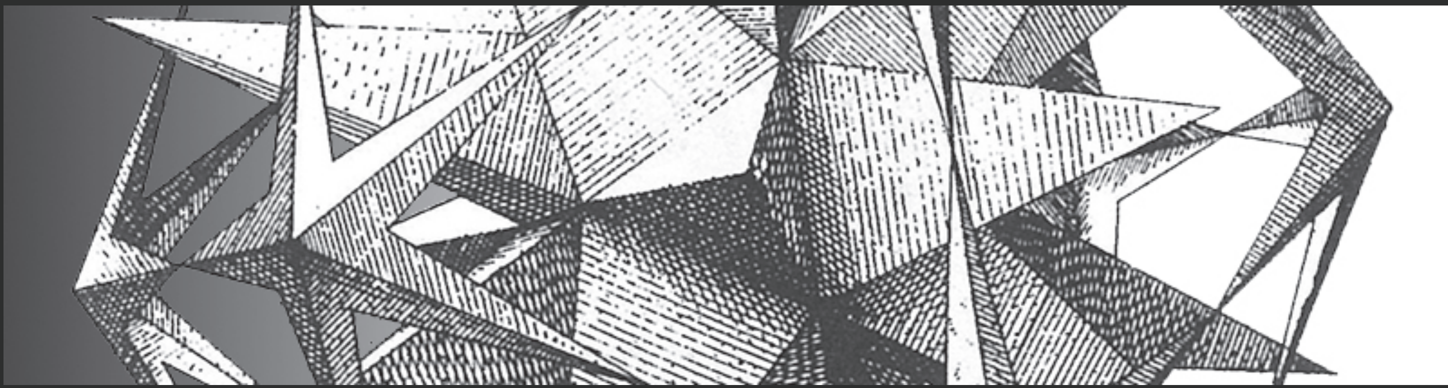

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05, 06, 07 . September. 2019 | Faculdade de Ciências da Universidade do Porto

GEOMETRIAS'19: BOOK OF ABSTRACTS

Printed Edition: ISBN 978-989-98926-9-9.
Digital Edition : ISBN 978-989-98926-8-2 | doi:10.24840/978-989-98926-8-2.

U.PORTO
FC FACULDADE DE CIÊNCIAS UNIVERSIDADE DO PORTO
CENTRO DE MATEMÁTICA UNIVERSIDADE DO PORTO
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GEOMETRIAS'19: BOOK OF ABSTRACTS
Printed Edition: ISBN 978-989-98926-9-9.
Digital Edition : ISBN 978-989-98926-8-2 | https://doi.org/10.24840/978-989-98926-8-2.

Overall literary edition and graphic edition by Vera Viana.

Front cover image: Jamnitzer. W., 1568. *Perspectiva Corporum Regularium*.
Retrieved from http://aproged.pt/biblioteca/jamnitzer.pdf

Published by:
APROGED - ASSOCIAÇÃO DOS PROFESSORES DE GEOMETRIA E DE DESENHO
Rua Major David Magno, 139 | 4000-191 PORTO PORTUGAL

Printed edition:
Viana, V. (Ed.). (2019). *Geometrias'19: Book of Abstracts*. Porto: Aproged. ISBN 978-989-98926-9-9
Depósito Legal: 460487/19
Printed by Nozzle, Lda. (nozzle@sapo.pt)

Digital edition:
Viana, V. (Ed.). (2019). *Geometrias'19: Book of Abstracts*. Porto: Aproged.
ISBN 978-989-98926-8-2. doi: 10.24840/978-989-98926-8-2
The digital edition of this Book of Abstracts is available at www.aproged.pt/geometrias19/g19bookofabstracts.pdf

Conference's homepage: www.aproged.pt/geometrias19.html
Conference Venue: Faculdade de Ciências da Universidade do Porto, Portugal.

The International Conference Geometrias'19 was organized by Aproged, Faculdade de Ciências da Universidade do Porto and Centro de Matemática da Universidade do Porto.

