proposition 5.13. *The nominal wage is convex in the spatial distribution.*

Proof. Using $F(w,h)$ to find d^2w/dh^2, we know that:

$$\underbrace{\frac{\partial^2 F}{\partial h^2}}_{0} + \frac{\partial^2 F}{\partial w^2}\left(\frac{dw}{dh}\right)^2 + \underbrace{\frac{\partial F}{\partial w}}_{>0}\frac{d^2 w}{dh^2} = 0,$$

from which we obtain:

$$\frac{d^2w}{dh^2} = -\frac{\frac{\partial^2 F}{\partial w^2}\left(\frac{dw}{dh}\right)^2}{\frac{\partial F}{\partial w}}. \tag{5.25}$$

We can compute $\partial^2 F/\partial w^2$ as:

$$\frac{\partial^2 F}{\partial w^2} = -\frac{w^{\sigma-1}P}{\left[w^{2\sigma} - (w+1)\phi w^{\sigma} + w\right]^3}, \tag{5.26}$$

where:

$$\begin{aligned} P = &- w^{4\sigma}(\sigma-1)\sigma\phi - w^{3\sigma}\left\{2\sigma(1-2\sigma) + \phi^2\left[\sigma^2+\sigma+(\sigma-2)(\sigma-1)w\right]\right\} + \\ &+ w^{2\sigma}\phi\left\{w\left[3(\sigma-3)\sigma + 2\left(\phi^2+2\right)\right] - 3(\sigma-1)\sigma\right\} + w^{\sigma}(\sigma-1)\left\{\phi^2\left[\sigma+(\sigma+4)w\right] + (2-4\sigma)w\right\} + \\ &+ (\sigma-2)(\sigma-1)w\phi, \end{aligned}$$

which is a 4th degree polynomial in w^{σ}. Therefore, $P(X = w^{\sigma})$ has either zero, two or four real roots. Since $w^{\sigma} = 0$ if and only if $w = 0$, $X = 0$ is one solution to $P(X) = 0$.

Let us focus on the case where $\sigma \geq 2$.[18] We have the following results concerning $P(X)$.

(i). Evaluating $P(X)$ at $X = \phi$ yields:

$$P(X=\phi) = \sigma\phi\left(1-\phi^2\right)\left\{2\sigma\phi^2 + \left[(\sigma-3)\phi^2 + 3(1-\sigma)\right]\phi^{1/\sigma}\right\},$$

which, by substituting $\sigma = 2$, becomes:

$$-2\phi^{3/2}\left(1-\phi^2\right)\left(3+\phi^2-4\phi^{3/2}\right),$$

[18] Empirically, we find this to be a more than reasonable assumption. See the discussion in Section 3.2 for more details.

which is negative since $3+\phi^2-4\phi^{3/2}>0$. Taking the derivative of $\left[(\sigma-3)\phi^2-3(\sigma-1)\right]\phi^{1/\sigma}+2\sigma\phi^2$ with respect to σ yields:

$$\frac{d\,(.)}{d\sigma}=-\frac{\left[(\sigma-3)\phi^2+3(1-\sigma)\right]\phi^{1/\sigma}\log\phi}{\sigma^2}-\left(3-\phi^2\right)\phi^{1/\sigma}+2\phi^2.$$

This derivative is negative, since

$$-\left(3-\phi^2\right)\phi^{1/\sigma}+2\phi^2<0,$$

and

$$-\left\{\left[(\sigma-3)\phi^2+3(1-\sigma)\right]\phi^{1/\sigma}\log\phi\right\}/\sigma^2<0.$$

Therefore, the term $\left[(\sigma-3)\phi^2-3(\sigma-1)\right]\phi^{1/\sigma}+2\sigma\phi^2$ is negative for all $\sigma\geq 2$, which implies that $P(X=\phi)<0$ for all $\sigma\geq 2$.

(ii). Evaluating $P(X)$ at $X=1$ gives us the following:

$$P(X=1)=2(1-\phi)^2(2\sigma+\phi-1)>0.$$

(iii). The derivative of $P(X)$ at $X=0$ is positive, since:

$$\left.\frac{dP(X)}{dX}\right|_{X=0}=(\sigma-1)\sigma\phi^2.$$

From (i) to (iii), we conclude that two more zeros exist for $X\in(0,1)$.

(iv). We have the following limit:

$$\lim_{X\to\infty}P(X)=-\infty,$$

because the coefficient of the term of highest order is negative. Using (ii) and (iv), we then know that the last root of $P(X)$ lies in the interval $(0,+\infty)$.

(v). Evaluating $P(X)$ at $X=1/\phi>1$ yields:

$$P\left(X=\frac{1}{\phi}\right)=-\sigma\left(1-\phi^2\right)\phi^{-\frac{1+3\sigma}{\sigma}}\underbrace{\left\{2\sigma\phi^2+\left[(\sigma-1)\phi^2-3\sigma+1\right]\phi^{\frac{1}{\sigma}}\right\}}_{\Omega}.$$

Substituting, as in (i), $\sigma = 2$ in Ω, we get:

$$\phi\left(4\phi + \phi^2 - 5\right) < 0.$$

Taking the derivative of Ω with respect to σ yields:

$$\frac{d\Omega}{d\sigma} = \underbrace{2\phi^2 + \phi^{1/\sigma}(\phi^2 - 3)}_{A} - \frac{\ln\phi}{\sigma^2}\underbrace{\left[(\sigma - 1)\phi^2 - 3\sigma + 1\right]}_{B}.$$

We have $B < 0$ and $A < 0$. Therefore, $d\Omega/d\sigma < 0$. As a result, $P\left(X = 1/\phi\right) > 0$, for all $\sigma \geq 2$. Thus, we conclude that the last root of $P\left(X\right)$ lies in the interval $(\phi^{-1}, +\infty)$, thus implying that $P(X) > 0$ for $X \in [1, \phi^{-1}]$.
(vi). The denominator in expression (5.26) is positive if:

$$w^{2\sigma} - (w+1)\phi w^{\sigma} + w > 0.$$

This statement is true if $w^{2\sigma} + w - (w+1)w^{\sigma} > 0$. Substituting X for w^{σ} we get

$$Q(X) \equiv X^2 - (w+1)X + w,$$

which has roots $X = 0$ and $X = 1$. Since $Q(X)$ is convex, it is positive for $X > 1$ implying that $w^{2\sigma} - (w+1)\phi w^{\sigma} + w > 0$ if $h \in [1/2, 1]$.

