Calibration of bi-planar radiography with minimal phantoms

Daniel C. Moura[1,2], Jorge G. Barbosa[2], João Manuel R. S. Tavares[3], and Ana M. Reis[4]

[1] INEB – Instituto de Engenharia Biomédica, Laboratório de Sinal e Imagem
FEUP, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
`daniel.moura@fe.up.pt`
[2] U. do Porto, Faculdade de Engenharia, Dep. de Engenharia Informática
Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
`jbarbosa@fe.up.pt`
[3] U. do Porto, Faculdade de Engenharia, Dep. de Eng. Mecânica e Gestão Industrial
Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
`tavares@fe.up.pt`
[4] SMIC – Serviço Médico de Imagem Computorizada, S. A
Rua Pedro Hispano, 881, 4250-367 Porto, Portugal
`docmaf@sapo.pt`

Abstract. In this paper we propose a method for the geometrical calibration of bi-planar radiography that aims at minimising the impact of calibration phantoms on the content of radiographs. These phantoms are required for determining scale or to estimate the geometrical parameters of the system. Unfortunately, they often overlap anatomical structures. For accomplishing this goal, we propose a small extension to conventional imaging systems: a distance measuring device that enables to estimate some of the geometrical parameters. This leads to a reduction of the search space of solutions, which makes possible reducing requirements of calibration phantoms.

The proposed method was tested on 17 pairs of radiographs of a phantom object of known dimensions. For calculating scale, only a reference distance of 40mm was used. Results show a RMS error of 0.36mm with 99% of the errors inferior to 0.85mm. Additionally, the requirements of the calibration phantom are very low when compared with other methods, but experiments with anatomical structures should be conducted to confirm these results.

Key words: computer-generated 3D imaging – radiography – calibration

1 Introduction

Nowadays, Computer Tomography (CT) is the gold standard for 3D reconstructions of bone structures. However, CT scans may not be used for accurate reconstructions of large bone structures, such as the spine, because of the high doses

of radiation that are necessary. Additionally, when compared to radiography, CT scans are more expensive, more invasive, less portable, and require patients to be lying down. Therefore, using radiography for obtaining 3D reconstructions and accurate measurements remains an interesting alternative to CT.

Currently, it is possible to do 3D reconstructions of the spine [1, 2], pelvis [3], distal femur [4] and proximal femur [5] with minimal radiation by subjecting the patient just to two radiographs. For achieving this, all these methods require a calibration procedure that must be executed for every examination in order to capture the geometry of the x-ray imaging system. Usually, this calibration is performed using very large phantoms that surround the patient and introduce undesirable objects into radiographs. These phantoms are neither practical nor affordable. Not surprisingly, efforts have been made to use smaller phantoms [6–8], or even to eliminate them at all [9, 10]. Currently, and to our knowledge, no method is capable of accurate reconstructions without using phantoms. Kadoury et al. were able to calculate angular measures from spine radiographs without using any phantom, but absolute measures scored very poor results and were considered to be unreliable [10]. As for methods that use small phantoms, reconstruction errors remain considerably higher than when using large phantoms, and a significant number of undesirable objects is still visible in radiographs.

The method proposed in this paper tries to show that it is possible to obtain accurate calibrations of bi-planar radiography using very small phantoms that produce minimal changes to radiographs. For helping accomplishing this goal, a small extension to conventional systems of x-ray imaging is proposed: a distance measuring device that provides good estimates of some of the geometrical parameters of the system. For accurately assessing this method, experiments were conducted on a phantom object of known geometry.