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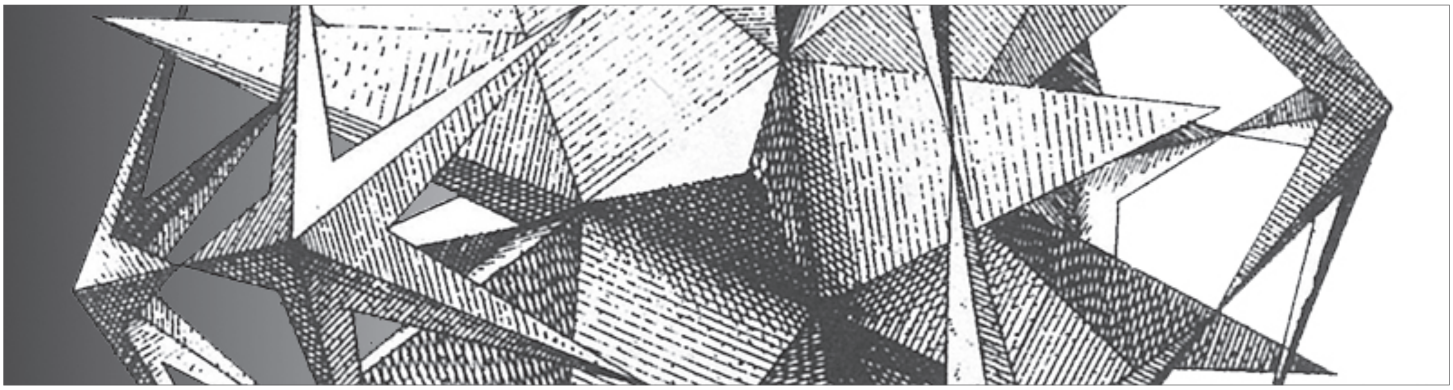

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SATURDAY, 07 SEPTEMBER 2019

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PAPERS' SESSION
PAPER 13

AN INTRODUCTION TO SOLID TESSELLATIONS WITH STUDENTS OF ARCHITECTURE

João Pedro Xavier[1], José Pedro Sousa[2], Alexandra Castro[3] and Vera Viana[4]

KEYWORDS: Architectonic Tessellations, Three-dimensional Software, Architectural Design.

INTRODUCTION

This research intends to describe a didactic experiment on the exploration of solid tessellations, accomplished in the Geometry course of the 1st year of the Architecture program's Master in the Faculty of Arquitecture of Porto's University (FAUP). In this didactic experiment, digital three-dimensional modelling was explored as a recurrent tool to study and research the topics proposed.

From the vast subject of solid tessellations and its possible applications, students were guided to focus their attention on uniform tessellations, in which regular-faced polyhedra fit together "to fill all space just once, so that every face of each polyhedron belongs to one other polyhedron" [01: p. 68] In uniformly tessellated polyhedra, all the vertices are equally surrounded and superimposable under symmetries onto any other. For the sake of concision and feasibility, only six of the thirteen "architectonic tessellations" [02] were explored by the students in collaborative assignments.

THE RESEARCH

For many years, the Geometry course in the Faculty of Arquitecture of Porto's University was entirely concerned with projective geometry based on the traditional systems of representation in architecture. However, since 2015, its syllabus has been updated with the introduction of three-dimensional modelling software. Besides allowing students to address a broader range of geometric themes (whose exploration through the traditional representational systems would be harder to accomplish) 3D modelling allows students to focus more on geometry itself, than in the representational procedures and its predicaments. When a student creates a virtual model, his/her attention focuses mostly in the geometry of the objects, rather than in the representational system. The fact that digital tools automatically provide an accurate representation of geometric objects, by itself, strongly enhances the students' abilities for spatial visualization, mental rotation and geometric reasoning.

The introduction to digital 3D modelling is structured in a 5-6 weeks teamwork, in which students explore Computer Aided Design (CAD) processes to study a selection of geometric subjects that might have a sturdy impact on architectural design. Each year, a different topic is proposed, aiming to introduce students into themes that are generally unexplored within the course and whose complexity clearly justifies the use of digital media.

