

## PREFACE

This special volume gathers a number of new contributions addressing various topics related to the field of optimal control theory and sensitivity analysis. The field has a rich and varied mathematical theory, with a long tradition and a vibrant body of applications. It has attracted a growing interest across the last decades, with the introduction of new ideas and techniques, and thanks to various new applications.

The idea for this special issue grew out of the European Marie-Curie ITN SADCO (Sensitivity Analysis and Deterministic Controller Design, <http://itn-sadco.inria.fr/>) on the occasion of its final conference NetCo (New trends in optimal Control, <http://netco2014.sciencesconf.org/>). Participants of NetCo and members of SADCO have been invited to contribute, thus guaranteeing an inflow of high quality papers in all areas covered by the issue. Moreover, the issue has been opened to contributions from outside SADCO project, as one of its aims is to reach the whole community of optimal control theory. The quality and the number of the contributions reveal the dynamism and intensity of the community's commitment in its development and its effective promotion.

Besides attesting the vitality of the optimal control theory, the volume aims at revealing the state of the art of some challenging issues in the targeted research area. The present volume contains 33 contributions that cover a wide range of topics. A partial list includes: state-constrained control problems, numerical methods and new applications in optimal control theory, Hamilton-Jacobi equations, mean field games, zero-sum repeated games, control problems on networks, stability of nonlinear systems.

While the papers are ordered in the the volume alphabetically by the last names of the first authors, we provide hereafter a short description of the contributions classified by their topics.

### **Theoretical optimal control.**

- *B. Bonnard, T. Combot and L. Jassionnesse* study the minimum time control of a chain three spins in quantum mechanics (coherence transfer problem). Using a lifting of the problem on  $SO(3)$ , they are able to provide an explicit parameterization of the extremals of the problem.

- *U. Boscain, G. Charlot, M. Gaye and P. Mason* study almost-Riemannian structures (that define a metric under Hörmander condition) in dimension three. In contrast with dimension two, abnormal extremals play a role and generic singularities are investigated.

- *A.V. Dmitruk and N. Osmolovskii's* paper is devoted to the Mayer problem in optimal control where the standard ode is replaced by a Volterra integral equation. The initial and final times are left free, and the authors provide necessary conditions for a suitable generalization of the notion of weak minimum.

- *E. Feleqi and F. Rampazzo* establish exact integral formulas for multi-flows of iterated brackets for a family of vector fields. Remarkably this is done for brackets of degree higher than two.

- *E. Goncharova and M. Staritsyn* add new results to optimal control problems with impulsive control. More precisely, they derive existence of solution and a new variant of the maximum principle for problems whose dynamics depends nonlinearly on impulsive controls.

- Stratified optimal control is investigated by *C. Hermosilla*, concerning ode's with discontinuous vector fields. Existence, uniqueness and robustness properties are studied in the framework of what the author refers to as stratified vector fields.

- By using metric regularity techniques, *R. Kipka and Y. Ledyev* derive optimality conditions for optimal control problems settled on a manifold, where the dynamics take the form of a general differential inclusion.

- *M. Palladino and R. Vinter* establish some results on characterization of relaxed and usual controls which are strong local minimizers, and proves that they satisfy some version of abnormal maximum principles (in an averaged form).

- *S.P.S. Santos, N. Martin and D.F.M Torres* investigate variational problems of Herglotz type with time delay. Necessary conditions are established, extending in particular the first Noether theorem.

#### **Hamilton-Jacobi equations. Mean field games.**

- *Y. Achdou and M. Laurière* investigate new results concerning the system of Fokker-Planck and Hamilton-Jacobi-Bellman equations arising from the finite horizon control of McKean-Vlasov dynamics. Based on this study, the authors propose some simple models for the motion of pedestrians and perform some numerical simulations in which they compare mean field games and mean field type control.

- *M. Assellaou, O. Bokanowski and H. Zidani* derive error estimates for second order Hamilton-Jacobi-Bellman equations when the value function is unbounded and when the final cost is discontinuous. This result is then used to study the approximation of probabilistic reachable sets.

- *P. Bettiol* addresses problems with an  $L^\infty$  cost and state constraints. The analysis relies on interpreting the optimal control problem as a static differential game and leads to the characterization of the value function as solution of a Hamilton-Jacobi equation with continuous Hamiltonian.

- A specific version of the problem “when does the meeting start?” has been considered by *F. Camilli, E. Carlini and C. Marchi*. In this contribution, the agents are constrained to move on a network and the problem is assumed to be of mean field type.

- *P. Cannarsa, M. Mazzola and C. Sinestrari* provide conditions for global propagation, which allow a higher dimensional state space. The previous known results in this area were of restrictive nature as they assumed that the state dimension is one.

- Semi-Lagrangian numerical schemes are already well known for the discretization of Hamilton-Jacobi-equations related to optimal control. *E. Carlini and F. Silva* show in their paper how such schemes can also be applied to for a degenerate second order Mean Field Game system.

