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## THE YIELD CURVE AND THE MACRO-ECONOMY ACROSS TIME AND FREQUENCIES

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# The yield curve and the macro-economy across time and frequencies

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**Abstract**

This paper assesses the relation between the yield curve and the main macroeconomic variables in the U.S. between early 1960s and 2009 across time and frequencies, using wavelet analyses. The shape of the yield curve is modelled by latent factors corresponding to its level, slope and curvature, estimated by maximum likelihood with the Kalman filter. The macroeconomic variables measure economic activity, unemployment, inflation and the fed funds rate. The cross wavelet tools employed – coherency and phase difference –, the set of variables and the length of the sample, allow for a thorough appraisal of the time-variation and structural breaks in the direction, intensity, synchronization and periodicity of the relation between the yield curve and the macro-economy. Our evidence establishes a number of new stylized facts on the yield curve-macro relation; and sheds light on several results found in the literature, which could not have been achieved with analyses conducted strictly in the time-domain (as most of the literature) or purely in the frequency-domain.

**Keywords:** Macro-finance; Yield curve; Kalman filter; Continuous wavelet transform; Wavelet coherency; Phase-difference.

**JEL classification**: C32; C49; E43; E44

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# 1 Introduction

The last 25 years have witnessed the development of a prolific literature on the dynamic relation between the shape (level, slope and curvature) of the sovereign yield curve and the main macroeconomic variables. Such relation – possibly bi-directional – is relevant for policymakers in a twofold sense: firstly, the information content of the yield curve may be valuable for the prediction of business cycles, inflation and monetary policy; second, the response of the yield curve may be informative about the transmission of monetary policy and, overall, the dynamic impact of shocks on the macro-economy.

This literature has, so far, been conducted in the time-domain, thus being essentially uninformative about the frequencies at which the relation between the yield curve components and the macroeconomic variables occur. Moreover, for the sake of feasibility, the empirical analyses have typically set a-priori a limited number of possible lead horizons for the dynamic relation between the yield curve and the macro variables and only infrequently have allowed for bi-directional dynamic relations. Further yet, a large part of this literature has used proxies for the level, slope and curvature that account in a rather ad-hoc manner and only partially for the shape of the yield curve.

The overall assessment of the literature clearly suggests that the co-movement between the yield curve shape and the main macroeconomic variables has been subject to time-variation and possibly structural breaks as regards its intensity, its direction, its synchronicity (the lead-lag horizons) and its frequency (the periodicity of the co-movements). Hence, the time-frequency framework is a natural econometric approach to progress in the study of this topic; and, within this approach, wavelets seem surely one of the most promising tools, as will be argued below.

Against this background, this paper studies the relation between the level, slope and curvature of the yield curve and macroeconomic activity, unemployment, inflation and the policy interest rate, in the U.S., across time and frequencies, using Wavelet Analysis. The econometric approach, the extent of the sample – 1961:6 through 2009:12 – and the set of variables, allow for a thorough assessment of the time-variation and breaks that have occurred in the intensity, direction and time-lags of the relation yield curve-macro-economy relation, at each cyclical periodicity.

To measure the shape of the yield curve, rather than using empirical proxies, we adopt a decomposition of the curve into 3 latent factors – level, slope and curvature – that has a long tradition in the finance literature, is model-based, accounts for the whole shape of the curve and is implemented by means of formal econometric techniques. Studying the yield curve-macro relation in the time-frequency domain with such a latent factors approach to the yield curve is another contribution of the paper.

The remaining of the paper is organized as follows. In the second section we describe the related literature, showing how its evolution motivates the use of time-frequency methods. In the third section we present the wavelet analyses tools that are used in the paper. In the fourth section we present the data, with a special focus on the modeling and estimation of the yield curve latent factors. In the fifth section we present and discuss the empirical results of our wavelet analyses. Finally, section six concludes the paper.

# 2 Literature overview

In this section we review the literature on the relation between the yield curve and the macro-economy. We first describe its evolution regarding the set of yield curve components as well as of macroeconomic variables. We then highlight how time-variation or structural breaks in the yield curve-macro relation became dominant of the literature and how it has remained uninformative about the frequency-domain aspects of the relation, thus establishing the motivations for our paper. We finally refer the literature that is closer to our paper, pointing out its limitations and clarifying our contributions.

## 2.1 The yield curve, output and inflation

The earlier literature on the relation between the yield curve and the macro-economy focused on the ability of the curve slope to predict output and inflation.

The ability of the yield curve slope to predict real activity or inflation has been assessed with two classes of regression models. On the one hand, discrete (binary) regression models, in which the dependent variable corresponds to a state of recession or expansion (or to a state of inflation pressure or no pressure); on the other hand, continuous dependent variable models, in which the dependent variable is the growth rate of real output (or changes in the rate of inflation). In some papers, both formulations have, alternatively, been tested – e.g. Estrella, Rodrigues and Schich (2003), Rudebusch and Williams (2009) – and their stability has been compared.

Theoretically, only the expectations component of the term spread should help to predict business cycles, as its term premium component reflects the demand for higher yields to compensate for the loss of liquidity and the risk associated to holding longer-term securities. However, Hamilton and Kim (2002) found that both components make statistically significant contributions, similar at short horizons but larger for the expectations component for predicting output more than two years ahead (with interest rate volatility explaining part of the contribution of the term premium component).[1]

[1] More recently, Estrella and Wu (2008) found that decomposing the spread into expectations and term premium components does not significantly enhance the predictive power of the yield curve. Decomposing the yield slope is beyond the scope of this paper.

Following the seminal paper by Harvey (1988), the term spread (the yield curve slope, typically – but not always – measured as the difference between zero-coupon interest rates of 3-month Treasury bills and 10-year Treasury bonds) has been considered relevant for forecasting business cycles. Stock and Watson (1989) found that interest rate spreads added value to their multivariate index of leading economic indicators. Evidence on the ability of the yield curve slope to predict real economic activity has then been put forth by, e.g., Estrella and Hardouvelis (1991) and Estrella and Mishkin (1998) for the U.S. and Estrella and Mishkin (1997) and Plosser and Rouwenhorst (1994) for several industrialized countries. More recently, it has been shown that the yield slope has a good record in forecasting recessions in real-time (see e.g. Estrella and Trubin, 2006) and has marginal predictive power for U.S. recessions over the Survey of Professional Forecasters (Rudebusch and Williams, 2009). The relevance of the yield slope has survived – and even been reinforced – in the context of more complex dynamic models and iterative forecasting procedures (see e.g. Kauppi and Saikonen, 2009).[2]

As regards inflation, Mishkin (1990a, 1990b and 1990c) and Jorion and Mishkin (1991) found that the difference between the n-month yield and the m-month yield helped to predict the change in inflation between n and m months ahead.

While most of the earlier literature has focused on the ability of the yield curve to predict real activity or inflation, in theory there could be influences in the opposite direction – see e.g. Estrella (2005) – essentially through the feed-back from the macro-economy to monetary policy and its impact on the yield curve. Empirical examination of such effects has been made by, e.g., Estrella and Hardouvelis (1991) and Estrella and Mishkin (1997). In the context of VAR models, Ang and Piazzesi (2003) found evidence that, in the U.S., macro variables explain a large part of the variation in yields, which Evans and Marshall (2007) confirmed and attributed mostly to the systematic reaction of monetary policy. In a similar context, Diebold, Rudebusch and Aruoba (2006) found that, in the U.S., the influence from macroeconomic activity to the yield curve is stronger than the opposite way around. Overall, there are theoretical and empirical results that are consistent with a bidirectional relationship.

The literature has recently evolved along two major paths. One has been the enhancement of the yield curve components used to forecast output and, more generally, the build of macro-finance models with a joint modeling of the yield curve components and the main macro variables. The other has been the explicit consideration of time-variation in the relation between the yield curve components and the macro variables. We now discuss these in turn, as both are

[2] In spite of the overwhelming evidence, there is still less theoretical agreement about why does the yield curve slope predict real output fluctuations. Traditional explanations rely either on the effects of monetary policy – see e.g. Estrella (2005 – or on movements of the real yield curve and their effect on expectations – see e.g. Harvey (1988). Recently, Adrian, Estrella and Shin (2010) suggested a new causal mechanism deriving from the balance sheet management of financial intermediaries who borrow short and lend long. Disentangling the theoretical linkages between the yield curve and the macro variables is, however, beyond the scope of this paper.

crucial motivations for this paper.

## 2.2 The yield curve latent components and the main macroeconomic variables

Following the seminal introduction of macroeconomic variables in the standard affine term structure framework by Ang and Piazzesi (2003), a number of no-arbitrage macro-finance models has been proposed. These have been used in the analysis of several topics, ranging, for example, from the prediction of the yield curve – e.g. Hordahl, Tristani and Vestin (2006) – to the role of inflation expectations in the modeling of long-term bond yields – e.g. Dewachter and Lyrio (2006) – and to the analysis of the monetary policy regime and the macroeconomic structure – e.g. Rudebusch and Wu (2008).[3] Closer to our purposes in this paper, Ang, Piazzesi and Wei (2006) showed that in such a macro-finance model including the two first principal components of the curve – corresponding closely to the short interest rate, a proxy for the curve level, and the term spread, a proxy for its slope – enhances the ability of the model to forecast growth.[4]

Parallel to the no-arbitrage literature, another branch has explored the parsimonious modeling of the yield curve suggested by Nelson and Siegel (1987). First, Diebold and Li (2006) showed how to estimate the Nelson-Siegel components as time-varying parameters that distill the entire yield curve shape period-by-period and interpreted them as the level, slope and curvature of the yield curve. Diebold, Rudebusch and Aruoba (2006) augmented the model with time-series of inflation, output and the policy interest rate, suggested a state-space representation for such macro-finance model and estimated it by maximum-likelihood with the Kalman filter.

Following Diebold, Rudebusch and Aruoba (2006), among others, it became relatively consensual to associate the yield curve level to inflation – especially at low frequencies, reflecting a possible link with inflation expectations –, and the slope to the business cycle. The slope-business cycle association is not, however, as consensual as the level-inflation association (Moench, 2008 found that innovations to the slope generate immediate but mild and insignificant responses of real output). As regards the curvature, while Dewachter and Lyrio (2006) suggested it is associated with monetary policy, its relation with the macro-economy has been harder to establish. Recently, it has been suggested that it could be a coincident indicator for economic activity (Modena, 2008)

and that positive innovations to the curvature (higher concavity) would impact in the level,

[3] See Diebold, Piazzesi and Rudebusch (2005) for a review of the recent evolution and challenges facing the macro-finance models. For the inclusion of a yield curve in the new-keynesian dynamic stochastic general equilibrium models that are currently used for monetary policy conduction and assessment see, e.g., De Graeve, Emiris and Wouters (2009).

[4] Wright (2006) confirmed that there is more information in the shape of the yield curve about the probability of recessions than that provided by the term spread, in the context of probit regression models for predicting U.S. recessions.

generate a hump-shaped increase in the slope (a flattening of the yield curve) and then a significant hump-shaped fall in real output (Moench, 2008).

More recently, Christensen, Diebold and Rudebusch (2009) have specified a generalized no-arbitrage Nelson-Siegel model of the yield curve, bridging the gap between the two above referred branches of the macro-finance literature (see also Rudebusch, 2010). However, as Diebold and Li (2006) and Diebold, Rudebusch and Aruoba (2006) state, it is not clear that arbitrage-free models are necessary or even desirable for forecasting exercises: if the data abides by the no-arbitrage assumption, then the parsimonious but flexible Nelson-Siegel curve should at least approximately capture it; if it's not, imposing it would depress the model's ability to forecast the yield curve and the macro variables. Motivated by these arguments, in this paper we follow the parsimonious Nelson-Siegel decomposition of the yield curve, as detailed in section 4.

