Culture social capital: creation of human capital and economic growth

Sistelo, Marta and Mazeda Gil, Pedro

Faculdade de Economia da Universidade do Porto, Portugal

2025

Culture & social capital: creation of human capital and economic growth*

Marta Sistelo† Pedro Mazeda Gil ‡

October 29, 2025

Abstract

Culture and social capital may be variables of particular interest when explaining economic growth. In recent years, policymakers and economists have increasingly considered their role in economic growth, yet cultural capital and social capital are analyzed separately. Despite being different concepts of capital, in this paper we argue that there is a link between cultural and social capital, and both need to be accounted for when analyzing economic growth and welfare. We develop a theoretical dynamic general-equilibrium model using a mainstream endogenous economic growth set-up (namely with human capital accumulation), incorporating cultural and social capital. We use the model to devise long-run and transitional-dynamics effects from the perspective of both economic growth and welfare, explicitly considering the interplay between cultural and social capital and other forms of capital. A detailed calibration of the model allows for the derivation of quantitative results, with an emphasis on policy implications.

Keywords: Endogenous Growth; Social Capital; Cultural Capital; Human Capital; Welfare.

JEL Codes: J24, O4, Z10

*This research has been financed by Portuguese public funds through FCT - Fundação para a Ciência e a Tecnologia, I.P., in the framework of the project with reference UIDB/04105/2020 (CEF.UP - Center for Economics and Finance at University of Porto). Marta Sistelo gratefully acknowledges support from CEF.UP and also the financial support of FCT and ESF - European Social Fund, through individual Ph.D. scholarship 2023.03673.BD. The views expressed in the paper are those of the authors and do not necessarily represent those of PLANAPP.

†Faculdade de Economia da Universidade do Porto, Portugal. Corresponding author. Address: Rua Dr. Roberto Frias, 4200-464, Porto, Portugal. Corresponding author: marta.sistelo@gmail.com

‡PLANAPP, Faculdade de Economia da Universidade do Porto, and CEF.UP.

1 Introduction

This paper develops an endogenous growth model that aims to provide an integrated theoretical view of the potential contribution of cultural and social capital to economic growth and welfare.

Recently, the notion of cultural capital has been brought to the fore as a potential source of economic growth (Throsby, 1999; Ulibarri, 2000; Shockley, 2004; Rizzo and Throsby, 2006). The most usual definition is from Throsby (2001), who presents cultural capital as an "asset which embodies, stores, or provides cultural value in addition to whatever economic value it may possess" (Throsby, 2001, p. 46).

The Throsbian cultural capital definition has an economic framework that enables one to equate it with other types of capital. Throsby distinguishes two types of cultural capital, the tangible and intangible cultural capital. The *tangible* cultural capital exists in the form of "buildings, structures, sites, and locations endowed with cultural significance (commonly called "cultural heritage") and artworks and artefacts existing as private goods, such as paintings, sculptures, and other objects" (Throsby, 1999, p. 7). These assets require the allocation of human and physical resources to their production. They can be consumed as a final good (e.g., a visit to a historic site) or used in the production of other goods or services, including new cultural capital (e.g., inspiring the creation of other artefacts). Cultural heritage, artworks and artefacts can be damaged or may wear off over time (similar to physical capital depreciation), so they require investment to maintain their economic and cultural value. However, even if they are damaged or partially destroyed, some public-good value may remain, so that there may be a nonzero residual value to these assets (e.g., ancient Roman ruins) (Throsby, 1999, 2001; Rizzo and Throsby, 2006). The *intangible* cultural capital is defined as "the set of ideas, practices, beliefs, traditions and values which serve to identify and bind together a given group of people, however the group may be determined, together with the stock of artworks existing in the public domain as public goods, such as literature and music" (Throsby, 1999, p.8). Intangible assets can contribute to the creation of other cultural goods or give rise to a flow of services which can constitute final consumption. The stock of intangible cultural capital can deteriorate or increase with new investment. The creation and maintenance of new intangible cultural capital requires human and physical resources (Throsby, 2001).

Like cultural capital, social capital has been considered a source of economic growth. Social capital is a concept that economics has imported from the economic sociology literature (Woolcock, 1998) and is being integrated into economic growth theory. But, despite the existing research about it in social, political, and economics studies, there are various meanings and definitions for social capital. The most usual definition is the one by Putnam (1993), who describes social capital as the "features of social organisation, such as trust, norms, and networks, that can improve the efficiency of society by facilitating coordinated actions". The concept of social capital has been used to explain a wide range of phenomena in economics (Durlauf and Fafchamps, 2005). Empirical studies consistently find a positive relationship between social capital and economic growth, although the magnitude and mechanisms vary. Knack and Keefer (1997) show that trust and civic cooperation have significant positive effects on economic activity; however, horizontal networks, as measured in Putnam (1993) through group membership, appear unrelated to trust, civic norms, or economic performance. Whiteley (2000) finds that social capital has an effect on economic growth as strong as that of human capital or education, while Rupasingha et al. (2002) report positive but more moderate effects at the county level in the United States. Beugelsdijk et al. (2004) confirm a positive association across European regions, though they emphasize that the estimated effects are sensitive to model specification and the choice of social capital measures. These social capital studies have been subject to criticism, with authors like, e.g., Durlauf (2002) and Bovenberg (2003) pointing out to the lack of development of the theoretical framework. Studies such as Beugelsdijk and Smulders (2009) try to fill this void. In their paper, social capital is modelled as participation in two distinct networks: a closed one, reflecting within-group relations (*bonding* social capital) and an open one, reflecting inter-community relations (*bridging* social capital). Using data from the European Values Survey, they show that the effect of social capital on economic growth depends on its nature: while *bonding* social capital has no robust or even negative impact on growth, higher levels of *bridging* social capital are positively associated with economic growth.

Following Bille (2024),[1] who shows empirically that participation in cultural activities can generate positive social effects, we conjecture that cultural capital can play a significant role in the formation of social capital, and that institutions, trust and networks (social capital) are closely linked to cultural capital. Institutions and networks facilitate the production, dissemination and accessibility of cultural capital, while participation in cultural activities reinforces social cohesion and trust, thereby enhancing social capital. To the best of our knowledge, no paper integrates cultural, social, and human capital alongside physical capital within a single framework. In this paper, we propose a dynamic general-equilibrium model that examines the relationships among these assets, aiming to provide a comprehensive theoretical perspective on their potential contributions to economic growth and welfare within an endogenous-growth framework that sustains both long-run and transitional-dynamics effects. In doing so, we contribute to the analysis of the channels and mechanisms through which culture and trust (social capital) are generated and connected, particularly in relation to human capital formation. Following Throsby (1999; 2001), we divide cultural capital into two types: tangible and intangible. In our model, tangible cultural capital contributes to the accumulation of

[1] See also references therein.

intangible cultural capital, which, in turn, fosters the accumulation of social and human capital. Additionally, both intangible cultural capital and social capital directly enhance individual utility.

We run a detailed calibration of the model, using, when possible, a mixture of data- and model-based procedures. In particular, we use European data from the Harmonised European Time Use Surveys (HETUS), which allows us to map out the different daily activities of the respondents across the various relevant categories of activities to calibrate the share of time allocated to the accumulation of human capital, social capital, and intangible cultural capital. This procedure also allows us to calibrate the elasticity of intangible cultural capital and social capital in the utility function. We devise a number of calibration scenarios, under which the steady-state equilibrium of the model features local indeterminacy, with either a 5- or a 6-dimensional stable manifold characterized by a dampened oscillatory behavior. As is well known, local indeterminacy raises the possibility of sunspot equilibrium trajectories for the macroeconomic variables, i.e., trajectories around their steady-state values that do not result from fundamentals.

Relying on this calibration, we perform numerical exercises to assess the impact of policy-induced shocks on, namely, the capital ratios, economic growth and welfare. We underline the following results regarding the long-run equilibrium:

- A shock to the efficiency of human capital in the education sector incentivizes the accumulation of human capital and, thereby, of all types of capital (through the respective accumulation laws), thus increasing the economic growth rate and the ratio of all types of capital to consumption.

- Shocks to the preference for social and intangible cultural capital in the utility function leverage the households' risk-aversion factor and, thus, reduce their willingness to move resources intertemporally, decreasing the ratio of all types of capital to consumption and the economic growth rate.

- As for the efficiency of a given type of capital – other than human capital – in the accumulation of some type of capital, the shocks have no impact on economic growth while, in general, decrease the ratios of capital to consumption: the accumulation of a given stock of capital becomes more attractive, but this return effect is usually dominated by the efficiency effect, according to which a smaller quantity of the stock of capital is needed to produce a given amount of output usable as consumption.

- In general, however, shocks to the key technological and preferences parameters increase human, social, and intangible cultural capital vis-à-vis physical capital – the exception being the parameters that control for the efficiency of human capital accumulation, which incentivize physical capital accumulation through the imperfect substitutability of human and physical production capital in the aggregate production function.

- A shock to the efficiency of human capital in the education sector is the only one with a positive impact on welfare regardless of the calibration scenario, by yielding positive level effects (through the impact on the ratios of social and intangible cultural capital to consumption) and dynamic effects (through the impact on the economic growth rate). All in all, our results suggest that the most cost-effective policies aiming to leverage welfare are those targeting the efficiency of human capital in the educational activities. Additionally, policies might target the impact of human and intangible cultural capital on social capital accumulation.

We emphasize the following results regarding transitional dynamics:

- There is a marked oscillatory behavior of physical capital, which is particularly exuberant over the short-to-medium run, as well as a smooth non-monotonic behavior of social, intangible cultural, and human capital. Also, it is noteworthy that, say, if all types of capital have initial values below the steady state, they display a slight short-run undershooting vis-à-vis the steady-state level, followed by significant short-to-medium run overshooting by physical capital and medium-to-long run overshooting by the other (intangible) types of capital.

- The dynamics of the economic growth rate is greatly determined by the time path of the marginal productivity of physical capital in the short-to-medium run and, hence, mirrors the time path of physical capital over that period – namely with a salient short-to-medium run oscillatory behavior that starts with an overshooting vis-à-vis the steady-state level. Afterwards, however, the marginal productivity effect pertaining to human capital dominates, resulting in a smooth downward path for the growth rate. Thus, in most of this later period, the economic growth rate and physical capital display a positive relationship, which contrasts to the typical neoclassical transitional-dynamics result.

- The pattern of the medium-to-long run transition of the key variables in the model is fairly robust to the arbitrary choice of the initial value for the control variables (given the local indeterminacy result), suggesting, for this time horizon, that the sensitiveness to the sunspot behavior is small. Yet, a distinct short-run behavior may arise for different initial values of the control variables: for instance, the different types of capital may exhibit no undershooting at all while the oscillatory behavior displayed by namely physical capital and the economic growth rate may be quite attenuated.

- A shock to the efficiency of some type of capital accumulation (other than the efficiency of human capital in the education sector) or to the preference for social and intangible cultural capital generates a short-to-medium run (flow) welfare gain, due to the overshooting behavior of consumption and also (although in a lower frequency) to that of social and intangible cultural capital. These effects are reverted in the medium-to-long run horizon, although possibly only partially (depending on the calibration scenarios) in the particular case of the shock to the efficiency of human and intangible cultural capital in social-capital accumulation.

- A shock to the efficiency of human capital in the education sector generates the opposite behavior: a short-to-medium run (flow) welfare loss occurs (although with high frequency oscillation), due to the undershooting behavior of consumption and of social and intangible cultural capital. These effects are entirely reverted over the medium-to-long run.

Therefore, since the welfare results in the long run and in the short-to-medium run may be quite different, policymakers face a (sub-optimal) intertemporal trade-off as regards the welfare-oriented policies in the model.

Despite the growing literature aimed at explaining how cultural and artistic production feeds into economic growth (especially in practical institutions such as the World Bank and development agencies; e.g., UNESCO, 2021), the casual relationships and interplay are not investigated in depth (Sacco and Segre, 2009; Bucci and Segre, 2011; Bucci et al., 2014; Throsby, 2017). A few theoretical developments have been proposed to bring cultural capital into economic models, namely by Bucci and Segre (2011) and Bucci et al. (2014). Based on Throsby's definition (2001), Bucci and Segre (2011) propose a two-sector endogenous growth model that integrates both human and cultural capital. In their model the channel through which culture can impact economic growth is the complementarity between the accumulation of human capital and cultural capital. They find that, in the long run, a higher growth rate of income per capita is associated with a higher complementarity between cultural and human capital. Bucci et al. (2014) develop a single-sector endogenous growth model in which growth is driven by the creative use of skills fostered by cultural and human capital. Rather than focusing on complementarities, this model emphasizes how cultural investment enhances total factor productivity and positively impacts growth, provided the economy is sufficiently "culture-intensive".

As regards social capital, Bartolini and Bonatti (2008), using an augmented Solow-Ramsey growth model, uncover a negative relationship between social capital and economic growth. Their results are consistent with Putnam (2000), who argues that social capital declined in the last decades of the 20th century in the US despite satisfactory GDP growth. Sequeira and Ferreira-Lopes (2011) develop an endogenous growth model of social, human and physical capital accumulation, but where households earn specific returns from social capital. The authors obtain a decreasing social-to-human capital ratio combined with a (slight) acceleration in the economic growth rate over transition to the steady-state. In contrast, Dinda (2008), using an AK-type model combining the accumulation of human and social capital, treats social capital as a positive externality of human capital accumulation. In this framework, human capital results from "productive consumption" (expenditure on education and health), which indirectly generates social capital. The author argues that human capital is essential for social capital formation and finds a positive relationship between human capital, social capital, and economic growth. Roseta-Palma et al. (2010) introduces social capital in an endogenous growth model with natural capital, physical capital and human capital. They conclude that the presence of an environmental asset implies distortions to human capital allocation. Human capital would be allocated to environmental protection and social capital rather than to school attendance or final-good production, having a negative effect on economic growth. Kim and Lee (2015) develop a theoretical endogenous growth model with social capital that can generate two possible outcomes: sustained economic growth or collapse. Their model focuses on the interaction between productive ideas – such as scientific ideas – and less productive ideas – such as social values. They argue that the complementarity between social values and scientific ideas in utility is crucial for a growth outcome. Otherwise, when social values and scientific ideas are substitutable the economy may collapse.

In any case, it is worth noting that these strands of the literature do not combine social and cultural capital within a single endogenous growth framework, and transitional dynamics are generally not explored (except in Sequeira and Ferreira-Lopes (2011) and Kim and Lee, 2015). Moreover, these studies illustrate their results with numerical examples rather than through full calibration, as we do in our paper.

The next section lays out the model. In Section 3, we analyze its equilibrium dynamics. Section 4 presents the numerical results, detailing the calibration of the model and analyzing the impact of policy on the key capital ratios, economic growth, and welfare both in the long-run BGP equilibrium and in transitional dynamics. Section 5 discusses, in light of the existing literature, a number of policies related to different types of capital. Finally, Section 6 concludes.

2 The Model

This section presents the basic structure of our endogenous growth model. We draw on the social capital and cultural capital literature to build an endogenus growth model that combines different types of capital: physical, human, social, and cultural capital. Physical production capital is only used in the production of the final good. Human capital has different uses: production of the final good, and the accumulation of human (i.e., school atendence), social, and cultural capital. Social capital is used in the accumulation of human and cultural capital, and in its own accumulation. Following Throsby (1999) and Throsby (2001), we divide cultural capital in two types: tangible and intangible cultural capital. Tangible cultural capital is a form of physical capital and is used in the accumulation of intangible cultural capital. Intangible cultural capital is used in the accumulation of human and social capital, and in its own accumulation. Finally, in our model, social capital and intangible cultural capital also bring utility to individuals. Following the literature, we consider that social and intangible cultural capital are non-rival in use (unlike physical and human capital).

2.1 Capital Accumulation and Production of the Final Good

2.1.1 Accumulation of cultural and physical capital

Following Throsby (1999, 2001), we define two types of cultural capital: tangible, $K_{C_T}(t)$, and intangible cultural capital, $K_I(t)$. And tangible cultural capital, together with production capital, $K_P(t)$, comprise physical capital, $K_Y(t)$. Thus, in this economy, the fraction $z(t)$ of physical capital is devoted to production capital and the remaining $1-z(t)$ is devoted to tangible cultural capital.

$$K_Y(t) = K_P(t) + K_{C_T}(t) = z(t)K_Y(t) + (1-z(t))\,K_Y(t). \tag{1}$$

Tangible cultural capital and production capital are jointly accumulated using the fraction of aggregate production which is not consumed, that is:

$$\dot{K}_Y(t) = Y(t) - C(t) - \delta_P z(t)K_Y(t) - \delta_{C_T}(1-z(t))\,K_Y(t), \tag{2}$$

where $Y(t)$ is the aggregate production of final goods, $C(t)$ represents consumption, δ_P is the depreciation rate of production capital, and δ_{C_T} is the depreciation rate of tangible cultural capital.

We assume that intangible cultural capital accumulates as follows:

$$\dot{K}_I(t) = \omega_I\,(1-u_Y(t)-u_H(t)-u_S(t))\,H(t) + \Omega_I K_I(t) + \pi_S K_S(t) + \pi_Y\,(1-z(t))\,K_Y(t), \tag{3}$$

where $(1-u_Y(t)-u_H(t)-u_S(t))$ is the share of human capital, $H(t)$, devoted to intangible cultural capital accumulation (for a detailed explanation see next section), ω_I is the effect of human capital in the production of intangible cultural capita, and $\Omega_I \lesseqgtr 0$ is the effect of intangible cultural capital on its own accumulation. When $\Omega_I > 0$, the gross effect of the stock of intangible cultural capital on its accumulation overweights the depreciation of this type of capital. When $\Omega_I < 0$, there is a net depreciation effect. In turn, $\pi_S > 0$ is the effect of social capital, $K_S(t)$, on the accumulation of intangible cultural capital and $\pi_Y > 0$ is the effect of tangible cultural capital on the accumulation of intangible cultural capital.

2.1.2 Accumulation of human capital

Human capital is the main driver of economic growth in this model, as will be shown below. We divide the human capital into working hours in the final good sector, $H_Y(t)$, school hours in the education sector, $H_H(t)$, and hours spent in accumulating social capital, $H_S(t)$, and intangible cultural capital, $H_I(t)$. Human capital (labour) is supplied inelastically by the households. Therefore, full employment requires:

$$H(t) = H_Y(t) + H_H(t) + H_S(t) + H_I(t) \tag{4}$$

Households devote a share $u_Y(t)$ of total human capital to the accumulation of $H_Y(t)$, a share $u_H(t)$ to $H_H(t)$, a share $u_S(t)$ to $H_S(t)$, and the remaining fraction $1-u_Y(t)-u_H(t)-u_S(t)$ is devoted to $H_I(t)$. This implies that $H(t) = u_Y(t)H(t) + u_H(t)H(t) + u_S(t)H(t) + (1 - u_Y(t) - u_H(t) - u_S(t))\, H(t)$ with $u_Y(t) = \frac{H_Y(t)}{H(t)}, u_H(t) = \frac{H_H(t)}{H(t)}, u_S(t) = \frac{H_S(t)}{H(t)}$.

Human capital is produced using human capital allocated to the education sector (as in Uzawa (1965) and Lucas (1988)), but also social capital and intangible cultural capital:

$$\dot{H}(t) = \xi u_H(t)H(t) + \alpha K_S(t) + \varphi K_I(t). \tag{5}$$

In (5), $\xi > 0$ is a parameter that measures the efficiency of human capital in educational activity, while $\alpha \geq 0$ and $\varphi \geq 0$ measure the (external) effect of, respectively, social and of intangible cultural capital on human capital accumulation. Thus, equation (5) formalizes the idea that both social capital and cultural capital are important to the production of human capital, and that skill acquisition occurs only in part through formal education in the schooling system. This echoes the writings by Coleman (1988) and Teachman et al. (1997), according to which social capital is important to the production of human capital, but also the model by Roseta-Palma et al. (2010), which considers social capital in the accumulation of human capital, and the models by Bucci and Segre (2011) and Bucci et al. (2014), which consider instead cultural capital. In our case, however, and differently from Bucci and Segre (2011) and Bucci et al. (2014), we only consider the intangible part of cultural capital in the accumulation of human capital, since tangible cultural capital enters as an input in final-good production (as will be shown below).

2.1.3 Accumulation of social capital.

Social capital accumulation requires human capital to be allocated to its production, while also benefiting from the existing stock of social capital and the stock of intangible cultural capital:

$$\dot{K}_S(t) = \omega_S u_S(t)H(t) + \Omega_S K_S(t) + \Phi K_I(t), \tag{6}$$

where ω_S measures the effect of human capital in the production of social capital and Ω_S

$\lesseqgtr 0$ measures the effect of social capital on its own accumulation. When $\Omega_S > 0$, the gross effect of the stock of social capital on its accumulation overweights the depreciation of this type of capital. When $\Omega_S < 0$, there is a net depreciation effect. In turn, $\Phi \geq 0$ measures the effect of intangible cultural capital in social capital accumulation. The positive connection between social capital and human capital accumulation is described by, e.g., Coleman (1988) and Teachman et al. (1997) in sociological research on secondary school attrition.

2.1.4 Production of the final good.

The aggregate output of the final good, $Y(t)$, is obtained using human and physical production capital. Assuming a Cobb-Douglas technology and constant returns to scale, we have the aggregate production function:

$$Y(t) = (z(t)K_Y(t))^{\beta} \cdot (u_Y(t)H(t))^{1-\beta} \; 0 < \beta < 1. \tag{7}$$

That is, physical capital contributes to final-good production in proportion to the respective share of production capital, $z(t)$, while only the share of human capital that is specifically dedicated to production, $u_Y(t)$, is considered in (7), since the remainder is used for different activities, as stated above.[2]

2.2 Households

Household preferences in our model specify social capital, intangible cultural capital and consumption as arguments of the intertemporal utility function. Thus, welfare depends not only on consumption but also on the quality of institutions and culture. The former reflects improvements in social trust, reciprocity and cooperation, with social trust empirically shown to be positively related to happiness (Bjørnskov, 2008). Other theoretical models have incorporated social capital in a similar way (e.g., Sequeira and Ferreira-Lopes (2013); Roseta-Palma et al. (2010)). The latter captures the positive effects of cultural participation and access to cultural institutions on individual life satisfaction and well-being (e.g., Baldin and Bille, 2023; Grossi et al., 2011). In this way, both institutional and cultural dimensions enter the utility function as non-material determinants of welfare, so that intertemporal utility can be expressed as:

[2] The literature has considered alternative versions of the aggregate production function that explicitly feature social capital as an input/externality; e.g., Roseta-Palma et al. (2010) and Sequeira and Ferreira-Lopes (2011). In our model, we stick to the view that this externality effect occurs in a more diffused and intertwined way in the economy. Thus, as explained earlier, we consider social capital as a non-rival input in the accumulation of human and intangible cultural capital and consider intangible cultural capital as a non-rival input in the accumulatoin of human and social capital, whereas human capital enters as a (rival) input in the aggregate production function (7).

$$U(C(t), K_S(t), K_I(t)) = \frac{1}{1-\tau}\int_0^{\infty}\left(C(t)K_S^{\phi}(t)K_I^{\psi}(t)\right)^{(1-\tau)}e^{-\rho t}dt \tag{8}$$

where τ is the intertemporal elasticity of substitution, ϕ controls for the preference for social capital and ψ for the preference for intangible cultural capital (both relative to consumption), and ρ is the intertemporal discount rate.

We are interested in the set of paths $\{C(t),\ K_Y(t),\ K_I(t),\ H(t),\ K_S(t),\ u_Y(t),\ u_H(t),\ u_S(t),\ z(t)\}$ that solve the following dynamical optimization problem:

$$\max U(C(t), K_S(t), K_I(t)) \tag{9}$$

subject to (1)-(7) and:[3]

$$K_Y(0) = K_{Y0},\ K_I(0) = K_{I0},\ H(0) = H_0,\ K_S(0) = K_{S0}, \tag{10}$$

$$\begin{aligned} &C(t) > 0,\ u_Y(t) \in (0,1), u_S(t) \in (0,1), u_H(t) \in (0,1),\ z(t) \in (0,1),\\ &\quad K_Y(t) > 0,\ K_I(t) > 0,\ H(t) > 0,\ K_S(t) > 0 \end{aligned} \tag{11}$$

Equations (2), (3), (4), (5), and (6) are the resource constraints faced by the economy, while (10) gives the initial conditions for the state variables $(K_Y(t), K_I(t), H(t), K_S(t))$. The time paths $\{C(t),\ K_Y(t), K_I(t),\ H(t),\ K_S(t),\ u_Y(t),\ u_H(t),\ u_S(t),\ z(t)\}$ that solve the optimization problem in (9) arise from the optimal choice of the time paths of the control variables $C(t)$, $z(t)$, $u_Y(t)$, $u_H(t)$, and $u_S(t)$, subject to the indicated constraints and initial conditions.

Before laying down the solution to this dynamic optimization problem, it will be convenient, in the next section, to discuss the conditions for the existence of a balanced growth path and some of its properties.

3 Equilibrium Dynamics

3.1 The balanced-growth path: existence and properties

This section lays down the properties of a balanced-growth path (BGP), as a representation of the long-run equilibrium of the model, and the conditions for its existence.

The BGP, denoted by *, is the path $\{C^*(t), K_Y^*(t), K_I^*(t), H^*(t), K_S^*(t), u_S^*(t),\ u_H^*(t), u_Y^*(t), z^*(t); t \geq 0\}$ such that the growth rates g_C^*, $g_{K_Y}^*$, g_H^*, $g_{K_I}^*$, $g_{K_S}^*$, $g_{u_S}^*$, $g_{u_H}^*$, $g_{u_Y}^*$ and g_z^* are constant.

From equation (6), we get the growth rate of social capital as $g_{K_S} \equiv \frac{\dot{K}_S(t)}{K_S(t)} = \omega_S u_S(t)\frac{H(t)}{K_S(t)}+$

[3]We consider, for simplification, that the households internalize all effects of social and cultural capital, as our objective is not the study of eventual distortions in the market economy; see, e.g., Sequeira and Ferreira-Lopes (2011).

$\Omega_S + \Phi \frac{K_I(t)}{K_S(t)}$. So, for the growth rate to be constant in the BGP, we must satisfy the (sufficient) conditions $g^*_{K_S} = g^*_H = g^*_{K_I}$ and $g^*_{u_S} = 0$.

From equation (5), we get the growth rate of human capital as $g_H \equiv \frac{\dot{H}(t)}{H(t)} = \xi u_H(t) + \alpha \frac{K_S(t)}{H(t)} + \varphi \frac{K_I(t)}{H(t)}$. Thus, for the growth rate to be constant in the BGP, we must satisfy the conditions $g^*_H = g^*_{K_I} = g^*_{K_S}$ and $g^*_{u_H} = 0$.

From equation (3), we have the growth rate of intangible cultural capital as $g_{K_I} \equiv \frac{\dot{K}_I(t)}{K_I(t)} = \omega_I \left(1 - u_Y(t) - u_H(t) - u_S(t)\right) \frac{H(t)}{K_I(t)} + \Omega_I + \pi_S \frac{K_S(t)}{K_I(t)} + \pi_Y \left(1 - z(t)\right) \frac{K_Y(t)}{K_I(t)}$. So, for the growth rate to be constant in the BGP, we must satisfy the conditions $g^*_{K_I} = g^*_H = g^*_{K_S} = g^*_{K_Y}$ and $g^*_{u_Y} = g^*_{u_H} = g^*_{u_S} = 0$.

Finally, from (2), we get the growth rate of physical capital as $g_{K_Y} \equiv \frac{\dot{K}_Y(t)}{K_Y(t)} = \frac{Y(t)}{K_Y(t)} - \frac{C(t)}{K_Y(t)} - \delta_P z(t) - \delta_{C_T}(1 - z(t))$. For the growth rate of physical capital to be constant in the BGP, we must satisfy $g^*_{K_Y} = g^*_Y = g^*_C$ and $g^*_z = 0$.

