The interaction between firms and Government in the context of investment decisions: a real options approach

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Abstract

Given the global financial crisis and, particularly, the European sovereign-debt crisis, European countries have the urgent need to promote output growth. However, due to the current financial constraints, it is difficult for policy makers to stimulate economic growth by directly increasing investment. Alternatively, the Governments can promote private investment, either by reducing the uncertainty and costs, or by subsidizing those investments. In this paper we try to analyze alternative solutions to promote investment, and hence economic growth, under a context of Government austerity. We develop a real options model in order to study optimal investment decisions, considering both the point of view of firms and Government. So, we incorporate the Government in the baseline real options model, and we use this extended model to drive the optimal behavior for firms and Government on their decision to invest and promote investment, respectively. To be more realistic, the model takes in account, not only inefficiencies (both concerning the implementation and management of the project), but also the economic benefits of investing, i.e., the investment multiplier effect in the economy. We also make a sensitivity analysis for the key parameters and define regions for different types of investment. Alternative solutions are also considered.Among the main conclusions we find that the probability of being optimal for the Government to subsidize private investment rather than investing directly is greater the larger the private investment multiplier effect, the tax rates, the private present value of the profit flows, the private cost of the investment and, also, the inefficiency level of the Government.

JEL codes: E22; E62; G31; G32; H32.

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1 Introduction

The global financial crisis that began in 2007 and most recently the European sovereign-debt crisis have penalized the economic growth in Europe. After years of easy and accessible credit, these crises led the European economy to the longest and deepest recession since the 1930s Great Depression and have increased the financial constraints, both for private and public sectors all over the world. With the lack of monetary resources and a high uncertainty, the investment of firms and the consumption of the families have been deferred and, consequently, the economic recovery has also been delayed. In fact, investment, either public or private, is indispensable to the economic growth and if there is a private investment reduction, the Keynesian school defends an increase in the public investment in order to offset the fall in private demand. However, this perspective overlooks two issues that may be critical: the government budget constraint and the presence of inefficiencies in public investment that can hinder their beneficial effects (Leeper et al. 2010).

Actually, on the one hand, governments need to promote economic growth but, on the other hand, they face severe liquidity constraints and an institutional environment that results in public investment inefficiencies. Given the budget deficits and debts accumulated over the years, some governments had to adopt austerity policies in order to promote public finances sustainability, which are hindering the adoption of measures to promote economic growth. In a context of few governmental resources, it is necessary to find alternative and profitable solutions to promote investment. Instead of the traditional public investment, this can be done by ensuring high levels of political and institutional environments, which means low risk and uncertainty, and by a policy of public incentives, namely a subsidizing policy. In some circumstances, for high value or very important projects, public incentives may hasten their implementation. These alternative policies can represent a smaller financial effort for the Government and can bring the same payoff or even more than directly investing, which means a higher capital rate of return.

The main goal of this paper is to explore alternative solutions to promote investment, as we think that it has a crucial role on firms evolution and economic performance. In fact, our motivation is to find a possible solution to promote economic growth with few Government resources. In order to do so, we study the interaction between firms and the Government in the context of investment decisions, taking as baseline a real options approach. The real options theory insists on the fact that uncertainty generates an option value for waiting, and tends to lead investors to postpone their investment decisions. Accordingly, we try to obtain the optimal behaviour both for firms and Government by managing some key parameters to reduce the critical value and thus to hasten the private investment. Although we know that immediate exercise of the option to invest may not be optimal for individual projects, we try to maximize aggregate welfare by promoting that investment. This can be particularly relevant during economic crisis, when it may be crucial to hasten the investment. By exploring the interaction between the Government and firms, and the concerns of both, we develop a real

options model which explores some key factors for decision-making. The outcome is a model that drives the optimal behaviour for firms and the Government on their decision to invest and promote investment, respectively. To be more realistic, the model will take in account, not only inefficiencies (both concerning the implementation and management of the project), but also the economic benefits of investing, i.e., the investment multiplier effect in the economy. We also make a sensitivity analysis for the key parameters and define regions for different types of investment. Alternative solutions are also considered.

Our main contribution to the literature, and also to the policy debate about fiscal stimulus, is related to the introduction of some macroeconomic aspects into the baseline real options model, namely the private and public investment multipliers, the efficiency degree of the agents (public and private) and taxes, in addition to the common aspects such as profit flows, uncertainty, risk-free rate and the cost of the investment. The model can be useful both for the private sector and for the policy maker. On the one hand, the private sector will know the conditions of negotiation with the Government, namely about the maximum subsidy that the policy maker is willing to give for each type of project. On the other hand, the Government will know the optimal values for firms and how to optimally stimulate the investment. This paper unfolds as follows. In section 2 we present a brief literature review. In section 3 we develop the model, presenting the value-functions and the triggers both for firms and for the Government. The optimal incentives that prompt investment are derived. The section 4 contains some case-scenario and statics analysis, and in section 5 we explore some alternative stimuli for investment. Finally, in section 6 we conclude.

2 Literature Review

Traditionally, in the context of investment decisions, the Net Present Value (NPV) is the main criterion for project selection and valuation. Based on this method, the decision will be to invest if the NPV is positive. In other words, if the present value of cash flows is greater than the present value of the costs of the investment, the project should be implemented. This approach has some limitations because it is suitable for stable and predictable environments.

In the presence of flexibility and uncertainty, the investment decision is different from the traditional approach. Even when the NPV is positive, firms may delay the projects implementation. This is because the project implementation is an option, and by investing now the company loses the option to invest later in the same project. Since the investment cost is mainly irreversible and considering that firms can wait for more information about the project, it is important for them to determine the optimal moment to invest.

Then, apart from the NPV, there is an option value for the project, and so, according to the real options theory[1], the total value is named as the value of the investment opportunity. That

[1]For further details please refer to McDonald and Siegel (1986), Dixit and Pindyck (1994) and Trigeorgis (1996).

value can be divided in two components: (i) its intrinsic value (the NPV); and, (ii) the time value (the value of delaying the investment). Total investment value increases with NPV but the time value component decreases until it disappears. When the value of waiting is null, there is no reason to postpone the project implementation. When the NPV is positive and the time value has been depleted, that is the optimal timing to exercise the option to invest. As the profit flows are unknown, and there is a sunk cost of the investment, we can take advantage of the flexibility to modify the project and wait until the value of the opportunity of the investment is optimal. Thus, the NPV underestimates the value of the investment opportunity because it does not take into account the option of waiting. Accordingly, in our work, we make use of the real options theory for a better approach, overcoming the limitations of the traditional rules.

According to the real options literature, the optimal timing to invest is highly dependent on the uncertainty surrounding the project. Everything else kept equal, the higher the uncertainty, the higher will be the optimal trigger to invest. The economic intuition is that, under (high) uncertainty, the investor tends to postpone the investment decision, for instance, waiting for better information about market conditions, instead of investing as soon as the NPV is positive. Note that this is particularly important in a crisis context, given that a crisis tends to increase uncertainty, thus increasing the real option value of delaying the investment (Inklaar and Yang 2012).

Lee et al. (2008) studied the value of real options investments under abnormal uncertainty, specially the case of Korean economic crisis. They show that the more flexible the investment project is, the more valuable it will be, so flexible projects like R&D accumulates more value under uncertainty than other investments such as advertising. Other important conclusion is about the size of firms, which is negatively correlated with the firm value because smaller firms could be more flexible and give a better response to uncertainty.

So, in order to hasten the investment, the Government should try to reduce uncertainty. Keeping political and economic stability and reducing institutional uncertainty will lower the volatility, but normally this is already reached in developed countries[2]. It is also possible for the Government to increase the opportunity cost of defer the investment with regulations that increases firms competition. On the other hand, in the context of the European and Monetary Union (EMU), the risk-free interest rate is given by European Central Bank (ECB), directly for the banks and indirectly for companies, so it is not a Government business. Even so, the existence of other institutions or mechanisms such as the European Investment Bank (EIB) is in some situations important[3].

In addition to try to reduce uncertainty, one of the ways to stimulate the output is by promoting the investment. The Governments decision on where and when to invest, given the budget constraint that confines its choices, needs to consider whether the private sector has

[2]Although the political and economical stability may be achieved, the Government can reduce volatility by ensuring the number of users or revenues.

