VRP with release dates and deadlines: a blood sample collection application

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Application Problem

- Collect blood samples from a set of geographically disperse healthcare centers and deliver them to a central laboratory for clinical analysis
  - Patients have blood samples extracted at healthcare centers
  - Healthcare centers have a time window during which blood is extracted
  - Healthcare centers store the extracted blood until collected
  - Blood samples are transported to a central laboratory for clinical analysis

(Uncapacitated) Vehicle Routing Problem with time windows (VRPTW)

Application Problem

- Blood is perishable – it must be analyzed within 150 minutes of extraction (lifespan):
  - Release date – blood extraction time
  - Deadline – release date plus lifespan (150 minutes) after which the blood is no longer viable

VRPTW with release dates and deadlines

Application Problem

- Dynamic VRPTW with release dates and deadlines – blood deadline is given by its release date plus its lifespan
  - Lifespan limits the time between healthcare center previous visit (opening time) and the delivery of the blood collected to the central laboratory in the subsequent (first) visit

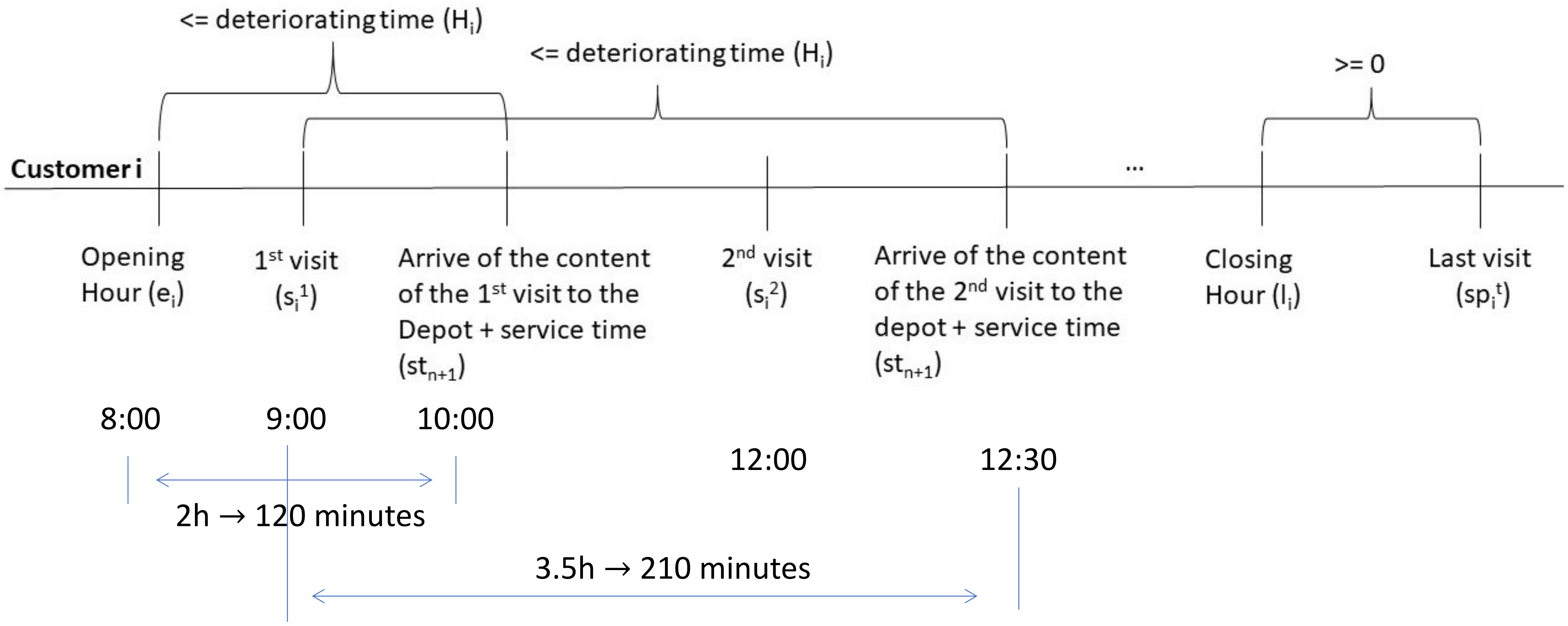
Dynamic VRPTW with release dates and deadlines

Application Problem

- Extraction time windows are larger than blood lifespan:
  - Multiple tours – each tour may visit any subset of the set of healthcare centers
  - Multiple visits to each healthcare center – the blood extraction time window is larger than the blood lifespan (viability time window)
  - Last visit to each healthcare center– must be after the end of the blood extraction time window

Dynamic VRPTW with release dates and deadlines and multiple tours and multiple visits

Dynamic VRPTW with release dates and deadlines and multiple tours and multiple visits



Problem characteristics.

