Matching contours in images using curvature information

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ABSTRACT: The work here described consists in searching for the optimum global matching between contours of two objects represented in images, which are sampled by equal or different number of points. Thus, to determine the optimum global matching between the points of two contours, it is used curvature information, that is totally invariant to rigid transformations. For the case of contours sampled by different numbers of points, two approaches are proposed to exclude, from the matching process, the additional points. In the last section of this paper, a method for the determination of the rigid transformation associated to two contours that is based in the same solution considered in our matching process is also described.

1 INTRODUCTION

The determination of correspondence between the data of two objects represented in images is a topic of raised importance and hard research in Computational Vision (Maciel 2002, Tavares 2001). In image processing and analysis, the applications that need the determination of correspondence between objects are numerous. Some examples that can be referred are: tracking and movement analysis (Bastos et al. 2006, Pinho et al. 2005, 2007, Tavares et al. 2000, Tavares 2001), 3D reconstruction (Azevedo et al. 2006, Maciel 2002), recognition (Tavares et al. 2000, Vasconcelos et al., 2006), registration (Vasconcelos et al., 2006), etc.

In this paper, a methodology to determine the matches between points of contours represented in images is presented. The referred methodology uses curvature information and optimization of the global matching cost.

2 MATCHING OF CONTOURS

2.1 *Equal number of points*

As it is well known, rigid transformations or, using the mathematical definition, transformations of similarity, can produce changes in an object, as in its position, scale or orientation, but they do not imply changes in its form. An invariant feature that persists between two objects, when one is obtained from the other by a rigid transformation, is the curvature information.

Being two objects to be matched represented by a set of points that define the polygon associated with the contour of each one, the curvature information along the same can be estimated using the angle defined for each collection of three consecutive points. To calculate the angle associated to each point, it is also important to consider the direction along the contour in which the curvature is being analyzed, otherwise, strange situations can happen, as exemplified in figure 1. Thus, the assumption that the points to be matched are correctly ordered is important, see figure 2.

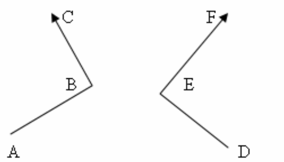

Figure 1: In our matching methodology the direction in which the contours are being analysed is considered and, therefore, considering the parts of two contour represented, we have $A\hat{B}C = 270^\circ$ and $D\hat{E}F = 90^\circ$; instead, if the corrected direction was not considered, we have $A\hat{B}C = 90^\circ = D\hat{E}F$.

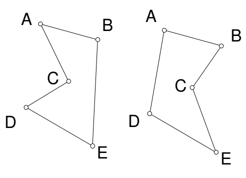

Figure 2: Two different contours configurations that are defined by the same set of points.

Let us considerer that to point i of a *contour 1* has associated the amplitude of angle α_i, and to point j

of a *contour 2* correspond the amplitude of angle θ_i. Consequently, along these two contours to be matched, it can be considered the association between each point and the angle of its curvature. It should be noted that the sequences of angles obtained for each contour depends only on its shape, and not on its scale or position in the original image.

The following step of our matching methodology consists in searching for the correspondences between angles of the two contours with the minor global matching cost associated. That is, the goal of this step is to minimize the sum of absolute values of the angular differences between matched points.

Thus, we have an assignment problem, but with a fundamental constraint that prevent the application of the traditional assignment algorithms: the order of the points that defines the two contours should be maintained. Therefore, the methodology proposed in (Maciel 2002), for instance, can not be used. For that reason, it is necessary to develop a new matching algorithm that we explain next.

Let us consider that the two contours to be matched are sampled by *n* points. Using our approach, we obtain two sequences of angular amplitudes associated with the curvature along each contour to be matched:

$$\alpha_1, \alpha_2, ..., \alpha_n \text{ and } \theta_1, \theta_2, ..., \theta_n.$$

The next step is to test the *n* matching hypotheses successively, figure 3, saving the associated matching values in the following matrix of costs:

$$\begin{bmatrix} |\alpha_1-\theta_1| & |\alpha_1-\theta_2| & \ldots & |\alpha_1-\theta_n| \\ |\alpha_2-\theta_2| & |\alpha_2-\theta_3| & \ldots & |\alpha_2-\theta_1| \\ \ldots & \ldots & \ldots & \ldots \\ |\alpha_n-\theta_n| & |\alpha_n-\theta_1| & \ldots & |\alpha_n-\theta_{n-1}| \end{bmatrix}.$$

Therefore, the sum of the elements of the first column of the matrix of costs represents the angular cost of the first global matching; the sum of the elements of the second column represents the angular cost of the second global matching, and so forth. With this approach, the best matching will be given by:

$$\min\left\{\sum_{i=1}^{n}|\theta_i-\alpha_i|; \sum_{i=1}^{n}|\theta_{i+1}-\alpha_i|; \ldots; \sum_{i=1}^{n}|\theta_{i+n-1}-\alpha_i|\right\},$$

with $\theta_{i+n}=\theta_i$.

