Decomposition Methods in Economics to Assess which Covariates Matter

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Data Science and Statistics Webinar

23rd February 2021

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23rd February 2021 1 / 29

- Motivation
- Oaxaca-Blinder Decomposition
- Gelbach Decomposition
- Empirical Exercise

Decomposition methods in Economics aim to answer questions such as (Firpo et al., 2011):

- What are the most relevant factors that explain pay differences between men and women? Natives and Immigrants?
- What are the most relevant factors that explain wage inequality increases?
- What are the sources of the wage losses of displaced workers?
- The seminal papers by Oaxaca (1973) and Blinder (1973) with the aim of understanding the sources of the gender wage gap, became notorious in labor economics

- How much of the gender wage gap could be attributed to differences in human capital (education and experience), occupational choices, industry?
- In this framework, the "unexplained" part of the gender gap is often attributed to labor market discrimination
- An extensive set of methodological papers aimed at refining and extending the OB decomposition (Freeman (1984), Juhn et al. (1993), DiNardo et al. (1996), Machado & Mata (2005), Firpo et al. (2009))

- Let w_{Ai} and \mathbf{x}_{Ai} be respectively the wage and vector of observable characteristics of individual *i* belonging to the reference group A.
- The wage equation relative to this group is:

$$\ln w_{Ai} = \mathbf{x}_{Ai} \boldsymbol{\beta}_A + u_{Ai} \tag{1}$$

• Similarly, the wage equation relative to group B is:

$$\ln w_{Bj} = \mathbf{x}_{Bj} \boldsymbol{\beta}_B + u_{Bj} \tag{2}$$

• The regression model implies that the raw wage differential can be written as:

$$\Delta \overline{w} = \overline{\ln w_A} - \overline{\ln w_B} = (\overline{\mathbf{x}}_A - \overline{\mathbf{x}}_B)\widehat{\boldsymbol{\beta}}_A + \overline{\mathbf{x}}_B(\widehat{\boldsymbol{\beta}}_A - \widehat{\boldsymbol{\beta}}_B)$$
(3)

- The first term of the decomposition, $(\overline{\mathbf{x}}_A \overline{\mathbf{x}}_B)\widehat{\boldsymbol{\beta}}_A$, represents the "explained" component of wage differences between groups, also called the *composition effect*.
- The second term, x
 _B(β
 _A β
 _B), represents the "unexplained" component, also called the *wage structure effect*. It measures, for group B, the differences of return to characteristics due to membership in this group.

- The portion explained by discrimination using the Blinder-Oaxaca method depends on the reference group chosen.
- If B is the reference group instead of A, the explained part of the differences in average values of the wage logarithms is (x
 ⁻_A - x
 ⁻_B)β
 ⁻_B.
- As $\overline{\ln w_A} \overline{\ln w_B}$ is independent of the decomposition, the unexplained part changes according to the reference group.
- If group B is the reference group instead of group A, then $\Delta \overline{w} = (\overline{\mathbf{x}}_A \overline{\mathbf{x}}_B)\widehat{\boldsymbol{\beta}}_B + \overline{\mathbf{x}}_A(\widehat{\boldsymbol{\beta}}_A \widehat{\boldsymbol{\beta}}_B).$

- It is widely common in empirical research to estimate multiple specifications of a linear regression model to evaluate the "robustness" of the coefficient on some variable of interest, X_1 , as various covariates, X_2 , are added to the base specification
- Or to account for the "effects" on the coefficient of X₁ as one moves from a base specification that has no covariates, X₂, to the full specification that includes both X₁ and X₂
- Example: cross-group gaps in wages (e. g., Altonji & Blank, 1999; Lang & Manove, 2006, Carneiro et al., 2012)

Immigrant-native wage gap in Portugal: POLS (Carneiro, Fortuna & Varejão, 2012, JPE)

