Control and Model Identification of a Mobile Robot's Motors based in Least Squares and Instrumental Variable Methods

André Scolari Conceição[1], Paulo J. Costa and A. Paulo Moreira

Department of Electrical and Computer Engineering
University of Porto - Porto - Portugal.
[scolari,paco,amoreira]@fe.up.pt

Abstract. When no exact measurements are available the task of solving the differential equations describing the dynamics of mobile robot can be hard and inefficient. In this paper we propose a procedure for dynamic model identification and control of the process (motor + reduction + encoder) of a omni-Directional Mobile Robot's Motors. Our techniques are based in Least Squares Methods and Instrumental Variable Methods for linear dynamic systems. Notably, the approach used here is applicable to any robot with DC motors, or, more generally, to any process with DC motor with encoder and PWM control made by a microcontroller.

Key Words. Dynamic model Identification, mobile robots, control systems.

1. INTRODUCTION

In this paper we focus attention on a omni-directional mobile robot with four motors, as shown in 1, built for the 5dpo-2005 Robotic Soccer team from the Department of Electrical and Computer Engineering at the University of Porto at Porto, Portugal. We are particularly interested in the motor+reduction+encoder process represented in Fig. 2. This process consists on the relation between the voltage and the velocities of the robot's motors. The dynamics of this process is complex and may be nonlinear due to the mechanic architecture of the robot which may add perturbations like attrition due to the interaction between the robot components. The task of solving the differential equations modeling the dynamics of this process may be a hard one or even be inefficient not only because of the nonlinearities but also because no exact measurements of the parameters are known. The design and simulation of controllers (as for example, the discrete PID controller implemented in the microcontroller of the robot) for the navigation of the robot require the identification of a dynamic model for this process. Here we present the identification of a discrete system (see Fig. 1) for the motor+reduction+ encoder process. This is a first step towards the dynamic model identification of the whole robot. We consider a discrete linear model and we use techniques based on Least Square Methods and Instrumental Variable methods to estimate a transfer function for the process. This procedure is not only confined to our mobile robot. It can be applied to any robot with DC motors, or, more generally, to any process with DC motor with encoder and PWM control made by a microcontroller.

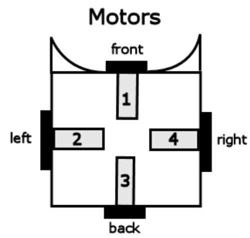

Figure 1: Mobile robot - motors.

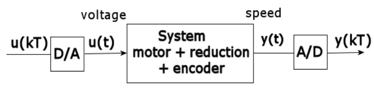

Figure 2: Discrete dynamic system.

The robot's control and communication structure is as sketched in Fig. 3. The computer (PC) controls all the actions of the robot. The communication with the microcontroller is done through the serial port (RS232). It controls the motors using signals of PWM and a drive of power. The

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software of control in PC works in a frequency of $25Hz$, from the camera. However, and in order to control efficiently the motors of the robot, the microcontroller has a discrete PID controller that works in a frequency of $100Hz$.

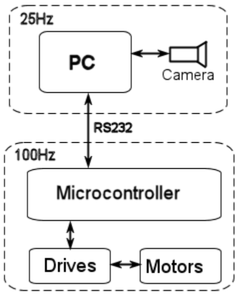


Figure 3: Robot's control and communication structure.

This paper is structured in the following way. In section 2. we give a brief description of a deterministic model and we determine a Least Square Estimator. In section 3. we apply the Least Squares estimator to the process for the motor 1 (front) of the mobile robot. An Instrumental variable estimator is presented in section 4.. The estimation results for motors 2, 3 and 4 of the mobile robot is presented in Section 5.. The design of PID controller for Robot's Motors is showed in section 6.. Finally, the conclusion and future works is drawn in section 7..