## 2.3 Time-variation in the relation between the yield curve and the main macroeconomic variables

The possible time-variation in the yield curve–macro relation has been receiving an increased attention. Initially, within bivariate models of a yield curve factor – generally the slope – and a macro variable, focusing on changes in the intensity and in the time-lags of the relations and used structural break tests; recently, within time-varying parameters models; and, very recently, in the context of macro-finance models.

Stock and Watson (1999) documented econometric instability in the cyclical behavior of a number of U.S. macroeconomic time-series, including the yield curve slope. Haubrich and Dombrosky (1996) found that the predictive ability of the yield spread, although very good, has changed over time. Dotsey (1998) showed that, in contrast to previous periods, the information content of the slope is not statistically significant between the beginning of 1985 and the end of 1997. Estrella, Rodrigues and Schich (2003) tested for structural breaks in models of the slope and real output or inflation, for discrete and continuous dependent variable regressions; overall, they found that models of real output are more stable than models of inflation, and that discrete regression models are more stable than continuous models of the growth rate of output or the inflation rate. Using several alternative measures for the yield slope and multiple structural break tests, Giacomini and Rossi (2006) found a significant breakdown in the forecasting performance of the slope in 1974-76 and in 1979-87. Kucko and Chinn (2009) compared the ability of the yield slope to forecast industrial production growth in samples before and after 1999, finding that overall the predictive ability of the yield slope has decreased after 1998. As Hamilton (2010) refers, recent anecdotal evidence of instability in the yield curve–macro relation is the well-known episode of the summer of 2006 when an inverted yield curve was not followed by a recession, possibly because of the very low level of the curve.

A second line of literature has modeled time-variation with more sophisticated methods, but has overall remained focused on a single component of the yield curve – the slope – and its relation to one macro variable. Using Bayesian time-varying parameters VARs with stochastic volatility, Benati and Goodhart (2008) detected changes in the marginal predictive power of the yield slope for output growth at several forecast horizons in a number of countries, which have not always followed the same pattern for alternative forecast horizons. Time-varying parameter models relating the yield slope and output growth, with ex-post and real-time data, have been used by De Pace (2009), to find a decrease in the marginal predictive power in the recent years in the U.S. and U.K. and a marked instability of the relation in continental European countries. Chauvet and Potter (2005) allowed for time-varying parameters and for auto-correlated errors (to account for the duration of business cycles) in a discrete regression model of the yield slope and output growth in the U.S., finding that once such time-variation is considered, inversions of the yield curve forecast high probability of recessions. Chauvet and Senyuz (2009) found evidence of time-variation and breaks in the forecast-horizon at which yields predict output growth, in a dynamic bi-factor model that extracts from the data a yield curve cycle and a business cycle, each following its own two-state Markov switching process.

Very recently, some papers have allowed for time-varying dynamic relations within macro-finance models, rather than bivariate models. While some have done so imposing no-arbitrage restrictions – e.g. Ang, Boivin, Dong and Loo-Kung (2009) – others have pursued versions of the Nelson and Siegel (1987) parsimonious yield curve model – e.g. Mumtaz and Surico (2008) and Bianchi, Mumtaz and Surico (2009).

Overall, our review of the literature shows that allowing for time-variation in the intensity and time-lags of the yield-curve–macro relation is currently a key issue.

## 2.4 The Wavelets approach

Having devoted most of the effort in tackling time–variation issues, the literature has remained uninformative about the frequencies (cyclical periodicity) at which the relation between the yield curve components and the macroeconomic variables occurs. Yet, given the changes in the structure of the economy and in the monetary policy regimes, there surely may have been frequency variations in the yield curve–macro relation.

Overall, progress in the study of the yield curve–macro relation may be pursued with a framework that (i) considers the whole yield curve shape (level, slope, curvature), (ii) allows for time-varying sensitivity and lead/lags, and (iii) allows for time-varying frequencies. The continuous time-frequency framework thus emerges as an approach with unique advantages to study this topic; and, within this approach, wavelets are the most promising tool.

Against this background, we use wavelet analysis tools – previously employed by Aguiar-

Conraria, Soares and Azevedo (2008) and Aguiar-Conraria and Soares (forthcoming) – to disentangle the time-frequency relations between the 3 Nelson-Siegel latent factors of the yield curve (level, slope and curvature) and 4 macroeconomic variables (unemployment, an index of macroeconomic activity, inflation and the monetary policy interest rate) in the U.S. Following analyses using the wavelet power spectrum, we then compute the cross-wavelet transform and coherence as well as the phase difference. For each pair formed by a yield curve factor and a macroeconomic variable, these tools give us quantified indications of, respectively, the similarity of power between each time series and a measure of the lead-lags between their oscillations, at each time and frequency. These wavelet tools provide a thorough vision of the inter-relation between the yield curve components and the macro variables that is almost impossible to obtain with purely time-domain or frequency-domain analysis.

# 3 Wavelets Analysis

## 3.1 The Wavelet

The minimum requirements imposed on a function $\psi$ to qualify for being a mother (admissible or analyzing) wavelet are that $\psi$ is a square integrable function and also that it fulfills a technical condition, usually referred to as the *admissibility condition*.

For most of the applications, the wavelet $\psi$ must be a well localized function, both in the time domain and in the frequency domain, in which case the admissibility condition reduces to requiring that $\psi$ has zero mean, i.e. $\int_{-\infty}^{\infty} \psi(t)\, dt = 0$. This means that the function $\psi$ has to wiggle up and down the $t-$axis, i.e. it must behave like a wave; this, together with the assumed decaying property justifies the choice of the term wavelet to designate $\psi$.

### 3.1.1 The Continuous Wavelet Transform

Starting with a mother wavelet $\psi$, a family $\psi_{\tau,s}$ of "wavelet daughters" can be obtained by simply scaling and translating $\psi$:

$$\psi_{\tau,s}(t) := \frac{1}{\sqrt{|s|}} \psi\left(\frac{t-\tau}{s}\right), \quad s, \tau \in \mathbb{R}, s \neq 0, \tag{1}$$

where $s$ is a scaling or dilation factor that controls the width of the wavelet (the factor $1/\sqrt{|s|}$ being introduced to guarantee preservation of the unit energy, $\left\|\psi_{\tau,s}\right\| = 1$) and $\tau$ is a translation parameter controlling the location of the wavelet. Scaling a wavelet simply means stretching it (if $|s| > 1$) or compressing it (if $|s| < 1$), while translating it simply means shifting its position in time. Given a time series $x(t)$, its continuous wavelet transform (CWT) with respect to the wavelet $\psi$ is a function of two variables, $W_x(\tau, s)$ :

$$W_x(\tau, s) = \langle x, \psi_{\tau,s} \rangle = \int x(t) \frac{1}{\sqrt{|s|}} \overline{\psi} \left( \frac{t-\tau}{s} \right) dt, \tag{2}$$

where the bar denotes complex conjugation.

In practice, we deal with a discrete time-series $x = \{x_n,\ n = 0, \ldots, T-1\}$ of $T$ observations with a uniform time step $\delta t$, which we can take as the unity ($\delta t = 1$), the integral in (2) has to be discretized and is, therefore, replaced by a summation over the $T$ time steps; also, it is convenient, for computational efficiency, to compute the transform for $T$ values of the parameter $\tau$, $\tau = m\,;\ m = 0, \ldots, T-1$. In practice, naturally, the wavelet transform is computed only for a selected set of scale values $s \in \{s_k, k = 0, \ldots, F-1\}$ (corresponding to a certain choice of frequencies $f_k$). Hence, our computed wavelet spectrum of the discrete-time series $x$ will simply be a $F \times T$ matrix $W_x$ (wavelet spectral matrix) whose $(k, m)$ element is given by

$$W_x(k, m) = \frac{1}{\sqrt{s_k}} \sum_{n=0}^{T-1} x_n \overline{\psi} \left( \frac{(n-m)}{s_k} \right);\ m = 0, \ldots, T-1,\ k = 0, \ldots, F-1. \tag{3}$$

### 3.1.2 The Choice of the Mother Wavelet

There are several types of wavelet functions available with different characteristics, such as, Morlet, Mexican hat, Haar, Daubechies, etc. Since the wavelet coefficients $W_x(s, \tau)$ contain combined information on both $x(t)$ and $\psi(t)$, the choice of the wavelet is an important aspect to be taken into account, which will depend on the particular application one has in mind.

If quantitative information about phase interactions between two time-series is required, continuous, rather than discrete, and complex wavelets provide the best choice. When the wavelet $\psi(t)$ is chosen as a complex-valued function, the wavelet transform $W_x(\tau, s)$ is also complex-valued. In this case, the transform can be separated into its real part, $\Re(W_x)$, and imaginary part, $\Im(W_x)$, or in its amplitude, $|W_x(\tau, s)|$, and phase, $\phi_x(\tau, s) : W_x(\tau, s) = |W_x(\tau, s)|\, e^{i\phi_x(\tau,s)}$. The phase-angle $\phi_x(\tau, s)$ of the complex number $W_x(\tau, s)$ can be obtained from the formula: $\tan(\phi_x(\tau, s)) = \frac{\Im(W_x(\tau,s))}{\Re(W_x(\tau,s))}$, using the information on the signs of $\Re(W_x)$ and $\Im(W_x)$ to determine to which quadrant the angle belongs to. For real-valued wavelet functions, the imaginary part is zero and the phase is undefined. Therefore, to separate the phase and amplitude information of a time-series, it is necessary to use complex wavelets.

In such case, it is convenient to choose $\psi(t)$ to be progressive (or analytic), i.e. to be such that $\hat{\psi}(f) = 0$, for $f < 0$. In fact, when $\psi$ is analytic and $x(t)$ is real, reconstruction formulas involving only positive values of the scale parameter $s$ are available.[5] Analytic wavelets are ideal

[5]In particular, if $0 < K_\psi := \int_0^\infty \frac{\overline{\hat{\psi}(f)}}{f} df < \infty$, one can use the reconstruction formula, given by $x(t) =$

for the analysis of oscillatory signals, since the continuous analytic wavelet transform provides an estimate of the instantaneous amplitude and instantaneous phase of the signal in the vicinity of each time/scale location $(\tau, s)$. In the rest of this paper, it is always implicitly assumed that the scaling parameter $s$ takes only positive values.

Therefore, for our applications it is essential to choose a complex analytic wavelet, as it yields a complex transform, with information on both the amplitude and phase, crucial to study the cycles synchronism. Examples of popular analytic wavelets are the Paul, Gaussian, Morlet, and Shannon mother wavelets.

The Morlet wavelet has another important property: it has optimal joint time-frequency concentration. Theoretically, the time-frequency resolution of the continuous wavelet transform is bounded by the Heisenberg box, which describes the trade-off relationship between time and frequency. The area of the Heisenberg box is minimized with the choice of the Morlet wavelet.[6]

For all these reasons we will use the Morlet wavelet, first introduced in Goupillaud et al. (1984):

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}. \tag{4}$$

All our numerical results are obtained with the particular choice $\omega_0 = 6$. For this parametrization of the Morlet wavelet, there is an inverse relation between wavelet scales and frequencies, $f \approx \frac{1}{s}$, greatly simplifying the interpretation of the empirical results. Thanks to this very simple one-to-one relation between scale and frequency we can use both terms interchangeably.

## 3.2 Wavelet Tools

In analogy with the terminology used in the Fourier case, the (local) **wavelet power spectrum** (sometimes called scalogram or wavelet periodogram) is defined as

$$(WPS)_x(\tau, s) = |W_x(\tau, s)|^2. \tag{5}$$

This gives us a measure of the variance distribution of the time-series in the time-scale/frequency plane.[7]

The concepts of cross wavelet power, wavelet coherency and phase-difference are natural generalizations of the basic wavelet analysis tools that enable us to deal with the time-frequency dependencies between two time-series. The cross-wavelet transform of two time-series, $x(t)$ and

$2\Re\left[\frac{1}{K_\psi}\int_0^\infty W_x(t,s)\frac{ds}{s^{3/2}}\right]$.