Dependent variable: log of the real hourly wage								
	(1)	(2)	(3)					
Male sample (N=7,706,548)								
Immigrant	-0.28643	-0.19083	-0.08249					
	(0.00164)	(0.00167)	(0.00152)					
Years Since Migration	0.01423	0.01089	0.00530					
	(0.00051)	(0.00048)	(0.00047)					
Years Since Migration squared	-0.00009	-0.00003	-0.00004					
	(0.00002)	(0.00002)	(0.00002)					
Concentration of Immigrants		-0.24440	-0.18876					
R-squared	0.438	0.490	0.590					
Female sample (N=6,060,863)								
Immigrant	-0.17755	-0.10620	-0.04398					
	(0.00168)	(0.00167)	(0.00144)					
Years Since Migration	0.00561	0.00422	0.00087					
	(0.00056)	(0.00052)	(0.00047)					
Years Since Migration squared	0.00019	0.00019	0.00013					
	(0.00003)	(0.00003)	(0.00003)					
Concentration of Immigrants		-0.22967	-0.14982					
		(0.00283)	(0.00245)					
R-squared	0.500	0.549	0.637					

Pooled OLS wage regression (selected estimates), 2003-2008

Notes: Worker-cluster robust standard errors in parentheses. Specification 1 includes controls for the worker's age (and age squared), immigrant status, years since migration, schooling achievement, and time and region effects. Specification 2 further controls for three employer characteristics: size, industry, and immigrant concentration at the workplace. Specification 3 adds controls for worker tenure (and tenure squared), and skill-categories.

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- As pointed out by Gelbach (2016, p. 529): "Comparing coefficient estimates from specifications in which covariates are added sequentially to a base model does not generally identify population parameters of interest.";
- In an insightful paper published in JOLE, Gelbach (2016) proposes a detailed decomposition based on the omitted variable bias formula that allows to unambigously disentangle the contibution of each ommited variable for the change in the coefficient of the variable of interest.

• Consider we are interested in estimating the gender wage gap

• We start by estimating the following model

In wage
$$_{it}=eta_{o}+eta_{1}^{base}$$
 female $_{i}+u_{it}$

•
$$eta_1^{\mathit{base}} = \mathsf{E}(\mathsf{ln} \ \mathit{wage}_{\mathit{it}} \mid \mathit{female} = 1) - \mathsf{E}(\mathsf{ln} \ \mathit{wage}_{\mathit{it}} \mid \mathit{female} = \mathsf{0})$$

 This may correspond to a biased measure of the gender wage gap as males and females may differ in other characteristics that also affect wages. • Let's add those factors to the base model

In wage
$$_{it} = eta_o + eta_1^{\mathit{full}}$$
female $_i + eta_2^{\mathit{full}}$ edu $c_i + eta_3^{\mathit{full}}$ expe $r_{it} + u_{it}$

- $\beta_1^{\textit{base}} \neq \beta_1^{\textit{full}}$
- How much of the change in β_1 can be attributed to *exper* and how much can be attributed to *educ*?
- How much of the male-female wage gap is explained by gender differences in experience and education?

Gelbach Decomposition - Gender Wage Gap

Researchers often solve this problem by estimating an intermediate model

In wage_{it} =
$$\beta_o + \beta_1^{educ}$$
 female_i + β_2^{educ} educ_i + u_{it}

- and then attribute the difference $\beta_1^{base}-\beta_1^{educ}$ to differences in education across the groups
- \bullet and the difference $\beta_1^{educ}-\beta_1^{full}$ to differences in labor market experience
- This is incorrect, as the order in which covariates are added can affect the accounting order/sequence dependence
- Only if the variable *female* is orthogonal to *educ* and *exper* will the order be irrelevant

Gelbach Decomposition - Omitted variable bias formula

- Gelbach (2016) provides a solution based on the omitted variable bias formula:

$$\widehat{\beta}_{2}^{\textit{full}}\widehat{\Gamma}_{\textit{educ}} + \widehat{\beta}_{3}^{\textit{full}}\widehat{\Gamma}_{\textit{exper}} = \widehat{\beta}_{1}^{\textit{base}} - \widehat{\beta}_{1}^{\textit{full}}$$