From (i) to (vi) we can thus conclude that $\partial^2 F/\partial w^2 < 0$ for $h \in [1/2, 1]$. Hence, from expression (5.25) we know that $d^2 w/dh^2 > 0$ for $h \in [1/2, 1]$. In other words, the nominal wage is convex in the spatial distribution when L is the largest country. □

Proposition 5.14. *The nominal wage w is decreasing in the freeness of trade when L is the largest country ($h > 1/2$).*

Proof. From $F(h, w) = 0$ defined above, differentiating with respect to ϕ, we get:

$$\frac{\partial F}{\partial \phi} + \frac{\partial F}{\partial w}\frac{dw}{d\phi} = 0 \iff$$
$$\frac{dw}{d\phi} = -\left.\frac{\partial F}{\partial \phi}\right/\frac{\partial F}{\partial w}.$$

Using (5.10) we get:

$$\frac{\partial F}{\partial \phi} = \frac{w^{\sigma+1}\left(w^{2\sigma}-1\right)}{\left[w^{2\sigma}-(w+1)\phi w^{\sigma}+w\right]^{2}},$$

which is positive if $h > 1/2$, because the latter implies $w > 1$ (recall from (1) in the Appendix that w is increasing in h). As a result, we have $dw/d\phi < 0$. □

5.B - Utility differentials

In this Appendix we establish the sign of the derivative of the utility differential Δu with respect to h.

Proof of Proposition 5.2.

(1). General isoelastic utility with $\theta \in [0,1) \cup (1,+\infty)$.

Using the chain rule we can write the derivative of Δu with respect to h as follows:

$$\frac{d\Delta u}{dh} = \frac{\partial \Delta u}{\partial h} + \frac{\partial \Delta u}{\partial w}\frac{dw}{dh}.$$

First:

$$\frac{\partial \Delta u}{\partial h} = \frac{w^{1-\theta}\left(w^{1-\sigma}-\phi\right)\left[h\left(w^{1-\sigma}-\phi\right)+\phi\right]^{-\frac{\theta+\sigma-2}{\sigma-1}} + \left(1-\phi w^{1-\sigma}\right)\left[h\left(\phi w^{1-\sigma}-1\right)+1\right]^{-\frac{\theta+\sigma-2}{\sigma-1}}}{\sigma-1}.$$

We shall restrict to the case where $h \in (1/2, 1]$, i.e., $w \in (1, \phi^{-1/\sigma}]$. We have $w^{1-\sigma} - \phi > 0$, because $w^{1-\sigma}$ is decreasing in w, and $w^{1-\sigma}(w = 1) = 1$ and $w^{1-\sigma}(w = \phi^{-1/\sigma}) = \phi^{(\sigma-1)/\sigma} > \phi$. The terms $h\left(w^{1-\sigma}-\phi\right)+\phi$ and $h\left(\phi w^{1-\sigma}-1\right)+1$ are both positive because $1 - h > 0$. Finally, we have $1 - \phi w^{1-\sigma} > 0$ because $\phi w^{1-\sigma}(w = 1) = \phi < 1$, $\phi w^{1-\sigma}(w = \phi^{-1/\sigma}) = \phi^{(2\sigma-1)/\sigma} < 1$ and $\phi w^{1-\sigma}$ is increasing in w. Therefore, $\partial\Delta u/\partial h > 0$ for $\theta < 1$.

Second, we have:

$$\frac{\partial \Delta u}{\partial w} = \phi\left\{(1-h)w^{-\theta}\left[h\left(w^{1-\sigma}-\phi\right)+\phi\right]^{-\frac{\theta+\sigma-2}{\sigma-1}} + \frac{h\left[h\left(\phi w^{1-\sigma}-1\right)+1\right]^{\frac{1-\theta}{\sigma-1}}}{(1-h)w^{\sigma}+hw\phi}\right\}.$$

Given the preceding arguments it follows also that $\partial\Delta u/\partial w > 0$. Since, from Appendix A, we know that $dw/dh > 0$, we thus conclude that $d\Delta u/dh > 0$.

(2). Log-utility ($\theta = 1$), $u_i = \ln C_i$:

$$\frac{d\Delta u}{dh} = \frac{d(u_L - u_R)}{dh},$$

which is given by the following expression:

$$\frac{d\Delta u}{dh} = \frac{h^2(\sigma-1)\phi w^2 w' + w^{\sigma+1}\left[2(1-h)h(\sigma-1)\phi^2 w' - \left(\phi^2-1\right)w\right] + (h-1)^2(\sigma-1)\phi w^{2\sigma} w'}{(\sigma-1)w\left[h\phi w + (1-h)w^\sigma\right]\left[hw + (1-h)\phi w^\sigma\right]}.$$

Since $w' > 0$, it follows that $d\Delta u/dh > 0$. This concludes the final part of the proof.$\square$

5.C - Stability of symmetric dispersion

In this Appendix we compute the stability condition for symmetric dispersion.

Proof of proposition 5.4. For $\theta \in [0,1) \cup (0,+\infty)$, using the indirect utility in (5.13), we reach:

$$\frac{d\Delta u}{dh} = -\frac{w^{-(\theta+\sigma)}\left\{w^\theta\left(h\phi w^{1-\sigma} - h + 1\right)^{-\frac{\theta+\sigma-2}{\sigma-1}}\left[-h(\sigma-1)\phi w' - w^\sigma + \phi w\right] + \left(hw^{1-\sigma} - h\phi + \phi\right)^{-\left(1+\frac{\theta}{\sigma-1}\right)}\Omega\right\}}{(\sigma-1)}, \tag{5.27}$$

where:

$$\Omega = -w\left(hw^{1-\sigma} - h\phi + \phi\right)^{\frac{1}{\sigma-1}}\left[-h(\sigma-1)w' - \phi w^\sigma + w\right] - (\sigma-1)w^\sigma w'\left(hw^{1-\sigma} - h\phi + \phi\right)^{\frac{\sigma}{\sigma-1}},$$

and $w' = dw/dh$. From Appendix A, we know that:

$$w' = -\frac{w^{-\sigma}\left[w^{2\sigma} - (w+1)\phi w^\sigma + w\right]^2}{(\sigma-1)\phi + (\sigma-1)\phi w^{2\sigma} + \left(-2\sigma + \phi^2 + 1\right)w^\sigma}.$$

