Pontryagin's Maximum Principle for Impulsive Control Problems with Mixed Constraints: Controlling System Jumps

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Our work is devoted to the investigation of Pontryagin's Maximum Principle [1] for constrained impulsive control problems with nonclassical impulsive controls that incorporate the so called adjoint regular controls, i.e., controls of the conventional type that enable to steer the state trajectory of the dynamical system as the jump develops.

Our problem formulation includes mixed constraints. Important issues in impulsive control problems with mixed constraints have not been satisfactorily investigated. To the best of our knowledge, there are no general results for nonlinear and nonconvex smooth problems in the literature, and, in particular, no necessary conditions of optimality have been derived in this context. In our work we propose theorems that generalize previous results and fill this gap.

To be more precise, we focus on a simple and complete proof of the Maximum Principle for an impulsive control problem with generalized mixed constraints of the type $R(x, u, t) \in C$, where C is a closed convex set, and no convexity assumptions on the dynamics are assumed. The proof of the main result, Theorem 1, is based on Ekeland's variational principle and on a certain specific penalty function method. In Theorem 2 we prove a Maximum Principle, in which the regularity assumption on the data of the problem is considerably weakened. In the proof of Theorem 2, we use the so called method of discontinuous time-variable change initially proposed in [2].

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