3D FE MODELLING STRATEGIES FOR ANCIENT MASONRY BELL TOWERS. UPDATING OF MECHANICAL PARAMETERS AND BOUNDARY CONDITIONS

D. Li Rosi[1], L. Contrafatto[2] & H. Varum[3]

[1] Università degli Studi di Catania, Catania, Italia, davide.lirosi@phd.unict.it

[2] Università degli Studi di Catania, Catania, Italia

[3] CONSTRUCT-LESE, Universidade do Porto, Faculdade de Engenharia, Porto, Portugal

Abstract: *Masonry towers show recurrent dynamic behaviour, such as the ordered sequence of modes and the range of frequencies. Modal parameters depend on geometrical and mechanical characteristics, as showed by the literature, and architectural features that cause specific vulnerabilities. During the centuries these towers underwent modifications implying different materials, construction techniques and mechanical properties. In many cases the tower is built after the church. Therefore, the structural connection with the adjacent church is not fully effective. This has a main role in determining the tower's response and must be considered in a correct modelling. The bell tower of San Giuseppe in Aci Castello (Italy) is small and simple, but still interesting because it is representative of the cited recurring features and structural issues. Because of this simplicity, it allows a deeper and precise analysis, whose results are useful to improve the approach to the FE modelling of bell towers in general. Photogrammetric techniques provided the accurate geometry of the tower, while direct observation of the masonry texture, materials and construction techniques allowed to determine the masonry type. Sensitivity analyses to architectural and geometric details and mechanical parameters were implemented to choose relevant input parameters. E, G, ν and γ were selected and modal frequencies from AVT were used as control variables for the model calibration. The first-step values of the mechanical parameters of the masonry were determined through qualitative analysis of the masonry, literature data comparison and application of empirical formulas from the European and Italian codes. Eigenvalue analyses on different FE models were performed and the mechanical parameters were updated until the best matching between experimental and numerical results was achieved. The importance of the accurate choice and distribution of mechanical parameters and boundary conditions was evident. An oversimplified FE model was analysed. Then, removing some starting simplifying hypothesis, the geometry was refined and an orthotropic material was adopted. This allowed to adjust the torsional frequencies without affecting the flexural ones, just varying only the shear stiffness.*

1. Introduction

Religious buildings are an important part of architectural heritage, especially in Europe. In more than a millennium of history, several chef-d'oevre have been outstanding examples of human creativity in terms of architectural and engineering genius. Nevertheless, most of these masonry buildings show inadequate seismic performance and remain prone to severe damages caused by horizontal actions due to dynamic loads, such as intense ambient vibrations or earthquakes. Because of this, dynamic identification and seismic vulnerability evaluation are major issues for such structures. Furthermore, as ancient buildings, during centuries of life the bell towers often underwent several modifications, reconstruction, and restoration. These interventions imply the use of different materials and construction techniques along the height of the tower and, therefore, different mechanical properties. Among the others, the most common interventions due to both human or natural causes are: the building of the bell tower after the church; the increase of the height of the bell tower adding a new bell chamber, as soon as the financial conditions were available; the partial or total rebuilding of the bell chamber or the upper crown, which is the part mainly vulnerable to earthquake or lightning damages. As a consequence, the structural connection between the tower and the adjacent church, or between the parts of the tower itself dating back to different ages, is usually far from being fully effective. Since the connections' effectiveness (and in general the boundary conditions) have a great influence in determining the tower's response, especially in the dynamic and non-linear field, it should be taken into account in a correct modelling (Valente et al. 2018, Torelli et al. 2020, Shabani et al 2022, De Angelis et al. 2022). In Sicily, the explicit require of putting stone blocks having connection purpose between old and new building parts can be found in XVIII century building specifications, with reference to the addition of bell towers to existing churches. Even if this request has ever been fulfilled by the builders, a full structural link could not have been achieved. Masonry bell towers show some typical and recurrent modal characteristics, such as the ordered sequence of two flexural modes along orthogonal directions followed by a torsional one, and a limited range of modal frequencies, roughly about 0.5-10.0 Hz. The modal parameters mainly depend on the height and dimensions in plan, as showed by the wide literature on this topic (Ivorra and Pallarés 2006, Ivorra et al.2011). Obviously, this is related to recurring and typical architectural features that make these buildings prone to some specific vulnerabilities (Shakya et al. 2018).

A Finite Elements model is a useful tool to carry out preventive analyses about static and dynamic behaviour of the building and the effectiveness of reinforcement interventions. The experimental dynamic identification of the structure is a smart method to improve the effectiveness of the numerical FE model in representing the actual dynamic behaviour. Operational Modal Analysis (OMA) techniques are based on Ambient Vibration Tests (AVT), i.e., the instrumental measurement of velocities or accelerations of the structure subject to seismic noise. OMA is non-destructive, low cost, and characterised by simple applicability, thanks to the instrumentation's high handling guaranteed by low weight and small dimensions. Therefore, it is particularly suitable for heritage towers. The values of modal parameters experimentally obtained can be used as control parameters to calibrate a FE model. These values can be compared to those determined through FEM eigenvalue analysis so that the input parameters of the FE model can be iteratively adjusted to reduce differences. OMA techniques are also useful for continuous monitoring and structural damage detection and assessment (Cabboi et al. 2017) through stiffness degradation observation. Finally, many studies about OMA dynamic identification have been carried out in the last decade, demonstrating the relevance of FEA for model updating and swinging bells interaction, and the effectiveness of this approach (Ivorra and Pallarés 2006, Ivorra et al. 2011).