Knowing that $w = 1$ for $h = 1/2$, evaluating the derivative simplifies to:

$$w'(1/2) = \frac{8\sigma}{2\sigma + \phi - 1} - 4.$$

Using $h = 1/2$ and $w = 1$ in (5.27) yields:

$$\left.\frac{d\Delta u}{dh}\right|_{h=\frac{1}{2}} = -\frac{\left\{\frac{1+\phi}{2}^{-\left(1+\frac{\theta}{\sigma-1}\right)}\Omega\left(\frac{1}{2}\right) - \left(\frac{1+\phi}{2}\right)^{-\frac{\theta+\sigma-2}{\sigma-1}}\left[\frac{1}{2}(\sigma-1)\phi w'\left(\frac{1}{2}\right) - \phi + 1\right]\right\}}{\sigma-1},$$

with:

$$\Omega\left(\tfrac{1}{2}\right)=\left(\frac{1+\phi}{2}\right)^{\frac{1}{\sigma-1}}\left[\frac{1}{2}(\sigma-1)w'\left(\frac{1}{2}\right)+\phi-1\right]-(\sigma-1)\left(\frac{1+\phi}{2}\right)^{\frac{\sigma}{\sigma-1}}w'\left(\frac{1}{2}\right).$$

Finally, we use $w'(1/2)$ to substitute for the expression above, which yields:

$$\left.\frac{d\Delta u}{dh}\right|_{h=\frac{1}{2}}=\frac{4(2\sigma-1)(1-\phi)\left(\frac{\phi}{2}+\frac{1}{2}\right)^{\frac{1-\theta}{\sigma-1}}}{(\sigma-1)(2\sigma+\phi-1)}.$$

Since $d\left[t(h)-t(1-h)\right]/h=t'(h)+t'(1-h)$, symmetric dispersion is stable if:

$$\frac{2(2\sigma-1)(1-\phi)\left(\frac{1+\phi}{2}\right)^{\frac{1-\theta}{\sigma-1}}}{(\sigma-1)(2\sigma+\phi-1)}<t'\left(\tfrac{1}{2}\right).$$

If $\theta=1$, then the inequality above simplifies to:

$$\frac{2(2\sigma-1)(1-\phi)}{(\sigma-1)(2\sigma+\phi-1)}<t'\left(\tfrac{1}{2}\right),$$

which concludes the proof. □

5.D - Partial agglomeration

Proof of Proposition 5.6. Taking the derivative of (5.13) with respect to h yields the expression in (5.27) from Appendix C.

Substituting for w' using (5.24) from Appendix A, and given that any partial equilibrium h^* must satisfy the short-run equilibrium condition given by (5.10), we reach:

$$\left.\frac{d\Delta u}{dh}\right|_{h=h^*}=\zeta\left\{\varphi\left[\frac{w^{\sigma-1}+w^{-\sigma}-(1+w^{-1})\phi}{1-\phi^2}\right]^{\frac{\theta-1}{\sigma-1}}-\psi\left[\frac{w^{2\sigma}-(w+1)\phi w^{\sigma}+w}{w\left(1-\phi^2\right)}\right]^{\frac{\theta+\sigma-2}{\sigma-1}}\right\},$$

with:

$$\begin{aligned}
\zeta&=\frac{w^{-(\theta+\sigma+1)}}{(\sigma-1)\left[(\sigma-1)\phi+(\sigma-1)\phi w^{2\sigma}+\left(-2\sigma+\phi^2+1\right)w^{\sigma}\right]};\\
\varphi&=w\left\{\phi\left[\sigma+(\sigma-1)w\right]w^{\sigma}-2\sigma w+w\right\}\left[w^{2\sigma}-(w+1)\phi w^{\sigma}+w\right];\\
\psi&=\left(1-\phi^2\right)w^{\theta+\sigma+1}\left[(2\sigma-1)w^{\sigma}-\sigma(w+1)\phi+\phi\right].
\end{aligned}$$

Since partial agglomeration is stable if $d\Delta u/dh<d\left[t(h)-t(1-h)\right]/dh$ at $h=h^*$,

partial agglomeration is stable if:

$$\zeta\left\{\varphi\left[\frac{w^{\sigma}+w^{1-\sigma}-(w+1)\phi}{w\left(1-\phi^{2}\right)}\right]^{\frac{\theta-1}{\sigma-1}}-\psi\left[\frac{w^{2\sigma}-(w+1)\phi w^{\sigma}+w}{w\left(1-\phi^{2}\right)}\right]^{\frac{\theta+\sigma-2}{\sigma-1}}\right\}<t'\left(h^{*}\right)-t'\left(1-h^{*}\right),$$

which completes the proof. □

Proof of Proposition 5.7. Any interior long-run equilibrium $h^{*}\in(1/2,1)$ must satisfy the following system of equations:

$$\begin{aligned} F(h,w) &\equiv f(w)-h=0 \\ G(h,w) &= \Delta u(h,w)+t(1-h)-t(h)=0, \end{aligned} \tag{5.28}$$

where $f(w)$ is given by the expression in (5.10) and $\Delta u(h,w)$ is utility differential given by (5.13). For points (w_0,h_0,ϕ_0) satisfying (5.28), if $F(w,h,\phi)$ and $G(w,h,\phi)$ have continuous partial derivatives in a neighbourhood of (w_0,h_0,ϕ_0) and:

$$\det\begin{pmatrix} \dfrac{\partial F}{\partial h} & \dfrac{\partial F}{\partial w} \\[2ex] \dfrac{\partial G}{\partial h} & \dfrac{\partial G}{\partial w} \end{pmatrix}\neq 0,$$

then there exists a neighbourhood B of $\phi_0\in\mathbb{R}$ and a continuous differentiable function $\mathcal{W}:B\mapsto\mathbb{R}$ such that $F\left(\phi,\mathcal{W}\left(\phi\right)\right)=0$ and $G\left(\phi,\mathcal{W}\left(\phi\right)\right)=0$ for all $\phi\in B$. Then, using Cramer's rule, we can compute the following derivative:

$$\frac{dh^{*}}{d\phi}=\frac{\dfrac{\partial F}{\partial w}\dfrac{\partial G}{\partial \phi}-\dfrac{\partial G}{\partial w}\dfrac{\partial F}{\partial \phi}}{\dfrac{\partial F}{\partial h}\dfrac{\partial G}{\partial w}-\dfrac{\partial F}{\partial w}\dfrac{\partial G}{\partial h}}. \tag{5.29}$$