2. DETERMINISTIC MODEL AND LEAST SQUARES ESTIMATOR

In general, a linear time-invariant discrete-time system with input sequence $u(k)$ and output sequence $y(k)$ can be represented by an nth-order difference equation relating the input and output ([3]),

$$y(k) = -\sum_{i=1}^{na} a_i y(k-i) + \sum_{i=0}^{nb} b_i u(k-i), \quad (1)$$

where k is the time variable, and n is a fixed integer called the *order* of the difference equation. The Z-transform of the difference equation (1) leads to

$$\frac{U(z)}{Y(z)} = \frac{(1 + a_1 z^{-1} + ... + a_n z^{-n})}{(b_o + b_1 z^{-1} + ... + b_n z^{-n})}, \quad (2)$$

Multiplying both sides of (2) by z^n and rearrange it to obtain the transfer function of the discrete-time system,

$$H(z) = \frac{Y(z)}{U(z)} = \frac{B(z)}{A(z)}, \quad (3)$$

where the numerator polynomial $B(z)$ and denominator polynomial $A(z)$ are defined as:

$$B(z) = b_0 z^n + b_1 z^{n-1} + ... + b_n, \quad (4)$$

$$A(z) = z^n + a_1 z^{n-1} + ... + a_n. \quad (5)$$

For the design of a computer-controlled system like the one in Fig. 2, the model (1) must describe the dynamical behavior of the control loop between the input of the D/A converter and the output of the A/D converter. A general model for a large class of single-input, single-output systems proposed in [6] and [4], is

$$y(k) = H_1(z)u(k) + H_2(z)\xi(k), \quad (6)$$

where $y(k)$ and $u(k)$ are the output and input sequences, respectively, and $\xi(k)$ is a gaussian white noise sequence with variance σ^2 and zero mean. Parameterizing $H_1(z)$ and $H_2(z)$ respectively as $\frac{B(z)}{A(z)}$ and $\frac{1}{A(z)}$ where $B(z)$ and $A(z)$ are defined in (4) and (5), (1) can be expressed as,

$$A(q^{-1})y(k) = B(q^{-1})u(k) + \xi(k) \quad (7)$$

where q^{-1} is the operator of unit delay $q^{-1}y(k) = y(k-1)$ yielding

$$\begin{aligned} y(k) &= -a_1 y(k-1)... - a_{na} y(k-na) + ... \\ &\quad b_0 u(k)... + b_{nb} u(k-nb) + \xi(k) \\ &= x(k)^T \theta + \xi(k) \quad (8) \end{aligned}$$

where

$$\begin{aligned} \theta^T &= (a_1, ..., a_{na}, b_0, ..., b_{nb}) \quad (9) \\ x(k)^T &= (-y(k-1), ..., -y(k-na)... \\ &\quad u(k), ..., u(k-nb)). \quad (10) \end{aligned}$$

Equation (8) can be expressed in vector form for N samples, as

$$Y = X\theta + \Xi \quad (11)$$

where

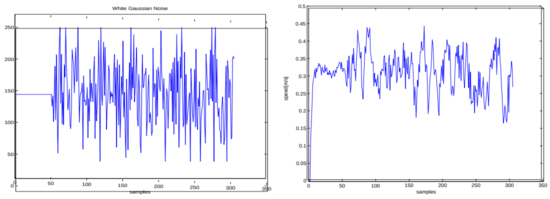
$$Y^T = [y(1), ..., y(N)], \quad (12)$$
$$X^T = [x(1), ..., x(N)], \quad (13)$$
$$\Xi^T = [\xi(1), ..., \xi(N)]. \quad (14)$$

Applying the Least Square Method to (11) suggests that the resulting estimator for θ is,

$$\widehat{\theta} = [X^T X]^{-1} X^T Y. \quad (15)$$

3. APPLICATION OF LEAST SQUARES ESTIMATOR

Now we apply the excitation signal in Fig. 4(a) to the process to obtain the curve of speed of robot's motor 1 (front), in meters per second, as is shown in Fig. 4(b).



(a) Signal 1 - Gaussian white noise. (b) Curve of speed - motor 1.

Figure 4: Signal of excitation 1 and response of motor 1.