In this paper the proposed methodology is adopted to implement a refined FEM modelling of a historical masonry tower after dynamic identification through OMA techniques. This approach is presented through the case-study of the church of San Giuseppe in Aci Castello (Sicily, Italy). Its tower is small and simple, but still interesting because it is representative of a series of the cited recurring features and structural issues in bell towers. Because of this simplicity, it allows a more deep and precise modelling and analysis, whose results are useful to improve the approach to the FE modelling of bell towers in general. An accurate digital survey through photogrammetric techniques provided the precise and detailed geometry of the tower. Furthermore, an on-site survey campaign consisted in the direct observation of the masonry texture and construction details, materials, and techniques. This allowed to determine the masonry type and understand how to make a structural model that could interpret the real behaviour of the tower with a good reliability. Young's normal elasticity modulus E, shear modulus G, Poisson's ratio ν and specific weight γ characterize the construction materials. Sensitivity analyses to architectural features, geometric details and mechanical parameters were carried out with the aim to identify the most relevant input parameters (Gentile et al. 2015). Modal frequencies experimentally obtained through AVT were used as control variables for the model calibration. The first-step values of the mechanical parameters were determined through comparison with available literature data (Liberatore 2000), and the implementation of empirical formulas from the European code EC6 and the Italian building code (Ministero delle Infrastrutture e dei Trasporti 2018, NCE 2005) for similar types of masonry, in the attempt to describe the actual masonry layout and quality. Eigenvalue FEM analyses on a series of different FE models were performed and the resulting modal response was compared to experimental ones. Through this comparison, the mechanical parameters selected as input parameters were updated until the best matching between homologue values was reached. A deep influence of the modelling choices on the modal parameters has been revealed, showing the importance of an accurate choice of mechanical parameters and boundary conditions.

A model of an ideal tower was used to carry out sensitivity analyses on both architectural features of the tower and mechanical parameters of the building materials. Furthermore, a first-step, oversimplified FE model of the tower of San Giuseppe, with a squared plan and perfectly vertical facades, has been analysed with an uniform, isotropic material. Then, these starting simplifying hypothesis have been gradually removed, refining the geometry of this model and adopting a more realistic orthotropic material. This choice allowed to vary G values on an equal value of E, γ and ν to adjust, as expected, the values of the frequencies associated with torsional mode shapes with higher accuracy, without affecting the ones related to flexural mode shapes. Finally, the goal of taking into account the non-uniformity of the material and the structural imperfection of the connections has been accomplished. The first task has been reached varying the mechanical properties of the different parts of the tower. The second one has been reached varying the boundary conditions both changing the constraints and modelling, or not, the adjacent church with a direct approach.

2. The bell tower of the church of San Giuseppe in Aci Castello (Catania, Italy)

2.1. History and description of the building

Aci Castello is a small former-walled town north of Catania. It grew around the XI-XIII century castle imposingly standing on a great rock along the coastal cliff almost isolated by the sea, in a place in which there were earlier fortified structures, maybe dating back to the VII-VI century A.D. The church of San Giuseppe is located about 50 m from the coastal cliff, opposite the castle, with GPS coordinates 37.554186, 15.148149 (Figure 1).


(a)

(b)

Figure 1. Church of San Giuseppe in Aci Castello. General view from the castle in which the coastal cliff close to the apse of the church can be seen (a) and north view of the tower (b).

The church is cited for the first time in a document in 1748 and it was probably built on the ruins of an earlier church dating back to the XVI century and probably destroyed by an earthquake in 1547 (Castorina 2002). It has not been possible to find any information about this earthquake, but only about another two ones in 1537 and in 1542.

The church presents a single nave with apse, rectangular plan of about 20.00 m x 7.60 m and a burial crypt beneath its entire floor. Documental information about the damages caused by past earthquakes in the area, literature data and historic drawings and photographs of the church have been studied. This historical research allowed to recognise the construction phases and achieve a better and deeper understanding of the structural link existing between building parts belonging to different phases (Figure 2).


(a)

(b)

Figure 2. "Capo d'Aci dal versante orientale", watercolour by Jean Houel, 1776-1779 (image from VV.AA. 1989. La Sicilia di Jean Houel all'Ermitage. Sicilcassa, Palermo) (a). Photograph dating back to the late XIX century, anonymous (b). In both the pictures the ancient spire of the tower can be clearly recognised.

The 15.15 m high bell tower has a rectangular-plan shaft erected upon a battered-wall basement 2.35 m high. The facades of the tower are divided in four orders by a major lava stone horizontal cornice that crowns the basement, an intermediate simple lava stone cornice and a plaster cornice at the bottom of the bell chamber. The tower is crowned by a terminal plaster cornice. The sides of the tower have a width of 4.40 m x 4.00 m at the ground level, reducing to 3.90 m x 3.27 m at a height of 2.35 m, upon the lower cornice. The corner quoins of the first three orders give way to plaster pilasters at the corners of the bell chamber. The tower is currently accessible from a door within the church, but in the past there was another external door in the northeastern side, with a triangular arch, that has been bricked up and transformed in an internal niche. A square-shafted internal staircase on rampant vaults gives access to the bell chamber. The floors of the penultimate and the last floor lays on barrel vaults. The bell chamber has three windows with round arches. A fourth window, on the south-eastern side, has been bricked up. The northeastern and the northwestern windows host a bell with the yoke fixed on the projecting stone elements at the impost of the arch. One of the most particular peculiarities of this tower is that the southeastern wall of the bell chamber lays on a round arch, parallel to the northeastern wall of the church and completely detached from it, springing from the two side walls of the shaft. This could suggest that the bell chamber was built in a later construction phase and that the builders had the aim of keeping the bell chamber totally independent from the church from a structural point of view. The bell tower was crowned by a pyramid spire lined with ceramic tiles (Figure 2) which was demolished in 1908 because of the concerns of collapse due to its poor condition. The spire has been replaced by the current pyramidal roof with wooden structure and earthenware roof-tiles. According to some scholars, some architectural features of the bell tower, like the battered wall of the bottom and the corner quoins, prove that at least the lower part the tower belongs to a more ancient construction phase (Castorina 2002). The well-squared quoins of the basement appear to be a bit different from the rusticated corner quoins of the shaft of the tower. This comment could prove that the basement is a pre-1547 earthquake structure and the rest of the tower is a later structure. Nevertheless, the evidence acquired through deeper surveys seems to suggest that the bell tower may have been erected when at least the lower part of the walls of the nave was already built. This hypothesis could be demonstrated by the joints between the walls of the tower and the adjacent wall of the nave, through which traces of refined mortar can be seen both from the outside (Figure 3) and the interior. It would make sense, since it was a recurrent practice to build the bell tower after the rest of the church, depending on the availability of financial resources. Further details can be found in Mondello et al. (2019).

