

Recent Results on Approximate Optimization Methods for the Unit Commitment Problem

LUÍS A.C. ROQUE

Instituto Superior de Engenharia do Porto
LIAAD-INESC-TEC, Department of Mathematics
4200-072 Porto, Portugal
lar@isep.ipp.pt

DALILA B.M.M. FONTES

Universidade do Porto
LIAAD-INESC-TEC, Faculdade de Economia
4200-464 Porto, Portugal
fontes@fep.up.pt

FERNANDO A.C.C. FONTES

Universidade do Porto
ISR-Porto, Faculdade de Engenharia
4200-465 Porto, Portugal
faf@fe.up.pt

Abstract: This work provides an account of recently proposed methods to address the Unit Commitment (UC) problem. In the UC problem, the goal is to schedule a subset of a given group of electrical power generating units and also to determine their production output in order to meet energy demands at minimum cost. In addition, the solution must satisfy a set of technological and operational constraints. Here, computational results are reported for the most effective methodologies. Amongst the problems chosen to report the computational results are the most frequently used benchmark problems, due to Kazarlis, Bakirtzis and Petridis. In the problems considered, the units, which can be up to 100, have to be scheduled for 24-hour period.

Key-Words: Unit Commitment, Heuristics, Metaheuristics, Electrical Power Generation.

1 Introduction

The study and operation of power systems involves solving many different optimization problems and is critical since the commodity involved is essential to everyday life [15]. Amongst power systems related problems, the Unit Commitment (UC) problem is a key problem since it involves planning and operating the generating units.

The UC problem is an optimization problem where one wishes to determine the on/off status of the generating units at minimum operating costs. In addition, the production of the committed units, which also has to be determined, must be such that it meets demand and spinning reserve requirements. Furthermore, a large set of technological constraints are also imposed on generating units.

The UC problem is a combinatorial optimization problem that has multi-period characteristics and involves nonlinearities thus, solving it is a hard optimization task. Therefore, the solution of real sized systems is highly computational demanding. Most methodologies proposed to address it look for an approximated solution. Optimal solutions can only be obtained for small sized problem instances by solving the corresponding Mixed Integer Quadratic Programming (MIQP) model. However, computational time

requirements are enormous and, usually, increase exponentially with the problem size, even for efficient software packages (such as the CPLEX), as will be seen in the results section. Some authors have tried to improve the performance of the MIQP model by reformulating the UC problem as a mixed integer linear programming model by means of piece-wise linear approximations of the cost function (see, e.g., [9, 28]).

Several heuristic methodologies, based on exact and on approximate algorithms have been reported. In the past, several traditional heuristic approaches have been proposed, based on exact methods such as Dynamic Programming, Branch and Bound, and Lagrangian Relaxation. Most of the recently developed approaches are based on approximations and metaheuristics (see, e.g., [29, 25, 4, 13, 17, 21, 12]). In general, these latter algorithms have led to better results, particularly the metaheuristics.

Lagrangian Relaxation (LR) is capable of solving large scale UC problems in a fast manner, however the solutions obtained are, usually, suboptimal. Based on the LR approach, the UC problem can be approximated by joining the coupling constraints and the cost function using Lagrange multipliers. The resulting relaxed problem is to minimize the so-called Lagrangian subject to the unit constraints. LR was first applied to

solve the UC problem without considering ramp constraints [18]. Recently, in [9] an effective Lagrangian relaxation approach for the UC problem has been proposed. This approach relies on an exact algorithm for solving the single-unit commitment problem proposed in [8]. More recently, in [5] two lagrangian relaxation methods are proposed: one based on subgradient optimization and the other based on cutting planes. They were tested on several problem instances generated by the authors with a simpler and linear cost function, but not on the usual benchmark ones. Therefore, no comparisons with alternative methods were possible. From the tests performed, it was concluded that the subgradient method yields better results.

