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MULTIOBJECTIVE DESIGN OF AUTOMOBILE COMPOSITE APPLICATIONS BASED ON FEASIBILITY ROBUSTNESS

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ABSTRACT

In the proposed multi-objective optimization approach the weight and the determinant of the variance-covariance matrix of the response of composite structures used in automobile applications are considered as performance and robustness functions, respectively. The Pareto front is built using a modified version of previously proposed hierarchical genetic algorithm with co-evolution of populations here denoted by MOGA-2D.

Keywords: Multi-objective optimization, automobile composite structures, weight, feasibility robustness, robust design.

INTRODUCTION

In theoretical developments of RDO, both the robustness of design objectives and the robustness of design constraints are usually studied, conceptually denoted by performance robustness and feasibility robustness. The goal of robust design is to optimize the mean performance commonly known as optimality, and minimize the variability of the performance function known as robustness (Zaman et al., 2011 and Ragavajhala and Mahadevan, 2013). In the proposed multi-objective optimization approach the weight and the determinant of the variance-covariance matrix of the response of composite structures used in automobile applications are considered as performance and robustness functions, respectively. The Pareto front is built using a modified version of previously proposed hierarchical genetic algorithm with co-evolution of populations (Conceição António, 2013). In this version, denoted by MOGA-2D, the evolution is based on the exchange data between two populations: a short population using local dominance and elitism and an enlarged population to store the non-dominated solutions.

BI-OBJECTIVE OPTIMIZATION BASED ON ROBUSTNESS FEASIBILITY

The fundamental objective of robust design is to improve the structural performance and to stabilise response performances by minimising the effects of the propagation of uncertainties. In the proposed approach applied to composite plate/shell structures the variability of both the maximum displacement

$$\bar{u} = Max(u_1, ..., u_r) \quad , \quad r = 1, ..., N_{dis} \tag{1}$$

and of the most critical *Tsai number*

$$\bar{R} = Max(R_1, ..., R_j) \quad , \quad j = 1, ..., N_{str} \tag{2}$$

being N_{dis} the total number of displacements and N_{str} the total number of points where the stress vector is evaluated on the composite structure. The stress analysis is performed using the *Tsai number* R_j calculated as the ratio between the failure (or maximum allowable) stress and the actual stress at the *j-th* point of the structure where the stress vector is evaluated. The *Tsai number* R_j is a function of the actual stresses and it is obtained by solving the interactive quadratic failure criterion of Tsai-Wu (S. Tsai 1987) as follows

$$\left(F_{ik}\, s_i\, s_k\right)R_j^2 + \left(F_i\, s_i\right)R_j = 1 \qquad i,k = 1,2,6 \tag{3}$$

where s_i is the *i-th* component of the stress vector, F_{ik} and F_i are strength parameters associated with unidirectional reinforced laminate defined from the macro-mechanical point of view (S. Tsai 1987).

The critical measures of the structural response considered in Equations (1) and (2) are included in the vector $\boldsymbol{\varphi} = \left(\overline{u}, \overline{R}\right)$. Since the displacement and stress constraints must be considered on optimal design formulation defining the feasibility of design space, the variability of both the critical values $\overline{u}$ and $\overline{R}$ are measures of feasibility robustness (Conceição António C and Hoffbauer L. 2007, 2008, 2009). So, in this work the evaluation of the response uncertainty is done in a simple and systematic way using the determinant of variance-covariance matrix $\mathbf{C}_{\boldsymbol{\varphi}}$ of structural response defined by

$$\mathbf{C}_{\boldsymbol{\varphi}} = \begin{bmatrix} \mathrm{var}(\overline{u}) & \mathrm{cov}(\overline{u},\overline{R}) \\ \\ \mathrm{cov}(\overline{u},\overline{R}) & \mathrm{var}(\overline{R}) \end{bmatrix} \tag{4}$$

In the proposed approach for robust design optimization of composite structures, the feasibility robustness of the system is searched together the minimization process of performance/cost. The goal is to minimise the sensitivity of the optimal performance/cost of the system associated with the response to the uncertainty on the feasibility of constraints. A bi-objective optimization is performed by considering the following objective functions: a) a function describing the performance/cost of the structural composite structure and b) a function describing the feasibility robustness of constraints related to the variability of the structural response.