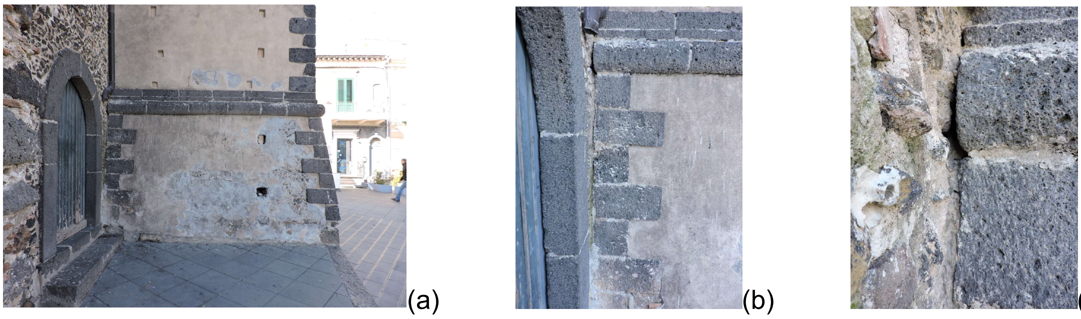
(a) (b) (c)

Figure 3. View of the battered wall at the bottom of the tower on the eastern side (a). The detail of the interface between the quoins of the battered wall of the tower and the surface of the wall of the church (b) could demonstrate the absence of structural connection. It is clearer in the detail picture of a single stone block of the lower cornice (c).

2.2. Description of the structure and materials of the tower

The main part of the walls is covered by plaster and only some degraded areas give a limited view of texture and materials, so that an effective direct survey of these features was not possible. Only the interior of the bell chamber is not lined with plaster and can be completely observed. Nevertheless, the building technique and materials of the tower can be considered analogous to that of the church, being that a recurrent and typical technique in the area of the Etna volcano between XVI and XVIII centuries. The bearing walls of the church are built with lava stone rubble, with heterogeneous dimensions from about 15 cm x 20 cm x 30 cm up to 26 cm x 26 cm x 52 cm, disposed on horizontal layers and linked with lime mortar and volcanic sand. Stone scales and broken earthenware tiles are used to fill the gaps between the irregular stones and there are mortar horizontal regularization layers every 60÷100 cm. The tower was assumed to have the same type of masonry. The thickness of the walls of the tower shaft ranges from 70 to 80 cm, while those of the church are about

65÷70 cm thick. The masonry of the bell chamber, whose texture has been widely surveyed, appears to be slightly thinner, about 70÷75 cm, and of a very worse quality, maybe due to its reconstruction with reused materials after lightning damages in 1840. The bell chamber wall texture is plenty of small lava stone scale, pieces of calcarenite stone slabs, maybe from a former decorative apparatus, and extended parts built with bricks belonging to the repairs in 1840. The corner quoins have quite regular dimensions, about l=52 cm x h=26 cm x w=26 cm, and their presence improves the connection between orthogonal walls, even if their size is not enough to be fully effective if compared with the wall thickness. The lava stone used for the masonry ranges from "compact" to "highly porous". The porous one (called “pumice” in Sicilian) is used in the arches and the vaults because of its reduced specific weight according to a typical building technique of this geographical area. A series of passing-through putlog-holes of about 17 cm x 12 cm can be seen in each facade of the tower from the bottom to the top. The putlog-holes of the bell chamber are covered by plaster on the exterior, but they are visible on the interior and in the historical photographs dating back to the late XIX century and the early XX century. The putlog-holes were realised in correspondence of each regularization layer. The fact that the putlog-holes of the tower are not horizontally aligned with those of the church could confirm that the construction works did not happen simultaneously.

No chance to directly analyse the cross section of the wall is given. The observation through the putlog-holes gives only a local, non-representative information. The cross section can be supposed to be analogous to that of other local examples dating back to about the same period. The ruins of the church of the Misericordia of Mompilieri (in Mascalucia, a Municipality of the Province of Catania) or of the church of Campanarazzu (in Misterbianco, a Municipality of the Province of Catania) could be assumed as a typical cross section of this type of masonry. In these examples, irregular or rough-cut stones with average width of about more than a half of the wall thickness are arranged in sub-horizontal layers. There are no, or a few, blocks passing through the entire wall cross section as a “diatono”. The lime mortar could be mainly concentrated in the nearby of the external leaves of the wall, and a widespread presence of voids could characterise the entire wall or only localised parts of it.

The foundation of the tower probably consists in a deepening of the walls in the ground of about 1.5 m. A change of the wall thickness of the apse is indeed present in correspondence of the external side. This horizontal mark extends along the entire northern facade of the church and is clearly recognisable as the head of the foundation at the bottom of the tower.

A direct survey and measurement campaign was carried out to support photogrammetric survey and photomodelling, leading to the precise geometry of the tower (Figure 4a, b). The sections obtained through photomodelling were the basis to generate the FE model (Figure 4c, d, e).

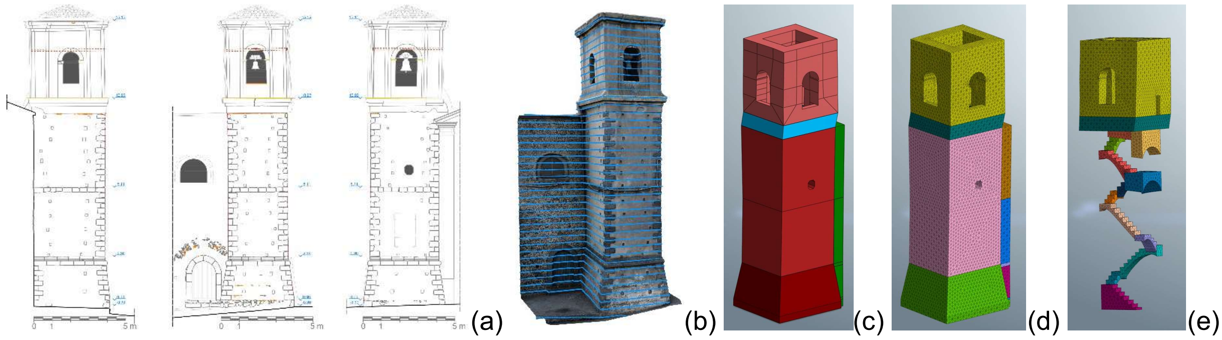

Figure 4. Elevation drawings (a) and 3D textured model from photomodelling with automatically generated horizontal sections (b). High geometric detail 3D model of the tower (c). The 3D model discretized in tetrahedral mesh (d). The mesh of the internal stairs (e).

