INTEGRATED ANALYSIS AND OPTIMAL DESIGN

By

Alfredo V. Soeiro

A THESIS PRESENTED TO THE GRADUATE SCHOOL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF ENGINEERING

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Cochairman: Dr. Marc I. Hoit
Major Department: Civil Engineering

Structural optimization generally involves a two-stage procedure that in most cases is a repetitive one. It includes in succession an analysis and then a design optimization. The configuration described here is a closed form where the two phases are integrated, avoiding their separation and the consequent cycling. It creates a general technique valid for all structures submitted to external loadings.

The proposed method is based on the theoretical formulation of the Augmented Lagrangian function using updated lagrangian multipliers. The compatibility equations are handled as equality constraints. The limits of the global displacements are represented by a group of inequality constraints. The function to be minimized is the volume of the structure, while satisfying all

the constraints that are previously imposed to the behavior of the structure.

To examine the performance of the method, some simple structures were tested. The results are presented, discussed and analysed. The principal conclusion is that the method has promising future applications and enhancements that will improve the efficiency of structural optimization.

Chairman

CHAPTER 1

STRUCTURAL OPTIMIZATION

1.1 Overview of Structural Optimization

The structural design activity is a multifaceted task, with knowledge requirements in a number of technical areas. The design process must be guided by clearly defined objectives. They can be simple ones, like cost, reliability, and weight or it can be a combination of these. These objectives can be a function of the design variables as well as of the behavior constraints like stresses or displacements. These constraints generally have the form of equalities, inequalities or limits for the variables. The whole of these conditions, in a explicit formulation, creates what is called a structural optimization problem. With an implicit formulation, it is recognized as an analysis question that the designer must solve based upon information resulting from his design experiences.

The common design methodology for the implicit formulation involves numerous tedious computations. This is due to the fact that the adequacy of the cross sections can only be confirmed after a structural analysis of an assumed model is performed. Generally, an experienced designer is essential to the process or the final design will be an oversized structure. To overcome these problems, research has been pursued on the explicit formulation of the design.

1.2 Structural Optimization Examples

There are two major types of problems in structural design where optimization is used, each requiring a different method of solution. The first type is the optimization of the basic elements of a structure on an uncoupled basis. An example of this type is the optimization of the reinforcing steel of a concrete beam [1]. The second is the search for the optimal global configuration or sizing of the basic elements of the structure. A commonly used model of this second type of problems is the planar trusses, where the goal is to find the optimal bar sizes [2].

It is possible to implement methods on small computers that can address the first problem. The result is a computer program that is easy to use. It requires as much data as the designer would have to know in a traditional implicit design. Fast and accurate designs may be obtained, because shortcuts can be implemented as a consequence of the specific nature of the problem. These programs generally obtain a design required to be efficient, capable of withstanding the external forces, which complies with the applicable codes and other designing limitations such as the possibility of being executed. These requirements are generally postulated within a numerical optimization framework. This framework leads to the minimization of an objective function while satisfying a set of algebraic functional constraints. The advantage of using an explicit formulation in element design methods lies in the ability to produce the desired information simply and rapidly. The main disadvantage is the assumption that the structure and element behaviors are uncoupled, which cannot be validated for most structural designs.

Considering the second type of optimization problem, one of the most studied areas is truss design. The commonly used objective function is the minimum weight, assuming that the cost of the truss is proportional to its weight. The explicit design problem consists of finding the optimal cross section area for each truss member. The constraints may represent design limitations such as bounds on member sizes or behavior requirements like allowable stresses and displacements. This must be achieved while satisfying the equilibrium and compatibility conditions. The conventional truss design method is extended to an explicit design problem by resizing the bars through the use of optimization methods. The size of the problem is sometimes reduced by a technique called variable linking, consisting of making linear combinations of the design variables. A constraint deletion may also be used to reduce the problem size when avoiding noncritical constraints during the future iteration cycles [3].

Another highly researched problem of the second type is in the area of rigid frames. The iterative process of analyzing an initially assumed design, member resizing and reanalysis has been used successfully. However, it has required many simplifying assumptions that restrain the general application. One common approach is to extract the resulting member forces from a standard analysis. These forces are then used in a separate optimization step where the elements are designed and resized independently of the structure. This method uses the global structure to distribute the member forces and then an uncoupled approach to optimize the particular element [4].

Another area where the explicit formulation of the design problem is useful is the optimization of the topologic configuration. Methods have been studied where the most economical geometric form is a function of the number of bays, spans, frames and frame spacings. They are identified in order to satisfy the requirements of floor area, site dimensions, useable headroom, existence of internal columns, and other architectural demands [5].

1.3 Optimization Techniques in Structural Analysis

1.3.1 General Problem Formulation

The general mathematical statement formulating an optimization problem in structural design is

Minimize (or Maximize) $f(\underline{x})$

$$\underline{x} = (x_1, x_2, \ldots, x_n)$$

Subject to

$$g_j(\underline{x}) \leq 0 \quad , \quad j = 1,m$$

$$h_k(\underline{x}) = 0 \quad , \quad k = 1,p$$

$$\underline{x} \geq 0$$

The objective function is f and depends on the design variable vector $\underline{x}$. It represents some chosen criterion of merit of the design which may be cost, efficiency, benefit, etc.. The g_j inequalities and the h_k equalities are the constraints with the vector $\underline{x}$ as an argument. In the case of structural element design, the vector $\underline{x}$ may represent the configuration of the element, such as bar areas, depths, bar spacings, etc. The constraints g_j are the expression defining the upper and lower bounds of stresses, dimensions of the structural elements, deflections, or other limiting code provisions. These constraints are generally highly nonlinear. The constraints h_k are the definition of binding values for the design variables, obliging the design vector to satisfy the equality constraints [6].

1.3.2 Search Methods

These optimization approaches are based on the presumption that there is no information about the optimum. They explore the behavior of the objective function and constraints, and sometimes the functional derivatives, when different design points are tested in order to reach the optimal point. These methods always involve a reanalysis of the structure after an optimization cycle. The cycle is repeated until an optimal solution is obtained.

The penalty function method is the most robust for solving the constrained problem as a sequence of unconstrained optimizations. This method uses a penalty parameter, p, that multiplies the constraints violations. These factored constraints are then added to the objective function, $\underline{f}$, in such a way that a violation of any constraint leads to a very high value of the augmented objective function. Unconstrained optimization is then used to find a minimum of the augmented objective function for a particular value of p. Then p is updated such that when the unconstrained optimization is repeated, the objective function value is reduced as well as any constraint violations. The optimal solution will satisfy the original constraints and approximate the minimum of f. Several techniques are directly based on this formulation such as the interior penalty function and the exterior penalty function. Both present discontinuities near the constraints. Various attempts have been made to minimize this problem of discontinuities such as the extended linear and quadratic penalty functions formulations [7].

Another common method involves using the gradient of the objective function. The methods using gradients may achieve considerable enhancement if the derivatives of the equality constraints,supported by the stiffness matrix, are easy to obtain, like in truss bars. In the case of a frame element, additional effort is required since first moments and areas in the stiffness matrix are themselves some function of the design variables. Also, the gradients of the constraints require the intermediate calculation of the gradient of the displacement with respect to the design variables, since most of the constraints are directly related to the displacements. Several methods emerged from the use of the gradients like the conjugate gradient method and the variable metric method, widely used in some recent algorithms [6].

A great improvement in search methods can be achieved through the use of the derivatives of the constraint functions with respect to the design variables. This is called design sensitivity analysis and is very useful because it gives information of the constraint behavior during the search method. Several approaches have been used: behavior space, design space and virtual load [1]. They all give the same results, but have different generality and efficiency. The behavior space approach creates an adjoint relationship to express the effect of the changes in the displacement vector in terms of the variation of the design variables. The design space approach assumes that the dependent behavior variables, displacements, are expressed in terms of the independent design variables. The virtual load approach is formulated identically to the design space approach, but uses a virtual load to simplify the related calculations [2].

1.3.3 Optimality Criteria Methods

These methods try to establish conditions for the uniqueness of the solution at the beginning of the process. These conditions characterize the optimum of a problem and distinguish it from all other possible solutions. The methods then attempt to devise a scheme which iteratively satisfies the optimality criteria conditions while searching for the optimum. These methods have two problems: first, in most cases there is no absolute criterion to distinguish a global optimum from any other local optimum ; second, the resizing schemes (based on the optimality criteria) are only approximate and need careful programming to yield good results [8]. The Kuhn-Tucker conditions are often used to define the optimality criteria. They introduce a new type of variables, u_j, called the lagrangian multipliers. Although the size of the problem increases, this formulation is advantageous because it creates helpful requirements and information about the optimum [9]. Considering a nonlinear programming problem, the optimality conditions assure that $\underline{x}$ is an optimal point if :

$$\text{grad } f(\underline{x}) - \text{Sum}_j\ (\underline{u}_j \text{ grad } g_j(\underline{x})) - \text{Sum}_k\ (\underline{v}_k \text{ grad } h_k(\underline{x})) = 0$$

$$g_j(\underline{x}) \geq 0 \qquad j = 1,\ldots,m$$

$$h_k(\underline{x}) = 0 \qquad k = 1,\ldots,l$$

$$\underline{u}_j\ g_j(\underline{x}) = 0 \qquad j = 1,\ldots\ldots,m$$

$$u_j \geq 0 \qquad k = 1,\ldots\ldots,l$$

This equation shows that the gradients of the constraints multiplied by the respective adequate lagrangian multipliers will form a linear combination that will nullify the gradient of the objective function. The other equations represent the conditions requiring feasibility and zero values for the lagrangian multipliers for which the inequality constraints are not binding ($g_j < 0$).

1.3.4 Description of Structural Examples

In trusses, the number of variables and constraints is very large. However, the regular mathematical structure of the problem can be used as an advantage in using simpler methods. The objective function is linearly dependent on the design variables, but the constraints are nonlinear. The dual problem is the one that can be defined in association with the initial problem (also designated as primal). The important feature of this association is that a solution to one is a solution to the other. Considering the dual, the problem can be mapped into another space where the objective function is nonlinear and the constraints are linear [10]. This problem may be solved by making a sequence of linear approximations to the nonlinear objective function, which are solved by linear programming methods. It must be emphasized that these methods are simply numerical search methods which make no assumptions about the nature of the optimum.

The optimum rigid plastic design of frames may also be mapped in a problem of linear programming. This has the considerable advantage that very large problems can be solved quickly and efficiently. However, the formulation of the linear programming model may be difficult since all possible collapse mechanisms must be known and analysed. For large frames, the number of these mechanisms can be very large. Some research has been done in order to make full use of the duality between the static and kinematic theorems of plasticity, trying to minimize the correspondent formulation effort [11].

Optimal design of elastic frames leads to an optimization problem similar to that of a truss-sizing. However, members with flexural and axial loads do not easily map into the dual problem. One popular method finds the optimum design by using the virtual load method to formulate displacement constraints and a force matrix approach to reduce the problem size. This method still generates a nonlinear objective function with linear constraints. In order to solve the problem, direct search methods are required. Some of the direct methods that have been used are feasible directions, generalized reduced gradient and the gradient projection method [12]. All of these methods are difficult to implement due to the size of the problems and its nonlinear nature. Mixtures of these methods have been tried depending on the solution strategy adopted. In problems where the definition of some of the constraints is not well posed, such as concrete strength, fuzzy logic has been applied to control the uncertainty of the material and structural behavior [13].

Another technique used in large scale systems uses decomposition, or substructuring, in the problem and then performs optimization for each subproblem until convergence is obtained. The energy method is also used to generate a function which can be maximized, representing the amount of energy absorbed by the structure when loaded and deflected. The consideration of the problem case where structural frames are submitted to dynamic loading has been studied by a few researchers [14]. The importance of this type of optimization, due to the large amount of surplus materials involved in the design of the related structures that are not optimal, has not been a sufficient reason to overwhelm the complexity of these calculations. Most of the research studies involve simplifications for specific problems that reduce their chance of application.

1.4 Conclusions

All these structural optimization techniques were developed later than those for structural analysis. They appeared like an extension of structural analysis methods using experiences from other optimization areas. For that reason there is a real separation between these two areas in structural design. Sometimes that is the main reason why the structural optimization techniques are specific for some type of problems or are difficult to use.

There is currently a trend to abandon imported optimization methods from other areas and try to develop adequate techniques for structural optimization. This trend is precisely the opposite of what happened in the past where the structural optimization problems were limited, and sometimes adapted, to suit the optimization techniques. For instance, as a major consequence of previous research there are no general structural optimization techniques.

What is described in the next chapters is the research done to obtain a universal structural optimization method that at the same time integrates analysis and optimal design. The technique is based in the Augmented Lagrangian Multipliers method and the type of structures tested are planar frames with displacement constraints. To test the performance of the method two simple structures were submitted to different load cases: a cantilever beam and a one bay frame.

CHAPTER 2

DUALITY IN OPTIMIZATION

2.1 Duality Theory

2.1.1 General Description

This theory is very important for all mathematical programming problems,linear and nonlinear. An example of this formulation applied to a linear programming problem is described as follows.

Given the primal (or initial) problem

$$\begin{aligned} \text{maximize} \quad & \underline{c}\,\underline{x} \\ \text{subject to} \quad & \underline{A}\,\underline{x} \leq \underline{b} \\ & \underline{x} \geq 0 \end{aligned}$$

Then, the dual problem is defined as

$$\begin{aligned} \text{minimize} \quad & \underline{w}\,\underline{b} \\ \text{subject to} \quad & \underline{w}\,\underline{A} \geq \underline{c} \\ & \underline{w} \geq 0 \end{aligned}$$

The new variables $w_1, w_2, \ldots\ldots, w_n$, that compose the row vector $\underline{w}$, are called the dual variables and $\underline{w}\ \underline{b}$ is now the dual function. It is easy to prove that the minimum of the dual function is the maximum of the primal problem [15]. Given the primal and dual constraints

$$\underline{A}\ \underline{x} \leq \underline{b} \quad \text{and} \quad \underline{w}\ \underline{A} \geq \underline{c} \quad ,$$

premultiplying and postmultiplying by the vector $\underline{w}$ and $\underline{x}$,respectively we obtain

$$\underline{w}\ \underline{A}\ \underline{x} \leq \underline{w}\ \underline{b} \quad \text{and} \quad \underline{w}\ \underline{A}\ \underline{x} \geq \underline{c}\ \underline{x}$$

and therefore,

$$\underline{c}\ \underline{x} \leq \underline{w}\ \underline{A}\ \underline{x} \leq \underline{w}\ \underline{b}$$

or

$$\underline{c}\ \underline{x} \leq \underline{w}\ \underline{b}$$

The assumption is that the primal problem is well defined, meaning that it has a solution and is bounded. Then the dual problem is also feasible and has a bounded solution. This implies that the final inequality must be strictly satisfied by an equality. The maximum of the primal function and the minimum of the dual function have exactly the same optimal value.

All these conclusions are in great part a consequence of two important theorems in duality theory: Strong duality and Weak duality [10]. The formulation of these theorems for a linear programming problem are defined in the next subchapters.

2.1.2 Strong Duality Theorem

Let $\underline{x}^*$ be the optimal solution for the primal problem, X^n the design space with n dimensions for the primal problem and $\underline{w}^*$ the optimal solution for the dual problem.

Suppose that for $\underline{x}^*$ from X^n,

$$\underline{A}\ \underline{x}^* \leq \underline{b}$$

$$\text{and} \quad \underline{x}^* \leq 0.$$

Then if for the dual problem there is $\underline{w}^*$ such that

$$\underline{w}^*\ \underline{A} \geq \underline{c}$$

and $\underline{w}^* \geq 0$, the following holds

$$\max\ \underline{c}^t\ \underline{x}^* = \min\ \underline{w}^{t*}\ \underline{b}$$

2.1.3 Weak Duality Theorem

Let $\underline{x}$ be a feasible solution for the primal problem. Let $\underline{w}$ be also a feasible solution to the dual problem.

Then, the following statement relating both solutions, $\underline{x}$ and $\underline{w}$, holds

$$\underline{c}^t \, \underline{x} \leq \underline{w}^t \, \underline{b}.$$

The proof of these statements may be found in several references [9] and [10]. The extension of these theorems to nonlinear programming problems and integer problems is straightforward to derive [15].

2.2 Lagrangian Dual Problem

Given a primal formulation of a nonlinear programming problem defined by

$$\begin{aligned} &\text{minimize } f(\underline{x}) \\ &\text{subject to } h_i(\underline{x}) = 0 \;, \quad i = 1,\ldots\ldots,l \\ &g_i(\underline{x}) \leq 0 \;, \quad i = 1,\ldots\ldots,m \\ &\underline{x} \text{ in the set } X^n \end{aligned}$$

where at least one the functions $\underline{f}$,$\underline{g}$ or $\underline{h}$, is nonlinear, several duality formulations may be derived from the primal problem. A commonly used one is related with the derivation of the optimality conditions for optimization methods based on optimality criteria. They are used to assure the Kuhn-Tucker conditions. One of the most important and useful is the Lagrangian dual problem [16]. It has been widely used in the creation of successful algorithms to solve linear, nonlinear and integer programming problems. The formulation is the following

$$\text{maximize } L(\underline{u},\underline{v})$$
$$\text{subject to } \underline{u} \geq 0$$

and if inf is the greatest lower bound of the set then

$$L(\underline{u},\underline{v}) = \inf \{ f(\underline{x}) + \text{grad}\,(u_i g_i(x)) + \text{grad}\,(v_i h_i(x)) \}$$

This function L is updated until we find a stationary point, with solution $\underline{x}^*$, $\underline{u}^*$ and $\underline{v}^*$, corresponding to a minimum of $f(\underline{x})$. The Lagrangian dual function has no lower bound since the lagrangian multiplier u_i ,associated with inequalities, are defined to be positive or null. The other lagrangian multiplier v_i, corresponding to the equalities, is unrestricted in sign.