The design and uncertainty rules of the proposed RDO approach are controlled by following classes of variables and parameters: the vector of deterministic design variables, $\mathbf{d} \in \mathbf{R}^k$, the vector of random design variables, $\mathbf{z} \in \mathbf{R}^m$, and the vector of random parameters, $\boldsymbol{\pi} \in \mathbf{R}^p$. The nominal values of random design and random parameters are taken to be the expected values $\boldsymbol{\mu}_{\mathbf{z}}$ and $\boldsymbol{\mu}_{\boldsymbol{\pi}}$, respectively, and the associated uncertainties are given by the corresponding standard deviations. No probability distribution functions are considered in the present analysis.

The design variables intervening in the optimization procedure are the deterministic design variables, $\mathbf{d}$, and the nominal/expected values $\boldsymbol{\mu}_{\mathbf{z}}$ of the random design variables, $\mathbf{z}$. The standard deviation of $\mathbf{z}$ is kept constant during the optimization procedure.

The performance/cost of the composite structure is given by its weight $W(\mathbf{d}, \boldsymbol{\mu}_\mathbf{Z})$. The functional $V(\mathbf{d}, \boldsymbol{\mu}_\mathbf{Z}, var(\bar{u}), var(\bar{R}), cov(\bar{u},\bar{R}))$ is a measure of feasibility robustness, which is concerned with ensuring that the constraints are adequately satisfied under uncertainty (Salazar and Rocco, 2007, Ragavajhala and Mahadevan, 2013). The bi-objective optimization problem can then be established as

$$\underset{\text{over } \mathbf{d},\, \boldsymbol{\mu}_\mathbf{Z}}{\text{Minimise}} \quad OBJ\left(\mathbf{d}, \boldsymbol{\mu}_\mathbf{Z}, \mathbf{C}_\boldsymbol{\varphi}\right) = \left(f_1, f_2\right) \tag{5}$$

with

$$f_1 = W(\mathbf{d}, \boldsymbol{\mu}_\mathbf{Z}) \quad \text{and} \quad f_2 = V(\mathbf{d}, \boldsymbol{\mu}_\mathbf{Z}, var(\bar{u}), var(\bar{R}), cov(\bar{u},\bar{R})) = det\ \mathbf{C}_\boldsymbol{\varphi}$$

subject to

$$g_1(\mathbf{d}, \boldsymbol{\mu}_z) = \frac{\bar{u}(\mathbf{d}, \boldsymbol{\mu}_z)}{u_a} - 1 \le 0$$

$$g_2(\mathbf{d}, \boldsymbol{\mu}_z) = 1 - \frac{\bar{R}(\mathbf{d}, \boldsymbol{\mu}_z)}{R_a} \le 0 \tag{6}$$

and

$$d_j^l \le d_j \le d_j^u, \quad j = 1, \ldots, \bar{N}_\mathbf{d}$$

$$\mu_{z_j}^l \le \mu_{z_j} \le \mu_{z_j}^u, \quad j = 1, \ldots, \bar{N}_\mathbf{Z} \tag{7}$$

being $\bar{u}$ and $\bar{R}$ the critical displacement and critical Tsai number both of them defined by Equation (1) and Equation (2), respectively. These critical values are compared with the allowable values u_a and R_a for displacement and Tsai number, respectively. In this approach the feasibility robustness of composite structures is associated with the variability of the structural response, V defined as the determinant of variance-covariance matrix $\mathbf{C}_\boldsymbol{\varphi}$ of the system defined on Equation (4) of propagation of uncertainties. In the inequalities (7) $\bar{N}_\mathbf{d}$ and $\bar{N}_\mathbf{Z}$ are the number of deterministic and random design variables, respectively.

The performance/cost $W(\mathbf{d}, \boldsymbol{\mu}_z)$ depends on deterministic design variables and/or random design variables (throughout their nominal/expected values). The feasibility robustness associated with the variability of the structural response, $V(\mathbf{d}, \boldsymbol{\mu}_\mathbf{Z}, var(\bar{u}), var(\bar{R}), cov(\bar{u},\bar{R}))$ depends on both deterministic/random design variables and also on random parameters of the system.