Only tour duration is time constrained

	Doerner et al. 2008	Anaya-Arenas et al. 2016	Toschi et al. 2018	Anaya-Arenas et al. 2021	**Ours**
Biological degradation during transport	Yes	Yes	Yes	Yes	**Yes**
Biological degradation waiting in the center	Yes	No	No	No	**Yes**
Number of vehicles	infinite	infinite	infinite	infinite	**one**
Number of pickups at each center	fixed	fixed	variable	fixed	**variable**
Opening and closing time in the centers	defined	defined	undefined	undefined	**defined**
Tour time limit	No	Yes	Yes	Yes	**Yes**
Time limit between consecutive visits	No	Yes	Yes	Yes	**Yes**

Problem characteristics.

Tours are not interconnected: can start and finish at any time

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Problem characteristics.

Predefined and not optimized

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Opening and closing time in the centers	defined	defined	undefined	undefined	**defined**
Trip time limit	No	Yes	Yes	Yes	**Yes**
Time limit between consecutive visits	No	Yes	Yes	Yes	**Yes**

Problem characteristics.

Imposed after getting a solution

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Time limit between consecutive visits	No	Yes	Yes	Yes	**Yes**

Problem characteristics.

Not major changes, but may make sense drivers and service

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Time limit between consecutive visits	No	Yes	Yes	Yes	**Yes**

Problem formulation – notation

Sets and indices:

- $V = \{1, \dots, n\}$ - Set of *n* customer, indexed by *i*, *j* and *l*;
- $V' = V \cup \{0\}$ - Set of *n* customer and departure depot, indexed by *i*, *j* and *l*;
- $V'' = V \cup \{n + 1\}$ - Set of *n* customer and arrival depot, indexed by *i*, *j* and *l*;
- $W = V \cup \{0, n + 1\}$ - Set of *n* customers and departure and arrival depot (departure and arrival depots are the same), indexed by *i*, *j* and *l*;
- $A = \{(i, j) : i \in V', j \in V'', i \neq j\}$ - Set of arcs;
- T - Set of tours $\{1, \dots, U\}$ ($T' = T \cup \{0\}$), indexed by *k*, *s* and *t*;

Problem formulation – notation

Parameters:

- H - Product lifespan (time between production and delivery to the depot);
- st_i - Service time, customer i ∈ W (st_0 = 0);
- e_i - Production starting / opening time, customer i ∈ V ;
- l_i - Production ending / closing time, customer i ∈ V ;
- t_{ij} - Travel time of arc *(i, j)* ∈ *A*;
- d_{ij} - Travel distance of arc *(i, j)* ∈ *A*;
- D – Maximum tour duration;
- F_i – Maximum time between consecutive visits to customer i ∈ V ;
- $U = \sum_{i \in V} \left\lceil \frac{l_i - e_i}{H} \right\rceil$
- M – a sufficiently large integer.

Problem formulation – notation

Decision Variables & Auxiliary Variables:

- s_i^t - Pickup time, $i \in W$ in tour $t \in T'$, 0 if customer i is not visited in tour t (real);
- x_{ij}^t - set to 1 if $j \in V''$ is visited immediately after $i \in V'$ in tour $t \in T$; 0 otherwise;
- y^t - set to 1 if tour $t \in T'$ is used; 0 otherwise;
- v_i^{st} - set to 1 if tours $s \in T'$ and $t \in T$, $s < t$ are consecutive visits to $i \in V$; 0 otherwise;
- w_i^t - set to 1 if customer e $i \in V$ is visited in tour $t \in T$; 0 otherwise;
- z_i^t - set to 1 if tour t $\in T$ is the last visit to customer $i \in V$; 0 otherwise;
- sp_i^t - Last pickup time, $i \in V$ if tour $t \in T$ is its last tour, 0 otherwise (real).

Problem formulation – objective function

- Minimize the total traveled distance

$$\text{Minimize} \sum_{t \in T} \sum_{i \in V'} \sum_{j \in V''} x_{ij}^{t} \times d_{ij}$$

- Other possibilities
  - Minimize the total traveled time
  - Minimize the time of the last arrival to the central laboratory (how long the vehicle/driver is used)

Problem formulation – Tours

(1) $\sum_{i \in V} x_{0i}^{t} \leq 1,$ $\forall t \in T,$

(2) $\sum_{i \in V} x_{0i}^{t} = \sum_{j \in V} x_{j,n+1}^{t},$ $\forall t \in T,$

(3) $\sum_{j \in V'':j \neq i} x_{ij}^{t} \leq 1,$ $\forall \mathrm{i} \in V', t \in T,$