2.3. Deformation pattern and crack pattern of the tower

Except for a few, minor cracks, there is no appreciable crack pattern, also because of the recent restorations. Nevertheless, certainly the church suffered damage caused by the earthquakes of 1818, 1908 and 1990 and by a lightning that struck the bell chamber in 1840. The tower lost its verticality with about a 0.3°÷0.9° tilt angle, increasing to 1.5° for the bell chamber and resulting in more than 9 cm of displacement at the top in the southern direction.

3. SENSITIVITY ANALYSES OF MODAL PARAMETERS OF HISTORICAL MASONRY TOWERS TO ARCHITECTURAL FEATURES AND MECHANICAL PARAMETERS

Some architectural features were analysed to verify their influence on frequencies of an ideal tower with a square plan of 3.00 m x 4.00 m and 15.00 m high. Tilted walls, plan deformations of the bell chamber, putlog-holes, presence of ashlars or stone coatings or other massive elements, presence of heavy bells, inner voids of the walls, different mechanical properties of the masonry of different parts or epochs were separately taken into account with respect to a simple, ideal tower. Tilted walls and torsional deformation affected frequencies for a mere 0.1÷0.2%, while through putlog holes showed a greater influence, reaching a 2.4% gap in torsional frequency. In fact, the regular and diffuse presence of through putlog-holes reduces the torsional stiffness of the horizontal sections. The influence of the internal stairs was also analysed. In the linear-elastic field it could behave as a torsional stiffener. As expected, sensitivity tests showed no influence of the specific weight of the stairs upon the value of the frequencies, while its elastic modulus becomes more relevant for the frequency of the torsional mode. Finally, a comparison between the model of the bell tower of San Giuseppe with oversimplified geometry and the model of the same tower with accurate, detailed geometry, with the same boundary conditions and mechanical properties, showed a decrease of the frequencies of about 5%÷9% in the detailed model.

In a second set of sensitivity analyses the hypothesis of isotropic behaviour of masonry was removed and an orthotropic material was considered. Sensitivity tests were carried by varying E, G, ν, γ (normal elasticity modulus, shear elasticity modulus, specific weight, Poisson's ratio) and using the natural frequencies B1 (bending mode, X-direction), B2 (bending mode, Y-direction), T1 (torsional mode, around vertical Z-direction) as control responses. The results are summarized in Figure 5.

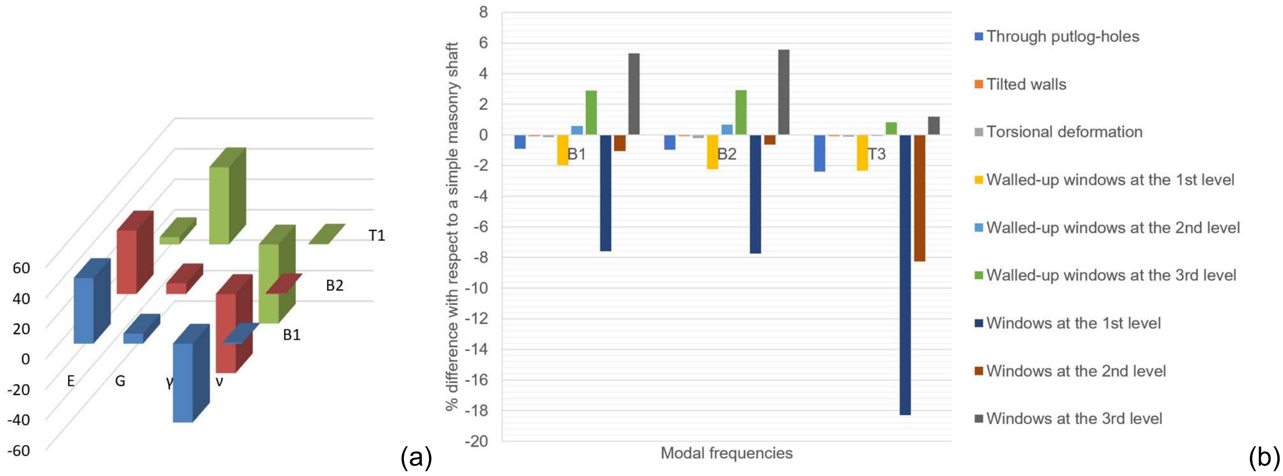

Figure 5. Sensitivity coefficients for orthotropic material mechanical parameters (a) and for architectural details of the model (b) for an ideal tower. B1 is the first bending frequency, B2 is the second bending frequency and T1 is the first torsional frequency.

It can be easily seen that E mainly affects the two flexural frequencies, G the torsional one. The effect of γ on natural frequencies is greater and uniform, while ν changes are not relevant in any case. Thus, the way to fix the gap between the value of the torsional frequency obtained through OMA and the value of the torsional frequency obtained through FE eigenvalue analysis is the updating of G value according to an orthotropic behaviour of masonry.

4. APPLICATION OF OMA TO SAN GIUSEPPE BELL TOWER

Tromino, by Micromed SPA, is a three-axial velocimeter that measures the environmental vibrations in a frequency range from 0.1 and 200 Hz. It is small and light, with 10 cm x 14 cm x 8 cm size and a mass of 1.1 kg and it is completely wireless, with a battery power supply and an internal 1 Gb memory, avoiding every mechanical or electronic noise. It is suitable for narrow places, such as a small tower shaft, in which it is difficult to move and to install wired systems due to the absence of electricity installation. It has three velocimetric channels which measure environmental vibrations with a speed value until about ± 1.2 mm/s with a minimum resolution of 23 bit and an accuracy of about 10^{-4} on the spectral components over 0.1 Hz. Six different Tromino