Uncertainties in different groups of random variables and/or random parameters show distinct behaviours and importance on structural response variability during RDO search (Conceição and Hoffbauer, 2007, 2008, 2009). In particular, the definition of feasibility robustness depends on the groups of random design variables and/or random parameters considered on optimization process loop. At the end of the RDO optimization process, the Pareto front representing the frontier of the trade-off between the “performance” and the “robustness” functions is obtained.

The multi-objective optimization search is performed using on a new proposed approach based on two levels of dominance concepts (Deb, 2001 and Conceição, 2013) denoted by Bi-level Dominance Multi-Objective Genetic Algorithm (MOGA-2D). The Pareto front is built and such a challenge is performed here using a modified version of previously proposed hierarchical genetic algorithm with co-evolution of two populations (Conceição António, 2013). The approach proposed in this work uses a mixture of developed techniques (Conceição António, 2009, 2013) and new techniques in order to find multiple Pareto-optimal solutions in parallel using two populations (short and enlarged). The principal aspects are: (i) the use of the concept of Pareto dominance in order to assign scalar fitness values to individuals; (ii) the clustering through the co-evolution of a short population to reduce the number of non-dominated solutions stored without destroying the characteristics of the Pareto-optimal front; and (iii) the storage of the obtained Pareto-optimal solutions in an enlarged population; (iv) exchange of information between short and enlarged populations through the crossover operator.

The evolutionary process of MOGA-2D is performed by four genetic operators: mutation, crossover, replacement due to genetic similarity and selection (Conceição António, 2013). The binary code format is used to encoding the phenotype of design variables. The stopping criterion is based on reaching the minimum number of generations without improvement of Pareto front of enlarged population. The algorithm performs using the concept of local dominance at short population (SP) and storing the new generated non-dominated individuals/solutions (rank 1) from SP sorting, into an enlarged population (EP). The enlarged population is continuously updated based on global dominance concepts and has two principal functionalities: to build the global Pareto front and to transmit its best member's genetic properties to the next populations of the evolutionary process.

RESULTS AND ANALYSIS

To study the capability of the proposed approach for bi-objective optimization based on feasibility robustness, an engine hood of a car built using a shell laminated structure is considered. The shell structure is considered symmetric being a half part represented in Figure 1. So, symmetry boundary conditions are applied on linear side (AB) of the structure. The nodes belonging to the elements 41, 42, 51 and 52 of the engine hood shell are supporting vertical loads of mean value $P_k = 100\,N$. The non-linear side (CD) is constrained in the z-axis direction.

The structure is divided into eight macro-elements, grouping all elements, and there is one laminate per each macro-element. The laminate distribution of the structure is shown in Table 1. The balanced angle-ply laminates with eight layers and the stacking sequence $\left[+a/+a/-a/-a\right]_S$ are considered in the symmetric composite construction. Ply angle, a, is a design variable and is referenced to the x-axis of the reference axis, as detailed in Figure 1. The design variable h_i, denotes the laminate thickness and four laminates are considered in this example. A smoothing procedure is followed at the boundary of laminates to guarantee the continuity of structure.

The structural analysis of laminated composite structures is based on the shell finite element model developed by Ahmad with further improvements. This shell element is obtained from a 3D finite element using a degenerative procedure. It is an isoparametric element with eight nodes and five freedom degrees per node based on the Mindlin shell theory (Conceição António C and Hoffbauer L. 2007, 2008, 2009).