[3]In troubled times, it is common to exist a high difference between the risk-free interest rate for Banks and for companies.

an incentive to undertake the project, because *without considering the private sectors incentives correctly, state investments are likely to waste money* (Warner 2013). So, the Government should make a criterious selection of the projects, and also of the policy tools available to guarantee the implementation of the selected projects, which range from the direct provision by the public sector to, for example, a partnership or a subsidizing policy. However we must take in account that even the best projects often does not yield the expected improvement in outcomes. This can be explained, following Rajkumar and Swaroop (2008), by two factors: (i) public investment can crowd out private investment, reducing the global impact on output, and thus the investment multiplier; and, (ii) the ineffectiveness associated to public investment, both at implementation and management levels.

Since Aschauer (1989b) seminal work on the productivity of public capital, there has been a growing strand of the literature concerned in measuring the effects of public investment on aggregate economic activity.[4] One key issue in these studies is whether public investment crowds in or crowds out private investment. Aschauer (1989b) has found a clearly crowding in effect for the US economy, concluding that public and private investment are complementary. However, since then, the results in the literature are ambiguous. While empirical studies by Seitz (1994), Argimon et al. (1997) and, for example, Pereira (2001) also support the existence of a crowding in effect of private investment, others conclude, either for the crowding out hypothesis (Everhart and Sumlinski (2001), Voss (2002), Zou (2006), Cavallo and Daude (2011)), or for mixed results (Afonso and St. Aubyn 2009, 2010).

This ambiguous relationship is usually justified by two opposing forces. On the one hand, public investment tends to increase the productivity of private factors, including fixed capital (Aschauer 1989a, Barro 1990). By increasing resources and infrastructures on the economy, costs for the private sector are reduced. On the other hand, by increasing demand for funds in the financial markets public investment causes an upward pressure in interest rates, discouraging private investment (Afonso and St. Aubyn 2009). There may be, also, a credit deviation from the private to the public sector, thus reducing the available credit for the private sector (Cavallo and Daude (2011)). Therefore, while productive public investment has a positive and significant impact on private investment and output, spending resources on unproductive investments could have a null or even a negative impact on the economy.[5]

In fact, the impact of the investment on GDP, measured by the multiplier effect, is very useful to infer about the importance of stimulating the investment implementation. Despite the necessity of promoting investment, it is not easy to know which of the public or private investment is better. Erden and Holcombe (2006), considering that investment is important for economic growth, have studied the connections between public and private investment. They say: *"Overall, the empirical evidence from the US and from developing countries suggests that private*

[4]For comprehensive surveys on this empirical literature see, among others, Romp and De Haan (2007) and Pereira and Andraz (2010).

[5]See, for example, Blanchard and Perotti (2002) and Mountford and Uhlig (2009).

capital is more productive than public investment, and that although public investment contributes to the productivity of private capital, it does not explain the major part of the variation in output growth" (Erden and Holcombe 2006, p.479). Afonso and St. Aubyn (2009, 2010) studied the rates of return of public and private investment in seventeen countries, fourteen from the European Union (EU) and also USA, Canada and Japan, for the period 1960-2005. They found that the effects of impulses to public investment are never statistically significant at the 95 per cent level, while the impulses to private investment have mostly a positive and significant impact on output. Furthermore, the output elasticity of private investment is always positive and higher than the output elasticity of public investment. They also found that the impact of a unitary increase in investment on GDP is, on average, 0.73 and 1.47, respectively for public investment and private investment. This means that the private investment multiplier is twice as much of the public investment multiplier for this sample.

Nevertheless, the economic outcome of public investment is also related with the quality and (in)efficiency of projects implementation and management. Afonso et al. (2005) shows an international comparison of public sector efficiency. According to them, *"Most studies conclude that public spending could be much smaller and more efficient than today. However, for this to happen, governments should adopt better institutions and should transfer many non-core activities to the private sector"* (Afonso et al. 2005, p.321).

Leeper et al. (2010) emphasized the importance of inefficiencies introduced by some implementation delays associated with public projects, which can produce small or even negative effects on output. These delays, related with the speed at which public investment occurs, can be severe due to the fact that many projects require a strong coordination among several levels of the public administration (national, regional and local governments), which implies that every project implementation *"have to go through a long process of planning, bidding, contracting, construction, and evaluation"* (Leeper et al. 2010, p.1001).

Dabla-Norris et al. (2011) highlighted, among others, some inefficiencies in the public management process, related with the absence of clear organizational arrangements, regular reporting and monitoring frameworks, that typically result in chronic under-execution of public investment budgets, waste and leakage of resources, rent seeking and corruption. Another source of management inefficiency is derived from the fact that public investment decisions can be influenced by political economy motives, like the pressure of interest groups, rather than simple economic efficiency considerations.

Chakraborty and Dabla-Norris (2009) have studied the quality of public investment across countries, by developing a growth model in order to study how the inefficiency and corruption of the public investment services affects the productivity of private capital, the specialization and the economic growth. Synthetically, the paper suggests that the economic growth resulting from public investment depends critically on the quality and efficiency of public capital. It also suggests that the quality of public institutions influences GDP more than the differences of capital across countries. This may represent that the efficiency of the public projects and

investment is vital for the growth and evolution of the countries. They have also concluded that weak public institutions decreases the productivity and efficiency of public investment and it results in a lower rate of return of the private investment, less specialization and thus in lower economic growth.

Nevertheless, public investment is sometimes crucial for the productivity of private capital and, consequently, for a good growth performance. Besides the quality of investment, other factors are also important for the investment decision. Since we look to how the Government can increase the investment, we have to identify political and economic instruments that fulfill their own objectives. Keeping political and economic stability and reducing institutional uncertainty will lower the volatility, but normally this is already reached in developed countries . It is also possible for the public institution to increase the opportunity cost of defer the investment with regulations that increases firms competition. On the other hand, in the context of a Monetary Union in Europe, the risk-free interest rate is given by European Central Bank, directly for the Banks and indirectly for companies, so it is not a Government business. Even so, the existence of other institutions or mechanisms such as the European Investment Bank (EIB) is in some situations important . Finally, we can hasten the investment by decreasing the cost of the investment. Therefore this is possible by two ways: subsidize directly a percentage of the investment or indirectly via public investment that reduces the cost of the investment for firms, and here it is crucial a high quality of the investment to ensure the success of this measure.

All these questions about whether public investment actually stimulates output, lead us to explore other fiscal policy tools available to guarantee the implementation of the relevant projects. One of these alternative policies is a private investment subsidizing policy. There is some literature about the effect of subsidies in the investment decisions. The subsidy reduces the cost of the investment, and turns the implementation of the project more attractive, by creating *"a gap between the return as perceived by the private sector and the true return"* (Warner 2013, p.15). If the Government increases taxation in order to offset the budget impact of the subsidy, ensuring an expected zero cost, it may reduce firms profits, but also it may ensure the implementation of the project, because the ratio between the profits and the cost of the investment is higher than in the original situation, and ensures the expected rate of return of the optimal situation. Pennings (2000) showed that the Government can hasten the investment decisions under uncertainty with a zero expected cost stimulus. Specifically, the author proposes that a subsidy policy can decrease the trigger value, until it equals the current level of the profit flows. This means that, from the firms point of view, it will be optimal to invest now, and so they will hasten the investment. At the same time, with a proper taxation of the profit flows of the project, the Government can recover the subsidy, and so it is possible a zero expected cost. Nevertheless, some may criticize the fact that this policy lowers the value of the project and then the value of the firm. Maoz (2011) has shown this fact, but also that this program for stimulating the investment, in this conditions, instead of offered, has to be enforced on firm because of the lost of value. This means that this program penalizes the firms that delay their

investments.

Although some may criticize that this is not optimal for the individual project because of the value reduction, we argue that it could be beneficial for both, the firms and the Government. In fact, even if the project has a lower value now, it can stimulate the economy and increase the value of other projects, thus generating positive externalities that maximize aggregate welfare and encouraging the selection of the best projects. Even with this, the firms will take the same capital rate of return, because there is not only a profits reduction, but also a reduction of the cost of the investment supported by firms. Therefore, if firms were given the necessary stimuli and some incentives, they may decide to hasten the investments. The Government will take also a better condition inasmuch as there is an increase in taxes that will pay the subsidy cost and could be also an improvement in economic situation.