- Gelbach (2016) provides a solution based on the omitted variable bias formula:
 - Estimate the full model by OLS and obtain: $\hat{\beta}_1^{full}$, $\hat{\beta}_2^{full}$, $\hat{\beta}_3^{full}$
 - 2 Use OLS to estimate two auxiliary regressions of *educ* on *female* and of *exper* on *female* and then recover the estimates $\hat{\Gamma}_{educ}$ and $\hat{\Gamma}_{exper}$, respectively.

$$\widehat{\beta}_2^{\textit{full}} \widehat{\Gamma}_{\textit{educ}} + \widehat{\beta}_3^{\textit{full}} \widehat{\Gamma}_{\textit{exper}} = \widehat{\beta}_1^{\textit{base}} - \widehat{\beta}_1^{\textit{full}}$$

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 - Estimate the full model by OLS and obtain: $\hat{\beta}_1^{full}$, $\hat{\beta}_2^{full}$, $\hat{\beta}_3^{full}$
 - 2 Use OLS to estimate two auxiliary regressions of *educ* on *female* and of *exper* on *female* and then recover the estimates $\hat{\Gamma}_{educ}$ and $\hat{\Gamma}_{exper}$, respectively.
 - The male-female wage gap attributable to educ is $\hat{\beta}_2^{full} \hat{\Gamma}_{educ}$ and to experience is $\hat{\beta}_3^{full} \hat{\Gamma}_{exper}$

$$\widehat{\beta}_2^{\textit{full}}\widehat{\Gamma}_{\textit{educ}} + \widehat{\beta}_3^{\textit{full}}\widehat{\Gamma}_{\textit{exper}} = \widehat{\beta}_1^{\textit{base}} - \widehat{\beta}_1^{\textit{full}}$$

Gelbach Decomposition - Omitted variable bias formula

- Hence, the contribution of each variable are the mean male-female gap in *educ* and *exper* ($\hat{\Gamma}_{educ}$ and $\hat{\Gamma}_{exper}$), scaled by each regressor's impact on wages ($\hat{\beta}_2^{full}$ and $\hat{\beta}_3^{full}$), respectively.
- Consider the population regression model:

$$Y = X_1\beta_1 + X_2\beta_2 + \varepsilon$$

- Assuming that $E[\varepsilon \mid X] = 0$, where $X = [X_1, X_2]$
- Under these assumptions, the OLS estimator for the full model is $\widehat{\beta} = (X'X)^{-1}X'Y$ is consistent

Gelbach Decomposition - Omitted variable bias formula

 Now, consider the coefficient on X₁ from the base specification that ignores X₂. The estimator of β₁ is

$$\widehat{eta}_{1}^{\textit{base}} = (X_{1}^{'}X_{1})^{-1}X_{1}^{'}Y$$

whose probability limit is

$$p\lim\widehat{\beta}_{1}^{base} = \beta_{1} + p\lim(X_{1}'X_{1})^{-1}X_{1}'X_{2}\beta_{2}$$

or
$$\beta_1^{base} = \beta_1 + \Gamma \beta_2 = \beta_1 + \delta$$

where Γ is the matrix of coefficients from projecting the columns of X₂ on the columns of X₁.