For methods based on metaheuristics there is recent literature reporting results on evolutionary programming [14], particle swarm optimization [29], quantum evolutionary algorithms [13, 17], memetic algorithms [27], and genetic algorithms [16, 1, 25, 4, 21, 22]. Just *et al.* [14] employ evolutionary programming in which populations of individuals are evolved through random changes, competition, and selection. The UC schedule is coded as a string of symbols and viewed as a candidate for reproduction. Initial populations of such candidates are randomly produced to form the basis of subsequent generations. An improved particle swarm optimization (IPSO) with adoption of the orthogonal design for generating the initial population scattered uniformly over a feasible solution space is introduced in [29]. The good results produced were recently outperformed [13, 17]. In these latter works, Quantum-inspired Evolutionary Algorithms (QEAs) are proposed. The QEA is based on the concept and principles of quantum computing, such as quantum bits, quantum gates and superposition of states. QEA employs quantum bit representation, which has better population diversity compared to other representations used in evolutionary algorithms, and uses quantum gates to drive the population towards the best solution. The mechanism of QEA can inherently treat the balance between exploration and exploitation, thus incorporating a sort of local search. In [13, 17] the UC problem is divided into two subproblems: 1) unit status schedule and 2) units power production. In both works, repair mechanisms are used to accelerate the solution quality and to ensure that unit schedules generated by QEA are feasible. In [13] the conventional QEA is improved by introducing a simplified rotation gate for updating Q-bits and a decreasing rotation angle approach for determining the magnitude of the rotation angle. At the The results obtained were at the time the best known results, which have been improved in [22]. A very recent type of evolutionary algorithm, the Imperialist Competition Algorithm (ICA), has been applied to the

UC problem in [12]. In it a population consists of a set of countries, all divided between imperialist countries and colonies, based on the imperialistic power, which is inversely proportional to its cost function for a minimization problem. Then the colonies move toward their relevant imperialist country and the position of the imperialists is updated if necessary. In the next stage, the imperialistic competition among the empires begins, and through this competition, the weak empires are eliminated. The imperialistic competition will gradually lead to an increase in the power of dominant empires and a decrease in the power of weakest ones, until only one empire remains. The authors were able to improve upon literature results, but only for the problem instance with 10 generating units. Very recently, Hybrid Biased Random Key Genetic Algorithm (HBRKGA) has been proposed to address the UC problem [22]. This approach is based on the framework provided by [10], which has been used in other important applications in an effective and efficient way (see e.g. [6, 11, 7]). Biased Random Keys GAs (BRKGAs) are a variation of the random key genetic algorithms, first introduced by [3]. A BRKGA differs from a random key GA in the way parents are selected for mating and also in the probability of inheriting chromosomes from the best parent. Repair mechanisms are also included therefore, all the individuals considered for evaluation are feasible. The BRKGA is hybridized with a local search procedure in order to intensify the search close to good solutions. The resulting HBRKGA was capable of improving the best known solution for most of the benchmark problems commonly use in the literature.

More details on these methods and other developed applications for the UC problem can be found in the extensive and comprehensive bibliographic surveys published over the years ([24, 19, 20, 23]), the most recent one being from 2007. In this paper, we concentrate on reporting the results for the most recent and effective methods.

2 The UC Problem Formulation

A solution to the Unit Commitment problem must provide the status of each generating unit (on-line and off-line), as well as the the output level for the on-line units for a given time horizon. The decisions must be such that the operational and technological constraints are satisfied at minimum cost.

Consider the following parameters and decision variables:

Indexes:

t: Time period index;

j: Generating unit index;

Decision Variables:

$y_{t,j}$: Power generation of unit j at time t , in $[MW]$;

$u_{t,j}$: Status of unit j at time t (1 if it is on; 0 otherwise);

Auxiliary Variables

$T_j^{on/off}(t)$: Number of time periods for which unit j has been continuously on/off-line until time t , in $[hours]$;

Parameters:

T : Number of time periods (hours) of the horizon;

N : Number of generating units;

R_t : System spinning reserve requirements at time t , in $[MW]$;

D_t : Load demand at time period t , in $[MW]$;

$Y_{min,j}$: Minimum generation limit of unit j , in $[MW]$;

$Y_{max,j}$: Maximum generation limit of unit j , in $[MW]$;

$T_{c,j}$: Cold start time of unit j , in $[hours]$;

$T_{min,j}^{on/off}$: Minimum uptime/downtime of unit j , in $[hours]$;

$S_{H/C,j}$: Hot/Cold start-up cost of unit j , in $[\$]$;

$\Delta_j^{dn/up}$: Maximum allowed output level decrease/increase in consecutive periods of unit j , in $[MW]$;

The model has two types of decision variables. Binary decision variables $u_{t,j}$, which are either set to 1, meaning that unit j is committed at time period t ; or otherwise are set to zero. Real valued variables $y_{t,j}$, which indicate the amount of energy produced by unit j at time period t . Such decisions are limited by two types of constraints: load constraints, consisting of demand and spinning reserve constraints; and technological constraints. The objective of the UC problem is the minimization of the total operating costs over the scheduling horizon.