Then, from (7), after taking logs and time-differentiating, we obtain the growth rate of aggregate output as:

$$g_Y = \beta \cdot (g_z + g_{K_Y}) + (1 - \beta) \cdot (g_{u_Y} + g_H) \tag{12}$$

Recalling that $g^*_{K_Y} = g^*_Y$, $g^*_{u_Y} = 0$, and $g^*_z = 0$, we can substitute in (12) to get $g^*_Y = g^*_H$. To sum up, on the BGP, the following conditions apply:

$$g^* \equiv g^*_Y = g^*_H = g^*_{K_S} = g^*_{K_I} = g^*_{K_Y} = g^*_C \tag{13}$$

$$g^*_z = g^*_{u_Y} = g^*_{u_H} = g^*_{u_S} = 0 \tag{14}$$

Using the Caballé and Santos (1993) decomposition, it will be convenient to transform (stationarize) the variables $C(t)$, $K_Y(t)$,$K_I(t)$, $H(t)$, $K_S(t)$ and $Y(t)$ as follows:

$$\tilde{C}(t) = C(t) e^{-g^* \cdot t} \tag{15}$$

$$\tilde{K_Y}(t) = K_Y(t) e^{-g^* \cdot t} \tag{16}$$

$$\tilde{K_I}(t) = K_I(t) e^{-g^* \cdot t} \tag{17}$$

$$\tilde{H}(t) = H(t) e^{-g^* \cdot t} \tag{18}$$

$$\tilde{K_S}(t) = K_S(t) e^{-g^* \cdot t} \tag{19}$$

$$\tilde{Y}(t) = Y(t)e^{-g^* \cdot t} = \left(z(t)\tilde{K}_Y(t)\right)^{\beta} \cdot \left(u_Y(t)\tilde{H}(t)\right)^{1-\beta} \tag{20}$$

In this redefinition of the variables, we have used their respective growth rates on the BGP as a discounting factor. Thus, the modified variables $\tilde{C}(t)$, $\tilde{K}_Y(t)$, $\tilde{K}_I(t)$, $\tilde{H}(t)$ and $\tilde{K}_S(t)$ remain constant along the BGP, with $\tilde{C}^*$, $\tilde{K}_Y^*$, $\tilde{K}_I^*$, $\tilde{H}^*$ and $\tilde{K}_S^*$ denoting their steady-state values.

3.2 Dynamic Optimization

To study the dynamics of this model, we rewrite the optimization problem in equations (9)-(11), considering (15)-(20):

$$\max \frac{1}{1-\tau} \int_0^{\infty} \left(\tilde{C}(t)\tilde{K}_S^{\phi}(t)\tilde{K}_I^{\psi}(t)\right)^{(1-\tau)} e^{-[\rho-(1-\tau)\cdot(1+\phi+\psi)\cdot g^*]t} dt \tag{21}$$

subject to

$$\dot{\tilde{K}}_Y(t) = \tilde{Y}(t) - \tilde{C}(t) - \delta_P z(t)\tilde{K}_Y(t) - \delta_{C_T}\left(1 - z(t)\right)\tilde{K}_Y(t) - g^*\tilde{K}_Y(t), \tag{22}$$

$$\dot{\tilde{K}}_I(t) = \omega_I\left(1 - u_Y(t) - u_H(t) - u_S(t)\right)\tilde{H}(t) + \Omega_I\tilde{K}_I(t) + \pi_S\tilde{K}_S(t) + \pi_Y\left(1 - z(t)\right)\tilde{K}_Y(t) - g^*\tilde{K}_I(t), \tag{23}$$

$$\dot{\tilde{H}}(t) = \xi u_H(t)\tilde{H}(t) + \alpha\tilde{K}_S(t) + \varphi\tilde{K}_I(t) - g^*\tilde{H}(t), \tag{24}$$

$$\dot{\tilde{K}}_S(t) = \omega_S u_S(t)\tilde{H}(t) + \Omega_S\tilde{K}_S(t) + \Phi\tilde{K}_I(t) - g^*\tilde{K}_S(t), \tag{25}$$

$$\tilde{K}_Y(0) = \tilde{K}_{Y_0},\ \tilde{K}_I(0) = \tilde{K}_{I_0},\ \tilde{H}(0) = \tilde{H}_0,\ \tilde{K}_S(0) = \tilde{K}_{S_0}, \tag{26}$$

$$\begin{gathered}\tilde{C}(t) > 0,\ u_Y(t) \in (0,1),\ u_S(t) \in (0,1),\ u_H(t) \in (0,1),\ z(t) \in (0,1),\\ \tilde{K}_Y(t) > 0,\ \tilde{K}_I(t) > 0,\ \tilde{H}(t) > 0,\ \tilde{K}_S(t) > 0\end{gathered} \tag{27}$$

where $\tilde{K}_Y$, $\tilde{K}_I$, $\tilde{H}$, and $\tilde{K}_S$ are the (stationarized) state variables, and $\tilde{C}$, z, u_Y, u_H, and u_S are the (stationarized) control variables.

By solving the optimization problem in equations (21) to (27), we get the following dynamical system in the plane $\left\{\tilde{C}(t), \tilde{K}_Y(t), \tilde{K}_I(t), \tilde{H}(t), \tilde{K}_S(t), \frac{z(t)}{u_Y(t)}\right\}$, which we present for a given vector of sectoral shares $\{u_Y(t), u_H(t), u_S(t)\}$ to be arbitrarily set by the households (as explained below)[4]:

[4] As an alternative, we could let the share $z(t)$ be set arbitrarily in the vector of sectoral shares instead of $u_Y(t)$. As shown in Appendix E.1.1, however, the long-run equilibrium of the model is unstable in (almost) all our calibration scenarios under that case. See also Proposition 3.3.

$$\dot{\tilde{K}}_Y(t) = \tilde{K_Y}(t)^{\beta} u_Y(t) \left(\frac{z(t)}{u_Y(t)}\right)^{\beta} \tilde{H}(t)^{1-\beta} - \tilde{C}(t) - \delta_P u_Y(t) \left(\frac{z(t)}{u_Y(t)}\right) \tilde{K}_Y(t) - \delta_{C_T} \left[1 - u_Y(t) \left(\frac{z(t)}{u_Y(t)}\right)\right] \tilde{K}_Y(t) - g^* \tilde{K}_Y(t), \tag{28}$$

$$\dot{\tilde{K}}_I(t) = \omega_I \left(1 - u_Y(t) - u_H(t) - u_S(t)\right) \tilde{H}(t) + \Omega_I \tilde{K}_I(t) + \pi_S \tilde{K}_S(t) + \pi_Y \left[1 - u_Y(t) \left(\frac{z(t)}{u_Y(t)}\right)\right] \tilde{K}_Y(t) - g^* \tilde{K}_I(t), \tag{29}$$

$$\dot{\tilde{H}}(t) = \xi u_H(t) \tilde{H}(t) + \alpha \tilde{K}_S(t) + \varphi \tilde{K}_I(t) - g^* \tilde{H}(t), \tag{30}$$

$$\dot{\tilde{K}}_S(t) = \omega_S u_S(t) \tilde{H}(t) + \Omega_S \tilde{K}_S(t) + \Phi \tilde{K}_I(t) - g^* \tilde{K}_S(t), \tag{31}$$

$$\dot{\tilde{C}}(t) = \frac{\tilde{C}(t)}{\tau} \cdot \left[\phi (1-\tau) \frac{\dot{\tilde{K}}_S(t)}{\tilde{K}_S(t)} + \psi (1-\tau) \frac{\dot{\tilde{K}}_I(t)}{\tilde{K}_I(t)} + \right.$$

$$\left. + g^* \cdot (\phi + \psi - \tau (1 + \phi + \psi)) + \frac{\beta \cdot \left[\tilde{K_Y}(t)^{\beta} u_Y(t) \left(\frac{z(t)}{u_Y(t)}\right)^{\beta} \tilde{H}(t)^{1-\beta}\right]}{\tilde{K}_Y(t)} - \delta_P - \rho \right] \tag{32}$$

$$\dot{\left(\frac{z(t)}{u_Y(t)}\right)} = \left(\frac{\dot{\tilde{H}}(t)}{\tilde{H}(t)} - \frac{\dot{\tilde{K}}_Y(t)}{\tilde{K}_Y(t)}\right) \cdot \left(\frac{z(t)}{u_Y(t)}\right) \tag{33}$$

The details of the derivation of equations (32) and (33) can be found in Appendix A, whereas equations (28)-(31) are essentially a repetition of (22)-(25). Note that, as also shown in Appendix A, the dynamical system is indeterminate regarding the sectoral share variables. This results from linearity among alternative allocations across sectors (physical capital, intangible cultural capital, human capital, and social capital) in the model. Yet, it should be emphasized that the dynamics of the ratio of the share variables in the final-good production function, $\frac{z(t)}{u_Y(t)}$, is not indeterminate (see 33). Then, e.g., for a given $u_Y(t)$, equation (33) implicitly determines the dynamics of $z(t)$. In this case, $(u_Y(t),\ u_H(t),\ u_S(t))$ are to be regarded as exogenous from the point of view of the households' optimization problem.

3.3 Steady state

In this section, we present the steady state of the model, as a representation of its long-run equilibrium consistent with the BGP conditions derived in Section 3.1, above.

Proposition 1. The long-run economic growth rate is:

$$g^* = \frac{\xi - \rho}{\tau (1 + \phi + \psi) - \phi - \psi}. \tag{34}$$

Proof. See Appendix B.

As is common in the literature (e.g., Barro and Sala-i Martin (2004)), we let $\tau > 1$, which means that $\xi > \rho$ is a sufficient condition for positive long-run economic growth, i.e., $g^* > 0$. Thus, being ξ the efficiency of human capital employed in the education sector (recall equation (5)), we have a human-capital driven endogenous growth model.

We will be able to further examine the steady state analytically under the assumption $\delta_P = \delta_{C_T} = \delta$ and, alternatively, under $\delta_P \neq \delta_{C_T}$ and $\beta = 0.5$, as shown in Appendix B. In this section, we present the former case, while the later is only treated in Appendix B.2.

Thus, let us assume $\delta_P = \delta_{C_T} = \delta$, so that the following proposition obtains:

Proposition 2. Let $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$, $\mathbf{B} > 0$. In the steady-state, the economy features the following ratios of state and control variables:

$$\frac{\tilde{K}_Y^*}{\tilde{C}^*} = \frac{1}{\mathbf{B}}, \tag{35}$$

$$\frac{\tilde{K}_I^*}{\tilde{C}^*} = \frac{\mathbf{A_3}}{\mathbf{B}}, \tag{36}$$

$$\frac{\tilde{H}^*}{\tilde{C}^*} = \frac{\mathbf{A_1} \cdot \mathbf{A_3}}{\mathbf{B}}, \tag{37}$$

$$\frac{\tilde{K}_S^*}{\tilde{C}^*} = \frac{\mathbf{A_2} \cdot \mathbf{A_3}}{\mathbf{B}}, \tag{38}$$

$$\frac{z^*}{u_Y^*} = \mathbf{A_1} \cdot \mathbf{A_3} \cdot \left(\frac{\delta + \xi}{\beta}\right)^{\frac{1}{\beta - 1}}, \tag{39}$$

where: $\mathbf{A}_1 \equiv \frac{\alpha\Phi + \varphi \cdot (g^* - \Omega_S)}{\left(g^* - \xi u_H^*\right) \cdot (g^* - \Omega_S) - \alpha \omega_S u_S^*}$, $\mathbf{A}_2 \equiv \frac{\omega_S u_S^* \mathbf{A_1} + \Phi}{g^* - \Omega_S}$, $\mathbf{A_3} \equiv \frac{\pi_Y (1 - z^*)}{(g^* - \Omega_I) - \omega_I \left(1 - u_Y^* - u_H^* - u_S^*\right) \mathbf{A_1} - \pi_S \mathbf{A_2}}$, $\mathbf{B} \equiv z^* \cdot \left(\frac{\delta + \xi}{\beta}\right) - \delta - g^*$, and

$$\left(\frac{\omega_I \beta}{(1 - \beta)\, \pi_Y}\right)^{\beta - 1} = \frac{\delta + \xi}{\beta}. \tag{40}$$

Proof. See Appendix B.1.

Lemma 1. The levels of the sectoral share variables, $z = z^*$, $u_Y = u_Y^*$, $u_H = u_H^*$, and $u_S = u_S^*$ are indeterminate. But, for a given $u_Y = u_Y^*$ (respectively, $z = z^*$), equation (39) implicitly determines the level of $z = z^*(u_Y = u_Y^*)$.

As shown in Appendix B.1, the levels of all transformed (stationarized) variables are indeterminate. Using $\tilde{C}^*$ as the (arbitrary) reference variable, Proposition 3.3 establishes that all state variables are determined given $\tilde{C}^*$ and the level of the sectoral share variables and the structural parameters, as shown in (35)-(38). In parallel, e.g., for a given $u_Y = u_Y^*$, equation (39) implicitly determines the level of $z = z^*$. Finally, it is noteworthy that equation 40 requires that, in the steady state, the returns to z^* relative to u_Y^*, for a given $\frac{\tilde{H}^*}{\tilde{K}_Y^*}$ (see Appendix B.1), matches a measure of (gross) returns to human capital accumulation. This arises as a knife-edge condition because of the referred to indeterminacy regarding the sectoral share variables in the model.

Note that, in light of the Caballé and Santos (1993) transformation (equations (15)-(19)), the results in (35)-(38) also apply to the ratios of the original variables, $\frac{K_Y^*}{C^*}$, $\frac{H^*}{C^*}$, $\frac{K_I^*}{C^*}$, and $\frac{K_s^*}{C^*}$, since $\frac{\tilde{K_Y}^*}{\tilde{C}^*} = \frac{K_Y^*}{C^*}$, $\frac{\tilde{H}^*}{\tilde{C}^*} = \frac{H^*}{C^*}$,$\frac{\tilde{K_I}^*}{\tilde{C}^*} = \frac{K_I^*}{C^*}$, and $\frac{\tilde{K_S}^*}{\tilde{C}^*} = \frac{K_s^*}{C^*}$.

Corollary 1. There is an economically admissible steady state if the conditions $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$, $\mathbf{B} > 0$ are satisfied. This implies that the admissible sets of values for g^*, z^*, u_H^*, u_S^*, and u_Y^* are:

$$\max\left(\Omega_S, \pi_S \mathbf{A_2} + \mathbf{\Omega}_{\mathrm{I}}\right) < g^* < \frac{\delta + \xi}{\beta} - \delta \tag{41}$$

$$z^* > \frac{\beta\left(\delta + g^*\right)}{\delta + \xi} \tag{42}$$

$$u_S^* < \frac{\left(g^* - \xi u_H^*\right) \cdot \left(g^* - \Omega_S\right)}{\alpha \omega_S} \tag{43}$$

$$u_H^* < \frac{g^*}{\xi} \tag{44}$$

$$1 - u_Y^* - u_H^* - u_S^* < \frac{\left(g^* - \Omega_I\right) - \pi_S \mathbf{A_2}}{\omega_I \mathbf{A_1}} \tag{45}$$

Proof. See Appendix B.1.

Constraint (41) provides the admissible range of values for g^*. In (42), we have the minimum admissible value for z^*, and in (43), (44), and (45), we establish the maximum admissible values for u_H^*, u_S^*, and $1 - u_Y^* - u_H^* - u_S^*$ (the share of human capital devoted to intangible cultural capital accumulation), respectively.

Proposition 3. Let us determine the ratio $\frac{z}{u_Y}$ for a given u_Y in the 6-dimensional dynamical system 28-33 and consider the set of values of the parameters and of the sectoral shares u_Y,u_H, and u_S selected in the calibration of the model (to be shown in Section 4.1). The steady state of the model features local indeterminacy, with either a 5- or a 6-dimensional stable manifold characterized by a dampened oscillatory behavior.

Proof. See Appendix E.1.2.

Local indeterminacy means that there exists a continuum of perfect-foresight equilibria in which $\tilde{K}_Y(t)$, $\tilde{K}_S(t)$, $\tilde{K}_I(t)$, $\tilde{H}(t)$, $\tilde{C}(t)$ and $\frac{z(t)}{u_Y(t)}$ converge asymptotically to the steady-state equilibrium. That is, since there are two jump variables in our setup, $\tilde{C}(t)$ and $\frac{z(t)}{u_Y(t)}$, and there is only one or no real positive eigenvalues (and no eigenvalues with null real part), depending on the calibration scenario (as shown in Appendix E.1.2), either a 5- or 6-dimensional stable manifold will arise. Under one real positive eigenvalue, there exists a neighbourhood around the steady state so that, for given predetermined $\tilde{K}_Y(0)$, $\tilde{K}_S(0)$, $\tilde{K}_I(0)$, $\tilde{H}(0)$ and for any initial $\tilde{C}(0)$ (respectively, $\frac{z(0)}{u_Y(0)}$) chosen by the households, there is an initial $\frac{z(0)}{u_Y(0)}$ (respectively, $\tilde{C}(0)$) such that the trajectories of $\tilde{K}_Y(t)$, $\tilde{K}_S(t)$, $\tilde{K}_I(t)$, $\tilde{H}(t)$, $\tilde{C}(t)$ and $\frac{z(t)}{u_Y(t)}$ implied by (28)-(33) converge asymptotically to that steady-state. Under no real positive eigenvalues , there exists a neighbourhood around the steady-state so that, for given predetermined $\tilde{K}_Y(0)$, $\tilde{K}_S(0)$, $\tilde{K}_I(0)$, $\tilde{H}(0)$ and for any $\tilde{C}(0)$ and $\frac{z(0)}{u_Y(0)}$ chosen by the households, the trajectories of all variables converge asymptotically to that steady-state. As is well known, this raises the possibility of sunspot equilibrium trajectories for the macroeconomic variables, i.e., trajectories around their steady-state values that do not result from fundamentals. In Appendix E.1.2, we also show that the system features either one or two pairs of complex eigenvalues with negative real parts, which, per se, determines an oscillatory transitional behavior.

4 Numerical Results

4.1 Calibration of the model

In this section, we present and discuss the calibrated values for the parameters of the model, as well as the sectoral share variables, which, for the purpose of the numerical exercise, we treat as exogenous variables.[5] The values are depicted by Tables 1 and 2.

In Table 1, we display three sets of parameters/exogenous variables: literature-based and data-based (external calibration), and model-based (internal calibration). As regards the first set, the following parameters in our model are standard in the growth literature: ρ, δ (e.g., Barro and Sala-i Martin, 2004), and τ (Jones et al., 2000). We draw ω_S from Roseta-Palma et al. (2010) and Sequeira and Ferreira-Lopes (2013), while, in the absence of a reference for ω_I, and given the similitude in nature with ω_S, we let the former take the value of the latter. For Ω_S and Ω_I, we consider two alternative values, one negative and one positive. The negative value represents the (net) depreciation of the respective type of capital and is directly taken from Roseta-Palma et al. (2010), Sequeira and Ferreira-Lopes (2013), and Sequeira and Ferreira-Lopes (2011). Alternatively, the positive value represents a (net) positive effect of the existing stock of capital in the

[5]Please recall from Section 3.2 that the dynamical system of the model is indeterminate regarding the level of the sectoral share variables.

accumulation of the same type of capital. This value is based on the same literature, but taking in consideration the constraint $g^* > \Omega_S$ in our model (please recall 41).

Regarding the data-based parameters, the value for β comes from the capital share of income in the Euro Area (19 countries) in 2022, as reported by AMECO.[6] The value for $1-z^*$ is obtained by considering the share of the gross investment in tangible non-current assets by companies of the Cultural and Creative sectors and the share of government gross fixed capital formation in Recreation, Culture, and Religion. We used the available data from the Eurostat (Switzerland and Finland, only) for 2021.[7] For the computation of u_Y^*, u_H^*, and u_S^* , we used information (available for 10 countries) from the Harmonised European Time Use Surveys (HETUS), Eurostat, for 2010.[8] We mapped out the different daily activities of the respondents across the various relevant categories of activities (for the categories and the complete list of activities considered, please see Appendix C), so that the value of each variable represents the share of the corresponding category of activities in the set of relevant activities in an individual's day. The value of u_Y^* was adjusted downwards by a percentage corresponding to the proportion of the population employed in 'Education' and in 'Arts, Entertainment, and Culture', based on data from the Eurostat.[9] This portion was then added to the share of time allocated to the accumulation of human capital and of intangible cultural capital, respectively. This methodology was followed for all countries, so that the final value of each variable corresponds to a population-weighted average based on data from the World Bank.[10] For ϕ and ψ, the elasticities of K_S and K_I in the utility function (8), we used as reference the values for u_S^* and $1-u_Y^*-u_H^*-u_S^*$ adjusted by also considering the share of time devoted to consumption. The recalculated values of u_S^* and $1-u_Y^*-u_H^*-u_S^*$ were then normalized by the share of time devoted to consumption (for more details see Appendix C). Next, we set ξ so that, given (34), the steady-state economic growth rate, g^*, matches the real GDP per capita average growth rate in the Euro Area (19 countries) between 2000-2022. This procedure resulted in a value for ξ which is within the range commonly used in the human capital literature (e.g., Funke and Strulik, 2000).

Finally, as regards the model-based (internal) calibration, π_Y was computed from equation (40) taking into account the value of all the relevant parameters mentioned above.

---

[6] The data on the capital share is available online at: https://dashboard.tech.ec.europa.eu/qs_digit_dashboard_mt/pu eea7-4d17-abf0-ef20f6994336/sheet/2f9f3ab7-09e9-4665-92d1-de9ead91fac7/state/analysis.

[7] The data on gross investment tangible non-current assets is available online at: https://ec.europa.eu/eurostat/databrowser/view/sbs_ovw_act___custom_11922604/default/table. The data on gross fixed capital formation is available online at: https://ec.europa.eu/eurostat/databrowser/view/gov_10a_exp/default/table?lang=en.

[8] The list of countries used in the calculation is in Appendix C. The HETUS 2010 is available online at: https://ec.europa.eu/eurostat/databrowser/view/tus_00age/default/table?lang=en&category=livcon.tus.

[9] Employment data is available online at: https://ec.europa.eu/eurostat/databrowser/view/htec_emp_nat2___custom

[10] Population data is available online at: https://databank.worldbank.org/reports.aspx?source=2&series=SP.POP.TOT

Literature-based parameters			
Parameter		Value	Literature references
ρ	Intertemporal discount rate	0.02	Barro and Sala-i Martin (2004)
δ	Depreciation rate of K_P	0.05	Barro and Sala-i Martin (2004)
τ	Intertemporal elasticity of substitution	1.25	Jones et al. (2000)
ω_S	Effect of H on the accum. of K_S	0.01	Roseta-Palma et al. (2010); Sequeira and Ferreira-Lopes (2013)
ω_I	Effect of H on the accum. of K_I	0.01	Equal to ω_S
Ω_S	Effect of K_S on its own accum.	-0.01/0.005	Roseta-Palma et al. (2010); Sequeira and Ferreira-Lopes (2011); Sequeira and Ferreira-Lopes (2013)
Ω_I	Effect of K_I on its own accum.	-0.01/0.005	Equal to Ω_S
Data-based parameters/exogenous variables			
Parameter		**Value**	**Empirical target**
β	Share of production capital	0.443	Capital share of income
$1-z^*$	Share devoted to the accum. of K_{C_T}	0.025	Share of investment on tangible cultural assets
u_Y^*	Share of H devoted to Y prod.	0.61	Share of time spent on production
u_H^*	Share of H devoted to H accum.	0.12	Share of time spent on education
u_S^*	Share of H devoted to K_S accum.	0.21	Share of time spent on social capital accum.
ψ	Preference for K_I	0.11	$1-u_Y^*-u_H^*-u_S^*$ as proxy (normalized by cons. time)
ϕ	Preference for K_S	0.35	u_S^* as proxy (normalized by cons. time)
ξ	Efficiency of H in education sector	0.03365	Real GDP pc average growth rate ($g^*=0.01$)
Model based-parameters			
Parameter		**Value**	**Model relationships/constraints**
π_Y	Effect of K_{C_T} on the accum. K_I	0.000399	$\left(\frac{\omega_I \beta}{(1-\beta)\pi_Y}\right)^{\beta-1} = \frac{\delta+\xi}{\beta}$

Table 1: Baseline calibration of the model. Note: 'accum.' refers to accumulation, 'prod' refers to production, and 'cons.' referes to consumption . See text for details.

The remaining parameters are α, π_S, Φ, and φ. These parameters are specific and key to our model (particularly when related to K_I) and/or have limited empirical or theoretical discussion in the literature. Therefore, we used a hybrid methodology, combining references from the literature (external calibration) and own empirical targets (for internal calibration) across a number of alternative scenarios. The explanation of the different scenarios and the corresponding values of the parameters are displayed in Table 2.[11]

As regards the literature, we built on Roseta-Palma et al. (2010), Sequeira and Ferreira-Lopes (2013), and Sequeira and Ferreira-Lopes (2011). The parameter φ takes the value 0.01, analogous to the value of α in this literature, in scenarios II and III under $\Omega_I = 0.005$, and in scenarios V and VI under $\Omega_I = -0.01$, as both parameters represent the effect of some form of intangible capital on the accumulation of human capital. Similarly, π_S takes the value of α in scenarios I, II, IV, and V, as both parameters represent the effect of social capital on the accumulation of some form of immaterial capital. As

[11] An alternative approach would have been to solve a high-dimensional system that considered simultaneously all the empirical targets described below. Yet, our computations show that such a system did not have a sufficient number of independent equations in order to converge to a feasible solution.

empirical targets, we use the data on the ratios of intangible cultural capital to physical capital, as well as the share of time allocated to production. To compute the intangible cultural capital-to-physical capital ratio, we first calculate the intangible cultural capital-to-output ratio. The value for the stock of intangible cultural capital in 2022 was obtained by applying the perpetual inventory method to the data on government expenditure on Culture from 1996 to 2022 (deflated using the IHPC for Culture), considering the two alternative values for Ω_I in Table 1. The value for the stock of intangible cultural capital was then divided by the real GDP for 2022 for the Euro Area. Hence, bearing in mind the obtained ratio of intangible cultural capital-to-physical capital, we used equations (36) and (35) to set the target $\left(\frac{\tilde{K}_I}{\tilde{K}_Y}\right)^* = 0.21$ under $\Omega_I = 0.005$ and $\left(\frac{\tilde{K}_I}{\tilde{K}_Y}\right)^* = 0.13$ under $\Omega_I = -0.01$. Finally, we considered also as an empirical target the value for the share of time spent on production, using equation (39) solved with respect to u_Y^*.

Data-based / Literature-based parameters							
		I		II		III	
Parameter		**Value**	**Emp Tg/Lit Ref**	**Value**	**Emp Tg/Lit Ref**	**Value**	**Emp Tg/Lit Ref**
φ	Effect of K_I on the accum. of H	0.0005	(2)	0.01	Equal to α (I)	0.01	Equal to α (I)
Φ	Effect of K_I on the accum. of K_S	0.0005	Equal to φ	0.01	Equal to φ	0.01	Equal to φ
π_S	Effect of K_S on the accum. of K_I	0.01	Equal to α	0.0011	Equal to α	0.0022	(1) and (2)
α	Effect of K_S on the accum. of H	0.01	*	0.0011	(2)	-0.0036	
* Literature References: Roseta-Palma et al. (2010); Sequeira and Ferreira-Lopes (2011); Sequeira and Ferreira-Lopes (2013) (1) Intagible cultural capital to physical capital ratio: $\left(\frac{\tilde{K}_I}{\tilde{K}_Y}\right)^* = 0.21\,(\Omega_I = 0.005)$; (2) Share of time spent on prod.: $u_Y^* = 0.61$							
		IV		V		VI	
Parameter		**Value**	**Emp Tg/Lit Ref**	**Value**	**Emp Tg/Lit Ref**	**Value**	**Emp Tg/Lit Ref**
φ	Effect of K_I on the accum. of H	0.0192	(2)	0.01	Equal to α (IV)	0.01	Equal to α (IV)
Φ	Effect of K_I on the accum. of K_S	0.0192	Equal to φ	0.01	Equal to φ	0.01	Equal to φ
π_S	Effect of K_S on the accum. of K_I	0.01	Equal to α	0.0172	Equal to α	0.0346	(1) and (2)
α	Effect of K_S on the accum. of H	0.01	*	0.0172	(2)	-0.0112	
* Literature References: Roseta-Palma et al. (2010); Sequeira and Ferreira-Lopes (2011); Sequeira and Ferreira-Lopes (2013) (1) Intagible cultural capital to physical capital ratio: $\left(\frac{\tilde{K}_I}{\tilde{K}_Y}\right)^* = 0.13\,(\Omega_I = -0.01)$; (2) Share of time spent on prod.: $u_Y^* = 0.61$							

Table 2: Calibration of the model. Scenarios I, II, and III were calculated under $\Omega_I = 0.005$. Scenarios IV, V, and VI were calculated under $\Omega_I = -0.01$. In scenarios III and VI, a system of equations was used to calculate the parameters π_S and α. Note: 'Emp Tg' refers to empirical target, 'accum' refers to accumulation, and 'Lit ref' refers to literature referece. See text for more details.