The value added of this paper is the aggregated analysis, as we consider the perspective of firms and the perspective of the Government too, by studying public investment in a real options model. To understand how the Government can improve economic performance, increasing GDP through investment stimulus is particularly interesting. In the next section we are going to develop the model and derive the equilibrium.

3 The Model

Consider the existence of a project (or a group of projects) important to the economy. This project can be implemented either by the private sector or by the public sector. However, as we saw, the public investment can be less efficient and have a lower impact in the output, so the Government may have benefits in promoting the private investment. If the optimal investment trigger for the private companies has not been yet achieved, the Government may have two alternatives. The first one is to undertake the investment himself, instead of the private sector; the second one is to modify the investment conditions for the private sector in order to make the project more attractive, inducing the private sector to undertake investment immediately.

We are going to study the following three alternatives in detail: private investment, public investment and private investment with public subsidy, analyzing the situations where the alternatives are more appropriate. When the immediate implementation is optimal for the private companies, there is no need for any direct intervention of the Government. However, if this is not the case, the Government can invest directly in the project or, in alternative, encourage the private sector to do that. In next sections, we will address the perspectives of both of the sectors (public and private), determine decision rules and quantify the incentives needed for prompting the investment decision. We do this by taking into account the (in)efficiencies and the impacts on the economy that results from each alternative.

3.1 Firms Motivation

The firms seek for investment projects to create or expand supply, to improve their competitive situation or also to reduce their production costs. Beyond the position facing the competition and eventual market share objectives, one of the main benefits of the investment implementation to the firms relates to the value creation. In fact, firms have interest to find the best investment projects in order to maximize the benefits extracted from those projects.

The implementation of positive NPV projects brings gains to the firm, but this is not enough to decide the immediate investment. A good example of this is the current economic conjuncture in many countries of EU. This happens because the negative economic conjuncture, besides the great uncertainty about the future evolution and eventual liquidity constraints, make firms delaying the projects while waiting for more information, with the objective of minimizing the risk. This highlights the NPV insufficiency in the value determination and the consequent importance of the timing of the investment decisions for a better evaluation. The real options approach improves the analysis of this problem, by introducing a new component to the value determination. In fact, the real options allow to determinate the optimal timing to invest, i.e., identifies the trigger value for which is optimal to invest.

In order to analyze companies motivation for investment, we use the real options approach.[6] Some key parameters of the model are the volatility of the profit flows (σ), the risk-free interest rate (r), the cost of defer the investment (δ), the cost of the investment (I) and the present value of the pre-tax profit flows (V). We assume that the present value of pre-tax profit flows (V) follow a Geometric Brownian Motion (GBM):

$$dV = \alpha V dt + \sigma V dz \tag{1}$$

where α is the expect profit flows drift, σ is the expect volatility and dz the increment of the Wiener process.

The value of the investment opportunity, $F(V)$, according to the standard contingent claim analysis must satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 F''(V) + (r-\delta)VF'(V) - rF(V) = 0 \tag{2}$$

The solution for $F(V)$ must satisfy, also, the following boundary conditions:

$$F(0) = 0 \tag{3}$$

$$F(V^*) = V^* - I \tag{4}$$

$$F'(V^*) = 1 \tag{5}$$

where V^* represents the optimal investment threshold. The first condition says that the option

[6] Please refer to Dixit and Pindyck (1994)

to invest in projects that do not produce profit flows has no value. The second condition results from the notion that when the investment is optimal, there is no option value to defer. Thus, the value of the investment opportunity equals to NPV. Finally, the last one is the smooth pasting condition, that ensures the value function is continuously differentiable along V. The solutions are as follows:

$$F(V) = \begin{cases} (V^* - K)\left(\frac{V}{V^*}\right)^{\beta} & \text{for } V < V^* \\ V - K & \text{for } V \geq V^* \end{cases} \tag{6}$$

where:

$$\beta = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$$

and the optimal trigger for investing is:

$$V^* = \frac{\beta}{\beta - 1} K$$

This means that the company should only implement the project when the gross project value is large enough, specifically $\beta/(\beta-1)$ bigger than the cost of the investment. In technical terms, this happens when the value of the option to defer becomes worthless. Any investment being undertaken before that is considered suboptimal.

3.2 The inclusion of the Government

The goals of firms are different from those of the Government, but both want to manage efficiently the resources in order to maximize their own objectives. The firms want to maximize profits with minimum resources, while the Government has two main objectives: to stabilize economic cycles and to promote economic growth, also with minimal resources. This means that to achieve objectives with a neutral or even positive impact in the present value of public deficit and debt. Generally, the Government can try to stimulate GDP by increasing public consumption, public investment, or transfers to the private sector, or even by reducing taxes.

Perotti (2004) considers that there is no evidence that public investment shocks are more effective than other alternatives, but productive investment can have greater impact in some cases. In fact, in some countries with current account imbalances, a higher available income can exacerbate the imbalances and have a reduced impact on GDP, while in others it can have positive effects. In addition, investment decisions are crucial for the evolution and growth of firms.

Therefore, we are going to study the Government motivation to increase the investment, by analyzing the interaction between public and private investment in order to find which is better in an aggregated analysis.

Basically, if we consider a list of projects that can be implemented in some economy, in-

creasing the investment means to implement some of these projects. However, if for most of them the optimal has not been achieved, the public authority may intend to hasten some of those investments in order to improve economic situation. But how can they do it? In order to increase the aggregated investment, the Government can anticipate some of their projects to increase the investment now. This can be done, either by directly assuming the investments (public investment), or by promoting, for instance by subsidizing, the private investment. But is this efficient? In spite of returning a smaller profit value, the implementation of the investment opportunity now can bring a positive externality to the economy and improve the value of other projects too. In fact, both the private sector and the Government can benefit with the existence of a subsidy. To the private firms, the investment gets optimal, whereas the Government can promote the economic growth, and increase the collected taxes, and this can be possible without deteriorating public finances. This leads to a higher performance of the economy, because as more projects are implemented the better the economic situation and, as a consequence, other projects increase in value and are implemented sooner as well. Furthermore, it can be expected an increase in GDP and an unemployment reduction. This can be crucial if the Government is able to promote the implementation of the most productive projects, which allow the implementation of other type of projects, complementary to these, but more sophisticated. This represents an evolutionary process, as it potentiates economic growth and economic and social development. So it is crucial to implement projects with high economic value added.

Instead of being only done by the Public Sector, the investment can also be made by private firms, which can do it naturally or by hastening the investment decision. In the first case it is not necessary much attention of the Government, but this is quite different in the second case. When the projects implemented normally by firms are sufficient, the economy grows naturally, but when this is not enough there must be some interference. In this case, if the public institution wants to increase the private investment in the economy, assuming that all institutional and political incentives are achieved, it implies to reduce the trigger value of the private investments, namely by subsidizing them. In fact, the firms must have incentives to implement the investment earlier, but the Government must also take some benefits on than. In some circumstances the Government may have benefits by investing itself, but in other cases it would be better to subsidize. Based on the real options model shown previously we are going to present a model to try to solve this questions.

Including the government, there are new parameters and new conclusions. There will be thus two trigger values: for government (V_g^*) and for private firms (V_p^*). There is also the multiplier of private investment (λ_p) and of the public investment (λ_g), which measures the impact of each kind of investment in the output. On the other hand, we also consider the existence of two tax rates: one over capital, the capital income tax rate (t_c), and another for the rest of the economy, the normal average tax rate of the economy (t_n). Beyond that we also consider the comparative inefficiency of public sector facing the private, both in the investment cost (γI), and in the current value of the profits flows (γV). This means that we will have two investment

costs, for private (I_p) and for the Government (I_g), and also two present values of the profit flows, respectively, V_p and V_g. In this stage we will consider that motivations of V_g^* and V_p^* are similar, because this kind of investment is not a public good. Nevertheless, the payoffs are different. With a normal private investment the firms payoff will be the after-tax profit flows net of the investment cost:

$$V_p(1 - t_c) - I_p \tag{7}$$

With this investment, for payoffs for the Government are:

$$V_p t_c + t_n \lambda_p I_p \tag{8}$$

We consider that the benefits of the investment are not only the profit flows, but also all the positive effects in the economy. With a project implementation, there is a purchase of goods from others companies, wages for the employers and a panoply of external services and all of them pay taxes, thus we consider a normal average tax that focus on this indirect effect.