2   Methods

2.1   Radiography calibration

In a bi-planar x-ray system, the projection of a 3D point in each of the two radiographs may be calculated as:

$$\begin{bmatrix} w_i \cdot u_i \\ w_i \cdot v_i \\ w_i \end{bmatrix} = M_i \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \text{for } i = 1, 2 \tag{1}$$

where for each acquisition i, M is the calibration matrix that describes the projection of the 3D point (X, Y, Z) into image coordinates (u, v) subjected to a scaling factor w. For flat x-ray detectors, M may be modelled as:

$$M_i = \begin{bmatrix} f_i/s & 0 & u_{p_i} & 0 \\ 0 & f_i/s & v_{p_i} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathcal{R}_i & t_i \\ 0^T & 1 \end{bmatrix} \tag{2}$$

where f is the focal distance (the distance between the x-ray source and the detector), s is the known sampling pitch of the detector, (u_p, v_p) is the principal point (2D projection of the x-ray source in the image), and $\mathcal{R}$ and t define the geometrical transformation that aligns the object coordinate system with the source coordinate system. More precisely, t is a translation that may be decomposed in (t_x, t_y, t_z) and $\mathcal{R}$ is a 3×3 rotation matrix that depends of three angles: an α rotation around the X axis, a β rotation around the Y axis, and γ rotation around the Z axis.

The goal of the calibration procedure is to find the optimum values of the calibration parameters:

$$\xi_i = (f_i, u_{p_i}, v_{p_i}, t_{x_i}, t_{y_i}, t_{z_i}, \alpha_i, \beta_i, \gamma_i) \quad \text{for } i = 1, 2. \tag{3}$$

When not using calibration objects, this is usually done by minimising the retro-projection error of a set of point matches marked in the two images [9, 10]. This problem may be formulated as a least-squares minimisation:

$$\min_{\xi_1^*, \xi_2^*} \left(\sum_{i=1}^{2} \sum_{j=1}^{n} \| p_{ij} - prj(\xi_i, tri(\xi_1, \xi_2, p_{1j}, p_{2j})) \|^2 \right), \tag{4}$$

where n is the number of marked point matches, p_{ij} is the jth point marked in image i, prj is the 2D projection of a 3D point as defined in equation 1, tri is a triangulation operation that calculates the 3D coordinates for a given point match, and (ξ_1^*, ξ_2^*) denotes the best parameters estimate found.

Calibration may be accomplished using a standard nonlinear least-squares minimisation algorithm. This class of algorithms needs an initial solution for the calibration parameters, which are then iteratively updated towards reducing the sum of squared distances between the marked and retro-projected points. Unfortunately, the search space of solutions is very large and this procedure gets easily trapped in local minima.

2.2 Narrowing the search space of solutions

In order to reduce the search space of solutions without introducing phantoms, a distance measuring device was attached to the x-ray machine that allows to estimate the focal length (f) and the distance between the object and the x-ray source (t_z). Figure 1 illustrates how the device is attached to the x-ray machine. The device is only capable of measuring the distance between the x-ray machine and the table (d_m). In order to calculate f, parameters d_s and d_d are determined using a procedure described elsewhere[5]. This procedure uses a set of radiographs of a phantom object with known geometry that are acquired at different distances (d_m) while keeping the rest of the parameters constant. Then, for each radiography, the optimum calibration parameters are found for different values of d_s, which enables to determine the optimum d_d for every

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[5] Paper already submitted, waiting for notification.

tested d_s. Finally, data from all radiographs is crossed, which enables to find the values of d_s and d_d that are similar for all setups. Both parameters are fixed for a given system independently of the setup and therefore this procedure only needs to be executed once for that system.

Knowing d_s and d_d enables to accurately calculate f and the distance between the x-ray source and the table. This last distance is used to estimate the initial value of t_z – the distance between the x-ray source and the object (which is usually located near the table).
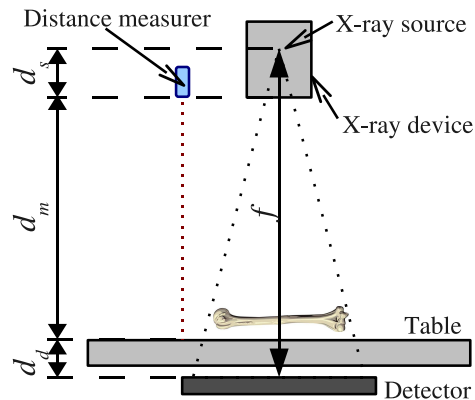

Fig. 1. Illustration of a conventional radiographic imaging system with a measuring device attached.