(4) $\sum_{j \in V'':j \neq i} x_{ij}^{t} = \sum_{j \in V':j \neq i} x_{ji}^{t},$ $\forall \mathrm{i} \in V, t \in T,$

(5) $y^{t} = \sum_{i \in V} x_{0i}^{t},$ $\forall t \in T,$

(6) $y^{t} \leq y^{t-1},$ $\forall t \in T: \mathrm{t} \geq 2,$

(7) $y^{1} = 1.$

Problem formulation – visiting times

(8) $s_0^1 \geq 0,$

(9) $s_0^t \leq \mathrm{M}\, y^t, \qquad \forall\, t \in T : \mathrm{t} \geq 2,$

(10) $s_0^t \geq s_{n+1}^{t-1} + st_{n+1} - M(2 - y^{t-1} - y^t), \quad \forall\, t \in T : t \geq 2,$

(11) $s_i^t \geq e_i - M\left(1 - w_i^t\right), \qquad \forall \mathrm{i} \in V, t \in T,$

(12) $s_j^t \geq s_i^t + st_i + t_{ij} - M\left(1 - s_{ij}\right), \qquad \forall i \in V', j \in V'', i \neq j, t \in T,$

(13) $s_i^t \leq \mathrm{M}\ w_i^t, \qquad \forall \mathrm{i} \in V, t \in T.$

Problem formulation – last visit to each customer

(14) $$\sum_{t \in T} z_i^t = 1, \qquad \forall i \in V,$$

(15) $$\sum_{t \in T} sp_i^t \geq l_i, \qquad \forall i \in V,$$

(16) $$sp_i^t \leq s_i^t - M(z_i^t - 1), \qquad \forall i \in V, t \in T,$$

(17) $$\sum_{t \in T} sp_i^t \geq s_i^t, \qquad \forall i \in V, t \in T,$$

(18) $$sp_i^t \leq M \times z_i^t, \qquad \forall i \in V, t \in T.$$

Problem formulation – goods delivered within lifespan

(19) $s_{n+1}^{t} + st_{n+1} - s_i^{s} \leq \mathrm{H} - \mathrm{M}(v_i^{st} - 1), \quad \forall i \in V, s \in T', t \in T, s < t,$

(20) $s_i^0 = e_i, \quad \forall i \in V,$

(21) $w_i^t = \sum_{j \in V'': j \neq i} x_{ij}^t, \quad \forall i \in V, t \in T,$

(22) $w_i^0 = 1, \quad \forall i \in V,$

(23) $v_i^{st} + \sum_{k=s+1}^{t} w_i^k \geq w_i^s + w_i^t, \quad \forall i \in V, s \in T', t \in T, s < t.$

Problem formulation – other time limitations and variables domain

(24) $s_{n+1}^{t} - s_{0}^{t} \leq D, \quad \forall t \in T,$

(25) $s_{i}^{t} - s_{i}^{s} \leq F_{i} - \mathrm{M}(v_{i}^{st} - 1), \quad \forall i \in V, s \in T', t \in T, s < t,$

(26) $s_{i}^{t}, sp_{i}^{t} \geq 0, \quad \forall i \in V, t \in T,$

(27) $x_{ij}^{t}, v_{l}^{st}, w_{l}^{s}, y^{t} \in \{0, 1\}, \quad \forall i \in V, j \in V'', l \in V, s \in T', t \in T.$