In the case of a public investment the payoffs for the Government are:

$$V_g(1 - t_c) - I_g + V_g t_c + t_n \lambda_g I_g \tag{9}$$

or simply:

$$V_g - I_g + t_n \lambda_g I_g \tag{10}$$

Notice that the capital income tax rate is a neutral parameter for the Government, when public investment is undertaken.

The action of the Government reduces the payoff for firms, as a part of the profit flows takes a form of taxes. However, the public institution in addition to capture taxes directly from the profit flows of the investment, also receives taxes from the multiplier effect of the investment in the economy. Since the private investment is usually higher than the public investment multiplier (as we have seen in section 2, Afonso and St. Aubyn (2009) estimated a private multiplier that is, on average, twice as large as the public multiplier), it can be better for the Government to subsidize the private investment instead of directly implement some of the projects.

About the question if the public investment pays for itself, Perotti (2004) argues that there is no strong evidence on it. However, Pennings (2000) have concluded that the Government could subsidize private investment with a zero expected cost. Stimulate the private investment is less costly to the Government (the subsidy is only a part of the investment instead of the total cost), the multiplier effect in the economy may be higher, the private efficiency may be higher and it is possible to implement this policy with a zero expected cost to the Government.

3.3 Performance and Efficiency

Some literature has shown that in some countries the cost of the investment and project managing leaded by the Government are inefficient. As we saw, in the presence of inefficiencies, public investment may be important only to some extent, that is, to ensure a sufficiently high productivity of private capital and basic needs. In fact, the institutions weakness and corruption can exacerbate the decreasing of marginal productivity of the investment and confine it to the vital functions. In this case, increasing public investment crowds-out private investment and decreases economic performance. So, as the private one has a predominant role, the Government should transfer non-core activities to the private sector. However, if we have a better efficiency and quality of the public investment it can extends to other areas and increase the productivity of the economy, so it is a key factor to choose between public and private investment. Naturally, the decision will be different from country to country as it depends on the parameters. Accordingly, Chakraborty and Dabla-Norris (2009) consider that due to the political cycles, clientelism, voting behavior, corruption and mismanagement, in some countries the cost of the investment for the Government is bigger than the cost of the private investment and respectively, the present value of the profit flows are smaller. Afonso et al. (2005) estimate values for two kinds of inefficiencies across countries the public sector performance (PSP), and the public sector efficiency (PSE) concluding that some countries are less inefficient while others are more inefficient. Furthermore, we will consider an inefficiency parameter that could be 0 in some efficient countries or superior in inefficient countries, and we define V_g and I_g as:

$$V_g = (1-\gamma_V)V_p, \text{ where: } 0 \leq \gamma_V < 1 \tag{11}$$

$$I_g = (1+\gamma_I)I_p, \text{ where: } \gamma_I \geq 0 \tag{12}$$

where γ_V represents the percentage of inefficiency of profit flows and γ_I the percentage of inefficiency about the cost of investment. On the one hand, the percentage of inefficiency of profit flows means that the management capacity of the Government is usually worse than the management capacity of the private sector, thus the Government could have lower profit. On the other hand, the percentage of inefficiency of the cost of the investment means that the cost of implementing a project could be higher in the case of the public investment.

For certain levels of Government inefficiency it would be better a subsidy policy. In fact, if the public projects are not as efficient as the private ones and knowing that the cost of subsidizing is smaller than the cost of the investment, in some cases would be better the existence of private investment with Government aid. Under this point of view, the firms can hasten some of the better projects that have more profitability and more value added, leading to an output expansion and it is possible that it has a null or even positive impact in the present value of the public budget account. In this situation, the payoffs for firms are:

$$V_p(1-t_c) - (I_p - s) \tag{13}$$

where s is the total amount of subsidy for the project. The payoff for Government is:

$$V_p t_c + t_n \lambda_p I_p - s \tag{14}$$

We resume all the payoffs in the following table:

	Payoffs for Firm	Payoffs for Government
Private Investment	$V_p(1-t_c) - I_p$	$V_p t_c + t_n \lambda_p I_p$
Private Investment with subidy	$V_p(1-t_c) - (I_p - s)$	$V_p t_c + t_n \lambda_p I_p - s$
Public Investment	–	$V_g - I_g + t_n \lambda_g I_g$

Table 1: Payoffs for Firm and Government with and without subsidy.

The trigger value of the profit flows for the firms is given by:

$$V_p^* = \frac{\beta}{\beta - 1} \frac{I_p - s}{1 - t_c} \tag{15}$$

and for the Government is given by:

$$V_g^* = \frac{\beta}{\beta - 1}(1 - t_n \lambda_g) I_g \tag{16}$$

Thus, the value of the investment opportunity for firms is:

$$F(V_p) = \begin{cases} \left[V_p^*(1-t_c) - (I_p - s)\right] \left(\frac{V_p}{V_p^*}\right)^\beta & \text{for } V_p < V_p^* \\ V_p(1-t_c) - (I_p - s) & \text{for } V_p \geq V_p^* \end{cases} \tag{17}$$

and for the Government is:

$$G(V_g) = \begin{cases} \left(V_g^* - I_g + t_n \lambda_g I_g\right) \left(\frac{V_g}{V_g^*}\right)^\beta & \text{for } V_g < V_g^* \\ V_g - I_g + t_n \lambda_g I_g & \text{for } V_g \geq V_g^* \end{cases} \tag{18}$$

After determining the trigger values and the value of the investment opportunity for firms and for the Government, we can now study equilibrium solutions.

3.4 Equilibrium

In this section, we are going to analyze the interaction between public and private investments. If it is necessary to promote the investment and if the natural private investment is insufficient, the Government can increase public investment or stimulate private investment, as we have seen before. The firms will only hasten the investment if it is optimal to do so and the public authority

will only subsidize if it is better than implement public investment. The optimal subsidy for the firm, s_{opt}, is the amount that turns the trigger value of the profit flows equal to its current value, $V_p^* = V_p$:

$$s_{opt} = I_p - \frac{\beta - 1}{\beta} V_p(1 - t_c) \tag{19}$$

which compensates the firm for losing the option to defer the project implementation.

On the other hand, as the Government will only subsidize if this decision is better than or, at least, equals the public investment decision, the maximum subsidy will be:

$$s_{max} = t_c V_p + t_n(\lambda_p I_p - \lambda_g I_g) - (V_g - I_g) \tag{20}$$

Consequently the maximum subsidy depends positively on the direct taxes of the private investment, $t_c V_p$, negatively on the direct payoffs of the public investment $(V_g - I_p)$. We will consider that depends positively also on the difference of indirect taxes, $t_n \lambda_p I_p - \lambda_g I_g$, because the private investment multiplier is usually higher than the public one, $\lambda_p > \lambda_g$.

After introducing and explaining some Governments inefficiencies, we will analyze their options. The Government will only subsidize if the maximum amount they accept to give is higher than the optimal subsidy for firms:

$$s_{max} \geq s_{opt} \tag{21}$$

If it is inferior, then the public authority will not subsidize, since it will not prompt the investment for the private sector, and can realize the investment by itself or waiting for a better moment. Accordingly, there are three types of investment. The first is the normal private investment with zero subsidies, which has more value; the second is the private investment with subsidy policy; and the third type is the public investment. Higher volatility, σ, higher cost of the investment, I, higher risk-free interest rate, r or lower cost of defer the investment, δ, increases the trigger for which is optimal to invest. This means that when the parameters vary in this way, ceteris paribus, we go from investment region of type one to investment region of type two and in some cases to region of type three.

It is easy to understand that in the same circumstances the trigger value for the public investment, V_g^*, is smaller than that for the private sector, V_p^*, because the Government has more payoffs coming from the taxes. Nevertheless, with some different parameters it is possible to obtain a different solution. In fact, the optimal trigger for the public investment is smaller than the optimal trigger for the private one, $V_g^* < V_p^*$, when:

$$t_n \lambda_g > 1 - \frac{1 - \psi}{(1 - t_c)(1 + \gamma_t)} \tag{22}$$

where $\psi = s/I_p$, the percentage of subsidy over total investment. This means that the normal average tax rate in the economy multiplied by the public investment multiplier has to be higher than one minus the percentage of the investment paid by privates $(1 - \psi)$, which is divided by

the multiplication between the percentage of profit flows owned by privates $(1 - t_c)$, and the level of inefficiency of the public investment $(1 + \gamma_I)$.