instruments were used to measure the velocities due to environmental vibrations and then identify the modal parameters of the tower through Operational Modal Analysis: one red Tromino TEP ENGY, one grey Tromino TRZ ZERO, four yellow Tromino TEN ENGINEERING. The instruments have been placed at the corners of the landings of the internal stairs and of the window-sills, to obtain three instrumented levels so that both flexural and torsional modes were identified. The instruments were oriented with their north-south axis coinciding with the transversal axis of the church, that is, the geographic northeast-southwest direction and the Y-axis of the FE model (Figure 6, Table I).
A series of 30-minutes recordings with a sampling rate of 512 Hz was carried out. The operations of pre-treatment and analysis of the recorded data and the extraction of the modal parameters from ambient vibration measurements were carried out with autonomously implemented MATLAB codes (Imposa et al. 2023). Only one velocimeter, the red one, had a GPS receiver. Thus, it was necessary to align the recorded data in order to study them in tandem. The alignment procedure consisted of computing the cross-correlation functions between every pair of signals, normalizing them, so that their maximum value is equal to unity, and finding the location of the maximum values that indicate lead or lag times. Then, all the aligned signals were cut, considering a final length of 10 minutes. Therefore, the filtering of data before the modal identification is necessary to leave-out undesired frequency contents. Accordingly, low-pass and high-pass filtering and decimation, respectively, to 20 Hz and 0.5 Hz were performed on the recorded data. The lower limit value of 0.5 Hz allowed to exclude instrumental errors, since the instrumental precision was about 0.1 Hz. The upper limit value of 20 Hz allowed to include the relevant frequency range for a civil structure, at least for the first three modes.

Table I. Type and location of the used instruments.

Virtual instrumented level	Tromino	Type	Height [m]	Average height of the virtual instrumented level [m]
1	C1	TEP ENGY (Red)	11.40	11.35
	C2	TRZ ZERO (Grey)	11.30	
2	C3	TEN ENGINEERING (Yellow)	7.60	7.05
	C4	TEN ENGINEERING (Yellow)	6.50	
3	C5	TEN ENGINEERING (Yellow)	4.00	3.35
	C6	TEN ENGINEERING (Yellow)	2.70	

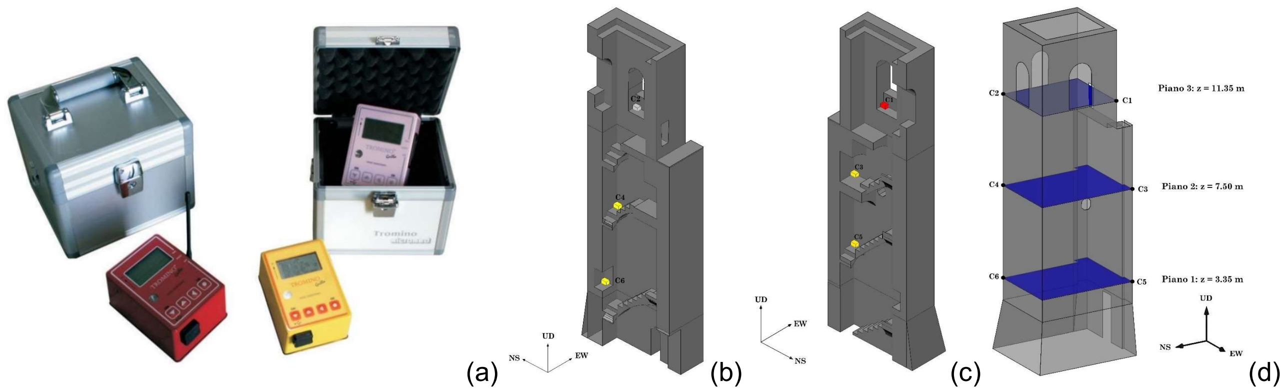

Figure 6. The three types of Tromino used for measurement of the velocity of ambient vibrations (a). The location along two vertical alignments on the opposite corners of the shaft of the tower (b), (c). The identified instrumented levels (d).

The PSD matrix was estimated using the Welch Method, using time windows of 30 seconds, a Hanning window with an overlap of 50%. The extraction of modal parameters from ambient vibration data was carried out by using two different output-only techniques: Frequency Domain Decomposition (FDD) and Basic Frequency Domain (BFD). The natural frequencies were estimated by inspecting, in parallel, the first Singular Value line of the PSD matrix, for the FDD method, and the trace of the PSD matrix, for the BFD method. The first three modes were identified: bending mode 1 (B1), in the X-axis direction, with f_1=3.94 Hz; bending mode 2 (B2) in the Y-axis direction, with f_2=4.50 Hz; torsional mode 1 (T1), with f_3=9.78 Hz.

5. NUMERICAL EIGENVALUE ANALYSES OF THE FE MODELS OF SAN GIUSEPPE BELL TOWER

Midas FEA *NX* software was used in the FE analyses. Several FE models of the tower were tested, starting from different hypotheses about the effectiveness of the connection between the walls of the church and the tower. Fixed boundary condition were imposed at the ground level. Two sets of models were tested. The first characterised by a simplified geometry, the second one with a detailed geometry taking into account the irregularities of the tower. The simplified models were discretized in tetrahedral, 4-nodes solid elements, with 300 mm average element size. This size also allowed a good polygon approximation of the geometry of the tower. The element size was reduced to 200 mm in the geometrically refined models to obtain a better meshing of the irregular tower sections. The characteristics of the implemented models are summarized in Table II, while the three different hypotheses of connection between the church and the tower are graphically explained in Figure 7.

Table II. FE models analysed. In the Model 6 series, a 0.001% $E_{connection}/E_{masonry}$ ratio corresponds to an isolated tower model with three connected walls (Model 2), while a 100% $E_{connection}/E_{masonry}$ ratio corresponds to a tower model with a full connection to the church (Model 1).

Model	Type	Geometry	Structural connection with the adjacent church
Model 1	Tower+adjacent church	Oversimplified	Full
Model 2	Isolated tower with four connected walls	Oversimplified	Partial
Model 3	Isolated tower with three connected walls	Oversimplified	None
Model 4	Isolated tower with four connected walls	Detailed	Partial
Model 5	Isolated tower with three connected walls	Detailed	None
Model 6 series	Tower+adjacent church	Oversimplified	Variable ($E_{connection}/E_{masonry}$=0.01%÷100%)

In Model 1, the connection between the tower and the church is fully effective. In Model 2, the part of the wall shared with the church, on the southeastern side of the tower, is regarded as belonging to the tower from a structural point of view, while the rest of the wall of the church is regarded as non-effectively connected and is not modelled. In Model 3, the wall on the southeastern side is regarded as belonging to the church from a structural point of view and completely disjointed from the tower, so that it is not modelled. Model 6 coincides with model 1, but a thin wall layer in inserted that simulates an interface between the tower and the church. Different values of the elasticity modulus were assigned to the joint element, ranging from 1% to 100% of the elasticity modulus of the masonry, in order to simulate different levels of the connection degree (Table II). The models 2, 3, 4 and 5 will be discussed.