An example of the relation between primal and dual formulations may be illustrated by the geometric interpretation that can be done for a simple mathematical programming problem [9].

Considering the primal problem defined as minimize $f(\underline{x})$, subject to $\underline{x}$ from X^n and $g(\underline{x}) \leq 0$ where, in the plane (y,z), $y = g(\underline{x})$ and $z = f(\underline{x})$ for $\underline{x}$ of X^n. This generates the set K, representing all the possible solutions, as shown in figure 2.1.

The primal problem demands a point in K that is in the negative halfspace limited by the z axis. The optimum point is obviously A. Considering the dual problem, for a given multiplier u, the goal is to find L (u) such that the value $f(\underline{x}) + u.g(\underline{x})$ is minimum for all x of X^n. This means we want to minimize z + u.y over all points in K. This is the equation of a straight line with slope -u. This line intercepts the z axis at an ordinate c. Maximizing this ordinate we obtain the value u_1 leading to an optimal c_1, which is the ordinate of the optimal point for the primal problem.

One attractive feature of the Lagrangian function is that it is a concave function. That means that any local optima, if it exists, is a global optima. However, the main difficulty in the use of the Lagrangian function is the fact that the solution is not explicitly available. The function can be evaluated only at the end of each minimization subproblem [17].

The proof of the concavity of the Lagrangian function is based on the fact that the value of Lagrangian function for a point $\underline{Q}_3$, that is a linear combination of two other points $\underline{Q}_1$ and $\underline{Q}_2$, is greater or equal to the sum of the values of the Lagrangian function at those points $\underline{Q}_1$ and $\underline{Q}_2$. For the sake of simplicity, the combination of vectors $\underline{u}$ and $\underline{v}$ is

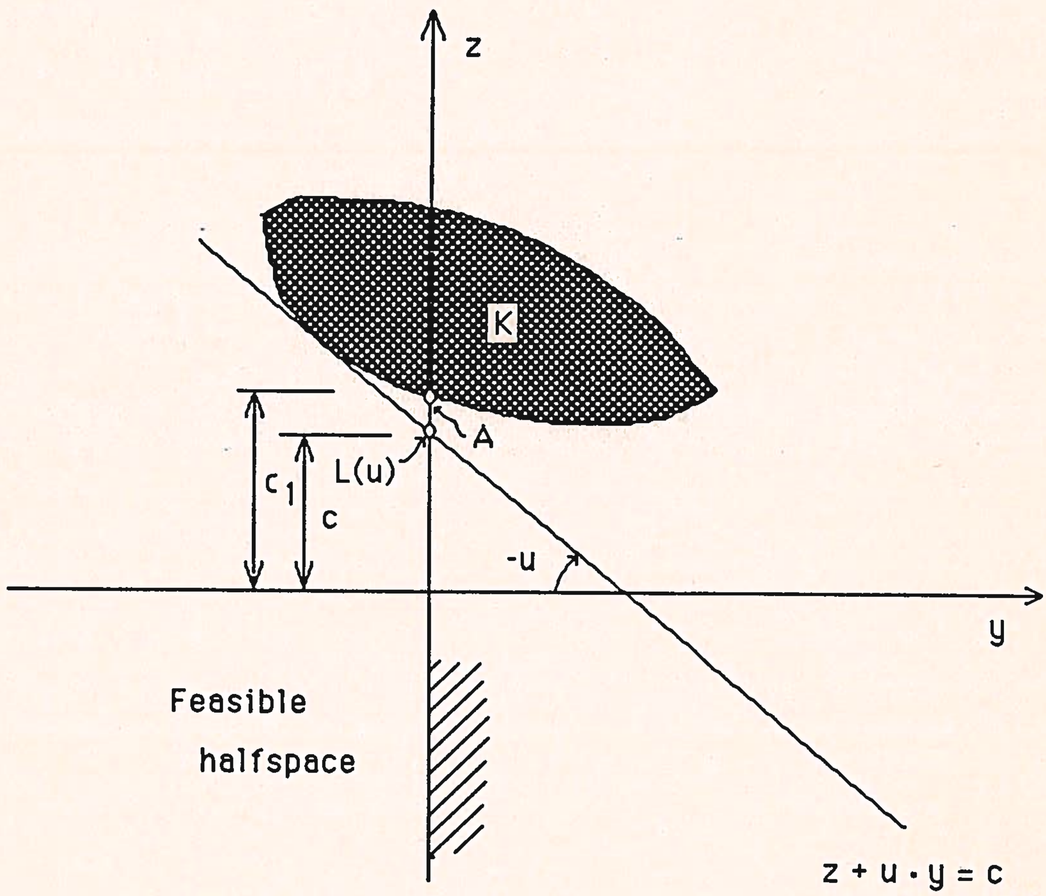

Figure 2.1 Geometric interpretation of dual problem.

designated as t and of the functions $\underline{h}$ and $\underline{g}$ as j. Assuming that f and j are continuous functions and that the design set is compact, then L is finite. Let c be a scalar between 0 and 1 and inf the infimum of the considered set.

$$L (c \underline{Q}_1 + (1-c) \underline{Q}_2) =$$
$$\inf\{ f(\underline{x}) + [c \underline{Q}_1 + (1-c) \underline{Q}_2]^t \underline{j}(\underline{x}) \} =$$
$$\inf\{ c [f(\underline{x}) + \underline{Q}_1{}^t \underline{j}(\underline{x})] + (1-c) [f(x) + \underline{Q}_2 \underline{j}(\underline{x})] \} \geq$$
$$c \inf\{ f(\underline{x}) + \underline{Q}_1{}^t \underline{j}(\underline{x}) \} + (1-c) \inf\{f(\underline{x}) + \underline{Q}_2{}^t \underline{j}(\underline{x}) \} =$$
$$c L (\underline{Q}_1) + (1-c) L (\underline{Q}_2)$$

2.3 Augmented Lagrangian Multipliers

Based on the conclusions described above in the previous subchapters, a general algorithm was developed to solve the primal nonlinear problem. This enhancement is due to the research studies of Schultdt [18], presented by Hestenes. The method is based in the formulation of an augmented penalty function based on the dual problem.

Given a typical nonlinear problem defined as in 2.1, the problem may be redefined using the augmented function and the formulation is

$$\text{minimize } L (\underline{x},\underline{u},\underline{v}) = f(\underline{x}) + \underline{u}^t \underline{h}(\underline{x}) + \underline{v}^t \underline{g}(\underline{x})$$
$$\text{subject to } \underline{g}(\underline{x}) \leq 0 , \underline{h}(\underline{x}) = 0 \text{ and } \underline{v} \geq 0,$$

where $\underline{f}$,$\underline{h}$ and $\underline{g}$ are assumed to be differentiable and $\underline{f}$ has a constrained minimum, meaning that it has a minimum and at the same time is feasible considering the constraints.

The algorithm introduces a penalty factor P, positive and large, to penalize the constraints violations [19]. The new form of the function is

$$\text{minimize } L(\underline{x},\underline{u},\underline{v}) = f(\underline{x}) + W(\underline{x},\underline{u},P) + V(\underline{x},\underline{v},P)$$

The scalar functions W and V are defined as

$$W(\underline{x},\underline{u},P) = \underline{u}^t\,\underline{h}(\underline{x}) + P\,\underline{h}(\underline{x})^t\,\underline{h}(\underline{x})$$

$$V(\underline{x},\underline{v},P) = \underline{v}^t\,\underline{g}(\underline{x}) + P\,\underline{g}_m(\underline{x})^t\,\underline{g}_m(\underline{x})$$

$$\text{subject to } P > 0$$

$$\underline{u} \geq 0,$$

where $\underline{g}_m(\underline{x})$ is the vector with the j^{th} component defined as

$$g_{jm}(\underline{x}) = \max\{ -v_j / (2 * P),\ g_j(\underline{x}) \}$$

The authors of the method suggest that the penalty constant P should be kept constant along the iteration process, although some other developers and users of the algorithm advise an increasing penalty factor.

The function L is minimized as an unconstrained function, updating the lagrangian multipliers at the end of each cycle. Since the convergence rate also depends on the initial values of the lagrangian multipliers, it is suggested that for the first cycle they should be set to zero. This will make the Lagrangian function, L, take the shape of an exterior augmented penalty function, with the terms involving the lagrangian multipliers equal to zero. In this case the value of the j^{th} component of the truncated inequality is

$$g_{jm}(\underline{x}) = \max \{ g_j(\underline{x}) \, , \, 0 \}.$$

On the next cycles the rules for updating the lagrangian multipliers in the k^{th} iteration are

$$\underline{u}^{k+1} = \underline{u}^{k} + 2 * P \, \underline{h}(\underline{x})$$
$$\underline{v}^{k+1} = \underline{v}^{k} + 2 * P \, \underline{g}_m(\underline{x})$$

These updating rules guarantee that the inequality lagrangian multipliers are positive or null [20]. They also assure that when updating the lagrangian multipliers at the end of each cycle, the value of the Augmented Lagrangian function will increase. This is caused by the fact that the lagrangian multipliers have nondecreasing values. Finally, it must be stressed that the objective of this method is to find a stationary point of the design set where $\underline{x}$, $\underline{u}$ and $\underline{v}$ are optimal, at the end of a cycle of iterations.

It can be proved that at the beginning of each unconstrained minimization cycle, the Augmented Lagrangian function value will not decrease when changing the value of the multipliers obeying to these established updating rules[21]. Thus

$$W(\underline{x},\underline{u}^{k+1},P) - W(\underline{x},\underline{u}^{k},P) = 2 * P\ \underline{h}^{t}(\underline{x})\ \underline{h}(\underline{x}) \geq 0$$

$$V(\underline{x},\underline{v}^{k+1},P) - V(\underline{x},\underline{v}^{k},P) = \underline{v}^{k+1}\ \underline{g}_m{}^{k+1}(\underline{x}) - \underline{v}^{k}\ \underline{g}_m{}^{k}(\underline{x}) + P\ [\underline{g}_m{}^{tk+1}(\underline{x})\ \underline{g}_m{}^{k+1}(\underline{x}) - \underline{g}_m{}^{tk}(\underline{x})\ g_m{}^{k}(x)]$$

Dropping the argument $\underline{x}$, the proof will be done for related scalars with similar results as if it was done with vectors [18].

<u>Case a</u> : $v^k = 0$ and $g \leq 0$ (end of a cycle)

$$g_m{}^k = 0\ ,\ v^{k+1} = 0 \text{ and } g_m{}^{k+1} = 0$$

$$\text{Then : } v^{k+1} - v^k = 0$$

<u>Case b</u> : $v^k = 0$ and $g^k > 0$

$$g^k{}_m = g^k\ ,\ v^{k+1} = 0 + 2 * P * g^k > 0 \text{ and } g^{k+1}{}_m = g^k$$

$$\text{Then : } v^{k+1} - v^k = 2 * P * g^{k2} > 0$$

<u>Case c</u> : $v^k > 0$ and $g^k - v^k / (4 * P)$

$$g^k{}_m = g^k\ ,\ v^{k+1} = v^k - 2 * P * g^k \text{ and } g_m{}^{k+1} = g^k$$

$$\text{Then : } v^{k+1} - v^k = 2 * P\ g^{k2} > 0$$

<u>Case d</u> : $v^k > 0$ and $- v^k / (4 * P) > g^k - v^k / (2 * P)$

$$g^k_m = g^k \ , \ v^{k+1} = v^k - 2 * P * g^k \ \text{and}$$

$$g^{k+1}_m = - v^{k+1} / (2 * P)$$

Then : $v^{k+1} - v^k = - v^{k2} / (2 * P) - 2 * g^k *$

$$v^k + 2 * P * g^{k2} > 0$$

<u>Case e</u> : $v^k > 0$ and $g^k < v^k / (2 * P)$

$$g^k_m = v^k / (2 * P) \ , \ v^{k+1} = 0 \ \text{and} \ g^{k+1}_m = 0$$

Then : $v^{k+1} - v^k = v^{k2} / (4 * P) > 0$

The outline of the method may be summarized in the following steps:

a) The initial constrained objective function is replaced by a series of unconstrained augmented functions.

b)In each cycle, the unconstrained functions are minimized with relation to <u>x</u>, holding <u>u</u> and <u>v</u> constants. The solution is then used for the starting point of the next cycle.

c)The multipliers are updated in accordance with the rules described above.

d)The process stops when a convergence of the vectors <u>x</u>, <u>u</u> and <u>v</u> is obtained.

CHAPTER 3

METHODOLOGY AND IMPLEMENTATION

3.1 Introduction

Generally, in optimization problems, the set of variables regarding the physical properties is disjoint from the set that guarantees compatibility of the deformed structure. This implies a cycle of analysis-optimization operations.

As described in the first chapter, general structural optimization techniques include analysis and design stages, most of the time done recursively, to achieve the desired results. Exceptions were made to small or very specific problems, where particular methods were developed [22].

The concept behind the procedure implemented in this work is not limited to planar frames. It is a general formulation for the optimization of any structure with a linear and elastic behavior submitted to static loads. It integrates analysis and design in the optimization process.

This method is based on the unconstrained optimization using equilibrium equations as equality constraints in the

Augmented Lagrangian function. The intention is to provide a good final solution and obtain a better convergence rate, avoiding the separation of the phases, therefore with greater efficiency. It is a global procedure where the satisfaction of the equilibrium conditions and the optimal constrained frame are found simultaneously.

3.2 Description of Formulation

The specific problem is to minimize the weight of planar frames with linear elements submitted to static loads. The behavior of the frame is assumed linear and elastic. The objective function is the weight of the structure. The design variables are the areas and inertias of each element and the displacements of the global degrees of freedom of the structure. The beam element considered has six degrees of freedom, three for each node. The number of degrees of freedom for each joint are three, horizontal and vertical displacements and rotation.

There are additional constraints imposed to the physical properties that must have positive value. The behavior of the structure was constrained imposing displacement limits for the global degrees of freedom. These generated the set of the inequality constraints.

The compatibility of displacements is assured by the equilibrium between the vector of the external loads and the internal forces. This is given by the product of the stiffness matrix and the vector of displacements, creating the set of the equality constraints to be satisfied.

The method of the Augmented Lagrangian function is used to create an integrated formulation of this problem. The Lagrangian function was then optimized as an unconstrained function using the Pattern Search method. The step by step description of this method is presented in subchapter 3.3. The statement of the problem may be given for a given a planar frame structure with a fixed topology and

- $\underline{n}$ structural elements,
- $\underline{m}$ global degrees of freedom,
- $\underline{R}$ vector of static external loads,
- $\underline{D}$ vector of limit for the displacement vector $\underline{m}$,

one can define the following:

a) variable design vector

$$\underline{x}\ (x_1, x_2, .., x_{2n}, x_{2n+1}, .., x_{2n+m}),\ \text{where}$$

x_k, k odd , k = 1,3,....,2n-1 -- area of element (k+1)/2

x_j, j even , j = 2,4,...,2n ---- inertia element j/2

x_i, i = 2n+1,2n+2,..,2n+m --- global displacements

b) objective function

$$f(\underline{x}) = \underline{L}_p^{\,t} * \underline{x}_k,\ k = 1,3,..,2n-1,$$

where k is defined as in a), p = 1,2,...,n and $\underline{L}_p$ is the vector defining the length of the $\underline{n}$ elements.

c) equality constraints

$\underline{h}(\underline{x}) = [\ \underline{K}\ ]\ \underline{x} - \underline{R},\ i = 2n+1, 2n+2, .., 2n+m,$

where subscript i is defined as in a) and [$\underline{K}$] is the global stiffness matrix.

d) inequality constraints

$\underline{g}(\underline{x}) = \underline{x} - \underline{D} \leq 0,\ i = 2n+1, 2n+2, ..., 2n+m,$

where i is the same as in a).

e) Augmented Lagrangian function

$$L\ (\ \underline{x}, \underline{u}, \underline{v}\ ) = f(\underline{x}) + \underline{u}^t\ \underline{h}(\underline{x}) + P\ \underline{h}^t(\underline{x})\ \underline{h}(\underline{x}) + \underline{v}^t\ \underline{g}_m(\underline{x}) + P\ \underline{g}^t{}_m(\underline{x})\ \underline{g}_m(\underline{x}),$$

where $\underline{g}_m(\underline{x})$ is defined as max { $\underline{g}(\underline{x})$, $- \underline{v}$ / (2 * P) } and $\underline{u} \geq 0$.

3.3 Step by Step Description of Pattern Search Method

Given parameters alp, fcinc, fcdec and maxi, set the initial failure counter iter to zero.

Step 1. Choose a starting point $\underline{x}^0 = (\ x^0{}_1, ..., x^0{}_n\ )$.

Step 2. Set i = 1.

Step 3. Calculate a new design point $\underline{x}^1 = \underline{x}^0 + alp\ x^0{}_i$. Set iter = iter + 1. If iter > maxi go to step 6.

Step 4. If $L(\underline{x}^1) < L(\underline{x}^0)$, continue in the same presented. Appendix A has the complete listing of the computer code. Appendix B has a detailed description of the format of input data, Table B.1, and an example of the output data, Table B.2.

3.4.2 Main Program: Princi

This program calls subroutines Datini and Optimi.