Calculated from the empirical targets, the values for φ and Φ vary between 0.0005 and 0.0192 (we let Φ take the value of φ in all scenarios, as both parameters represent the effect of intangible cultural capital on the accumulation of some form of immaterial capital). In the case of scenario IV, the value of those parameters is very close to the one we considered from the literature (as mentioned above, by analogy with α, under scenarios II, III, V, and VI). The value for π_S ranges between 0.0011 and 0.0172. The value for α ranges between -0.0036 and 0.0172. In scenario V, it is fairly in line with the reference value we used from the literature, whereas in scenarios III and VI a negative value obtains (thus implying a negative effect of social capital on human capital accumulation) . Yet, since there is no evidence in the literature supporting a negative value for α , we will restrict our analysis to scenarios I, II, IV, and V in the following sections.

4.2 Comparative statics

In this section, we carry out a comparative-statics analysis for the calibrated version of the model to study the relationship between a number of key structural parameters and the steady-state (or BGP) economic growth rate and key ratios of state and control variables.

As shown in Table (3), the steady-state economic growth rate, g^*, as well as the steady-state capital-to-consumption ratios $\frac{\tilde{K}_Y^*}{\tilde{C}^*}$, $\frac{\tilde{H}^*}{\tilde{C}^*}$, $\frac{\tilde{K}_S^*}{\tilde{C}^*}$, and $\frac{\tilde{K}_I^*}{\tilde{C}^*}$, are positively related to the efficiency of human capital allocated to the education sector, ξ, but negatively to the parameters that control for the preference for social and intangible cultural capital, ψ and ϕ, in the households' utility function. The human-capital efficiency parameter ξ incentivizes the accumulation of human capital and, thereby, of all types of capital (through the respective accumulation laws), thus positively impacting economic growth (recall equation (34)) and the ratio of all types of capital to consumption in long-run equilibrium. The preference parameters ψ and ϕ leverage the households' risk-aversion factor, $\tau > 1$ and, thus, dampen their willingness to move resources across time, hurting capital accumulation of all types (vis-à-vis consumption) and economic growth.

As for the remaining key (technological) parameters, they do not affect g^*but reduce $\frac{\tilde{K}_Y^*}{\tilde{C}^*}$, $\frac{\tilde{H}^*}{\tilde{C}^*}$, $\frac{\tilde{K}_S^*}{\tilde{C}^*}$, and $\frac{\tilde{K}_I^*}{\tilde{C}^*}$ in almost all cases considered. These are the parameters that control for the efficiency of a given type of capital regarding the accumulation of itself or of another type of capital (except for the efficiency of human capital in the accumulation of itself, controlled by ξ). In general, a higher value of one of this parameters makes the accumulation of a given stock of capital more attractive (because of the higher marginal return), but also makes the use of the stock of capital in order to generate resources ultimately usable as consumption more efficient (i.e., a smaller quantity of the stock of capital is necessary to produce a given amount of output usable as consumption). Given the absence of a dynamic long-run return effect (because $g^*$is not affected), the efficiency channel tends to dominate and the capital-to-consumption ratios are negatively impacted. Given our calibration of the model, the exception is the positive impact of parameters Φ and ω_S (the efficiency of intangible cultural and human capital, respectively, in the accumulation of social capital) in the ratio $\frac{\tilde{K}_S^*}{\tilde{C}^*}$, under the calibration scenarios II and IV.

Finally, the steady-state ratio of sectoral share variables $\frac{z^*}{u_Y^*}$ is negatively impacted by ξ and positively by the remaining parameters. This ratio directly reflects the behavior of $\frac{\tilde{H}^*}{\tilde{K}_Y^*}$, given the condition (40) (see Appendix B.1 and recall Proposition 2; the comparative-statics results regarding the ratio $\frac{\tilde{H}^*}{\tilde{K}_Y^*}$ are displayed in Table (4), below).

	g^*	$\frac{\tilde{K}_Y^*}{\tilde{C}^*}$	$\frac{\tilde{H}^*}{\tilde{C}^*}$	$\frac{\tilde{K}_S^*}{\tilde{C}^*}$	$\frac{\tilde{K}_I^*}{\tilde{C}^*}$	$\frac{z^*}{u_Y^*}$
ξ	$+$	$+$	$+$	$+$	$+$	$-$
ψ	$-$	$-$	$-$	$-$	$-$	$+$
ϕ	$-$	$-$	$-$	$-$	$-$	$+$
α	0	$-$	$-$	$-$	$-$	$+$
Φ	0	$-$	$-$	$-$ (I, V); $+$(II, IV)	$-$	$+$
φ	0	$-$	$-$	$-$	$-$	$+$
ω_S	0	$-$	$-$	$-$ (I, V); $+$(II, IV)	$-$	$+$
ω_I, π_Y	0	$-$	$-$	$-$	$-$	$+$
π_S	0	$-$	$-$	$-$	$-$	$+$

Table 3: Comparative statics: analysis of the relationship between the key variables of the model in steady state (economic growth rate and ratios of state and control variables) and the key structural parameters. The results correspond to calibration scenarios I, II, IV, and V (see Tables 1 and 2), except when explicitly indicated. In order to satisfy constraint (40), we consider: (i) a proportional change in parameters ω_I and π_Y ; (ii) a variation in ξ accompanied by a variation in δ of the same magnitude but with opposite sign.

In turn, Table (4) reports, by calibration scenario, the comparative-statics results for the ratio of each type of intangible capital, $\tilde{H}^*$, $\tilde{K}_S^*$, and $\tilde{K}_I^*$, to physical production capital, $\tilde{K}_Y^*$. This analysis allows us to filter the effect of the key structural parameters on consumption and retain solely the effect on the different types of capital. Given the focus of our paper on the intangible types of capital, we emphasize their behavior relative to physical production capital.

	I			II			IV			V		
	$\frac{\tilde{H}^*}{\tilde{K}_Y^*}$	$\frac{\tilde{K}_S^*}{\tilde{K}_Y^*}$	$\frac{\tilde{K}_I^*}{\tilde{K}_Y^*}$	$\frac{\tilde{H}^*}{\tilde{K}_Y^*}$	$\frac{\tilde{K}_S^*}{\tilde{K}_Y^*}$	$\frac{\tilde{K}_I^*}{\tilde{K}_Y^*}$	$\frac{\tilde{H}^*}{\tilde{K}_Y^*}$	$\frac{\tilde{K}_S^*}{\tilde{K}_Y^*}$	$\frac{\tilde{K}_I^*}{\tilde{K}_Y^*}$	$\frac{\tilde{H}^*}{\tilde{K}_Y^*}$	$\frac{\tilde{K}_S^*}{\tilde{K}_Y^*}$	$\frac{\tilde{K}_I^*}{\tilde{K}_Y^*}$
ξ	−	−	−	−	−	+	−	−	+	−	−	+
ψ	+	+	+	+	+	−	+	+	−	+	+	−
ϕ	+	+	+	+	+	−	+	+	−	+	+	−
α	+	+	−	+	−	−	+	−	−	+	−	−
Φ	+	+	+	+	+	+	+	+	+	+	+	+
φ	+	+	−	+	−	−	+	−	−	+	−	−
ω_S	+	+	+	+	+	+	+	+	+	+	+	+
ω_I, π_Y	+	+	+	+	+	+	+	+	+	+	+	+
π_S	+	+	+	+	+	+	+	+	+	+	+	+

Table 4: Comparative statics: analysis of the relationship between the ratios of intangible-to-physical production capital and the key structural parameters., for calibration scenarios I, II, IV and V (see Tables 1 and 2). In order to satisfy constraint (40), we consider: (i) a proportional change in parameters ω_I and π_Y ; (ii) a variation in ξ accompanied by a variation in δ of the same magnitude but with opposite sign.

In general, the positive effect on the intangible-to-physical production capital ratios dominates across the calibration scenarios and the key structural parameters considered. In particular, the parameters that control for the efficiency of social and intangible cultural capital accumulation, Φ, ω_S, ω_I, π_Y, and π_S (recall (3) and (6)) always feature a positive impact on the three intangible-to-physical capital ratios. Directly or indirectly, these parameters always incentivize the accumulation of all types of intangible capital vis-à-vis physical capital.

The exception are the parameters that control for the efficiency of human capital accumulation, ξ, α, and φ (recall equation (5)), which tend to induce a negative effect on the ratios – especially $\frac{\tilde{H}^*}{\tilde{K}_Y^*}$ and $\frac{\tilde{K}_S^*}{\tilde{K}_Y^*}$, as regards ξ, and $\frac{\tilde{K}_S^*}{\tilde{K}_Y^*}$ and $\frac{\tilde{K}_I^*}{\tilde{K}_Y^*}$, as regards α and φ. These results emerge because these parameters incentivize human capital accumulation, but, through the imperfect substitutability of human and physical production capital in the aggregate production function (7), also incentivize physical capital accumulation. This efficiency-enhancing effects provide the motivation for the policy experiments and welfare analysis presented in the next section.

4.3 Long-run welfare analysis

In this section, we analyze the effect of policy on welfare in the long-run (steady-state/BGP) equilibrium. Thus, we rewrite the intertemporal utility function in equation (21) as:

$$U^* = \frac{1}{1-\tau}\left[\tilde{C}^*\left(\tilde{K}_S^*\right)^{\phi}\left(\tilde{K}_I^*\right)^{\psi}\right]^{(1-\tau)}\int_0^{\infty} e^{-[\rho-(1-\tau)g^*(1+\phi+\psi)]t}dt \tag{46}$$

Solving the integral, dividing and multiplying by $\left(\tilde{C}^*\right)^{\phi+\psi}$ and, considering, without loss of generality, that $\left(\tilde{C}^*\right)^{1+\phi+\psi} = 1$, we get:

$$U^* = \frac{\left[\left(\frac{\tilde{K}_S^*}{\tilde{C}^*}\right)^{\phi}\left(\frac{\tilde{K}_I^*}{\tilde{C}^*}\right)^{\psi}\right]^{(1-\tau)}}{(1-\tau)\left[\rho-(1-\tau)\cdot g^*\left(1+\phi+\psi\right)\right]} \qquad \begin{array}{l}\left.\vphantom{\Bigg[}\right\}\text{level effect}\\ \left.\vphantom{\Big[}\right\}\text{dynamic effect}\end{array} \tag{47}$$

As shown in, respectively, the numerator and denominator of equation (47), there is a static (or level) effect, as well as a dynamic (or slope) effect influencing welfare on the long-run equilibrium. The static effect translates into a higher U^* due to a larger $\frac{\tilde{K}_S^*}{\tilde{C}^*}$ or $\frac{\tilde{K}_I^*}{\tilde{C}^*}$ with elasticities $\phi(1-\tau)$ and $\psi(1-\tau)$, respectively,. The dynamic effect occurs as a higher U^* due to a smaller effective discount rate $\rho-(1-\tau)\cdot g^*\left(1+\phi+\psi\right)$.

Since several policies regarding the different types of capital (namely the social and intangible cultural capital) are discussed in the literature (as detailed in Section 5, below), we now carry out a counterfactual numerical exercise to assess the effect of hypothetical policy-induced shocks to key structural parameters of the model on the welfare along the long-run equilibrium – measured as a shift in the steady-state intertemporal utility, U^*, in equation (47). For the sake of quantitative comparison, we translate each policy measure into an equivalent of a 10% subsidy, as usual in the literature.[12] Table (5) summarizes the results regarding the elasticity of U^* with respect to each parameter of interest, considering a 10% variation in the latter.[13]

[12] As a robustness check, we also experiment with a 20% subsidy-equivalent shock to the key parameters of the model. The results are presented and discussed in Appendix D.

[13] In order to simplify the analysis, we do not explicitly consider a government sector in the model, but we adopt the usual (underlying) assumption that the government balances its budget every period by levying the necessary amount of lump-sum taxes.

I		II		IV		V	
	Elasticity		**Elasticity**		**Elasticity**		**Elasticity**
$\xi_{(1\%)}$	3.48(+)	ξ	0.72(+)	ξ	0.80(+)	ξ	0.98(+)
α	0.33	φ	0.09	ϕ	0.14	ϕ	0.16
ω_S	0.23	Φ	0.03(+)	φ	0.07	α	0.09
ϕ	0.14	ψ	0.029	ψ	0.056	φ	0.056
Φ	0.07	α	0.028	α	0.055	ψ	0.05
π_S	0.06	π_S	0.017	π_S	0.03	π_S	0.042
φ	0.03	ω_S	0.014(+)	ω_I, π_Y	0.009	Φ	0.024
ψ	0.02	ω_I, π_Y	0.009	Φ	0.002(+)	ω_S	0,022
ω_I, π_Y	0.01	ϕ	0.003(+)	ω_S	0.001(+)	ω_I, π_Y	0.009

Table 5: Elasticity of the steady-state intertemporal utility, U^*, with respect to each parameter of interest, considering as a rule a 10% variation in the latter, under the calibration scenarios I, II, IV and V in Tables (1) and (2). The parameters are ordered from the highest to the lowest modulus of the utility elasticity. A positive sign (+) indicates that the parameter has a positive impact on U^*; otherwise, the impact is negative. In order to satisfy constraint (40), we consider: (i) a proportional change in parameters ω_I and π_Y ; (ii) a variation in ξ accompanied by a variation in δ of the same magnitude but with opposite sign. The elasticity of ξ in scenario I was calculated assuming a 1% subsidy for computational reasons and, thus, is not strictly comparable with the other values.

The parameter that controls for the efficiency of human capital allocated to the education sector, ξ, is the only one with a positive impact on welfare regardless of the calibration scenario. As depicted by Table 3, in Section 4.2, ξ impacts positively on $\frac{\tilde{K}_S^*}{\tilde{C}^*}$, $\frac{\tilde{K}_I^*}{\tilde{C}^*}$, and g^*, thus yielding positive level and dynamic effects on U^* – recall equation (47) and note that $\frac{\tilde{K}_S^*}{\tilde{C}^*}$ and $\frac{\tilde{K}_I^*}{\tilde{C}^*}$ increase the numerator of that equation, while g^* decreases the effective discount rate, $\rho - (1-\tau) \cdot g^* (1 + \phi + \psi)$, in the denominator).[14] In addition to ξ, the parameters that measure the effect of human and intangible cultural capital on social capital accumulation, ω_S and Φ, and the parameter that controls for the preference for social capital in the households' utility function, ϕ, also have a positive impact on welfare, although this result depends on the calibration scenario. ω_S and Φ have a positive impact in scenarios II and IV, the cases in which they generate a positive static effect (because the ratios $\frac{\tilde{K}_S^*}{\tilde{C}^*}$ and $\frac{\tilde{K}_I^*}{\tilde{C}^*}$ are positively impacted; recall Table 3) while producing no dynamic effect (the effective discount rate, in the denominator in (47), is not affected). In turn, ϕ does so only in scenario II, the case in which it produces a positive static effect (through the elasticity of $\frac{\tilde{K}_S^*}{\tilde{C}^*}$, which countervails the decrease in this ratio) and a positive dynamic effect (through a direct negative impact on the effective discount rate, which countervails the impact of opposite sign due to the decrease in g^*). Parameter ψ is not able to produce the same (positive) effect on welfare than ϕ because, under our calibration of the model (see Table 1), ψ is fairly smaller than ϕ and, thus, a 10% variation

[14]Starting with a baseline long-term growth rate of 0.01, a 10% increase in ξ results in the long-term growth rate increasing to 0.0125, while welfare increases by an average of 8.36% across calibration scenarios II, IV, and V. In the case of a 20% variation in ξ, the long-term growth rate increases to 0.0149 and welfare rises by an average of 22.83% across scenarios II, IV, and V (see Appendix D).

amounts also to a much smaller absolute shift in the former. All other parameters have a negative effect on welfare under all calibration scenarios, as they generate a negative static effect – they negatively impact the ratios $\frac{\tilde{K}_S^*}{\tilde{C}^*}$ and $\frac{\tilde{K}_I^*}{\tilde{C}^*}$, as shown in Table 3 –, while producing no dynamic effect (the effective discount rate is not affected, neither directly nor through g^*).

Overall, we emphasize the following results concerning the parameters and their positive impact on welfare:

- The parameter with the highest elasticity is ξ across all scenarios, with an average value of 0.84 across scenarios II, IV and V (the ones that pertain to a 10% variation in ξ);
- The parameters Φ and ω_S exhibit their highest (positive) elasticity in scenario II. The average elasticity of Φ is 0.017 and that of ω_S is 0.008 in scenarios II and IV.

To conclude, our results suggest that the most effective policies aiming to leverage welfare in long-run equilibrium would be those targeting the efficiency of human capital in educational activities (controlled by ξ in our model). Additionally, depending on the calibration scenario, policies might target the impact of intangible cultural capital and of human capital on social capital accumulation (controlled by Φ, and ω_S) or the households' preference for social capital (controlled by ϕ), although these would be less cost-effective (in real terms) than targeting the efficiency of human capital in the education sector.

4.4 Transitional dynamics

4.4.1 Transition paths for given initial conditions

In order to analyze the transitional dynamics of the model, we build on the solution of the linearized system obtained from the dynamical system (28)-(33) to perform a numerical illustration of the transition paths that emerge from given initial conditions off the steady state/BGP . We consider the model calibration from Section 4.1 and the normalization $\tilde{C}^* = 1$, bearing in mind that, under the conditions of Proposition 3, the steady state features local indeterminacy, with either a 5- or a 6-dimensional stable manifold characterized by a dampened oscillatory behavior.

Thus, for the purpose of illustration, Figure 1 depicts the results for initial values of all types of capital (the state variables), $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$, set 10% below their steady-state values and for the calibration scenario IV, under which a 5-dimensional stable manifold emerges and, thus, for which we consider arbitrarily either a high initial consumption ($\tilde{C}(0) > \tilde{C}^*$) or a low initial consumption ($\tilde{C}(0) < \tilde{C}^*$) with a 10% deviation vis-à-vis the steady state.[15]

[15] For results under a calibration scenario in which a 6-dimensional stable manifold emerges, and thus both $\tilde{C}(0)$ and $\frac{z(0)}{u_Y(0)}$ may be set arbitrarily, see Appendix E.2.

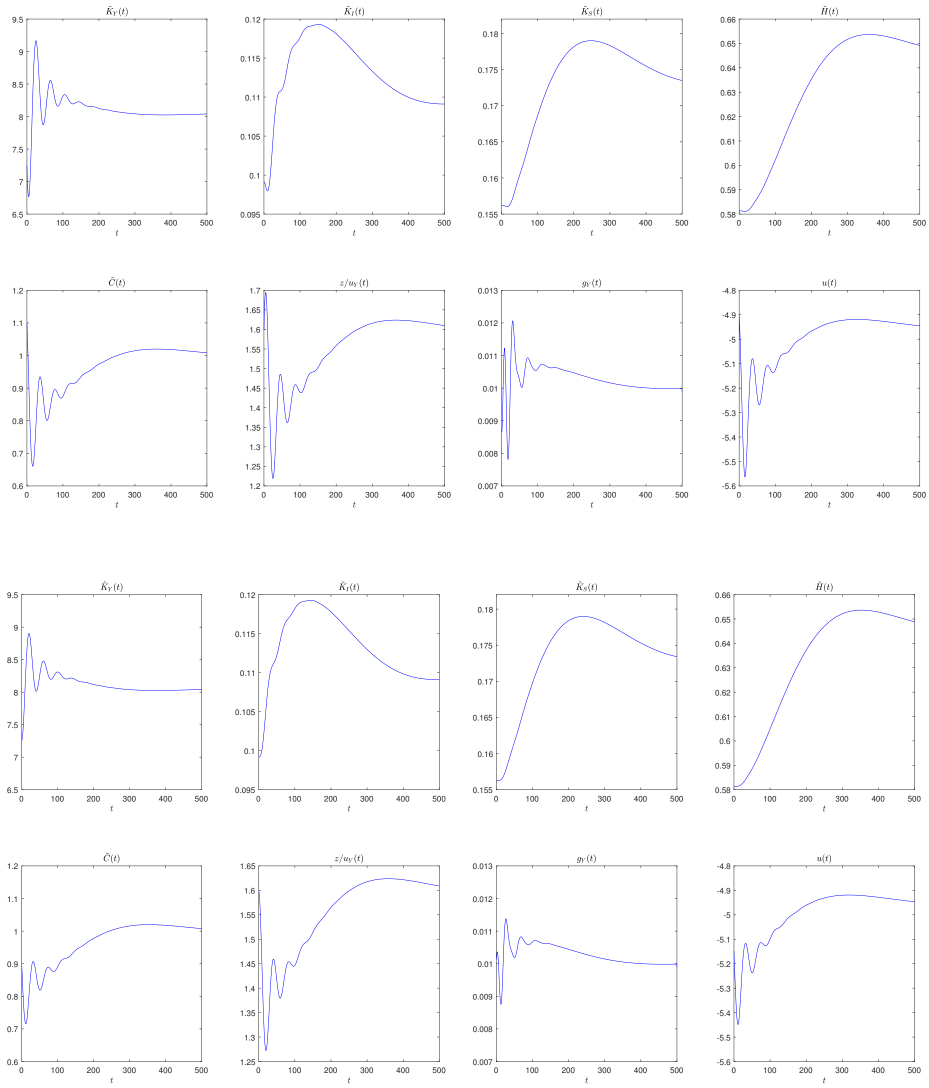

Figure 1: Transitional dynamics of selected macroeconomic variables for given initial conditions. Top panel: high initial consumption, $\tilde{C}(0)$ (+10% off the steady-state value). Bottom panel: low initial consumption, $\tilde{C}(0)$ (−10% off the steady-state value). All initial values of the state variables, $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$, are set 10% below their respective steady state ($\tilde{C}^* = 1$; calibration scenario IV; $u(t)$ denotes the instantaneous utility in (21), $u(t) = \frac{1}{1-\tau}\left[\tilde{C}(t)\left(\tilde{K_S}(t)\right)^{\phi}\left(\tilde{K_I}(t)\right)^{\psi}\right]^{(1-\tau)}$; $\frac{z}{u_Y}$ is determined for a given u_Y).

Examining the transition dynamics for the case $\tilde{C}(0) > \tilde{C}^*$(top panel of Figure 1), it is interesting to observe the marked oscillatory behavior of physical capital, $\tilde{K}_Y(t)$, which is particularly exuberant up to $t = 50$, as well as the smooth non-monotonic behavior of intangible cultural capital, $\tilde{K}_I(t)$, social capital, $\tilde{K}_S(t)$, and, to a lesser extent, human capital, $\tilde{H}(t)$. Also, it is noteworthy that all types of capital display a slight short-run undershooting vis-à-vis the steady-state level (more noticeable for $\tilde{K}_Y(t)$ and less visible for $\tilde{H}(t)$), followed by significant short-to-medium run overshooting by $\tilde{K}_Y(t)$ and medium-to-long run overshooting by the other (intangible) types of capital. Interestingly, the inverted-U shaped time-path displayed by social capital, and the fact that it coincides with an upward behavior by human capital over most part of the transition to the steady-state, is in line with the empirical patterns suggested by the literature for the 20th century (e.g., Bjørnskov, 2008; Tamura, 2006; Putnam, 2000).

The dynamics of the growth rate of aggregate output, $g_Y(t)$ (recall equation (12)), is greatly determined by the time path of the marginal productivity of physical production capital roughly up to $t = 100$ and, hence, $g_Y(t)$ mirrors the time path of $\tilde{K}_Y(t)$ over that period – namely with a salient short-to-medium run oscillatory behavior that starts with an overshooting vis-à-vis the steady-state level (in contrast to the initial undershooting by $\tilde{K}_Y(t)$). Afterwards, however, the marginal productivity effects pertaining to $\tilde{H}(t)$ and $\frac{z(t)}{u_Y(t)}$ dominate, resulting in a smooth downward path for $g_Y(t)$. Thus, it is interesting to observe that, in most of this later period, $g_Y(t)$ and $\tilde{K}_Y(t)$ display a positive relationship, (in contrast to the typical neoclassical transitional-dynamics result) while $g_Y(t)$ relates negatively with $\tilde{H}(t)$ and $\frac{z(t)}{u_Y(t)}$.

On the other hand, after a significant short-run downward adjustment, $\tilde{C}(t)$ follows the transition pattern of $\tilde{K}_Y(t)$ and $\frac{z(t)}{u_Y(t)}$ roughly up to $t = 100$ and that of $\frac{z(t)}{u_Y(t)}$ afterwards, as these two variables crucially determine the behavior of the level of aggregate output (given the smoother transition pattern of $\tilde{H}(t)$) and, therefore, that of aggregate income. Furthermore, the instantaneous utility, $u(t)$, follows from close the transition pattern of $\tilde{C}(t)$, as the shifts in the latter dominate over the shifts in $\tilde{K}_I(t)$ and $\tilde{K}_S(t)$ (observe the clearly lower order of magnitude of the variations in the latter variables than those in $\tilde{C}(t)$). As a result, $u(t)$ first exhibits significant oscillation, with a marked short-run undershooting, and, then, a smooth medium-to-run upward path followed by only a slight decline towards the steady state.

For the case $\tilde{C}(0) < \tilde{C}^*$(bottom panel of Figure 1), the pattern of the medium-to-long run transition is similar to that described above for all variables, suggesting, for this time horizon, that the sensitiveness to the sunspot behavior in consumption is small. It is noteworthy, however, the distinct short-run behavior that arises when initial consumption is low relative to its steady state: the different types of capital exhibit no short-run undershooting vis-à-vis the steady-state level and the short-run oscillatory behavior displayed by $\tilde{K}_Y(t)$, $\tilde{C}(t)$, $\frac{z(t)}{u_Y(t)}$, $g_Y(t)$, and $u(t)$ is quite attenuated.[16]

[16]As is well-known in the literature (e.g., Mazeda Gil et al., 2016), a linearized dynamical system characterized by a multidimensional stable manifold, such as ours, may feature time-varying speeds of convergence, even in the absence of oscillatory behavior, due to the interaction of the multiple state variables over transition. The speeds of convergence may also be different across variables. The graphical representation of the speeds of convergence for selected macroeconomic variables in our model are depicted by Figure 4, in Appendix (E.2).

4.4.2 Transition paths under policy-induced shocks

In this section, we focus on the transitional-dynamics effects originated by hypothetical policy-induced shocks to key structural parameters in the model. For practical reasons, we consider calibration scenario IV with the normalization $\tilde{C}^* = 1$, and select three representative cases of policy-induced shocks bearing in mind the comparative-statics results in Table 3, i.e., a shock to: (i) the effect of social capital on human capital accumulation, α, in which case the steady-state values of all the capital-to-consumption ratios *decrease* as the parameter increases;[17] (ii) the effect of intangible cultural capital on social capital accumulation, Φ, in which case the steady-state values of all capital-to-consumption ratios *decrease* with the parameter, except for the social capital ratio, which *increases*;[18] and (iii) the efficiency of human capital in the education sector, ξ, in which case the steady-state values of all capital-to-consumption ratios (and of the economic growth rate) *increase* with the parameter. As before, we translate each policy measure into an equivalent of a 10% subsidy (see Section 4.3). Figure (2) displays the case with low initial consumption ($\tilde{C}(0) < \tilde{C}^*$), and the initial values for $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$ corresponding to their steady-state levels under no subsidy.[19][20]

[17] Recall that the other key parameters that generate the same result are ψ, ϕ, φ,ω_I, π_Y, and π_S.

[18] The other parameter that generates the same result is ω_S.