We test the efficiency of the Least Square Estimator (15) for eight different transfer functions(TF). Table 3 shows the results of the Least Squares estimation for those eight TFs: the estimation error, value of the gain, poles and zeros are presented for each TF.

Fig.5(a) shows the measured and estimated speed for robot's motor 1, with the transfer function type FTb. The estimation values can be seen in table 3. The error in table 3 is calculated by MSE (Mean Square Error),

$$MSE() = \frac{\sum_1^N [(\widehat{\Phi} - \Phi)^2]}{N}, \qquad (16)$$

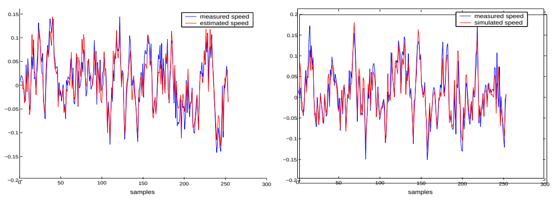
It calculates the sum of squared errors between the vector of estimated speeds($\widehat{\Phi}$)

$$\widehat{\Phi} = [\widehat{y}(1), ..., \widehat{y}(N)]^T,$$

and the vector of measured speeds (Φ) of the motor,

$$\Phi = [\overline{y}(1), ..., \overline{y}(N)]^T.$$

where N is the number of samples.

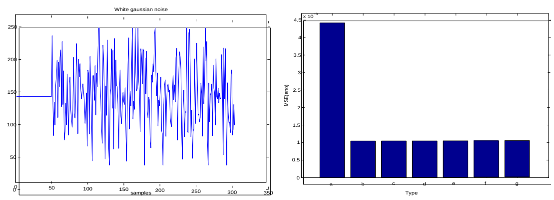


(a) Measured and estimated speeds with FTb - motor 1. (b) Measured and estimated speeds with TFb - (Validation).

Figure 5: Speeds - motor 1.

3.1. VALIDATION

To validate the Least square estimation, we apply another excitation signal to the process and estimated transfer functions, shown in Fig 6(a). In Table 1 and Fig.6(b) we present the MSE of error for the eight TFs for the second excitation signal. Fig.5(b) shows the curves of measured speed and simulated speed, for the transfer function of type FTb.



(a) Signal 2 - Gaussian white noise (b) Results of validation for eight TFs

Figure 6: Signal of excitation 2 and results of validation.

Type TF	order	MSE(error)
FTa	1	0.0042402
FTb	2	0.0010013
FTc	2	0.00099887
FTd	3	0.00099609
FTe	4	0.00099436
FTf	5	0.00099183
FTg	6	0.00099442

Table 1: Results of validation for eight TFs.

Analyzing the errors in the validation in table 1, we conclude that a TF type TFb, order two, is a good approximation of the process, because the system in Fig. 2 has one delay from the loop of communication, represented by the pole at the origin in transfer function TFb. The process of DC motor can be approximated by one first-order system, considering inductance of motor null. Notice that the error with TFg is greater that with TFf, so the best order is 5. The TFb has order two, hence it is simpler to use when designing the controller in general. Moreover the difference between the error of TFb and the others TFs is small. Taking all this into account, we chose TFb. Observe that the transfer function TFa does not have a satisfactory result. The TFd was tested to verify the estimation with two delays (two poles at the origin), but the estimation error is greater than the TFs with one delay.

4. INSTRUMENTAL VARIABLE

The Least Squares estimators are not in general consistent when the sequence $\xi(k)$, in (8), is correlated. Since Instrumental variable estimators are weakly consistent (see [6]and [4]) we implemente it and compare with results from the Least Squares Estimator.