The main function of the program is to input the initial data. It begins with the general information about the structure: number of elements, number of global degrees of freedom, number of displacement constraints, module of elasticity and the number of degrees of freedom per element. It reads the behavior properties of the frame: external forces, displacement constraints, length and direction cosines for each element and the assembly location matrix for the global degrees of freedom. It follows with the parameter options for the optimization cycles: maximum number of iterations per cycle of unconstrained minimization, penalty factor, factor of increase for the penalty factor, parameter for the control of convergence, constant for the perturbation of the pattern search method, parameters for the decrease and increase of the search, the initial values of the design variables and the tolerance to close each iteration cycle with the same lagrangian multipliers.

At the end of the process, it writes to a data file the final results: total number of iterations, value of the Lagrangian function, the values of the design variables, values of the equalities and inequalities constraints.

3.4.3 Subroutine Datini

This subroutine is called by program Princi. The values of the lagrangian multipliers and of scaling factors for the constraints and the objective function are initialized. Also the parameters that are kept constant during the procedure and that also depend on the input data are derived on this subroutine.

3.4.4 Subroutine Optimi

This subroutine is called by Princi and calls Hoojee and Lagfun. The subroutine begins with the scaling of the objective function, inequality and equality constraints. It updates the lagrangian multipliers and the penalty factor. This updating is made after the unconstrained minimization of the Lagrangian function.

It checks if the values of the equalities are less than a relative value of the maximum external force. In the affirmative case, it returns to main program and the program stops running.

3.4.5 Subroutine Hoojee

This subroutine is called by Optimi and calls Lagfun. The subroutine performs the procedure prescribed in the Pattern Search or Hooke and Jeeves method. It is a zero order method based on a cycle of searches along the directions defined by the design variables. At the end of each cycle of search on the set of design variables, it tries a pattern move along a line defined by the initial and the final points. This exploratory move is regulated by some heuristic rules that seek to accelerate the convergence.

At the end of each cycle it checks if the change in the augmented lagrangian function, was less than the imposed tolerance.

3.4.6 Subroutine Lagfun

This subroutine is called by Optimi and Hoojee and calls Equcon, Inecon and Valobf. This subroutine computes the value of the Lagrangian function for each vector $\underline{x}$ in accordance with the formulation presented in subchapter 2.3, after obtaining the values of the objective function, equality constraints and inequality constraints.

3.4.7 Subroutine Equcon

This subroutine is called by Lagfun and calls Glosti. In each evaluation of the Lagrangian function, calculates the vector whose components are the product of each row of the global stiffness by the displacement vector. Then, it finds the value of each equality constraint. This is equal to each component of the vector obtained by subtracting the vector of the external forces from the vector obtained in the multiplication.

3.4.8 Subroutine Inecon

This subroutine is called by subroutine Lagfun. It determines for each global degree of freedom that is restrained, the difference between the actual displacement and the respective imposed limit.

3.4.8 Subroutine Valobf

This subroutine is called by subroutine Lagfun. At each evaluation of the Lagrangian function it evaluates the volume of material in the frame.

3.4.9 Subroutine Glosti

This subroutine is called by subroutine Equcon. The subroutine uses a closed form to evaluate, for each element, the element global stiffness matrix.

3.4 Program Structure

The diagrams of the program and subroutines of the implemented code are presented on figures 3.1 trough 3.7.

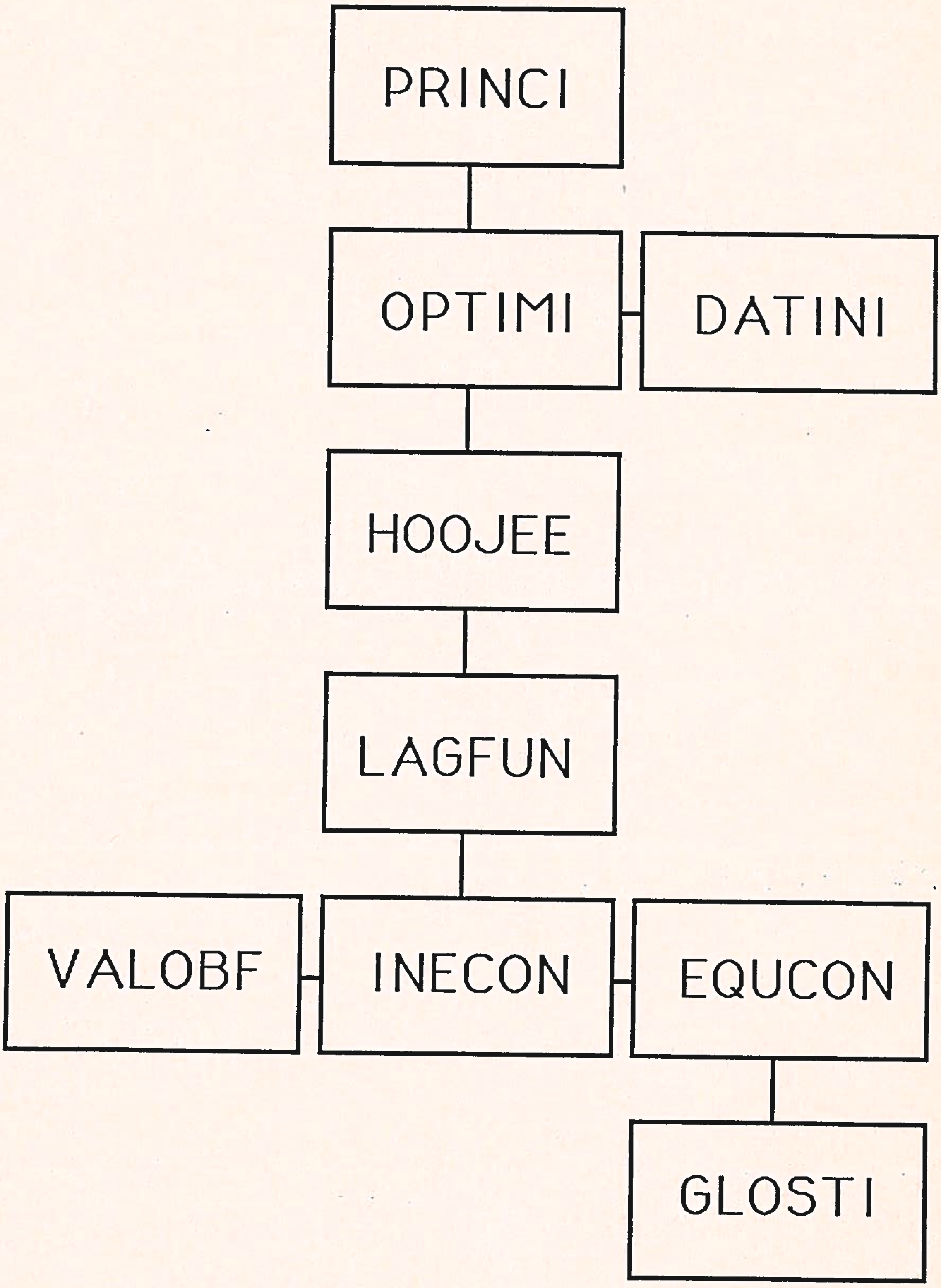

Figure 3.1 Structure of the computer program.

Read number of elements, number of equatilities, number of inequalities, number of iterations, modulus of elasticity and number of degrees of freedom :
N, IQH, IQG, E, ND

Read external global forces **R : (1,IQH)**

Read displacement constraints **D : (1,IQG)**

Read length and direction cosines for each structural element **CL, COS1, COS2 : (1,N)**

Read location matrix for all elements
LM : (1,N (1,ND))

Read maximum number of cycles and penalty factor : **NUMCY, RP**

Read increase factor, decrease factor and update constant for penalty factor : **FCINC, DECFC, GA**

Read perturbation factor and tolerance for convergence : **ALP1, TOL**

Read initial guesses for design variables
X : (1, 2 * N + IQH)

Write input data to file : **FINRES**

Call subroutine : **DATINI**

Call subroutine : **OPTIMI**

Write final data to file : **FINRES**

Figure 3.2 Main program: Princi.

Initialize lagrangian multipliers for equality constraints CLAH : (1,IQH)

Initialize lagrangian multipliers for inequality constraints CLAG : (1,IQG)

Initialize scaling factors for equality constraints CH : (1,IQH)

Initialize scaling factors for inequality constraints CG : (1,IQG)

Initial scaling factor for objective function : CV = 1.0

Evaluate total value of equality constraints
Call subroutine : EQUCON

Do for K = 1,IQH

TVAH = TVAH + CLAH * VAH * CH + RP * VAH * CH * VAH * CH

Evaluate total value of inequality constraints
Call subroutine : INECON

Do for K = 1,IQG

PSI = min [VAG * CG , CLAG / (2 * RP)]
TVAG = TVAG + PSI * (CLAG + RP * PSI)

Call subroutine : VALOBF

TVAG = CV * VOF + TVAH + TVAG

Figure 3.3 Subroutines Datini and Lagfun.

Call subroutine LAGFUN

Scale equality constraints :
DO for K = 1, IQH
    CV = 1 / || VAH ||

Scale inequality constraints :
DO for K = 1, IQG
    CG = 1 / || VAG ||

Scale objective function : CV = 1 / VOF

Control the error in equality constraints and number of loops
QUOT = max || VAH(K) / R(K) || for K = 1, IQH
While QUOT > 0.0025 or number of loops < 10
DO

    Call subroutine HOOJEE

    Update lagrange multipliers for equality constraints
    DO for K = 1, IQH
        CLAH = CLAH + 2 * RP * VAH * CH

    Update lagrange multipliers for inequality constraints
    DO for K = 1, IQG
        PSI = min [CLAG / (2 * RP) , VAG]
        CLAG = CLAG - 2 * RP * PSI

    Update the penalty factor : RP = RP * GA

    LOOP = LOOP + 1

Figure 3.4 Subroutine Optimi.

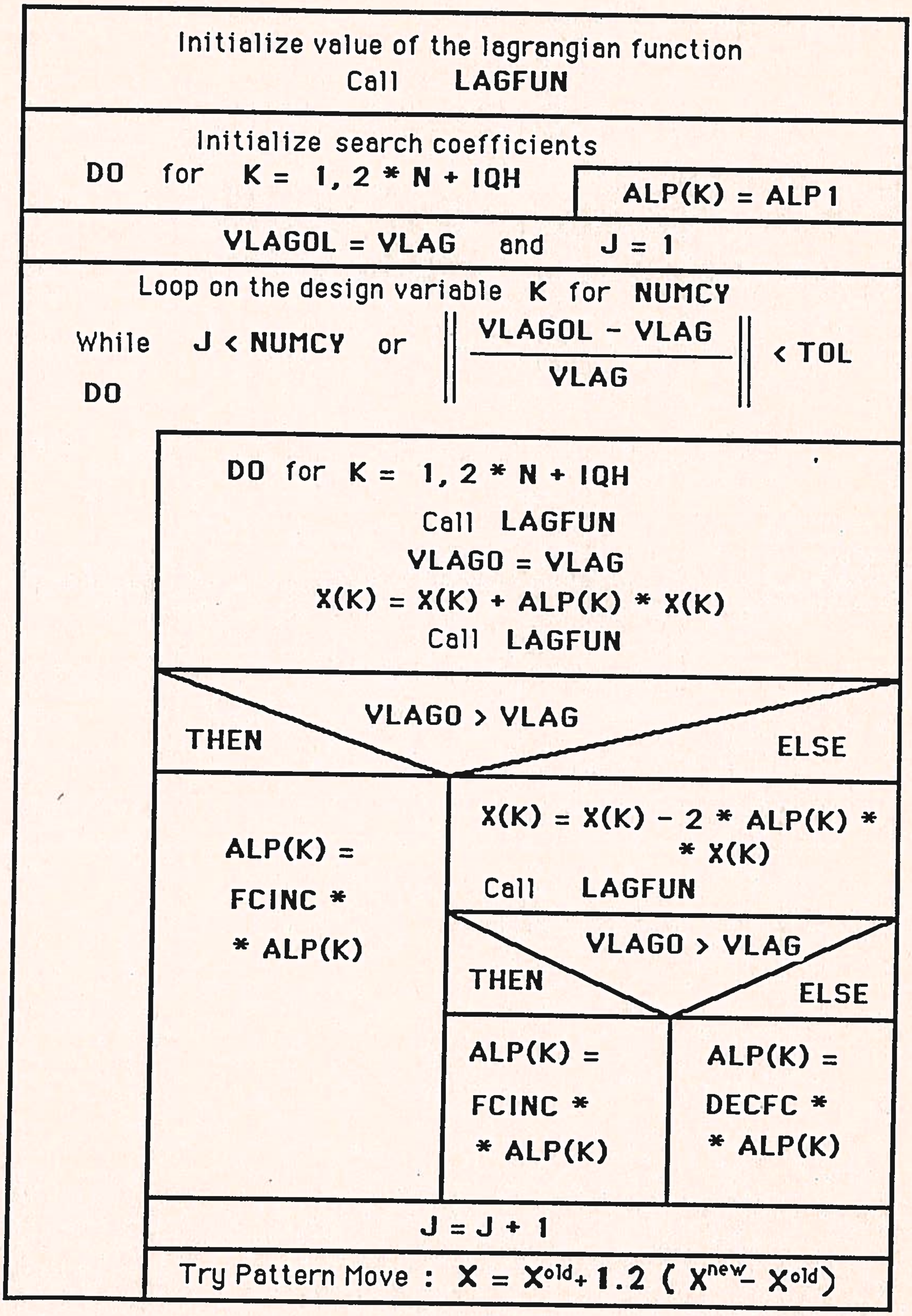

Figure 3.5 Subroutine Hoojee.

```
Loop over all elements for global stiffness evaluation
DO   for   J = 1, N
    Call subroutine   GLOSTI
    DO   for   L = 1,ND  :  K = LM ( L,J )
        DO   for   LL =  1, ND
                 M = LM ( L,J )
            VAH(K) = VAH(K) + CK( L,LL ) *
                            * X( 2 * N + M )
DO   for   K = 1, 2 * N + IQH
        VAH(K) = VAH(K) -  R(K)
```

```
DO   for   K = 1, IQG
        VAG(K) = D(K) - X( 2 * N + K )
```

```
DO   for   K = 1, N
        VOF = VOF + X( 2 * K - 1 ) * CL(K)
```

Figure 3.6 Subroutines Equcon, Inecon and Valobf.

```
A = E * X( 2 * J ) / CL(J) ** 3
B = X( 2 * J - 1 ) * CL(J) ** 2 / X( 2 * J )
G1 = A * ( B * COS1(J) ** 2 + 12 * COS2(J) ** 2 )
G2 = A * COS1(J)* COS2 (J)* ( B -12 )
G3 = A * ( B * COS2 (J)** 2 + 12 * COS1 (J) ** 2)
G4 = - 6 * A * CL(J) * COS2(J)
G5 = 6 * A * CL(J) * COS1 (J)
G6 = 4 * A * CL(J) ** 2
G7 = 2 * A * CL(J) ** 2

CK(1,1) = G1        CK(1,2) = G2        CK(1,3) = G4
CK(1,4) = -G1       CK(1,5) = -G2       CK(1,6) = G4
CK(2,2) = G3        CK(2,3) = G5        CK(2,4) = -G2
CK(2,5) = -G3       CK(2,6) = G5        CK(3,3) = G6
CK(3,4) = -G4       CK(3,5) = -G5       CK(3,6) = G7
CK(4,4) = G1        CK(4,5) = G2        CK(4,6) = -G4
CK(5,5) = G3        CK(5,6) = -G5       CK(6,6) = G6

        MATRIX  CK  IS  SYMMETRIC
```

Figure 3.7 Subroutine Glosti.

CHAPTER 4

TESTING EXAMPLES

4.1 Introduction

The methodology and the computer program described in the previous chapters need a practical evaluation of their performance. Some simple structures with different loading cases were used to assess the validity of the assumptions made. The data results are presented in Appendix B and the analysis and discussion are presented in the following subchapters.

4.2 Procedure Description

The examples chosen to test the program and its performance were a cantilever beam and a one bay rectangular frame. The reason for this choice was that the optimality conditions were simple to determine in the first case and the behaviors of both structures easy to assess and understand. The loadings for the cantilever beam were external forces acting individually on each global degree of freedom and a combination of the three forces. For the frame, three load cases were tested. One was symmetric: a vertical uniformly distributed load on the horizontal element. One of the asymmetric loadings was an horizontal load applied axially on the horizontal element. Finally, the third loading case was the combination of the previous two.

The displacement constraints imposed on the behavior of the structures varied for the examples tested. Regarding the cantilever beam, as it is an isostatic structure, the imposed displacements will condition the physical properties in a explicit form. For the frame, the results were compared to exact structural analysis. The analysis was made using the direct stiffness method for a frame with the cross dimensions obtained in the optimization process. The displacements obtained this way were compared with the constraints and the values obtained in the optimization procedure. The units used for the tests were kilopounds for the forces and inches for the dimensions. The material chosen was steel, with a module of elasticity of 30,000 kilopounds per square inch. In both structures, to avoid the possibility of having an area or moment of inertia with zero or negative value, a minimum of 0.1 square inch and 0.1 inch elevated to the fourth power, respectively, were established.

4.3 Cantilever Beam

The figure 4.1 indicates the geometric definition as well as the definition of the global degrees of freedom. For the cases tested, the global external forces are defined precisely as the global displacements.

The results of the four cases tested are presented with the initial, the final results and the exact solution in Tables B.3, B.4, B.5 and B.6. The optimal exact solution for some values is not available, since it is not dependent of the objective function. For instance, on the second loading case, Table B.4, there is no optimal area since the optimization iteration is going to be made around the inertia of the element. In these cases, the expected theoretical value in the tables is undefined.