Computing all partial derivatives in (5.29), and evaluating at h^{*} using (5.10) yields:

$$\frac{dh^{*}}{d\phi}=\frac{(2\sigma-1)\left(w^{2\sigma}-1\right)}{\Theta},$$

where:

$$\begin{aligned}\Theta =&(\sigma-1)\left[(\sigma-1)\phi+(\sigma-1)\phi w^{2\sigma}+\left(-2\sigma+\phi^2+1\right)w^{\sigma}\right]\left[t'\left(h^*\right)+t'\left(1-h^*\right)\right]+\\&+w^{-(\sigma+1)}(2\sigma-1)\left[w^{2\sigma}-(w+1)\phi w^{\sigma}+w\right]^2.\end{aligned}$$

The numerator is positive since $w^{2\sigma}>1$ for $h>1/2$. As for Θ, it is negative if (5.18) holds, since we are assuming that:

$$\left[(\sigma-1)\phi+(\sigma-1)\phi w^{2\sigma}+\left(-2\sigma+\phi^2+1\right)w^{\sigma}\right]<0.$$

Therefore, when agglomeration is stable, we have $dh^*/d\phi<0$, implying that partial agglomeration becomes more symmetric as the freeness of trade increases. This concludes the proof. □

5.E - Logit and linear heterogeneity

Proof of Proposition 5.9. When $t(x)=\mu\ln x$, the RHS of expression (5.17) is given by:

$$t'\left(h^*\right)-t'\left(1-h^*\right)=\mu\left[\frac{1}{h(1-h)}\right].$$

Replacing in (5.17) and rearranging, partial agglomeration is stable if:

$$-\frac{w^{-\sigma-1}\left[w^{2\sigma}-(w+1)\phi w^{\sigma}+w\right]^2\Gamma}{(\sigma-1)\left(w^{\sigma}-\phi\right)\left(1-\phi w^{\sigma}\right)\left[(\sigma-1)\phi+(\sigma-1)\phi w^{2\sigma}+\left(-2\sigma+\phi^2+1\right)w^{\sigma}\right]}<0, \quad (5.30)$$

where

$$\begin{aligned}\Gamma=&\,\phi\left[\mu(\sigma-1)^2-2\sigma+1\right]+\phi\left[\mu(\sigma-1)^2-2\sigma+1\right]w^{2\sigma}-\\&-w^{\sigma}\left\{\phi^2\left[-(\mu+2)\sigma+\mu+1\right]+(2\sigma-1)\left[\mu(\sigma-1)-1\right]\right\}.\end{aligned}$$

The numerator of (5.30) except Γ is positive, and so is $\left(w^{\sigma}-\phi\right)\left(1-\phi w^{\sigma}\right)$. From Appendix A, we know that:

$$(\sigma-1)\phi+(\sigma-1)\phi w^{2\sigma}+\left(-2\sigma+\phi^2+1\right)w^{\sigma}<0.$$

Therefore, (5.17) holds if and only if $\Gamma < 0$, which gives:

$$\mu > \mu_p \equiv \frac{(2\sigma - 1)(w^\sigma - \phi)(1 - w^\sigma \phi)}{(\sigma - 1)\left[\left(2\sigma - \phi^2 - 1\right) w^\sigma - (\sigma - 1)\phi w^{2\sigma} - (\sigma - 1)\phi\right]}.$$

The condition for partial agglomeration in (5.30) holds for any interior equilibrium including symmetric dispersion $h^* = 1/2$. At symmetric dispersion we have $w = 1$ and the condition above simplifies to:

$$\mu > \mu_d \equiv \frac{(2\sigma - 1)(1 - \phi)}{(\sigma - 1)(2\sigma + \phi - 1)}.$$

The derivative of μ_p with respect to $X \equiv w^\sigma$ equals:

$$\frac{\partial \mu_p}{\partial X} = \frac{\sigma(2\sigma - 1)\left(X^2 - 1\right)\phi\left(\phi^2 - 1\right)}{(\sigma - 1)\left((\sigma - 1)\left(X^2 + 1\right)\phi - 2\sigma X + X\phi^2 + X\right)^2} < 0.$$

We can also see that μ_p approaches zero as w^σ approaches ϕ^{-1} (i.e., as partial agglomeration tends to full agglomeration in L). Therefore, we conclude that $0 < \mu_p < \mu_d$. Under the assumption that only one partial agglomeration equilibrium $h^* \in (1/2, 1)$ exists, we have two possibilities: (i) if $\mu \in (\mu_p, u_d)$, partial agglomeration is the only stable equilibrium; and (ii) if $\mu > \mu_d$, symmetric dispersion is the only stable equilibrium. Assume now, by way of contradiction, that $\mu \in (0, \mu_p)$. This would imply that both dispersion and partial agglomeration are unstable. Since the state space is one dimensional, this would require agglomeration to be stable. However, we know that agglomeration is always unstable, which implies that either partial agglomeration or dispersion are stable. Hence, $\mu \notin (0, \mu_p)$.
Therefore, we can sum up the two previous cases by saying that symmetric dispersion is the only stable equilibrium if $\mu > \mu_d$; otherwise, partial agglomeration is the only stable equilibrium. This concludes the proof. □

Proof of Proposition 5.10. Considering Proposition 5.4 and $t(h) = \mu h$, dispersion is stable if:

$$-\frac{2\left[\mu(\sigma - 1)(2\sigma + \phi - 1) - 2(2\sigma - 1)(1 - \phi)\right]}{(\sigma - 1)(2\sigma + \phi - 1)} < 0.$$

Rewriting the inequality in terms of μ we get:

$$\mu > \mu_b \equiv \frac{2(2\sigma - 1)(1 - \phi)}{(\sigma - 1)(2\sigma + \phi - 1)}.$$

Using Proposition 5.3, together with $t(h) = \mu h$, we conclude that agglomeration is stable if:

$$\mu < \mu_s \equiv -\frac{(2\sigma - 1)\ln \phi}{\sigma(\sigma - 1)}.$$

If $\mu_s > \mu_b$, there exists a $\mu \in (\mu_b, \mu_s)$ for which both agglomeration and dispersion are simultaneously stable. We have:

$$\mu_s - \mu_b = -\frac{(2\sigma - 1)\left[-2\sigma(1 - \phi) + (2\sigma + \phi - 1)\log \phi\right]}{(\sigma - 1)\sigma(2\sigma + \phi - 1)} > 0.$$

Therefore, $\mu_s > \mu_b$. We thus conclude that if $\mu < \mu_b$, only agglomeration is stable. If $\mu \in (\mu_b, \mu_s)$, both dispersion and agglomeration are simultaneously stable. Finally, for $\mu > \mu_s$, only dispersion is stable. $\square$

Chapter 6

Conclusions and discussion

We now briefly discuss the generality of our results, namely what plausible different assumptions are likely to qualitatively change our predictions and what could be done in the future as a follow-up of the present work.