Our Instrumental variable estimator is

$$\overline{\theta} = [Z^T X]^{-1} Z^T Y$$

the matrix Z being constructed using the auxiliary model

$$Z(k)^T = [-\widehat{y}(k-1), -\widehat{y}(k-2), ... \qquad (17)$$
$$-\widehat{y}(k-n), u(k), u(k-1), ..., u(k-n)] \qquad (18)$$

where

$$\widehat{A}(z)\widehat{y}(k) = \widehat{B}(z)u(k)$$

In the above $\widehat{A}(z)$, $\widehat{B}(z)$ are polynomials in z^{-1}. $\widehat{A}(z)$ and $\widehat{B}(z)$ are obtained from an initial least squares fit. Table 2 presents results of estimation with Instrumental variable(IV) and Least squares(LS), for the transfer function type TFb. After three iterations, the values of poles and gain stabilize.

FTs	LS	$IV\ 3^a\ Iteration$
$b1$	0.00081626	0.00081538
$z(z+a1)$	$z(z-0.6827)$	$z(z-0.7051)$
$MSE(error)$	0.0010013	0.00098799

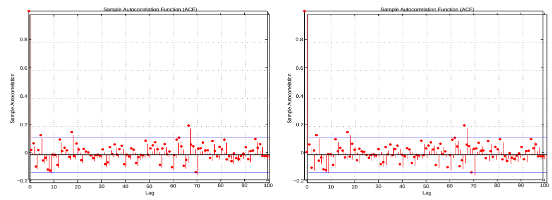
Table 2: Estimated values.

Analyzing the auto-correlation of error, calculated as

$$\widehat{y}(k) = -\widehat{a1}\overline{y}(k-1) + \widehat{b1}u(k-2), \quad (19)$$

$$Erro_r(k) = \widehat{y}(k) - \overline{y}(k). \quad (20)$$

where $\widehat{y}(k)$ is the output estimated, $\overline{y}(k)$ is the output measured and $u(k)$ the input signal, we can calculate the correlation of residuals. In Fig.7 we present the auto-correlation for the estimation with Least Squares and Instrumental Variable, for 100 samples. For the Least Square estimator the mean of de error $Erro_r(k)$, (see (20)) is $5.2879e-4$ and for Instrumental variable estimator is $5.2554e-4$. It is possible to verify that the auto-correlation of error $Erro_r$ for the Least Squares estimator is similar to auto-correlation of the a white noise. This is the reason why it does not have a significant improvement when we use the Instrumental variable estimator.



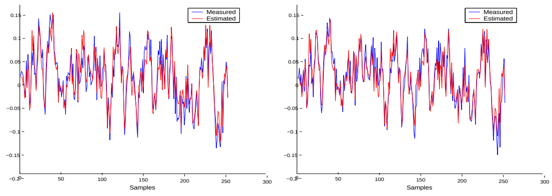
(a) Least Squares (b) Instrumental variable

Figure 7: Auto-correlation of error.

5. OTHERS RESULTS WITH ROBOT'S MOTORS

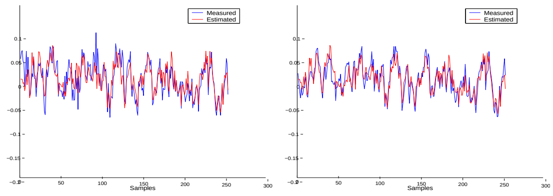
In the previous sections we show the procedure for transfer function estimation of the motor 1 of the mobile robot. Now, we present the estimation results of the motors 2, 3 and 4 of the mobile robot. For these motors the estimation results using LS and IV estimators are shown in table 4. The mobile robot into consideration has left and right wheels (2 and 4) larger than the front and back (1 and 3) wheels. The estimated gains of motors 2 and 4 in table 4 reflect this difference.

Differences between robot's motors estimation are not surprising. They are due to irregular distribution of the weight on base of mobile robot. The transfer function of the motor 4 has a slower pole. This is because of the position of battery on the base of the robot which adds weight to wheel 4. In Fig. 8 and 9 we present the measured and estimated speeds for robot's motors using the transfer function type TFb.



(a) Motor 1 - front wheel (b) Motor 3 - back wheel

Figure 8: Motors 1 and 3.



(a) Motor 2 - left wheel (b) Motor 4 - right wheel

Figure 9: Motors 2 and 4.