In Table B.3, with the final data from the case with axial load and limits for all degrees of freedom, it is clear that the optimal results were found and equilibrium was obtained. The area determined was the minimum value necessary to have the maximum allowable displacement under the axial external force. The inertia takes the minimum value prescribed, because it is not conditioned by any constraint or by the objective function.

Tables B.4 and B.5, with the results of vertical loading and applied moment, respectively, having constraints on all

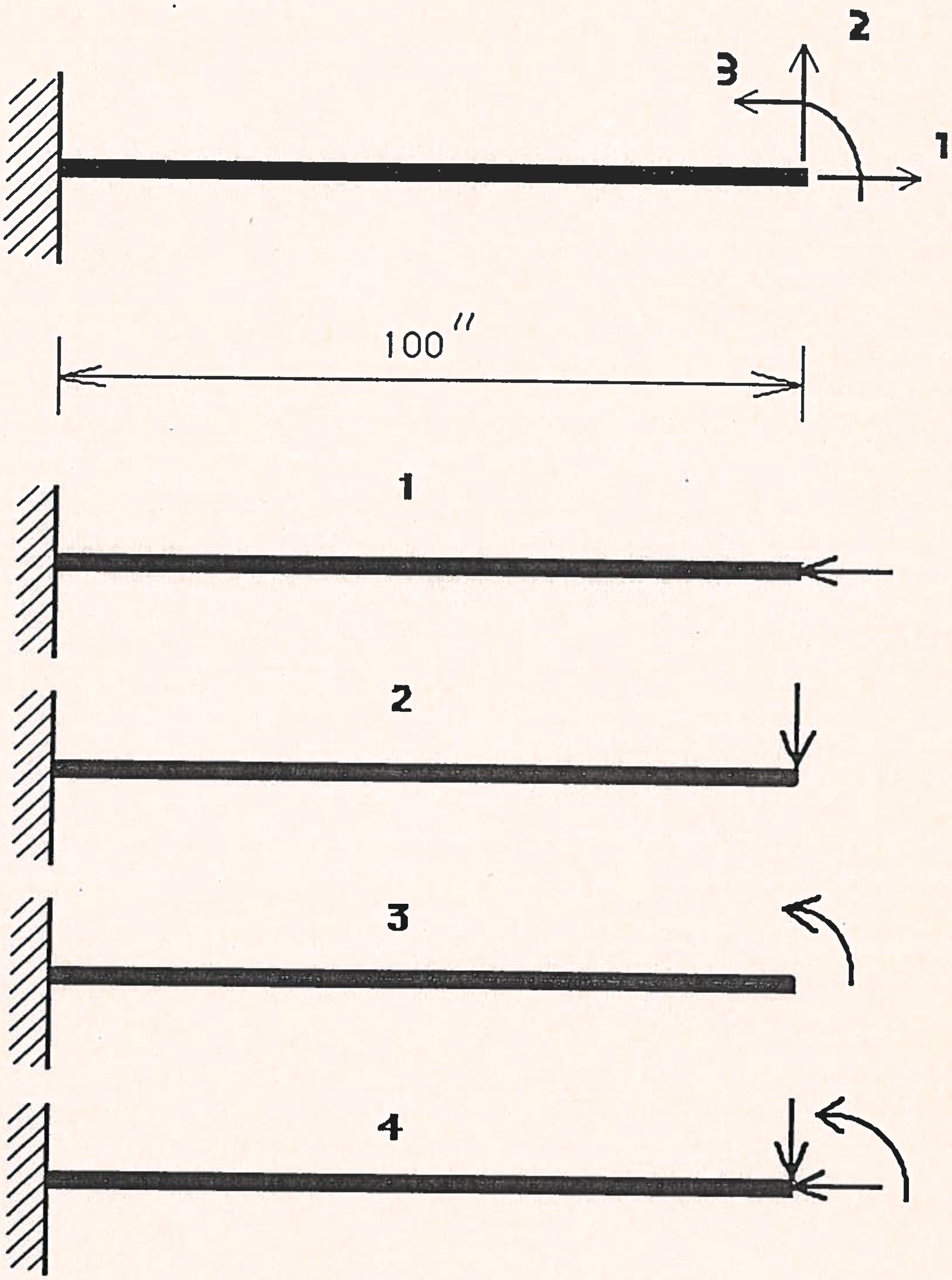

Figure 4.1 Cantilever beam and load cases.

degrees of freedom, show without doubt that the method guarantees equilibrium, represented by the equality constraints. However, the minimum value of inertia is not found because, as design variables, the moment of inertia doesn't participate in the objective function.

Table B.6, with the data of the case with the three types of external loading and constraints on all degrees of freedom, presents final results fulfilling all requirements: the minimum of the objective function has been found; the final displacements are bounded by the preimposed limits; the equality constraints, guaranteeing equilibrium, are also satisfied.

As conclusion of the analysis of these results, the final values are excellent. For the cases where the moment of inertia is not connected with the Augmented Lagrangian function, except for the inequality constraints, the value founded nevertheless satisfies both equilibrium and displacement limitations.

4.4 One Bay Frame

The geometric definition of the frame tested is presented in the figure below. Also, the definition of the global displacements, related to the global degrees of freedom, and global forces are indicated in figure 4.2.

This one bay frame has been tested with three loading cases. The results from these tests are presented in Tables B.7, B.8, B.9 and B.10 of Appendix B. As the optimal solutions couldn't be derived explicitly, like in the cantilever beam, a different methodology was used to verify the final results.

To evaluate if the final areas were the optimal, the program was passed again using as starting points the results to verify if there was a significant improvement on the total volume of the frame.

Regarding the equilibrium constraints, the final values of the areas and inertias were used in an exact analysis of the frame. The program Stan using direct stiffness method, was used. It verified if the displacements from this analysis matched the final displacements found in the optimization procedure.

The displacement constraints were straightforward to check once found the final values of the displacements.

Tables B.7 and B.8, have the results of the first and second loading case, respectively. The comparison with the exact structural analysis show that equilibrium is obtained. Sequential runs didn't present any significant decrease in the total volume of the structure.

Table B.9 has the results of the frame with the third loading case. It shows clearly no equilibrium when the exact structural analysis is performed, although the inequality constraints are satisfied. Table B.10 presents the final results obtained using as starting data the final values from the previous run as shown in the preceding Table. With these physical properties, equilibrium is obtained. Sequential runs don't have a significant improvement of the total volume of the structure.

In conclusion, the computer program performed very well for the one bay frame, with the exception of the third loading case. However, with a better starting point, in this case the data from the previous attempt, the efficiency was acceptable.

4.5 Analysis and Discussion

As can be concluded from the comparisons between the results from the computer runs and from the exact structural analysis, the results are very encouraging. For the beam, the results are very close and are comparable to the exact solutions. The results of the frame test, can only be compared to the accuracy of the exact analysis that was made with the physical properties found in the optimization cycles.

Although only a small number of tests are presented, they represent the final product of a series of computer runs. These will create a enormous volume of information. The results of Tables B.3 trough B.10 are sufficient to illustrate the performance of the method. However, some conclusions may be drawn from this set of runs that is omitted. These conclusions are briefly described below.

4.5.1 Penalty Factor

The penalty factor is kept constant during the process. This is due to the reason that the objective function and constraints are initially scaled to a common value. The value chosen was ten, one order of magnitude bigger than the initial values of constraints and objective function. The update scalar for the penalty factor was one. When large values of the penalty factor were tried, in most cases, this caused slow or no convergence. This is a common source of failure in optimization problems, because it often creates an ill conditioned problem.

4.5.2 Scaling

The current method to use for scaling still needs further study. The various expressions that are compared have very different orders of magnitude. Scaling is imperative if convergence is desired without the dominance of a single constraint, or type of constraints. Several attempts without scaling were a complete failure. The adopted method consists of taking the scale factors as the inverse of the values of the equalities, inequalities and objective function. This causes the equations and inequalities to have the same order of magnitude at the beginning of the optimization. Another method that was tested was scaling through the used gradients. The gradients were computed numerically using finite differences, but the performance for the tests was slower than the method chosen.

4.5.3 Initial Guesses

Some of the cases tested for the beam were done with initial physical guesses close to the exact ones. The results were no better than with random numbers or unity values. This is probably due to the simplicity of the tested structure. It is undoubtedly true that initial values closer to the final values will give a better scaling. Unfortunately there isn't a explicit way of obtaining the optimal values of the one bay frame, except by trial and error or using the values from the previous attempt like is shown in Tables B.9 and B.10. Those are a good test for the importance of the initial values for the physical properties. Regarding the displacements, as the limits are known in advance there is no reason why the initial values shouldn't be correct.

4.5.4 Perturbation Factor

During the search procedure, the value of the design variable may be very small compared to the optimal one. Then, the value of scalar that causes the perturbation of the design variable value, perturbation factor, is important to obtain convergence. If it is too small and the Lagrangian function has a small slope at that point, it will be difficult to move in the correct direction. Several tests were run to check these values. It seems that as the Augmented Lagrangian function is well conditioned, the several values tested didn't create significant changes on the behavior of the function. An exception is made to the number of iterations of the unconstrained minimization cycle. The larger the perturbation factor, the higher the number of iterations per design variable needed.

CHAPTER 5

CONCLUSIONS AND SUGGESTIONS

5.1 Final Considerations

The main conclusion from the work described is that the hypothesis of using the Augmented Lagrangian Multipliers method for simultaneous analysis and optimal design, has been verified. Also, the principal goal of obtaining a method that is as general as possible within the structural optimization, has been respected. The results of the two tested structures, cantilever beam and one bay frame, submitted to several loading patterns prove that assumption.

The cantilever beam was a good structure to test because the analytical solution of the optimum was easy to evaluate. The reason was that the convergence to the minimum volume of the structure could be derived in an explicit form. Another advantage of this structure was that any changes in the parameters of the program were promptly examined and evaluated, like the several different values of the penalty factor or number of cycles. The final data from the test of the cantilever beam, as described in Chapter 4, show clearly that convergence and equilibrium were met for all different loading cases. At the end, the equality constraints were almost zero, as required, the displacements were within the imposed limits and the minimum material was always obtained.

The one bay frame tested with the three loading cases, gave final results that verified the equilibrium constraints, satisfied the displacement constraints and no significant diminishment of the total volume was found with sequential runs, as presented in chapter 4. The optimization and analysis performed very well for the two first loading cases. The minimum volume of the structure was found and constraints were satisfied. However, on the third loading case, to obtain the final equilibrium of the structure a second run of the program was needed, using the final results of the first one. This example of the one bay frame tested with the third loading case, leads to the conclusion that improvements in the program are necessary for more complicated loading cases, due certainly to numerical problems or bad initial guesses for the design variables.

Nevertheless, the results are promising, giving a substantial basis for further research. Possible improvements are described in the following subchapters. The first set of improvements are those that won't change the basic structure of the computer program, although increasing the performance. The second group are major changes that allow the extension of the computer program to other features.

5.2 Improvements to the Actual Structure of the Program

The main task proposed is related to the unconstrained minimization search. The method adopted was a zero order technique, where only the pattern search movement uses information from previous attempts. Even so, this movement was a very empirical one requiring extra calculations of the function value. It didn't use any information about the behavior of the objective function or any of the constraints. The test examples had a small number of variables that could be handled by this minimization technique. For frames where the number of design variables is significant, the method will probably need some improvement. The suggestion is to use more efficient minimization methods of first or second order, with a quicker rate of convergence and less function evaluations. These powerful techniques could be used individually or in an association of the several techniques available. Of course, the derivatives of the objective function and constraints will have to be calculated numerically, but that can be done with sufficient accuracy if proper numerical methods are used.

Another critical aspect is the scaling of the objective function, equalities and inequalities. After running a long series of test on various scaling techniques, the type of scaling adopted was the one that performed best. Like the previous proposition, scaling using first or second order information about the behavior of those functions, may bring a significant improvement to the optimization scheme.

The objective function adopted, considered only the weight of the structural elements, is a function of the cross area. The inertias of the elements were independent of the minimization process. This is not a real situation and a correlation between both types of design variables must be established. For the case of rectangular sections the relation is very easy to implement. For I, T or U shape elements only approximate formulas may be used, unless a database is installed. The objective function will then be explicitly a function of both types of design variables.

Another enhancement, involves the speeding up of the computational operations. This can be done by changing only, at each iteration step, the values of the objective function, equalities and inequalities that are related to the changing design variable. This will save computational time involved in the evaluation of all the other equalities and inequalities whose values don't change.

Another improvement is related with the initial guess of the design variables. The example of the third load case of the one bay frame, is very clear. The second run used initial guesses of the design variables that were the results of the first run. Not only was the convergence criteria satisfied, but also the number of iterations needed to achieve the solution were less than the first run. Good initial values will improve the performance of the method if they satisfy simultaneously the equilibrium conditions and the displacement constraints. This can be done with a previous exact analysis of a frame where the structural elements will have sufficiently large dimensions in order to satisfy the displacement constraints.

5.3 Major Additions to the Program Structure

Further enhancements are possible to this technique. The most important, and also the main concern in the origin of this formulation, is the possibility of using any type of structural element with the very same optimization technique. For instance, the optimization of a set of structural finite elements can be performed simply with the substitution of the subroutine for the generation of the global stiffness and minor changes in the rest of the program. These changes will have to do with the dimensions and format of input and output. Similarly, the same can be made for truss elements. This allows the use of any combination of these different types of elements.

Another significant addition is the implementation of stress constraints. These will create another criterion for the minimization scheme. This will imply the recovery of the element forces and the evaluation of maximum stresses along each element. Instead of the constraints based only on allowable maximum displacements, we can condition the performance of the structure to the maximum stresses on the structural material. The inclusion of this type of constraints allows a more realistic approach, since a great part of structural designs are based on the satisfaction of stress constraints limitations. This type of constraint may be used alone or with the displacement constraints. The program has been created in such a way that this addition will be easy to implement.

Other major modifications need to be studied. The first one is the addition of elements that represent the performance of reinforced concrete. It should include the ductility of the element and the nonlinear properties of the concrete and reinforcing steel. The second one is the consideration of dynamic loads, more specifically earthquake loadings. These improvements are the subject of a research proposal to the National Science Foundation. The transcription of that research proposal is presented in the Appendix C.