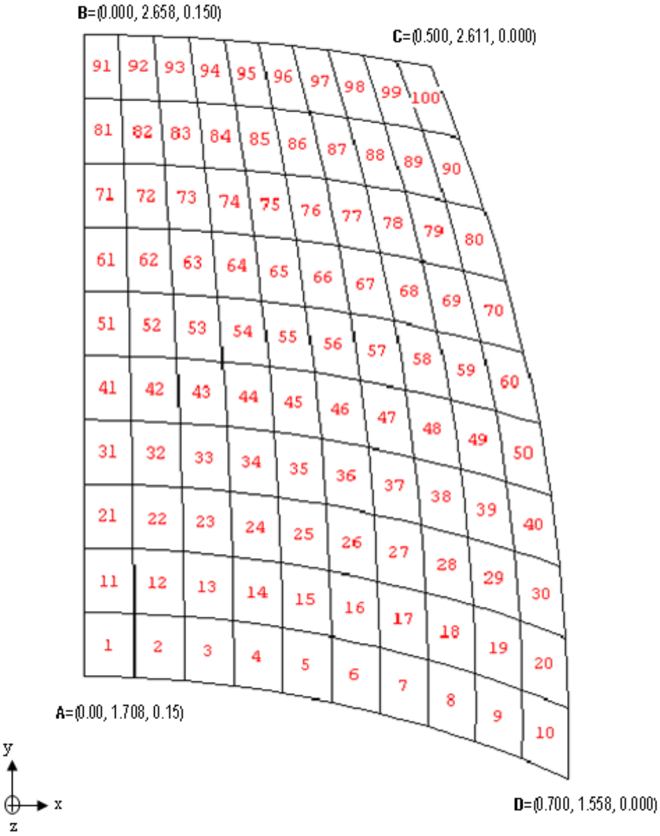

Fig. 1 - Geometric definition and discretization of engine hood composite shell

Table 1 - Laminates definition for engine hood composite shell

Laminates	Elements (in Fig. 1)
1	1/2/11/12/21/22/31/32/41/42/51/52/61/62/71/72
2	3/4/5/6/13/14/15/23/24/33
3	7/8/9/10/16/17/18/19/20/25/26/27/34/35/36/43/44/ 45
4	28/29/30/37/38/39/40/46/47/48/49/50
5	56/57/58/59/60/67/68/69/70/78/70/80
6	53/54/55/64/65/66/75/76/77/86/87/88/89/90/99/10 0
7	63/73/74/83/84/85/93/94/95/96/97/98
8	81/82/91/92

A composite system, the glass/epoxy composite Scotchply 1002 (S. Tsai 1987) is used in the presented analysis. This is a unidirectional glass long fibres aggregated in a epoxy matrix. The macro mechanics mean values of the elastic and strength properties of the ply material used in the symmetric laminate construction of the composite structure are presented in Table 2.

The elastic constants of the orthotropic ply are the longitudinal elastic modulus E_1, the transversal elastic modulus E_2, the in-plane shear modulus G_{12}, and the in-plane Poisson's ratio ν_{12}. The ply strength properties are the longitudinal strength in tensile, X, and in compression, X', the transversal strength in tensile, Y, and in compression, Y', and the shear strength, S.

Table 2 - Mean values of mechanical properties of composite layers

Material	E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	ν_{12}
Scotchply 1002	38.60	8.27	4.14	0.26
	X ; X' [MPa]	Y ; Y' [MPa]	S [MPa]	ρ □[kg/m3]
Scotchply 1002	1062 ; 610	31 ; 118	72	1800

The design variables are encoded using a binary code format with different number of digits. The genetic parameters used at short population evolution and the design variables constraint intervals are defined in Table 3.

Table 3 - Genetic parameters and design variables constraint intervals

Population size	21
Elite group size	7
Mutation group size	4
Number of generations	300
Code format (digits nr.) / size constraint interval, for ply angle a	4 / [0º, 90º]
Code format (digits nr.) / size constraint interval, for laminate thickness, h_i , $i = 1, \cdots, 4$	5 / [0.005m , 0.015m]

The RDO problem based on weight minimization and feasibility robustness maximization formulated from Equation (5) to Equation (7) is solved using the MOGA-2D approach proposed. The optimization process evolves along 300 generations. The allowable values in the constraints on displacement and Tsai number are $u_a = 5.0 \times 10^{-2} m$ and $R_a = 1.$, respectively.

In this studied case, the variance properties of the response of engine hood composite shell structures are associated with two sources of uncertainty: on random design variables $\mathbf{z}$ and on random parameters $\boldsymbol{\pi}$ of the structural system. They are organized in following four groups with allowable tested variations:

Group 1 of the mechanical properties, **m** defined as random parameters;

Group 2 of the ply angle, a on laminates, defined as random design variable;

Group 3 of the laminate thicknesses, $\mathbf{h}$ defined as random design variable;

The mechanical properties group, **m** includes the following random parameters: longitudinal Young's modulus $E_{1,j}$, transversal modulus $E_{2,j}$, transversal tensile strength Y_j, and shear strength S_j, where subscript j denotes the laminate number. Thirty two mechanical properties are considered as random parameters with uncertainty in this analysis: $E_{1,j}$, $E_{2,j}$, Y_j, S_j, j=1, …, 8. This random parameters are aggregated in vector $\boldsymbol{\pi}$.