6. PID CONTROLLER FOR ROBOT'S MOTORS

To choose appropriated values for parameters of the PID controller (K_c, Ti and Td), we use the close-loop pole locations for an nth-order plant using prototype Bessel systems (see [3]). Equation (21) shows the transfer function chosen to process, obtained from the IV estimator, detailed in section 4..

$$G(z) = \frac{0.00081538}{(z-0.7051)} = \frac{b1}{z-a1} \quad (21)$$

The pole of the origin is ignored since it represents one delay from the loop of communication. The equivalent continuous of the process (21) can be calculated as in is [3]. It is

$$G(s) = \frac{K}{\tau s + 1},$$

for

$$K = \frac{b1}{1+a1}$$

$$\tau = \frac{1}{|ps|}$$

$$pz = e^{Tps}.$$

where ps is the pole in S-plane and pz is the pole in Z-plane. The continuous transfer function is

$$G(s) = \frac{0.002765}{0.1145s + 1} \quad (22)$$

A PI (Proportional + Integral) controller is represented as

$$Gc(s) = \frac{K_c(Tis + 1)}{Tis} \quad (23)$$

We choose a PI controller ($Td = 0$), because the process has characteristics of a first-order system.

The open-loop system G_{MA} is

$$G_{MA}(s) = G_c(s).G(s) = \frac{K_c(Tis + 1)}{Tis}.\frac{K}{\tau s + 1} \quad (24)$$

The close-loop system with unit feedback, from $G_{MA}(s)$, is

$$G_{MF}(s) = \frac{\frac{KK_c}{T_i\tau} + \frac{KK_cs}{\tau}}{s^2 + \frac{(1+KK_c)s}{\tau} + \frac{KK_c}{T_i\tau}} \quad (25)$$

With the transfer function of the close-loop system, which is a second-order system, the procedure for choosing close-loop pole locations is as follows. Let the desired settling time be called T_s. We determine the desired settling time ($T_s = 0.6seg$) of the close-loop system based on performance of robot and taking into account the limitations of the system hardware. Considering the table of normalized Bessel polynomials[3], we divide the roots of the second-order polynomials $p_{1s} = -4.0530 \pm j2.3400$ by T_s to obtain the desired close-loop s-plane pole locations. This yields poles at $p_{0.6s} = -6.7550 \pm j3.9000$.

The value for K_c is given by,

$$-(p_1 + p_2) = \frac{(1 + K * K_c)}{\tau}$$

$$K_c = \frac{-\tau * (p_1 + p_2) - 1}{K} \quad (26)$$

and for T_i,

$$(p_1p_2) = \frac{KK_c}{T_i\tau}$$

$$T_i = \frac{KK_c}{\tau p_1 p_2} \quad (27)$$

where $p_1 = -6.7550 + j3.9000$ and $p_2 = -6.7550 - j3.9000$. The result TF is given by(28),

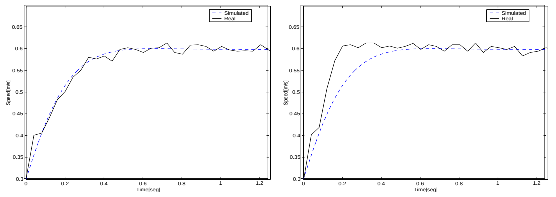
$$G_{MF}(s) = \frac{4.775s + 0.3747}{s^2 + 13.51s + 60.84}. \quad (28)$$

Equations (29) and (30) are, respectively, the continuous PI transfer function and the discrete PI transfer function, invariant to step responses (ZOH-zero-order hold) [1], for a sample period of $10ms$:

$$Gc(s) = \frac{197.68(0.07848s + 1)}{0.07848s} \quad (29)$$

$$Gc(z) = \frac{197.68(z - 0.8726)}{(z - 1)} \quad (30)$$

In Fig.10(a) we show the results of PI controller with the desired settling time ($T_s = 0.6(seg)$). In order to improve this result, we introduce a *Feedforward* gain f from the reference input to the process input. This gain cannot affect the stability of the control system because it does not alter the close-loop poles (see [3]). However, this gain may improve the transient response of the system. We chose the value of the parameter f to be 200, the choice being made based on simulations. Fig.10(b) shows the response of system with gain f. The reference input is a step with amplitude of $0.3(m/s)$ (from $0.3(m/s)$ to $0.6(m/s)$).