[6] Unlike, for example the Paul wavelet, which has good time localization, but, at the same time, is known for its poor frequency localization.

[7] Sometimes the wavelet power spectrum is averaged over time for comparison with classical spectral methods. When the average is taken over all times, we obtain the global wavelet power spectrum, $(GWPS)_x(s, \tau) = \int_{-\infty}^{\infty} |W_x(\tau, s)|^2 \, d\tau$.

$y(t)$, is defined as

$$W_{xy}(\tau, s) = W_x(\tau, s)\,\overline{W_y}(\tau, s), \tag{6}$$

where $W_x$ and $W_y$ are the wavelet transforms of $x$ and $y$, respectively. We define the **cross wavelet power**, as $|W_{xy}(\tau, s)|$. The cross-wavelet power of two time-series depicts the local covariance between two time-series at each time and frequency. Therefore, cross-wavelet power gives us a quantified indication of the similarity of power between two time-series. When compared with the cross wavelet power, the **wavelet coherency** has the advantage of being normalized by the power spectrum of the two time-series. In analogy with the concept of coherency used in Fourier analysis, given two time-series $x(t)$ and $y(t)$ one defines their wavelet coherency:

$$R_{xy}(\tau, s) = \frac{|S(W_{xy}(\tau, s))|\,|}{\sqrt{S(|W_{xx}(\tau, s)|)\,S(|W_{yy}(\tau, s)|)}},$$

where $S$ denotes a smoothing operator in both time and scale.

Although there is some work done on the theoretical distribution of the wavelet power (Ge, 2007) and on the distribution of cross wavelets (Ge, 2008), the available tests imply null hypotheses that are too restrictive to deal with economic data. Therefore, we will rely on Monte Carlo simulations for statistical inference.

As we have discussed, one of the major advantages of using a complex-valued wavelet is that we can compute the phase of the wavelet transform of each series and thus obtain information about the possible delays of the oscillations of the two series as a function of time and scale/frequency, by computing the phase difference. The **phase difference** can be computed from the cross wavelet transform, by using the formula

$$\phi_{x,y}(s, \tau) = \tan^{-1}\left(\frac{\Im(W_{xy}(s, \tau))}{\Re(W_{xy}(s, \tau))}\right), \tag{7}$$

and information on the signs of each part to completely determine the value of $\phi_{xy} \in [-\pi, \pi]$. A phase-difference of zero indicates that the time series move together at the specified frequency; if $\phi_{xy} \in (0, \frac{\pi}{2})$, then the series move in phase, but the time-series $y$ leads $x$; if $\phi_{xy} \in (-\frac{\pi}{2}, 0)$, then it is $x$ that is leading; a phase-difference of $\pi$ (or $-\pi$) indicates an anti-phase relation; if $\phi_{xy} \in (\frac{\pi}{2}, \pi)$, then $x$ is leading; time-series $y$ is leading if $\phi_{xy} \in (-\pi, -\frac{\pi}{2})$.

With the phase difference one can calculate the instantaneous time lag between the two time-series:

$$\Delta T(s, \tau) = \frac{\phi_{x,y}(s, \tau)}{2\pi f(\tau)}, \tag{8}$$

where $f(\tau)$ is the frequency that corresponds to the scale $\tau$.

# 4 Data and Estimation

In this section we present the data used in our wavelet analyses. In a first subsection we describe the source of the zero-coupon yield data and then the modeling choices made to estimate the latent factors that define the shape of the yield curve at each moment. In a second subsection we present the macroeconomic data. For each of our seven time-series we provide and analyze their Wavelet power spectrum, which proves to be very useful preliminary information.

## 4.1 Yield data and the yield curve latent factors

At each point in time, the yield curve is the set of yields of zero-coupon Treasury securities for each residual maturity. As, in practice, the Treasury issues a limited number of securities with different maturities and coupons, obtaining the yield curve at each moment requires estimation, i.e. inferring what the zero-coupon yields would be across the whole maturity spectrum. Yield curve estimation requires the assumption of some model for the shape of the yield curve, so that the gaps may be filled in by analogy with the yields seen in the observed maturities. Once a model is selected, estimates of its coefficients are chosen so that the weighted sum of the squared deviations between the actual prices of Treasury securities and their predicted prices is minimized. Once the values for the coefficients are estimated, they may be straightforwardly used to obtain the notional zero-coupon yields for the residual maturities absent from the raw data.[8]

In this paper we use U.S. yield curve data for 1961:6-2009:12 publicly made available by Gurkaynak, Sack and Wright (2007b).[9] These data are estimated with an approach that follows the extension by Svensson (1994) of the functional form originally suggested by Nelson and Siegel (1987). The online database Gurkaynak, Sack and Wright (2007b) provides regularly updated daily zero-coupon yields for all yearly maturities from 1 to 30 years and has been increasingly used in recent research (see e.g. De Graeve, Emiris and Wouters, 2009, and Chauvet and Senyuz, 2009).

Given our purpose of relating the yield curve with macro variables, we are not interested in daily yield curves but rather in monthly yield curve data. In the literature, there is a large heterogeneity as regards the choice between beginning-of-month, end-of-month or monthly average zero-coupon yields, depending on the objectives of the analysis. On the one hand, studies focusing on the forecasting relations between the yield curve and the macro-economy typically use monthly averages of zero-coupon yields (e.g. Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1997, 1998, Estrella, Rodrigues and Schich, 2003, and Rudebusch and Williams,

[8]Typically, yield curve estimation further requires filtering out some issues that have insignificant liquidity, due to small outstanding amounts or residual life. See Gurkaynak, Sack and Wright (2007a) for additional details.

[9]The data are daily estimates and are available at http://www.federalreserve.gov/pubs/feds/2006/

2009). On the other hand, papers focusing on macro-finance structural modeling tend to choose beginning-of-month or end-of-month yield data, according to the timing assumptions in the model (e.g. Diebold and Li, 2006, Diebold, Rudebusch and Aruoba, 2006 and Rudebusch and Wu, 2008). Given our empirical framework and purposes, we follow Estrella and Hardouvelis (1991, p. 558) and use average monthly yield data.

In view of our purposes and following the literature, we are not interested in the very-long end of the yield curve (maturities above 10 years), while, in contrast, we are interested in a richer set of yield curve points for short and medium term residual maturities than those present in Gurkaynak, Sack and Wright (2007b). Accordingly, we use the appropriate formulae and parameters in Gurkaynak, Sack and Wright (2007a, 2007b) and compute the implied zero-coupon yields for a set of additional relevant intra-year maturities. We end up with monthly-average time-series of zero-coupon yields for the 17 maturities considered in Diebold, Rudebusch and Aruoba (2006): 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months.

We then used these yield curve data to estimate the yield curve latent factors – level, slope and curvature –, following the parsimonious Nelson and Siegel (1987) approach to the modelling of the yield curve used by e.g. Diebold and Li (2006) and Diebold, Rudebusch and Aruoba (2006). Our choice of not following an arbitrage-free approach is motivated by the arguments set out by Diebold and Li (2006, pp. 361-362) and Diebold, Rudebusch and Aruoba (2006, pp. 333) and briefly described above in section 2.

The yield curve is modeled with the three-component exponential approximation to the cross-section of yields at any moment in time proposed by Nelson and Siegel (1987),

$$y(\tau) = \beta_1 + \beta_2 \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_3 \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \tag{9}$$

where $y(\tau)$ denotes the set of (zero-coupon) yields and $\tau$ denotes the corresponding maturity.[10]

Following Diebold and Li (2006) and Diebold, Rudebusch and Aruoba (2006), the Nelson-Siegel representation is interpreted as a dynamic latent factor model where $\beta_1, \beta_2$ and $\beta_3$ are time-varying parameters that capture the level (L), slope (S) and curvature (C) of the yield curve

[10] The above mentioned Svensson (1994) extension of this functional form that is adopted by Gurkaynak, Sack and Wright (2007a) to generate the zero-coupon yields may be, analogously, written as follows: $y(\tau) = \beta_1 + \beta_2 \left( \frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} \right) + \beta_3 \left( \frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau} \right) + \beta_4 \left( \frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau} \right)$. Svensson's (1994) model includes an additional $\beta$ and an additional $\lambda$, used to determine its loading. $\beta_4$ is meant to capture humps occurring at the very long end of the curve – typically around the 20-years maturity. As mentioned in the text, such very long maturities are typically skipped in macro-finance analysis, given that they add little relevant information to the 10-years maturities in which economics agents' typically base their longer-term decisions and, moreover, in view of the rather low liquidity of most of their issues (for instance, before the 1980s, Gurkaynak, Sack and Wright (2007a, 2007b) restrict $\beta_4$ to zero, for the longest maturities – thus imposing the original Nelson and Siegel, 1987, formulation).

at each period t, while the terms that multiply the factors are the respective factor loadings:

$$y(\tau) = L_t + S_t \left( \frac{1 - e^{-\lambda\tau}}{\lambda_\tau} \right) + C_t \left( \frac{1 - e^{-\lambda\tau}}{\lambda_\tau} - e^{-\lambda\tau} \right). \tag{10}$$

$L_t$ may be interpreted as the overall level of the yield curve, as its loading is equal for all maturities; $S_t$ has a maximum loading (equal to 1) at the shortest maturity, which then monotonically decays through zero as maturities increase; $C_t$ has a loading that is null at the shortest maturity, increases until an intermediate maturity and then falls back to zero as maturities increase. Hence, $S_t$ and $C_t$ may be interpreted as the short-end and medium-term latent components of the yield curve, with the coefficient $\lambda$ ruling the rate of decay of the loading towards the short-term factor and the maturity where the medium-term factor has maximum loading.[11]

As in Diebold, Rudebusch and Aruoba (2006) we assume that $L_t$, $S_t$ and $C_t$ follow a vector autoregressive process of first order, which allows for casting the yield curve latent factor model in state-space form and using the Kalman filter to obtain maximum-likelihood estimates of the hyper-parameters and the implied estimates of the time-varying parameters $L_t$, $S_t$ and $C_t$.

The state-space form of the model comprises the transition system

$$\begin{bmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{bmatrix} + \begin{bmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{bmatrix}, \tag{11}$$

where $t = 1, \ldots .T$ is the sample period, $\mu_L$, $\mu_S$ and $\mu_C$ are estimates of the mean values of the three latent factors, and $\eta_t(L)$, $\eta_t(S)$ and $\eta_t(C)$ are innovations to the autoregressive processes of the latent factors.