4 Numerical Example and Comparative Statics

In this section we are going to simulate the model and study the parameters sensitivity . We consider an investment opportunity with a present value below to the trigger value, so the optimal decision is to defer the project implementation. The basic case inputs are as follows:

Parameter	Value	Description
I_p	100	Investment cost for private firm
γ_I	0.25	Public inefficiency regarding the investment cost
I_g	125	Investment cost for the government
λ_p	2	Multiplier of private investment
λ_g	1	Multiplier of public investment
t_c	0.25	Capital income tax rate
t_n	0.25	Average tax rate on the economy
r	0.05	Risk-free interest rate
δ	0.06	Dividend-yield
σ	0.20	Instantaneous volatility of V
V_p	200	Present value of future cash flows for private investment
γ_V	0.25	Public inefficiency for running the project
V_g	160	Present value of future cash flows for public investment

Table 2: The base case parameters.

Outputs	Value	Description
V_p^*	222.22	Optimal trigger for private firm
$F(V_p)$	51.23	Value of investment opportunity for private firm
V_g^*	156.25	Optimal trigger for public investment
$G(V_g)$	66.25	Value of investment opportunity for Government
s_{max}	43.75	Maximum subsidy
s_{opt}	10.00	Optimal subsidy
V_p^* with s_{opt}	200.00	Optimal trigger for private firm with subsidy
$F(V_p)$ with s_{opt}	60.00	Value of investment opportunity for private firm with subsidy
Π_g	66.25	Payoff for the Government (public investment)
$\Pi_{gp(s)}$	90.00	Payoff for the Government (private investment with subsidy)

Table 3: The base case parameters.

With these inputs, the trigger value for the public investment, V_g^*, is nearly 156.25, while the trigger value for the private investment, V_p^*, is about 222.22. We can see this representation in Figure 1, below.

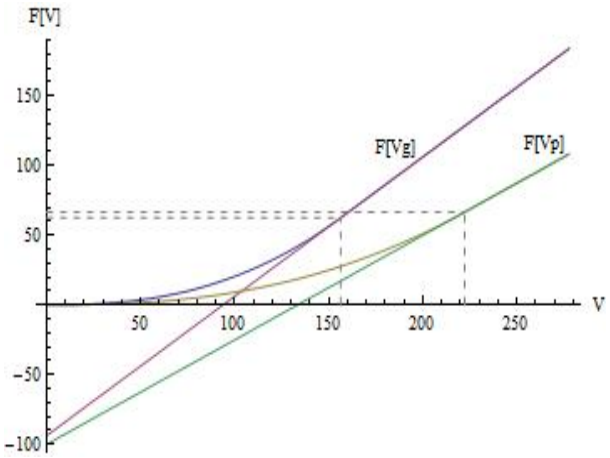

Figure 1: Value of the investment opportunity for Government and firm.

In this example, private firms will not invest until the present value of the profit flows equals the trigger value, V_p^*. It is possible to decrease this trigger value by managing some key parameters, as we have seen before. However, even with these parameters the Government can promote this investment by a subsidy policy. Actually, the public institution intends to put V_p^* equal to V_p . The optimal subsidy in this case is 10, which is inferior to the maximum subsidy (that is 43.75). So, with a subsidy percentage of $\psi = 10\%$, the cost of the investment for firms turns to be 90 and the trigger value V_p^* decreases to 200, as intended. This means that with a subsidy of 10 the firms will hasten the project implementation and invest now.

On the other hand, the investment can also be implemented by the Government. In this case, as the percentage of inefficiency is 25% both for the cost of the investment, as for the present value of the profit flows, I_g is 125 and V_g is 150. The trigger value, V_g^*, is 156.25, so it is not yet optimal to invest. Thus, the Government must compare the payoffs to choose the best decision. The payoff of public investment is 56.25, the direct part results from the difference between V_g and I_g, which is 25, and the indirect part results from $t_n \lambda_g I_g$, which is 31.25. Notwithstanding, the payoff of subsidy policy is 90, resulting from Equation (14). So, in this circumstances will be better from the public purse and for the economy subsidy the investment, because with fewer resources the output will be much superior. Considering the payoff for the firm, which is 50, the total profit flows of the project is 140 with an investment of 100, which compares with a total payoff of public investment of 56.25 with an investment of 125. This means that in some circumstances it is better to choose a subsidy policy. Furthermore, the capital rate of return for the Government of subsidizing is 900%, much higher than invest directly, 45%. This means that with fewer resources the Government can have the same or even more payoff by subsidizing.

It is obvious that the result depends on the parameters, so we are going to analyze now the parameters sensitivity.

4.1 Sensitivity of the percentage of inefficiency

If we consider the same investment with a percentage of inefficiency of 0%, ceteris paribus, V_g^* decreases to 125, as it is shown in tables Tables A.1 and A.2 in the Appendix. In this situation, the Government will never subsiding private investment based on budget account, as the optimal subsidy is higher than the maximum subsidy. Nevertheless, if the private investment multiplier is superior, the impact on GDP is also higher. With a percentage of inefficiency of 50%, ceteris paribus, V_g^* changes for 187.5. Thus, in spite of investing the Government will always opt to subsidize, ensuring that it is profitable for the Government. With these numerical simulations we can confirm that the inefficiency has a great impact in the value of the public investment opportunity and in the maximum subsidy that the Government intends to accept. Therefore, it is crucial for choose between investing and subsidize. The augmenting of the inefficiency increases V_g^* and decreases the present value of the investment opportunity, which means a bigger s_{max}. Accordingly, a bigger inefficiency leads to more subsidy policy instead of public investment.

4.2 Sensitivity of the multipliers

The multipliers have a great impact in the Government parameters. As we can see in Table A.3 in the Appendix, if there is no indirect impact on GDP (case 1) the trigger value for the public investment is higher than the base case situation, because there are no additional taxes. Increasing the public investment multiplier, λ_g, will reduce V_g^* and increase the value of the investment opportunity, so the investment will be implemented earlier. Ceteris paribus, this will also increase the public investment hypothesis. Contrariwise, increasing the private investment multiplier, λ_p, increases the payoff of the Government of subsidize and increases this hypothesis too.

With an improvement in λ_g the public investment becomes more favorable and, in other way, an improvement in λ_p turns the subsidy policy more propitious. In addition, the multipliers have no impact on the trigger value of the private investment, V_p^*.

4.3 Sensitivity of tax rates

Assuming that there are no taxes, as in case 1 in Table A.4 in the Appendix, the investment might be already implemented, because the trigger value for the private investment is below the present value. In case 2 the investment can be implemented now, as $V_p = V_p^*$, but t_c must be 16,66%. If t_c rises to 25%, V_p^* changes to 222.22 and it is necessary a subsidy of 10 to invest now. As we can see in the third part, only this tax rate has impact in the private investment decision. By the opposite, only t_n has impact in the public investment decision. Nevertheless, both have impact on the subsidize decision.

It is obvious that higher tax rates leads to an increase in the profits of the Government and it improves their investment conditions. Nevertheless, increasing the capital income tax rate

will reduce the payoff for firms, so they will delay the investment execution. This means that the Government must keep this tax rate in a moderate level.

4.4 Sensitivity of the interest rate

Table A.5 in the Appendix shows the influence of the interest rate in the investment decision. The interest rate affects almost all outputs. If interest rate passes from 5% to 3%, the firm will be able to invest now. By the opposite, if it changes to 7% the amount of subsidy necessary for hasten the investment will duplicate.

Figure 2 below shows, in a descending order, the trigger value of the investment with taxes (IPtc), without taxes (IP), for the Government (IG), with taxes and a subsidy of 50% (IPstc) and finally with taxes and a subsidy of 75% (IP0.75stc). As we can see, the rise in interest rate increases the trigger value for both public and private investment. Thus, there is a delay of the investment projects, and so public authorities must ensure that the interest rate remains low.