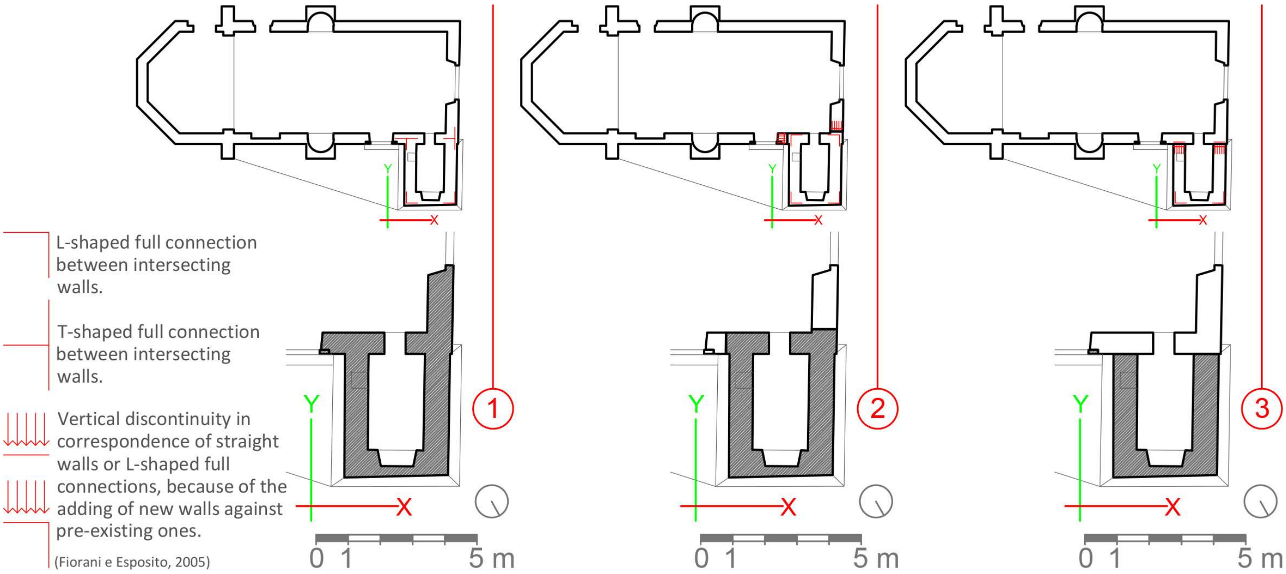

Figure 7. The three different hypotheses about the structural connection between the walls of the tower and the church. (1): Model 1; (2): Model 2; (3): Model 3.

A monolithic behaviour of the walls was assumed, as only modal analyses were carried out. The results of the analyses on the simplified models tested under isotropic material behaviour (Table 3) and on the detailed models tested under orthotropic material (Table 4) are reported. In the detailed model different mechanical parameters were assigned to different parts of the tower, with regard to the visual inspection of the masonry quality and materials. Starting from literature data (Ministero delle Infrastrutture e dei Trasporti 2018, Liberatore 2000, Baronio et al. 2003, Binda et al. 1999, Saisi et al. 2001), E=1500 MPa, γ=18 kN/m^3, ν=0.15 were assigned as first-step parameters of the uniformly isotropic material. After calibration, the values shown in Table 3 were achieved for the different models. The table also shows the frequency values obtained in each case.

Table 3. Calibrated values of the mechanical parameters of uniformly isotropic simplified models and the modal frequencies. The error with respect to the experimental frequencies is also reported.

Model	γ [kN/m^3]	E [MPa]	ν [-]	G=E/2(1+ν) [MPa]	G/E [-]	f_1 [Hz]	Err% f_1	f_2 [Hz]	Err% f_2	f_3 [Hz]	Err% f_3
Model 2	19	2800	0.3	1077	~0.38	3.68	-6.60	4.47	-0.67	11.60	+18.6
Model 3	19	4600	0.3	1769	~0.38	3.94	-0.75	4.25	-5.55	9.76	-0.20

The calibrated E values are in the range 1500÷5000 MPa, almost coincident with the extreme values assigned by the Italian building code (Ministero delle Infrastrutture e dei Trasporti 2018) for this type of stone masonry. They are also compatible with most literature data examined (Liberatore 2000) about experimental double flat jack tests carried out on analogous ancient masonry walls in the same geographical area of the tower. Starting from literature data in eastern Sicily, ν can be considered as ranging between 0.05 and 0.15 for calcarenite stone masonry (Baronio et al. 2003, Binda et al. 1999) and between 0,1 and 0.35 for lava stone masonry (Saisi et al. 2001, Liberatore 2000). For an isotropic material, it is well known that G=E/2(1+ν), so 0<ν=E/2G-1<0.5 and G is inversely proportional to ν. This means that G can be controlled (although in a limited way) through ν and that G can take only the values allowed by the theoretical limits of ν, so that 0.33E<G<0.5E. If ν=0.15, it is G~0.43E, that meets the G=0.4E correlation proposed by the Italian building code and EC6 (Ministero delle Infrastrutture e dei Trasporti 2018, NCE 2005). Actually, G/E should be considerably less than 0.4, decreasing also until 0.1÷0.25 (Wilding et al. 2021). It is worthy to note that, for some models, increasing ν over certain values causes an increase, albeit slight, of the torsional frequency, instead of a decrease of it.

After this, the orthotropic material has been assigned to Model 4 and Model 5, assuming the first-step value of the shear modulus as G=0.4E. The calibration of the model based on the orthotropic material model is reported in Table 4. The calibrated value of G resulted in the range 0.37E÷0.38E, in agreement with theoretical previsions from EC6.

Table 4. Calibrated values of the mechanical parameters of the non-uniform, orthotropic geometrically refined models and modal frequencies. The error with respect to the experimental frequencies is also reported.