2.3 Correcting scale

In our experiments, the proposed calibration procedure as presented so far, only enables to determine up to scale solutions for the 3D coordinates. For correcting scale, a reference measure is needed. Such measure may be obtained by a small phantom composed by only two radiopaque objects placed at a known distance, which should be attached to the patient and be visible in both radiographs. The scaling factor may be calculated as the ratio between the real distance between the two radiopaque objects and the distance between the reconstructed 3D coordinates of the same objects.

3 Experiments and Results

For evaluating the proposed method, we used a phantom made of stainless steel (AISI 304) with dimensions of 380x380x1mm and laser cut squares of 20.0 ± 0.1mm (figure 2). Eight radiographs of the phantom where taken, with the phantom at different positions and with different orientations. Distance

d_m was the same for all setups ($d_m = 909$mm), resulting in a focal length of $f = 1257.7$mm and in a distance between the x-ray source and the table of 1183.0mm. Distance d_m was measured with a laser measuring device (typical error of ±1.5mm, maximum error of ±3.0mm, and range of operation of 0.05–50m), which was also used in the procedure for determining d_d and d_s. Film size was of $14''\times17''$ (355.6 $\times$ 431.8mm), scanned with a sampling pitch of 175.0 μm/pixel, resulting in images with a resolution of 2010 $\times$ 2446 pixels.

The eight radiographs were combined in a total of 17 pairs (out of 28 possible combinations). Only pairs of radiographs with considerably different pose were considered. Pairs of radiographs with near pose were discarded because are less tolerable to triangulation errors (when pose is similar triangulation lines tend to intersect at infinity because they are close to parallel).

As stated previously, the minimisation algorithm requires an initial estimation of the calibration parameters. Parameters (u_p, v_p) were initialised with the 2D coordinates of the image centre, and t and $\mathcal{R}$ where roughly estimated in the following way:

- t_x and t_y were always initialised with zero (we assumed that the object was roughly centred in the radiography);
- t_z was initialised with $d_s + d_m$ when its centre was near the table; when the centre was farther away from the table due to its pose, half of the object width was subtracted;
- α, β, and γ were roughly provided in a $10°$ resolution scale.

Figure 2 shows three examples of the initial guesses for three radiographs, and table 1 summarises the errors for all of them. These errors were calculated by comparing the initial guess for each radiography with the parameters that achieved optimal solutions (minimum projection error) when projecting a 3D model of the phantom on the respective radiography.
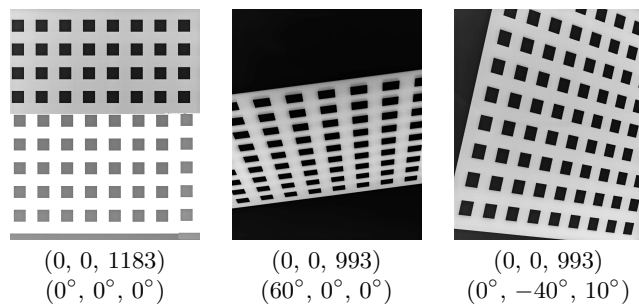

(0, 0, 1183)   (0, 0, 993)   (0, 0, 993)
$(0°, 0°, 0°)$ $(60°, 0°, 0°)$ $(0°, -40°, 10°)$

Fig. 2. Example of three radiographs of the phantom (1, 4 and 6) and the initial guess for (t_x, t_y, t_z) in mm and (α, β, γ) in degrees.

As stated in the previous section, the calibration process tries to minimise the retro-projection errors of a set of point matches marked in a pair of images.

Table 1. Absolute errors for position (t_x, t_y, t_z) and orientation (α, β, γ) of the initial guess for each radiography.