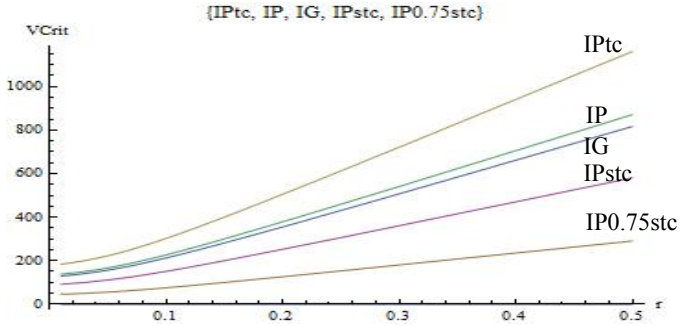

Figure 2: Evolution of the trigger value by changing the interest rate.

4.5 Sensitivity of the dividend yield

Increasing the dividend yield, δ, represents a higher cost of delay the investment, which means that the investment will be implemented earlier. In Table A.6 in the Appendix we see that the reduction of δ to 4% triplicates the amount of subsidy needed for implement the investment now, s_{opt}, and an increase to 8% allows that the private investment is performed now without public intervention.

Increasing the cost of delay the investment, δ, also increases the payoff for the Government of the private investment execution. From δ near to 8% it is not necessary a subsidizing policy and the maximum payoff for the Government is 100, that is when $V_p^* = 200$ (without public intervention). In this case, without any intervention, the public institution obtains the maximum taxes, so this is an optimal hypothesis. Nevertheless, even with $\delta = 6\%$ the payoff can reach 90 and a capital rate of return of 900% (the investment is a subsidy of 10). Figure 3 below shows the relation between the cost of delay the investment, δ, and the trigger value of the investment,

V_{crit}. It is evident that increasing this cost reduces the trigger value for all kind of investments. This graphic also shows, in a descending order, the trigger value of the investment with taxes (IPtc), without taxes (IP), for the Government(IG), with taxes and a subsidy of 50% (IPstc), and finally with taxes and a subsidy of 75% (IP0.75stc).

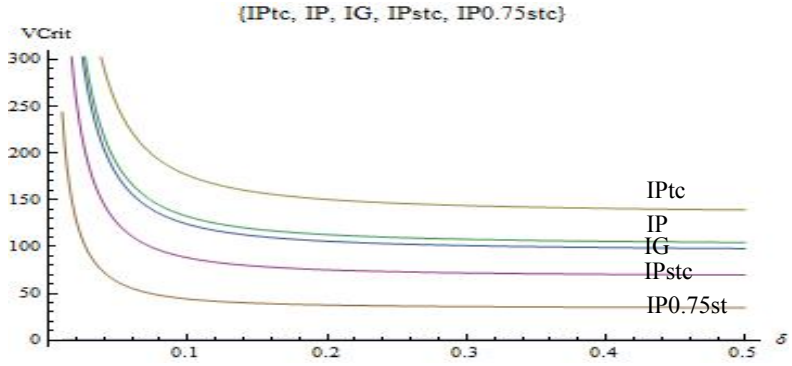

Figure 3: Evolution of the trigger value by changing the dividend yield.

4.6 Sensitivity of the volatility

Testing volatility for 10%, 20% and 30%, we see in Table A.7 in the Appendix that, with other parameters constant, only for $\sigma = 10\%$ the investment is realized now. In fact, the present value of the investment is equal to the trigger value only when the volatility is near to 16%.

An increase in the volatility turns the trigger value of the investment higher, which means that the investment will be implemented later or the subsidy cost to invest now is higher. This way, the Government should ensure good economic, political and institutional indicators to minimize the volatility.

Figure 4 below shows exactly the increase of the trigger value of the investment when there is an increase on the volatility, but mainly it shows that the increase of V_{crit} is higher with capital income tax rate (t_c) and lower with subsidy, as we see, respectively, an higher slope on the first curve and a lower slope on the last curve. This is because, in a descending order, the first curve (IPtc) shows a situation where firms pay 100% of the cost of the investment and pay taxes too. The last two curves (IPstc and IP0.75stc) shows a situation where even paying taxes, firms only pay a part of the investment, because of the subsidizing policy. This way, as the investment of the firms is only a percentage of the total investment, they support only a part of the risk, which means a lower trigger value of the investment, because they will invest earlier.

Instead of the subsidizing policy, there is other ways to promote private investment. In the next section we will explore other solutions.

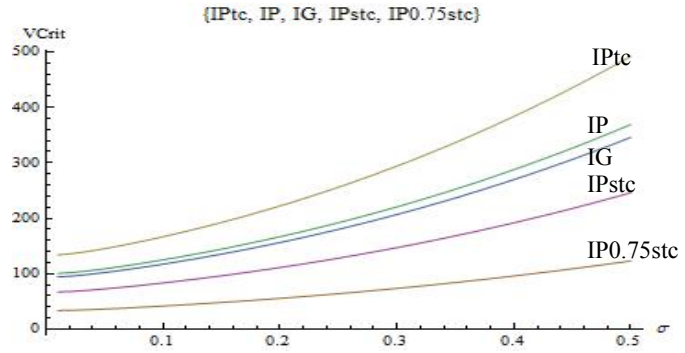

Figure 4: Evolution of the trigger value by changing the volatility.

5 Alternative Stimulus

In addition to the cost of the investment, the sensitivity of other parameters is also important, so the Government must try to optimize all this parameters before introducing a subsidy policy, in order to use it only when it is crucial.

By testing the parameters sensitivity we can see that the investment can be implemented now by many ways: if the capital tax is reduced to 16.67%; if the interest rate goes to 3%; if the dividend yield ups to near 8%; or if the volatility is reduced to about 16%. This means that instead of using a subsidy policy, the Government can use other solutions.

5.1 Subsidy versus Tax policy

One of the other solutions is the tax policy. Instead of using a subsidy policy, the Government can change the tax rates. Figure 5 below shows a relation between the tax rate, t_c, and the percentage of subsidy which ensures that the investment is implemented now. As we have seen before, with a zero subsidy, t_c must be nearly 16.67% to ensure the project execution. An increase in t_c turns the investment less desirable, so it is necessary a subsidy policy to encourage it. Now, we need to understand if to hold the tax rate and to create a subsidy policy make sense or if it is advantageous manage the tax rate.

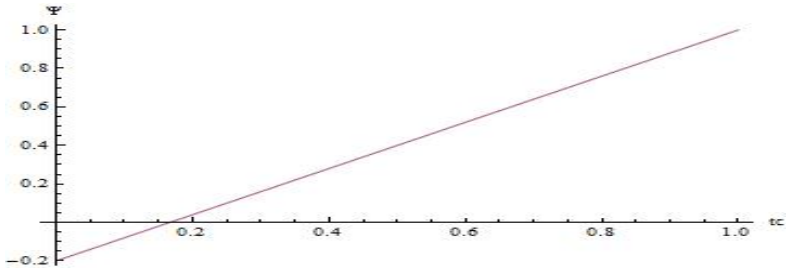

Figure 5: Percentage of subsidy for different levels of tax rate.

As we know that for hasten the investment is necessary a subsidy of 10 or a reduction of the tax rate to nearly 16,67%, we will compare the payoff for the Government in both cases. Using Equation (14) we find easily that the payoff of subsidy is 90 and the payoff of the tax reduction is 83.33.

In fact, it is better to subsidy if the cost or the tax reduction is higher that the optimal subsidy:

$$V_p(t_c - t_{c_{opt}}) > s_{opt} \tag{23}$$

where t_c is the capital income tax of the economy and $t_{c_{opt}}$ is the capital income tax that promotes the immediate investment. If this happens it is better a subsidy policy. However, if the Government have low resources at a given time, implementing a tax reduction for some projects could be better.

5.2 Subsidy versus interest rate

Beyond the subsidy policy, other variables can influence the timing of the investment too. An interest rate reduction may influence firms to hasten the investment. Therefore, if there is an institutional cooperation, instead of the Government implementing a subsidy policy, the Central Bank can implement an interest rate reduction. Exemplifying, in this case a reduction of the interest rate to 3% is sufficient to hasten the investment without any other incentive.