Model	Part	γ [kN/m³]	E [MPa]	ν [-]	G [MPa]	G/E [-]	f_1 [Hz]	Err% f_1	f_2 [Hz]	Err% f_2	f_3 [Hz]	Err% f_3
Model 4	Basement and shaft	18	3850	0.3	810	~0.21	3.93	-0.25	4.72	+4.89	9.78	0.00
	Bell chamber	17	1900	0.2	600	~0.31						
	Barrel vaults and arches	11	3850	0.3	810	~0.21						
	Rampant vaults of the stairs	17	1900	0.3	600	~0.31						
Model 5	Basement and shaft	18	5100	0.3	1950	~0.38	3.95	-0.25	4.38	-2.67	9.78	0.00
	Bell chamber	17	3200	0.2	1200	~0.37						
	Barrel vaults and arches	11	5100	0.3	1950	~0.38						
	Rampant vaults of the stairs	17	3200	0.3	1200	~0.37						

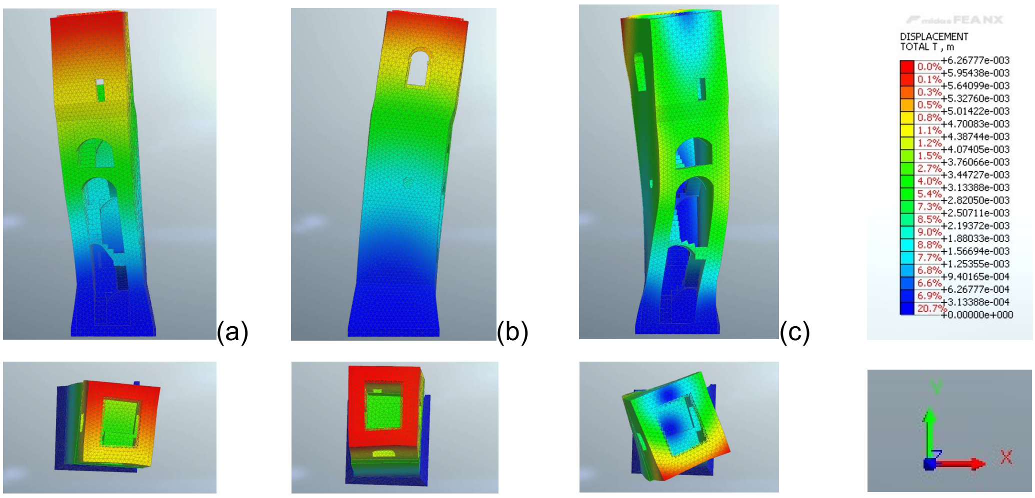

Figure 8 Modal shapes for Model 5. Mode B1, f_1=3.95 Hz (a); Mode B2, f_2=4.38 Hz (b); Mode T1, f_3=9.78 Hz (c). (Figure 9).

The detailed model was calibrated changing E and G for the different parts of the tower: the basement and the shaft, the bell chamber, the stairs, the vaults. The stiffness of the stairs and the vaults and arches was considered lower than the one of masonry, to take into account both the different materials and the non-perfect effectiveness of the connection between this building elements. In Model 5 with orthotropic materials, a very good agreement between the experimental frequencies and modal shapes and the numerical ones was reached. The results are better than the ones given by the model with isotropic materials, which in general is not able to fit all the three considered frequencies. In fact, in this case in Model 3 the second bending frequency results underestimated, while in Model 1 G is overestimated, causing an undesired increase of the only torsional frequency. This increase can not be balanced even by increasing ν to its upper limit value, ν=0.49, since in this case a G/E=0.26<0.33 ratio is required, corresponding to an impossible ν=0.93>0.5. The different materials attributed to the different parts of the tower in Model 5 are shown in Figure 9.

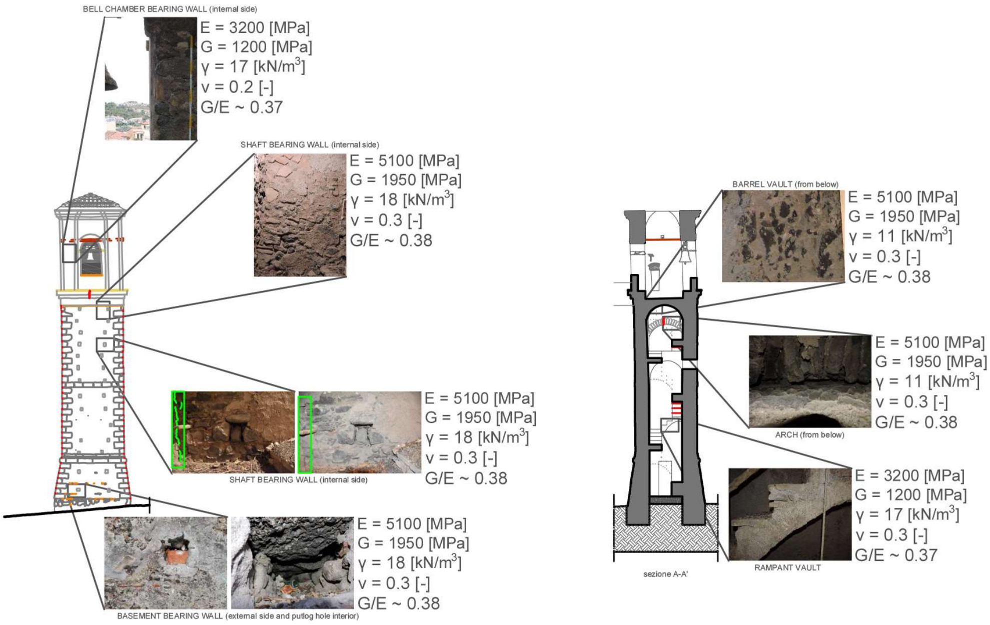

Figure 9. Mechanical parameters attributed to the different parts in the calibrated detailed Model 5, without connection to the church.