Setup	t_x (mm)	t_y (mm)	t_z (mm)	α (deg)	β (deg)	γ (deg)
1	26.2	6.2	40.1	0.1	0.2	0.0
2	14.5	22.1	40.3	0.1	0.3	2.8
3	30.5	20.4	14.6	0.1	3.3	1.9
4	9.6	34.1	13.1	3.2	3.9	6.9
5	11.0	11.5	58.6	0.1	2.4	4.8
6	16.8	26.1	8.8	3.8	2.7	2.5
7	17.5	4.9	0.1	0.1	0.3	0.1
8	15.9	43.2	0.0	0.1	0.3	0.2
Mean	17.7	21.0	21.9	0.9	1.7	2.4
SD	7.2	13.4	21.6	1.6	1.6	2.5
Max	30.5	43.2	58.6	3.8	3.9	6.9

We used the corners of the squares of the phantom that are visible in both images, which in average originated 199 point matches. Corners coordinates where extracted semi-automatically with the Camera Calibration Toolbox for Matlab[6] and then optimised with OpenCV[7].

For determining the scaling factor, small phantoms were simulated using reference distances of 40mm, which correspond to two consecutive squares of the phantom grid. For the first experiment we tested scaling with 50 reference distances uniformly distributed by the part of the phantom that was visible in both radiographs. First, the 3D coordinates of every point match were calculated using the optimised parameters. Then, the 3D points were scaled (using a reference distance) and aligned with a 3D model of the phantom. Finally, 3D errors were computed as the Euclidean distance between the calculated point and the corresponding point in the model. Figure 3 shows an histogram of the errors for the complete experiment and figure 4 presents the relation between errors and cumulative observations. RMS error was 0.36mm and 99% of the errors were inferior to 0.85mm.

A second experiment was carried out for testing the proposed procedure in less optimal conditions of point matches identification. This was done by adding uniformly distributed noise to the 2D coordinates of the phantom's corners on every radiography. Then, the previous experiment was repeated (17 pairs of images × 50 reference distances) starting with no noise, and adding up to ±15 pixels to each coordinate of every point. We decided not adding noise to the points that defined the reference distance because in real cases these points would be represented by two radiopaque objects that would be easy to identify accurately. Results are presented in figure 5 showing the relation between pixel noise and mean 3D error.

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[6] http://www.vision.caltech.edu/bouguetj/calib_doc/
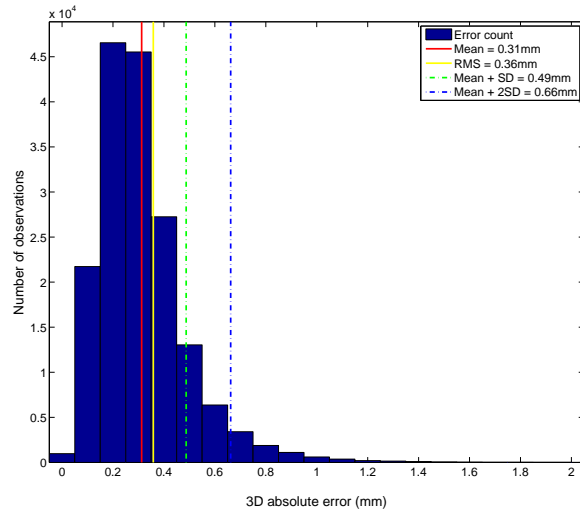[7] http://www.intel.com/technology/computing/opencv/

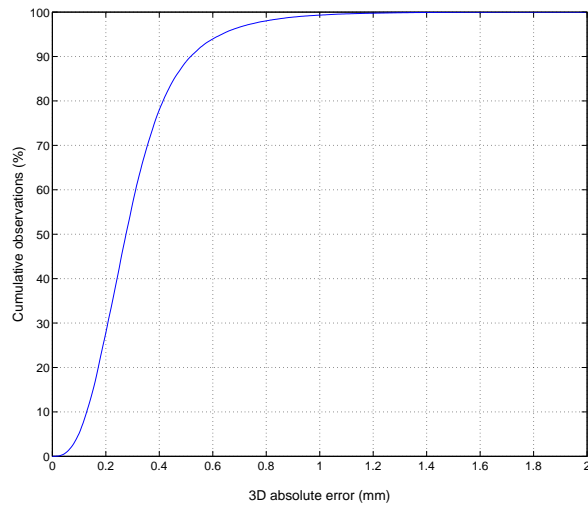**Fig. 3.** Histogram of the 3D errors of the phantom reconstruction.