Case Study

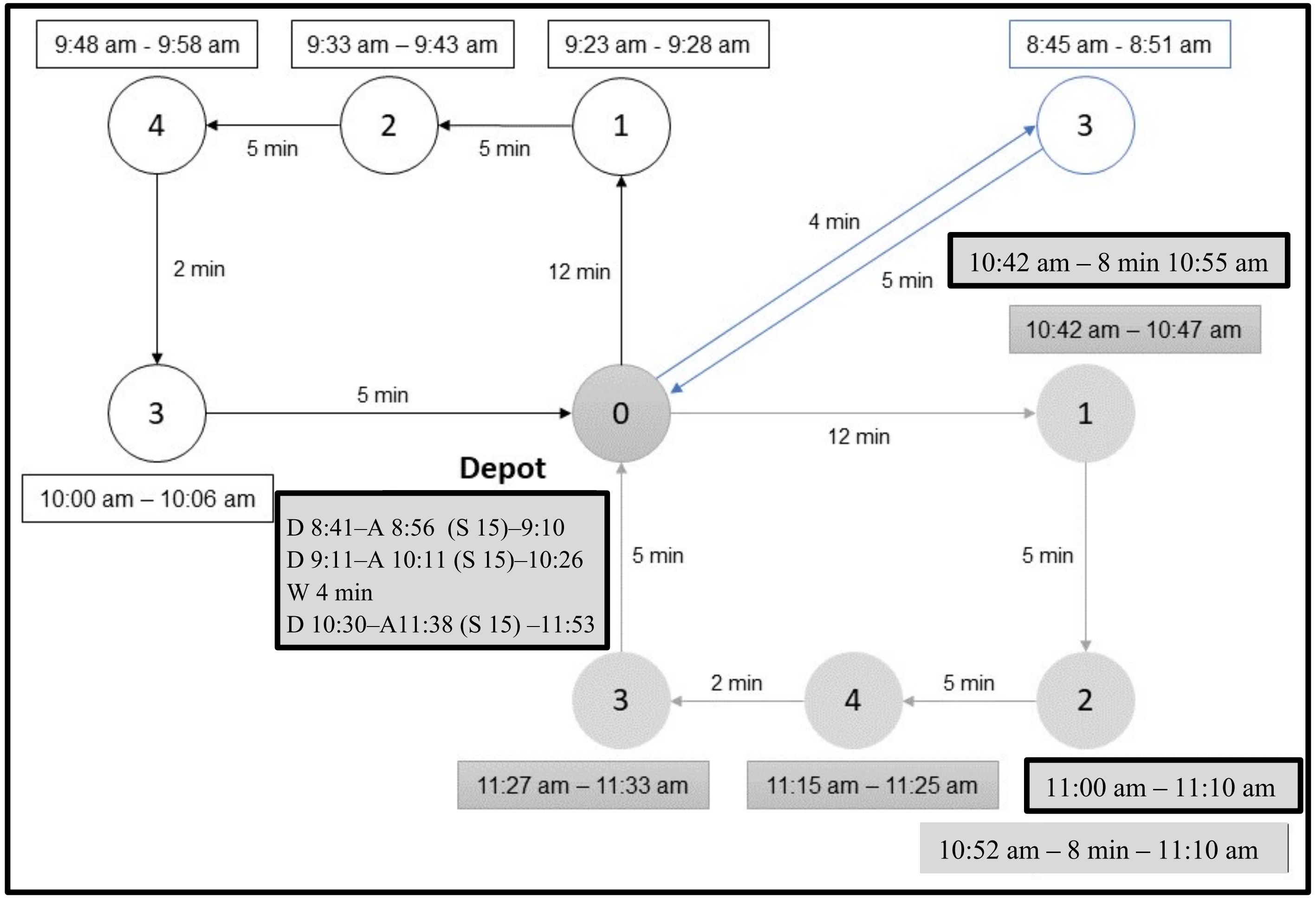
Problem faced in North of Portugal by a collection of community healthcare centers and a hospital in the region named Unidade Local de Saúde de Matosinhos (ULSM)

Implemented and solved in CPLEX

Working hours, service time (min) and travel time/distance (min/Km)

Health centers	Opening time	Closing time	Service time		Health center 1	Health center 2	Health center 3	Health center 4	Central Laboratory
Health center 1 São Mamede de Infesta	08:00	10:30	5		0	5	11	10	13
Health center 2 USF Porta do Sol	08:00	11:00	10		5	0	8	5	11
Health center 3 Matosinhos	07:30	11:00	6		12	10	0	5	5
Health center 4 Sra. Hora	08:00	11:00	10		10	6	2	0	4
Central Laboratory HPH	00:00	23:59	15		12	10	4	5	0

Solution of the case study



Centers	C1	C2	C3	C4
Open	08:00	08:00	07:30	08:00
Visit			08:45	
Delivery			09:11	
Time elapsed			09:11 – 07:30 101 < 150	
Visit	09:23	09:33	10:00	09:48
Delivery	10:26	10:26	10:26	10:26
Time passed	10:26 – 08:00 146 < 150	10:26 – 08:00 146 < 150	10:26 – 08:45 101 < 150	10:26 – 08:00 146 < 150
Closing	10:30	11:00	11:00	11:00
Visit	10:42 + 8 10:42	11:00 10:52 + 8	11:15	11:27
Delivery	11:53	11:53	11:53	11:53
Time passed	11:53 – 09:23 150	11:53 – 09:33 140	11:53 – 10:00 113	11:53 – 09:48 125

Minimum total travel distance: 67 km

Conclusions

- This work addresses a current and relevant problem with several applications in healthcare (collection of blood and/or other biological products).
- Extends previous works by considering release dates, deadlines, multiple tours, multiple visits, and (dynamic) time limitations.
- We propose a MILP model and solve a small case study.
- We are in the process of gathering other partners with similar problems in other application areas.
- A metaheuristic is being developed so that large instances can be solved.

Thanks for Your Attention

Q/A

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