6. CONCLUSIONS

A FE model of the bell tower of the church of San Giuseppe in Aci Castello was calibrated by comparison of Eigenvalue FEM analyses and OMA results. Different modelling detail levels were tested and several sensitivity analyses were carried out, both on mechanical parameters and geometric or architectural details. The influence of different modelling choices on the results of FEM eigenvalue analysis was highlighted. In particular, it was shown that E has a major influence on the frequencies associated with the bending modes, while G strongly influences the torsional ones. γ seems to have a higher, but uniform, influence on modal frequencies than E. Changing ν in its admissible range of values, a slight influence on modal parameters is obtained. In the case of isotropic masonry models, it is difficult to calibrate the mechanical parameters by fitting both flexural and torsional frequencies due to the dependency between G and E. The choice of an orthotropic constitutive law of the material, instead of the unrealistic hypothesis of an isotropic behaviour, improves the accuracy of the model. The presence of windows or large voids in the walls plays an important role in determining the mass distribution and the stiffness of the tower, while the diffuse presence of putlog holes can affect the torsional stiffness. As expected, their relevance increases with the increase of the ratio between voids volume due to the presence of the holes on total wall volume. Putlog-holes mainly affect the first torsional frequency (2.5% for a small tower). When the masses of the roof and the bells represents a small percentage of the total weight of the tower (less than 0.5% in the case of San Giuseppe, against more than 95% of the walls), and given their neutral contribution to stiffness, the modelling of these details is not relevant to the numerical determination of the values of the modal parameters (about 0.1-0.5% of influence on the first frequencies). Just like the roof and the bells, the internal stairs have not enough mass and stiffness, if compared to those of the entire tower (about 2÷3% of the total mass in the case of San Giuseppe), to relevantly affect the values of the modal parameters. Nevertheless, a greater influence (of about 7% on the first torsional frequency) is shown in the case of a three-walls tower. For the case of San Giuseppe, it is shown that the switch from an oversimplified-geometry 3D model to an accurate-geometry model cause variation in the order of 5÷9% in the frequencies, depending on the model type. Precisely, the detailed geometry produced a decrease of almost 8% of the first two flexural frequencies and of about 9% of the first torsional frequency in a four-walls tower model with uniform, isotropic material. A decrease of almost 8% and more than 4%, respectively for the first and the second flexural frequency, and of more than 5% for the first torsional frequency, was produced in a three-walls tower model with non-uniform material. The boundary conditions and the interaction with neighbouring buildings have the main relevance on modal properties. The mere presence of four fully connected walls, instead of three, in an oversimplified-geometry model makes the Young's modulus decrease of about 40%. A connection to the adjacent church by means of a thin masonry element with a mere $E_{connection}$=0.01% of $E_{masonry}$ Young's modulus gives a change of about 4÷8% in the first three modal frequencies. With a $E_{connection}$=0.1÷1% of $E_{masonry}$ the modes of the tower start to be no more uniquely recognisable, and their natural succession is lost. Such a high variability gives high uncertainty about the actual values of E and G. Plausible values, although sometimes a bit higher than expected (E~2900 MPa for the four-walls Model 1, E~5000 for the three-walls Model 5, still within their likely range of values for such type of masonry), were obtained for all the calibrated models. Thus, only an on-site flat-jack tests and sonic tests campaign could confirm their realistic range of value. Moreover, borescope inspections could clarify the connection ratio between the walls and the actual boundary conditions.

7. References

Baronio G., Binda L., Tedeschi C., Tiraboschi C. (2003). Characterisation of the materials used in the construction of the Noto Cathedral. *Construction and building materials* 17:557-571.

Binda L., Baronio G., Gavarini C., De Benedictis R., Tringali S. G. (1999). Investigation on materials and structures for the reconstruction of the partially collapsed Cathedral of Noto (Sicily). *Transactions on the Built Environment* 39:323-332.

Cabboi A., Gentile C., Saisi A. (2017). From continuous vibration monitoring to FEM-based damage assessment: Application on a stone-masonry tower. *Construction and Building Materials* 156: 252–265.

Castorina S. (2002). Sotto il piano del Castello. Quattromila anni di presenza umana nella benna dell'escavatore. *Agorà* IX: 16-20.

CEN (2005). *EN 1996-1:2005. Eurocode 6: Design of masonry structures*, Comité Européen de Normalisation, Brussels.

De Angelis A., Lourenço P.B., Sica S., Pecce M. R. (2022). Influence of the ground on the structural identification of a bell-tower by ambient vibration testing. *Soil Dynamics and Earthquake Engineering* 155:1-14.

Gentile C., Saisi A., Cabboi A. (2015). Structural Identification of a Masonry Tower Based on Operational Modal Analysis, *International Journal of* Architectural *Heritage*, 9(2): 98-110.

Imposa S., Cuomo M., Contrafatto L., Mineo S., Grassi S., Li Rosi D., Barbano M.S., Morreale G., Galasso M., Pappalardo G. Engineering Geological and Geophysical Studies Supporting Finite Element Analysis of Historical Buildings after Dynamic Identification. *Geosciences*. 2023; 13(3): 84

Ivorra S., Pallarés F. J. (2006). Dynamic investigations on a masonry bell tower, *Engineering Structures*, 28: 660–667.

Ivorra S., Pallarés F. J., Adam J. M. (2011). Masonry bell towers: dynamic considerations, *Structures and Buildings*, 164(1): 3-12.

Liberatore D. (ed.) (2000). *Progetto Catania: indagine sulla risposta sismica di due edifici in muratura,* CNR-Gruppo Nazionale per la Difesa dai Terremoti, Roma.

Ministero delle Infrastrutture e dei Trasporti (2018). *D.M. 17 Gennaio 2018. Norme Tecniche per le Costruzioni.* Roma.

Mondello A., Garozzo R., Salemi A., Santagati C., 2019, HBIM for the seismic vulnerability assessment of traditional bell towers, *Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci.*, XLII-2/W15: 791-798.

Saisi A., Binda L., Zanzi L. (2001). Diagnostic investigation of the stone pillars of S. Nicolò l'Arena, Catania. *The 9th Canadian masonry symposium*, Canada Fredericton (New Brunswick).

Shakya M., Varum H., Vicente R., Costa A. (2018). Seismic vulnerability assessment methodology for slender masonry structures, *International Journal of Architectural Heritage*, 12(7-8): 1297-1326.

Shabani A., Feyzabadi M., Kioumarsi M. (2022). Model updating of a masonry tower based on operational modal analysis: The role of soil-structure interaction, *Case Studies in Construction Materials* 16: 1-18.

Torelli G., D'Ayala D., Betti M., Bartoli G. (2020). Analytical and numerical seismic assessment of heritage masonry towers, *Bulletin of Earthquake Engineering* 18: 969-1008.

Valente M. and Milani G. (2018). Effects of Geometrical Features on the Seismic Response of Historical Masonry Towers. *Journal of Earthquake Engineering* 22: 2-34.

Wilding B. V., Godio M., Beyer K. (2020). The ratio of shear to elastic modulus of in-plane loaded masonry. *Materials and structures* 53(40): 1-18