On the academic year of 2016/2017, the teamwork focused on solid tessellations, with the following as main purposes:

- to introduce students into polyhedral geometry and in the subject of space-filling polyhedra and solid tessellations, and their possible relations with architecture;
- to overcome the characteristic abstraction of their underlying geometrical structures, by regarding them from an architectural - spatial - standpoint, and trying to explore, in the design process, their intrinsic spatiality;
- to explore 3D computer modelling as a support (and driving force) to study, represent and communicate a geometric design assignment.

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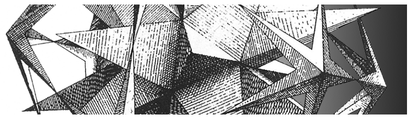
ISBN: 978-989-98926-8-2 | https://doi.org/10.24840/978-989-98926-8-2



POLYHEDRA AND SOLID TESSELLATIONS

Studying and modelling polyhedra in the educational context stands as one of the most interesting subjects through which geometric concepts can be explored, given not only their tangibility, but the myriad of mathematical contents and branches they allow students to be introduced into [03: 135]. For higher education courses in which modelling and materializing space is a topmost concern such as Architecture, researches on polyhedra might be regarded as a valuable subject matter. This is particularly true for under-graduate students, especially if they are given the possibility to model polyhedra virtually with three-dimensional modelling software, through which students can gain further knowledge on geometric concepts and operations, while exploring them as if "directly in space" [04, 507]. The full extent through which students' geometric reasoning can be stimulated and developed in a CAD educational environment is yet to be determined, and the experiment we are currently presenting is but a minor contribution to this discussion.

Understanding the conditions through which polyhedra can fill space around a vertex, and exploring the concepts of "plesiohedra" [05], space-filling polyhedra, primary paralelohedra and the twenty-eight possibilities [06, 07] through which polyhedra outline uniform solid tessellations or honeycombs, is of particular interest for students of architecture, given their potential for architectural design, for instance, as constructive modules or spaceframes. In this regard, six (Fig. 1) of the thirteen honeycombs considered as analogues to the semiregular plane tessellations, that Conway et al. designate as "architectonic tessellations" [02: 292-298], were selected as subject matter for a teamwork.