[19] To facilitate comparison of transition patterns across the different shocks, we normalize these initial values to unity.

[20] Results for the case $\tilde{C}(0) > \tilde{C}^*$ are depicted by Figure (7), in Appendix E.2.

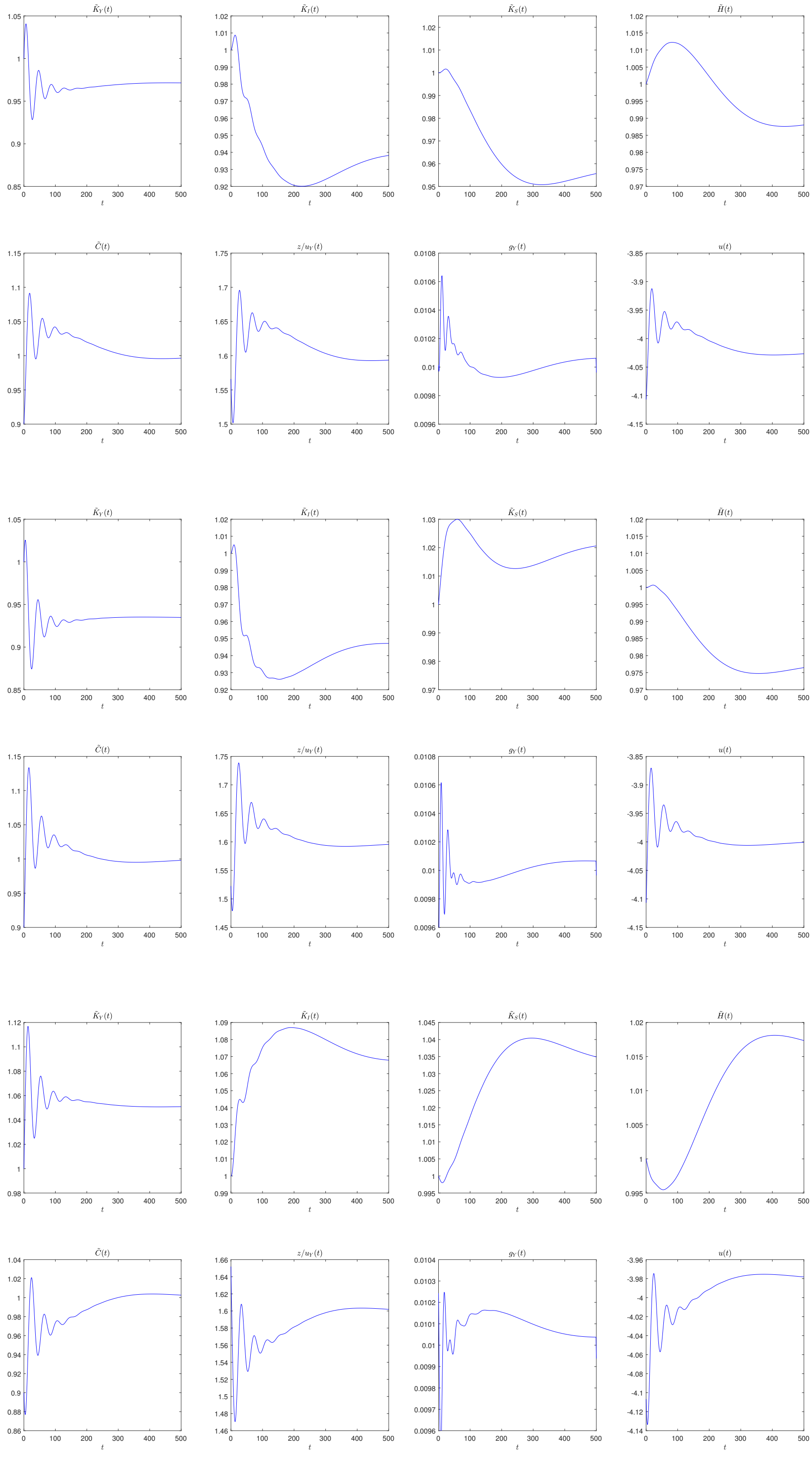

Figure 2: Transitional dynamics of selected macroeconomic variables under policy-induced shocks. Three representative cases are shown, a shock to: α (upper panel); Φ (middle panel); and ξ (bottom panel). Initial values for the state variables, $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$, correspond to their steady-state levels under no subsidy, normalized to unity. The initial value $\tilde{C}(0)$ is set 10% below its steady-state level ($\tilde{C}^* = 1$; calibration scenario IV; $u(t)$ denotes the instantaneous utility in (21), $u(t) = \frac{1}{1-\tau}\left[\tilde{C}(t)\left(\tilde{K_S}(t)\right)^{\phi}\left(\tilde{K_I}(t)\right)^{\psi}\right]^{(1-\tau)}$; $\frac{z}{u_Y}$ is determined for a given u_Y).

As said, the shocks to α and to Φ decrease the steady-state values of the capital-to-consumption ratios (except for social capital in the latter case), implying that the state variables start the transition to the new steady state from *above*. Thus, the transitional dynamics triggered by these shocks features a pattern that, in general, is the opposite of that described for the comparable case in Figure 1, lower panel (transition for given initial conditions, with the initial capital ratios and consumption set *below* their steady-state values). The top and middle panel of Figure (2) show, like in the previous section, a salient oscillatory behavior of physical capital, $\tilde{K}_Y(t)$, especially up to $t = 50$, and a non-monotonic behavior of intangible cultural capital, $\tilde{K}_I(t)$, social capital, $\tilde{K}_S(t)$, and human capital, $\tilde{H}(t)$. However, now, the trajectories of all types of capital are marked by a slight short-run overshooting vis-à-vis the steady-state level (more noticeable for $\tilde{K}_Y(t)$), followed by significant short-to-medium run undershooting by $\tilde{K}_Y(t)$ and medium-to-long run undershooting by the other (intangible) types of capital. In particular, the time-path of $\tilde{K}_S(t)$ under the shock to Φ (in the middle panel) features a quite regular low frequency, medium-to-long run oscillation because this variable is converging to a higher steady state due to the shock, whereas the remaining types of capital are converging to a lower steady state.

The growth rate of aggregate output, $g_Y(t)$, is greatly determined, first, by the behavior of the marginal productivity of $\tilde{K}_Y(t)$ in aggregate production and then by the marginal productivity effects pertaining to $\tilde{H}(t)$ and $\frac{z(t)}{u_Y(t)}$. Again, in the later phase of adjustment, $g_Y(t)$ and $\tilde{K}_Y(t)$ display a positive relationship, while $g_Y(t)$ relates negatively with $\tilde{H}(t)$ and $\frac{z(t)}{u_Y(t)}$. Overall, $g_Y(t)$ features a short-to-medium run oscillatory behavior that starts with an overshooting vis-à-vis the steady-state level and, in the medium-to-long run, a smooth U-shaped time path. Notice that $g_Y(t)$ converges to a steady-state value equal to that previous to the shock, since α and Φ do not affect g_Y^* (recall Table 3).

As before, the instantaneous utility, $u(t)$, follows from close the transition pattern of $\tilde{C}(t)$ (which, in turn, follows that of $\tilde{K}_Y(t)$ and $\frac{z(t)}{u_Y(t)}$, as they command the dynamics of aggregate output), since the shifts in the latter dominate over the shifts in $\tilde{K}_I(t)$ and $\tilde{K}_S(t)$. As a result, $u(t)$ first exhibits significant oscillation, with a marked short-run overshooting, and, then, a smooth medium-to-run downward path followed by a slight increase towards the steady state. All in all, $u(t)$ converges to a steady-state level that is lower than that previous to the shock, in the case of α (because both ratios $\frac{\tilde{K}_S^*}{\tilde{C}^*}$ and $\frac{\tilde{K}_I^*}{\tilde{C}^*}$ are negatively impacted; see the numerator in equation 47 and recall Section 4.3), but that is higher in the case of Φ (because the ratio $\frac{\tilde{K}_S^*}{\tilde{C}^*}$ is positively impacted, which outweighs the negative impact on $\frac{\tilde{K}_I^*}{\tilde{C}^*}$).[21]

As expected, the transitional dynamics triggered by the shock to ξ (which, as said,

[21]Notice that the steady-state instantaneous utility, u^*, is only affected by static effects – which amount to the numerator in equation (47) –, while the intertemporal utility, U^*, is affected by static and dynamic effects - the denominator and the numerator in (47).

increases the steady-state values of the capital-to-consumption ratios) features a pattern that is fairly similar to that described for the comparable case in Figure 1, lower panel. Yet, reflecting the magnitude of the shock, the short-run oscillation in $\tilde{K}_Y(t)$, $\tilde{C}(t)$, $\frac{z(t)}{u_Y(t)}$, $g_Y(t)$, and $u(t)$ is much more intense now than that in Figure 1, while $\tilde{K}_S(t)$ and $\tilde{H}(t)$ now display a salient short-run undershooting vis-à-vis the steady-state level.[22] Also note that, in the case of the shock to ξ, the variables $g_Y(t)$ and $u(t)$ eventually converge to a steady-state value that is higher than that previous to the shock (as regards $u(t)$, because the ratios $\frac{\tilde{K}_S^*}{\tilde{C}^*}$ and $\frac{\tilde{K}_I^*}{\tilde{C}^*}$ are both positively impacted).

To conclude, our model suggests that, a policy-induced shock to a given capital-accumulation parameter other than ξ or to the parameters that control for the preference for social and intangible cultural capital, ϕ and ψ, generates a short-to-medium run (flow) welfare gain, due to the overshooting behavior of consumption and also (although in a lower frequency) to that of social and intangible cultural capital. These effects are reverted in the medium-to-long run horizon, although only partially (under calibration scenarios II and IV) for the parameters controlling for the efficiency of intangible cultural and human capital in social-capital accumulation, Φ and ω_S. A policy-induced shock to the efficiency of human capital allocated to the education sector, controlled by ξ, generates the opposite behavior: a short-to-medium run (flow) welfare loss occurs (although with high frequency oscillation), due to the undershooting behavior of consumption and of social and intangible cultural capital. These effects are entirely reverted over the medium-to-long run.

5 Policy discussion

In this section, we discuss, with the support of the literature, several policies related to different types of capital: social, cultural (intangible), and human. We begin by discussing policies related to (the impact of) social capital (parameters π_S, α, ϕ respectively, in equations 3, 5, and 8,), followed by intangible cultural capital (parameters Φ and ψ, respectively, in equations 6 and 8), and finally human capital (parameters ω_I, ξ, and ω_S respectively, in equations 3, 5, and 6).

Preference for social capital, ϕ, can increase through encouragement of participation in leisure activities such as sports centers, parks, and community centers. To promote such participation, Chou (2006) proposes that there should be adequate government funding, along with the possibility of tax incentives, for sports clubs and other associations. This approach would not only enhance social capital but also foster stronger community en-

[22] These differences arise from the fact that the 10% subsidy-equivalent policy-induced shock to ξ positions the dynamical system further away from the (new) steady state than in the case in which the state variables are set 10% below their steady-state values. The high elasticity of the steady-state value of intertemporal utility with respect to ξ, shown in Table 5, serves as an illustration of this point.

gagement and cohesion. As regards policy measures associated with the impact of social capital on human capital accumulation, α, e.g., Chou (2006) suggests measures directed towards parents, encouraging them to spend more time with their children and to be actively involved in their formal education. In practical policy terms, Chou (2006) proposes offering financial rewards and other incentives for parents to join parent-teacher associations. Regarding measures associated with the impact of social capital on intangible cultural capital, π_S, policies should not only encourage communities to engage with cultural institutions, but also bring art and culture directly to communities. This highlights the importance of physical infrastructure (such as museums, libraries, etc.), but also the need for community-based programs that foster stronger ties within local communities.

Policy measues associated with the preference for intangible cultural capital, ψ, are those related to key factors in cultural participation, such as improving accessibility, proximity, and reducing the cost of the cultural "offer". An example of such a policy could be the allocation of vouchers that citizens can use to attend museums, performances, or go to the theater with a discount or for free (e.g. Van der Ploeg, 2006). Considering the impact of intangible cultural capital on social capital, Φ, and as noted by Hammonds (2023), those who engage with culture and art tend to be more involved in community and civic activities, such as participating in neighborhood associations. In this sense, policies that promote the design of cultural programs encouraging participation in groups—such as theater groups, music composition (like singing in a choir), performing arts activities, dance, or painting—are well-suited.

In the case of the impact of human capital on the accumulation of social capital, ω_S, the role of civic education in school curricula and citizenship programmes is particularly noteworthy. For example, Briole et al., 2025 present evidence on the causal positive effects of civic-specific education on pupils' behaviour and civic outcomes. The study also reveals that citizenship programmes help pupils to develop their network of friends (specifically, the programme has a sizeable, positive impact on the number of "socially different" friends (as measured by gender, social origin or geographical origin)). As regards the impact of human capital on intangible cultural capital, ω_I, notable policies include those that integrate art and culture into the learning process. Additionally, policies fostering collaboration among schools, artists, and cultural organizations can create valuable synergies, enhancing both educational outcomes and cultural engagement (Hammonds, 2023). Regarding the efficiency of human capital on its own accumulation, ξ, the literature has been largely consistent in highlighting the importance of educational policies focused on the early years of life and the provision of quality preschool education (e.g., Heckman et al., 2010; Heckman et al., 2013; Attanasio, 2015). Moreover, evidence emphasizes that adolescence can also be a critical period for development (e.g., Blakemore and Mills, 2014), suggesting that human capital policies should address this stage as well (Cunha and Heckman, 2007; Attanasio, 2015; Heckman and Kautz, 2013). From a policy perspective, both the quantity and the quality of education matter, as evidence shows that low-quality education often fails to translate into human capital accumulation (e.g., Pritchett, 2001; Hanushek and Woessmann, 2007).

6 Concluding remarks

Culture and social capital may be particularly relevant variables in explaining important dimensions of the economic growth phenomenon. In recent years, policymakers and economists have increasingly considered the role of these immaterial types of capital in economic growth. Yet, cultural capital and social capital have been analyzed separately in the growth literature. We argue that there is a link between institutions, trust, and networks (social capital) and cultural capital, as well as a link between culture and the formation of social capital. In this paper, we propose a theoretical model that examines the relationships among these assets, aiming to provide a comprehensive theoretical perspective on their potential contributions to economic growth and welfare within an endogenous growth framework.

We develop a dynamic general-equilibrium model using an endogenous economic growth set-up (namely with human capital accumulation) and incorporating cultural and social capital. We use the model to devise long-run and transitional-dynamics effects from the perspective of both economic growth and welfare, explicitly considering the interplay between cultural and social capital and other forms of capital. A detailed calibration of the model using European data yields quantitative results with relevant policy implications. Under the model calibration, the steady-state is locally indeterminate, which makes possible the existence of sunspot equilibrium trajectories. In addition, we show that some variables follow oscillatory dampened trajectories towards the long-run equilibrium.

Relying on this calibration, we perform a counterfactual numerical exercise to assess the impact of diverse policy-induced shocks on, namely, the capital ratios, the economic growth rate and welfare. In particular, our long-run equilibrium results suggest that the most cost-effective policies aiming to leverage welfare would be those targeting the efficiency of human capital in the educational activities. Additionally, policies might target the impact of human and intangible cultural capital on social capital accumulation. However, the transitional-dynamics results point in a different direction. A shock to the efficiency of human capital in the education sector generates a short-to-medium run (flow) welfare *loss* (although with high frequency oscillation), due to the undershooting behavior of consumption and of social and intangible cultural capital. In contrast, a shock to the efficiency of some type of capital accumulation (other than the efficiency of human capital in the education sector) or to the preference for social and intangible cultural capital generates a short-to-medium run (flow) welfare *gain*, due to the overshooting behavior of consumption and of social and intangible cultural capital. Thus, these results suggest policymakers face a (sub-optimal) dilemma when weighing in the different time horizons of the policy effects.

Taking into account the role of the government in the accumulation of these types of capital (cultural and social), it would be interesting in the future to extend the model to explicitly incorporate the presence of the government and consider public debt. Furthermore, given the challenges related to the calibration of the parameters related to intangible types of capital – namely, intangible cultural capital and social capital – it would be valuable in the future to conduct a detailed empirical estimation of these forms of capital and their dynamics in the data, bearing in mind their multidimensional nature.

References

Attanasio, O. P. (2015). The determinants of human capital formation during the early years of life: Theory, measurement, and policies. *Journal of the European Economic Association 13*(6), 949–997.

Baldin, A. and T. Bille (2023). The lost value for users of cultural institutions during the COVID-19 pandemic: a life satisfaction approach. *International Review of Economics 70*(2), 257–281.

Barro, R. J. and X. Sala-i Martin (2004). *Economic Growth* (2nd ed.). Cambridge, MA: The MIT Press.

Bartolini, S. and L. Bonatti (2008). Endogenous growth, decline in social capital and expansion of market activities. *Journal of Economic Behavior & Organization 67*(3-4), 917–926.

Beugelsdijk, S., H. L. De Groot, and A. B. Van Schaik (2004). Trust and economic growth: a robustness analysis. *Oxford Economic Papers 56*(1), 118–134.

Beugelsdijk, S. and S. Smulders (2009). Bonding and bridging social capital and economic growth. Discussion Paper Series No. 2009-27, CentER, Tilburg University.

Bille, T. (2024). The values of cultural goods and cultural capital externalities: state of the art and future research prospects. *Journal of Cultural Economics 48*(3), 347–365.

Bjørnskov, C. (2008). Social capital and happiness in the United States. *Applied Research in Quality of Life 3*(1), 43–62.

Blakemore, S.-J. and K. L. Mills (2014). Is adolescence a sensitive period for sociocultural processing? *Annual Review of Psychology 65*(1), 187–207.

Bovenberg, L. (2003). Unity produces diversity: The economics of Europe's social capital. In *The Cultural Diversity of European Unity*, pp. 403–419. Brill.

Briole, S., M. Gurgand, E. Maurin, S. McNally, J. Ruiz-Valenzuela, and D. Santín (2025). The making of civic virtues: a school-based experiment in three countries. *American Economic Journal Association: Economic Policy 17*(3), 496–521.

Bucci, A., P. L. Sacco, and G. Segre (2014). Smart endogenous growth: cultural capital and the creative use of skills. *International Journal of Manpower 35*(1/2), 33–55.

Bucci, A. and G. Segre (2011). Culture and human capital in a two-sector endogenous growth model. *Research in Economics 65*(4), 279–293.

Caballé, J. and M. S. Santos (1993). On endogenous growth with physical and human capital. *Journal of Political Economy 101*(6), 1042–1067.

Chou, Y. K. (2006). Three simple models of social capital and economic growth. *The Journal of Socio-Economics 35*(5), 889–912.

Coleman, J. S. (1988). Social capital in the creation of human capital. *American Journal of Sociology 94*, S95–S120.

Cunha, F. and J. Heckman (2007). The technology of skill formation. *American Economic Review 97*(2), 31–47.

Dinda, S. (2008). Social capital in the creation of human capital and economic growth: A productive consumption approach. *Journal of Socio-Economics 37*(5), 2020–2033.

Durlauf, S. N. (2002). On the empirics of social capital. *Economic Journal 112*(483), F459–F479.

Durlauf, S. N. and M. Fafchamps (2005). Social capital. In *Handbook of Economic Growth*, pp. 1639–1699. Elsevier.

Funke, M. and H. Strulik (2000). On endogenous growth with physical capital, human capital and product variety. *European Economic Review 44*(3), 491–515.

Grossi, E., P. L. Sacco, G. T. Blessi, and R. Cerutti (2011). The impact of culture on the individual subjective well-being of the Italian population: An exploratory study. *Applied Research in Quality of Life 6*(4), 387–410.

Hammonds, W. (2023). Culture and democracy: the evidence - how citizens' participation in cultural activities enhances civic engagement, democracy and social cohesion - lessons from international research. Technical report, Publications Office of the European Union.

Hanushek, E. A. and L. Woessmann (2007). The role of education quality for economic growth. *World Bank Policy Research Working Paper* (No. 4122).

Heckman, J. and T. Kautz (2013). Fostering and measuring skills: Interventions that improve character and cognition. *Human Capital and Economic Opportunity Global Working Group Working Paper* (2013-019).

Heckman, J., S. H. Moon, R. Pinto, P. Savelyev, and A. Yavitz (2010). Analyzing social experiments as implemented: A re-examination of the evidence from the HighScope Perry Preschool Program. *Quantitative Economics 1*(1), 1–46.

Heckman, J., R. Pinto, and P. Savelyev (2013). Understanding the mechanisms through which an influential early childhood program boosted adult outcomes. *American Economic Review 103*(6), 2052–2086.

Jones, L. E., R. Manuelli, and H. E. Siu (2000). Growth and business cycles. Working Paper No. w7633, NBER.

Kim, Y. J. and C.-I. Lee (2015). Social values and economic dynamics. *Journal of Economic Dynamics and Control 53*, 69–84.

Knack, S. and P. Keefer (1997). Does social capital have an economic payoff? a cross-country investigation. *Quarterly Journal of Economics 112*(4), 1251–1288.

Lucas, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics 22*(1), 3–42.

Mazeda Gil, P., A. Almeida, and S. B. Castro (2016). Rich transitional dynamics, physical capital, and technology intensity. *Oxford Economic Papers 69*(3), 678–706.

Pritchett, L. (2001). Where has all the education gone? *The World Bank Economic Review 15*(3), 367–391.

Putnam, R. D. (1993). *Making Democracy Work: Civic Traditions in Modern Italy.* Princeton University Press.

Putnam, R. D. (2000). *Bowling alone: The collapse and revival of American community.* Simon and Schuster.

Rizzo, I. and D. Throsby (2006). Cultural heritage: economic analysis and public policy. *Handbook of the Economics of Art and Culture 1*, 983–1016.

Roseta-Palma, C., A. Ferreira-Lopes, and T. N. Sequeira (2010). Externalities in an endogenous growth model with social and natural capital. *Ecological Economics 69*(3), 603–612.

Rupasingha, A., S. J. Goetz, and D. Freshwater (2002). Social and institutional factors as determinants of economic growth: Evidence from the United States counties. *Papers in Regional Science 81*(2), 139–155.

Sacco, P. L. and G. Segre (2009). Creativity, cultural investment and local development: a new theoretical framework for endogenous growth. In *Growth and Innovation of Competitive Regions*, pp. 281–294. Springer.

Sequeira, T. N. and A. Ferreira-Lopes (2011). An endogenous growth model with human and social capital interactions. *Review of Social Economy 69*(4), 465–493.

Sequeira, T. N. and A. Ferreira-Lopes (2013). Social capital and investment in R&D: new externalities. *Journal of Business Economics and Management 14*(1), 77–97.

Shockley, G. E. (2004). Government investment in cultural capital: a methodology for comparing direct government support for the arts in the US and the UK. *Public Finance and Management 4*(1).

Tamura, R. (2006). Human capital and economic development. *Journal of Development Economics 79*(1), 26–72.

Teachman, J. D., K. Paasch, and K. Carver (1997). Social capital and the generation of human capital. *Social Forces 75*(4), 1343–1359.

Throsby, D. (1999). Cultural capital. *Journal of Cultural Economics 23*(1), 3–12.

Throsby, D. (2001). *Economics and Culture.* Cambridge University Press.

Throsby, D. (2017). Culturally sustainable development: theoretical concept or practical policy instrument? *International Journal of Cultural Policy 23*(2), 133–147.

Ulibarri, C. A. (2000). Rational philanthropy and cultural capital. *Journal of Cultural Economics 24*(2), 135–146.

UNESCO (2021). *Cities, culture, creativity: Leveraging culture and creativity for sustainable urban development and inclusive growth.* Paris:UNESCO.

Uzawa, H. (1965). Optimum technical change in an aggregative model of economic growth. *International Economic Review 6*(1), 18–31.

Van der Ploeg, F. (2006). The making of cultural policy: A European perspective. In *Handbook of the Economics of Art and Culture*, pp. 1183–1221. Elsevier.

Whiteley, P. F. (2000). Economic growth and social capital. *Political Studies 48*(3), 443–466.

Woolcock, M. (1998). Social capital and economic development: Toward a theoretical synthesis and policy framework. *Theory and Society 27*(2), 151–208.