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APPENDIX A

COMPLETE PROGRAM LISTING

```
      PROGRAM  PRINCI
      IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
      CHARACTER*40 TITLE
      DIMENSION X(100), R(40), CL(30), COS1(30), COS2(30),
     *  D(20), CLAH(40), CLAG(20), LM(6,30), CK(6,6), VAG(20),
     *    VAH(40), CH(40), CG(40), ALP(100), XOL(100)
      COMMON /PARR/ DECFC,FCINC,PHLIM,CV,ALP1,E,RP
      COMMON /PARI/ TOL,ITER,NUMCY,NITER,GA,IQH,IQG,ND,N,NTOT
      OPEN ( 7,FILE='data',form='formatted' )
      REWIND 7
      OPEN ( 8,FILE='finres',form='formatted' )
      REWIND  8
C
C-----------------READ NAME OF PROBLEM----------------------------
C
      READ ( 7,120 ) TITLE
C
C---READ # ELE,# EQU,# INE,# ITE,MODULUS ELAS,# OF D. FREEDOM---
C
      READ ( 7,* ) N, IQH, IQG, NITER, E, ND
      NTOT = N + N + IQH
C
C-----------------READ THE EXTERNAL FORCES-----------------------
C
      DO  100  K = 1,IQH
         READ ( 7,* ) R(K)
  100 CONTINUE
C
C---------------READ THE DISPLACEMENT CONSTRAINTS---------------
C
      DO  200  K = 1,IQG
         READ ( 7,* ) D(K)
  200 CONTINUE
C
C----------READ LENGTH AND DIRECTION COSINES FOR EACH ELEMENT---
C
      DO  300  K = 1,N
         READ ( 7,* ) CL(K), COS1(K), COS2(K)
  300 CONTINUE
C
C--------------READ LOCATION MATRIX FOR ALL ELEMENTS------------
C
      DO  400 K = 1,N
         READ ( 7,* ) (LM(KK,K), KK = 1,ND)
  400 CONTINUE
C
C-------------READ MAXIMUM NUMBER OF CYCLES AND PENALTY FACTOR--
C
      READ ( 7,* ) NUMCY, RP
C
C------------INPUT INCREASING FACTOR,DECREASING FACTOR,GAMMA----
C
```

```
      READ ( 7,* ) FCINC, DECFC, GA
C
C-------------INPUT PERTURBATION FACTOR AND TOLERANCE -----------
C
      READ ( 7,* ) ALP1, TOL
C
C-------------INPUT INITIAL GUESSES FOR VARIABLES---------------
C
      N2 = N + N
      N21 = N2 + 1
      NTOT = N2 + IQH
      READ ( 7,* )  ( X(I), I = 1,N2 )
      READ ( 7,* )  ( X(I), I = N21,NTOT )
C
C-------------------WRITE ALL INPUT DATA-------------------------
C
      WRITE ( 8,190 ) TITLE
      WRITE ( 8,110 )
      WRITE ( 8,130 ) N
      WRITE ( 8,140 ) IQH
      WRITE ( 8,150 ) IQG
      WRITE ( 8,160 ) NITER
      WRITE ( 8,170 ) E
      WRITE ( 8,180 ) ND
      WRITE ( 8,240 )
      DO  500  K = 1,IQH
         WRITE ( 8,250 ) K, R(K)
  500 CONTINUE
      WRITE ( 8,260 )
      DO  600  K = 1,IQG
         WRITE ( 8,270 ) K, D(K)
  600 CONTINUE
      WRITE ( 8,350 )
      DO 700  K = 1,N
          WRITE ( 8,360) K, CL(K), COS1(K), COS2(K)
  700 CONTINUE
      WRITE ( 8,380 )
      WRITE ( 8,390 )
      DO  750  K = 1, N
          WRITE ( 8,370 ) K, ( LM ( I,K ), I = 1,6 )
  750 CONTINUE
      WRITE ( 8,640 ) NUMCY
      WRITE ( 8,650 ) RP
      WRITE ( 8,730 ) FCINC
      WRITE ( 8,740 ) DECFC
      WRITE ( 8,760 ) GA
      WRITE ( 8,840 )
      DO  800  I = 1,N
         NE = 2 * I
         NO = NE - 1
         WRITE ( 8,850 ) I, X(NO), X(NE)
  800 CONTINUE
```

```
      WRITE ( 8,860 )
      DO   900   I = N21, NTOT
         K = I - N2
         WRITE ( 8,870 ) K, X(I)
  900 CONTINUE
C
C-----------------CALL SUBROUTINE FOR DATA INITIALIZATION--------
C
      CALL  DATINI ( CLAH, CLAG, CH, CG )
C
C-----------------CALL SUBROUTINE OPTIMIZATION-------------------
C
      CALL  OPTIMI ( VLAG, R, X, CL, COS1, COS2, LM, D, CLAH,
     *        CLAG, VAH, VAG, CK, CH, CG, ALP, XOL )
C
C-----------------WRITE FINAL DATA-------------------------------
C
      WRITE ( 8,880 )
      WRITE ( 8,890 ) ITER
      WRITE ( 8,910 ) VLAG
      WRITE ( 8,840 )
      DO  1000  K = 1,N
      NE = 2 * K
      NO = NE - 1
         WRITE ( 8,850 ) K, X(NO), X(NE)
 1000 CONTINUE
      WRITE ( 8,960 )
      DO 1150  K = N21, NTOT
          I = K - N2
          WRITE ( 8,970 ) I, X(K)
 1150 CONTINUE
      WRITE ( 8,920 )
      DO  1100  K = 1,IQH
         WRITE ( 8,930 ) K, VAH(K)
 1100 CONTINUE
      WRITE ( 8,940 )
      DO  1200  K = 1,IQG
         WRITE ( 8,950 ) K, VAG(K)
 1200 CONTINUE
C
C-----------------OUTPUT FORMATTING------------------------------
C
  110 FORMAT ( //,10X,
     *  '************ INITIAL  VALUES **********',/ )
  120 FORMAT (  A25 )
  130 FORMAT ( /,10X,'NUMBER OF ELEMENTS = ',I3 )
  140 FORMAT ( /,10X,'NUMBER OF EQUALITY CONSTRAINTS = ',I3 )
  150 FORMAT ( /,10X,'NUMBER OF INEQUALITY CONSTRAINTS = ',I3 )
  160 FORMAT ( /,10X,'NUMBER OF ITERATIONS PER CYCLE = ',I5 )
  170 FORMAT ( /,10X,'MODULUS OF ELASTICITY = ',E14.8 )
  180 FORMAT (/,10X,'NUMBER OF GLOBAL DEGREES OF FREEDOM =',I2)
  190 FORMAT ( //,10X,A25,// )
```

```
240   FORMAT ( //,10X,'GLOBAL DEGREE OF FREEDOM',10X,
     *        'EXTERNAL FORCE' )
250   FORMAT ( /,20X,I2,22X,E14.8 )
260   FORMAT ( //,10X,'GLOBAL DEGREE OF FREEDOM',5X,
     *   'DISPLACEMENT CONSTRAINT' )
270   FORMAT ( /,20X,I3,19X,E14.8 )
350   FORMAT ( //,10X,'ELEMENT',8X,'LENGTH',9X,'COS',7X,'SIN')
360   FORMAT ( /,12X,I3,5X,E14.8,3X,F8.5,5X,F8.4 )
370   FORMAT ( /,10X,I3,14X,3I4,3X,3I4 )
380   FORMAT ( //,10X,
     *     'LOCATION MATRIX FOR GLOBAL DEGREES OF FREEDOM ' )
390   FORMAT ( /,10X,' ELEMENT ',10X,' NODE I',10X,'NODE J' )
640   FORMAT ( /,10X,'MAXIMUM NUMBER OF CYCLES = ',I3 )
650   FORMAT ( /,10X,'PENALTY FACTOR = ',E14.8 )
730   FORMAT ( /,10X,'FACTOR OF INCREASE = ',E14.8 )
740   FORMAT ( /,10X,'FACTOR OF DECREASE = ',E14.8 )
760   FORMAT ( /,10X,'PENALTY FACTOR MULTIPLIER = ',E14.8 )
840   FORMAT ( /,10X,'ELEMENT',13X,'AREA',16X,'INERTIA' )
850   FORMAT ( /,15X,I2,8X,E14.8,8X,E14.8 )
860   FORMAT ( /,10X,'GLOBAL DEGREE OF FREEDOM',5X,
     *        'INITIAL GUESS' )
870   FORMAT ( /,20X,I2,18X,E14.8 )
880   FORMAT ( ////,10X,
     *    '************ FINAL VALUES ************',/// )
890   FORMAT ( /,10X,'NUMBER OF ITERATIONS = ',I3 )
910   FORMAT ( /,10X,'VALUE OF LAGRANGIAN FUNCTION = ',E14.8 )
920   FORMAT ( /,10X,'EQUALITY',17X,'FINAL VALUE' )
930   FORMAT ( /,12X,I3,17X,E14.8 )
940   FORMAT ( /,10X,'INEQUALITY',15X,'FINAL VALUE' )
950   FORMAT ( /,12X,I3,17X,E14.8 )
960   FORMAT ( /,10X,'DISPLACEMENT',14X,'FINAL VALUE' )
970   FORMAT ( /,12X,I3,17X,E14.8 )
      STOP
      END
```

```
      SUBROUTINE DATINI ( CLAH,CLAG,CH,CG )
      IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
      DIMENSION CLAH(IQH),
      CLAG(IQG), CH(IQH), CG(IQG) COMMON /PARR/
      DECFC,FCINC,PHLIM,CV,ALP1,E,RP COMMON /PARI/
      TOL,ITER,NUMCY,NITER,GA,IQH,IQG,ND,N,NTOT
C
C---------INITIALIZE  LAGRANGIAN MULT. FOR EQUALITY CONST.------
C
      DO  100  K = 1,IQH
  100 CLAH(K) = 0.0
C
C-------INITIALIZE OF LAGRANGIAN MULT. FOR INEQUALITY CONST.----
C
      DO  200  K = 1,IQG
  200 CLAG(K) = 0.0
C
C---------INITIALIZE SCALING FACTORS FOR EQUALITY CONST.-----
C
      DO  300  K = 1,IQH
  300 CH(K) = 1.0
C
C---------INITIALIZE SCALING FACTORS FOR INEQUALITY CONST.-----
C
      DO  400  K = 1,IQG
  400 CG(K) = 1.0
C
C------------PHYSICAL LIMITS FOR AREA AND INERTIA---------------
C
      PHLIM = 0.1
C
C------------SCALING FACTOR FOR OBJECTIVE FUNCTION--------------
C
      CV = 1.0
      RETURN
      END
```

```
      SUBROUTINE  OPTIMI ( VLAG, R, X, CL, COS1, COS2, LM, D,
     *     CLAH, CLAG, VAH, VAG, CK, CH, CG, ALP, XOL )
      IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
      DIMENSION R(IQH), X(NTOT), CL(N), COS1(N), COS2(N),
     * D(IQG), ALP(NTOT), CLAG(IQG), LM(ND,N), VAG(IQG),
     * VAH(IQH), CH(IQH),XOL(NTOT), CLAH(IQH), CG(IQG)
      COMMON /PARR/ DECFC,FCINC,PHLIM,CV,ALP1,E,RP
      COMMON /PARI/ TOL,ITER,NUMCY,NITER,GA,IQH,IQG,ND,N,NTOT
      I = 0
      ITER = 0
 220   CALL LAGFUN ( VLAG, TVAH, R, X, CL, COS1, COS2, LM, D,
     *  CLAH, CLAG, VAG, CK, VAH, CH, CG, VOF )
C
C----------SCALE EQUALITY CONSTRAINTS--------------------------
C
      DO  530  K = 1, IQH
         CH(K) = DABS ( 1./VAH(K) )
 530  CONTINUE
C
C-----------SCALE INEQUALITY CONSTRAINTS-----------------------
C
      DO  540  K = 1,IQG
         CG(K) = DABS ( 1./VAG(K) )
 540  CONTINUE
C
C-------------SCALE OBJECTIVE FUNCTION-------------------------
C
      CV = 1./VOF
 300  CALL HOOJEE ( TVAH, VLAG, R, VAH, VAG, X, CL, COS1,
     *   COS2, LM, D, CLAH, CLAG, CK, CH, CG, ALP, XOL )
      I = I + 1
      ITER = ITER + 1
C      WRITE ( 8,1500 ) I
C      WRITE ( *,1500 ) I
C 1500  FORMAT ( 'END OF  LOOP = ',I3)
C
C-------------CONTROL MAXIMUM NUMBER OF ITERATIONS------------
C
        IF  ( I.GT.10 ) THEN
          RETURN
        ENDIF
      RP2 = RP + RP
C
C-----------UPDATE LAGRAGIAN MULTIPLIERS  FOR EQUALITY CONS.--
C
      DO  100  K = 1,IQH
        CLAH(K) = CLAH(K) + RP2 * VAH(K) * CH(K)
 100  CONTINUE
C
C-------UPDATE LAGRAGIAN MULTIPLIERS  FOR INEQUALITY CONS.----
C
      DO  200  K = 1,IQG
```

```
            V = VAG(K) * CG(K)
            CLAGK = CLAG(K)
            Z = CLAGK / RP2
            PSI = DMIN1 ( V,Z )
            CLAGK = CLAGK - RP2 * PSI
            CLAG(K) = CLAGK
  200   CONTINUE
C
C-----------UPDATE PENALTY FACTOR-----------------------------
C
        RP = GA * RP2
C
C-----------CONTROL OF ERROR IN EQUALITY CONSTRAINTS----------
C
        CONTE = 0.0
        DO  700  K = 1,IQH
            IF ( R(K).EQ.0 ) THEN
              GO  TO  700
            ENDIF
          QUOT = DABS ( VAH(K) / R(K) )
            IF ( CONTE.LT.QUOT )  THEN
              CONTE = DABS ( VAH(K) / R(K) )
            ENDIF
  700   CONTINUE
           IF ( CONTE.LT.0.0025 ) THEN
             RETURN
           ENDIF
        GO TO 300
        END
```

```
      SUBROUTINE  HOOJEE ( TVAH, VLAG, R, VAH, VAG, X, CL,
     *    COS1, COS2, LM, D, CLAH, CLAG, CK, CH, CG, ALP, XOL )
      IMPLICIT  DOUBLE  PRECISION ( A-H,O-Z )
      DIMENSION X(NTOT), CL(N), COS1(N), COS2(N), LM(ND,N),
     *    D(IQG), CLAH(IQH), CLAG(IQG), R(IQH), VAH(IQH),
     *    VAG(IQG), CH(IQH), CG(IQG), ALP(NTOT), XOL(NTOT)
      COMMON /PARR/ DECFC,FCINC,PHLIM,CV,ALP1,E,RP
      COMMON /PARI/ TOL,ITER,NUMCY,NITER,GA,IQH,IQG,ND,N,NTOT
C
C----------INITIALIZE VALUE OF LAGRANGIAN FUNCTION---------------
C
      CALL  LAGFUN ( VLAG, TVAH, R, X, CL, COS1, COS2,
     *    LM, D, CLAH, CLAG, VAG, CK, VAH, CH, CG, VOF )
      VLAGO = VLAG
      VLAGOL = VLAG
      KL = 0
C
C-----------INITIALIZE OPTIMIZATION COEFFICIENTS-----------------
C
  150 KL = KL + 1
      DO  110  K = 1,NTOT
  110 ALP(K) = ALP1
C      WRITE ( *,1500 ) KL
C      WRITE ( 8,1500 ) KL
C 1500  FORMAT(/,5X,'CYCLE # =',I1,/)
      N2 = N + N
C
C-----LOOP ON VARIABLE # K FOR THE SPECIFIED # OF ITERATIONS----
C
      DO  200  J = 1,NITER
        DO  115  K = 1,NTOT
  115      XOL(K) = X(K)
              DO  100  K = 1,NTOT
                XO = X(K)
                XK = X(K)
C
C----------VALUE OF LAGRANGIAN FUNCTION FOR INITIAL VALUES------
C
  510           CONTINUE
                Z = ALP(K) * XO
                X(K) = X(K) + Z
                  IF ( K.LE.N2 ) THEN
                     X(K) = DMAX1 ( X(K),PHLIM )
                  ENDIF
C
C------------CALCULATE NEW VALUE OF LAG. FUNCTION---------------
C
               CALL  LAGFUN ( VLAG, TVAH, R, X, CL, COS1, COS2,
     *        LM, D, CLAH, CLAG, VAG, CK, VAH, CH, CG, VOF )
                  IF ( VLAGO.GT.VLAG ) THEN
C
C----INCREASE OF VARIABLE WITH SUCCESS IN DECREASING FUNCTION----
```

```
C
                              ALP(K) = ALP(K) * FCINC
                              VLAGO = VLAG
                              XO = X(K)
                              GO  TO  510
                           ELSE
C
C----DIRECTION REVERSED WITH FAILURE IN DECREASING FUNCTION----
C
                              ALP(K) = - ALP(K)
                              X(K) = XO + ALP(K) * XO
                           ENDIF
                           IF ( K.LE.N2 ) THEN
                              X(K) = DMAX1 ( X(K),PHLIM )
                           ENDIF
                           CALL  LAGFUN ( VLAG, TVAH, R, X, CL, COS1,
     * COS2, LM, D, CLAH, CLAG, VAG, CK, VAH, CH, CG, VOF )
                           IF ( VLAGO.GT.VLAG ) THEN
C
C-----INCREASE OF VARIABLE W/ SUCCESS IN DECREASING FUNCTION----
C
                              ALP(K) = ALP(K) * FCINC
                              VLAGO = VLAG
                              XO = X(K)
                              GO  TO  510
                           ELSE
C
C------DECREASE OF VARIABLE W/ FAILURE IN DECREASING FUNCTION--
C
                              ALP(K) = ALP(K) * DECFC
                              X(K) = XO
                           ENDIF
                           X(K) = XO
  100           CONTINUE
  190       CONTINUE
C
C------------ATTEMPT OF PATTERN MOVE--------------------------------
C
            DO  170  K = 1,NTOT
              X(K) = 1.2 * X(K) - 0.2 * XOL(K)
              IF ( K.LE.N2.AND.X(K).LT.PHLIM ) THEN
                 X(K) = PHLIM
              ENDIF
  170       CONTINUE
            CALL  LAGFUN ( VLAG, TVAH, R, X, CL, COS1, COS2,
     *            LM, D, CLAH, CLAG, VAG, CK, VAH, CH, CG, VOF )
            IF ( VLAGO.GT.VLAG ) THEN
               VLAGO = VLAG
               GO  TO  190
            ELSE
               DO  180  K = 1,NTOT
  180            X(K) = ( X(K) + 0.2 * XOL(K) ) / 1.2
```

```
            ENDIF
            CALL  LAGFUN ( VLAG, TVAH, R, X, CL, COS1, COS2,
     *              LM, D, CLAH, CLAG, VAG, CK, VAH, CH, CG, VOF )
  200     CONTINUE
C
C---------CHECK FOR CONVERGENCE-----------------------------------
C
          DMAX = DABS ( ( VLAGOL - VLAGO ) / VLAGO )
            IF ( DMAX.LT.TOL ) THEN
              RETURN
            ELSE
              VLAGOL = VLAGO
            ENDIF
            IF ( KL.LT.NUMCY ) THEN
              GO TO 150
            ELSE
              RETURN
            ENDIF
          END
```

```
      SUBROUTINE  LAGFUN ( VLAG, TVAH, R, X, CL, COS1, COS2, LM,
     *     D, CLAH, CLAG, VAG, CK, VAH, CH, CG, VOF )
      IMPLICIT  DOUBLE  PRECISION  ( A-H,O-Z )
      DIMENSION  X(NTOT), CL(N), COS1(N), COS2(N), LM(ND,N),
     *      D(IQG), CLAH(IQH), CLAG(IQG), VAG(IQG), R(IQH),
     *      VAH(IQH), CH(IQH), CG(IQG)
      COMMON /PARR/ DECFC,FCINC,PHLIM,CV,ALP1,E,RP
      COMMON /PARI/ TOL,ITER,NUMCY,NITER,GA,IQH,IQG,ND,N,NTOT
      TVAH = 0.0
      TVAG = 0.0
      RP2 = RP + RP
C
C---------EVALUATE TOTAL VALUE OF EQUALITY CONSTRAINTS----------
C
      CALL EQUCON ( E, IQH, N , ND, NTOT, VAH, X, CL, COS1,
     *        COS2, LM, R, CK )
      DO  100  K = 1,IQH
         VAHK = VAH(K) * CH(K)
         CLAHK = CLAH(K)
         TVAH = TVAH + DABS ( CLAHK * VAHK ) + RP * VAHK * VAHK
  100 CONTINUE
C
C----------EVALUATE TOTAL VALUE OF INEQUALITY CONSTRAINTS-------
C
      CALL INECON ( IQG, N, NTOT, VAG, X, D )
      DO 200 K = 1,IQG
          V = VAG(K) * CG (K)
          CLAGK = CLAG (K)
          Z = CLAGK / RP2
          PSI = DMIN1 ( V,Z )
          TVAG = TVAG + DABS ( CLAGK * PSI ) + PSI * PSI * RP
  200 CONTINUE
C
C--------EVALUATE TOTAL VALUE OF OBJECTIVE FUNCTION-------------
C
      CALL VALOBF ( N, NTOT, VOF, X, CL )
      AVOF = CV * COF
      VLAG = AVOF + TVAH + TVAG
      RETURN
      END
```

```
      SUBROUTINE  EQUCON ( E, IQH, N, ND, NTOT, VAH, X, CL,
     *          COS1, COS2, LM, R, CK )
      IMPLICIT  DOUBLE  PRECISION ( A-H,O-Z )
      DIMENSION  VAH(IQH), X(NTOT), CL(N), COS1(N), COS2(N)
      DIMENSION  CK(6,6), LM(ND,N), R(IQH)
      NEL2 = N + N
      DO 100 K = 1,IQH
  100 VAH (K) = 0.0
C
C---LOOP OVER ALL ELEMENTS FOR GLOBAL STIFFNESS EVALUATION------
C
      DO  700  J = 1,N
        CALL  GLOSTI ( E, J, N, IQH, NTOT, CL, CK, X, COS1,
     *      COS2, ND )
        DO  300  L = 1,ND
          K = LM ( L,J )
          VAHT = 0.0
          IF ( K.EQ.0 )  GO  TO  300
          DO  200  LL = 1,ND
            M = LM ( LL,J )
            IF ( M.EQ.0 )  GO  TO  200
            JJ = NEL2 + M
            VAHT = VAHT + CK( L,LL ) * X(JJ)
  200     CONTINUE
          VAH(K) = VAH(K) + VAHT
  300   CONTINUE
  700 CONTINUE
C
C---------SUBTRACTION OF THE EXTERNAL GLOBAL FORCES-------------
C
      DO  500  K = 1,IQH
          VAH(K) = VAH(K) - R(K)
  500 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE  GLOSTI ( E, J, N, IQH, NTOT, CL, CK, X, COS1,
     *      COS2, ND )
      IMPLICIT  DOUBLE  PRECISION ( A-H,O-Z )
      DIMENSION CL(N), X(NTOT), CK(6,6), COS1(N), COS2(N)
      C1 = COS1(J)
      C2 = COS2(J)
      C12 = C1 * C1
      C22 = C2 * C2
      JI = J + J
      JA = JI - 1
      CL1 = CL(J)
      CL2 = CL1 * CL1
      CL3 = CL2 * CL1
      A = E * X(JI) / CL3
      B = X(JA) * CL2 / X(JI)
      G1 = A * ( B * C12 + 12. * C22)
      G2 = A * C1 * C2 * ( B - 12. )
      G3 = A * ( B * C22 + 12. * C12 )
      G4 = - A * 6. * CL1 * C2
      G5 = A * 6. * CL1 * C1
      G7 = A * 2. * CL2
      G6 = G7 + G7
      CK(1,1) = G1
      CK(2,1) = G2
      CK(3,1) = G4
      CK(4,1) = - G1
      CK(5,1) = - G2
      CK(6,1) = G4
      CK(1,2) = G2
      CK(2,2) = G3
      CK(3,2) = G5
      CK(4,2) = - G2
      CK(5,2) = - G3
      CK(6,2) = G5
      CK(1,3) = G4
      CK(2,3) = G5
      CK(3,3) = G6
      CK(4,3) = - G4
      CK(5,3) = - G5
      CK(6,3) = G7
      CK(1,4) = - G1
      CK(2,4) = - G2
      CK(3,4) = - G4
      CK(4,4) = G1
      CK(5,4) = G2
      CK(6,4) = - G4
      CK(1,5) = - G2
      CK(2,5) = - G3
      CK(3,5) = - G5
      CK(4,5) = G2
      CK(5,5) = G3
      CK(6,5) = - G5
```

```
      CK(1,6) = G4
      CK(2,6) = G5
      CK(3,6) = G7
      CK(4,6) = - G4
      CK(5,6) = - G5
      CK(6,6) = G6
      RETURN
      END
```

```
      SUBROUTINE  VALOBF  ( N, NTOT, VOF, X, CL )
      IMPLICIT  DOUBLE  PRECISION  ( A-H,O-Z )
      DIMENSION  X(NTOT), CL(N)
      VOF = 0.0
      DO 100 K = 1,N
         J = K + K - 1
100   VOF = VOF + X(J) * CL(K)
      RETURN
      END
```

```
      SUBROUTINE  INECON  ( IQG, N, NTOT, VAG, X, D )
      IMPLICIT  DOUBLE  PRECISION  ( A-H,O-Z )
      DIMENSION  VAG(IQG), X(NTOT), D(IQG)
      NEL2 = N + N
      DO 100 K = 1,IQG
         J = NEL2 + K
         VAG(K) = DABS (D(K)) - DABS (X(J))
100   CONTINUE
      RETURN
      END
```