Thus, in order to use our method, the points of each contour to be matched should be previously ordered, and the two contours should be analyzed in a common direction. Thus, reflections between contours to be matched are not possible.

Notice that the point *i* of the *contour 1* does not have to correspond to the point *i* of the *contour 2*. The only condition that has to be respected is that, for example, if point *i* of *contour 1* corresponds to point *j* of *contour 2*, then its subsequent neighbour in *contour 1* (point *i+1*) has to correspond to subsequent neighbour of point *j* in *contour 2* (point *j+1*). Thus, this restriction of order reduces the hypothesis of our matching problem from $n!$ to *n*.

In figure 3, a graphical representation of our matching searching methodology is made. The associated idea is the following: the two contours are "overlapped" and one is "rotated" successively in order that the best fit between the curvature angles is found.

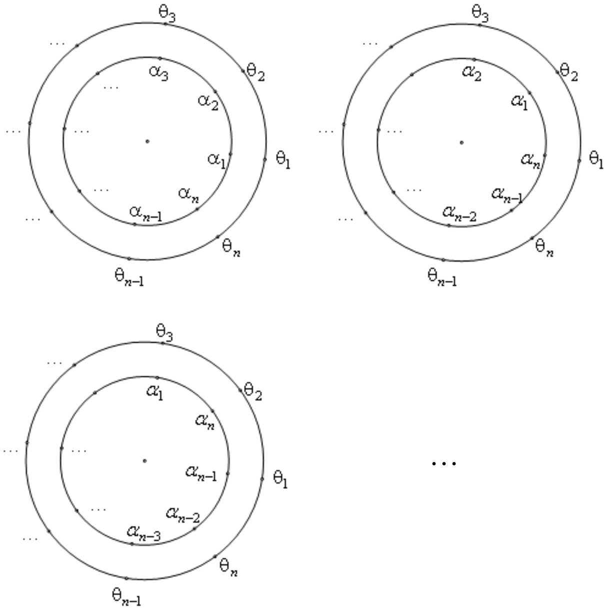

Figure 3: Example of searching for the best global matching between two contours sampled by equal number of points.

In figure 4, we can see an adequate matching found between two contours using our methodology. In this example, one of the contours is rotated 175º relatively to the other one.

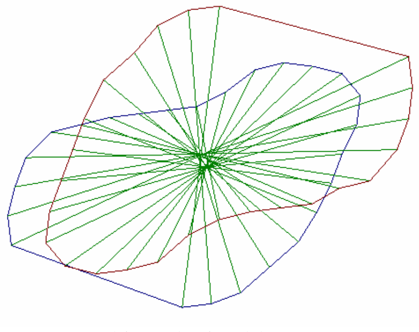

Figure 4: Matching obtained between two contours that have a rotation of 175º between them (lines between contours represent the matches found).

2.2 *Different number of points*

If a contour has more points than the other one, for example, *contour 1* has only *n* points and *contour 2* has *m* points, with $n < m$, then an approach to reject the $m-n$ extra elements from the angles' sequence of *contour 2*, $(\theta_i)_{i=1,2,...,m}$ is necessary.

In this work, we developed two approaches to discard the angles associated with the exceeded points. In both, we use the same methodology described previously to get the angular differences between the possible correspondences.

Thus, first, the matrix of angular costs for the m possible global matches is obtained:

$$\begin{bmatrix} |\alpha_1 - \theta_1| & |\alpha_1 - \theta_2| & \ldots & |\alpha_1 - \theta_{m-n}| & \ldots & |\alpha_1 - \theta_m| \\ |\alpha_2 - \theta_2| & |\alpha_2 - \theta_3| & \ldots & |\alpha_2 - \theta_{m-n+1}| & \ldots & |\alpha_2 - \theta_1| \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ |\alpha_n - \theta_n| & |\alpha_n - \theta_{n+1}| & \ldots & |\alpha_n - \theta_m| & \ldots & |\alpha_n - \theta_{n-1}| \end{bmatrix}.$$

Once again, the main idea is to imagine that the sequences of angles are "overlapped" and one of them is "rotating", finding, this way, the best global correspondence between them, figure 5.