Five random design variables are considered in vector $\mathbf{z}$ for this case study: one ply angle a for all symmetric laminates with the stacking sequence $\left[+a/-a/+a/-a\right]_S$, and the laminate thicknesses h_i , $i=1,\cdots,8$. So, it can be written,

$$\mathbf{z}=\left(a,\ h_1,\cdots,\ h_8\right) \tag{8}$$

The variability is referred to the expected values $\boldsymbol{\mu}_{\mathbf{z}}$ corresponding to the design solution value obtained at each generation of the optimization procedure. However, a prescribed and fixed standard deviation is allowed for these random design variables.

Since the expected values $\boldsymbol{\mu}_{\mathbf{z}}$ are not fixed during the optimization process, prescribed fixed standard deviations are used to consider the uncertainty in random design variables $\mathbf{z}$. On contrary, the coefficients of variation $\mathrm{CV}(\boldsymbol{\pi})$ are used to prescribe the uncertainty of the random parameters $\boldsymbol{\pi}$ having means and standard deviations fixed at the beginning of the optimization process. Thus, the variability in input variables/parameters are prescribed as follows:

- Group 1: The mechanical properties group (**m**), with the prescribed coefficient of variation, $\mathrm{CV}(\mathrm{m_i})=6\%$, $i=1,\cdots 16$;
- Group 2: The ply angle group (a), with the prescribed standard deviation, $\sigma(\alpha)=5^o$;
- Group 3 The laminate thickness group $(\mathbf{h})$, with the prescribed standard deviation, $\sigma(h_i)=1.2\times10^{-3}m$, $i=1,\cdots,8$;

The RDO problem formulated from equation (29) to equation (31) is solved using the proposed MOGA-2D approach. In this case the RDO problem is formulated as:

$$\underset{\text{over } \boldsymbol{\mu}_{\mathbf{z}}}{\text{Minimise}} \quad OBJ\left(\boldsymbol{\mu}_{\mathbf{z}},\mathbf{C}_{\boldsymbol{\varphi}}\right)=\left(f_1,\ f_2\right) \tag{9}$$

with

$$f_1=W(\boldsymbol{\mu}_{\mathbf{z}}) \quad \text{and} \quad f_2=V(\boldsymbol{\mu}_{\mathbf{z}},\ var(\bar{u}),\ var(\bar{R}),\ cov(\bar{u},\bar{R}))=det\ \mathbf{C}_{\boldsymbol{\varphi}}$$

subject to

$$g_1(\boldsymbol{\mu}_{\mathbf{z}}) \ = \ \frac{\bar{u}(\boldsymbol{\mu}_{\mathbf{z}})}{u_a}-1 \ \le \ 0$$

$$g_2(\boldsymbol{\mu}_{\mathbf{z}}) \ = \ 1-\frac{\bar{R}(\boldsymbol{\mu}_{\mathbf{z}})}{R_a} \ \le \ 0 \tag{10}$$

and

$$\mu^{l}_{z_j} \leq \mu_{z_j} \leq \mu^{u}_{z_j} \ , \quad j = 1,\dots,\overline{N}_{\mathbf{z}} \tag{11}$$

The robustness feasibility functional depends on the expected values of random design variables vector $\boldsymbol{\mu}_{\mathbf{z}}$, and on the derivatives of $\boldsymbol{\varphi} = \left(\overline{u}, \overline{R}\right)$ in order to random design variables and random parameters also calculated at expected value vector $\boldsymbol{\mu}_{\mathbf{z}}$, as follows:

$$det\,\mathbf{C}_{\boldsymbol{\varphi}} = det\left(\mathbf{S}\ \mathbf{C}_{x}\ \mathbf{S}^{T}\right) = f_2\left(\boldsymbol{\mu}_{\mathbf{z}},\ \partial\overline{u}/\partial\,\mathbf{z}\Big|_{\boldsymbol{\mu}_{\mathbf{z}}}, \partial\overline{R}/\partial\,\mathbf{z}\Big|_{\boldsymbol{\mu}_{\mathbf{z}}}, \partial\overline{u}/\partial\,\boldsymbol{\pi}\Big|_{\boldsymbol{\mu}_{\mathbf{z}}}, \partial\,\overline{R}/\partial\,\boldsymbol{\pi}\Big|_{\boldsymbol{\mu}_{\mathbf{z}}}\right) \tag{12}$$