The state-space form further comprises the measurement system, relating a set of $N$ observed zero-coupon yields of different maturities to the three latent factors by

$$\begin{bmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{bmatrix} = \begin{bmatrix} 1 & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda_{\tau_1}}\right) & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda_{\tau_1}} - e^{-\lambda\tau_1}\right) \\ 1 & \left(\frac{1-e^{-\lambda_2}}{\lambda_{\tau_2}}\right) & \left(\frac{1-e^{-\lambda\tau_2}}{\lambda_{\tau_2}} - e^{-\lambda\tau_2}\right) \\ \vdots & \vdots & \vdots \\ 1 & \left(\frac{1-e^{-\lambda\tau_3}}{\lambda_{\tau_3}}\right) & \left(\frac{1-e^{-\lambda\tau_N}}{\lambda_{\tau_N}} - e^{-\lambda\tau_N}\right) \end{bmatrix} \begin{bmatrix} L_t \\ S_t \\ \vdots \\ C_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{bmatrix}, \tag{12}$$

where $t = 1, \ldots, T$, and $\varepsilon_t(\tau_1), \varepsilon_t(\tau_2), \ldots \varepsilon_t(\tau N)$ are measurement errors, i.e. deviations of the observed yields at each period t and for each maturity $\tau$ from the implied yields defined by the

[11] Diebold and Li (2006) assume $\lambda = 0.0609$, which corresponds to a maximum of the curvature at 29 months, while Diebold, Rudebusch and Aruoba (2006) estimate $\lambda = 0.077$ for the US 1970-2001, with Fama-Bliss zero-coupon yields, which corresponds to a maximum of the curvature at 23 months.

shape of the fitted yield curve. In matrix notation, the state-space form of the model may be written, using the transition and measurement matrices $A$ and $\Lambda$, as

$$f_t - \mu = A\left(f_{t-1} - \mu\right) + \eta_t \tag{13}$$

$$y_t = \Lambda f_t + \varepsilon_t \tag{14}$$

For the Kalman filter to be the optimal linear filter, it is assumed that the initial conditions set for the state vector are uncorrelated with the innovations of both systems: $E\left(f_t \eta_t^T\right) = 0$ and $E\left(f_t \varepsilon_t^T\right) = 0$.

Following Diebold, Rudebusch and Aruoba (2006) we assume that the innovations of the measurement and of the transition systems are white noise and mutually uncorrelated

$$\begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix} \sim WN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & H \end{bmatrix}\right), \tag{15}$$

where the matrix of variance-covariance of the innovations to the transition system $Q$ is unrestricted, while the matrix of variance-covariance of the innovations to the measurement system $H$ is assumed to be diagonal.

Given a set of starting values for the parameters (the three latent factors) and for the hyper-parameters (the coefficients that define the statistical properties of the model, such as, e.g., the variances of the innovations), the Kalman filter may be run from $t = 2$ through $t = T$ and the one-step-ahead prediction errors and the variance of the prediction errors may be used to compute the log-likelihood function. The function is then iterated on the hyper-parameters with standard numerical methods and at its maximum yields the maximum-likelihood estimates of the hyper-parameters and the implied estimates of the time-series of the time-varying parameters $L_t$, $S_t$ and $C_t$. The latent factors are then recomputed with the Kalman smoother, which uses the whole dataset information to obtain the factors at each period from $t = T$ through $t = 2$ (see Harvey, 1989, for details on the Kalman filter and the fixed-interval Kalman smoother).

The resulting time-series, which are depicted in Figure 1, are those subject to the cross wavelet analyses of the next section, jointly with the macroeconomic data described in the next sub-section. [12]

The figure further includes the power spectrum of each of the latent factors, which is, by itself, very informative (it is apparent that the time-frequency decomposition of the three factors

[12] Recall that, by construction, negative values of the slope correspond to the typical upward sloping yield curve (positive values of the slope correspond to inverted yield curves) and thus most of the times the slope factor is negative. In turn, positive, null and even (in most cases) small negative values of the curvature correspond to the typical concave yield curve; below some negative value (which is a function of the value of the slope) the yield curve turns into a convex curve. In short, the lower the value of the slope, the steeper is the yield curve; the lower the value of the curvature, the less concave is the yield curve.

is quite different).[13]

In the case of the level, the most intense action occurs until 1990, at a frequency band centered on the 12 years and encompassing frequencies from around 10 to a bit more than 14 years. Overall, the power is significant for frequency bands from 8 to 16 years throughout the whole sample period. In the 1980s, there is a region of significant power at a frequency band between 3 and slightly more than 4 years, clearly associated with the intense short-run variability of the overall level of interest rates during the disinflation initiated with the Volcker monetarist experiment of 1979-82.

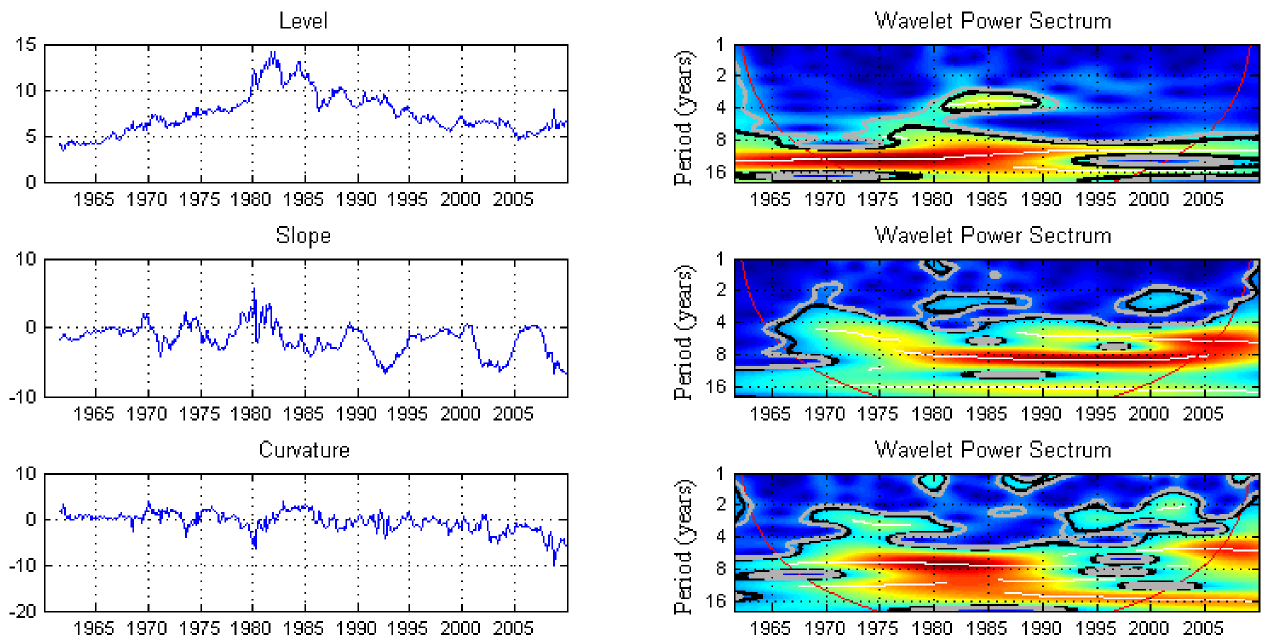


Figure 1: Level, Slope and Curvature of Yield Curve, U.S. 1961:6-2009:12

In the case of the slope, the time-frequency regions with higher energy have occurred since the mid-1970s at frequencies around 8 years, with a spread to higher frequencies – centered on the 6 years – since the early-2000s. The power is significant at 5 percent for frequency bands corresponding to periods above 4 years throughout most of the sample, thus comprising variability at business cycles frequencies, as expected. The power spectrum detects the high sort-run variability of the slope during the monetarist episode, which is associated with a frequency-band of 1∼1.5 years and another of 2∼4 years – with the latter spreading through the whole 1980s. It is furthermore worth notice a high power of the slope in the late 1990s and early 2000s.

[13]Our Wavelet figures throughout the paper depict the power at each time-frequency region associating colder colors (in the extreme, blue) with low power and hotter colors (in the extreme, red) with higher power. The dark lines represent regions of statistically significant powers at 5 percent, while grey lines delimit regions significant at 10 percent. Significance levels have been obtained by bootstraping with 5000 replications. The white stripes show the maxima of the undulations of the wavelet power spectrum.

The peaks of energy are rather less concentrated, in the case of the curvature. In fact, a very high energy is detected during the 1970s and 1980s at a frequency band of between 6 and 8 years, but also, since mid-1970s, at a band centered on 12 years – which then gradually moves to a band that approaches the 16 year frequency and is visible until the end of the sample. Moreover, in the second half of the 2000s there is a very high energy in the 4∼6 frequency band. As regards the time-frequency regions with significant power, the curvature displays a pattern somehow similar to the slope – all regions tend to be significant for frequencies lower than 4 years – but with the difference that there are more important regions of significance for higher frequencies, namely at the 2∼4 band during the 1970s and 1990s, at the 1.5∼3 band in the early 2000s and at the 1∼1.5 band in the second half of the 1980s and first half of the 1990s. The Volcker monetarist episode is also captured as a region of significant power in the 1∼1.5 years band around 1980.

## 4.2 Macroeconomic data

Our macroeconomic data include the civilian unemployment rate, a measure of economic activity, a measure of price inflation (yearly per cent changes of the Consumer Price Index) and a measure of the monetary policy interest rate (the Effective Federal Funds Rate). With the exception of the economic activity index the data have been downloaded from the St Louis Fed website (FRED database).

In order to complement the information given by the unemployment rate, with a measure of the business cycle (filtered out of natural rate changes), several indexes of economic activity could be used. The most obvious would be GDP growth or an output gap, but these have the very important disadvantage of only being available on a quarterly basis. A possible monthly indicator of economic activity would be the industrial production index. However this indicator is rather limited in its coverage of the overall economic activity – especially as the U.S. increasingly developed non-industrial activities since the 1980s. Another possible indicator would be the Chicago Fed National Activity Index (CFNAI). Unfortunately, there is no data for this indicator before 1967: using it would imply missing an important amount of data. All considered, we chose to use the Aruoba Diebold Scotti Index (ADS index, see Aruoba, Diebold and Scotti, 2009), available in the Philadelphia Fed website. The correlation between the CFNAI and the ADS is above 93%, suggesting that they are close substitutes.

Figure 2 presents the time-series of our macroeconomic variables, as well as their Wavelet power spectrum, showing their different behavior in the time-frequency space.

The wavelet power spectrum of the unemployment rate is, overall, statistically significant during most of the sample period for cycles of periodicity higher than 4 years. Around the first oil shock the power is significant also at cycles of smaller periodicity, a pattern that is even more evident in the first half of the 1980s, during the disinflation. As regards the time-frequency regions with higher in energy, one can identify a peak in the 8∼12 frequency band, from 1970 until 2000. Around the 6 year frequency, we observe two peaks, one in the middle and the second half of the 1970s, coinciding with the oil crises, and another in the second half of the first decade of the new millennium. Hence, with the exception of the abnormal episodes of the 1970s, it seems that the unemployment rate has most of its variability in frequencies marginally lower than the business cycle.