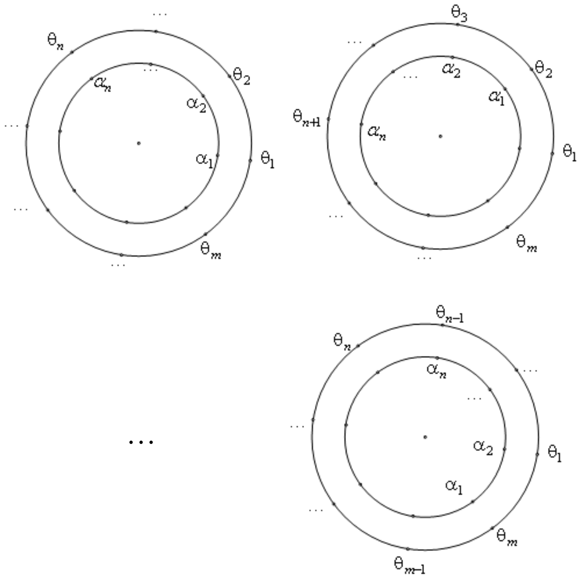

Figure 5: Example of searching for the best global matching between two contours sampled by different number of points.

If necessary, in the matching procedure adopted, points can be eliminated from the contour that presents more data points. This elimination, and consequent angles recalculation, might be necessary, because, the angles' amplitudes can be rather different between the two contours, and therefore, the probability of bad matches can be increased considerably. Thus, the geometry of the contour that presents more points will be simplified until a good global matching can be found. Two approaches to perform this elimination are described in the next subsections.

In both approaches, in case of a good matching to be verified before eliminating all exceeded points, the process will be interrupted.

2.2.1 *Approach I*

Let us consider as the best possible global matching between the angles' amplitude of the *contour 1* with the ones of *contour 2*, the case for which is minimum the global angular cost for the n angles matched in the m possible hypotheses; that is:

$$\min\left\{\sum_{i=1}^{n}|\theta_i - \alpha_i|;\sum_{i=1}^{n}|\theta_{i+1} - \alpha_i|;\ldots;\sum_{i=1}^{n}|\theta_{i+m-1} - \alpha_i|\right\},$$

where $\theta_{i+m} = \theta_i$.

In the next step, from the sequence of angles $(\theta_i)_{i=1,2,\ldots,m}$ is considered the angle that provokes major angular difference and has at the same time an amplitude of around 180º. This last condition guarantees that the curvature of the contour in that point is irrelevant; that is, that point is in an almost straight part of the contour and, because of that, it can be considered insignificant for the description of the associated object's shape. Thus, the point of *contour 2* that corresponds to that angle is discarded from the same one.

Until the second angles' sequence has length equal to n, the search for the optimum matching is repeated, discarding each data point in the conditions previously enunciated. In this searching process, for the best matching possible the one that has the minor total angular cost is selected again, figure 3.

2.2.2 *Approach II*

In this approach, it is considered for the best matching case the one that has more pairs of angles correctly matched. Being considered as correctly matched, if the angular difference between the angles involved is less than a predefined threshold value. Next, from the result of this matching process, we find the set of angles wrongly matched. From this set, the angle that is closest to 180º is considered, which corresponds to a point of the contour wrongly matched and has curvature that can be considered as negligible. Then, this point is discarded.

Until the second angles' sequence has a length equal to n, the process used to search for the best global matching is repeated and each point found in the conditions previously enunciated is discarded. Finally, the best matching is selected by the minor global angular cost.

In figure 6, an example can be visualized in which the exceeded points, that were not matched, are precisely the points whose curvature is small. In this example, one contour is sampled by 25 points and the other by 28. In the same figure, the arrows drawn indicate the 3 data points that were excluded from the matching process.

After this matching process, the points in excess can be matched easier using a second matching criterion.

2.3 *Observations*

In the case of matching two contours sampled by equal number of points, the developed methodology always makes the one-to-one correspondence that minimizes the global angular cost and generally obtains good results.

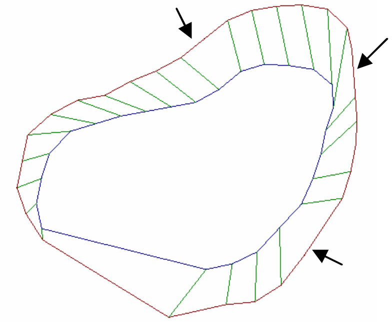

Figure 6: Result of the proposed matching approach with two contours sampled by a different number of points.

If the contours to be matched are sampled by different numbers of points, both approaches considered for the exclusion of the exceeded points, had shown adequate efficient. In the several experimental testes done, the second approach proposed has demonstrated to be slightly more efficient.