The bi-objective optimization problem based on minimizations of weight and variability appears to have contradictory objectives. The proposed approach considering weight minimization and feasibility robustness maximization (minimum variability) show its effectiveness, with the solutions shared along the optimal Pareto front as shown in Figure 2. The same picture shows the coefficient of variation of the critical displacement, $CV(u)$, along the optimal Pareto front.

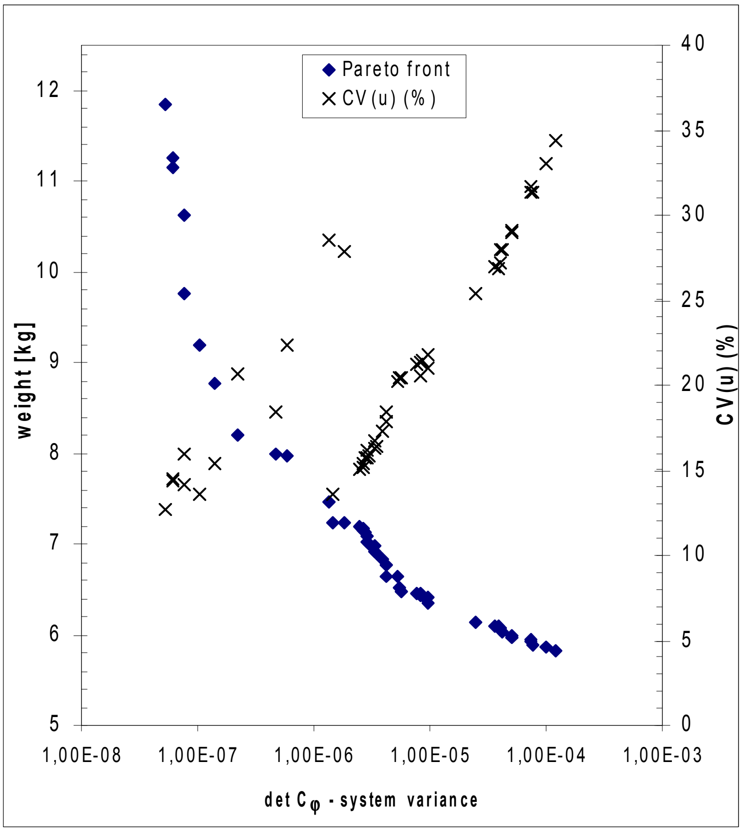

Fig. 2 - Optimal Pareto front and $CV(u)$ for critical displacement

The analysis shows that the proposed MOGA-2D approach is a powerfully tool to help designers to make decision establishing the priorities between performance and robustness.

CONCLUSIONS

The evaluation of the response uncertainty is done in a simple and systematic way using the variance-covariance matrix of structural response of composite shell structures. Uncertainties in different groups of random design variables and/or random parameters show distinct behaviours and importance on structural response during robust design optimization (RDO) search of composite structures. RDO searches for minimum weight (performance) and safe structural systems with minimal variability in the response defined as feasibility robustness, when subjected to uncertainties at the input design variables and/or input parameters.

The Multi-objective optimization search is based on a proposed Bi-level Dominance Multi-Objective Genetic Algorithm (MOGA-2D), which uses two levels of dominance concepts and two populations with exchange of data. At the end of the optimization process the Pareto front representing the frontier of the trade-off between the “performance” and the “feasibility robustness” functions is obtained. The combination of uncertainty sources is very important for design rules established from optimal Pareto front. In particular, for a fixed weight/cost the best minimum system variability can increases in several orders of magnitude when combining the uncertainty sources.

Finally, the analysis shows that the proposed MOGA-2D approach is a powerfully tool to help designers to make decision establishing the priorities between performance and robustness.

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