APPENDIX B

DATA FORMATS AND TEST RESULTS

Table B.1 User's manual and example of input form.

USER'S MANUAL

Heading line

This line is used to write the title of the problem

N, IQH, IQG, NUMAX, E, ND

where N is the number of elements of the frame
IQH is the number of equality constraints
IQG is the number of displacement constraints
NUMAX is the number of trial points for each design variable in each cycle
E is the module of elasticity of the material
ND is the number of global degrees of freedom

R (IQH lines)

where R is the external global force

D (IQG lines)

where D is the displacement constraint

CL, COS1, COS2 (N lines)

where CL is the length of each element
COS1 is the cosine of the angle between the element and the horizontal
COS2 is the sine of the angle defined above

LM (I,J), I = 1,ND and J = 1,N (N lines)

where LM (I,J) is the global degree of freedom corresponding to the i^{th} element degree of freedom of the j^{th} element

MAXCY, P

where MAXCY is number of cycles with the same lagrangian multipliers
P is the penalty factor

Table B.1 Continued.

FCINC, DECFA, GA

where	FCINC	is the increasing factor
	DECFC	is the decreasing factor
	GA	is the updating scalar for the penalty factor

ALP, TOL

where	ALP	is the perturbation factor
	TOL	is the minimum allowable change for the lagrangian function without the updating of the lagrangian multipliers

X (N + N elements, one line)

where	X	is the initial values of areas and moments of inertia; odd values are areas; even values are inertias

X (number of global degrees of freedom, one line)

where	X	is the initial values of global displacements

Table B.1 Continued.

```
Frame : first loading
3, 6, 6, 30, 30000, 6
0
1000
0
0
0
0
0.5
0.5
0.5
0.5
0.5
0.5
180, 0, 1
240, 1, 0
180, 0, 1
0, 0, 0, 1, 2, 3
1, 2, 3, 4, 5, 6
0, 0, 0, 4, 5, 6
5, 100
1.2, 0.7, 5
100, 0.0000000001
1, 1, 1, 1, 1, 1
0.4, 0.4, 0.4, 0.4, 0.4, 0.4
```

A) <u>First line</u>

Title of example

B) <u>Second line</u>

3, 6, 6, 30, 30000, 6

(number of elements, number of equality constraints, number of inequality constraints, number of trial points for each design variable, modulus of elasticity and number of degrees of freedom per element ; dimensions of modulus of elasticity are psi)

C) <u>Third to eigth line</u>

0

Table B.1 Continued.

1000
0
0
0
0

(one line for each global external force ; the only force applied has the direction and sense of the second global degree of freedom ; the global external forces are ordered as the global degrees of freedom ; the dimensions of forces are kips)

D) Nineth to thirtieth line

0.5
0.5
0.5
0.5
0.5
0.5

(absolute values of each of the global displacement constraint ; the global displacement constraints are ordered as the global degrees of freedom ; the dimensions of displacement constraints are inches)

E) Fifteenth to seventeenth line

180, 0, 1
240, 1, 0
180, 0, 1

(the lines are ordered per element ; the first value of each line is the length of the element ; the second value is the cosine of the angle between the horizontal axis and the structural element measured counterclockwise ; the third value is the sine of that angle ; the dimensions of length are inches)

F) Eighteenth to twentieth line

0, 0, 0, 1, 2, 3
1, 2, 3, 4, 5, 6
0, 0, 0, 4, 5, 6

(location matrix : for each element relates the local degrees of freedom with the global degrees of freedom ; first line states that for the first node in the element

Table B.1 Continued.

there are no global degrees of freedom and for the second node, the fourth local degree of freedom is the same as the first global degree of freedom and so forth ; the second line defines the relations for the second element and the third line for the third)

G) Twenty first line

5, 100

(first value defines the number of cycles with the same lagrangian multipliers; second value is the penalty factor)

H) Twenty second line

1.2, 0.7, 5

(first value is the increase stepsize value for the design variable ; the second is the decrease stepsize value ; the third value is the updating scalar for the penalty factor at the end of each cycle)

I) Twenty third line

100, 0.0000000001

(first value is the scalar that multiplies the value of each design variable at the first trial point ; the second is the minimum admissible relative change in the lagrangian function at each loop of cycles)

J) Twenty fourth line

1, 1, 1, 1, 1, 1

(guesses for the physical properties of the frame ; the first, third and fifth values are the initial areas for the first, second and third elements ; the others are the respective inertias)

K) Twenty fifth line

0.4, 0.4, 0.4, 0.4, 0.4, 0.4

(initial values of the global displacements, assumed feasible in the inequality constraints domain)

Table B.2 Example of output form.

TEST OF A BEAM

************ INITIAL VALUES **********

NUMBER OF ELEMENTS = 1

NUMBER OF EQUALITY CONSTRAINTS = 3

NUMBER OF INEQUALITY CONSTRAINTS = 3

NUMBER OF ITERATIONS PER CYCLE = 50

MODULUS OF ELASTICITY = 0.30000000E+05

NUMBER OF GLOBAL DEGREES OF FREEDOM = 6

GLOBAL DEGREE OF FREEDOM	EXTERNAL FORCE
1	0.10000000E+04
2	0.00000000E+00
3	0.00000000E+00

GLOBAL DEGREE OF FREEDOM	DISPLACEMENT CONSTRAINT
1	0.50000000E+00
2	0.50000000E+00
3	0.50000000E+00

ELEMENT	LENGTH	COS	SIN
1	0.10000000E+03	1.00000	0.0000

LOCATION MATRIX FOR GLOBAL DEGREES OF FREEDOM

ELEMENT	NODE I			NODE J		
1	0	0	0	1	2	3

MAXIMUM NUMBER OF CYCLES = 10

Table B.2 Continued.

PENALTY FACTOR = 0.10000000E+04

FACTOR OF INCREASE = 0.12000000E+01

FACTOR OF DECREASE = 0.70000000E+00

PENALTY FACTOR MULTIPLIER = 0.10000000E+01

ELEMENT	AREA	INERTIA
1	0.10000000E+01	0.10000000E+01

GLOBAL DEGREE OF FREEDOM	INITIAL GUESS
1	0.10000000E+01
2	0.10000000E+01
3	0.10000000E+01

************ FINAL VALUES ************

NUMBER OF ITERATIONS = 1

VALUE OF LAGRANGIAN FUNCTION = 0.27482949E-11

ELEMENT	AREA	INERTIA
1	0.66682531E+01	0.10000000E+00

DISPLACEMENT	FINAL VALUE
1	0.49988107E+00
2	-.21501374E-05
3	-.37650328E-07

EQUALITY	FINAL VALUE
1	0.36692930E-04
2	-.96343543E-08
3	-.64779215E-06

Table B.2 Continued.

INEQUALITY	FINAL VALUE
1	0.11893492E-03
2	0.49999785E+00
3	0.49999996E+00

Table B.3 Cantilever beam: first loading

Load Vector .... $\underline{R}$ = (1000, 0, 0)
(kips and kip.in)

Displacement Constraint Vector .... $\underline{D}$ = (0.5, 0.5, 0.5)
(inches and radians)

	Initial	Final	Exact solution
Area (in^2)	1.	6.6471	6.6667
Inertia (in^4)	1.	0.1	(undetermined)
Horizontal Displacement (in)	0.4	0.5009	0.5
Vertical Displacement (in)	0.4	0.0363	(undetermined)
Rotational Displacement (rad)	0.4	0.0006	(undetermined)

Table B.4 Cantilever beam: second loading

Load Vector............................ $\underline{R}$ = (0, 1000, 0)
(kips and kip.in)

Displacement Constraint Vector ......$\underline{D}$ = (0.5, 0.5, 0.5)
(inches and radians)

	Initial	Final	Exact solution
Area (in^2)	1.	0.1	(undetermined)
Inertia (in^4)	1.	34400.2	(undetermined)
Horizontal Displacement (in)	0.4	0	0
Vertical Displacement (in)	0.4	0.3232	0.3230
Rotational Displacement (rad)	0.4	0.0048	0.0048

Note : Exact values of displacements determined with final area and inertia.

Table B.5 Cantilever beam: third loading.

Load Vector .... R = (0, 0, 10000000)
(kips and kip.in)

Displacement Constraint Vector....... D = (0.5, 0.5, 0.5)
(inches and radians)

	Initial	Final	Exact solution
Area (in^2)	1.0	0.1	(undetermined)
Inertia (in^4)	1.0	58369.609	(undetermined)
Horizontal Displacement (in)	0.4	0	0
Vertical Displacement (in)	0.4	0.2856	0.2856
Rotational Displacement (rad)	0.004	0.0057	0.0057

Note : Exact values of displacements determined with final area and inertia.

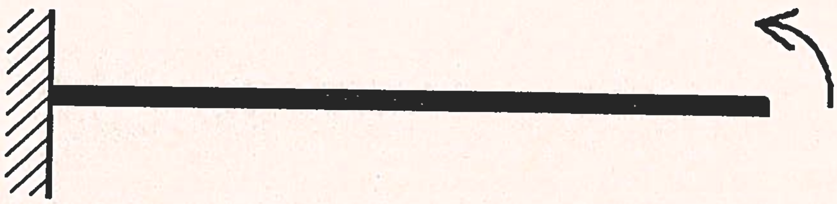


Table B.6 Cantilever beam: fourth loading.

Load Vector...................... $\underline{R}$ = (1000, 1000, 100000)
(kips and kip.in)

Displacement Constraint Vector..... $\underline{D}$ = (0.5, 0.5, 0.5)
(inches and radians)

	Initial	Final	Exact solution
Area (in^2)	1.0	6.649133	6.666667
Inertia (in^4)	1.0	78625.13	(undetermined)
Horizontal Displacement (in)	0.4	0.5009	0.5000
Vertical Displacement (in)	0.4	0.3534	0.3524
Rotational Displacement (rad)	0.4	0.0064	0.0064

Note : Exact values of displacements determined with final area and inertia

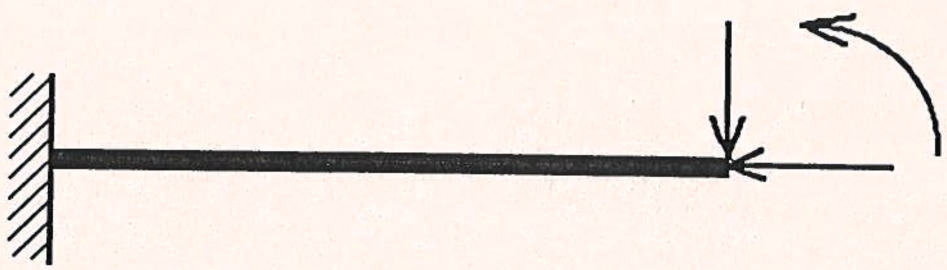


Table B.7 One bay frame: first loading.

Load Vector.........$\underline{R}$ = (0, -1000, -1000, 0, -1000, 1000)
(kips and kips.inch)

Displacement
Constraint Vector....$\underline{D}$ = (0.5, 0.05, 0.5, 0.5, 0.05, 0.5)
(kips and radians)

Physical properties

	Initial	Final
Element 1-2		
Area (in^2)	1.0	119.7756
Inertia (in^4)	1.0	0.20446
Element 2-3		
Area (in^2)	1.0	0.10000
Inertia (in^4)	1.0	8.71938
Element 3-4		
Area (in^2)	1.0	119.8357
Inertia (in^4)	1.0	0.19978

Table B.7 Continued.

Global Displacements

	Initial	Final	Exact
Degree of freedom # 1 (inch)	0.4	0.4764	0.4675
Degree of freedom # 2 (inch)	0.04	-0.0500	-0.0501
Degree of freedom # 3 (radians)	0.4	-0.4321	-0.4320
Degree of freedom # 4 (inch)	0.4	0.4376	0.4287
Degree of freedom # 5 (inch)	0.04	-0.0500	-0.0501
Degree of freedom # 6 (radians)	0.4	0.4321	0.4321

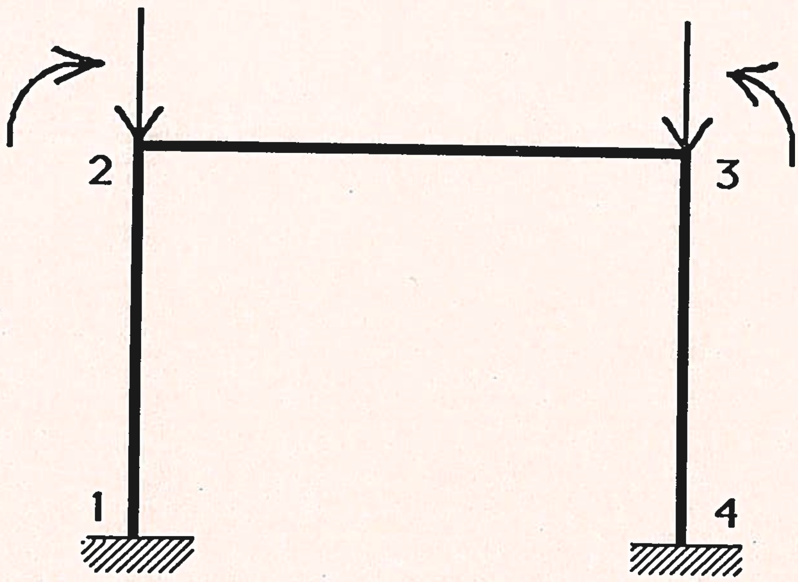


Table B.8 One bay frame: second loading.