(a) Close-loop system without the *Feedforward* gain f.
(b) Close-loop system with the *Feedforward* gain f.

Figure 10: Response of Close-loop system.

7. CONCLUSION AND FUTURE WORKS

In this paper we identify a discrete system, shown in Fig. 2, of the a mobile robot. We use Least Squares and Instrumental Variable estimator. These estimations permit the selection of appropriate values for PI controller, implemented in the mobile robot. This is the first step for the identification of a dynamic model for the whole mobile robot considering it as a multi-variable system and using the dynamic models estimated in this work.

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FTs	FTa	FTb	FTc	FTd
order	1	2	2	3
	$\frac{b1}{z+a1}$	$\frac{b2}{z(z+a1)}$	$\frac{b1z+b2}{z(z+a1)}$	$\frac{b2z+b3}{z^2(z+a1)}$
gain,zero, pole	$\frac{-5.73e-6}{(z-0.7033)}$	$\frac{8.16e-4}{z(z-0.6827)}$	$\frac{7.58e-6(z+107.7)}{z(z-0.6823)}$	$\frac{8.12e-4(z-0.1077)}{z^2(z-0.7295)}$
MSE(error)	0.0034488	0.00036275	0.00036142	0.00036471

FTs	FTe	FTf
order	3	4
	$\frac{b1z^2+b2z+b3}{z(z^2+a1z+a2)}$	$\frac{b1z^3+b2z^2+b3z+b4}{z(z^3+a1z^2+a2z+a3)}$
gain,zero, pole	$\frac{6.25e-6(z+129.6)(z+0.2702)}{z(z-0.7541)(z+0.4024)}$	$\frac{5.28e-6(z+153.5)(z^2+0.206z+0.053)}{z(z-0.6946)(z+0.457)(z-0.183)}$
MSE(error)	0.00035635	0.00032468

FTs	FTg
order	5
	$\frac{b1z^4+b2z^3+b3z^2+b4z+b5}{z(z^4+a1z^3+a2z^2+a3z+a4)}$
gain,zero, pole	$\frac{5.6e-6(z+143.8)(z-0.1982)(z^2+0.4159z+0.1107)}{z(z-0.6526)(z-0.4272)(z^2+0.6709z+0.1383)}$
MSE(erro)	0.00031945

FTs	FTh
order	6
	$\frac{b1z^5+b2z^4+b3z^3+b4z^2+b5z+b6}{z(z^5+a1z^4+a2z^3+a3z^2+a4z+a5)}$
gain,zero, pole	$\frac{9.50e-6(z+84.98)(z^2-0.6704z+0.2717)(z^2+0.8872z+0.3613)}{z(z-0.6577)(z^2-0.728z+0.2254)(z^2+0.9745z+0.3502)}$
MSE(error)	0.00031737

Table 3: Results of estimation for TFs with Least Squares.

Least Squares	Motor 1 front	Motor 2 left	Motor 3 back	Motor 4 right
$\frac{b1}{z(z+a1)}$	0.0008163 / z(z-0.6827)	0.00044521 / z(z-0.6425)	0.00075587 / z(z-0.677)	0.00037482 / z(z-0.7448)

Instrumemtal Variable	Motor 1 front	Motor 2 left	Motor 3 back	Motor 4 right
$\frac{b1}{z(z+a1)}$	0.00081538 / z(z-0.7051)	0.00044513 / z (z-0.6734)	0.00075507 / z (z-0.6953)	0.0003748 / z (z-0.7565)

Table 4: Estimation with Least Squares and Instrumental Variable for Robot's Motors.