Throughout Chapters 3 and 4, we have sought to further contribute to the understanding of spatial distributions that may arise in a multi-regional setting. As we have seen, the results for the n-region FE model do not stray far from the results of the original 2-region model of Forslid and Ottaviano (2003). Conversely, adding more regions to the QLLog model by Pflüger (2004) alters the qualitative structure of the model substantially. This allows us to conjecture that, in some frameworks, many results seem to carry over from the pre-existing 2-region setting to the multi-regional setting for a class of Core-Periphery models, but this is not universally the case.

As discussed in Chapter 2, a lot of effort has been dedicated to investigate what persistent features are likely to render most 2-region Core-Periphery "isomorphic" (Robert-Nicoud, 2005). Similarly, a significant number of papers have extracted conclusions from various multi-regional models under a variety of different assumptions. What is then perhaps lacking is a deeper comprehension on why some models produce results that remain by and large invariant under the number of regions, while other models show significant changes when more regions are considered.

Answering these questions is far from trivial, but there are some obvious starting points. One such starting point has to do with how the number of regions actually relates to the geographical structure. In this respect, one main concern relates to the internal dimension of regions themselves, so that they are not mere dimensionless points in space. The other is related with the transport cost structure, which is the embodiment of geography in NEG and whose complexity is likely to co-vary with the number of existing regions. On the other hand, this is far from an easy task, because most NEG models are inherently difficult to study analytically and numerically, which makes it difficult to compare between different spatial topologies.

To be more clear on this matter, it is worth mentioning the case of the FE model studied in Chapter 3. The FE model is an analytically solvable version of the original Core-Periphery model whose results are identical to the latter (i.e., it is convenient because it allows to confirm them analytically). However, the fact that we have obtained a tractable multi-regional model is more likely the result of pair-wise equidistance than that of considering a Footloose Entrepreneur structure. A few studies have shown that results from 2-region models carry over to their multi-regional extensions when regions are equidistant. This is the case, e.g., of the paper by Bosker *et al.* (2010) discussed in Chapters 2, 4 and 5, which uses a multi-regional version of Puga's (1999) model with equidistant regions. Equidistance is key here, as it limits the potential geographical role of new regions, whose interdependencies greatly depend on the distances between one another. In other words, including more regions while keeping the assumption of equidistance only marginally increases the dimensionality in NEG, because differences between each pair of regions are reflected symmetrically across all pair of regions. In spite of this, equidistance is also extremely useful because it greatly simplifies multi-regional analysis. Other geometries, such as equally spaced regions around a circumference (racetrack economy) or along a line segment, albeit simple by construction, still render the analysis significantly complicated and have no clear benefit in terms of empirical realism other than the fact that they account for some (very stylized) asymmetry. On the other hand, the fact that these geometries introduce regional asymmetries means that varying the number of regions is likely to produce a more significant impact on the resulting spatial distributions. To conclude, there exists a clear trade-off between more complex geometries and the ability to extract meaningful and intelligible insights.

We may raise yet another issue with respect to the introduction of complex asymmetries. As we have seen in Chapter 4, the extension of the QLLog model to an arbitrary number of equidistant regions has allowed us to obtain qualitatively different results compared to the original 2-region model. Therefore, in spite of some of the previous arguments, a complex spatial structure may arise even under a simple geographical structure. In this instance, equidistance is not only useful but also warranted, because it allows us to understand what causes different spatial structures without the interference of complex exogenous asymmetries. The challenge in this particular case is then quite the opposite: to understand whether a more complex geographical structure for the QLLog model allows to extract additional meaningful insights.

In Chapter 5, we have integrated market-driven factors with Hotelling-type preferences. This way of modelling consumer heterogeneity in an otherwise very simple NEG setup has allowed us to provide a reinterpretation of how consumers make their location choices and, hence, how different spatial distributions arise. Assuming a uniform distribution of preferences for consumers along a line allows us to rank each consumer with respect to his willingness towards residing in a region or another. This separates our framework from the widely used probabilistic migration setting in the following sense. Even though the population distributions in the long-run are the same, for equivalent formulations in both settings,[1] Hotelling-type preferences enable us to identify which consumers choose which region. This is not possible under probabilistic migration because the probability that an individual chooses a region over another depends on a random utility component that is unobservable and assumed to follow some statistical distribution.

Although it transcends the scope of this dissertation, being able to identify the consumers in a given region may be useful in future work for a number of reasons. Suppose we introduce other dimensions of consumer heterogeneity. For instance, consumers are heterogeneous not only in their preferences for location but also regarding their skill levels. In that case, a ranking of consumers by preferences, and by skill levels, would allow us to say more about the empirically observed spatial sorting of heterogeneous workers in different regions. As discussed in Chapter 2, such an analysis would certainly confer richness to the study of urban hierarchy formations.

We have also seen that Hotelling-type preferences induce a dispersive force (home-sweet-home effect) whose long-run implications on spatial distributions are similar to those of urban congestion costs. However, the interpretation is different. In the first case, all consumers have a different preference towards residing in one region or another, so we know which consumers become increasingly willing to migrate to a region where income levels progressively rise. In the latter case, all consumers in a region are equally and negatively affected by increases in population sizes (due to increases in income levels). Therefore, just as with probabilistic migration, congestion costs alone do not allow us to tell anything about the spatial sorting of agents. However, some agents are environmentally more conscientious than others, while some people are more sensitive to commuting costs or housing prices than others. In other words, congestion costs are also likely to affect individuals differently. This adds yet another level of heterogeneity which could be encompassed in Hotelling-type heterogeneity.

Along the lines of these suggestions we conclude that the role of multiple regions and individual heterogeneity may be yet enriched in several ways that could help NEG provide a better description of the causes underlying the spatial distribution of economic activities.

[1] Recall the Logit discrete choice model in a probabilistic migration setting and its equivalent form under Hotelling-type preferences studied in Chapter 5.

Bibliography

Aghion, P., Howitt, P. (1992), "A Model of Growth through Creative Destruction.", *Econometrica*, 60, 323–351.