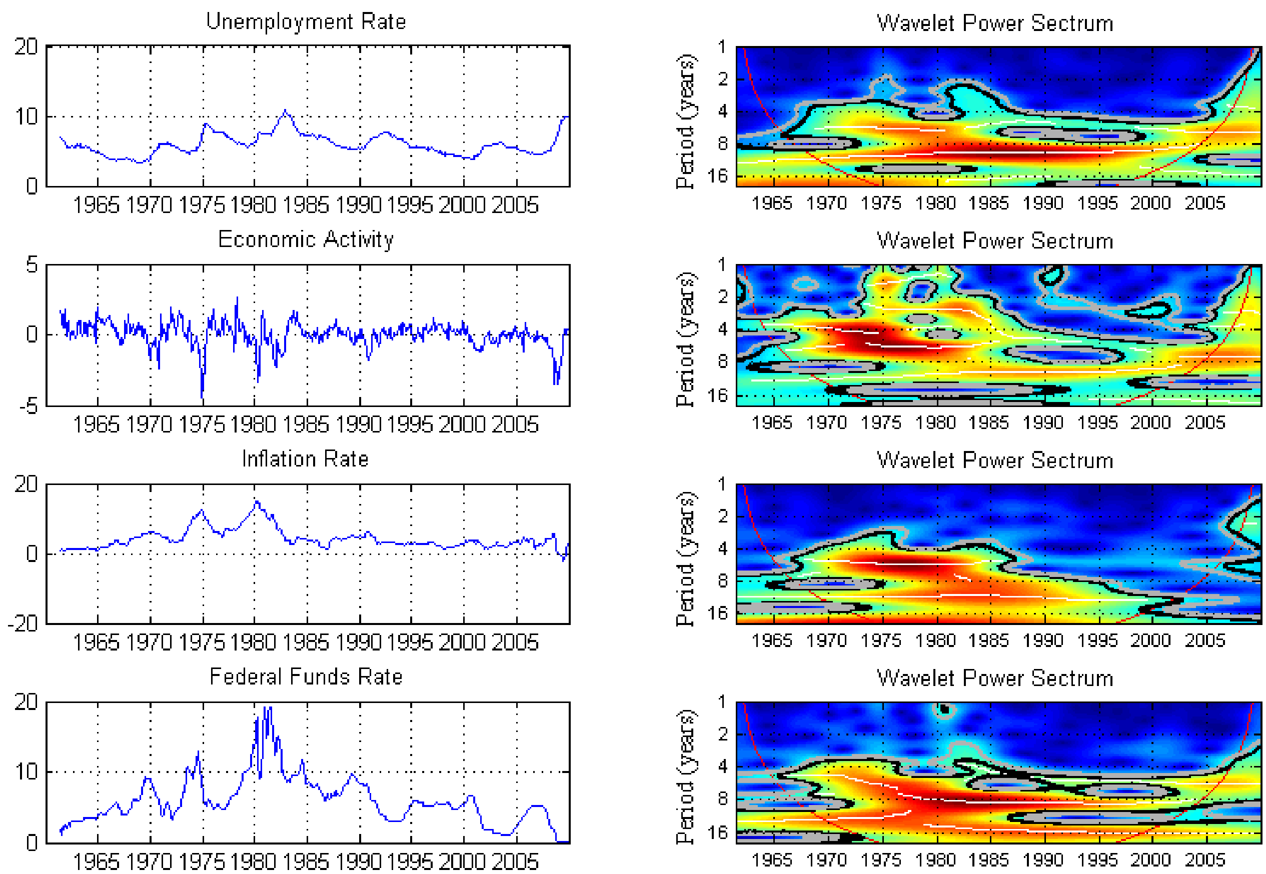


Figure 2: Macroeconomic Variables, U.S. 1961:6-2009:12

The peaks of energy in the wavelet power spectrum of the index of economic activity ADS are quite consistent with the history of U.S. business cycles. They show the stability of the 1960s, followed by the cyclical turmoil of the 1970s and early 1980s – first the oil shocks, then the effects of the disinflationary policy – which is identified as cycles in the frequency-band of 4∼8 years, with longer cycles being more predominant in the final part of this period; they then show the Great Moderation, as no comparable peaks occur after 1983/84 and the oscillations of the late 1980s and early 1990s are allocated to a frequency between the 9 and 12 years; finally, they show the increased cyclical volatility in the second half of the 2000s, identifying high power at frequencies in the 6 to 8 years. The effects of the oils shock and the disinflationary policy are also shown in significant powers at frequencies corresponding to shorter cycles, especially between 1973 and 1985; a similar significance region is also apparent in the late 2000s, corresponding to the recent financial and economic crisis.

As expected, the time-frequency decompositions of inflation and the fed funds rate are broadly similar and are very informative about the monetary history of the U.S.

The power spectrum of inflation clearly shows the buildup of inflation since the 1960s, with an expansion of the regions with significant power, and the inflationary effect of the oil shocks – with visible peaks in the 4∼8 frequency-band during the 1970s and early 1980s. It then shows the gradual control of inflation after the mid-1980s, with a decrease in the regions of significance and a gradual shift of the energy peaks to cycles of longer period – frequency-bands of more than 8 years – until the disappearance of peaks since the early 1990s, consistent with the recent control of inflation. There is evidence that this control includes long-run inflation – and thus inflation expectations – as the peak in frequencies corresponding to more than 16 years that existed since the 1960s also disappears around 1990.

In the second half of the 1960s, as monetary policy reacted to increasing inflation, the power spectrum of the fed funds rate starts showing peaks at frequency bands around the 12 years. These peaks then extend through cycles of higher frequency and join a region of peaks that appeared during the early 1970s at the 4∼8 frequency-band, to form a region of very strong power in frequencies between 6 and 16 years in the second half of the 1970s. The 1979-82 monetarist experiment is clear in the island of significant power at the 1∼1.5 frequency band and the whole disinflationary policy is apparent in the peaks that occur in a broad range of frequencies. The power spectrum then evolves to a concentration of energy around two poles, the 8 and the 16 years frequencies. These eventually fade out during the 1990s, which suggests that monetary policy has been far less active thereafter, with the above mentioned control of inflation. At the end of the sample there are signs of a resurrection of monetary policy, especially at business cycles frequencies (around 6 years) clearly in response to the financial and economic crisis.

# 5 Empirical Results

In this section we present the results of the Wavelet analyses of our time-series – the coherence and the phase difference between each pair of yield curve latent factor (level, slope, curvature) and macro variable (unemployment, economic activity, inflation and fed funds rate). Our tools give quantified indications of the similarity of power between each time series (including significance values generated by bootstrapping) and a measure of the lead-lags of their oscillations at each frequency and each point in time.[14]

### 5.1 Unemployment and the yield curve

In this sub-section we take on the unemployment rate as an indicator of macroeconomic activity and assess its relation to the latent factors of the yield curve. Based on the literature, we would expect the slope – and possibly the curvature, in view of recent research – to relate significantly with unemployment, at least during some important sub-sample periods.

Most regions of high coherency between the yield curve level and the unemployment rate occur between 1970 and 2000 and in cycles of periodicity in the 4∼12 years frequency band. It is visible a gradual shift from shorter-run frequencies (with period cycle closer to 4 years) to longer-run frequencies, with period closer to 12 years.

It is also worth notice a high coherency in the 1∼1.5 years frequency band during the 1979/82 monetary regime and in the 2∼4 years band during the ensuing (1980-1985) disinflation.

Independently of the shift in coherency across frequencies, we observe a rather stable phase relationship in all the frequency bands involved (1∼4, 4∼8 and 8∼12 years): for most of the time, the phase difference is between -$\pi/2$ and 0, reasonably indicating that the yield curve level leads the unemployment rate and that an increase in the level of yields is associated with an increase in unemployment.

The lead of the level over unemployment in the 1∼4 frequency band between the late 1970s and 1985 occurs at a horizon of 1 to 2 quarters. In the 4∼8 years band the lead varies strongly and includes a structural break at the beginning of the Greenspan mandate, but overall suggests a lead-horizon between 3 and 6 quarters. In the 8∼12 frequency band the time horizon of the lead falls gradually along 1975-2000, from about 7 quarters to around 5 quarters.

There are important regions of statistically significant coherency between the yield curve slope and the unemployment rate, namely between 1965 and 1985 and, then, between 1990 and the end of the 2000s.

[14] Overall, we will not mention often results about the coherency and phase differences at the higher frequencies – cycles of 1 month∼1 year – as they are frequently noisy and, as such, rather uninformative.

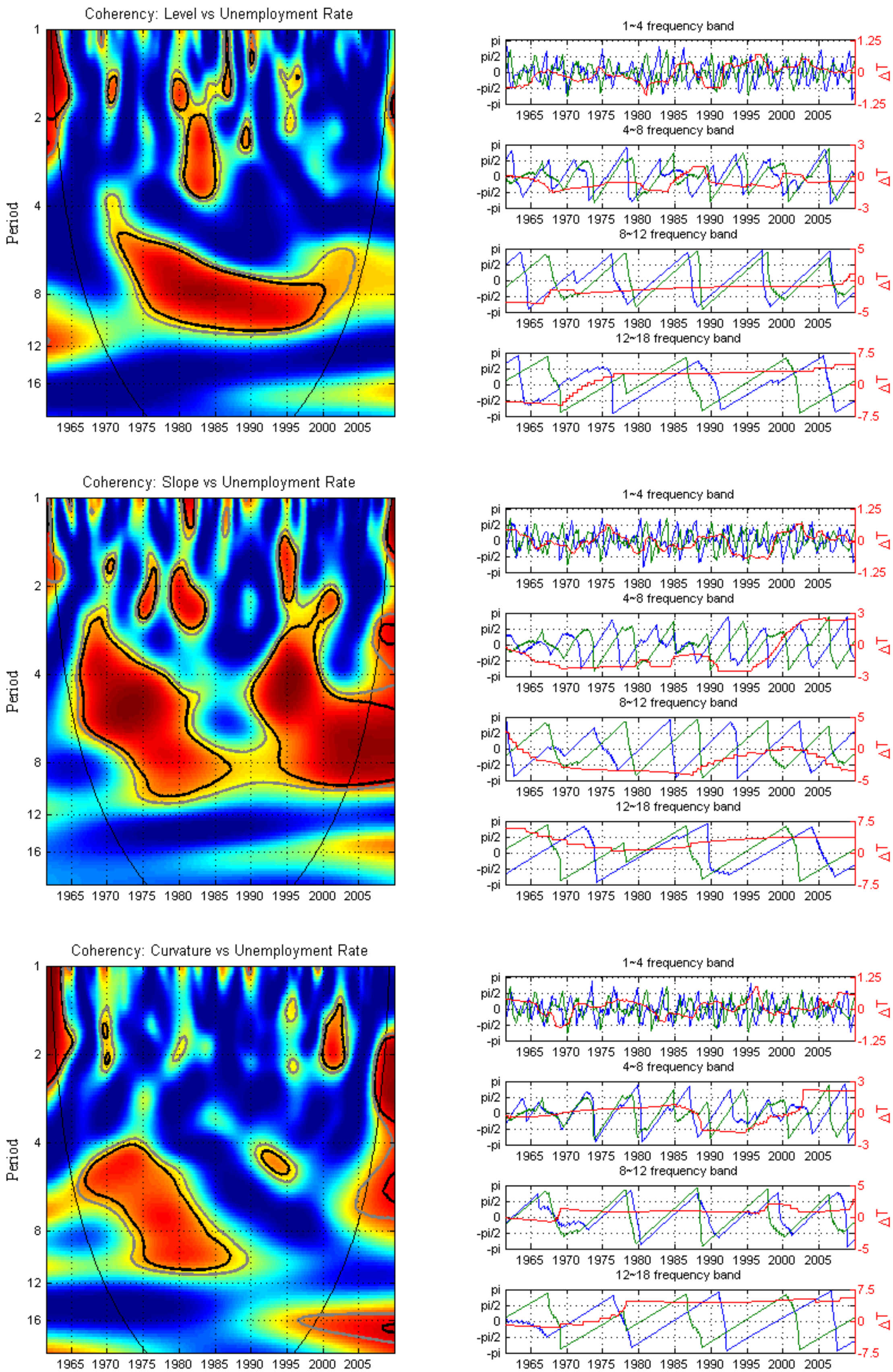


Figure 3: Unemployment and the Yield Curve, U.S. 1961:6-2009.12

The lack of coherency in 1985-1990 at all frequency bands is consistent with the structural break in continuous regressions of the yield slope on output growth (with a forecast breakdown) detected, with alternative methods and/or data, by, e.g., De Pace (2009), Chauvet and Potter (2002, 2005), Haubrich and Dombrosky (1996), and Dotsey (1998).[15] Such consistency, however, would require the proper support from the phase difference analysis, with a change from phase differences between $-\pi/2$ and 0 – which would indicate, as expected, that an increase in the slope (a flattening of the yield curve) anticipated an increase in unemployment – to other phase differences.