Appendices

A Dynamic Optimization Problem - First Order Conditions

Considering the dynamical optimization problem in equations (21) to (27) in the main text, we then write the corresponding Hamiltonian function, as follows:

$$
\begin{gathered}
\mathcal{H} = \frac{1}{1-\tau}\left[\left(\tilde{C}(t)\tilde{K}_S^{\phi}(t)\tilde{K}_I^{\psi}(t)\right)^{(1-\tau)} e^{[(1-\tau)g^*(1+\phi+\psi)-\rho]t}\right] + \\
+\lambda_Y(t)\left[\left(z(t)\tilde{K}_Y(t)\right)^{\beta}\left(u_Y(t)\tilde{H}(t)\right)^{1-\beta} - \tilde{C}(t) - \delta_P z(t)\tilde{K}_Y(t) - \delta_{C_T}\left(1-z(t)\right)\tilde{K}_Y(t) - g^*\tilde{K}_Y(t)\right] + \\
+\lambda_I(t)\left[\omega_I\left(1-u_Y(t)-u_H(t)-u_S(t)\right)\tilde{H}(t) + \Omega_I\tilde{K}_I(t) + \pi_S\tilde{K}_S(t) + \pi_Y\left(1-z(t)\right)\tilde{K}_Y(t) - g^*\tilde{K}_I(t)\right] + \\
+\lambda_H(t)\left[\xi u_H(t)\tilde{H}(t) + \alpha\tilde{K}_S(t) + \varphi\tilde{K}_I(t) - g^*\tilde{H}(t)\right] + \\
+\lambda_S(t)\left[\omega_S u_S(t)\tilde{H}(t) + \Omega_S\tilde{K}_S(t) + \Phi\tilde{K}_I(t) - g^*\tilde{K}_S(t)\right]
\end{gathered}
$$

Considering the control variables $C(t)$, $z(t)$, $u_Y(t)$, $u_H(t)$, $u_S(t)$, the first order conditions yield:

$$\frac{\partial\mathcal{H}}{\partial\tilde{C}(t)}=0\Leftrightarrow\frac{\left(\tilde{C}(t)\tilde{K}_S^{\phi}(t)\tilde{K}_I^{\psi}(t)\right)^{(1-\tau)}}{\tilde{C}(t)}e^{[g^*(1-\tau)(1+\phi+\psi)-\rho]t}=\lambda_Y(t) \tag{48}$$

$$\frac{\partial\mathcal{H}}{\partial z(t)}=0\Leftrightarrow\lambda_Y(t)\left[\beta z(t)^{\beta-1}\tilde{K}_Y(t)^{\beta}\left(u_Y(t)\tilde{H}(t)\right)^{1-\beta}-\tilde{K}_Y(t)\left(\delta_P-\delta_{C_T}\right)\right]=\lambda_I(t)\pi_Y\tilde{K}_Y(t)$$

$$\Leftrightarrow\lambda_I(t)=\lambda_Y(t)\frac{\beta z(t)^{\beta-1}\tilde{K}_Y(t)^{\beta}\left(u_Y(t)\tilde{H}(t)\right)^{1-\beta}-\tilde{K}_Y(t)\left(\delta_P-\delta_{C_T}\right)}{\pi_Y\tilde{K}_Y(t)}$$

$$\Leftrightarrow\lambda_I(t)=\lambda_Y(t)\frac{\beta z(t)^{\beta-1}\tilde{K}_Y(t)^{\beta-1}\left(u_Y(t)\tilde{H}(t)\right)^{1-\beta}-\delta_P+\delta_{C_T}}{\pi_Y} \tag{49}$$

$$\frac{\partial\mathcal{H}}{\partial u_Y(t)}=0\Leftrightarrow\lambda_Y(t)\left(z(t)\tilde{K}_Y(t)\right)^{\beta}(1-\beta)\,u_Y(t)^{-\beta}\tilde{H}(t)^{1-\beta}=\lambda_I(t)\omega_I\tilde{H}(t)$$

$$\Leftrightarrow\lambda_I(t)=\frac{\lambda_Y(t)\left(z(t)\tilde{K}_Y(t)\right)^{\beta}(1-\beta)\,u_Y(t)^{-\beta}\tilde{H}(t)^{1-\beta}}{\omega_I\tilde{H}(t)}$$

$$\Leftrightarrow\lambda_I(t)=\lambda_Y(t)\cdot\frac{\left(z(t)\tilde{K}_Y(t)\right)^{\beta}(1-\beta)\left(u_Y(t)\tilde{H}(t)\right)^{-\beta}}{\omega_I}$$

$$\Leftrightarrow\lambda_I(t)=\lambda_Y(t)\cdot\frac{(1-\beta)\,\tilde{Y}(t)}{u_Y(t)\tilde{H}(t)\omega_I}\Leftrightarrow\lambda_Y(t)=\lambda_I(t)\cdot\frac{u_Y(t)\tilde{H}(t)\omega_I}{(1-\beta)\,\tilde{Y}(t)} \tag{50}$$

$$\frac{\partial\mathcal{H}}{\partial u_H(t)}=0\Leftrightarrow-\lambda_I(t)\omega_I\tilde{H}(t)+\lambda_H(t)\xi\tilde{H}(t)=0\Leftrightarrow\frac{\lambda_I(t)}{\lambda_H(t)}=\frac{\xi}{\omega_I}\Leftrightarrow \tag{51}$$

$$\Leftrightarrow\lambda_I(t)=\frac{\xi}{\omega_I}\lambda_H(t)\Leftrightarrow\lambda_H(t)=\lambda_I(t)\frac{\omega_I}{\xi}$$

$$\frac{\partial\mathcal{H}}{\partial u_S(t)}=0\Leftrightarrow-\lambda_I(t)\omega_I\tilde{H}(t)+\lambda_S(t)\omega_S\tilde{H}(t)=0\Leftrightarrow\frac{\lambda_I(t)}{\lambda_S(t)}=\frac{\omega_S}{\omega_I}\Leftrightarrow \tag{52}$$

$$\Leftrightarrow\lambda_I(t)=\lambda_S(t)\frac{\omega_S}{\omega_I}\Leftrightarrow\lambda_S(t)=\lambda_I(t)\frac{\omega_I}{\omega_S}$$

$$\frac{\partial\mathcal{H}}{\partial\tilde{K}_Y(t)}=-\dot{\lambda_Y}(t)\Leftrightarrow\lambda_Y(t)\cdot\left[\beta z(t)^{\beta}\tilde{K}_Y(t)^{\beta-1}\left(u_Y(t)\tilde{H}(t)\right)^{1-\beta}-\delta_P z(t)-\delta_{C_T}\left(1-z(t)\right)-g^*\right]+$$

$$+\lambda_I(t)\pi_Y\left(1-z(t)\right)=-\dot{\lambda}_Y(t) \tag{53}$$

$$\frac{\partial\mathcal{H}}{\partial\tilde{K}_I(t)}=-\dot{\lambda_I}(t)\Leftrightarrow\frac{\psi\left(\tilde{C}(t)\tilde{K}_S^{\phi}(t)\tilde{K}_I^{\psi}(t)\right)^{(1-\tau)}}{\tilde{K}_I}e^{g^*(1-\tau)(1+\phi+\psi)-\rho t}+$$

$$+\lambda_I(t)\left(\Omega_I-g^*\right)+\lambda_H(t)\varphi+\lambda_S(t)\Phi=-\dot{\lambda_I}(t) \tag{54}$$

$$\frac{\partial\mathcal{H}}{\partial\tilde{H}(t)}=-\dot{\lambda}_H(t)\Leftrightarrow\lambda_Y(t)\cdot\left[\frac{(1-\beta)\left(z(t)K_Y(t)\right)^{\beta}\cdot\left(u_Y(t)H(t)\right)^{1-\beta}}{\tilde{H}(t)}\right]+ \tag{55}$$

$$+\lambda_I(t)\cdot\omega_I\cdot\left(1-u_Y(t)-u_H(t)-u_S(t)\right)+\lambda_H(t)\cdot\left(\xi u_H(t)-g^*\right)+\lambda_S(t)\omega_S u_S(t)=-\dot{\lambda}_H(t)$$

$$\frac{\partial\mathcal{H}}{\partial\tilde{K}_S(t)}=-\dot{\lambda}_S(t)\Leftrightarrow\frac{\phi\left(\tilde{C}(t)\tilde{K}_S^{\phi}(t)\tilde{K}_I^{\psi}(t)\right)^{(1-\tau)}}{\tilde{K}_S}e^{g^*(1-\tau)(1+\phi+\psi)-\rho t}+$$

$$+\lambda_I(t)\pi_S+\lambda_H(t)\alpha+\lambda_S(t)\left(\Omega_S-g^*\right)=-\dot{\lambda}_S(t) \tag{56}$$

$$\frac{\partial\mathcal{H}}{\partial\lambda_Y(t)}=\dot{\tilde{K}}_Y\Leftrightarrow\dot{\tilde{K}}_Y=\tilde{Y}(t)-\tilde{C}(t)-\delta_P z(t)\tilde{K}_Y(t)-\delta_{C_T}\left(1-z(t)\right)\tilde{K}_Y(t)-g^*\tilde{K}_Y(t)$$

$$\frac{\partial\mathcal{H}}{\partial\lambda_I(t)}=\dot{\tilde{K}}_I\Leftrightarrow\dot{\tilde{K}}_I=\omega_I\left(1-u_Y(t)-u_H(t)-u_S(t)\right)\tilde{H}(t)+\Omega_I\tilde{K}_I(t)+\pi_S\tilde{K}_S(t)+\pi_Y\left(1-z(t)\right)\tilde{K}_Y(t)-g^*\tilde{K}_I(t)$$

$$\frac{\partial\mathcal{H}}{\partial\lambda_H(t)}=\dot{\tilde{H}}\Leftrightarrow\dot{\tilde{H}}=\xi u_H(t)\tilde{H}(t)+\alpha\tilde{K}_S(t)+\varphi\tilde{K}_I(t)-g^*\tilde{H}(t)$$

$$\frac{\partial\mathcal{H}}{\partial\lambda_S(t)}=\dot{\tilde{K}}_S\Leftrightarrow\dot{\tilde{K}}_S=\omega_S u_S(t)\tilde{H}(t)+\Omega_S\tilde{K}_S(t)+\Phi\tilde{K}_I(t)-g^*\tilde{K}_S(t)$$

From (48), after taking logs and deriving with respect to time, we get:

$$\frac{\dot{\lambda_Y}}{\lambda_Y} = -\tau \frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} + \phi\left(1-\tau\right)\frac{\dot{\tilde{K}}_S(t)}{\tilde{K}_S(t)} + \psi\left(1-\tau\right)\frac{\dot{\tilde{K}}_I(t)}{\tilde{K}_I(t)} + (1-\tau)\cdot(1+\phi+\psi)\cdot g^* - \rho. \quad (57)$$

From (53) with (49), we have:

$$\frac{\dot{\lambda_Y}}{\lambda_Y} = g^* - \beta z(t)^{\beta-1}\cdot\tilde{K}_Y(t)^{\beta-1}\cdot\left(u_Y(t)\tilde{H}(t)\right)^{1-\beta} + \delta_P \Leftrightarrow \frac{\dot{\lambda_Y}}{\lambda_Y} = g^* - \frac{\beta\tilde{Y}(t)}{z(t)\tilde{K}_Y(t)} + \delta_P. \quad (58)$$

Then, combining the (57) and (58), we get the expression for the growth rate of consumption:

$$\frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \left(\frac{1}{\tau}\right)\left[\phi\left(1-\tau\right)\frac{\dot{\tilde{K}}_S(t)}{\tilde{K}_S(t)} + \psi\left(1-\tau\right)\frac{\dot{\tilde{K}}_I(t)}{\tilde{K}_I(t)} + g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right) + \frac{\beta\tilde{Y}(t)}{z(t)\tilde{K}_Y(t)} - \delta_P - \rho\right], \quad (59)$$

which can be easily shown to be equivalent to equation (32) in the main text.

Starting with (55) and using the relations in (51), (52), (53) and (49), we get:

$$-\dot{\lambda}_H(t) = \lambda_H\cdot\xi\cdot u_Y(t) + \lambda_H(t)\cdot\xi\cdot(1-u_Y(t)-u_H(t)-u_S(t)) + \lambda_H(t)\cdot(\xi u_H(t)-g^*) + \lambda_H(t)\cdot\xi\cdot u_S(t) \Leftrightarrow$$

$$\Leftrightarrow \frac{\dot{\lambda}_H(t)}{\lambda_H(t)} = g^* - \xi. \quad (60)$$

Considering (54) and using (51) and (52), we have:

$$-\dot{\lambda}_I(t) = \frac{\psi\cdot\left(\tilde{C}(t)\tilde{K}_S^{\phi}(t)\tilde{K}_I^{\psi}(t)\right)^{(1-\tau)}}{\tilde{K}_I}e^{(1-\tau)g^*(1+\phi+\psi)-\rho t} + \lambda_I(t)\cdot(\Omega_I-g^*) + \lambda_I(t)\cdot\frac{\omega_I}{\xi}\cdot\varphi + \lambda_I(t)\cdot\frac{\omega_I}{\omega_S}\cdot\Phi$$

Knowing that $\left(\tilde{C}(t)\tilde{K}_S^{\phi}(t)\tilde{K}_I^{\psi}(t)\right)^{(1-\tau)}e^{[(1-\tau)g^*(1+\phi+\psi)-\rho]t}$ equals $\lambda_Y(t)\cdot\tilde{C}(t)$ from equation (48) and having (53), we have:

$$\frac{\dot{\lambda}_I(t)}{\lambda_I(t)} = g^* - \frac{\psi\omega_I u_Y(t)\tilde{H}(t)\tilde{C}(t)}{(1-\beta)\tilde{Y}(t)\tilde{K}_I(t)} - \frac{\omega_I}{\xi}\varphi - \frac{\omega_I}{\omega_S}\Phi - \Omega_I. \quad (61)$$

Starting with (56) and using the relations in (52) and (51) we get

$$-\dot{\lambda}_S(t) = \frac{\phi\left(\tilde{C}(t)\tilde{K}_S^{\phi}(t)\tilde{K}_I^{\psi}(t)\right)^{(1-\tau)}}{\tilde{K}_S}e^{[(1-\tau)g^*(1+\phi+\psi)-\rho]t} + \lambda_S(t)\cdot\frac{\omega_S}{\omega_I}\cdot\pi_S + \lambda_S\cdot\frac{\omega_S}{\xi}\cdot\alpha + \lambda_S(t)\cdot(\Omega_S-g^*)$$

Knowing that $\left(\tilde{C}(t)\tilde{K}_S^{\phi}(t)\tilde{K}_I^{\psi}(t)\right)^{(1-\tau)} e^{[(1-\tau)g^*(1+\phi+\psi)-\rho]t}$ equals $\lambda_Y(t)\tilde{C}(t)$ from equation (48) and having (53) and (52) we have:

$$\frac{\dot{\lambda}_S(t)}{\lambda_S(t)} = g^* - \frac{\phi\omega_S u_Y(t)\tilde{C}(t)\tilde{H}(t)}{(1-\beta)\,\tilde{Y}(t)\tilde{K}_S(t)} - \frac{\omega_S}{\omega_I}\pi_S - \frac{\omega_S}{\xi}\alpha - \Omega_S. \tag{62}$$

Starting with (50), after taking logs and deriving with respect to time, we obtain:

$$\frac{\dot{z}(t)}{z(t)} = \frac{\dot{\tilde{H}}(t)}{\tilde{H}(t)} - \frac{\dot{\tilde{K}}_Y(t)}{\tilde{K}_Y(t)} + \frac{\dot{u}_Y(t)}{u_Y(t)} + \frac{1}{\beta}\frac{\dot{\lambda}_I(t)}{\lambda_I(t)} - \frac{1}{\beta}\frac{\dot{\lambda}_Y(t)}{\lambda_Y(t)}. \tag{63}$$

Considering (49), also after taking logs and deriving with respect to time, we get:

$$\frac{\dot{u}_Y(t)}{u_Y(t)} = \frac{1}{1-\beta}\left(\frac{\dot{\lambda}_I(t)}{\lambda_I(t)} - \frac{\dot{\lambda}_Y(t)}{\lambda_Y(t)}\right) + \frac{\dot{z}(t)}{z(t)} + \frac{\dot{\tilde{K}}_Y(t)}{\tilde{K}_Y(t)} - \frac{\dot{\tilde{H}}(t)}{\tilde{H}(t)}. \tag{64}$$

Combining equations (63) and (64) we obtain:

$$\frac{1}{\beta\,(1-\beta)}\frac{\dot{\lambda}_Y(t)}{\lambda_Y(t)} = \frac{1}{\beta\,(1-\beta)}\frac{\dot{\lambda}_I(t)}{\lambda_I(t)},$$

which means that:

$$\frac{\dot{\lambda}_Y(t)}{\lambda_Y(t)} = \frac{\dot{\lambda}_I(t)}{\lambda_I(t)}. \tag{65}$$

Substituting this relationship in (63) or in (64) we get:

$$\frac{\dot{z}(t)}{z(t)} - \frac{\dot{u}_Y(t)}{u_Y(t)} = \frac{\dot{\tilde{H}}(t)}{\tilde{H}(t)} - \frac{\dot{\tilde{K}}_Y(t)}{\tilde{K}_Y(t)}, \tag{66}$$

which, by rearranging terms, can be easily shown to be equivalent to equation (33) in the main text. Observe that the dynamical system derived from the first-order conditions laid out above is indeterminate regarding the sectoral share variables, being only possible to derive a dynamical equation for the ratio $\frac{z(t)}{u_Y(t)}$, as shown in equation (66).

B Steady state

In this appendix, we derive the steady state analytically.

Having equations (60) and (51), after taking logs and deriving with respect to time, we get:

$$\frac{\dot{\lambda}_I(t)}{\lambda_I(t)} = \frac{\dot{\lambda}_H(t)}{\lambda_H(t)}. \tag{67}$$

Considering (50), after taking logs and deriving with respect to time, we get:

$$\frac{\dot{\lambda}_Y(t)}{\lambda_Y(t)} = \frac{\dot{\lambda}_I(t)}{\lambda_I(t)} + \frac{\dot{u}_Y(t)}{u_Y(t)} + \frac{\dot{\tilde{H}}(t)}{\tilde{H}(t)} - \frac{\dot{\tilde{Y}}(t)}{\tilde{Y}(t)}.$$

In the steady state, $\frac{\dot{u}_Y(t)}{u_Y(t)} = 0$, but also, since $\tilde{H}(t)$ and $\tilde{Y}(t)$ are constant, too, it follows that $\dot{\tilde{H}}(t) = \dot{\tilde{Y}}(t) = 0$, and, thus:

$$\frac{\dot{\lambda}_Y(t)}{\lambda_Y(t)} = \frac{\dot{\lambda}_I(t)}{\lambda_I(t)}. \tag{68}$$

Bearing (68) in mind and recalling (67), together with equations (60) and (57), we have:

$$\begin{aligned} &\frac{\dot{\lambda}_H(t)}{\lambda_H(t)} = \frac{\dot{\lambda}_Y(t)}{\lambda_Y(t)} \Leftrightarrow \\ &\Leftrightarrow g^* - \xi = -\tau \frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} + \phi\,(1-\tau)\,\frac{\dot{\tilde{K}}_S(t)}{\tilde{K}_S(t)} + \psi\,(1-\tau)\,\frac{\dot{\tilde{K}}_I(t)}{\tilde{K}_I(t)} + (1-\tau)\cdot(1+\phi+\psi)\cdot g^* - \rho. \end{aligned} \tag{69}$$

But, in the steady state, $\frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \frac{\dot{\tilde{K}}_S(t)}{\tilde{K}_S(t)} = \frac{\dot{\tilde{K}}_I(t)}{\tilde{K}_I(t)} = 0$; so we get:

$$g^* = \frac{\xi - \rho}{\tau\,(1+\phi+\psi) - \phi - \psi}, \tag{70}$$

which is equation (34) in the main text.

Then, considering equations (49) and (50), we have:

$$\frac{\left(z(t)\tilde{K}_Y(t)\right)^{\beta} \cdot (1-\beta) \cdot u_Y(t)^{-\beta} \cdot \tilde{H}(t)^{-\beta}}{\omega_I} = \frac{\beta \cdot z(t)^{\beta-1} \cdot \tilde{K}_Y(t)^{\beta-1} \cdot \left(u_Y(t)\tilde{H}(t)\right)^{1-\beta} - \delta_P + \delta_{C_T}}{\pi_Y} \Leftrightarrow$$

$$\Leftrightarrow \frac{\pi_Y}{\omega_I} \cdot (1-\beta) \cdot \left(\frac{z(t)\tilde{K}_Y(t)}{u_Y(t)\tilde{H}(t)}\right) + (\delta_P - \delta_{C_T}) \cdot \left(\frac{z(t)\tilde{K}_Y(t)}{u_Y(t)\tilde{H}(t)}\right)^{1-\beta} = \beta. \tag{71}$$

At this point, we will split our analysis. We will be able to further examine the steady-state analytically under $\delta_P = \delta_{C_T} = \delta$ and under the alternative case when $\delta_P \neq \delta_{C_T}$ and $\beta = 0.5$.

B.1 Steady state under $\delta_P = \delta_{C_T} = \delta$

In this section, we will analyze the steady-state analytically for the case when $\delta_P = \delta_{C_T} = \delta$, , in order to prove Proposition 2, in the main text.

To determine $\frac{z^*}{u_Y^*}$ from (71) and assuming $\delta_P = \delta_{C_T} = \delta$. we get:

$$\frac{z^*}{u_Y^*} = \frac{\omega_I \beta}{(1-\beta)\,\pi_Y} \cdot \frac{\tilde{H}^*}{\tilde{K}_Y^*}. \tag{72}$$

To determine $\tilde{H}^*$, from (24) and having $\dot{\tilde{H}}(t) = 0$, we get:

$$\xi u_H(t)\tilde{H}(t) + \alpha \tilde{K}_S(t) + \varphi \tilde{K}_I(t) - g^* \tilde{H}(t) = 0 \Leftrightarrow$$

$$\Leftrightarrow \tilde{H}^* = \frac{\alpha \tilde{K}_S^* + \varphi \tilde{K}_I^*}{g^* - \xi u_H^*}. \tag{73}$$

To determine $\tilde{K}_S^*$, from (25) and having $\dot{\tilde{K}}_S = 0$, we get:

$$\omega_S u_S(t)\tilde{H}(t) + \Omega_S \tilde{K}_S(t) + \Phi \tilde{K}_I(t) - g^* \tilde{K}_S(t) = 0 \Leftrightarrow$$

$$\Leftrightarrow \tilde{K}_S^* = \frac{\omega_S u_S^* \tilde{H}^* + \Phi \tilde{K}_I^*}{g^* - \Omega_S}. \tag{74}$$

From equation (73) and substituting $\tilde{K}_S^*$ with equation (74), we obtain:

$$\tilde{H}^* = \frac{\alpha \cdot \left(\frac{\omega_S u_S^* \tilde{H}^* + \Phi \tilde{K}_I^*}{g^* - \Omega_S}\right) + \varphi \tilde{K}_I^*}{g^* - \xi u_H^*} \Leftrightarrow$$

$$\Leftrightarrow \tilde{H}^* = \tilde{K}_I^* \mathbf{A}_1, \tag{75}$$

where: $\mathbf{A}_1 \equiv \frac{\alpha\Phi + \varphi\cdot(g^* - \Omega_S)}{\left(g^* - \xi u_H^*\right)\cdot(g^* - \Omega_S) - \alpha\omega_S u_S^*}$. To satisfy $\tilde{H}^* > 0$, we must have $\mathbf{A_1} > 0$. We'll be back to this issue at the end of the section.

From equation (74) and substituting $\tilde{H}^*$ with equation (75), we obtain:

$$\tilde{K}_S^* = \tilde{K}_I^* \cdot \frac{\omega_S u_S^* \mathbf{A}_1 + \Phi}{g^* - \Omega_S}. \tag{76}$$

To detemine $\tilde{K}_I^*$, from (23) and having $\dot{\tilde{K}}_I = 0$, we get:

$$\omega_I \left(1 - u_Y(t) - u_H(t) - u_S(t)\right) \tilde{H}(t) + \Omega_I \tilde{K}_I(t) + \pi_S \tilde{K}_S(t) + \pi_Y \left(1 - z(t)\right) \tilde{K}_Y(t) - g^* \tilde{K}_I(t) = 0 \Leftrightarrow$$

$$\Leftrightarrow \tilde{K}_I^* = \frac{\omega_I \left(1 - u_Y^* - u_H^* - u_S^*\right) \tilde{H}^* + \pi_S \tilde{K}_S^* + \pi_Y \left(1 - z^*\right) \tilde{K}_Y^*}{g^* - \Omega_I}. \tag{77}$$

In the previous equation, if we substitute $\tilde{H}^*$ and $\tilde{K}_S^*$ with (75) and (76), we obtain:

$$\tilde{K}_I^* = \tilde{K}_Y^* \cdot \mathbf{A_3}, \tag{78}$$

where: $\mathbf{A}_2 \equiv \frac{\omega_S u_S^* \mathbf{A_1}+\Phi}{g^*-\Omega_S}$ and $\mathbf{A_3} \equiv \frac{\pi_Y(1-z^*)}{(g^*-\Omega_I)-\omega_I\left(1-u_Y^*-u_H^*-u_S^*\right)\mathbf{A_1}-\pi_S\mathbf{A_2}}$. To satisfy $\tilde{K}_I^* > 0$, we must have $\mathbf{A_2} > 0$, and $\mathbf{A_3} > 0$. We'll be back to this issue at the end of the section.

To determine $\tilde{K}_Y^*$, from (22) and having $\dot{\tilde{K}}_Y(t) = 0$, we get:

$$z(t)^\beta \tilde{K}_Y(t)^\beta \left(u_Y(t)\tilde{H}(t)\right)^{1-\beta} - \tilde{C}(t) - \delta\tilde{K}_Y(t) - g^*\tilde{K}_Y(t) = 0 \Leftrightarrow$$

$$\Leftrightarrow z(t)\cdot\left[\left(\frac{z(t)}{u_Y(t)}\cdot\frac{1}{\tilde{H}(t)}\right)^{\beta-1}\cdot\tilde{K}_Y(t)^\beta\right] - \delta\tilde{K}_Y(t) - g^*\tilde{K}_Y(t) = \tilde{C}(t).$$

Substituting $\frac{z(t)}{u_Y(t)}$ with equation (72), we then have:

$$\tilde{K}_Y^* = \frac{\tilde{C}^*}{\mathbf{B}} \Leftrightarrow \frac{\tilde{K}_Y^*}{\tilde{C}^*} = \frac{1}{\mathbf{B}}, \tag{79}$$

which is equation (35) in the main text, where: $\mathbf{B} \equiv z^*\cdot\left(\frac{\omega_I\beta}{(1-\beta)\pi_Y}\right)^{\beta-1} - \delta - g^*$. To satisfy $\tilde{K}_Y^* > 0$ for a given $\tilde{C}^* > 0$, we must have $\mathbf{B} > 0$. We'll be back to this issue at the end of the section.

If we replace (74) and (79) in (77), we obtain :

$$\tilde{K}_I^* = \frac{\tilde{C}^*}{\mathbf{B}}\mathbf{A_3} \Leftrightarrow \frac{\tilde{K}_I^*}{\tilde{C}^*} = \frac{\mathbf{A_3}}{\mathbf{B}}, \tag{80}$$

which is equation (36) in the text.

Now, substituting (80) in equations (73) and (74), we have:

$$\tilde{H}^* = \frac{\tilde{C}^*}{\mathbf{B}}\cdot\mathbf{A_1}\cdot\mathbf{A_3} \Leftrightarrow \frac{\tilde{H}^*}{\tilde{C}^*} = \frac{\mathbf{A_1}\cdot\mathbf{A_3}}{\mathbf{B}}, \tag{81}$$

which is equation (37) in the main text, and:

$$\tilde{K}_S^* = \frac{\tilde{C}^*}{\mathbf{B}}\cdot\mathbf{A_2}\cdot\mathbf{A_3} \Leftrightarrow \frac{\tilde{K}_S^*}{\tilde{C}^*} = \frac{\mathbf{A_2}\cdot\mathbf{A_3}}{\mathbf{B}}, \tag{82}$$

which is equation (38) in the main text.

Having equations for $\tilde{K}_Y^*$ (79) and $\tilde{H}^*$ (81) we can determine $\frac{z^*}{u_Y^*}$ from (72):

$$\frac{z^*}{u_Y^*} = \mathbf{A_1}\cdot\mathbf{A_3}\cdot\frac{\omega_I\beta}{(1-\beta)\,\pi_Y}, \tag{83}$$

which is equation (39) in the main text.

To find $\tilde{C}^*$, recall (59) with $\dot{\tilde{C}}(t) = 0$, to get:

$$\left(\frac{\tilde{C}(t)}{\tau}\right)\cdot\left[\phi\,(1-\tau)\,\frac{\dot{\tilde{K}}_S(t)}{\tilde{K}_S(t)} + \psi\,(1-\tau)\,\frac{\dot{\tilde{K}}_I(t)}{\tilde{K}_I(t)} + g^*\cdot(\phi+\psi-\tau\,(1+\phi+\psi)) + \frac{\beta\tilde{Y}(t)}{z(t)\tilde{K}_Y(t)} - \delta - \rho\right] = 0 \Leftrightarrow$$

$$\Leftrightarrow \phi\left(1-\tau\right)\frac{\dot{\tilde{K}}_S(t)}{\tilde{K}_S(t)}+\psi\left(1-\tau\right)\frac{\dot{\tilde{K}}_I(t)}{\tilde{K}_I(t)}+g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)+\frac{\beta\tilde{Y}(t)}{z(t)\tilde{K}_Y(t)}-\delta-\rho=0. \quad (84)$$

Since, in the steady-state, $\dot{\tilde{K}}_S(t)=\dot{\tilde{K}}_I(t)=0$, we get:

$$g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)+\frac{\beta\tilde{Y}}{z\tilde{K}_Y}-\delta-\rho=0.$$

Now, substituting $\tilde{Y}(t)$ from equation (22), with $\dot{\tilde{K}}_Y(t)=0$, we obtain:

$$g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)+\frac{\beta\left(\tilde{C}(t)+\delta z(t)\tilde{K}_Y(t)+\delta\left(1-z(t)\right)\tilde{K}_Y(t)+g^*\tilde{K}_Y(t)\right)}{z(t)\tilde{K}_Y(t)}-\delta-\rho=0\Leftrightarrow$$

$$\Leftrightarrow \tilde{C}^*=\tilde{K}_Y^*\cdot\left\{\frac{z^*}{\beta}\cdot\left[\delta+\rho-g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)\right]-\delta-g^*\right\}. \quad (85)$$

Substituting $\tilde{K}_Y^*$ with equation (79), we get:

$$\tilde{C}^*=\frac{\tilde{C}^*}{z^*\left[\left(\frac{\omega_I\beta}{(1-\beta)\pi_Y}\right)^{\beta-1}\right]-\delta-g^*}\cdot\left\{\frac{z^*}{\beta}\cdot\left[\delta+\rho-g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)\right]-\delta-g^*\right\}\Leftrightarrow$$

$$\Leftrightarrow \tilde{C}^*\cdot\left\{\frac{z^*\cdot\left[\left(\frac{\omega_I\beta}{(1-\beta)\pi_Y}\right)^{\beta-1}\right]-\delta-g^*}{\frac{z^*}{\beta}\cdot\left[\delta+\rho-g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)\right]-\delta-g^*}\right\}=\tilde{C}^*. \quad (86)$$

Continuing the derivation of equation (86), we have:

$$z^*\cdot\left(\frac{\omega_I\beta}{\left(1-\beta\right)\pi_Y}\right)^{\beta-1}=\frac{z^*}{\beta}\cdot\left[\delta+\rho-g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)\right]$$

Now, substituting g^* with (70) we get, for $z^*\neq 0$::

$$z^*\cdot\left(\frac{\omega_I\beta}{\left(1-\beta\right)\pi_Y}\right)^{\beta-1}=\frac{z^*}{\beta}\cdot\left[\delta+\rho-\frac{\xi-\rho}{\tau\left(1+\phi+\psi\right)-\phi-\psi}\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)\right]\Leftrightarrow$$

$$\Leftrightarrow\left(\frac{\omega_I\beta}{\left(1-\beta\right)\pi_Y}\right)^{\beta-1}=\frac{\delta+\xi}{\beta} \quad (87)$$

Given 87, we can rewrite 83, so that:

$$\frac{z^*}{u_Y^*}=\mathbf{A_1}\cdot\mathbf{A_3}\cdot\left(\frac{\delta+\xi}{\beta}\right)^{\frac{1}{\beta-1}}. \quad (88)$$

But also, we can rewrite $\mathbf{B}$ given 87 as:

$$\mathbf{B} \equiv z^* \cdot \left(\frac{\delta+\xi}{\beta}\right) - \delta - g^*.$$

Because we must have $\mathbf{B} > 0$, we obtain:

$$z^* \cdot \left[\left(\frac{\omega_I \beta}{(1-\beta)\,\pi_Y}\right)^{\beta-1}\right] > \delta + g^* \Leftrightarrow z^* > \frac{\delta + g^*}{\left(\frac{\omega_I \beta}{(1-\beta)\pi_Y}\right)^{\beta-1}}. \tag{89}$$

Now, substituting (87) in (89), we get:

$$z^* > \frac{\beta\,(\delta + g^*)}{\delta + \xi}. \tag{90}$$

Because we must satisfy $\mathbf{A_1} > 0$, we have that:

$$(g^* - \xi u_H^*) \cdot (g^* - \Omega_S) - \alpha \omega_S u_S^* > 0 \Leftrightarrow$$

$$\Leftrightarrow u_H > \frac{g^*\,(g^* - \Omega_S) - \alpha\omega_S u_S^*}{\xi\,(g^* - \Omega_S)} \Leftrightarrow u_S < \frac{(g^* - \xi u_H^*) \cdot (g^* - \Omega_S)}{\alpha \omega_S}. \tag{91}$$

Because we must satisfy $\mathbf{A_2} > 0$, we have that:

$$g^* - \Omega_S > 0 \Leftrightarrow g^* > \Omega_S. \tag{92}$$

Because we must satisfy $\mathbf{A_3} > 0$, we have that:

$$(g^* - \Omega_I) - \omega_I\,(1 - u_Y^* - u_H^* - u_S^*)\,\mathbf{A_1} - \pi_S \mathbf{A_2} > 0 \Leftrightarrow$$

$$\Leftrightarrow 1 - u_Y^* - u_H^* - u_S^* < \frac{(g^* - \Omega_I) - \pi_S \mathbf{A_2}}{\omega_I \mathbf{A_1}}. \tag{93}$$

Finally, in equation 90, for the right-hand side to be smaller than 1, we must ensure that $g^* < \frac{\delta+\xi}{\beta} - \delta$. In 91, for the right-hand side to be positive, given $g^* > \Omega_S$, we must have $g^* > \xi u_H^*$. In turn, to have the right-hand side of 93 larger than 0, we must impose that $g^* > \pi_S \mathbf{A_2} + \mathbf{\Omega}_\mathrm{I}$.