- $\delta = \Gamma \beta_2$ is the omitted variable bias from excluding X_2 when estimating β_1 , and is non-zero if $E[X_1X_2] \neq 0$
- It also suggests a natural decomposition of the difference between the base and full model coefficients on X₁

$$\begin{split} \delta &= & \Gamma_1 \beta_{2,1} + \ldots + \Gamma_k \beta_{2,k} \\ \delta &= & \beta_1^{base} - \beta_1 \end{split}$$

- . *base model*
- . reg lwage fem

Source	SS	df	MS		r of obs	=	4,165 491.72
Model Residual	93.6914807 793.213421	1 4,163	93.6914807 .190538895	5 R-squ	> F ared	= =	0.0000 0.1056
Total	886.904902	4,164	.212993492	5	-squared MSE	=	0.1054 .43651
lwage	Coef.	Std. Err.	t	P> t	[95% Cor	nf.	Interval]
fem _cons	4744661 6.729774	.0213967 .00718	-22.17 937.29	0.000 0.000	516415 6.715697		4325171 6.74385

Image: Image:

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- . *fulll model*
- . reg lwage fem ed exp

Source	SS	df	MS		er of obs 4161)	=	4,165 698,84
Model Residual	297.147867 589.757034	3 4,161	99.049289 .14173444	1 Prob 7 R-sq	,	=	0.0000 0.3350 0.3346
Total	886.904902	4,164	.21299349	5		=	.37648
lwage	Coef.	Std. Err.	t	P> t	[95% Cor	nf.	Interval]
fem	4358154	.0185376	-23.51	0.000	4721589)	3994719
ed	.0752953	.0021448	35.11	0.000	.0710903	3	.0795003
exp	.0118283	.0005476	21.60	0.000	.0107547	7	.0129019
_cons	5.523389	.0324625	170.15	0.000	5.459745	5	5.587033

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Gelbach Decomposition - Stata Codes

Decomposition of changes in coefficients on x1 vars:

fem

into parts due to these groups:

g1 = ed

g2 = exp

	lwage	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
fem							
	g1	0008109	.0102937	-0.08	0.937	0209862	.0193644
	g2	0378397	.0065713	-5.76	0.000	0507192	0249602
	TC	0386507	.0109806	-3.52	0.000	0601721	0171292

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De	Decomposition of the Gender Wage Gap (Gelbach, 2016)									
	Decomposition of									
	the chang									
Base	Model	Full Model	Change	educ	exper					
Coef. e	estimate Co	oef. estimate	(1)-(2)							
(1)	(2)	(3)	(4)	(5)					
_	0.47447	-0.43581	-0.038651	-0.00081	-0.03784***					
				(0.01029)	(0.00657)					

Note: *** significant at the 1% level.

Immigrant-native wage gap in Portugal: Gelbach Decomposition (Carneiro, Fortuna & Varejão, 2012, JPE)

Immigrants at new destinations: how they fare and why

 Table 2
 Decomposition of the immigrant-native wage gap variation between specifications 1 and 3 (contribution of covariates omitted in specification 1)

	Male sample		Female sample)	
	Coefficient	SD	Coefficient	SD	
Concentration of immigrants	-0.0436*	(0.0005)	-0.0278*	(0.0005)	
Size	-0.0001	(0.0003)	-0.0020*	(0.0003)	
Industry	-0.0300*	(0.0003)	-0.0161*	(0.0004)	
Tenure	-0.0631*	(0.0005)	-0.0491*	(0.0003)	
Qualification levels	-0.0670*	(0.0005)	-0.0388*	(0.0006)	
Total	-0.2038*	(0.0009)	-0.1337*	(0.0010)	

Notes: The figures reported in the last row ('Total') are estimates of the difference of the wage gap between immigrants and natives obtained with specifications 1 and 3 (and the corresponding robust standard errors); they are equal to the arithmetic difference between the estimated coefficients of the Immigrant variable reported in Table 1. All other figures in the coefficients' columns are interpreted as the absolute contribution of the corresponding covariate for the observed change in the estimated coefficient of the Immigrant variable from specifications 1 to 3. For details, see Gelbach (2009)

*Significant at 1%

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- Sample (*Quadros de Pessoal* data set):
 - Treatmet group: 25 cohorts of workers who lost their jobs between 1988 and 2014 due to firm closure