The objective function has three cost components: generation costs, start-up costs, and shut-down costs. The generation costs, also known as the fuel costs, are conventionally given by the following quadratic cost function.

$$F_j(y_{t,j}) = a_j \cdot (y_{t,j})^2 + b_j \cdot y_{t,j} + c_j, \quad (1)$$

where a_j, b_j, c_j are the cost coefficients of unit j .

The start-up costs, that depend on the number of time periods during which the unit has been off, are given by

$$S_{t,j} = \begin{cases} S_{H,j}, & \text{if } T_{min,j}^{off} \leq T_j^{off}(t) \leq T_{min,j}^{off} + T_{c,j}, \\ S_{C,j}, & \text{if } T_j^{off}(t) > T_{min,j}^{off} + T_{c,j}, \end{cases} \quad (2)$$

where $S_{H,j}$ and $S_{C,j}$ are the hot and cold start-up costs of unit j , respectively. The shut-down costs are here not considered since they typically are disregarded in the literature.

Therefore, the cost incurred with an optimal scheduling is given by the minimization of the total costs for the whole planning period,

$$\text{Min} \sum_{t=1}^T \sum_{j=1}^N \{F_j(y_{t,j}) \cdot u_{t,j} + S_{t,j} \cdot (1 - u_{t-1,j}) \cdot u_{t,j}\}. \quad (3)$$

The constraints are divided into two sets: the demand constraints and the technical constraints. The first set of constraints can be further divided into load requirements and spinning reserve requirements.

1) Load Requirement Constraints: The total power generated must meet the load demand, for each time period.

$$\sum_{j=1}^N y_{t,j} \cdot u_{t,j} \geq D_t, t \in \{1, \dots, T\}. \quad (4)$$

2) Spinning Reserve Constraints: The spinning reserve is the total amount of real power generation available from on-line units net of their current production level.

$$\sum_{j=1}^N Y_{max,j} \cdot u_{t,j} \geq R_t + D_t, t \in \{1, \dots, T\}. \quad (5)$$

The second set of constraints includes limits on the unit output range, on the maximum output variation allowed for each unit (ramp rate constraints), and on the minimum number of time periods that the unit must be continuously in each status (on-line or off-line).

3) Unit Output Range Constraints: Each unit has a maximum and minimum production capacity.

$$Y_{min,j} \cdot u_{t,j} \leq y_{t,j} \leq Y_{max,j} \cdot u_{t,j}, \\ \text{for } t \in \{1, 2, \dots, T\} \text{ and } j \in \{1, 2, \dots, N\}. \quad (6)$$

4) Ramp rate Constraints: Due to the thermal stress limitations and mechanical characteristics the output variation levels of each on-line unit for consecutive periods are restricted by ramp rate limits.

$$-\Delta_j^{dn} \leq y_{t,j} - y_{t-1,j} \leq \Delta_j^{up}, \\ \text{for } t \in \{1, 2, \dots, T\} \text{ and } j \in \{1, 2, \dots, N\}. \quad (7)$$

5) Minimum Uptime/Downtime Constraints: The unit cannot be turned on or turned off instantaneously once it is committed or decommitted. The minimum uptime/downtime constraints impose a minimum number of time periods that must elapse before the unit can change its status.

$$T_j^{on}(t) \geq T_{min,j}^{on} \text{ and } T_j^{off}(t) \geq T_{min,j}^{off}, \\ \text{for } t \in \{1, 2, \dots, T\} \text{ and } j \in \{1, 2, \dots, N\}. \quad (8)$$

3 Mixed integer quadratic programming

The UC problem can be casted as a mixed-integer nonlinear program (MINLP) with real and binary variables. Despite the ever-increasing availability of cheap computing power and the advances in off-the-shelf software for MINLP, solving (UC) by general-purpose software, even using the most advanced approaches available, is not feasible when the number of units and/or the length of the time horizon becomes large [9].

Here we formulate the UC problem as a MIQP model, which is then solved by the commercial software CPLEX. In order to so new auxiliary binary variables need to be defined:

$l_{t,j}$: indicates whether unit j has been started-up or not at time period t (1 if it has been started-up; 0 otherwise);

$h_{t,j}$: indicates the cold status of the off-line unit j at time t (1 if the unit is cold; 0 otherwise);

$v_{t,j}$: indicates whether unit j has had a cold start-up or not at time period t (1 if it had; 0 otherwise).