B.2 Steady-state under $\delta_P \neq \delta_{C_T}$ with $\beta = 0.5$

As explained in Section 3.3, we will further examine the steady-state analytically in the alternative case under $\delta_P \neq \delta_{C_T}$ and $\beta = 0.5$. This case can be perceived as more general than the case under $\delta_P = \delta_{C_T}$, since $\beta = 0.5$ is fairly similar to the value we use in the calibration based on empirical data (see Table 1).

Considering $\beta = 0.5$ in equation (71), we have:

$$\frac{\pi_Y}{2\omega_I}\cdot\left(\frac{z(t)\tilde{K}_Y(t)}{u_Y(t)\tilde{H}(t)}\right)+(\delta_P-\delta_{C_T})\cdot\left(\frac{z(t)\tilde{K}_Y(t)}{u_Y(t)\tilde{H}(t)}\right)^{0.5}=\frac{1}{2}.$$

If we make the following variable substitution:

$$x\equiv\left(\frac{z}{u_Y}\right)^{0.5}\Leftrightarrow x^2\equiv\frac{z}{u_Y}, \tag{94}$$

we obtain:

$$\frac{\pi_Y}{2\omega_I}\cdot\frac{\tilde{K}_Y(t)}{\tilde{H}(t)}x^2+\left(\frac{\tilde{K}_Y(t)}{\tilde{H}(t)}\right)^{0.5}(\delta_P-\delta_{C_T})\,x-\frac{1}{2}=0$$

Utilizing the quadratic formula, we get the solution:

$$x=\frac{-\left(\frac{\tilde{K}_Y(t)}{\tilde{H}(t)}\right)^{0.5}\cdot(\delta_P-\delta_{C_T})\pm\sqrt{\left[\left(\frac{\tilde{K}_Y(t)}{\tilde{H}(t)}\right)^{0.5}\cdot(\delta_P-\delta_{C_T})\right]^2+\left(\frac{\pi_Y}{\omega_I}\frac{\tilde{K}_Y(t)}{\tilde{H}(t)}\right)}}{\frac{\pi_Y}{\omega_I}\frac{\tilde{K}_Y(t)}{\tilde{H}(t)}}.$$

Given that $x^2=\frac{z}{u_Y}$, we then have:

$$\frac{z^*}{u_Y^*}\equiv x^2=\left[\frac{-\left(\frac{\tilde{K}_Y^*}{\tilde{H}^*}\right)^{0.5}\cdot(\delta_P-\delta_{C_T})\pm\sqrt{\left[\left(\frac{\tilde{K}_Y^*}{\tilde{H}^*}\right)^{0.5}\cdot(\delta_P-\delta_{C_T})\right]^2+\left(\frac{\pi_Y}{\omega_I}\frac{\tilde{K}_Y^*}{\tilde{H}^*}\right)}}{\frac{\pi_Y}{\omega_I}\cdot\frac{\tilde{K}_Y^*}{\tilde{H}^*}}\right]^2\Leftrightarrow$$

$$\Leftrightarrow\frac{z^*}{u_Y^*}=\left(\frac{\tilde{H}^*}{\tilde{K}_Y^*}\right)\left[\frac{-\omega_I\cdot(\delta_P-\delta_{C_T})\pm\sqrt{\omega_I^2\cdot(\delta_P-\delta_{C_T})^2+\left(\frac{\pi_Y}{\omega_I}\right)\omega_I^2}}{\pi_Y}\right]^2. \tag{95}$$

This result indicates that there are two solutions for $\frac{z^*}{u_Y^*}$, suggesting the possibility of two steady-state equilibria.

To determine $\tilde{K}_Y^*$, from (22) and considering $\dot{\tilde{K}}_Y=0$, we get:

$$z(t)^{0.5}\tilde{K}_Y(t)^{0.5}\left(u_Y(t)\tilde{H}(t)\right)^{0.5}-\tilde{C}(t)-\delta_P z(t)\tilde{K}_Y(t)-\delta_{C_T}\left(1-z(t)\right)\tilde{K}_Y(t)-g^*\tilde{K}_Y(t)=0\Leftrightarrow$$

$$\Leftrightarrow z(t)\cdot\left[\left(\frac{z(t)}{u_Y(t)}\cdot\frac{1}{\tilde{H}(t)}\right)^{-0.5}\tilde{K}_Y(t)^{0.5}-(\delta_P-\delta_{C_T})\,\tilde{K}_Y(t)\right]-(\delta_{C_T}+g^*)\,\tilde{K}_Y(t)=\tilde{C}(t).$$

Substituting $\frac{z(t)}{u_Y(t)}$ with equation (95), we have:

$$\tilde{K}_Y^* = \frac{\tilde{C}^*}{\mathbf{B}'} \Leftrightarrow \frac{\tilde{K}_Y^*}{\tilde{C}^*} = \frac{1}{\mathbf{B}'}, \tag{96}$$

where: $\mathbf{B}' = z^* \left[\frac{1}{\frac{-\omega_I \cdot \left(\delta_P - \delta_{C_T}\right) \pm \sqrt{\omega_I^2 \cdot \left(\delta_P - \delta_{C_T}\right)^2 + \pi_Y \omega_I}}{\pi_Y}} - (\delta_P - \delta_{C_T}) \right] - \delta_{C_T} - g^*$ which takes two values. To satisfy $\tilde{K}_Y^* > 0$ for a given $\tilde{C}^* > 0$, we must have $\mathbf{B}' > 0$. We'll be back to this issue at the end of the section.

To determine $\tilde{H}^*$, we consider (24), with $\dot{\tilde{H}}(t) = 0$, and (74), to get:

$$\tilde{H}^* = \tilde{K}_I^* \mathbf{A_1}, \tag{97}$$

where, as in case A, $\mathbf{A}_1 \equiv \frac{\alpha\Phi + \varphi \cdot (g^* - \Omega_S)}{\left(g^* - \xi u_H^*\right) \cdot (g^* - \Omega_S) - \alpha \omega_S u_S^*}$.

In the previous equation, substituting $\tilde{K}_I^*$ with (102), we get:

$$\frac{\tilde{C}^*}{\mathbf{B}'} \cdot \mathbf{A_1} \cdot \mathbf{A_3} \Leftrightarrow \frac{\tilde{H}^*}{\tilde{C}^*} = \frac{\mathbf{A_1} \cdot \mathbf{A_3}}{\mathbf{B}'}, \tag{98}$$

where, as in case A, $\mathbf{A}_2 \equiv \frac{\omega_S u_S^* \mathbf{A_1} + \Phi}{g^* - \Omega_S}$ and $\mathbf{A_3} \equiv \frac{\pi_Y (1 - z^*)}{(g^* - \Omega_I) - \omega_I \left(1 - u_Y^* - u_H^* - u_S^*\right) \mathbf{A_1} - \pi_S \mathbf{A_2}}$.

To determine $\tilde{K}_S^*$, from (25), with $\dot{\tilde{K}}_S = 0$, and (75), we get:

$$\tilde{K}_S^* = \tilde{K}_I^* \cdot \mathbf{A_2}. \tag{99}$$

In the previous equation, substituting $\tilde{K}_I^*$ with (102), we then get:

$$\tilde{K}_S^* = \frac{\tilde{C}^*}{\mathbf{B}'} \cdot \mathbf{A_2} \cdot \mathbf{A_3} \Leftrightarrow \frac{\tilde{K}_S^*}{\tilde{C}^*} = \frac{\mathbf{A_2} \cdot \mathbf{A_3}}{\mathbf{B}'}. \tag{100}$$

To determine $\tilde{K}_I^*$, from (23) with $\dot{\tilde{K}}_I = 0$, and (75) and (76), we obtain:

$$\tilde{K}_I^* = \tilde{K}_Y^* \cdot \mathbf{A_3}. \tag{101}$$

In the previous equation, substituting $\tilde{K}_Y^*$ with (96), we get:

$$\tilde{K}_I^* = \frac{\tilde{C}^*}{\mathbf{B}'} \cdot \mathbf{A_3} \Leftrightarrow \frac{\tilde{K}_I^*}{\tilde{C}^*} = \frac{\mathbf{A_3}}{\mathbf{B}'}. \tag{102}$$

To determine $\frac{z^*}{u_Y^*}$, we recall (95), combined with (98) and (96), to get:

$$\frac{z^*}{u_Y^*} = \mathbf{A_1} \cdot \mathbf{A_3} \left[\frac{-\omega_I \cdot (\delta_P - \delta_{C_T}) \pm \sqrt{\omega_I^2 \cdot (\delta_P - \delta_{C_T})^2 + \pi_Y \omega_I}}{\pi_Y} \right]^2. \tag{103}$$

To compute $\tilde{C}^*$, we use (59) and $\dot{\tilde{C}}(t) = 0$, to get:

$$\left(\frac{\tilde{C}(t)}{\tau}\right)\cdot\left[\phi\left(1-\tau\right)\frac{\dot{\tilde{K}}_S(t)}{\tilde{K}_S(t)}+\psi\left(1-\tau\right)\frac{\dot{\tilde{K}}_I(t)}{\tilde{K}_I(t)}+g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)+\frac{\beta\tilde{Y}(t)}{z(t)\tilde{K}_Y(t)}-\delta_P-\rho\right]=0$$

But, since in the steady state, $\dot{\tilde{K}}_S(t)=0$ and $\dot{\tilde{K}}_I(t)=0$, we get the following expression:

$$g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)+\frac{\beta\tilde{Y}^*}{z^*\tilde{K}_Y^*}-\delta_P-\rho=0$$

Now, substituting $\tilde{Y}(t)$ from equation (22), with $\dot{\tilde{K}}_Y(t)=0$, we obtain:

$$\tilde{C}^*=\tilde{K}_Y^*\cdot\left\{\frac{z^*}{\beta}\cdot\left[\delta_P+\rho-g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)\right]-\delta_P z^*-\delta_{C_T}\left(1-z^*\right)-g^*\right\}.$$

In the previous equation, substituting $\tilde{K}_Y^*$ with (96), we get, for $z^*\neq 0$:

$$\tilde{C}^*\frac{\mathbf{B}'}{\frac{z}{\beta}\cdot\left[\delta_P+\rho-g^*\cdot\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)\right]-\delta_P z-\delta_{C_T}\left(1-z\right)-g^*}=\tilde{C}^*\Leftrightarrow \tag{104}$$

$$\Leftrightarrow z^*\left[\left(\frac{\delta_P+\xi}{\beta}\right)-\frac{1}{\frac{-\omega_I\cdot\left(\delta_P-\delta_{C_T}\right)\pm\sqrt{\omega_I^2\cdot\left(\delta_P-\delta_{C_T}\right)^2+\pi_Y\omega_I}}{\pi_Y}}\right]=0\Leftrightarrow$$

$$\Leftrightarrow\frac{1}{\frac{-\omega_I\cdot\left(\delta_P-\delta_{C_T}\right)\pm\sqrt{\omega_I^2\cdot\left(\delta_P-\delta_{C_T}\right)^2+\pi_Y\omega_I}}{\pi_Y}}=\frac{\delta_P+\xi}{\beta}. \tag{105}$$

Given 105, we can then rewrite 103 as:

$$\frac{z^*}{u_Y^*}=\mathbf{A_1}\cdot\mathbf{A_3}\left[\frac{1}{\frac{\delta_P+\xi}{\beta}}\right]^2. \tag{106}$$

And also, given 105, we can rewrite $\mathbf{B}'$as:

$$\mathbf{B}'=z\left[\frac{\delta_P+\xi}{\beta}-\left(\delta_P-\delta_{C_T}\right)\right]-\delta_{C_T}-g^*.$$

Taking stock of the results above, we write the following proposition.

Proposition 2.B. Let $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$, $\mathbf{B}'>0$. In the steady-state, the economy features the following ratios of state and control variables:

$$\frac{\tilde{K}_Y^*}{\tilde{C}^*}=\frac{1}{\mathbf{B}'}, \tag{107}$$

$$\frac{\tilde{K}_I^*}{\tilde{C}^*} = \frac{\mathbf{A_3}}{\mathbf{B}'}, \tag{108}$$

$$\frac{\tilde{H}^*}{\tilde{C}^*} = \frac{\mathbf{A_1} \cdot \mathbf{A_3}}{\mathbf{B}'}, \tag{109}$$

$$\frac{\tilde{K}_S^*}{\tilde{C}^*} = \frac{\mathbf{A_2} \cdot \mathbf{A_3}}{\mathbf{B}'}, \tag{110}$$

$$\frac{z^*}{u_Y^*} = \mathbf{A_1} \cdot \mathbf{A_3} \left[\frac{1}{\frac{\delta_P + \xi}{\beta}} \right]^2, \tag{111}$$

where: $\mathbf{B}^{'} = z^* \left[\frac{1}{\frac{-\omega_I \cdot \left(\delta_P - \delta_{C_T}\right) \pm \sqrt{\omega_I^2 \cdot \left(\delta_P - \delta_{C_T}\right)^2 + \pi_Y \omega_I}}{\pi_Y}} - \left(\delta_P - \delta_{C_T}\right) \right] - \delta_{C_T} - g^*$, and $\frac{1}{\frac{-\omega_I \cdot \left(\delta_P - \delta_{C_T}\right) \pm \sqrt{\omega_I^2 \cdot \left(\delta_P - \delta_{C_T}\right)^2 + \pi_Y \omega_I}}{\pi_Y}} = \frac{\delta_P + \xi}{\beta}$.

Again, all state variables are determined given $\tilde{C}^*$ and also the level of the sectoral share variables and the structural parameters, as shown in (107)-(110). In parallel, e.g., for a given $u_Y = u_Y^*$, equation (111) implicitly determines the level of $z = z^*$. And note again, from the Caballé and Santos (1993) transformation (equations (15)-(19)), that $\frac{\tilde{K}_Y^*}{\tilde{C}^*} = \frac{K_Y^*}{C^*}$, $\frac{\tilde{K}_I^*}{\tilde{C}^*} = \frac{K_I^*}{C^*}$, $\frac{\tilde{H}^*}{\tilde{C}^*} = \frac{H^*}{C^*}$, and $\frac{\tilde{K}_S^*}{\tilde{C}^*} = \frac{K_S^*}{C^*}$.

We also notice that in light of equation 107 and the parametric constrains in Proposition 2.B, $\frac{z^*}{u_Y^*}$ only admits one value and thus the steady-state of the model is unique.

Because we must satisfy $\mathbf{B}^{'} > 0$, we have that:

$$z^* \left[\frac{1}{\frac{-\omega_I \cdot \left(\delta_P - \delta_{C_T}\right) \pm \sqrt{\omega_I^2 \cdot \left(\delta_P - \delta_{C_T}\right)^2 + \pi_Y \omega_I}}{\pi_Y}} - \left(\delta_P - \delta_{C_T}\right) \right] - \delta_{C_T} - g^* > 0 \Leftrightarrow$$

$$\Leftrightarrow z^* > \frac{\delta_{C_T} + g^*}{\frac{1}{\frac{-\omega_I \cdot \left(\delta_P - \delta_{C_T}\right) \pm \sqrt{\omega_I^2 \cdot \left(\delta_P - \delta_{C_T}\right)^2 + \pi_Y \omega_I}}{\pi_Y}} - \left(\delta_P - \delta_{C_T}\right)} \tag{112}$$

Substituting (105) in (112), we get:

$$z^* > \frac{\beta \left(\delta_{C_T} + g^*\right)}{\delta_P + \xi - \beta \left(\delta_P - \delta_{C_T}\right)} \tag{113}$$

Since we must satisfy $\mathbf{A_1} > 0$, we have that:

$$\left(g^* - \xi u_H^*\right) \cdot \left(g^* - \Omega_S\right) - \alpha \omega_S u_S^* > 0 \Leftrightarrow$$

$$\Leftrightarrow u_H > \frac{g^* \left(g^* - \Omega_S\right) - \alpha \omega_S u_S^*}{\xi \left(g^* - \Omega_S\right)} \Leftrightarrow u_S^* < \frac{\left(g^* - \xi u_H^*\right) \cdot \left(g^* - \Omega_S\right)}{\alpha \omega_S} \tag{114}$$

Because we must have $\mathbf{A_2} > 0$, we have that:

$$g^* - \Omega_S > 0 \Leftrightarrow g^* > \Omega_S \tag{115}$$

Because we must have $\mathbf{A_3} > 0$, we have that:

$$\left(g^* - \Omega_I\right) - \omega_I \left(1 - u_Y^* - u_H^* - u_S^*\right) \mathbf{A_1} - \pi_S \mathbf{A_2} > 0$$

$$1 - u_Y^* - u_H^* - u_S^* < \frac{\left(g^* - \Omega_I\right) - \pi_S \mathbf{A_2}}{\omega_I \mathbf{A_1}} \tag{116}$$

In equation 113, for the right-hand side to be smaller than 1, we must ensure that $g^* < \frac{\delta_P + \xi + \beta\left(\delta_{C_T} - \delta_P\right)}{\beta} - \delta_{C_T}$. In 114, for the right-hand side to be positive, given that we have $g^* > \Omega_S$, we must have $g^* > \xi u_H^*$. In turn, to have the right-hand side of 116 larger than 0, we must impose that $g^* > \pi_S \mathbf{A_2} + \mathbf{\Omega}_\mathrm{I}$.

The following corollary gathers the results above.

Corollary 1.B. There is an economically admissible steady-state if the conditions $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$, $\mathbf{B}' > 0$ are satisfied. This implies that the admissible values for g^*, z, u_H, u_S, and u_Y are:

$$\max\left(\Omega_S, \pi_S \mathbf{A_2} + \mathbf{\Omega}_\mathrm{I}\right) < g^* < \frac{\delta_P + \xi - \beta\left(\delta_P - \delta_{C_T}\right)}{\beta} - \delta_{C_T} \tag{117}$$

$$z > \frac{\beta\left(\delta_{C_T} + g^*\right)}{\delta_P + \xi - \beta\left(\delta_P - \delta_{C_T}\right)} \tag{118}$$

$$u_H^* < \frac{g^*}{\xi} \tag{119}$$

$$u_S^* < \frac{\left(g^* - \xi u_H^*\right) \cdot \left(g^* - \Omega_S\right)}{\alpha \omega_S} \tag{120}$$

$$1 - u_Y^* - u_H^* - u_S^* < \frac{\left(g^* - \Omega_I\right) - \pi_S \mathbf{A_2}}{\omega_I \mathbf{A_1}} \tag{121}$$

In constraint 117, we have the admissible range of values for g^*. In 118, we have the minimum admissible value for z^*, and in 119, 120, and 121, we establish the maximum admissible values for u_H^*, u_S^*, and $1 - u_Y^* - u_H^* - u_S^*$ (the share of human capital devoted to intangible cultural capital accumulation), respectively.

C Data on time usage

This appendix serves as a complement to Section 4.1, as it provides details regarding the calibration of the model using data on time usage from the Harmonized European Time Use Survey, provided by the Eurostat.

For the calculation of u_Y^*, u_H^*, and u_S^* , we used the available information for 10 countries (Belgium, Germany, Greece, Spain, France, Italy, Luxembourg, Netherlands, Austria, and Finland) for 2010.[23] We mapped out the different daily activities of the respondents across the various relevant categories of activities, so that the value of each variable represents the share of the corresponding category of activities in the set of relevant activities in an individual's day. In Table 6, columns I, II, III, and IV list the categories considered in the calculations for the share of time spent on the production, the share of time spent on education (or the accumulation of human capital), the share of time spent on the accumulation of social capital, and the share of time spent on culture (or the accumulation of intangible cultural capital). Activities from the HETUS questionnaire related to personal care and consumption were not considered in these calculations. The value of u_Y^* was adjusted by a percentage corresponding to the proportion of the population employed in 'Education' and in 'Arts, Entertainment, and Culture', based on data from the Eurostat.[24] This portion was then added to the share of time allocated to the accumulation of human capital and of intangible cultural capital, respectively. This methodology was followed for all countries, so that the final value of each variable corresponds to a population-weighted average based on data from the World Bank.[25] This resulted in $u_Y^* = 0.61$, $u_H^* = 0.12$, and $u_S^* = 0.21$.

For the calculation of ψ and ϕ, in addition to the previously used categories, we included categories related to the time spent on consumption and leisure (see Table 6 column V) to calculate the share of time devoted to consumption.[26] In Table 6, we have the population-weighted average time spent on production, education, accumulation of social capital, culture, and consumption. The value for ψ was calculated using the share of time spent on culture, normalized by the share of time spent on consumption. The value for ϕ was calculated using the share of time spent on accumulating social capital, normalized by the share of time spent on consumption. This resulted in $\psi = 0.11$ and $\phi = 0.35$.

[23] HETUS (2010) is available online at:
https://ec.europa.eu/eurostat/databrowser/view/tus_00age/default/table?lang=en&category=livcon.tus.

[24] Employment data is available online at:
https://ec.europa.eu/eurostat/databrowser/view/htec_emp_nat2___custom_11273684/default/table?lang=en.

[25] Population data is available online at:
https://databank.worldbank.org/reports.aspx?source=2&series=SP.POP.TOTL&country=BEL#.

[26] The survey did not distinguish between consumption and leisure activities. Since separating the two was challenging, we include leisure time within consumption time.

	I	II	III	IV	V
Variable	**Share of time spent on prod.**	**Share of time spent on education**	**Share of time spent on accum. social capital**	**Share of time spent on culture**	**Share of time spent on consumpt. and leisure**
Categories	Activities related to employment and unspecified employment	School and university except homework	Organisational work	Entertainment and culture	Shopping and services
	Food management except dish washing	Homework	Informal help to other households	Reading books	Travel related to shopping and services
	Dish washing	Free time study	Participatory activities	Radio and music	Computing
	Household upkeep except cleaning dwelling	Teaching, reading and talking with child	Visiting and feasts	Unspecified travel	Hobbies and games except computing and computer games
	Cleaning dwelling	Travel related to study	Other social life		Computer games
	Handicraft and producing textiles and other care for textiles		Travel related to leisure, social and associative life		Reading, except books
	Laundry				Unspecified leisure
	Ironing				Tending domestic animals
	Construction and repairs				Caring for pets
	Household management and help family member				Walking the dog
	Main and second job and related travel				Gardening
	Travel to/from work				Walking and hiking
	Travel related to other household purposes				Travel related to shopping and services
					TV and video
					Unspecified leisure
					Sports and outdoor activities except walking and hiking
Time (hh:mm)	04:55	00:34	01:33	00:24	04:29

Table 6: Categories from the harmonised European time use surveys (HETUS, 2010) considered in the calibration of u_Y^*, u_H^*, u_S^*, ϕ, and ψ. Note: 'prod.' refers to production,' 'accum' refers to accumulation, 'consumpt' refers to consumption, and 'norm' refers to normalized. See text for details.

D Welfare analysis

D.1 The case of a 20% subsidy-equivalent shock

This appendix serves as a complement to Section 4.3. The main text presents the results of the numerical study for a 10% subsidy-equivalen- shock to the key parameters of the model. In this section, the results for a 20% subsidy-equivalent shock are provided, as shown in Table 7.

I		II		IV		V	
	Elasticity		**Elasticity**		**Elasticity**		**Elasticity**
$\xi_{(4\%)}$	5.88(+)	ξ	0.86(+)	ξ	1.492(+)	ξ	1.07(+)
α	0.36	φ	0.09	ϕ	0.14	ϕ	0.16
ω_S	0.24	Φ	0.03(+)	φ	0.07	α	0.1
ϕ	0.14	ψ	0.0284	α	0.057	φ	0.06
Φ	0.07	α	0.0280	ψ	0.056	ψ	0.03
π_S	0.06	π_S	0.017	π_S	0.03	π_S	0.04
φ	0.04	ω_S	0.014(+)	ω_I, π_Y	0.01	Φ	0.03
ψ	0.02	ω_I, π_Y	0.01	Φ	0.003(+)	ω_S	0.02
ω_I, π_Y	0.01	ϕ	0.004(+)	ω_S	0.001(+)	ω_I, π_Y	0.01

Table 7: Elasticity of the steady-state intertemporal utility, U^*, with respect to each parameter of interest, considering as a rule a 20% variation in the latter, under the calibration scenarios I, II, IV and V in Tables (1) and (2). . The parameters are ordered from the highest to the lowest modulus of the utility elasticity. A positive sign (+) indicates that the parameter has a positive impact on U^*; otherwise, the impact is negative. In order to satisfy constraint (40), we consider: (i) a proportional change in parameters ω_I and π_Y ; (ii) a variation in ξ accompanied by a variation in δ of the same magnitude but with opposite sign. The elasticity of ξ in scenario I was calculated assuming a 4% subsidy for computational reasons and, thus, is not strictly comparable with the other values.

Just like in the 10%-subsidy case, ξ is the only parameter with a positive impact on utility, regardless of the calibration scenario. Depending on the calibration scenario, Φ, ω_S, and ϕ may also contribute positively to utility. Φ and ω_S have a positive impact in scenarios II and IV, while ϕ does so in scenario II. All other parameters have a negative effect on utility.