Load Vector......................$\underline{R}$ = (1000, 0, 0, 0, 0, 0)
(kips and kips.inch)

Displacement
Constraint Vector.....$\underline{D}$ = (0.5, 0.05, 0.5, 0.5, 0.05, 0.5)
(kips and radians)

<u>Physical properties</u>

	Initial	Final
Element 1-2		
Area (in^2)	1.0	0.3769091
Inertia (in^4)	1.0	145164.85
Element 2-3		
Area (in^2)	1.0	0.5641342
Inertia (in^4)	1.0	5836.6834
Element 3-4		
Area (in^2)	1.0	0.9709580
Inertia (in^4)	1.0	76.810583

Table B.8 Continued.

Global Displacements

	Initial	Final	Exact
Degree of freedom # 1 (inch)	0.4	0.4764	0.4193
Degree of freedom # 2 (inch)	0.04	0.0479	0.0479
Degree of freedom # 3 (radians)	0.4	-0.0034	-0.0034
Degree of freedom # 4 (inch)	0.4	0.3860	0.3860
Degree of freedom # 5 (inch)	0.04	-0.0186	-0.0186
Degree of freedom # 6 (radians)	0.4	0.0012	0.0012

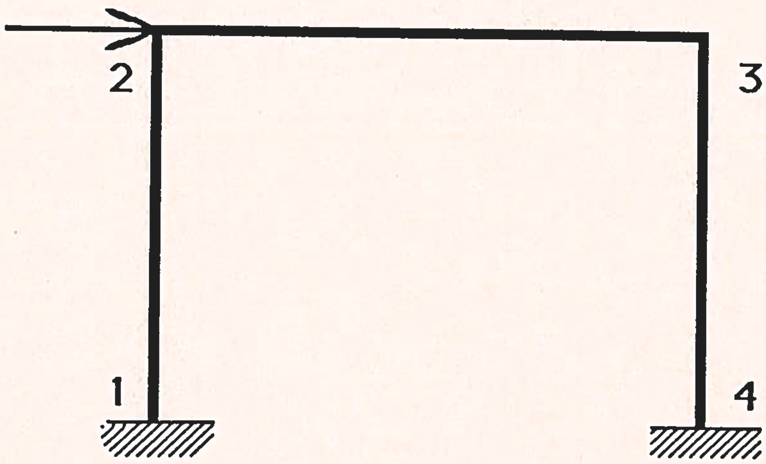


Table B.9 One bay frame: third loading (first run).

FIRST RUN (random initial guesses)

Load
Vector.......$\underline{R}$ = (1000, -1000, -100000, 0, -1000, 100000)
(kips and kips.inch)

Displacement
Constraint Vector......$\underline{D}$ = (0.5, 0.5, 0.5, 0.5, 0.5, 0.5)
(kips and radians)

Physical properties

	Initial	Final
Element 1-2		
Area (in^2)	1.0	25.547329
Inertia (in^4)	1.0	0.1000000
Element 2-3		
Area (in^2)	1.0	90.957438
Inertia (in^4)	1.0	590.32596
Element 3-4		
Area (in^2)	1.0	34.538765
Inertia (in^4)	1.0	150.23984

Table B.9 Continued.

Global Displacements

	Initial	Final	Exact
Degree of freedom # 1 (inch)	0.4	0.4499	86.152
Degree of freedom # 2 (inch)	0.4	0.2317	-0.1411
Degree of freedom # 3 (radians)	0.4	-0.4939	-0.4614
Degree of freedom # 4 (inch)	0.4	0.4149	86.064
Degree of freedom # 5 (inch)	0.4	-0.1728	-0.2435
Degree of freedom # 6 (radians)	0.4	0.5000	0.2434

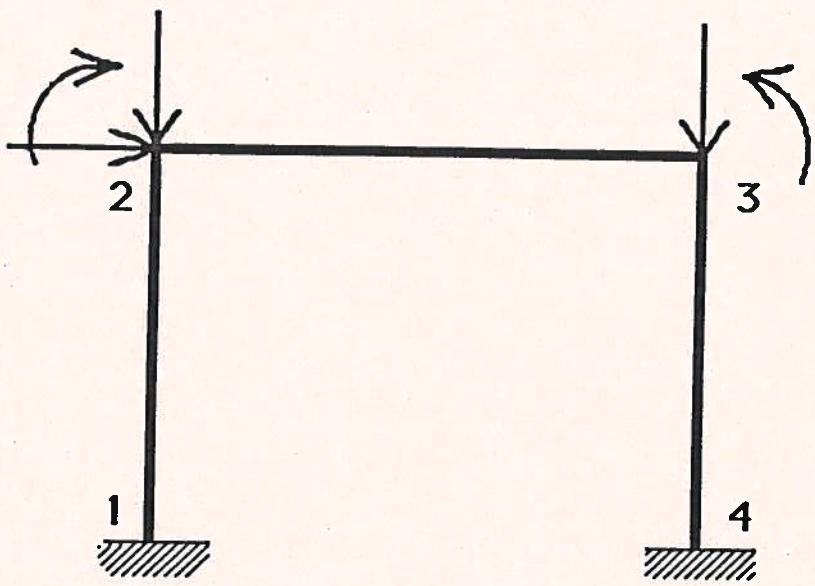


Table B.10 One bay frame: third loading (second run).

SECOND RUN (with results from first run)

Load
Vector........$\underline{R}$ = (1000, -1000, -100000, 0, -1000, 100000)
(kips and kips.inch)

Displacement
Constraint Vector.......$\underline{D}$ = (0.5, 0.5, 0.5, 0.5, 0.5, 0.5)
(kips and radians)

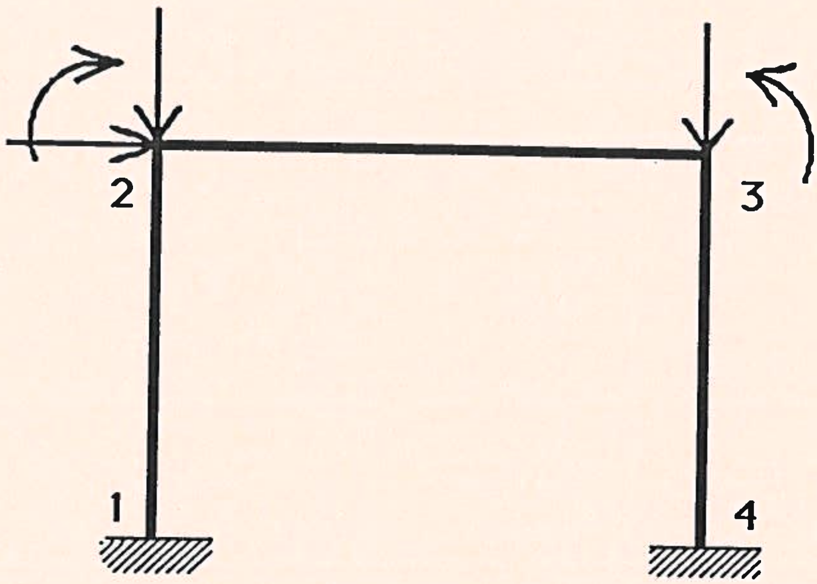
Physical properties

	Initial	Final
Element 1-2		
Area (in^2)	25.547329	25.414238
Inertia (in^4)	0.1000000	120224.43
Element 2-3		
Area (in^2)	90.957438	179.44512
Inertia (in^4)	590.32596	5912.0086
Element 3-4		
Area (in^2)	34.538765	35.071602
Inertia (in^4)	150.23984	17058.589

Table B.10 Continued.

Global Displacements

	Initial	Final	Exact
Degree of freedom # 1 (inch)	0.4499	0.4574	0.4642
Degree of freedom # 2 (inch)	0.2317	-0.2338	-0.2329
Degree of freedom # 3 (radians)	-0.4939	-0.0050	-0.0050
Degree of freedom # 4 (inch)	0.4149	0.4268	0.4241
Degree of freedom # 5 (inch)	-0.1728	-0.1728	-0.1734
Degree of freedom # 6 (radians)	0.5000	0.0049	0.0048



Proposal to the National Science Foundation

Integrated Equality Constraint Optimization

by

Marc Hoit, Fernando Fagundo and Alfredo Soeiro

SUMMARY

Optimization as a general design technique is an evolving area of study. In its current form, it is a varied collection of specialized methods and greatly simplified problems. In the area of structural optimization, these methods have not produced any clear direction for further research. The major result of current research is that it is not the optimization methods that need study, but the problem formulation. It has been shown that less than five percent of the effort in structural optimization is in the optimization strategies [1]. The rest of the effort is taken up by problem formulation.

It is for this reason that we propose to study a new problem formulation method. This method will directly develop the objective function as well as all constraint functions in the design variable space. This is different from current techniques which map portions of the design variables from one space to another. This mapping is performed so that standard techniques can be used to form the design variables. As an example, in structural analysis the design variables are usually stresses, moments of inertia and areas. Standard structural analysis uses a displacement-based analysis method. Therefore, the analysis problem is first solved in the displacement space. Then the displacements are mapped into the stress space by standard direct stiffness techniques. Optimization is performed on these resulting stresses [2].

The method we wish to study will use the equations of equilibrium as equality constraints of the optimization problem. The equality constraints will be satisfied through the use of Augmented Lagragian Multipliers. The displacements will now be considered as design variables. Although this will greatly increase the number of variables in the optimization problem, it should reduce the problem formulation effort. The overall effect will be to increase

solution accuracy while reducing numerical effort. In addition, it will shift the burden of solution to optimization methods, thereby taking full advantage of currently available search techniques.

Description of Research Project

General computational capabilities available to the designer have increased drastically in recent years. However, design methods still consist of a trial and error procedure. This is due in part to the lack of easily understood and used algorithms. Thus, current designs are in accordance with applicable design codes, but are generally not related to an optimal solution. The resulting structure is an unbalanced solution between the minimal total cost and acceptable performance due to external, gravity and lateral loading. The problem of designing reinforced concrete in areas exposed to seismic risks is of particular importance. The seismic forces acting on the structure are directly related to the structural configuration, stiffness and weight.

Until recently, the optimization of frames subjected to seismic loading dealt mainly with a homogeneous elastic material, namely steel. Reinforced concrete is made of two different materials, usually cast in place, and has both non-elastic behavior and varying external forces due to the seismic loading. This introduces a considerable amount of extra computation in the optimization procedure because of additional design variables and constraints. The fact that concrete cracks when subjected to tension creates a reduction in structural stiffness. This requires, without other simplifying assumptions, an iterative procedure to solve the non-linear optimization problem. Ductility variation is primarily important when the frame is submitted to seismic forces. The cyclic nature of seismic loading produces a generally more severe effect compared to the service design loads. All this creates a very complex problem in terms of formulation and computational strategy. As a result, the inclusion of non-linear materials has previously been ignored [3].

Proposed Formulation

The primary goal of this research is to study a new formulation method and the techniques required to efficiently solve the problem of the optimization of concrete structures subjected to seismic loading. The new formulation is a departure from current techniques in that the solution is carried entirely in the design space, using only optimization techniques. This formulation will use displacements, areas, moments of inertia and any related structural properties that are to be optimized as design variables. It is the simultaneous inclusion of the displacements with all other variables that is the basis of the new formulation.

Previously, researchers have used the displacements alone, properties alone or some very simplified combination of displacements and properties. None of the approaches were general enough to be usable as a basis for the optimization of general structures. Current techniques use general analysis programs to find the design variables (stresses). The optimization is then performed on the results of the analysis. This analysis and subsequent optimization is an iterative procedure that is repeated until the problem converges to an acceptable solution. It is exactly this iterative cycle that has hindered the progress of structural optimization [4].

Studies have shown that over 95% of the effort involved in current optimization methods is spent in problem formulation and in methods for information gathering. This means that almost all of the effort is involved in the traditional analysis portion of the problem. While traditional displacement-based techniques are relatively efficient, they still require large amounts of effort in generating the problem and reducing the results of the analysis.

The solution to this bottle neck is to formulate the entire problem as an optimization problem. This means removing the structural analysis step completely. The displacements will now be considered design variables. As a result, the normally separate displacement analysis and stress recovery phase of the optimization procedure is eliminated. The equilibrium equations used in traditional structural analysis will become equality constraints in the design space. The solution of equality constraints can be handled through the use of Augmented Lagrangian Multipliers. The use of this new formulation will shift the solution emphasis back to optimization techniques. This is due to the fact that the number of design variables and constraint equations will increase dramatically [5].

Research Objectives

The research project has three definite objectives. These objectives will be accomplished in succession over a two year period. The first objective is to implement and study the complete formulation. This consists of creating the pseudo-objective function relating the merit function, the equilibrium constraints and the behavioral constraints. The second is to research current optimization techniques and find the most applicable method or combination of methods for this new formulation. Third is to extend the technique to non-linear problems. The study will be limited to rigid frame structures. The inclusion of non-linear effects will be considered by using concrete as a structural material. These goals are the first step toward creating a general framework for the optimization of displacement-based analysis problems. The extension of these techniques can be applied to any problem currently using the finite element method as a solution technique. Further studies should include the extension to more general types of structural elements, structural systems and materials.

In order to accomplish the stated objectives, we have divided the procedure into five distinguishable steps. First, we will formulate the problem so that structural equilibrium will be satisfied through equality constraints. The behavioral constraints, like maximum allowable stresses and limits on the floor drifts, will be considered as inequality constraints. The use of the Augmented Lagrangian Multiplier method will derive a pseudo-objective function that can be minimized as an unconstrained function. This method will completely satisfy the equilibrium constraints while avoiding the re-analysis of the structure after each cycle of the optimization procedure.

Second, the concept of the forced-mode compliance will be studied in more depth. As an optimization algorithm, this method seems promising in the reduction of computation time. The method can be extended from the currently restricted formulation to more generalized optimality conditions. This will remove the restriction of using only shear building models. The inclusion of a more realistic approach to earthquake loading, rather than simple harmonic motion to a real ground motion, will be looked at.

Third, the extension of the formulation from linear elastic to non-linear inelastic structures will be studied. Research in optimization algorithms related to seismic resistant frames will create more general algorithms. We will specifically include the non-linear behavior of concrete and the inelastic performance of the structure due to deformations and cracking. This extension means a considerable increase in the computations, but it is necessary in creating more general optimization methods. Due to the characteristics of reinforced concrete, a possible way of accomplishing this is using the minimum dissipated energy concepts for seismic loading.

Fourth, we will study the simultaneous consideration of the service and seismic loads. These loading cases, due to their different nature, create different types of restrictions that must be made compatible. The different characteristics of the two load situations may bring some benefits to the optimization algorithm. The constraints and the optimality criteria will consider the stochastic nature of the seismic loads, opposed to the status of the service loads. The objective function will also consider this discrepancy with appropriate weighting of the effects of the two loadings. The algorithm will include specific criteria in the decision making process for each type of loading. The final solution must be a compromise between the sets of designs.

Finally, since optimization algorithms are iterative procedures that start with a solution, it is crucial to have a good initial design. The accuracy and reliability of the initial solution is important in the algorithm development and on the total computational time. Work will be concentrated on finding a procedure based on practical knowledge of experienced designers. This suggests the use of Expert Systems techniques to derive a good initial design that is both feasible and close to the optimal solution. This improvement in the preliminary estimate will incorporate knowledge from other disciplines, thus improvements on many optimization procedures are possible.

Review of Current Techniques

Practical uses of Optimization in Structural Analysis

There are two common types of problems in structural design in which optimization is used, each requiring a different method of solution. First is the uncoupled optimization of basic structural elements. Second is the optimization of the configuration and sizing of all the elements, considering the structure as a whole. Both procedures have been used in the sizing and detailing of beams, columns and slabs using steel, reinforced concrete or prestressed concrete.

Uncoupled optimization can be reasonably implemented on small computers. The result is a computer program that is easy to use and requires the designer to input the same data as required in a traditional, non-optimized design. This method can produce designs rather quickly, because shortcuts can be included due to the specialized nature of the problem. These programs generally have as goals a design that is relatively efficient, economic, capable of resisting the external forces, and compliant with other design code limitations, including buildability. These requirements are generally postulated within a numerical optimization framework of minimizing an objective function, while satisfying a set of algebraic functional constraints. The level of detail included in the optimization controls the ease of use. The value of element design methods lies in their ability to give useful information in a quick and easy manner. The problem is that uncoupling the elements often neglects critical parameters [6].

Optimization using a coupled approach is a much more complete method. It is the area in which most of the current research efforts have been concentrated. One of the most studied areas is the optimization of trusses. This is due to the simplicity of the problem formulation. Trusses are generally determinate and the functions of the design variables can be formed directly. The optimum design problem usually consists of finding a cross-sectional area for each truss member so that the total weight is minimized. The constraints are that the design loads are equilibrated with a satisfactory distribution of stresses and displacements. The conventional truss design method is extended to an optimization problem by re-sizing the bars through the use of optimization methods. There are procedures that can select the most appropriate discrete size for each truss member, thus producing efficient practical design [7].