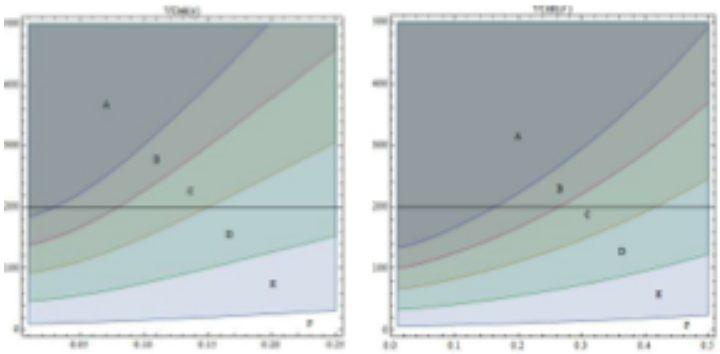

Figure 6: Regions for each type of investment by changing the interest rate and volatility.

Figure 6 (left panel) shows a relation between the interest rate, r, and the trigger value of the private investment, V_{Crit} or V_p^*. Each region represents one type of investment. The inferior limit of region B (curve between region B and C) is the curve of the trigger value for the private investment for each value of the interest rate without taxes and subsidies. The regions above represents a zero subsidy and an increase in t_c, while the regions below represents an increase in subsidy, which can be reconciled with the existence of the tax rate, t_c, if the subsidy rate is above the tax rate.

In fact, on region A and B, respectively with and without capital income tax rate, the private investment can be implemented without any subsidize, provided that the present value equals the trigger value of the investment opportunity. Thereby, with a tax rate of 25% and a subsidy rate of 50%, if it is optimal, the firms can invest both on region A and on region B and on region C. Nevertheless, if the subsidy rate ups to 75% firms can also invest on region D and if it goes to 95% firms can also invest on region E, but we know that they will invest only when the present value of the pre-taxes profit flows reaches the trigger value. However, to invest on region F firms will need a subsidy rate higher than 95%. This means that the Government can increase the regions of the private investment by reducing tax rate or increasing subsidy.

In this analysis we try to understand the different types of private investment, seeing how the trigger value varies, for each type, with interest rate changes. Keeping the present value of the profit flows $V_p = 200$, the investment will be implemented now only if the trigger value do not exceed the V_p which means in the regions below or equal to the line ($V_{Crit} = 200$).

With these parameters, with a tax rate of 25% and a subsidy of 0 (inferior limit of region A), firms will only invest if the interest rate does not exceed 3%. Without tax rate and subsidy (inferior limit of region B), firms will only invest if the interest rate does not exceed 8%. On region C, firms will invest until the interest rate equals 15%. On region D there is the possibility of investing until the interest rate is equal to 25%, but only on region E and F this is always possible, considering a maximum interest rate of 25%.

Table A.8 in the Appendix shows the conditions for the inferior and superior limits of each region and also some numerical values for the percentage of subsidy, ψ, and for the capital income tax rate, t_c, which allows the existence of those limits. Choosing one region, on region C must be a combination of ψ and t_c which ensure that $(1-\psi)(1-t_c)$ is 2/3 for the inferior limit and 1 for the superior limit. In this case, for example, the percentage of subsidy and the capital income tax rate could be, respectively, 50% and 25% for the inferior limit, but they have to be equal on the superior limit.

Table A.8 also shows that increasing the interest rate leads to a high necessity of Government intervention to ensure the execution of the investment. This means that the Government must try all the solutions to keep the interest rate in low levels before subsidizing directly the investment. Considering the independence of the central bank, if the official interest rate of the economy is not sufficient, the Government must try some credit lines in good conditions for important projects. One of the ways, in the European Union, is the financing of the European Investment Bank. Nevertheless, the Government can create some specific credit lines too. The importance of this way is to ensure low interest rates for positive projects.

5.3 Subsidy versus volatility

The volatility is another parameter with a large impact on investment decisions. Similarly to the interest rate, decreasing the volatility to about 16% would allow the implementation of the investment, in substitution of the subsidy rate of 10%. If we set the tax rate on 25%, the

volatility must be below 16%. As in the analysis of the interest rate, this means that the part of region A below $V_p* = 200$ ensures the investment implementation (Figure 6, right panel).

Then, with a decrease of the ratio $(1 - \psi)(1 - t_c)$, the region where the investment can be implemented immediately expands, as in the interest rate example. Table A.9 in the Appendix shows that an increase on volatility turns the investment more dependent of the Government support.

If we consider regions bellow F, firms will require more than 100% of the cost of the investment or, in other way, current transfers for implementing the project. This is a situation of projects with higher risk and lower or even negative values. Normally, these kinds of projects are implemented by the Government because the objectives, rather than its profitability, are other benefits for society.

It is not easy to understand whither the Government can incentive private investment when the objective is not the profits. Really, in this case the Government must weigh some important variables as the social welfare.

6 Conclusions

In this paper we have studied the interaction between the firms and the Government in the context of investment decisions. Following a real options approach, the firms will only invest when the present value of the investment reach the trigger value. However, the introduction of the Government, trough the existence of taxes, increases the trigger value of the investment, so this difficult the investment implementation. Anyway, in this paper we show that the Government can promote investment by many ways.

In fact, in this paper we have shown that the Government, even though looking to achieve their own objectives, can have also a very important role in the promotion of the private investment, particularly in what matters to their instant execution. Concretely, in the same way of Pennings (2000), we have shown that firms would like to invest immediately if there is a certain amount of subsidy which reduces the trigger value of the investment to the present value of the investment.

We focused on both perspectives, so it is possible to hasten the investment if both conditions are achieved: if the subsidy is sufficiently high for making firms want to invest and also if it is sufficiently low to be supported by the Government. Accordingly, if the optimal subsidy for firms is lower than the maximum one for the public authority. At this point, based on Afonso et al. (2005) and Chakraborty and Dabla-Norris (2009), we have studied the Government inefficiency hypothesis, and we have identified a direct relation between the level of inefficiency and the maximum subsidy that they may sustain. This is because inefficiency decreases their payoffs and so it makes the subsidy policy more attractive.

When the Government wants to promote economic growth and in the presence of inefficiency, it may be better to subsidy firms instead of invest by itself. This solution can have a great impact

on GDP, not only in the short-run because of the investment increasing, but also in the long-run because the substitution of some inefficient public investment by more productive private investment makes the economy more efficient and productive and it may results in a high level of GDP. For this to happen, it is crucial the selection of key sectors and the choice of the best projects. To keep the efficiency in a high level it is determinative to continue by selecting the most productive investments. This brings a higher multiplier effect of the investment and so a higher GDP and high payoffs for the Government. Notwithstanding, it is also crucial the existence of a basic stock of good public investment, implemented directly by the Government or indirectly via subsidy. This ensures the existence of suitable conditions for the private investment implementation.

By exploring the main parameters, we have shown that they have a great impact in investment decisions. This way, one of the worries of the Government must be to ensure low volatility and low interest rates, but also to keep the capital income tax rates in low levels. Nevertheless, we have found no evidence that reducing the tax rate is better than the subsidize policy. This could be possible for the Government if the reduction was temporary, which have a lower positive effect in the investment decisions of the private sector.

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A Appendix

Inputs	symbol	case 1	case 2	case 3
Private investment	I_p	100	100	100
Inefficiency of the public investment	γ_I	**0**	**0.25**	**0.50**
Public investment	I_g	**100**	**125**	**150**
Private investment multiplier	λ_p	2	2	2
Public investment multiplier	λ_g	1	1	1
Capital income tax	t_c	0.25	0.25	0.25
Normal average tax	t_n	0.25	0.25	0.25
Risk-free interest rate	r	0.05	0.05	0.05
Dividend yield	δ	0.06	0.06	0.06
Volatility	σ	0.20	0.20	0.20
Present value of private profit flows	V_p	200	200	200
Inefficiency of the public profit flows	γ_V	**0**	**0.25**	**0.50**
Present value of public profit flows	V_g	**200**	**150**	**100**

Figure A.1: Inputs for inefficiency sensitivity analysis

Outputs	symbol	case 1	case 2	case 3
Maximum subsidy	$s_{máx}$	**0***	**43.75**	**112.5**
Optimal subsidy	s_{opt}	10	10	10
Percentage of optimal subsidy	Ψ	10%	10%	10%
Private trigger value of profit flows	V_p^*			
• Without subsidy		222.22	222.22	222.22
• With subsidy		200	200	200
Public trigger value of profit flows	V_g^*	125	156.25	187.5
Value of the investment opportunity	$F(V)$			
• For firm without subsidy	$F(V_p)$	50	50	50
(capital rate of return %)	%	50%	50%	50%
• For firm with subsidy	$F(V_p s)$	–	60	60
(capital rate of return %)	%	–	66.67%	66.67%
• For Government	$F(V_g)$	**125**	**56.25**	**−12.50**
(capital rate of return %)	%	**125%**	**45%**	**−8.33%**
Government's subsidy payoff	$F(s)$	**90**	**90**	**90**
(capital rate of return %)	%	**900%**	**900%**	**900%**