Comparing the two cases, the average elasticity of each parameter is the same for both 10% and 20%, except for ξ, which has an average elasticity of 0.84 at 10% and 1.14 at 20% across scenarios II, IV, and V. It is worth noting there are similarities in the ranking of the parameters in both cases, namely:

- For all scenarios, ξ is the parameter with the highest elasticity;
- The parameters Φ and ω_S exhibit their highest (positive) elasticity in scenario II. The average elasticity of Φ is 0.017, and that of ω_S is 0.008 in scenarios II and IV.

D.2 Comparing quantitatively the 2 cases - 10% and 20% subsidy-equivalent shock

Given that our calibration encompasses four scenarios allowing for distinct numerical results, and considering the non-constant elasticity of welfare with respect to all parameters (because of the linear relationships in accumulation of the different types of capital), we wish to analyze the relationship between each parameter value and the elasticity obtained for the cases of a 10% and a 20% subsidy-equivalent shock. Figure (3) presents the relationship between the calibration values of parameters α, π_S, Φ, and φ, and the elasticity values under a 10% and a 20% subsidy. In the case of α and π_S, there is a positive relationship between the calibrated parameter value and the elasticity value. In the case of the parameters Φ and φ, no clear relationship is observed. The range of elasticities increases under the 20% case for all parameters.

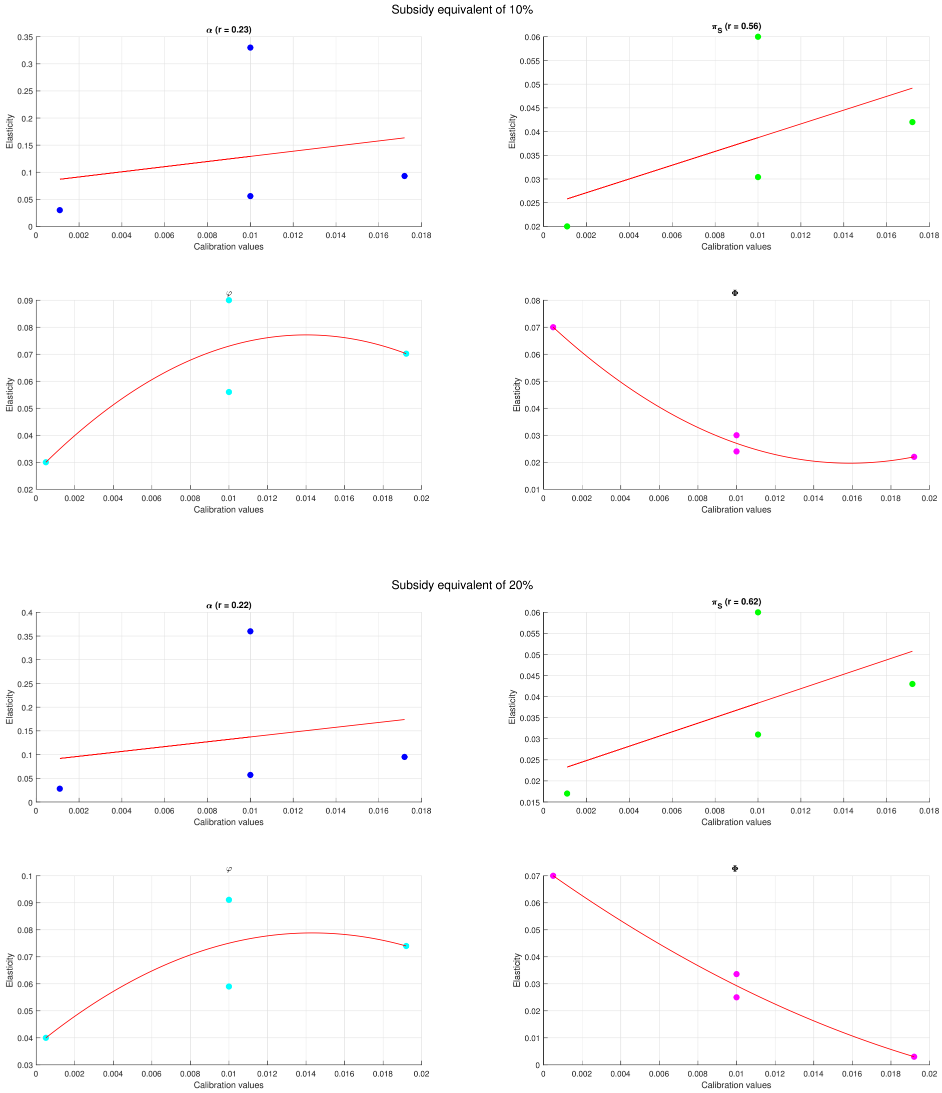

Figure 3: Calibration scenarios (I, II, IV and V) and their influence on the parameters' elasticities $(\alpha, \pi_S, \varphi, \Phi)$ in the context of a 10% and 20% subsidy-equivalent shock. For more details regarding the calibration, see Table (2) in the main text.

E Local dynamics and transitional analysis

This appendix serves as a complement to Sections 3.3 and 4.4 by characterizing the local dynamics of the model under two alternative cases – households determine $\frac{z}{u_Y}$ by arbitrarily setting z ('case A') or by arbitrarily settingu_Y ('case B') – and by providing further results for the transitional dynamics under the latter case. We show that the way $\frac{z}{u_Y}$ is determined impacts the local dynamics properties of the model.

E.1 Local-dynamics properties

E.1.1 Local-dynamics properties - Case A

This section of the appendix complements Section 3.3 by characterizing the local dynamics of the model with households arbitrarily setting z to determine $\frac{z}{u_Y}$. Accordingly, we rewrite the dynamic equations for $\tilde{K}_Y(t)$, in (28), and for $\tilde{K}_I(t)$, in (29), considering a given $z(t)$. We are able to characterise the local-dynamics properties in a neighbourhood of the steady-state equilibrium ($\tilde{K}_Y^*$, $\tilde{K}_I^*$, $\tilde{K}_S^*$, $\tilde{H}^*$,$\tilde{C}^*$, and $\frac{z^*}{u_Y^*}$), by studying the solution of the respective linearised system obtained from the dynamical system (28)-(33) rewritten by considering a given $z(t)$. The Jacobian matrix corresponding to the linearised system, evaluated at $\left(\tilde{K}_Y^*, \tilde{K}_I^*, \tilde{K}_S^*, \tilde{H}^*, \tilde{C}^*, \frac{z^*}{u_Y^*}\right)$, under the normalization $\tilde{C}^* = 1$ and the assumption $\delta_{C_T} = \delta_P = \delta$, is given by:

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{24} & J_{24} & J_{25} & J_{26} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} & J_{56} \\ J_{61} & J_{62} & J_{63} & J_{64} & J_{65} & J_{66} \end{pmatrix}, \tag{122}$$

where:

- Considering: $\dot{\tilde{K}}_Y(t) = \tilde{K}_Y(t)^\beta z(t) \left(\frac{z(t)}{u_Y(t)}\right)^{\beta-1} \tilde{H}(t)^{1-\beta} - \tilde{C}(t) - \delta\tilde{K}_Y(t) - g^*\tilde{K}_Y(t)$,

$$J_{11} = \frac{\partial\dot{\tilde{K}}_Y(t)}{\partial\tilde{K}_Y(t)} = \beta\left(\tilde{K}_Y^*\right)^{\beta-1} z^* \left(\frac{z^*}{u_Y^*}\right)^{\beta-1} \left(\tilde{H}^*\right)^{1-\beta} - \delta - g^*$$
$$J_{12} = \frac{\partial\dot{\tilde{K}}_Y(t)}{\partial\tilde{K}_I(t)} = 0$$
$$J_{13} = \frac{\partial\dot{\tilde{K}}_Y(t)}{\partial\tilde{K}_S} = 0$$
$$J_{14} = \frac{\partial\dot{\tilde{K}}_Y(t)}{\partial\tilde{H}(t)} = \frac{(1-\beta)\left(\tilde{K}_Y^*\right)^\beta z\left(\frac{z^*}{u_Y^*}\right)^{\beta-1}}{\left(\tilde{H}^*\right)^\beta}$$
$$J_{15} = \frac{\partial\dot{\tilde{K}}_Y(t)}{\partial\tilde{C}(t)} = -1$$
$$J_{16} = \frac{\partial\dot{\tilde{K}}_Y(t)}{\partial\frac{z(t)}{u_Y(t)}} = (\beta-1)\left(\tilde{K}_Y^*\right)^\beta z^* \left(\frac{z^*}{u_Y^*}\right)^{\beta-2} \left(\tilde{H}^*\right)^{1-\beta}$$

- Considering:$\dot{\tilde{K}}_I(t) = \omega_I \left(1 - z(t)\left(\frac{z(t)}{u_Y(t)}\right)^{-1} - u_H(t) - u_S(t)\right) \tilde{H}(t) + \Omega_I\tilde{K}_I(t) + \pi_S\tilde{K}_S(t) + \pi_Y\left(1 - z(t)\right)\tilde{K}_Y(t) - g^*\tilde{K}_I(t)$,

$$J_{21} = \frac{\partial\dot{\tilde{K}}_I(t)}{\partial\tilde{K}_Y(t)} = \pi_Y\left(1 - z^*\right)$$
$$J_{22} = \frac{\partial\dot{\tilde{K}}_I(t)}{\partial\tilde{K}_I(t)} = \Omega_I - g^*$$
$$J_{23} = \frac{\partial\dot{\tilde{K}}_I(t)}{\partial\tilde{K}_S} = \pi_S$$

$$J_{24} = \frac{\partial \dot{\tilde{K}}_I(t)}{\partial \tilde{H}(t)} = \omega_I \left(1 - z^* \left(\frac{z^*}{u_Y^*}\right)^{-1} - u_H^* - u_S^*\right)$$
$$J_{25} = \frac{\partial \dot{\tilde{K}}_I(t)}{\partial \tilde{C}(t)} = 0$$
$$J_{26} = \frac{\partial \dot{\tilde{K}}_I(t)}{\partial \frac{z(t)}{u_Y(t)}} = \omega_I z^* \left(\frac{z^*}{u_Y^*}\right)^{-2} \tilde{H}^*$$

- Considering: $\dot{\tilde{K}}_S(t) = \omega_S u_S(t)\tilde{H}(t) + \Omega_S \tilde{K}_S(t) + \Phi \tilde{K}_I(t) - g^* \tilde{K}_S(t),$

$$J_{31} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{K}_Y(t)} = 0$$
$$J_{32} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{K}_I(t)} = \Phi$$
$$J_{33} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{K}_S} = \Omega_S - g^*$$
$$J_{34} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{H}(t)} = \omega_S u_S^*$$
$$J_{35} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{C}(t)} = 0$$
$$J_{36} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \frac{z(t)}{u_Y(t)}} = 0$$

- Considering: $\dot{\tilde{H}}(t) = \xi u_H(t)\tilde{H}(t) + \alpha \tilde{K}_S(t) + \varphi \tilde{K}_I(t) - g^* \tilde{H}(t),$

$$J_{41} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{K}_Y(t)} = 0$$
$$J_{42} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{K}_I(t)} = \varphi$$
$$J_{43} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{K}_S} = \alpha$$
$$J_{44} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{H}(t)} = \xi u_H^* - g^*$$
$$J_{45} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{C}(t)} = 0$$
$$J_{46} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \frac{z(t)}{u_Y(t)}} = 0$$

- Considering:

$$\dot{\tilde{C}}(t) = \frac{\tilde{C}(t)}{\tau} \cdot \Bigg\{\phi\,(1-\tau)\left[\frac{\omega_S u_S(t)\tilde{H}(t)}{\tilde{K}_S(t)} + \Omega_S + \frac{\Phi \tilde{K}_I(t)}{\tilde{K}_S(t)} - g^*\right] +$$
$$+\psi\,(1-\tau)\left[\frac{\omega_I\left(1 - z(t)\left(\frac{z(t)}{u_Y(t)}\right)^{-1} - u_H(t) - u_S(t)\right)\tilde{H}(t)}{\tilde{K}_I(t)} + \Omega_I + \frac{\pi_S \tilde{K}_S(t)}{\tilde{K}_I(t)} + \pi_Y\,(1 - z(t))\,\frac{\tilde{K}_Y(t)}{\tilde{K}_I(t)} - g^*\right] +$$
$$+g^*\,(\phi + \psi - \tau\,(1 + \phi + \psi)) + \frac{\beta\left[\tilde{K}_Y(t)^\beta z(t)\left(\frac{z(t)}{u_Y(t)}\right)^{\beta-1}\tilde{H}(t)^{1-\beta}\right]}{\tilde{K}_Y(t)} - \delta - \rho\Bigg\},$$

$$J_{51} = \frac{\partial \dot{\tilde{C}}(t)}{\partial \tilde{K}_Y(t)} = \frac{1}{\tau}\left[\frac{\psi(1-\tau)\pi_Y(1-z^*)}{\tilde{K}_I^*} + \beta\,(\beta-1)\left(\tilde{K_Y^*}\right)^{\beta-2} z^* \left(\frac{z^*}{u_Y^*}\right)^{\beta-1}\left(\tilde{H}^*\right)^{1-\beta}\right]$$

$$J_{52} = \frac{\partial \dot{\tilde{C}}(t)}{\partial \tilde{K}_I(t)} = \frac{\phi(1-\tau)}{\tau}\frac{\Phi}{\tilde{K}_S^*} - \frac{\psi(1-\tau)}{\tau \tilde{K}_I^*}\left[\frac{\omega_I\left(1-z^*\left(\frac{z^*}{u_Y^*}\right)^{-1}-u_H^*-u_S^*\right)\tilde{H}^*}{\tilde{K}_I^*} + \frac{\pi_S \tilde{K}_S^*}{\tilde{K}_I^*} + \pi_Y\left(1-z^*\right)\frac{\tilde{K}_Y^*}{\tilde{K}_I^*}\right] =$$

$$\frac{\phi(1-\tau)}{\tau}\frac{\Phi}{\tilde{K}_S^*} - \frac{\psi(1-\tau)}{\tau \tilde{K}_I^*}\left[g^* - \Omega_I\right]$$

$$J_{53} = \frac{\partial \dot{\tilde{C}}(t)}{\partial \tilde{K}_S} = \frac{\phi(1-\tau)}{\tau}\left[-\frac{\omega_S u_S^* \tilde{H}^*}{\left(\tilde{K}_S^*\right)^2} - \frac{\Phi \tilde{K}_I^*}{\left(\tilde{K}_S^*\right)^2}\right] + \frac{\psi(1-\tau)}{\tau}\frac{\pi_S}{\tilde{K}_I^*}$$

$$J_{54} = \frac{\partial \dot{\tilde{C}}(t)}{\partial \tilde{H}(t)} = \frac{1}{\tau}\left[\phi\left(1-\tau\right)\frac{\omega_S u_S^*}{\tilde{K}_S^*} + \psi\left(1-\tau\right)\frac{\omega_I\left(1-z^*\left(\frac{z^*}{u_Y^*}\right)^{-1}-u_H^*-u_S^*\right)}{\tilde{K}_I^*} + \beta\left(1-\beta\right)\left(\tilde{K}_Y^*\right)^{\beta-1} z^*\left(\frac{z^*}{u_Y^*}\right)^{\beta-1}\left(\tilde{H}^*\right.\right.$$

$$J_{55} = \frac{\partial \dot{\tilde{C}}(t)}{\partial \tilde{C}(t)} = 0$$

$$J_{56} = \frac{\partial \dot{\tilde{C}}(t)}{\partial \frac{z(t)}{u_Y(t)}} = \frac{1}{\tau}\left[\psi\left(1-\tau\right)\omega_I z^*\left(\frac{z^*}{u_Y^*}\right)^{-2}\frac{\tilde{H}^*}{\tilde{K}_I^*} + \frac{\beta(\beta-1)\left(\tilde{K}_Y^*\right)^{\beta} z^*\left(\frac{z^*}{u_Y^*}\right)^{\beta-2}\left(\tilde{H}^*\right)^{1-\beta}}{\tilde{K}_Y^*}\right]$$

- considering:

$$\dot{\left(\frac{z(t)}{u_Y(t)}\right)} = \left[\frac{\xi u_H(t)\tilde{H}(t) + \alpha \tilde{K}_S(t) + \varphi \tilde{K}_I(t) - g^* \tilde{H}(t)}{\tilde{H}(t)}\right.$$
$$\left. - \frac{\tilde{K}_Y(t)^{\beta} z(t)\left(\frac{z(t)}{u_Y(t)}\right)^{\beta-1}\tilde{H}(t)^{1-\beta} - \tilde{C}(t) - \delta z(t)\tilde{K}_Y(t) - \delta\left(1-z(t)\right)\tilde{K}_Y(t) - g^*\tilde{K}_Y(t)}{\tilde{K}_Y(t)}\right]\cdot\left(\frac{z(t)}{u_Y(t)}\right),$$

$$J_{61} = \frac{\partial \dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial \tilde{K}_Y(t)} = \left[-\left(\beta-1\right)z^*\left(\frac{z^*}{u_Y^*}\right)^{\beta-1}\left(\tilde{K}_Y^*\right)^{\beta-2}\left(\tilde{H}^*\right)^{1-\beta} - \frac{1}{\left(\tilde{K}_Y^*\right)^2}\right]\cdot\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{62} = \frac{\partial \dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial \tilde{K}_I(t)} = \left(\frac{\varphi}{\tilde{H}^*}\right)\cdot\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{63} = \frac{\partial \dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial \tilde{K}_S} = \left(\frac{\alpha}{\tilde{H}^*}\right)\cdot\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{64} = \frac{\partial \dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial \tilde{H}(t)} = -\left[\alpha\frac{\tilde{K}_S^*}{\left(\tilde{H}^*\right)^2} + \varphi\frac{\tilde{K}_I^*}{\left(\tilde{H}^*\right)^2} + \frac{\tilde{K}_Y^{*\,\beta-1} z^*\left(\frac{z^*}{u_Y^*}\right)^{\beta-1}(1-\beta)}{\left(\tilde{H}^*\right)^{\beta}}\right]\left(\frac{z^*}{u_Y^*}\right) = \left(\frac{\xi u_H}{\tilde{H}^*} - \frac{g^*}{\tilde{H}^*} - \frac{\tilde{K}_Y^{*\,\beta-1} z^*\left(\frac{z^*}{u_Y^*}\right)^{\beta-1}(1-\beta)}{\left(\tilde{H}^*\right)^{\beta}}\right)\cdot$$
$$\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{65} = \frac{\partial \dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial \tilde{C}(t)} = \left(\frac{1}{\tilde{K}_Y^*}\right)\cdot\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{66} = \frac{\partial \dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial \frac{z(t)}{u_Y(t)}} = \left(1-\beta\right)\left(\tilde{K}^*{}_Y\right)^{\beta-1} z^*\left(\frac{z^*}{u_Y^*}\right)^{\beta-1}\left(\tilde{H}^*\right)^{1-\beta}$$

Given the high dimensionality of the Jacobian in (122), we resort to numerical analysis in order to compute the eigenvalues considering the calibration scenarios of the model presented in Section 4.1, in the main text, that yield a non-negative parameter α. Table (8) shows that scenario I has two positive eigenvalues, thus amounting to a saddle-path equilibrium, while scenarios II, IV, and V have three positive eigenvalues, implying that the equilibrium is unstable, since there are two jump variables in our setup, $\tilde{C}(t)$ and $\frac{z(t)}{u_Y(t)}$.

I	II	IV	V
0.1240	0.1238	0.1231	0.1234
7.7569e-04	-0.0138	-0.0352	-0.0334
-1.2000e-14	0.0047	0.0073	0.0051
-3.5126e-05	2.2117e-14	2.2864e-15	2.9352e-15
-0.0067	-8.9136e-05	-3.5283e-04	-3.3785e-04
-0.0099	-0.0065	-0.0167	-0.0167

Table 8: Eigenvalues associated with the Jacobian in (122). The four scenarios correspond to the calibration scenarios I, II, IV, and V (see Tables 1 and 2 in Section 4.1).

E.1.2 Local-dynamics properties - Case B

This section of the appendix provides the proof to Proposition 3.3, in Section 3.3, by characterizing the local dynamics of the model with households arbitrarily setting u_Y to determine $\frac{z}{u_Y}$. We are able to characterise the local-dynamics properties in a neighbourhood of the steady-state equilibrium ($\tilde{K}_Y^*$, $\tilde{K}_I^*$, $\tilde{K}_S^*$, $\tilde{H}^*$,$\tilde{C}^*$, and $\frac{z^*}{u_Y^*}$), by studying the solution of the respective linearised system obtained from the dynamical system (28)-(33). The Jacobian matrix corresponding to the linearised system, evaluated at $\left(\tilde{K}_Y^*, \tilde{K}_I^*, \tilde{K}_S^*, \tilde{H}^*, \tilde{C}^*, \frac{z^*}{u_Y^*}\right)$, under the normalization $\tilde{C}^* = 1$ and the assumption $\delta_{C_T} = \delta_P = \delta$, is given by:

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{24} & J_{24} & J_{25} & J_{26} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} & J_{56} \\ J_{61} & J_{62} & J_{63} & J_{64} & J_{65} & J_{66} \end{pmatrix}, \tag{123}$$

where:

- Considering: $\dot{\tilde{K}}_Y(t) = \tilde{K_Y}(t)^\beta u_Y \left(\frac{z(t)}{u_Y(t)}\right)^\beta \tilde{H}(t)^{1-\beta} - \tilde{C}(t) - \delta \tilde{K}_Y(t) - g^* \tilde{K}_Y(t)$,

$$J_{11} = \frac{\partial \dot{\tilde{K}}_Y(t)}{\partial \tilde{K}_Y(t)} = \beta \tilde{K}_Y^{*\beta-1} u_Y^* \left(\frac{z^*}{u_Y^*}\right)^\beta \tilde{H}^{*1-\beta} - \delta - g^*$$

$$J_{12} = \frac{\partial \dot{\tilde{K}}_Y(t)}{\partial \tilde{K}_I(t)} = 0$$

$$J_{13} = \frac{\partial \dot{\tilde{K}}_Y(t)}{\partial \tilde{K}_S} = 0$$

$$J_{14} = \frac{\partial \dot{\tilde{K}}_Y(t)}{\partial \tilde{H}(t)} = \frac{(1-\beta) \tilde{K}_Y^{*\beta} u_Y^* \left(\frac{z^*}{u_Y^*}\right)^\beta}{\left(\tilde{H}^*\right)^\beta}$$

$$J_{15} = \frac{\partial \dot{\tilde{K}}_Y(t)}{\partial \tilde{C}(t)} = -1$$

$J_{16} = \frac{\partial \dot{\tilde{K}}_Y(t)}{\partial \frac{z(t)}{u_Y(t)}} = \beta \tilde{K_Y^*}^{\beta} u_Y^* \left(\frac{z^*}{u_Y^*}\right)^{\beta-1} \tilde{H}^{*1-\beta}$

- Considering: $\dot{\tilde{K}}_I(t) = \omega_I \left(1 - u_Y(t) - u_H(t) - u_S(t)\right) \tilde{H}(t) + \Omega_I \tilde{K}_I(t) + \pi_S \tilde{K}_S(t) + \pi_Y \left(1 - u_Y(t) \left(\frac{z(t)}{u_Y(t)}\right)\right) \tilde{K}_Y(t) - g^* \tilde{K}_I(t)$,

$J_{21} = \frac{\partial \dot{\tilde{K}}_I(t)}{\partial \tilde{K}_Y(t)} = \pi_Y \left(1 - u_Y^* \left(\frac{z^*}{u_Y^*}\right)\right)$
$J_{22} = \frac{\partial \dot{\tilde{K}}_I(t)}{\partial \tilde{K}_I(t)} = \Omega_I - g^*$
$J_{23} = \frac{\partial \dot{\tilde{K}}_I(t)}{\partial \tilde{K}_S} = \pi_S$
$J_{24} = \frac{\partial \dot{\tilde{K}}_I(t)}{\partial \tilde{H}(t)} = \omega_I \left(1 - u_Y^* - u_H^* - u_S^*\right)$
$J_{25} = \frac{\partial \dot{\tilde{K}}_I(t)}{\partial \tilde{C}(t)} = 0$
$J_{26} = \frac{\partial \dot{\tilde{K}}_I(t)}{\partial \frac{z(t)}{u_Y(t)}} = -\pi_Y u_Y^* \tilde{K}_Y^*$

- Considering: $\dot{\tilde{K}}_S(t) = \omega_S u_S(t) \tilde{H}(t) + \Omega_S \tilde{K}_S(t) + \Phi \tilde{K}_I(t) - g^* \tilde{K}_S(t)$,

$J_{31} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{K}_Y(t)} = 0$
$J_{32} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{K}_I(t)} = \Phi$
$J_{33} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{K}_S} = \Omega_S - g^*$
$J_{34} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{H}(t)} = \omega_S u_S^*$
$J_{35} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \tilde{C}(t)} = 0$
$J_{36} = \frac{\partial \dot{\tilde{K}}_S(t)}{\partial \frac{z(t)}{u_Y(t)}} = 0$

- Considering: $\dot{\tilde{H}}(t) = \xi u_H(t) \tilde{H}(t) + \alpha \tilde{K}_S(t) + \varphi \tilde{K}_I(t) - g^* \tilde{H}(t)$,

$J_{41} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{K}_Y(t)} = 0$
$J_{42} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{K}_I(t)} = \varphi$
$J_{43} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{K}_S} = \alpha$
$J_{44} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{H}(t)} = \xi u_H^* - g^*$
$J_{45} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \tilde{C}(t)} = 0$
$J_{46} = \frac{\partial \dot{\tilde{H}}(t)}{\partial \frac{z(t)}{u_Y(t)}} = 0$

- Considering:

$$\dot{\tilde{C}}(t)=\frac{\tilde{C}(t)}{\tau}\cdot\left\{\phi\left(1-\tau\right)\left[\frac{\omega_S u_S(t)\tilde{H}(t)}{\tilde{K}_S(t)}+\Omega_S+\frac{\Phi\tilde{K}_I(t)}{\tilde{K}_S(t)}-g^*\right]+\right.$$

$$+\psi\left(1-\tau\right)\left[\frac{\omega_I\left(1-u_Y(t)-u_H(t)-u_S(t)\right)\tilde{H}(t)}{\tilde{K}_I(t)}+\Omega_I+\frac{\pi_S\tilde{K}_S(t)}{\tilde{K}_I(t)}+\pi_Y\left(1-u_Y(t)\left(\frac{z(t)}{u_Y(t)}\right)\right)\frac{\tilde{K}_Y(t)}{\tilde{K}_I(t)}-g^*\right]+$$

$$\left.+g^*\left(\phi+\psi-\tau\left(1+\phi+\psi\right)\right)+\frac{\beta\left[\tilde{K_Y}^{\beta}u_Y\left(\frac{z}{u_Y}\right)^{\beta}\tilde{H}^{1-\beta}\right]}{\tilde{K}_Y(t)}-\delta-\rho\right\},$$

$$J_{51}=\frac{\partial\dot{\tilde{C}}(t)}{\partial\tilde{K}_Y(t)}=\frac{1}{\tau}\left[\frac{\psi(1-\tau)\pi_Y\left(1-u_Y^*\left(\frac{z^*}{u_Y^*}\right)\right)}{\tilde{K}_I^*}+\beta^2\tilde{K_Y^*}^{\beta-1}u_Y^*\left(\frac{z^*}{u_Y^*}\right)^{\beta}\tilde{H}^{*1-\beta}\right]$$

$$J_{52}=\frac{\partial\dot{\tilde{C}}(t)}{\partial\tilde{K}_I(t)}=\frac{\phi(1-\tau)}{\tau}\frac{\Phi}{\tilde{K}_S^*}-\frac{\psi(1-\tau)}{\tau\tilde{K}_I^*}\left[\frac{\omega_I\left(1-u_Y^*-u_H^*-u_S^*\right)\tilde{H}^*}{\tilde{K}_I^*}+\frac{\pi_S\tilde{K}_S^*}{\tilde{K}_I^*}+\pi_Y\left(1-u_Y^*\left(\frac{z^*}{u_Y^*}\right)\right)\frac{\tilde{K}_Y^*}{\tilde{K}_I^*}\right]=$$
$$\frac{\phi(1-\tau)}{\tau}\frac{\Phi}{\tilde{K}_S^*}-\frac{\psi(1-\tau)}{\tau\tilde{K}_I^*}\left[g^*-\Omega_I\right]$$

$$J_{53}=\frac{\partial\dot{\tilde{C}}(t)}{\partial\tilde{K}_S}=\frac{\phi(1-\tau)}{\tau}\left[-\frac{\omega_S u_S^*\tilde{H}^*}{\left(\tilde{K_S^*}\right)^2}-\frac{\Phi\tilde{K}_I^*}{\left(\tilde{K_S^*}\right)^2}\right]+\frac{\psi(1-\tau)}{\tau}\frac{\pi_S}{\tilde{K}_I^*}$$

$$J_{54}=\frac{\partial\dot{\tilde{C}}(t)}{\partial\tilde{H}(t)}=\frac{1}{\tau}\left[\phi\left(1-\tau\right)\frac{\omega_S u_S^*}{\tilde{K}_S^*}+\psi\left(1-\tau\right)\frac{\omega_I\left(1-u_Y^*-u_H^*-u_S^*\right)}{\tilde{K}_I^*}+\beta\left(1-\beta\right)\left(\tilde{K_Y^*}\right)^{\beta-1}u_Y^*\left(\frac{z^*}{u_Y^*}\right)^{\beta}\left(\tilde{H}^*\right)^{-\beta}\right]$$

$$J_{55}=\frac{\partial\dot{\tilde{C}}(t)}{\partial\tilde{C}(t)}=0$$

$$J_{56}=\frac{\partial\dot{\tilde{C}}(t)}{\partial\frac{z(t)}{u_Y(t)}}=\frac{1}{\tau}\left[-\psi\left(1-\tau\right)\pi_Y u_Y^*\frac{\tilde{K}_Y^*}{\tilde{K}_I^*}+\frac{\beta^2\left(\tilde{K_Y^*}\right)^{\beta}u_Y^*\left(\frac{z^*}{u_Y^*}\right)^{\beta-1}\left(\tilde{H}^*\right)^{1-\beta}}{\tilde{K}_Y^*}\right]$$