The optimization of rigid frames is another highly researched area. The iterative process of analyzing an initially assumed design, then re-sizing and re-analyzing members has been used successfully. However, it has required many simplifying assumptions. One common approach is to extract the resulting member forces from a standard analysis. These forces are then used in a separate optimization step in which the elements are designed and re-sized independently of the structure. This method uses the global structure to distribute the member forces and then an uncoupled approach to optimize the particular element.

Another area in which optimization has shown to be useful is in choosing the structural configuration. Methods have been researched in which the most economical structural form, as a function of the number of bays, spans, frames and frame spacings, are identified to satisfy the requirements of floor area, site dimensions, usable headroom , existence of internal columns and others [8].

Optimization techniques

The general mathematical statement formulating an optimization problem is expressed as follows :

Minimize (or Maximize):	$f_0(x_i)$	$i=1,n$
Subject to:	$f_j(x_i) <$ or $=$ or > 0	$j=1,m$
	$x_i \geq 0$	$i=1,n$

Where f_0 is the objective function and is a function of the design variables x_i. It represents some chosen criterion of merit of a design which may be cost, efficiency, benefit, etc.. The constraints f_j are also functions of the variables x_i. In the case of structural element design, x_i may represent the configuration of the element; such as bar areas, depths or bar spacings [9]. The constraints f_j may also represent bending stresses, deflection, or other code provisions. These constraints are generally highly non-linear. Thus, this becomes one of the more difficult types of optimization problems to solve. Research has shown that element design problems are sufficiently small enough to make them solvable by a variety of non-linear, constrained optimization algorithms. Methods such as the generalized reduced gradient, Augmented Lagragian and penalty function are applicable for these types of problems [10].

The penalty function method is the most robust for solving the constrained problem as a sequence of unconstrained optimizations. This method uses a penalty parameter, P, that multiplies the constraints. These factored constraints are then added to the objective function (f_0) in such a way that a violation of any constraint leads to a very high value of the augmented objective function. Unconstrained optimization is then used to find a minimum of the augmented objective function for a particular value of P. The augmented objective function has P as part of the set of design variables. Now P is altered such that when the unconstrained optimization is repeated, the objective function value is reduced as is any constraint violations. The optimal solution is found to satisfy both the original design variables and P [11].

In trusses, the number of variables and constraints is very large. However, the regular mathematical structure of the problem can be used as an advantage in using simpler methods. The objective function is linearly dependent on the design variables, but the constraints are non-linear. Considering the dual of the problem, it can be mapped into another space where the objective functions (g_0) are non-linear and the constraints (g_1) are linear. This problem may be solved by making a sequence of linear approximations to the nonlinear g_0 and solving it by the sequential use of linear programming methods. It must be emphasized that these methods are simply numerical search methods which make no assumptions about the nature of the optimum of the problem [12].

Another common method of optimization is the one called optimality criteria. It attempts to establish conditions of uniqueness at the beginning of the process. These conditions characterize the optimum of a problem and distinguish it from all other possible solutions. The methods then attempt to devise a scheme which iteratively satisfies these criteria while searching for the optimum. This method has two problems. First, in most cases there is no absolute criterion to distinguish a global optimum from any other local optimum. Second, the re-sizing schemes (optimality criteria) are only approximate and need careful programming to yield good results [13].

Current frame optimization methods depend upon whether the problem is an elastic or a plastic design. The optimum rigid plastic design of frames turns out to be a problem of linear programming. This has the considerable advantage that very large problems can be solved quickly. However, the formulation of the linear programming model may be difficult since all possible collapse mechanisms must be known. For large frames, the number of these mechanisms can be enormous since the precise mechanism of collapse is not initially known. Some research has been done in order to make full use of the duality between the static and kinematic theorems of plasticity [14].

Optimal design of elastic frames leads to an optimization problem similar to that of truss-sizing. However, members with flexural and axial loads do not easily map into the dual problem. One popular method finds the optimum design by using the virtual load method to formulate displacement constraints and a force matrix approach to reduce the problem size. This method still generates a non-linear objective function with linear constraints. In order to solve the problem, direct search methods are required. Some of the direct methods that have been used are feasible directions, generalized reduced gradient and the gradient projection method. All of these methods are difficult to implement due to the size of the problems and their non-linear nature. Mixtures of these methods have been tried depending on the solution strategy adopted. In problems in which the definition of some of the constraints is not well posed (such as concrete strength), fuzzy logic has been applied to define the limitations [15].

Other methods described in the literature include the use of decomposition in the general problem and then iterative optimization between the parts. Computer-aided design techniques are used when the designer makes search direction designs. This technique could be extended through the use of Expert Systems. The energy method is also used to generate a function which can be maximized. This represents the amount of energy absorbed by the structure when loaded [16]. Optimization of the structural frames submitted to dynamic loading has been studied by a few researchers [17].

Summary of Current Optimization of Seismic Resistant Frames

In this section a summary of the current state-of-the-art information in structural optimization is presented. The research discussed here consists of the most promising methods that will be studied for use in solving the optimization problem in its new formulation. This presentation is the result of an intensive literature search on this subject. It is very difficult to synthesize the information presented due to the fact that each study represents the application of different techniques to different systems.

1) Optimum seismic resistant design of R/C frames

Zagajesky and Bertero present a method to optimally design ductile R/C moment-resisting frames for buildings subjected to severe earthquakes [18]. It is a five stage method grouped into two phases, a preliminary and a final phase. A weak girder - strong column assumption is adopted to simplify the analysis. The design moments for the girders are found by the minimization of the flexural reinforcement.

The preliminary phase is composed of a three step process: choose an initial design, analyze the structure, and optimize the structure. The first step consists of assuming the properties of the structure so that an analysis can be performed. The second step is the analysis of the initial design to find the story shear forces for a particular set of design conditions such as ground spectrum, damping ratio and displacement ductility factors. The seismic design forces are obtained using a modal analysis technique on a shear building model. The final step of the preliminary phase is to optimize the structure on a sub-assemblage basis. Each floor of the structure is considered a sub-assemblage and is independently optimized. The optimization is based on the gravity, wind loads, seismic story shear forces, critical load combinations and the mechanical characteristics of the materials. The objective is to find the optimum sizes and the optimal distribution of the reinforcements.

Equilibrium constraints are derived from the kinematic theorem of simple plastic theory. Serviceability constraints are imposed to prevent yielding, wide cracking and large deflections. Practical constraints were imposed to satisfy code requirements and to obtain a practical design consistent with the principles of seismic-resistant design. This optimization problem is solved for each story by a simplex method algorithm in a step-wise linear fashion. The proportioning of the members is made in accordance with the Uniform Building Code (UBC). Beams are designed to supply moment capacities at least equal to the results from the analysis. After the beams of an assemblage are designed, the columns are designed using joint equilibrium to develop the column moments.

The final design phase consists of two steps. First, the reanalysis of the optimized structure that resulted from the preliminary optimization of sub-assemblages (stories). The final step is to optimize again on a sub-assemblage basis using a more sophisticated sub-assemblage to formulate the optimization problem from which the final design is obtained. As a last check on the design, the final optimized structure is analyzed to insure overall reliability and guidelines for detailing to ensure ductile behavior.

The optimum values of the design moments are sensitive to the cost of steel. Since design moment variations affect the local inelastic demands, it is advisable to test different techniques for the objective function and design constraints. Also, the higher material total costs obtained with the optimization procedure should be evaluated under the perspective of the other benefits obtained with that same design.

The outlined method allows the inclusion of most of the important factors controlling the selection of design criteria, providing an efficient and rational basis for the seismic design. The procedure's versatility permits design constraints to be changed or added. It has the limitation that the structure's behavior has to be greatly simplified in order to reduce the problems to a reasonable size. This work is the most complete in the area of concrete structures subjected to seismic loading. It's procedures for forming constraints will be applied to the new formulation.

<u>2) Optimum earthquake design of shear buildings</u>

Kato, Nakamura and Anraku presented a method that optimizes a structure using constraint conditions based upon evaluation of the cost function, using either linear and non-linear programming [19]. Optimization of a structure subjected to seismic loads is very difficult because of the stochastic nature of the loads. As a result, a trial and error method is usually applied to the design. Prato considered the maximization of the natural frequency of the frame [20]. Clough et al investigated how ductility factors of members would be affected by stiffness distribution of a frame and the intensity of an earthquake [21].

The method described here includes the following four phases. First, a building is analyzed to obtain the elastic response, including member stresses. Second, sensitivity coefficients for story deflections and member stresses are calculated due to small changes of member sizes. Third, these sensitivity coefficients are used to form linearized constraint equations and an objective function. Finally, the optimum design is obtained by iterative stepwise linear programming.

In the first phase, a parametric study considering multistory frames with uniformly distributed mass was performed. The results showed an important influence of the stiffness distribution on the value of the generated objective function. The constraint condition required the stress to be smaller than the yield stress. Under this condition, the optimum design is the frame whose elastic shear strength corresponds to the shearing force of the response at each story level (fully stress design).

The last phase requires an optimization algorithm that seeks the optimum design with arbitrarily distributed story masses under either strength constraints, deflection constraints, or both constraints. Because reciprocals of the moment of inertia are used as design variables, the objective function and the constraint equations become nonlinear. A stepwise linear programming method is used. Linear approximations to the objective function and constraint equations are made using Taylor's series expansions. At each linearized step, the sensitivity coefficients and linear programming are used to modify the values of design variables.

A sensitivity coefficient consists of a static and dynamic factor. The dynamic factor represents the variation of the external forces due to member changes. It is obtained by taking the partial derivative of the natural periods and mode shapes with respect to the design variables.

The optimum design of shear-type frames under a given standard spectra proved to be unique. This was true even if the initial design variables were located in an unfeasible solution space. This result proved that the hyperplanes formed by constraint conditions are convex and smooth. In almost all cases, the optimum design corresponded to the fully stressed design. It was also found that the dynamic sensitivity coefficients were functions of changes of the eigen values and eigen vectors. Thus, the sensitivity coefficients could be modified by changing the member sizes of stories whose modal story deflections are large. The use of sensitivity coefficients may be beneficial to the solution of the general problem.

3) Optimum building design for forced mode compliance

Nakamura and Takewaki present a method with many new ideas useful in optimizing frame structures [22]. They studied a new system response quantity referred to as "forced-mode compliance". They found that it is a very good indicator for evaluating the overall dynamic compliance of a shear building model subjected to harmonic ground motion. This work extended the work of Icerman [23] by deriving a set of necessary and sufficient conditions for global optimality of a model subject to constraints on the forced-mode compliance, fundamental natural frequency and minimum stiffnesses. They also obtained the optimal story stiffnesses in closed form by taking advantage of the characteristics of the shear building model. Lastly, they developed formulas for directly controlling the base shear coefficients, the maximum mechanical energy level and the level of the maximum relative story displacement due to harmonic excitation.

The practical significance of the forced-mode compliance may still be argued by earthquake engineers since it has been defined with respect to a highly idealized excitation together with an artificial frequency constraint. However, just as the fundamental natural frequency has played the primary role the design of most engineering structures, the forced-mode compliance may also be equally significant in controlling the fundamental dynamic characteristics of building structures.

The forced-mode compliance is a measure for evaluating the overall dynamic compliance (or stiffness) of the shear building model. It is derived by minimizing the forced steady-state vibration (developed by utilizing Rayleigh's principle) and is subjected to a frequency constraint. The optimum design problem is formulated with constraints on forced-mode compliance, fundamental natural frequency and minimum stiffnesses.

The method has some disadvantages. First, the modeling of the structure as a shear frame excludes the effect of the moment distribution on the nodes due to the flexibility of the beams. This reduces the constraining moment of the story columns. Second, the method assumes that all the columns of a story have the same stiffness. Third, the method does not solve the case in which all or some of the minimum story stiffness constraints are active. This requires that a systematic search procedure be included in the procedure or that the optimality conditions be rederived. The paper states that the use of a harmonic ground motion as the simulation of an earthquake is questionable and further research is required.

4) Nonlinear optimum design of dynamic damped frames

Cheng and Botkin present research including effects previously neglected in other works by considering the structure to be non-linear [24]. In doing so, they included the following effects: viscous damping (formulated using Raleigh damping), P-delta effects, and masses lumped at the floor levels. Changes in the mass during the optimization were also included. The columns used axial forces due to static loads while the girders included the forces due to dynamic loads. The model allowed both flexible columns and girders. Any type of general dynamic load was allowed including impulses, harmonic motions or earthquakes.

The method of analysis is based on modal superposition where dynamic amplification factors are obtained from a shock spectrum. The ordinates of these factors are computed by a finite difference technique. The standard stepwise linear displacement method is used to recover moments and stresses from the displacements. As a result, the problem has a linear objective function and general non-linear constraints. The method of optimization used is the gradient search. The objective is to minimize the weight of the structure. This goal creates constraints that depend on the material density, the section modulus, and the member length. It is inversely proportional to the section depth. By choosing the section modulus as the design variable, the objective function becomes linear.

The method attempts to take a more general approach to the optimization problem. The use of an algorithm for optimization based on a systematic search procedure is, in this type of problems, capable of being very demanding on computation time in frames with a high number of design variables. Also, the objective function is developed for steel frames, when it considers parameters for the physical properties of the section, restricting the general use of this method for other frames.

5) Optimum structural design with design constraints

Cassis and Schmit's studies aim at devising an efficient method for the optimum design of structures subjected to dynamic loads by treating dynamic response constraints parametrically [25]. In order to reduce the complexity of the dynamic analysis and to achieve computational efficiency, this work employs several new concepts. Of particular interest is the use of a process called variable linking. This method reduces the number of design variables by finding relationships between them. A second innovation is the use of Taylor series expansions to obtain explicit approximations for dynamic response quantities in terms of the design variables. Finally, the dynamic analysis is organized to take advantage of the repetitive nature of the design process.

The investigation reveals that the feasible design space associated with structural optimization in the dynamic response regime is usually disjoint. This important feature leads to the use of an exterior penalty function formulation. Successful implementation of the exterior penalty function formulation is facilitated by the use of dummy constraint boundaries and a new approach to move the limits of the problem. The optimization limits need to be moved due to the use of approximate analysis techniques based on the Taylor series expansions. The concept of moving the limit involves the adaptive shrinking or expansion of the feasible design region. This makes it possible to use one set of constraints to serve the dual purpose of representing the approximate behavioral constraints and the moved limit constraints. This is necessary since these two regions are usually disjoint. This technique makes it possible to generate a sequence of non-critical designs using an exterior penalty function formulation whenever a feasible design is available.

The researchers conclude that the sinusoidal contributions in the relationship between design variables and dynamic response functions can cause minimums of the exterior penalty which could be unfeasible. The design procedure was applied to several frames and the numerical results illustrate the effectiveness and remarkable efficiency of the structural synthesis capability developed by this method.

The problem with this method is that it uses approximation concepts from a static optimization problem. Techniques such as variable linking, time parametric constraint deletion and first order Taylor expansion series were used to approximate the dynamic response functions in explicit form. It is an attempt to find a more efficient, but specific, method to find the optimal solution. Due to the specific nature of the problem, the method uses an exterior penalty function formulation for the decision making process to obtain better points in the design space.

6) Optimum seismic design of linear shear buildings

Rosenblueth and Asfura present a very simplified method for the design of multi-degree-of-freedom structures under earthquake excitation [26]. It is based on two main steps: First, create of a design spectra. Second, optimize to produce a structure that satisfies a set of minimum cost conditions. The structure is modeled as a shear building and is assumed to behave linearly. The rigid column and flexible beam assumption is used. Constraint conditions are written on an individual story basis. At each story there are two conditions to be met, maximum story deflection and column stress. Column area, section modulus and moment of inertia are related to each other in an unique way. The building is treated as a single bay, symmetric structure resulting in the column moments of inertia as the only unknowns. This creates one unknown value per story.

It is assumed that the story shear is equal to the square root of the sum of the squared modal shears for that story. The iterative optimization procedure is begun from an initial design by performing a modal analysis yielding the story shears and deformations. Next, the stiffness required to make the stress and deformation constraints active in each story is computed. It is assumed that the shears do not change. The larger of the two calculated stiffnesses are used to form a new structure. These stiffnesses are multiplied by a coefficient that is optimized. This optimal coefficient is found by a graphic procedure relating the design velocity spectrum and the fundamental period of vibration. The iterative method used to find the optimum is a replacement for standard search methods.

The process makes simplifications in order to improve the computational time. The first is to reduce the frame to a single bay structure. The second is to assume a linear behavior of the material, probably steel. A different philosophy is used compared to that in general procedures. An iterative procedure is used, instead of a search algorithm, to optimize the structure. This tries to make the maximum possible number of constraints active at a given time. This iterative procedure creates a cyclic process requiring a modal analysis at each cycle. The modal analysis takes into account the changes of the story stiffnesses for a better design, but does not reevaluate the necessary changes in the beams. The authors say that although the process involves a modal analysis in each cycle, it converges rapidly if the design velocity spectrum increases with the period in the neighborhood of the fundamental period.

Facilities

All of the research will be performed on the Civil Engineering Department's computer facilities. The equipment is a recent donation from the AT&T Corporation. The department is the only non-computer or electrical engineering department nation-wide to recieve a donation. The donation consisted of a 3B5 computer with a math processing unit. The computer has the same processing capabilities as a VAX 11/780 computer. The computer was donated expressly for research purposes only. As a result, there will be no computer time costs.

Since the system belongs to the Civil Department, the entire capabilities of the machine are available for this research. This will allow research into the complete formulation without regard to problem size or execution time. Since these two variables can be excluded, a true assesment of the evaluated techniques can be made.

The preliminary software required to perform the research already exists. This work is a direct result of previous research. The research involved methods for creating re-usable analysis programs. This allows for rapid creation of the tools required to study a specific problem.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a thesis for the degree of Master of Engineering.

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