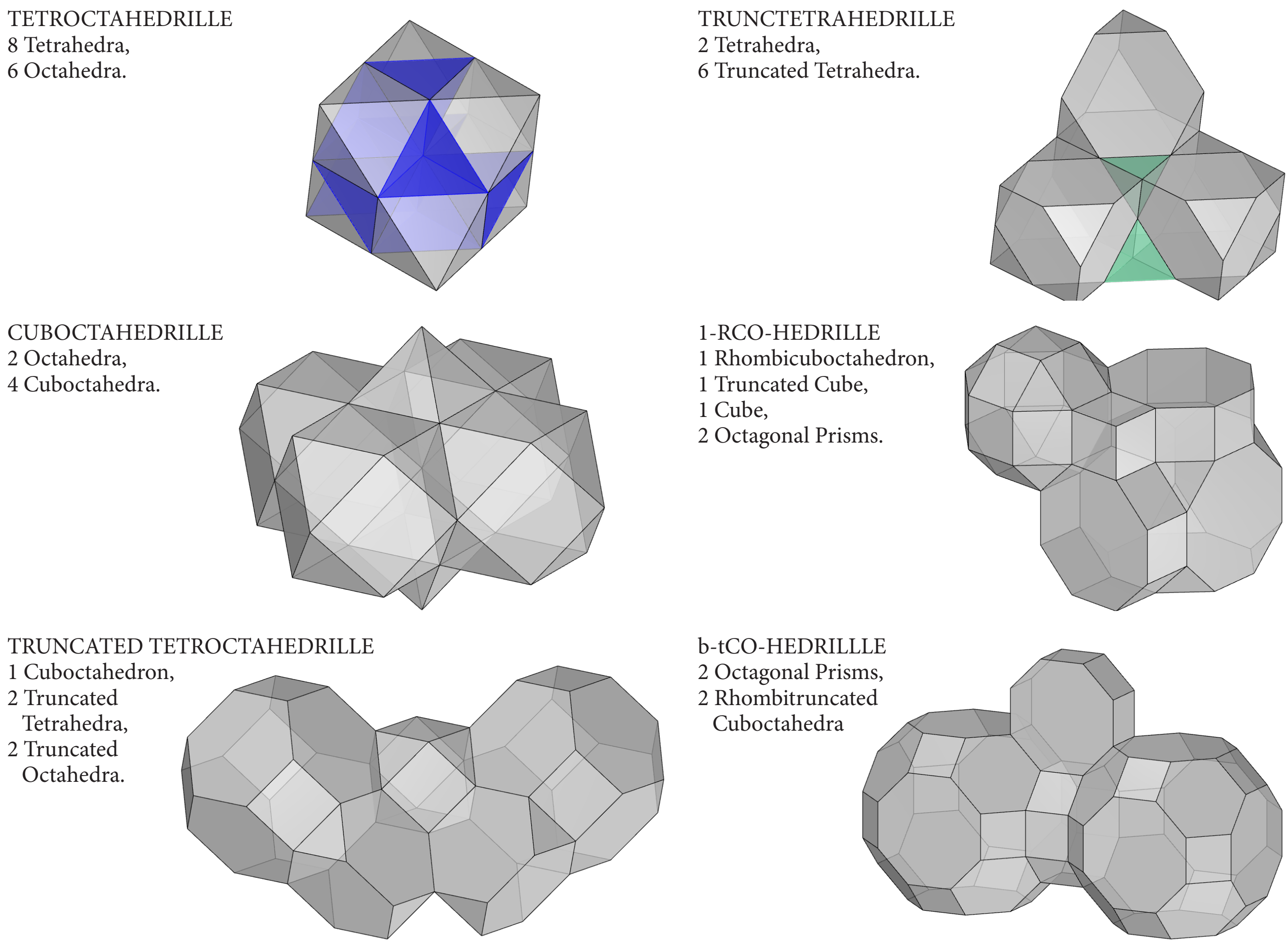


Fig. 1 - The six Architectonic Tessellations selected for the teamwork (designations according to [02]).

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THE DIDACTIC EXPERIMENT

The subject of Solid Tessellations was proposed as leitmotif for a teamwork, in which students, divided in groups of four, were asked to explore one architectonic tessellation, with the purpose of creating a spatial structure to be placed at one of the exterior spaces of the Faculty.

The time scheduled for the assignment was six weeks, and involved six classes with an average of 24 students each. The work was developed according to a methodology that comprised the following steps:

- The presentation of the theme and theoretical framework;
- A brief introduction to the software and demonstration of the commands that would be most relevant for the assignment;
- An understanding of the geometric concepts involved, through preliminary modeling experiments;
- The design and development of the Project itself,
- The presentation of the projects.

From the 28 different sets of convex uniform tessellations, six of the thirteen architectonic tessellations were selected, and one of them assigned to each of the six groups of the different classes. Each group was then proposed to manipulate virtual models of the different polyhedra of their tessellation, aiming to find out, by themselves, how each set would fill space. The task proposed afterwards aimed for students to diverge beyond the abstracting nature of the geometric structure under analysis, considering different possibilities of conceiving a habitable structured space to be installed in the garden's setting of the Faculty.

Following a theoretical introduction, the development of the work in CAD environment was structured sequentially in the following phases:

Phase 1 - Tessellation's modelling and research for its possibilities of multiplication through translation and rotations, exploring different operations of geometric transformations;

Phase 2 - Adapting the geometrical structure conceived to the location chosen for the assignment, by adjusting its scale and orientation, and considering the pos-sibility of including a cutting plane, as a disruptive and creative element, that would trim the entire structure or only part of it;

Phase 3 - With the geometric structure outlined, conceiving the materialization of the spatial structure, considering the following as rules: (1) structural bars in correspondence to the polyhedron's edges, leaving its faces open; (2) total or partial closure of some of the faces, using its vertices as a drawing reference;

Phase 4 - Preparing the communication of the design project, through a poster and a short movie or, alternatively, a mock-up.

The accomplishment of this task by the six classes resulted in the production of 33 geometric structures with six different convex uniform tessellations. The multiplicity and richness of the results, explicit on the different habitable structures imagined, and their spatial diversity are a clear testimony of the vast potential that solid tessellations (and, particularly, the architectonic) might bring as a creative leitmotif for students of architecture. This statement is particularly evident and interesting to look at, when we analyze different works developed from the same tessellation, that originated completely different spatial concepts, as shown in Figs. 2 and 3.