- Considering:

$$\dot{\left(\frac{z(t)}{u_Y(t)}\right)}=\left[\frac{\xi u_H(t)\tilde{H}(t)+\alpha\tilde{K}_S(t)+\varphi\tilde{K}_I(t)-g^*\tilde{H}(t)}{\tilde{H}(t)}\right.$$
$$\left.-\frac{\tilde{K_Y}^{\beta}u_Y\left(\frac{z(t)}{u_{Y(t)}}\right)^{\beta}\tilde{H}^{1-\beta}-\tilde{C}(t)-\delta z(t)\tilde{K}_Y(t)-\delta\left(1-z(t)\right)\tilde{K}_Y(t)-g^*\tilde{K}_Y(t)}{\tilde{K}_Y(t)}\right]\cdot\left(\frac{z(t)}{u_Y(t)}\right),$$

$$J_{61}=\frac{\partial\dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial\tilde{K}_Y(t)}=\left[-\left(\beta-1\right)u_Y^*\left(\frac{z^*}{u_Y^*}\right)^{\beta}\left(\tilde{K^*}_Y\right)^{\beta-2}\left(\tilde{H}^*\right)^{1-\beta}-\frac{1}{\left(\tilde{K}_Y^*\right)^2}\right]\cdot\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{62}=\frac{\partial\dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial\tilde{K}_I(t)}=\left(\frac{\varphi}{\tilde{H}^*}\right)\cdot\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{63}=\frac{\partial\dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial\tilde{K}_S}=\left(\frac{\alpha}{\tilde{H}^*}\right)\cdot\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{64}=\frac{\partial\dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial\tilde{H}(t)}=-\left[\alpha\frac{\tilde{K}_S^*}{\left(\tilde{H}^*\right)^2}+\varphi\frac{\tilde{K}_I^*}{\left(\tilde{H}^*\right)^2}+\frac{\left(\tilde{K}_Y^*\right)^{\beta-1}u_Y^*\left(\frac{z^*}{u_Y^*}\right)^{\beta}(1-\beta)}{\left(\tilde{H}^*\right)^{\beta}}\right]\left(\frac{z^*}{u_Y^*}\right)=\left(\frac{\xi u_H^*}{\tilde{H}^*}-\frac{g^*}{\tilde{H}^*}-\frac{\left(\tilde{K}_Y^*\right)^{\beta-1}u_Y^*\left(\frac{z^*}{u_Y^*}\right)^{\beta}(1-\beta)}{\left(\tilde{H}^*\right)^2}\right)\cdot$$
$$\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{65}=\frac{\partial\dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial\tilde{C}(t)}=\left(\frac{1}{\tilde{K}_Y^*}\right)\cdot\left(\frac{z^*}{u_Y^*}\right)$$

$$J_{66}=\frac{\partial\dot{\left(\frac{z(t)}{u_Y(t)}\right)}}{\partial\frac{z(t)}{u_Y(t)}}=-\beta\left(\tilde{K_Y^*}\right)^{\beta-1}u_Y^*\left(\frac{z^*}{u_Y^*}\right)^{\beta}\left(\tilde{H}^*\right)^{1-\beta}$$

Again, given the high dimensionality of the Jacobian in (123), we resort to numerical analysis in order to compute the eigenvalues considering the calibration scenarios of the model presented in Section 4.1, in the main text, that yield a non-negative parameter α. Table (9) shows that the Jacobian has at most one eigenvalue with positive real part (and no eigenvalues with null real part), which implies that the steady-state is locally indeterminate, since there are two jump variables in our setup, $\tilde{C}(t)$ and $\frac{z(t)}{u_Y(t)}$. Under one real positive eigenvalue, the stable manifold is 5-dimensional and, thus, there exists a neighbourhood around the steady state so that, for given predetermined $\tilde{K}_Y(0)$, $\tilde{K}_S(0)$, $\tilde{K}_I(0)$, $\tilde{H}(0)$ and for any initial $\tilde{C}(0)$ (respectively, $\frac{z(0)}{u_Y(0)}$) chosen by the households, there is an initial $\frac{z(0)}{u_Y(0)}$ (respectively, $\tilde{C}(0)$) such that the trajectories of $\tilde{K}_Y(t)$, $\tilde{K}_S(t)$, $\tilde{K}_I(t)$, $\tilde{H}(t)$, $\tilde{C}(t)$ and $\frac{z(t)}{u_Y(t)}$ implied by (28)-(33) converge asymptotically to that steady-state. Under no real positive eigenvalues, the stable manifold is 6-dimensional and, thus, there exists a neighbourhood around the steady-state so that, for given predetermined $\tilde{K}_Y(0)$, $\tilde{K}_S(0)$, $\tilde{K}_I(0)$, $\tilde{H}(0)$ and for any $\tilde{C}(0)$ and $\frac{z(0)}{u_Y(0)}$ chosen by the households, the trajectories of all variables converge asymptotically to that steady-state. Table (9) also shows that the system features either one or two pairs of complex eigenvalues with negative real parts, which determines a dampened oscillatory transitional behavior.

I	II	IV	V
-0.0300 + 0.1562i	-0.0302 + 0.1561i	-0.0305 + 0.1559i	-0.0302 + 0.1560i
-0.0300 - 0.1562i	-0.0302 - 0.1561i	-0.0305 - 0.1559i	-0.0302 - 0.1560i
9.5287e-16	-0.0049 + 0.0068i	-0.0330	-0.0336
-0.0010	-0.0049 - 0.0068i	4.2016e-15	-0.0059 + 0.0061i
-0.0043	-4.7130e-17	-0.0060 + 0.0089i	-0.0059 - 0.0061i
-0.0106	-0.0057	-0.0060 - 0.0089i	-2.4949e-18

Table 9: Eigenvalues associated with the Jacobian in (123). The four scenarios correspond to the calibration scenarios I, II, IV, and V (see Tables 1 and 2 in Section 4.1).

E.2 Transitional dynamics - Case B

E.2.1 Transition paths for given initial conditions

This section of the appendix serves as a complement to Section 4.4 by providing further results regarding the transitional dynamics under 'case B', i.e., under which u_Y is set arbitrarily to determine $\frac{z}{u_Y}$. As before, we build on the solution of the linearized system obtained from the dynamical system (28)-(33) to study the transition paths that emerge from given initial conditions off the steady state/BGP.

Figure 4 depicts the behavior of the speed of convergence of the variables of interest , measured as $-\dot{y}(t)/(y(t) - y^*)$ for a generic variable y with steady-state value y^* (e.g., Mazeda Gil et al., 2016), under calibration scenario IV and a low arbitrary initial consumption ($\tilde{C}(0) < \tilde{C}^*$). Interestingly, despite pertaining to the transition dynamics of a linearized system, the former is characterized by a speed of convergence that varies over time and across variables. This result reflects both the interaction of the multiple state variables over transition on the multidimensional stable manifold and the oscillatory behavior of the transition paths due the existence of complex eigenvalues.

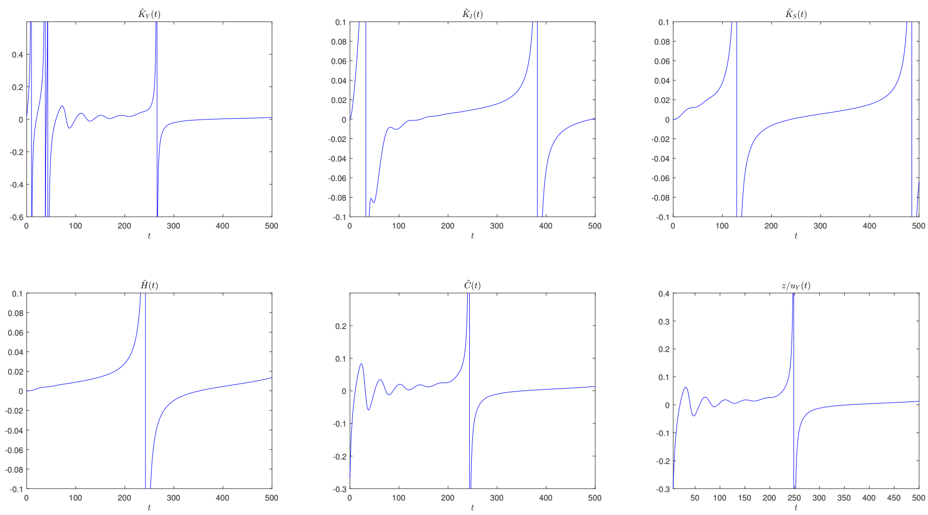

Figure 4: Time path of the speed of convergence of selected macroeconomic variables. Transitional dynamics for given initial conditions; all initial values of the state variables $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$, and also $\tilde{C}(0)$ are set 10% below the respective steady state ($\tilde{C}^* = 1$; calibration scenario IV; $\frac{z}{u_Y}$ determined for a given u_Y).

Figures 5 and 6 depict the transition paths under the calibration scenario V, in which a 6-dimensional stable manifold emerges and, thus, both $\tilde{C}(0)$ and $\frac{z(0)}{u_Y(0)}$ may be set arbitrarily. Therefore, for the sake of illustration, we consider the initial values of all types of capital (the state variables), $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$, set 10% below the respective steady-state values and: (i) either $\tilde{C}(0) > \tilde{C}^*$ and $\frac{z(0)}{u_Y(0)} < \frac{z^*}{u_Y^*}$ or $\tilde{C}(0) < \tilde{C}^*$ and $\frac{z(0)}{u_Y(0)} > \frac{z^*}{u_Y^*}$ (Figure 5); (ii) either $\tilde{C}(0) > \tilde{C}^*$ or $\tilde{C}(0) < \tilde{C}^*$, with $\frac{z(0)}{u_Y(0)} = \frac{z^*}{u_Y^*}$ Figure 6).

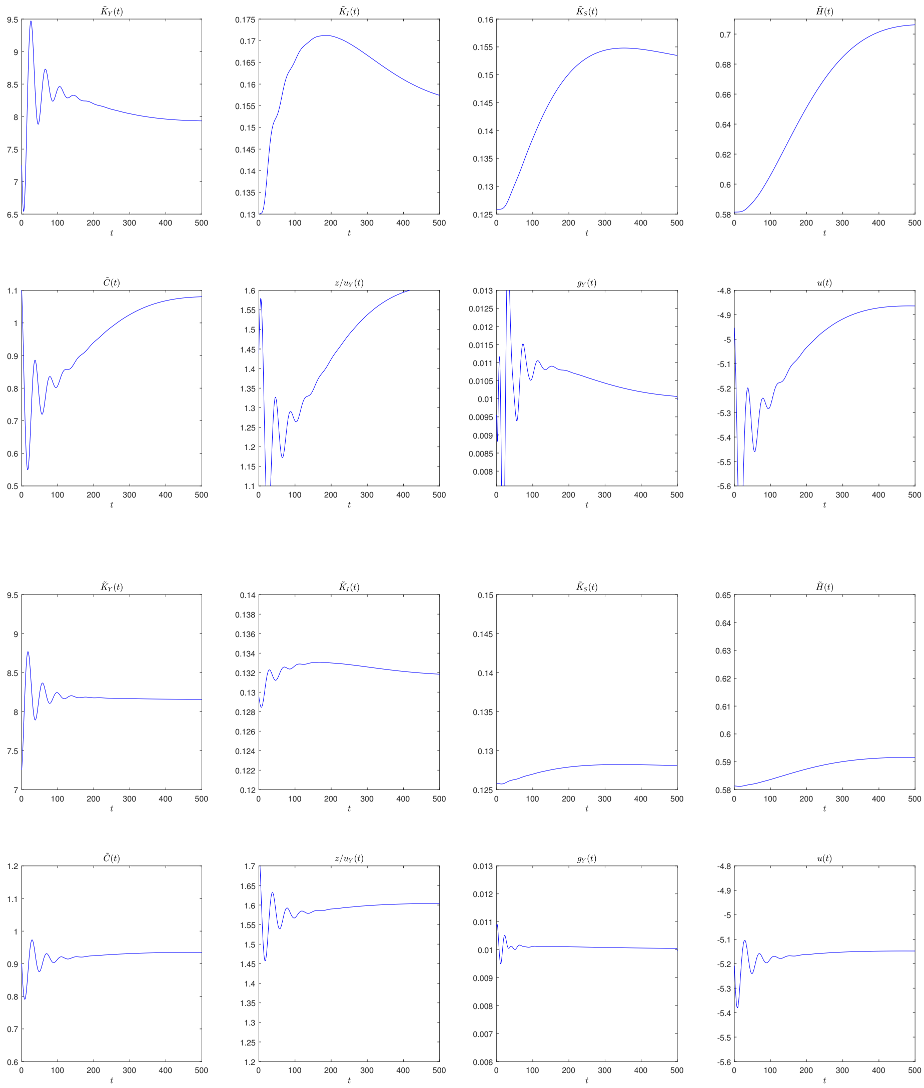

Figure 5: Transitional dynamics of selected macroeconomic variables for given initial conditions.. Top panel: high $\tilde{C}(0)$ ($+10\%$ off the steady-state value) and low $\frac{z(0)}{u_Y(0)}$ (-10% off the steady-state value). Bottom panel: low $\tilde{C}(0)$ (-10% off the steady-state value) and high $\frac{z(0)}{u_Y(0)}$ ($+10\%$ off the steady-state value). All initial values of the state variables, $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$, are set 10% below their steady state ($\tilde{C}^* = 1$; calibration scenario V; $u(t)$ denotes the instantaneous utility in (21), $u(t) = \frac{1}{1-\tau}\left[\tilde{C}(t)\left(K_{\tilde{S}}(t)\right)^{\phi}\left(K_{\tilde{I}}(t)\right)^{\psi}\right]^{(1-\tau)}$; $\frac{z}{u_Y}$ is determined for a given u_Y).

Overall, the transition patterns displayed by Figure (5), upper panel, are identical to those in Figure 1, upper panel, the cases that pertain to $\tilde{C}(0) > \tilde{C}^*$. Yet, there are two major differences in the case in Figure (5), where also $\frac{z(0)}{u_Y(0)}$ is set arbitrarily, with $\frac{z(0)}{u_Y(0)} < \frac{z^*}{u_Y^*}$: the short-to medium run oscillatory behavior – displayed by $\tilde{K}_Y(t)$, $\tilde{C}(t)$, $\frac{z(t)}{u_Y(t)}$, $g_Y(t)$, and $u(t)$ – is more exuberant, while the variables that feature a smooth non-monotonic behavior – $\tilde{K}_I(t)$, $\tilde{K}_S(t)$, and $\tilde{H}(t)$ – display no short-run undershooting vis-à-vis the steady-state level

In turn, the transition patterns depicted by Figure (5), lower panel, are similar to those in Figure 1, lower panel, the cases that pertain to $\tilde{C}(0) < \tilde{C}^*$, with an emphasis on the fact that the short-run oscillatory behavior displayed by $\tilde{K}_Y(t)$, $\tilde{C}(t)$, $\frac{z(t)}{u_Y(t)}$, $g_Y(t)$, and $u(t)$ is quite attenuated vis-à-vis the case of $\tilde{C}(0) > \tilde{C}^*$. However, the fact that, in Figure (5), $\frac{z(0)}{u_Y(0)}$ is also set arbitrarily, with $\frac{z(0)}{u_Y(0)} > \frac{z^*}{u_Y^*}$, induces some short-to-medium run oscillation in $\tilde{K}_I(t)$, while the variables that feature a smooth non-monotonic behavior – $\tilde{K}_S(t)$ and $\tilde{H}(t)$ – display a slight short-run undershooting vis-à-vis the steady-state level.

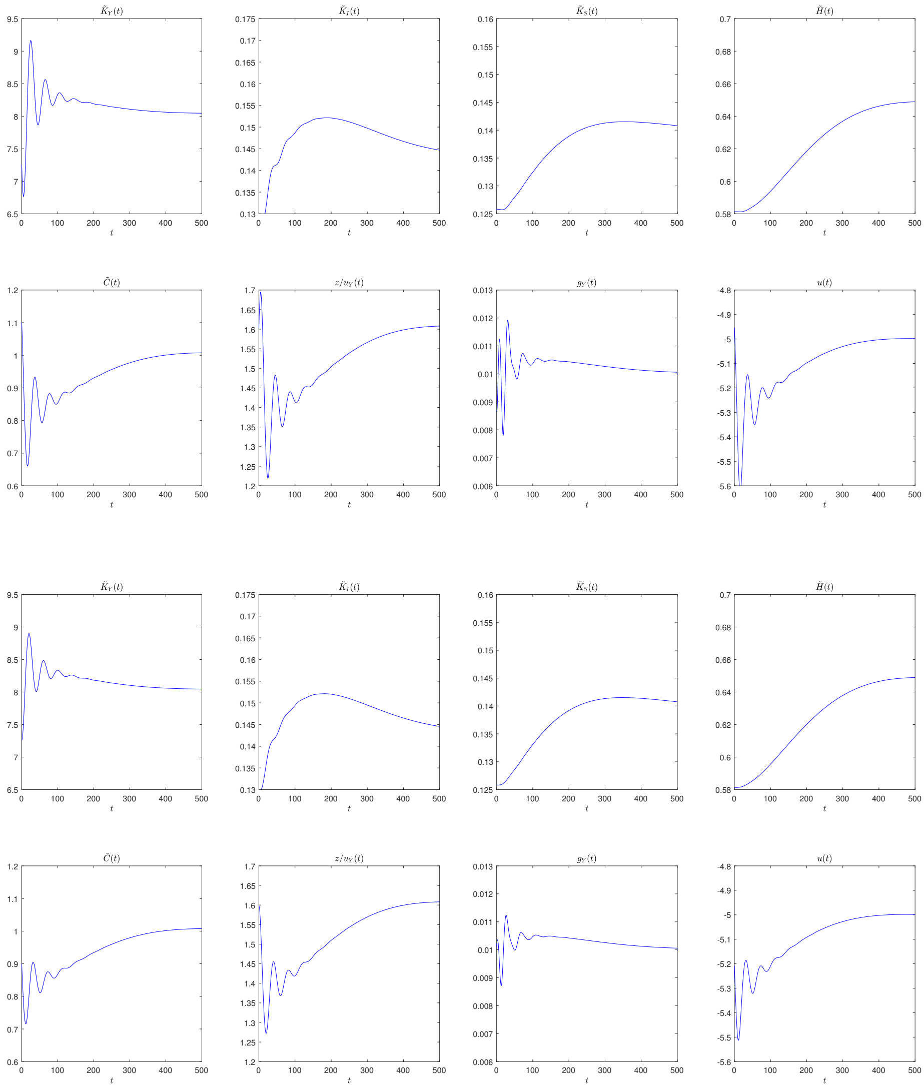

Figure 6: Transitional dynamics of selected macroeconomic variables for given initial conditions. Top panel: high $\tilde{C}(0)$ (+10% off the steady-state value). Bottom panel: low $\tilde{C}(0)$ (−10% off the steady-state value). $\frac{z(0)}{u_Y(0)}$ is set at its steady-state value in both cases. All initial values of the state variables, $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$, are set 10% below the respective steady state ($\tilde{C}^* = 1$; calibration scenario V; $u(t)$ denotes the instantaneous utility in (21), $u(t) = \frac{1}{1-\tau}\left[\tilde{C}(t)\left(\tilde{K_S}(t)\right)^{\phi}\left(\tilde{K_I}(t)\right)^{\psi}\right]^{(1-\tau)}$; $\frac{z}{u_Y}$ is determined for a given u_Y).

Figure (6) illustrates the cases when $\tilde{C}(0) > \tilde{C}^*$ or $\tilde{C}(0) < \tilde{C}^*$, with $\frac{z(0)}{u_Y(0)}$ set at its steady-state value. The transition patterns are identical to those obtained under calibration scenario IV with $\tilde{C}(0) > \tilde{C}^*$ or $\tilde{C}(0) < \tilde{C}^*$, in Figure 1.

E.2.2 Transition paths under policy-induced shocks

This section of the appendix complements the policy analysis presented in Section 4.4, by considering calibration scenario IV and a high arbitrary initial consumption ($\tilde{C}(0) > \tilde{C}^*$). Again, we focus on the transitional-dynamics effects originated by hypothetical policy-induced shocks to three representative key structural parameters in the model. translated as a 10% subsidy-equivalent policy measure, i.e., a shock to α, to Φ and to ξ, as explained in the main text.

In Figure (7), we display the case where $\tilde{C}(0) > \tilde{C}^*$ and the initial values for $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$ correspond to their steady-state levels under no subsidy.[27] Overall, the transition patterns displayed by Figure (7) are identical to those in Figure 2, where $\tilde{C}(0) < \tilde{C}^*$. Regarding the shocks to α and Φ (upper and middle panels), we find that, under a high $\tilde{C}(0)$, there is a less marked short-to-medium run oscillation behavior by $\tilde{K}_Y(t)$, $\tilde{C}(t)$, $\frac{z(t)}{u_Y(t)}$, $g_Y(t)$, and $u(t)$, and no short-run overshooting by $\tilde{K}_I(t)$ and $\tilde{K}_S(t)$, under the shock to α, and by $\tilde{K}_I(t)$ and $\tilde{H}(t)$, under the shock to Φ. Regarding the shock to ξ (lower panel), the time-paths are fairly identical to those depicted in Figure 2. All in all, these results suggest that the sensitiveness of the transitional dynamics triggered by the policy-induced shocks to the sunspot behavior in consumption is small.

[27] Again, to facilitate comparison of transition patterns across the different shocks, we normalize these initial values to unity.

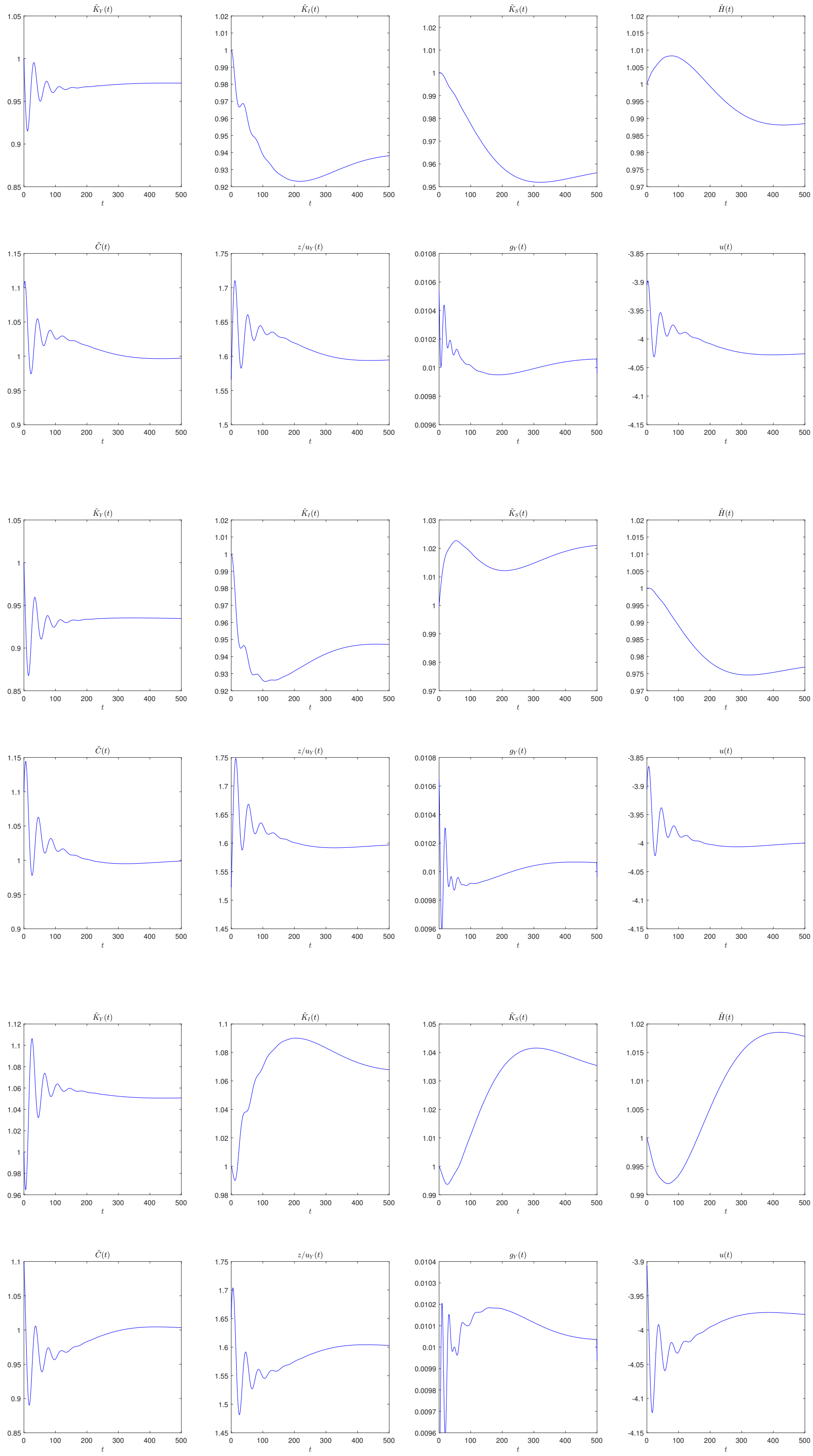

Figure 7: Transitional dynamics of selected macroeconomic variables under policy-induced shocks. Three representative cases are shown, a shock to: α (upper panel); Φ (middle panel); and ξ (bottom panel). Initial values for the state variables, $\tilde{K}_Y(0)$, $\tilde{K}_I(0)$, $\tilde{K}_S(0)$, and $\tilde{H}(0)$, correspond to their steady-state levels under no subsidy, normalized to unity. The initial value $\tilde{C}(0)$ is set 10% above its steady-state level ($\tilde{C}^* = 1$; calibration scenario IV; $u(t)$ denotes the instantaneous utility in (21), $u(t) = \frac{1}{1-\tau}\left[\tilde{C}(t)\left(\tilde{K_S}(t)\right)^{\phi}\left(\tilde{K_I}(t)\right)^{\psi}\right]^{(1-\tau)}$; $\frac{z}{u_Y}$ is determined for a given u_Y).