The objective function is now rewritten as

$$\text{Min} \sum_{t=1}^T \sum_{j=1}^N \{a_j \cdot (y_{t,j})^2 + b_j \cdot y_{t,j} + c_j \cdot u_{t,j} + S_{H,j} \cdot l_{t,j} + (S_{C,j} - S_{H,j}) \cdot v_{t,j}\}.$$

The power balance, the spinning reserve, the minimum and maximum production capacity and the ramp rate constraints are express as before, see equations (4) to (7) in Section 2.

The minimum up time constraints are nonlinear and thus are reformulated as

$$\sum_{k=t}^{t_{\max}^{on}(t,j)} u_{k,j} \geq (u_{t,j} - u_{t-1,j}) t_s^{on}(t,j), \quad (9)$$

for $t \in \{1, \dots, T\}$ and $j \in \{1, 2, \dots, N\}$,

where $t_s^{on}(t,j)$, the minimum number of time periods that unit j must be on given that it was switched on at time t , is given by

$$t_s^{on}(t,j) = \begin{cases} \min \{T_{\min,j}^{on}, T - t + 1\}, & \text{if } t > 1 \text{ or } (t = 1 \text{ and } I_0(j) < 0), \\ \max \{0, T_{\min,j}^{on} - I_0(j)\}, & \text{if } t = 1 \text{ and } I_0(j) > 0, \end{cases}$$

and $t_{\max}^{on}(t,j)$ is the last time period that unit j must be on given that it was switched on at time t and it is given as

$$t_{\max}^{on}(t,j) = \begin{cases} \min \{t + t_s^{on}(t,j) - 1, T\}, & \text{if } t_s^{on}(t,j) > 0, \\ T, & \text{otherwise.} \end{cases}$$

The minimum down time constraints are also nonlinear and thus are reformulated as given in equation

$$\sum_{k=t}^{t_{\max}^{off}(t,j)} (1 - u_{k,j}) \geq (u_{t,j} - u_{t-1,j}) t_s^{off}(t,j), \quad (10)$$

for $t \in \{1, \dots, T\}$ and $j \in \{1, 2, \dots, N\}$,

where $t_s^{off}(t,j)$ and $t_{\max}^{off}(t,j)$ are as follows

$$t_s^{off}(t,j) = \begin{cases} \min \{T_{\min,j}^{off}, T - t + 1\}, & \text{if } t > 1 \text{ or } (t = 1 \text{ and } I_0(j) > 0), \\ \max \{0, T_{\min,j}^{off} + I_0(j)\}, & \text{if } t = 1 \text{ and } I_0(j) < 0, \end{cases}$$

$$t_{\max}^{off}(t,j) = \begin{cases} \min \{t + t_s^{off}(t,j) - 1, T\}, & \text{if } t_s^{off}(t,j) > 0, \\ T, & \text{otherwise.} \end{cases}$$

Given the newly defined variables, we need to define the following coupling constraints for $t \in \{1, 2, \dots, T\}$ and $j \in \{1, 2, \dots, N\}$.

$$l_{t,j} \geq u_{t,j} - u_{t-1,j}, \quad (11)$$

$$l_{t,j} + h_{t,j} - 1 \leq v_{t,j}, \quad (12)$$

$$h_{t,j} \geq 1 - \sum_{k=t_{\min}}^t (u_{k,j}), \quad (13)$$

where $t_{lj} = T_{\min,j}^{off} + T_{c,j} + 1$ is the least time interval for which unit j has to be off-line for a cold start cost to be paid and $t_{min}(t,j)$ is the last time instant that unit j can be on-line for a cold start cost is to be paid.

$$t_{min} = \begin{cases} t - t_l, & \text{if } I_0(j) > 0 \text{ or } t > t_l \\ 1, & \text{if } I_0(j) < 0 \text{ and } t \leq t_l \\ \text{and } t - t_l - 1 - I_0(j) \geq 0, \end{cases}$$

with $t_l = T_{\min,j}^{off} + T_{c,j} + 1$.

Constraints (11) guarantee that unit j has been started at time t only if it is on at time t and has been off at time $t - 1$. In equation (12) it is ensured that the cold start costs are only paid if unit j is cold and has been just started. Finally, constraints (13) state that unit j is cold at time t if and only if it has not been started for at least T^{off} time periods.

CPLEX can be attractive in many situations since in addition to its robustness, it also allows for the incorporation of other constraints [9]. Nevertheless, even small sized problems require significant amounts of time and physical memory to be solved. Furthermore, the CPLEX cannot cope with more general cost functions, such as exponential start-up costs, as is the case of the problems proposed by [26] and [2].