We do not see such pattern in the phase differences for the frequency bands of 4∼8 and 8∼12 years; actually, most of the periods with significant coherency suggest that, at those frequencies, an increase in unemployment anticipates a decrease in the slope, i.e. a steepening of the yield curve – a pattern that is more consistent with a possible effect on the yield curve slope of a reaction of monetary policy to a deterioration of the economic conditions. Yet, in spite of its instability, there is some support for the hypothesis of a forecast breakdown in the frequency band of 1∼4 years. In fact, between 1965 and around 1973 as well as between around 1978 and 1987, the phase difference suggests that an increase in the slope – a flattening of the yield curve – anticipates an increase in unemployment by 1 to 2 quarters; and such pattern only reappears in around 1993 and ends by around 1998. These are the only signs of the above mentioned forecast breakdown that we detect.

Another result worth noticing is that at frequencies with period cycle between 4 and 8 years, from the early 2000s onward, the phase differences indicate that the yield curve slope leads unemployment by around 1.5 to 2 years, with – counter-intuitively – an increase in the slope (flattened curves) anticipating lower unemployment. This is consistent with the so-called conundrum of the summer of 2006, when the yield curve became inverted and yet no recession emerged – see e.g. Kucko and Chinn (2009). As Hamilton (2010) points out, it seems that the very low overall levels of interest rates recorded at the time has mitigated the recessionary signal given by the yield slope; our results confirm such conjecture and show that it relates to the ability of the yield curve to predict cycles of periodicity in the 4∼8 years frequency band.

We now turn to the analysis of the time-frequency relations between unemployment and the curvature, a factor that has received increasing attention in recent research.

There is a large region of statistically significant coherency in the 4∼8 frequency band (be-

[15] This forecasting breakdown of the yield slope has typically been associated with the beginning of the Great Moderation and with the change in monetary policy regime after the Volcker monetarist episode. However, the timing of the statistical disappearance of the forecast power of the slope has been estimated by others at different periods – see e.g. Giacomini and Rossi (2006), who have detected forecast breakdowns in the mid-1970s and in 1979-1987, i.e. during the Burns-Mitchell and Volcker regimes, rather than at the beginning of the Greenspan regime (actually stating, in page 794, that "during the early part of the Greenspan era the yield curve emerged as more reliable model to predict future changes in economic activity").

tween the late 1960s and the mid-1980s) and in the 8∼12 years band (between the mid-1970s and the late 1980s). The phase differences indicate, overall, that in these episodes the leading variable has been the unemployment rate, rather than the yield curve curvature. The same applies to the high coherency of the first half of the 1990s (frequencies between 4 and 6 years), as well as to various islands of high coherency at the end of the sample at different frequencies. The only exception to this pattern has been the first half of the 2000s, when the curvature and unemployment seem to have been in-phase at the frequency band of 1∼4 years.

Hence, with our data and methods, we do not find support for the hypothesis by Moench (2008) that the curvature is a leading indicator of economic activity, neither much for the one put forth by Modena (2008) that it is a contemporaneous indicator.

## 5.2 Economic activity and the yield curve

We now take the ADS index as the indicator of the overall economic activity. As in the previous sub-section, we would expect the slope – and possibly the curvature – to relate significantly with the index, at least during some sub-sample periods.

There is a large region of significant coherency between the level of the yield curve and the ADS index between 1975 and 2005, encompassing cycles within the frequency bands of 4∼8 and 8∼12 years. In contrast to what happens with unemployment for these time-frequency locations, the phase differences never suggest that an increase in the level anticipates a fall in economic activity. On the contrary, at the 4∼8 years band most of the time an increase in the level anticipates an increase in the ADS index and at the 8∼12 years band an increase in the ADS index anticipates a fall in the yield curve level – both rather unexpected results.

The only episode in which there is a leading role for the yield level, associated with reactions of the ADS index in the opposite direction, occurs in the 1960s, at the 8∼12 frequency band and with an exceedingly long lead.

At the 1∼4 years frequency band there are several regions of high and significant coherency in 1980-1985 and in 1990-2000, albeit located at different specific cycles. In all these time-frequency regions, an increase in economic activity anticipates an increase in the yield level, with a 1 to 3 quarters lead – a result that could be consistent with a monetary policy reaction causing changes in the whole range of interest rates.

Overall, our results indicate that the yield curve level–ADS index relation is quite different from the level–unemployment relation, with the latter more in line with our a-prioris.

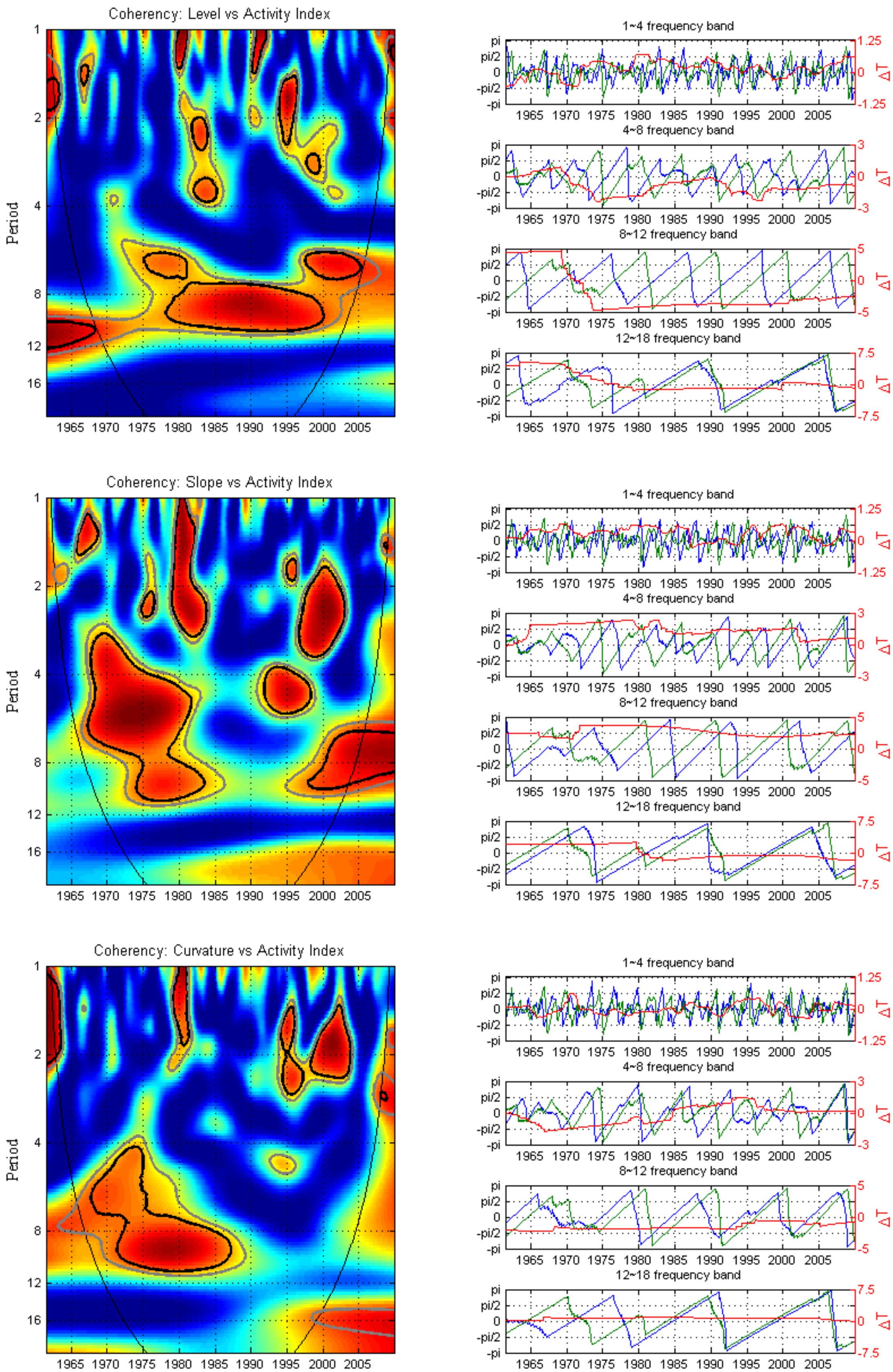


Figure 4: Economic Activity and the Yield Curve, U.S. 1961:6-2009:12

As happens in the slope-unemployment relation, there is no significant coherency at any frequency band between the yield curve slope and the ADS index in 1985-1990, which could be consistent with the structural break with a forecast breakdown mentioned in the previous sub-section. The phase differences in fact support such interpretation both for the 4∼8 and the 8∼12 years frequency bands (in contrast to what happens with unemployment). At the 4∼8 years band, in 1965-1985, and at the 8∼12 years band, in 1972-1985 (the regions of significant coherencies), the phase differences are between $\pi/2$ and $\pi$, meaning that an increase in the slope – a flattening of the curve – anticipated (as expected) falls in the ADS index; in 1990-2005, the phase differences are between 0 and $\pi/2$, meaning that increases in the ADS index anticipated a flattening of the yield curve – i.e. the slope fails to forecast economic activity.

There is also some evidence consistent with the conundrum of 2006 in our results for the 4∼8 frequency band, as there is a significant coherency at that time-frequency region suggesting that the slope and the ADS relate positively with each other and are almost in-phase – a flatter yield curve was associated to an economic expansion.

At higher frequencies (periods of 1∼4 years), there are significant regions of coherency in 1965-1985 and then between around 1992 and 2003. In both regions, the phase difference is between 0 and $\pi/2$, indicating that increases in economic activity anticipate increases in the slope, i.e. flatter yield curves, by 1 to 2 quarters. This result could be tentatively interpreted – especially since the late 1970s, when monetary policy rules consistent with inflation targeting emerged – as a sign of the role of real activity in the monetary policy rule: when an acceleration of activity is forecast, monetary policy contracts and the short-end of the yield curve moves upwardly, which, in the absence of an immediate reaction of the longer yields, flattens the yield curve.

There is a large time-frequency region of statistically significant coherency between the curvature of the yield curve and the ADS index, at the 4∼8 years frequency band (in 1965-1985) and at the 8∼12 years band (in 1970-1987). At this region, the phase differences are consistently located within -$\pi/2$ and 0, indicating that increases in the curvature (higher concavity) anticipate increases in economic activity.

Similar results occur at the 1∼4 years frequency band in a very limited episode around 1980 and in another between around 2002 and 2004. The significant coherency at that frequency band observed in other episodes (say, 1994-2001) displays phase differences in the 0 to $\pi/2$ range, indicating that an increase in economic activity anticipates a higher concavity of the yield curve.

Overall, the episodes of significant coherency are scarce and point to a positive relation between the curvature and economic activity, with no systematic leading role for any variable – results that do not appear to be in line with the recent literature on the yield curvature and the macro-economy referred above.

## 5.3 Inflation rate and the yield curve

In the literature, there are two main associations of inflation to the yield curve. On the one hand, its level is seen as reflecting the path of the nominal anchor of the economy (measured by inflation, as in, e.g., Diebold, Rudebusch and Aruoba, 2006, or by inflation expectations, as in, e.g., Mumtaz and Surico, 2008). On the other hand, its slope or changes between slopes computed at different horizons are seen as predictors of changes in inflation at such horizons (see, e.g., Mishkin, 1990a, 1990b, 1990c, and Estrella, Rodrigues and Schich, 2003). In this sub-section we assess the relation of the level, slope and curvature with inflation, in the time-frequency domain.[16]

The larger regions of high coherency between inflation and the yield curve level are situated in cycles in the frequency bands of 4∼8 and 8∼12 years and occur between the early 1970s and the early 1990s. Across these periods and frequencies, the phase difference is between 0 and $\pi/2$, suggesting that the yield curve level reacts, in the same direction, to changes in inflation, with lags of around 1 year in the 4∼8 band and around 1.5-2 years in the 8∼12 band.