Figure A.2: Results for inefficiency sensitivity analysis

Inputs	case 1	case 2	case 3	case4	Outputs	case 1	case 2	case 3	case 4
I_p	100	100	100	100	$s_{máx}$	**25**	**18.75**	**43.75**	**12.5**
γ_I	0.25	0.25	0.25	0.25	s_{opt}	10	10	10	10
I_g	125	125	125	125	Ψ	10%	10%	10%	10%
λ_p	**0**	**1**	**2**	**2**	V_p^*	222.22 200	222.22 200	222.22 200	222.22 200
λ_g	**0**	**1**	**1**	**2**	V_g^*	**208.33**	**156.25**	**156.25**	**104.17**
t_c	0.25	0.25	0.25	0.25	$F(V_p)$	50	50	50	50
t_n	0.25	0.25	0.25	0.25	%	50%	50%	50%	50%
r	0.05	0.05	0.05	0.05	$F(V_p s)$	60	60	60	60
δ	0.06	0.06	0.06	0.06	%	66.67%	66.67%	66.67%	66.67%
σ	0.20	0.20	0.20	0.20	$F(V_g)$	**25**	**56.25**	**56.25**	**87.5**
V_p	200	200	200	200	%	**20%**	**45%**	**45%**	**70%**
γ_V	0.25	0.25	0.25	0.25	$F(s)$	**40**	**65**	**90**	**90**
V_g	150	150	150	150	%	**400%**	**650%**	**900%**	**900%**

Figure A.3: Inputs and Results for sensitivity to multipliers

Inputs	c1	c2	c3	c4	Outputs	case 1	case 2	case 3	case 4
I_p	100	100	100	100	$s_{máx}$	–	–	**37.45**	**43.75**
γ_I	0.25	0.25	0.25	0.25	s_{opt}	–	0	10	10
I_g	125	125	125	125	Ψ	–	0%	10%	10%
λ_p	2	2	2	2	V_p^*	**166.67** –	**200** –	222.22 200	222.22 200
λ_g	1	1	1	1	V_g^*	**208.33**	**156.25**	**173.75**	**156.25**
t_c	**0**	**0.16**	**0.25**	**0.25**	$F(V_p)$	**100***	**66.66**	50	50
t_n	**0**	**0.25**	**0.166**	**0.25**	%	**100%***	**66.6%**	50%	50%
r	0.05	0.05	0.05	0.05	$F(V_p s)$	–	–	60	60
δ	0.06	0.06	0.06	0.06	%	–	–	66.67%	66.67%
σ	0.20	0.20	0.20	0.20	$F(V_g)$	**25**	**56.25**	**45.75**	**56.25**
V_p	200	200	200	200	%	**20%**	**45%**	**37%**	**45%**
γ_V	0.25	0.25	0.25	0.25	$F(s)$	–	–	**73.2**	**90**
V_g	150	150	150	150	%	–	–	**732%**	**900%**

Figure A.4: Inputs and Results for sensitivity to tax rates

Inputs	case 1	case 2	case 3	Outputs	case 1	case 2	case 3
I_p	100	100	100	$s_{máx}$	–	43.75	43.75
γ_I	0.25	0.25	0.25	s_{opt}	–	**10**	**20.18**
I_g	125	125	125	Ψ	–	**10%**	**20.18%**
λ_p	2	2	2	V_p^*	**200**	**222.22**	**250.55**
					–	200	200
λ_g	1	1	1	V_g^*	**140.625**	**156.25**	**176.17**
t_c	0.25	0.25	0.25	$F(V_p)$	50	50	50
t_n	0.25	0.25	0.25	%	50%	50%	50%
r	**0.03**	**0.05**	**0.07**	$F(V_p s)$	–	**60**	**70.18**
δ	0.06	0.06	0.06	%	–	**66.67%**	**87.92%**
σ	0.20	0.20	0.20	$F(V_g)$	56.25	56.25	56.25
V_p	200	200	200	%	45%	45%	45%
γ_V	0.25	0.25	0.25	$F(s)$	–	**90**	**79.82**
V_g	150	150	150	%	–	**900%**	**395%**

Figure A.5: Inputs and Results for sensitivity to interest rate

Inputs	case 1	case 2	case 3	Outputs	case 1	case 2	case 3
I_p	100	100	100	$s_{máx}$	43.75	43.75	–
γ_I	0.25	0.25	0.25	s_{opt}	31.05	10	–
I_g	125	125	125	Ψ	31.05%	10%	–
λ_p	2	2	2	V_p^*	**290.05**	**222.22**	**192.19**
				V_p^*s	200	200	–
λ_g	1	1	1	V_g^*	**203.95**	**156.25**	**135.13**
t_c	0.25	0.25	0.25	$F(V_p)$	50	50	45.93
t_n	0.25	0.25	0.25	%	50%	50%	44.14%
r	0.05	0.05	0.05	$F(V_p s)$	81.05	60	–
δ	**0.04**	**0.06**	**0.08**	%	117.54%	66.67%	–
σ	0.20	0.20	0.20	$F(V_g)$	**56.25**	**56.25**	**56.25**
V_p	200	200	200	%	**45%**	**45%**	**45%**
γ_V	0.25	0.25	0.25	$F(s)$	**68.95**	**90**	–
V_g	150	150	150	%	**222%**	**900%**	–

Figure A.6: Inputs and Results for sensitivity to dividend yield

Inputs	case 1	case 2	case 3	Outputs	case 1	case 2	case 3
I_p	100	100	100	$s_{máx}$	–	43.75	**43.75**
γ_I	0.25	0.25	0.25	s_{opt}	–	10	32
I_g	125	125	125	Ψ	–	10%	32%
λ_p	2	2	2	V_p^*	166.67	222.22	294.07
					–	200	200
λ_g	1	1	1	V_g^*	**117.19**	**156.25**	**206.77**
t_c	0.25	0.25	0.25	$F(V_p)$	25	50	50
t_n	0.25	0.25	0.25	%	25%	50%	50%
r	0.05	0.05	0.05	$F(V_p s)$	–	60	82
δ	0.06	0.06	0.06	%	–	66.67%	121%
σ	**0.10**	**0.20**	**0.30**	$F(V_g)$	56.25	56.25	56.25
V_p	200	200	200	%	45%	45%	45%
γ_V	0.25	0.25	0.25	$F(s)$	–	**90**	**68**
V_g	150	150	150	%	–	**900%**	**213%**

Figure A.7: Inputs and Results for sensitivity to volatility

Region	A	B	C	D	E	F
Inferior limit						
• $\frac{(1-\Psi)}{(1-t_c)}$	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{15}$	0
• e.g. Ψ	0	0	0.5	0.75	0.95	1
• e.g. t_c	0.25	0	0.25	0.25	0.25	0.25
Critical Interest rate	**0.03**	**0.08**	**0.15**	–	–	–
Superior limit						
• $\frac{(1-\Psi)}{(1-t_c)}$	∞	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{15}$
• e.g. Ψ	0	0	0	0.5	0.75	0.95
• e.g. t_c	1	0.25	0	0.25	0.25	0.25
Critical Interest rate	–	**0.03**	**0.08**	**0.15**	–	–

Figure A.8: Limits and critical values for each region of interest rate

Region	A	B	C	D	E	F
Inferior limit						
• $\frac{(1-\Psi)}{(1-t_c)}$	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{15}$	0
• e.g. Ψ	0	0	0.5	0.75	0.95	1
• e.g. t_c	0.25	0	0.25	0.25	0.25	0.25
Critical volatility	**0.163**	**0.264**	**0**	–	–	–
Superior limit						
• $\frac{(1-\Psi)}{(1-t_c)}$	∞	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{15}$
• e.g. Ψ	0	0	0	0.5	0.75	0.95
• e.g. t_c	1	0.25	0	0.25	0.25	0.25
Critical volatility	–	**0.163**	**0.264**	**0.15**	–	–

Figure A.9: Limits and critical values for each region of volatility