4 Numerical Results

This section provides an account of the best results reported in the literature for the most commonly used benchmark instances of the UC problem. Furthermore, the CPLEX (version 12.1) was used to solve the MIQP model presented in Section 3. This way, optimal solutions are obtained for some small problem instances, which allows to find out how close the reported results are to the optimum. Nevertheless, such comparisons are only possible for small sized problems, since the CPLEX is unable to solve larger problems due to the huge memory requirements.

This study comprises problems involving systems with 10 up to 100 generating units and considering, in each case, a scheduling horizon of 24 hours. The 10 generating unit system, the base case, was originally proposed in [16]. Problem instances involving 20, 40, 60, 80 and 100 units are obtained by replicating the base case system and the load demands are adjusted in proportion to the system size. In all cases the spinning reserve is kept at 10% of the hourly demand. The start up costs have one of two possible values depending on the number of time periods the

Table 1: Best results reported in literature.

Size						CPLEX
	IPSO	IQEA	QEA	ICA	HBRKGA	MIQP
10	563954	563977	563938	563938	563938	563938
20	1125279	1123890	1123607	1124274	1123955	1123297
40	2248163	2245151	2245557	2247078	2244345	2242634*
60	3370979	3365003	3366676	3371722	3363804	–
80	4495032	4486963	4488470	4497919	4485197	–
100	5619284	5606022	5609550	5617913	5605933	–

* Recall that this is the best known solution, although it may not be optimal.

Table 2: Computational time.

Size	IPSO	IQEA	QEA	ICA	HBRKGA	CPLEX
10	142	15	19	48	2	45
20	357	42	28	63	14	401
40	1100	132	43	151	90	1489
60	2020	273	54	366	301	–
80	3600	453	66	994	712	–
100	5800	710	80	1376	1503	–

unit has been off, as given in equation (2). These values are different for each generating unit. The shut down costs are disregarded. Details of how these benchmark problems were constructed and on the system and demand data can be found in [16].

For the problems in this study, the CPLEX was able to find an optimal solution to systems involving 10 and 20 units. For problems with 40 units, we report on the best solution found by the CPLEX before it crashed due to the excessive memory requirements. However, although the solution is not optimal, it is the best solution found so far. In Table 1 we compare the results obtained by the best performing methods in the literature (IPSO - [29]; IQEA - [13]; QEA - [17]; ICA - [12]; BRKGA - [22]). The best current solution, excluding the one by the CPLEX, is given in bold, for each of the problems. As it can be seen in Table 1, for all problem instances, except one of small dimension, the best results are due to [22]. Moreover, for the problem instances for which an optimal solution has been found by the CPLEX, it can be seen that the HBRKGA has been able to find an optimal solution in one case, while in the other case the solution found is within 0.06% of optimality.

Regarding the computational time, no exact comparisons may be done since, the values are obtained on different hardware. In addition, the results for the IPSO and for the IQEA may not be accurate, since the authors have reported them graphically. Nevertheless, they are shown in Table 2. It should be noticed that The IPSO has computational time requirements much larger than the other methods, while the QEA is the fastest method. The other three methods have similar computational requirements. The CPLEX computational time grows faster than the aother methods analysed.

5 Final remarks

The Unit Commitment Problem (UCP) is an important area of research which has attracted increasing interest from the scientific community due to the fact that even small savings in the operation costs for each hour can lead to major overall economic savings. In addition, the problem has been addressed by several approximate optimization methods, making it a good benchmark optimization problem.

In this paper, the performance of the best optimization algorithms have been tested on a set of UC benchmark problems commonly used in the literature.

From the results reported, we can see that, apart from very small dimension problems, meta-heuristic methods, mainly based on genetic algorithms, are the most competitive state-of-the-art methods for the UCP problem. From these, the genetic algorithm using biased random keys has, in general, shown to be able to obtain the best solutions, while using modest computational times. On the other hand, commercial off-the-shelf general-purpose optimizers, such as CPLEX, despite its large improvement in the recent years, are still unable to solve problems with more general cost functions or larger dimension on a PC platform.

Regarding future development in the field. Since very good quality solutions can already be obtained in reasonable time for instances in the order of 100 units, we foresee the development opportunities in variations of the original UCP problem, by including additional constraints, considering multi-objective performance criteria, including demand uncertainty, or including production uncertainty in renewable units.

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