At the very low frequencies (12∼18 years band), coherency between inflation and the yield curve level is overall low and only significant between 1980 and the early 1990s. At such frequencies, cycles in the level anticipate cycles of inflation in the opposite direction – which allows for interpreting this episode as the consolidation of the disinflation initiated in 1979, leading to a sustained anchoring of inflation.

A first lesson drawn from our analysis is that until the early 1990s the yield curve level has indeed mirrored the path of inflation, with some delay, in fluctuations of period within the standard concept of business cycles. This is consistent with the view that inflation determines the whole yield curve and is thus supportive of the above mentioned association between inflation (or expectations) and the yield level.

A second lesson is that the coherency between the yield curve level and inflation seems to have vanished in the 1990s and, largely, in the 2000s. The breakdown of the inflation-yield curve level relation seems to coincide with the consolidation of the low inflation regime typically associated with the FED chairmanship of Alan Greenspan, as well as with the intensification of the U.S. external imbalance in a context of a global savings glut – that has somehow detached the overall level of interest rates from macroeconomic conditions in the U.S.

[16] Truly, our assessment of the slope-inflation relation is not comparable to the others in the literature. First, most studies use empirical proxies for the yield spread, rather than a model-based one such as ours (and in many cases the proxy differs substantially from the empirical properties of ours). Second, the literature typically looks at regressions of the difference between inflation in period m and inflation in period n of the difference between the yield for maturity m and the yield for maturity n (e.g. Estrella, Rodrigues and Schich, 2003).

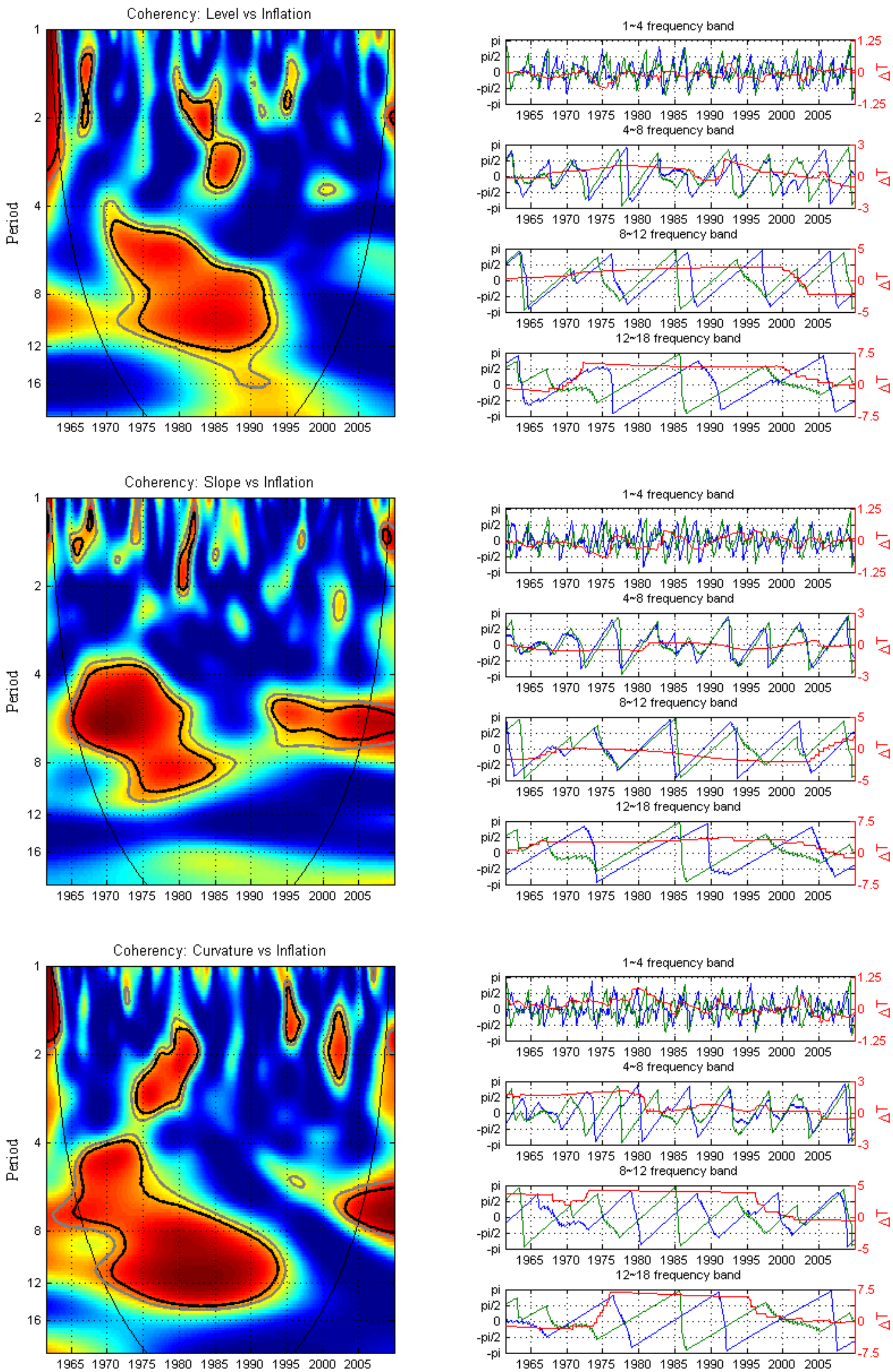

Figure 5: Inflation and the Yield Curve, U.S. 1961:6-2009:12

A first idea that emerges from our time-frequency analysis of inflation and the yield slope is that between around 1987 and around 1992 there is no significant coherency at any frequency band. Such result is consistent and complements the evidence of a structural break also found in the previous sub-sections regarding the relation between the slope and economic activity; and may explain the difficulties in estimating stable regressions reported in a large part of the literature (see, e.g., Mishkin, 1990a, 1990b, 1990c, and Estrella, Rodrigues and Schich, 2003).

There is no significant coherency at the lowest frequencies (frequency band of 12$\sim$18 years) and most of the coherency appears at the 4$\sim$8 years frequency band, which indicates that the relation between the slope and inflation relates to business cycles. At that frequency band, until the early 1980s the phase difference is located within the -$\pi/2$ and 0 interval, implying that an increase in the slope anticipates (by around 2 to 3 quarters) an increase in inflation. A similar leading relationship is seen, during that period, at the 1$\sim$4 years frequency band (albeit with some instability around the oil shocks). And at the 8$\sim$12 years frequency band the relation changes gradually from an in-phase relation in the 1970s – one of perfect synchronization of the slope and inflation – to a similar lead of inflation by some quarters in the 1980s. After the period of absence of coherency (1987-1992) the coherency resumes only within the 4$\sim$8 years frequency band, and the phase difference fluctuates around 0, indicating that in some periods the slope leads by 1 or 2 quarters, in others it's inflation that leads, while in others yet they evolve simultaneously.

Such results may seem hard to reconcile with the ability of increases in the slope to predict recessions – of which, as seen in the previous sub-.section, there is evidence until the mid-1980s – as recessions are typically associated to reductions in inflation. However, once the time-lags involved are taken into account a consistent history seems to emerge: when policy-makers forecast inflationary pressures and implement a tighter monetary policy, thus flattening the yield curve, a recession may arise as a side-effect of such policy, even if it is not entirely successful and inflation actually increases. Hence, the forecast breakdown since the mid-1980s may be seen as an indicator of success of monetary policy, in that its reaction to inflationary pressures seems to be less related to recessions thereafter and, additionally, seems to have been more effective in controlling inflationary pressures duly forecasted.

Overall, in the earlier part of the sample, the regions of significant coherency between the curvature of the yield curve and inflation correspond to phase differences within the interval between $\pi/2$ and $\pi$, indicating that an increase in the curvature (stronger concavity) anticipates reductions in inflation. This happens in the 4$\sim$8 years frequency band until 1980, in the 8$\sim$12 years band until 1995 and in the 12$\sim$18 years frequency band between 1975 and 1995. In the early 1980s a structural break occurs at the 4$\sim$8 years frequency band and then increases in inflation start anticipating increases in the curvature, as the phase differences turn into the interval between 0 and $\pi/2$; at that frequency band the significant coherency detected during the 2000s seem to indicate an in-phase relation between the curvature and inflation.

Finally, there are several regions of significant coherency at the 1$\sim$4 years frequency band, which suggest, until 1985, that inflation leads the curvature, while after the early 1990s it seems hard to establish a pattern other than that curvature and inflation are broadly in-phase at cycles of small period.

## 5.4 The Fed funds rate and the yield curve

The relation between the shape of the yield curve and the monetary policy interest rate may be twofold. On the one hand, monetary policy actions should affect the yield curve as part of its transmission mechanism, as most decisions by economic agents depend on medium-term or long-term interest rates; on the other hand, financial markets often anticipate the moves of monetary policy-makers and so the yield curve shape may lead the federal funds rate. In this sub-section, we assess the relation between the fed funds rate and the level, slope and curvature of the yield curve in the time-frequency domain.

As figure 6 shows, the relations are indeed strong.

There is a high coherency between the yield curve level and the fed funds rate (FFR) at frequencies corresponding to long cycles during the whole sample period. Reflecting the changes in the monetary policy regime (at the end of the 1970s and at the end of the 1980s) this high coherency moves to cycles of longer period after the late 1970s. In 1995 this coherency was concentrated in the 16 years period cycle and then it loses significance, at the 5 percent level (while not at the 10 percent) although remaining important until the end of the sample. The phase-differences at those frequencies (8$\sim$12 and 12$\sim$18 years) are overall located between 0 and $\pi/2$ but close to 0, indicating that increases in the FFR slightly anticipate increases in the level of the yield curve.

At the frequency band of 4$\sim$8 years, there are less periods of high coherency. Between 1970 and the late 1980s there is a statistically significant coherency, with the phase differences indicating that the FFR leads the level of the yield curve.

There are also further regions of high and often significant coherency in the 1$\sim$4 years frequency band, with a time-frequency pattern more diffuse but with a consistent result as regards phase differences: after erratic phase differences in the first part of the sample, from 1983 onwards and until the mid-2000s, the yield curve level leads the FFR. This result is possibly related to the development and sophistication of the financial markets since the 1980s, with the consequent increased ability of anticipating the short-term moves of the policy-maker.

The relation between the FFR and the yield slope is even stronger than with the level.