Ago, T., Isono, I., Tabuchi, T. (2006), "Locational disadvantage of the hub", *Annals of Regional Science*, 40 (4), 819-848.

Akamatsu, T., Takayama, Y. (2009), "A simplified approach to analyzing multi-regional core-periphery models", Technical report, University Library of Munich.

Akamatsu, T., Takayama, Y., Ikeda, K. (2012), "Spatial Discounting, Fourier, and Racetrack Economy: A Recipe for the Analysis of Spatial Agglomeration Models", *Journal of Economic Dynamics & Control*, 36, 1729-1759.

Albouy, D., Leibovici, F., Warman, C. (2013), "Quality of life, firm productivity, and the value of amenities across Canadian cities", *Canadian Journal of Economic*s, 46 (2), 379-411.

Allen, T., Arkolakis, C. (2014), "Trade and the Topography of the Spatial Economy", *The Quarterly Journal of Economics*, 129 (3), 1085-1140.

Anderson, J., van Wincoop, E. (2004), "Trade Costs", *Journal of Economic Literature*, 42, 691-751.

Anderson, S. P., De Palma, A., Thisse, J. F. (1992), *Discrete choice theory of product differentiation*, MIT Press.

Baldwin, R., Forslid, R., Ottaviano, G., Robert-Nicoud, F. (2004), *Economic Geography and Public Policy*, Princeton University Press.

Baldwin, R. (1999), "Agglomeration and endogenous capital", *European Economic Review*, 43 (2), 253-280.

Baldwin, R., Martin, P. (2004), “Agglomeration and regional growth”, *Handbook of Regional and Urban Economics*, 4, 2671-2711.

Baldwin, R., Okubo, T. (2006), “Heterogeneous firms, agglomeration and economic geography: spatial selection and sorting”, *Journal of Economic Geography*, 6 (3), 323-346.

Baldwin, R., Okubo, T. (2009), “Tax reform, delocation, and heterogeneous firms”, *The Scandinavian Journal of Economics*, 111 (4), 741-764.

Barbero, J., Zofío, J. L. (2012), “The multiregional core-periphery model: The role of the spatial topology, *Networks and Spatial Economics*, 1-28.

Behrens, K. (2005), “Market size and industry location: traded vs non-traded goods”, *Journal of Urban Economics*, 58 (1), 24-44.

Behrens, K., Gaigne, C., Ottaviano, G. I., Thisse, J.-F. (2006), “Is remoteness a locational disadvantage?”, *Journal of Economic Geography*, 6(3), 347-368.

Behrens, K., Murata, Y. (2007), “General equilibrium models of monopolistic competition: a new approach”, *Journal of Economic Theory*, 136 (1), 776-787.

Behrens, K., Ottaviano, G. I., Lamorgese, A. R., Tabuchi, T. (2004), “Testing the home market effect in a multi-country world: the theory”, CEPR Discussion Paper, 4468.

Behrens, K., Robert-Nicoud, F. (2011), “Tempora mutantur: in search of a new testament for NEG”, *Journal of Economic Geography*, 11 (2), 215-230.

Behrens, K., Thisse, J.-F. (2007), “Regional economics: A new economic geography perspective”, *Regional Science and Urban Economics*, 37(4), 457-465.

Berliant, M., Fujita, M. (2008), “Knowledge creation as a square dance on the Hilbert cube ”, *International Economic Review*, 49 (4), 1251-1295.

Berliant, M., Fujita, M. (2009), “Dynamics of knowledge creation and transfer: The two person case”, *International Journal of Economic Theory*, 5, 155-179.

Berliant, M., Fujita, M. (2011), “The dynamics of knowledge diversity and economic growth”, *Southern Economic Journal*, 77 (4), 856-884.

Berliant, M., Fujita, M. (2012), “Culture and diversity in knowledge creation”, *Regional Science and Urban Economics*, 42 (4), 648-662.

Berliant, M., Kung, F. (2009), “Bifurcations in Regional Migration Dynamics”, *Regional Science and Urban Economics*, 39, 714-720.

Blaug, M. (1997), *Economic theory in retrospect*, Cambridge university press.

Boschma, R. and K. Frenken (2006), “Why is economic geography not an evolutionary science? Towards an evolutionary economic geography”, *Journal of Economic Geography*, 6, 273-302.

Bosker, M., Brakman, S., Garretsen, H., Schramm, M. (2010), “Adding geography to the new economic geography: bridging the gap between theory and empirics”, *Journal of Economic Geography*, 10 (6), 793-823.

Brakman, S., Garretsen, H., Van Marrewijk, C. (2009), *The new introduction to geographical economics*, Cambridge University Press.

Castro, S.B.S.D., Correia-da-Silva, J., Mossay, P. (2012), “The Core-Periphery model with three regions and more”, *Papers in Regional Science*, 91 (2), 401-418.

Combes, P. P., Mayer, T., Thisse, J. F. (2008), *Economic geography: The integration of regions and nations*, Princeton University Press.

Commendatore, P., Kubin, I., Sushko, I. (2015a), “Typical bifurcation scenario in a three regional identical New Economic Geography model”, *Mathematics and Computers in Simulation*, 108, 63-80.

Commendatore, P., Filoso, V., Grafeneder-Weissteiner, T., Kubin, I. (2015b), “Towards a multiregional NEG framework: comparing alternative modelling strategies, in *Complexity and Geographical* Economics, 13-50, Springer.

Crozet, M. (2004), “Do migrants follow market potentials? An estimation of a new economic geography model”, *Journal of Economic Geography*, 4(4), 439-458.

Davis, D. R., Weinstein, D. E. (2002), “Bones, bombs, and break points: the geography of economic activity”, *The American Economic Review*, 92 (5), 1269-1289.

Dinopoulos, E., Segerstrom, P. (2010), “Intellectual property rights, multinational firms and economic growth”, *Journal of Development Economics*, 92 (1), 13-27.

Dixit, A. K., Stiglitz, J. E. (1977), “Monopolistic competition and optimum product diversity”, *The American Economic Review*, 67 (3), 297-308.

Duranton, G., Puga, D. (2001), “Nursery cities: Urban diversity, process innovation, and the life cycle of products”, *American Economic Review*, 1454-1477.

Duranton, G., Puga, D. (2004), “Micro-foundations of urban agglomeration economies”, *Handbook of Regional and Urban Economics*, 4, 2063-2117.