### **Ergodicity and Large deviation principle.**

- *M. Akian, S. Gaubert and A. Hochart* provide an extension of the mean ergodic theorem to the case of zero-sum repeated games. In particular, the authors show that the mean payoff is independent of the initial state for all state-dependent perturbations of the rewards if and only if an ergodicity condition is verified. The latter can be checked in polynomial time when the number of states is fixed.

- *M. Bardi, A. Cesaroni and D. Ghili* investigate the short time behaviour of stochastic systems affected by a stochastic volatility evolving at a faster time scale. By using viscosity theory and homogenization methods, the authors point out three regimes depending on how fast the volatility oscillates relative to the horizon length. A large deviation principle is derived for each regime and applied to asymptotics of option prices near maturity.

- The paper by *M. Motta and C. Sartori* considers optimal control problems of both finite and infinite horizons. The behavior of value function in the finite horizon problem is analyzed when the horizon becomes large. Moreover, the authors determine the limit of a discounted infinite horizon problem as the discount factor approaches 0.

### **Discretization issues and numerical methods in optimal control.**

- The paper by *O. Bokanowski, M. Falcone, R. Feretti, D. Kalise, L. Grüne and H. Zidani* focusses on the convergence of abstract fixed point approximation schemes using the general assumptions of invariance to the addition of constants,  $\epsilon$ -monotonicity and consistency. This study provides a unified framework for high-order semi-Lagrangian methods and to filtered schemes as well.

- *A. Bressan and F. Yu* analyze a class of conservation law models for traffic flow on a family of roads, near a junction. The focus is mainly on the dependence of the Riemann solvers both on the density of vehicles and on the parameters describing the drivers' turning preferences.

- For mechanical systems, variational integrators provide an attractive class of numerical schemes for the discretization in time. In their paper, *C. Campos, S. Ober-Blöbaum and E. Trélat* explain the use of high-order methods from this class for optimal control.

- *G. Colombo and T.T. Le* focus on the minimal time optimal control problem, and provide sufficient conditions for reaching a target, in terms of higher order Lie brackets. From this construction, they infer a suitable discretization method to approach the minimum time function.

- Stability and robustness properties of multistep MPC schemes are discussed in the paper by *L. Grüne and V. Palma*. The novelty, with respect to standard MPC schemes, is the use of a shrinking horizon strategy that, by re-optimising at each sampling time, enhances robustness of the scheme.

- While semi-Lagrangian methods typically use traditional finite element interpolation techniques, the paper by *O. Junge and A. Schreiber* introduces a new spatial approximation technique using radial basis functions.

- Numerical methods for the optimal control of PDEs particularly aim at constructing low dimensional approximations in order to reduce the computational effort. To this end, *K. Kunisch and M. Müller* propose and investigate a new variant of the POD model reduction method.

- *L.T. Paiva and F.A.C.C. Fontes* develop an adaptive time-mesh refinement for the numerical solution of optimal control with state constraints. As opposed to direct methods that increase uniformly the number of points on equidistant meshes, their approach relies on a clever use of information on the dual variables to perform this refinement.

### **Interactions and applications of optimal control.**

- Optimal control and optimization techniques can be used for computing Lyapunov functions in order to analyse stability properties of differential equations. The paper by *J. Björnsson, P.A. Giesl, S. Hafstein and C. Kellett* shows that this can be done even in the case when the system exhibits multiple local attractors.

- A multidisciplinary application is the analysis of consensus behaviour, modelling, e.g., flocking behaviour in social sciences as well as opinions in social sciences. An optimal control approach to this problem is considered by *M. Bongini, M. Fornassie and D. Kalise*.

- *M. Caponigro, A.C. Lai and B. Piccoli* introduce a model for multi-agent systems confined to the unit sphere, modeling in such a way the opinion formation in a group of people. They investigate different notions of equilibria (“consensus”) and address feedback stabilization issues, where the control stands for an external intervention.

- *H. Maurer and W. Semmler* explore in their paper how optimal control techniques can be used in order to design policies for the transition from non-renewable to renewable energies.

- Another non-engineering application of optimal control techniques is studied by *C.J. Silva and D.F.M. Torres* who consider optimal treatment strategies in a Tuberculosis-HIV/AIDS coinfection model.

- *H. Schättler and U. Ledzewicz* propose a field of extremals approach to perform a sensitivity analysis for multi-input bilinear optimal control problems. Such problems appear in cancer chemotherapy and broken extremals arise as concatenations of bang and singular arcs.

- The paper by *A. Zatezalo, D. Stipanovic* develops a probabilistic methodology for control systems design under uncertainty. The presented setting consists in a controlled deterministic dynamical system interacting with a stochastic system from which only noisy observations are available. In this context, the authors present a methodology based on the probabilistic characterization of the secondary system, for the computation of control strategies maximizing a certain payoff, subject to a set of observations.

Guest Editors:

Jean-Baptiste Caillau, Maria do Rosário de Pinho,  
Lars Grüne, Emmanuel Trélat and Hasnaa Zidani