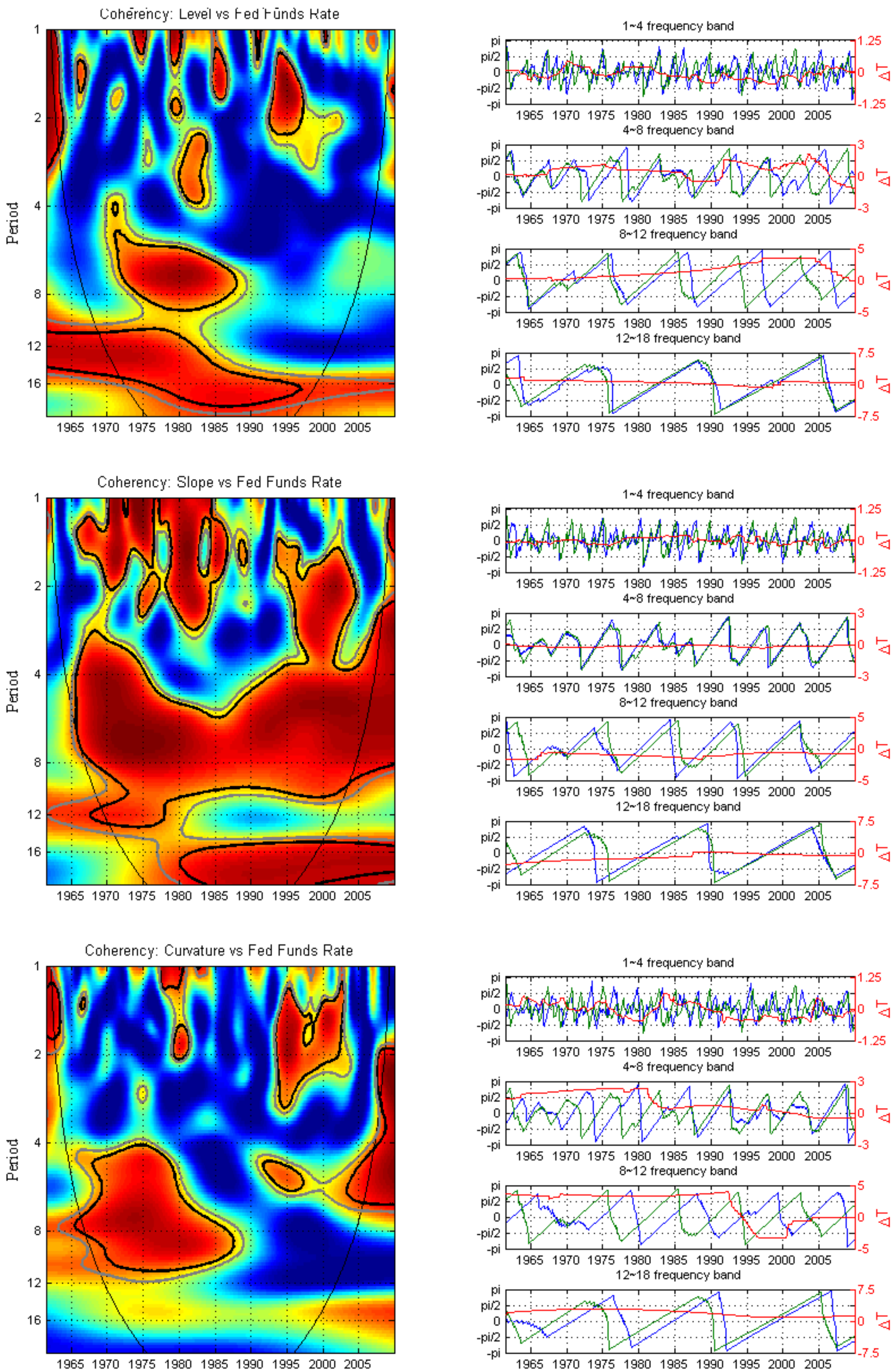


Figure 6: Fed Funds Rate and the Yield Curve, U.S. 1961:6-2009:12

There are regions of very high coherency at most of the time-frequency locations. Indeed, since 1965 there are statistically significant coherencies at the 1$\sim$4, 4$\sim$8 and 8$\sim$12 frequency bands, in spite of some change in the specific periodicity of the involved cycles. At the 12$\sim$18 years frequency band the coherencies are significant after 1980.

At the 4$\sim$8, 8$\sim$12 and 12$\sim$18 years frequency bands, the phase difference indicates that the yield curve slope leads the FFR, except around 1985 at the 4$\sim$8 years band and around 1990 at the 12$\sim$18 years band, where the variables appear to be in-phase. Truly, at the 4$\sim$8 years band, the lead is very small, but at the 8$\sim$12 years band it often amounts to 4 or 5 quarters. The relation is remarkably stable at a positive elasticity – increases in the slope (flattening of the yield curve) anticipate increases in the FFR – which is consistent with a monetary policy explanation of the changes in the yield curve shape.

The phase differences are somewhat more erratic in the case of the 1$\sim$4 years frequency band and we find it hard to reject the idea that the slope and the FFR are broadly in-phase as regards cycles with such period.

We draw two main conclusions from this analysis. First, tighter monetary policies have been associated with flatter yield curves throughout the whole sample period and across all the frequency bands. This means that monetary policy has impacted differently on the short-end of the yield curve than it has impacted on its long-end, irrespectively of the periodicity of the FFR and slope movements. Second, the yield curve slope is overall a good predictor of monetary policy for cycles of period above the standard business cycle definition (8$\sim$12 and 12$\sim$18 years) but less so for cycles of smaller period (1$\sim$4 and 4$\sim$8 years), in which the leading horizon is very small.

There is a first region of high and significant coherency between the yield curve curvature and the FFR at the 4$\sim$8 and the 8$\sim$12 years frequency bands between 1965 and the late 1980s. For these time-frequency areas, the phase difference is overall between $\pi/2$ and $\pi$, suggesting a negative relation with the yield curve curvature leading with a horizon that is considerable: increases in the curvature – i.e. higher degrees of concavity in the yield curve – anticipate lower FFRs. This result is consistent with the relation uncovered between the yield curve slope and the FFR, as more concave yield curves have a lower slope as they are steeper at the short-to-medium term segment of the interest rates. But it adds a new finding: the curvature seems to have had a predictive power for future monetary policy interest rates, until the late 1980s, at the 4$\sim$8 frequency band.

Yet, the ability of the curvature to anticipate the monetary policy interest rate seems to have disappeared since the 1990s. At the 4$\sim$8 years frequency band, during the 1990s increases in the FFR anticipated increases in the curvature, while during the 2000s increases in the curvature anticipated increases in the FFR. At the 1$\sim$4 years frequency band, since 1990 increases in the curvature anticipated increases in the FFR. And at the 8∼12 years frequency band there are no regions of statistically significant coherency, like in the 12∼18 years frequency band.

# 6 Conclusions

In this paper we have assessed the relation between the shape of the yield curve and the main macroeconomic variables in the U.S. between 1961:6 and 2009:12 across time and frequencies, using Wavelet tools. Following a well-established tradition in empirical finance, the shape of the yield curve is modelled with three time-varying latent factors corresponding to its level, slope and curvature. The macroeconomic variables are an index of overall economic activity, unemployment, inflation and the fed funds rate.

The time-frequency approach and the use of Wavelets, in particular, fills a gap in the literature, which has been conducted essentially in the time-domain dealing with structural breaks and time-variation in the intensity, direction and time-lags of the yield-macro relation. The cross-wavelet tools employed – coherency, with bootstrap confidence intervals, and phase difference –, the set of variables and the length of the sample, allow for a thorough appraisal of those time-variations and structural breaks at each frequency. The evidence we provide establishes a new set of stylized facts on the relation between the yield curve and the macro-economy, which, besides their immediate relevance, should prove useful for future research on this area. Moreover, it clarifies the reasons for a number of results in the literature.

After preliminary wavelet power spectrum analyses in sections 4.1 and 4.2, we assess the coherency and phase differences between our twelve pairs of yield curve–macroeconomic variables in the time-frequency domain, in section 5. Among the numerous results, a core set of findings may be summarized as follows.

There is a clear structural break in the second half of the 1980s in the relation between the yield curve slope and real economic activity – measured either by unemployment or the ADS index – which encompasses all frequency bands. The forecast breakdown that is reported in some time-domain literature – i.e. the ending of the ability of a flattening of yield curve to anticipate recessions, starting around the late 1980s – is detected, for unemployment, at the frequency band of 1∼4 years, while for the ADS index, at the 4∼8 band and (albeit less strongly) at the 8∼12 years frequency band. From the mid-2000s onward there is evidence consistent with the so-called conundrum of 2006 – when an inverted yield curve failed to predict a recession – at the 4∼8 years frequency band, with a flattening of the yield curve associated with a contemporaneous increase in the ADS index and a future reduction of unemployment. Our evidence is thus consistent with the hypothesis that the fall in the level of the yield curve in the last two decades has damaged the ability of its slope to predict real economic activity.

When economic activity is measured by the ADS index – but not by unemployment – there is some evidence that, until the mid-1980s, increases in the concavity of the yield curve anticipated increases in real economic activity at the 4∼8 and the 8∼12 frequency bands. Such leading role also existed at the 1∼4 frequency band until the early 1980s and in a limited years of the early 2000s. Yet, the time-frequency relation between the curvature and economic activity is limited and thus fails to endorse the attention given in some recent literature to the ability of the curvature to predict recessions.

Inflation has indeed – as usually referred in the literature – determined the level of the yield curve, but our evidence shows that such leading role has been confined to cycles of period in the frequency bands of 4∼8 and 8∼12 years and has disappeared in the early 1990s. The leading role of the level of the curve over inflation that has existed during the 1980s at the frequency band of 12∼18 years has also disappeared in the early 1990s. Both results indicate a structural change in the relation between the yield curve level and inflation that seems to be associated to the achievement of disinflation and the anchoring of inflation expectations, as well as to the savings glut that has detached the overall level of interest rates from the fundamentals (especially the foreign deficit) of the US economy.

As regards the other main yield curve–inflation relation in the literature, we find a clear structural break in the relation between the slope and inflation occurring in the late 1980s/early 1990s, when no coherency exists between the two time-series at any frequency band. Before the break, most of the significant relations occurred in the 4∼8 years frequency band – with many episodes also at the 1∼4 and 8∼12 years bands – and overall indicated that a flattening of the curve anticipated increases in inflation. After 1992 the relation resumed only at the 4∼8 years band – confirming the business cycle nature of the slope–inflation relation – and indicates a simultaneous co-movement. Overall, and taken together with the slope–activity results, we interpret this evidence as indicating an increase in the efficacy of monetary policy after the early 1990s (when the Greenspan mandate, the Great Moderation and the benign global conditions co-existed): flattened curves created by policy interest rate increases have not been significantly associated with recessions and yet have controlled inflation in the near future.

The time-frequency relations between the yield curve curvature and inflation – in particular when seen together with the economic activity–curvature relations – are overall not suggestive of any systematic pattern at any frequency band. This is especially clear after the early 1980s for the frequency band of 4∼8 years and since the early 1990s for the 8∼12 and 12∼16 years bands. We thus conclude, again, that the attention given in recent literature to the curvature of the yield curve seems unwarranted, especially for the current period of overall low interest rates.

The time-frequency evidence on the relation between the level of the yield curve and the federal fed funds rate (FFR) allows for two main conclusions. First, we detect the changes in the monetary policy regime – associated to Paul Volcker and then to Alan Greenspan – in a gradual shift of the leading role of the FFR over the yield level from cycles of lower periodicity to longer cycles, since the late 1970s; since the 1990s, with the successful anchoring of inflation and the global benign conditions, increases the FFR anticipate increases in the yield curve level only at cycles in the frequency band of 12$\sim$18 years. Second, we detect that at shorter-run cycles (in the frequency band of 1$\sim$4 years) since the mid-1980s the level of the yield curve anticipates the FFR, which we associate to the deregulation and development of financial markets and of their ability to predict – maybe influence – monetary policy.

Across the whole sample period, tighter monetary policies (increases in the FFR) have been associated with flatter yield curves, meaning that monetary policy has impacted differently on the short-end and on the long-end of the yield curve; this relation exists, broadly, at all frequency bands, but for cycles of very long period (equal to or above 16 years) is only statistically significant after the early 1980s, when the monetary policy regime changed and macroeconomic volatility fell. Our evidence further indicates that the yield curve slope has led the FFR, thus being, overall, a good predictor of monetary policy; yet, this predictive ability exists only for cycles of period above the standard business cycle definition (8$\sim$12 and 12$\sim$18 years), as in cycles of smaller period (1$\sim$4 and 4$\sim$8 years) the slope and the FFR are almost always virtually in-phase.

Finally, we detect that at the frequency bands of 4$\sim$8 and 8$\sim$12 years, increases in the concavity of the yield curve have anticipated reductions in the FFR with a considerable leading horizon; considered together with the slope-FFR relation that we uncovered, this evidence means that the curvature had an additional power, over the slope, to predict monetary policy in cycles of period within the standard business cycles concept. However, such predictive power vanished since the early 1990s and thus, once again, we conclude that the role that the curvature has had in recent research seems unwarranted.

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