- Sample (*Quadros de Pessoal* data set):
 - Treatmet group: 25 cohorts of workers who lost their jobs between 1988 and 2014 due to firm closure
 - Control group: individuals who were employed in firms that did not close in the 1986-2016 period

Methodology:

 High-dimensional FE model that accounts for worker, firm, job title, and match quality effects (returns to time-invariant characteristics of the worker-firm match)

$$w_{it} = \alpha_i + \lambda_{J(i,t)} + \theta_{F(i,t)} + \psi_{iF(i,t)} + \gamma_t + \beta \mathbf{X}_{it} + \sum_{k \ge -m} D_{it}^k \delta_k + \mu_{it}$$

Methodology:

 High-dimensional FE model that accounts for worker, firm, job title, and match quality effects (returns to time-invariant characteristics of the worker-firm match)

$$w_{it} = \alpha_i + \lambda_{J(i,t)} + \theta_{F(i,t)} + \psi_{iF(i,t)} + \gamma_t + \beta \mathbf{X}_{it} + \sum_{k \ge -m} D_{it}^k \delta_k + \mu_{it}$$

• Builds on Gelbach (2016) methodology, which appeals to the omitted variables bias formula, to compute a detailed decomposition of the contribution of each fixed effect to the change in the estimate of the displacement dummies.

Earnings Losses of displaced workers in Portugal: OLS Solution (Raposo et al., 2021, JHR)

	$\hat{\delta}_k^{ols}$	SE	$\hat{\delta}_{k}^{base}$	SE	δ_k^{full}	SE
	(1)	(.	2)	(3	3)
Specification 1						
D-10	-0.218	(0.003)	0.061	(0.002)	0.003	(0.001)
D.9	-0.178	(0.003)	0.061	(0.001)	0.008	(0.001)
D.8	-0.183	(0.003)	0.044	(0.001)	0.000	(0.001)
D.7	-0.135	(0.003)	0.042	(0.001)	0.005	(0.001)
D.6	-0.122	(0.003)	0.067	(0.001)	0.032	(0.001)
D-5	-0.159	(0.002)	0.032	(0.001)	-0.002	(0.001)
D.4	-0.171	(0.002)	0.024	(0.001)	-0.004	(0.001)
D-3	-0.176	(0.002)	0.021	(0.001)	-0.003	(0.001)
D-2	-0.181	(0.002)	0.018	(0.001)	-0.002	(0.001)
D-1	-0.185	(0.002)	0.015	(0.001)	-0.003	(0.001)
D ₀	-0.213	(0.002)	0.009	(0.001)	-0.008	(0.001)
D ₁	-0.156	(0.004)	-0.013	(0.001)	-0.024	(0.001)
D ₂	-0.219	(0.003)	-0.018	(0.001)	-0.011	(0.001)
D3	-0.241	(0.003)	-0.029	(0.001)	-0.005	(0.001)
D ₄	-0.240	(0.002)	-0.039	(0.001)	-0.002	(0.001)
Ds	-0.268	(0.003)	-0.048	(0.001)	0.001	(0.001)
D_6	-0.294	(0.003)	-0.056	(0.001)	0.002	(0.001)
D7	-0.334	(0.003)	-0.062	(0.001)	0.005	(0.001)
D ₈	-0.377	(0.003)	-0.072	(0.001)	0.006	(0.001)
D ₉	-0.373	(0.003)	-0.072	(0.001)	0.011	(0.001)
D ₁₀	-0.349	(0.003)	-0.067	(0.001)	0.013	(0.001)
R ²	0.11		0.89		0.92	
Specification 2						
Pre-displacement	-0.218	(0.003)	0.026	(0.000)	0.000	
Post-displacement	-0.178	(0.003)	-0.046	(0.000)	0.000	
R ²	0.11		0.89		0.92	
Specification 3						
Net	-0.106	(0.002)	-0.072	(0.000)	0.000	
R ²	0.11		0.89		0.92	
and EED)		D-6	CM/ab			01