Fabinger, M. (2015), “Cities as solitons: Analytic solutions to models of agglomeration and related numerical approaches”, *available at SSRN 2630599*.

Forslid, R., Ottaviano, G. (2003), “An analytically solvable core-periphery model”, *Journal of Economic Geography*, 3 (3), 229-240.

Forslid, R., Okubo, T. (2012), “On the development strategy of countries of intermediate size - an analysis of heterogeneous firms in a multi-region framework”, *European Economic Review*, 56(4), 747-756.

Frenken, K., Van Oort, F., Verburg, T. (2007), “Related variety, unrelated variety and regional economic growth”, *Regional studies*, 41 (5), 685-697.

Fujita, M. (2007), “Towards the new economic geography in the brain power society”, *Regional Science and Urban Economics*, 37 (4), 482-490.

Fujita, M., Krugman, P. (1995), “When is the economy monocentric?: von Thünen and Chamberlin unified”, *Regional Science and Urban Economics*, 25 (4), 505-528.

Fujita, M., Krugman, P., Mori, T. (1999), “On the evolution of hierarchical urban systems”, *European Economic Review*, 43 (2), 209-251.

Fujita, M., Krugman, P., Venables, A. (1999), *The Spatial Economy: Cities, Regions and International Trade*, Cambridge, MA: MIT Press.

Fujita, M., Mori, T. (2005), “Frontiers of the new economic geography”, *Papers in Regional Science*, 84 (3), 377-405.

Fujita, M., Thisse, J. F. (2002), *Economics of agglomeration: cities, industrial location, and globalization*, Cambridge university press.

Fujita, M., Thisse, J. F. (2003), "Does geographical agglomeration foster economic growth? And who gains and loses from it?", *Japanese Economic Review*, 54 (2), 121-145.

Fujita, M., Thisse, J. F. (2009), "New economic geography: an appraisal on the occasion of Paul Krugman's 2008 Nobel Prize in Economic Sciences", *Regional Science and Urban Economics*, 39 (2), 109-119.

Garretsen, H., Martin, R. (2010), "Rethinking (new) economic geography models: taking geography and history more seriously", *Spatial Economic Analysis*, 5 (2), 127-160.

Gaspar, J.M., Castro, S.B.S.D., Correia-da-Silva, J. (2013), "The Footloose Entrepreneur Model with 3 regions", *FEP Working Papers*, 496.

Guckenheimer, J., Holmes, P. (2002), *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag.

Harvey, D. (1985), "The geopolitics of capitalism", in: D. Gregory & J. Urry (eds) *Social Relations and Spatial Structure*s, 128-163, London, Macmillan.

Head, K., Mayer, T. (2004), "The empirics of agglomeration and trade", *Handbook of Regional and Urban Economics*, 4, 2609-2669.

Helpman, E. (1998), "The size of regions", in: Pines, D., Zilcha, I., *Topics in Public Economics*, Cambridge University Press.

Henderson, J. V. (1974), "The sizes and types of cities", *The American Economic Review*, 64 (4), 640-656.

Henderson, V., Thisse, J. F. (2004), *Handbook of Regional and Urban Economics: cities and geography,* 4, 2063-3073.

Hess, S., Rose, J. M. (2012), "Can scale and coefficient heterogeneity be separated in random coefficients models?", *Transportation*, 39 (6), 1225-1239.

Hotelling, H. (1929), "Stability in Competition", *Economic Journal*, 39 (157), 41-57.

Howitt, P. (1999), "Steady Endogenous Growth with Population and R&D Inputs Growing.", *Journal of Political Economy*, 107, 715–730.

Ikeda, K., Akamatsu, T., Kono, T. (2012), "Spatial period-doubling agglomeration of a core-periphery model with a system of cities", *Journal of Economic Dynamics and Control*, 36(5), 754-778.

Ikeda, K., Murota, K. (2014), *Bifurcation Theory for Hexagonal Agglomeration in Economic Geography*, Springer.

Isard, W. (1956), *Location and space-economy,* MIT Press, Cambridge MA.

Krugman, P. (1980), "Scale economies, product differentiation and the pattern of trade", *The American Economic Review*, 70 (5), 950-959.

Krugman, P. (1991), "Increasing Returns and Economic Geography", *Journal of Political Economy*, 99 (3), 483-499.

Krugman, P. (1993), "First nature, second nature, and metropolitan location", *Journal of Regional Science*, 33 (2), 129-144.

Krugman, P., Elizondo, R. L. (1996), "Trade policy and the third world metropolis", *Journal of Development Economics*, 49 (1), 137-150.

Krugman, P., Venables, A. (1995), "Globalization and the Inequality of Nations", *Quarterly Journal of Economics*, 110 (4), 857-880.

Lösch, A. (1954), *The Economics of Location*, New Haven, CT, Yale University Press.

Martin, R. (1999), "The new 'geographical turn' in economics: some critical reflections", *Cambridge Journal of Economics*, 23, 65-91.

Martin, P., Rogers, C. (1995), "Industrial location and public infrastructure", *Journal of International Economics*, 39 (3), 335-351.

Matsuyama, K. (1991), "Increasing returns, industrialization, and indeterminacy of equilibrium", *The Quarterly Journal of Economics*, 617-650.

McFadden, D. (1974), "The measurement of urban travel demand", *Journal of Public Economics*, 3 (4), 303-328.

Melitz, M. J., Ottaviano, G. I. (2008), “Market size, trade, and productivity”, *The review of economic studies*, 75 (1), 295-316.

Mossay, P. (2003), “Increasing returns and heterogeneity in a spatial economy”, *Regional Science and Urban Economics*, 33 (4), 419-444.

Mossay, P. (2006), “The core-periphery model: A note on the existence and uniqueness of short-run equilibrium”, *Journal of Urban Economics*, 59 (3), 389-393.

Mossay, P. (2013), “A theory of rational spatial agglomerations”, *Regional Science and Urban Economics*, 43 (2), 385-394.

Murata, Y. (2003), “Product diversity, taste heterogeneity, and geographic distribution of economic activities: market vs. non-market interactions”, *Journal of Urban Economics*, 53, 126-144.

Murata, Y., Thisse, J.-F. (2005), “A simple model of economic geography à la Helpman–Tabuchi”, *Journal of Urban Economics*, 58 (1), 137-155.

Neary, J. (2001), “Of hype and hyperbolas: introducing the new economic geography”, *Journal of Economic Literature*, 39 (2), 536-561.