3 ESTIMATION OF THE RIGID TRANSFORMATION

3.1 *Translation, scale and rotation*

For the rejection of the angles and associated points wrongly matched, a new process that is based on the rigid transformation involved between the contours is being developed. In this way, it will also be possible to consider as a matching criteria the distance between each pair of candidate points.

Essentially, the method in development is based on the following steps. The centers of the two contours to be matched are determined using a procedure in which the weight associated to each point depends on the incidence of contour's data in its neighborhood. Thus, for instance, in a region with high concentration of points, the weight associated to each one of those points is reduced.

Next, to determine the scale involved between the two contours, the respective perimeters are calculated, being the quotient the desired value. After that, both contours are centered in the origin and the scale determined is applied on the second contour. This way, two contours centered at the origin and with similar dimensions are obtained.

The problem consists now in finding the optimum estimator for the involved rotation angle. When the contours are sampled using a significant number of points, two sets of samples with equal dimension, one for each contour, are consider. With this procedure, the fact that local contour's details can have exaggerated influence in the global geometry is prevented, as it is only considered the polygon base of each contour. Later, using a process equivalent to the one described previously, the matching with minor global angular cost is obtained. From this matching, only the pairs of angles with angular difference smaller than a predefined threshold value are considered. Then, using each selected pair of points matched, the rotation angle that minimizes the distance between them is obtained; that is, considering the point (x_1, y_1) of *contour 1* and its matched point (x_2, y_2) of *contour 2*, we intended to determine β, such that:

$$f(\beta) = (x_1 - x_\beta)^2 + (y_1 - y_\beta)^2,$$

be minimum, with:

$$\begin{bmatrix} x_\beta \\ y_\beta \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}.$$

Later, the weighed average of the obtained rotation angles is considered as the rotation angle involved between the two contours; that is, if the angular difference between two angles considered matched is large, its weight in the final contribution for the global rotation angle is small. Finally, the rotation estimated is applied on *contour 2*.

In figure 7, some examples of experimental results obtained using the presented methodology for the estimation of the rigid transformation involved between two contours are presented.

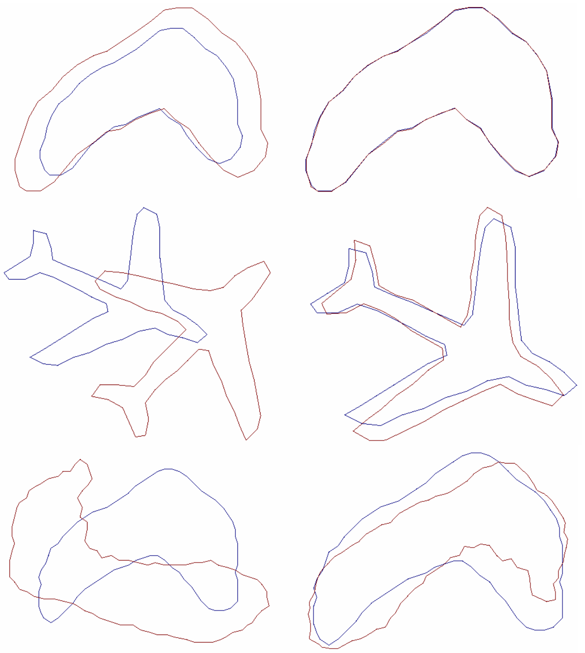

Figure 7: Some examples of results obtained in the determination of the rigid transformation involved between two contours (In each case, in the left are the original contours and in the right are the contours after applying an one of them the rigid transformation estimated.).

3.2 *Observations*

The process proposed in this paper for estimating the existing rigid transformation between two contours represented in images, has shown to be very efficient when the differences between them are essentially of rigid type.

The process proposed also shows to be very promising even when is accentuated the non-rigid component involved. In these cases, the process proposed allows the adequate estimation of the rigid part of the global transformation in cause, as it can also be verified from the examples presented in figure 7.

4 CONCLUSIONS

In this paper, a methodology that allows the adequate matching of contour that are represented in images and sampled by points was presented. The referred methodology is based on the curvature information along each contour and in the optimization of the global matching costs.

With the presented methodology, the matching of contours sampled by equal or different number of points is possible.

The experimental results obtained, some presented in this paper, validate the adequateness of the methodology proposed.

In this paper, a solution for the estimation of the rigid transformation involved between two contours, based also on the searching methodology for the minor global angular cost was also presented. The experimental results obtained with this solution are very promising, even in cases that the transformation involved has a significant non-rigid component.

As future works, beyond other possible tasks, we expect to apply the methodology proposed in contours represented along image sequences, to allow the matching of type “one with several” and “several with one” between points, and to quantify contours’ similarity.

5 ACKNOWLEDGEMENTS

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