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23rd February 2021 25 / 29

Earnings Losses of displaced workers in Portugal: OLS solution (Raposo et al., 2021, JHR)

			Decomp	osition of the wage loss into					
	Wage loss	Firm	n .	Match q	uality	Job ti	tle		
		FE		FE		FE			
	$\hat{\delta}_{k}^{base} - \hat{\delta}_{k}^{full}$	$\hat{\tau}_{k}^{\theta}$	SE	$\hat{\tau}_{k}^{\psi}$	SE	$\hat{\tau}_{k}^{\lambda}$	SE		
Specification 1									
D.10	0.058	0.016	(0.000)	0.014	(0.000)	0.028	(0.001)		
D.9	0.053	0.016	(0.000)	0.013	(0.000)	0.023	(0.000)		
D.,8	0.044	0.015	(0.000)	0.011	(0.000)	0.018	(0.000)		
D.7	0.037	0.013	(0.000)	0.011	(0.000)	0.013	(0.000)		
D.6	0.035	0.011	(0.000)	0.010	(0.000)	0.014	(0.000)		
D.5	0.034	0.012	(0.000)	0.009	(0.000)	0.012	(0.000)		
D.4	0.028	0.010	(0.000)	0.008	(0.000)	0.010	(0.000)		
D.3	0.024	0.008	(0.000)	0.007	(0.000)	0.008	(0.000)		
D.2	0.020	0.006	(0.000)	0.007	(0.000)	0.007	(0.000)		
D.1	0.018	0.005	(0.000)	0.007	(0.000)	0.006	(0.000)		
D ₀	0.017	0.005	(0.000)	0.006	(0.000)	0.005	(0.000)		
Di	0.011	0.014	(0.000)	-0.006	(0.001)	0.003	(0.000)		
D ₂	-0.007	0.006	(0.001)	-0.008	(0.001)	-0.004	(0.000)		
D ₃	-0.024	-0.003	(0.001)	-0.011	(0.000)	-0.010	(0.000)		
D4	-0.037	-0.011	(0.001)	-0.012	(0.000)	-0.014	(0.000)		
D ₅	-0.049	-0.017	(0.001)	-0.015	(0.000)	-0.017	(0.000)		
D ₆	-0.058	-0.023	(0.001)	-0.015	(0.000)	-0.020	(0.000)		
D ₇	-0.067	-0.026	(0.001)	-0.018	(0.000)	-0.023	(0.000)		
D ₈	-0.078	-0.031	(0.001)	-0.020	(0.001)	-0.028	(0.000)		
D ₉	-0.083	-0.031	(0.001)	-0.021	(0.001)	-0.031	(0.000)		
D ₁₀	-0.080	-0.027	(0.001)	-0.020	(0.001)	-0.033	(0.000)		
R ²		0.96		0.99		0.99			
Specification 2									
Pre-displacement	0.026	0.008	(0.001)	0.008	(0.001)	0.010	(0.001)		
Post-displacement	-0.046	-0.014	(0.002)	-0.015	(0.001)	-0.017	(0.001)		
R ²		0.96		0.99		0.99			
Specification 3									
Net	-0.072	-0.022	(0.001)	-0.023	(0.001)	-0.027	(0.001)		
R ²		0.96		0.99		0.99			

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23rd February 2021 26 / 29

The Sources of the Wage Losses of Displaced Workers (Raposo et al., 2021, JHR)

- Conclusions
 - Post-displacement monthly wages are, on average, 7.2 log points lower than pre-displacement wages;

The Sources of the Wage Losses of Displaced Workers (Raposo et al., 2021, JHR)

Conclusions

- Post-displacement monthly wages are, on average, 7.2 log points lower than pre-displacement wages;
- Sorting into lower paying job titles represents the largest component of the monthly wage loss of displaced workers, accounting for 37 percent of the total average monthly wage; firm and match effects account, respectively, for 31 and 32 percent of the total monthly wage loss;.

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