Niebuhr, A. (2006), “Market access and regional disparities”, *The Annals of Regional Science*, 40 (2), 313-334.

Nocke, V. (2006), “A gap for me: Entrepreneurs and entry”, *Journal of the European Economic Association*, 4 (5), 929-956.

Okubo, T., Picard, P. M., Thisse, J.-F. (2010), “The spatial selection of heterogeneous firms”, *Journal of International Economics*, 82 (2), 230-237.

Ottaviano, G. (2000), “Ad usum delphini: A Primer in" New Economy Geography"”, *Giornale degli Economisti e Annali di Economia*, 87-114.

Ottaviano, G. (2010), “‘New’new economic geography: firm heterogeneity and agglomeration economies”, *Journal of Economic Geography*, 11 (2), 231-240.

Ottaviano, G. (2012), “Agglomeration, trade and selection”, *Regional Science and Urban Economics*, 42 (6), 987-997.

Ottaviano, G., Peri, G. (2006), “The economic value of cultural diversity: evidence from US cities”, *Journal of Economic Geography*, 6 (1), 9-44.

Ottaviano, G., Peri, G. (2008), “Immigration and national wages: Clarifying the theory and the empirics”, National Bureau of Economic Research, 14188.

Ottaviano, G., Prarolo, G. (2009), “Cultural Identity and Knowledge Creation in Cosmopolitan Cities”, *Journal of Regional Sciences*, 49 (4), 647-662.

Ottaviano, G., Robert-Nicoud, F. (2006), “The ‘genome’of NEG models with vertical linkages: a positive and normative synthesis”, *Journal of Economic Geography*, 6 (2), 113-139.

Ottaviano, G., Tabuchi, T., Thisse, J. F. (2002), “Agglomeration and trade revisited”, *International Economic Review*, 43, 409-436.

Oyama, D. (2009), “Agglomeration under forward-looking expectations: Potentials and global stability”, *Regional Science and Urban Economics,* 39 (6), 696-713.

Pflüger, M. (2004), “A simple, analytically solvable, Chamberlinian agglomeration model”, *Regional Science and Urban Economics*, 34 (5), 565-573.

Pflüger, M., Südekum, J. (2008a), “A synthesis of footloose-entrepreneur new economic geography models: when is agglomeration smooth and easily reversible?”, *Journal of Economic Geography*, 8 (1), 39-54.

Pflüger, M., Südekum, J. (2008b), “Integration, agglomeration and welfare”, *Journal of Urban Economics*, 63, 544-566.

Picard, P. M., Tabuchi, T. (2010), “Self-organized agglomerations and transport costs”, *Economic Theory*, 43 (2), 565-589.

Puga, D. (1999), “The Rise and Fall of Regional Inequalities”, *European Economic Review*, 43, 303-334.

Puga, D. (2010), “The magnitude and causes of agglomeration economies”, *Journal of Regional Science*, 50 (1), 203-219.

Redding, S. J. (2016), “Goods trade, factor mobility and welfare”, *Journal of International Economics*, 101, 148-167.

Redding, S. J., Sturm, D. M. (2008), “The costs of remoteness: Evidence from German division and reunification”, *The American Economic Review*, 98 (5), 1766-1797.

Redding, S. J., Sturm, D. M., Wolf, N. (2011), “History and industry location: Evidence from German airports”, *Review of Economics and Statistics*, 93 (3), 814-831.

Robert-Nicoud, F. (2005), “The structure of simple ‘New Economic Geography’models (or, On identical twins)”, *Journal of Economic Geography*, 5 (2), 201-234.

Rodríguez-Pose, A., Ketterer, T. D. (2012), “Do Local Amenities affect the appeal of regions in Europe for migrants?”, *Journal of Regional Science*, 52 (4), 535-561.

Rosen, S. (2002), “Markets and diversity”, *The American Economic Review*, 92 (1), 1-15.

Scarpa, R., Thiene, M., Train, K. (2008), “Utility in willingness to pay space: a tool to address confounding random scale effects in destination choice to the Alps”, *American Journal of Agricultural Economics*, 90 (4), 994-1010.

Starrett, D. (1978), “Market allocations of location choice in a model with free mobility”, *Journal of Economic Theory*, 17 (1), 21-37.

Storper, M., Manville, M. (2006), “Behaviour, preferences and cities: Urban theory and urban resurgence”, *Urban Studies*, 43 (8), 1247-1274.

Tabuchi (2014), “Historical trends of agglomeration to the capital region and new economic geography”, *Regional Science and Urban Economics,* 44, 50-59.

Tabuchi, T., Thisse, J.-F. (2002), “Taste Heterogeneity, Labor Mobility and Economic Geography, *Journal of Development Economics*, 69, 155-177.

Tabuchi, T., Thisse, J.-F. (2011), “A new economic geography model of central places”, *Journal of Urban Economics*, 69 (2), 240-252.

Tabuchi, T., Thisse, J.-F., Zeng, D. Z. (2005), “On the number and size of cities”, *Journal of Economic Geography*, 5 (4), 423-448.

Tabuchi, T., Thisse, J.-F., Zhu, X. (2014), “Technological Progress and Economic Geography”, *CIRJE Discussion Papers*, 915.

Tabuchi, T., Thisse, J. F., Zhu, X. (2016), "Does technological progress magnify regional disparities?", *IDE Discussion Papers*, 599.

Takahashi, T. (2006), "Economic geography and endogenous determination of transport technology", *Journal of Urban Economics*, 60 (3), 498-518.

Tavassoli, S., Carbonara, N. (2014), "The role of knowledge variety and intensity for regional innovation", *Small Business Economics*, 43 (2), 493-509.

Tirole, J. (1988), *The theory of industrial organization*, MIT press.

Train, K. E. (2009), *Discrete choice methods with simulation*, Cambridge University Press.

Venables, A. J. (1996), "Equilibrium locations of vertically linked industries", *International Economic Review*, 341-359.

Von Ehrlich, M., Seidel, T. (2013), "More similar firms—More similar regions? On the role of firm heterogeneity for agglomeration", *Regional Science and Urban Economics*, 43 (3), 539-548.

Young, A. (1998), "Growth without Scale Effects", *Journal of Political Economy*, 106, 41–63.

Zhelobodko, E., Kokovin, S., Parenti, M., Thisse, J. F. (2012), "Monopolistic competition: Beyond the constant elasticity of substitution", *Econometrica*, 80 (6), 2765-2784.