Fig. 4. Relation between cumulative observations and the 3D error of the phantom reconstruction.
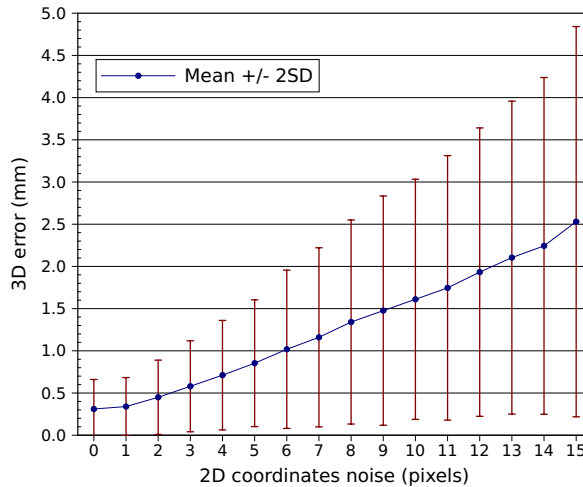
Fig. 5. 3D reconstruction errors vs. noise in landmarks location.

4 Discussion and Conclusions

This paper presents a method for bi-planar radiography calibration that uses a distance measuring device for narrowing the search space of solutions. This enhancement enables to improve the calibration performance of conventional x-ray imaging systems without affecting radiographs and with minor inconvenience for patients and technicians. Results show that this method is robust and offers sub-millimetric accuracy even when the initial guess of the calibration parameters is rather rough. Results also show that the quality of the calibration depends of the quality of point matches identification in radiographs. However, this dependence is close to linear for the tested range of noise, and when uniformly distributed noise of ± 5 pixels is added to each coordinate of every point, the RMS error remains inferior to 1mm. This demonstrates the robustness of the proposed method since it achieves acceptable errors even when using a pessimist distribution of noise where all values are equiprobable.

For achieving these results a small calibration object is needed (we used a 40mm reference distance in our experiments). Its only role is determining the scaling factor that should be applied to obtain real-world units. Thus, this object may be discarded if the goal is only to obtain shape, or angular and relative measurements. When compared to other works [6–8], the proposed dimensions of the calibration object are the smaller, as well as the impact in the content of the radiography. However, and in contrast with the other methods, the proposed method was not yet evaluated neither by *in vivo* nor by cadaveric experiments and therefore is not guaranteed that the calibration object requirements remain so low. For this same reason, reconstruction results may not be compared with these methods, which scored RMS errors superior to 2mm with real data (with

the exception of [8] that does not include 3D error evaluation). Even though, the proposed algorithm shows very encouraging results if we have in consideration the amount of noise in the initial guess of the parameters, and its behaviour to noise on point matches.

A possible disadvantage of this technique is that it requires an estimation of the rotation and translation parameters. This may not be a problem for some kind of examinations, such as spine radiography, where frontal and lateral radiographs are usually acquired. In these cases an initial estimation is very simple to obtain because the spine is typically centred on the radiography and the subject only experiences a 90° rotation around one of the axis. Either way, for best results the two acquisitions should not be taken with near poses for preventing triangulation errors.

In future work, we plan simulating spine radiography for determining the probable outcome of the proposed technique in this kind of examination, followed by experiments with anatomical structures (vertebrae) to assess the calibration quality in more realistic conditions.

Acknowledgements

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