CONCLUSION

Aiming to describe the whole experience in further detail, the full paper that this extended abstract intends to propose, will discuss and acknowledge the potential that the described didactic experiment embodied in the context of the Geometry's course to which it was proposed. This potential was confirmed, in many levels, by the students engagement with the challenge itself; through the contribution of the selected studies and researches on polyhedral geometry for the development of the students' geometrical knowledge and spatial intelligence; the potential of digital tools to support the development of the design process; and, more importantly, the way in which an improved knowledge on the geometric concepts and operations involved was achieved.

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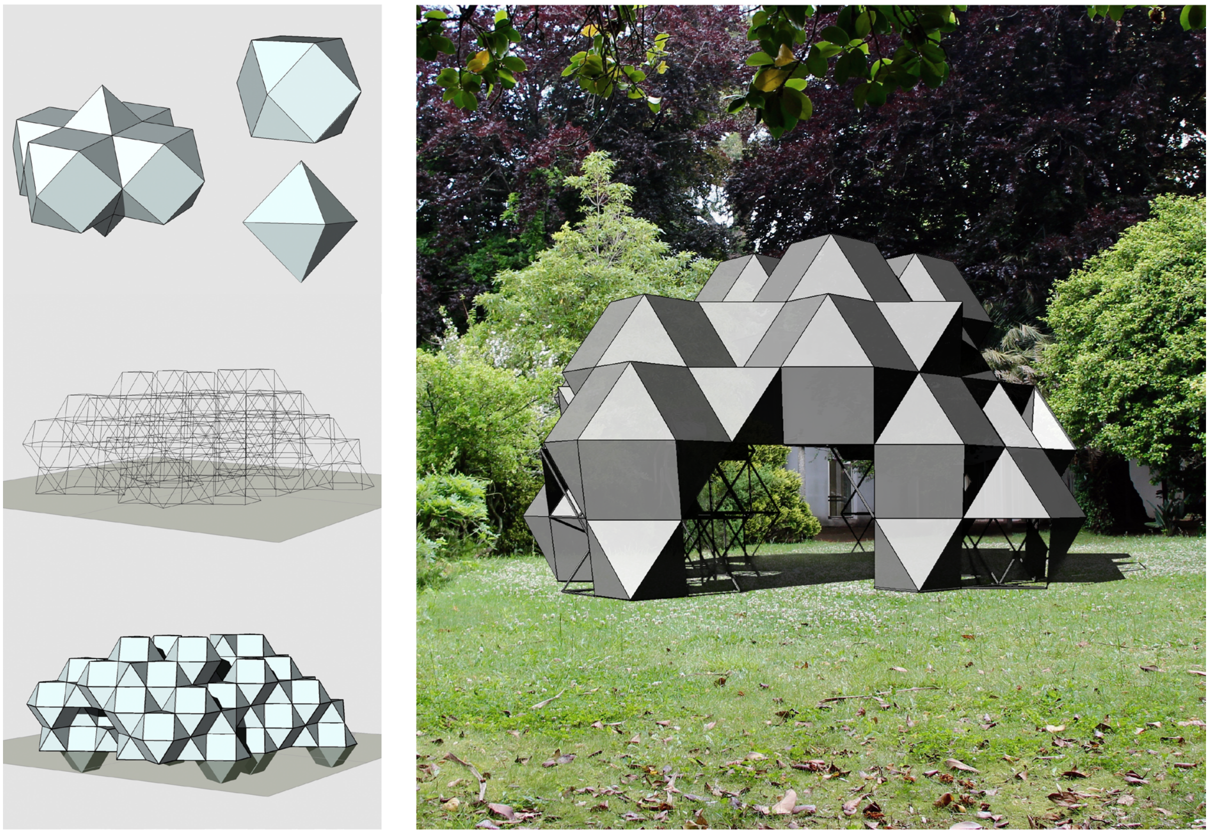

Fig. 2 - "(In)Tangível", Carolina Correia, Cynthia Machado, Nuno Delgado, Pedro Gouveia, 2016-17.

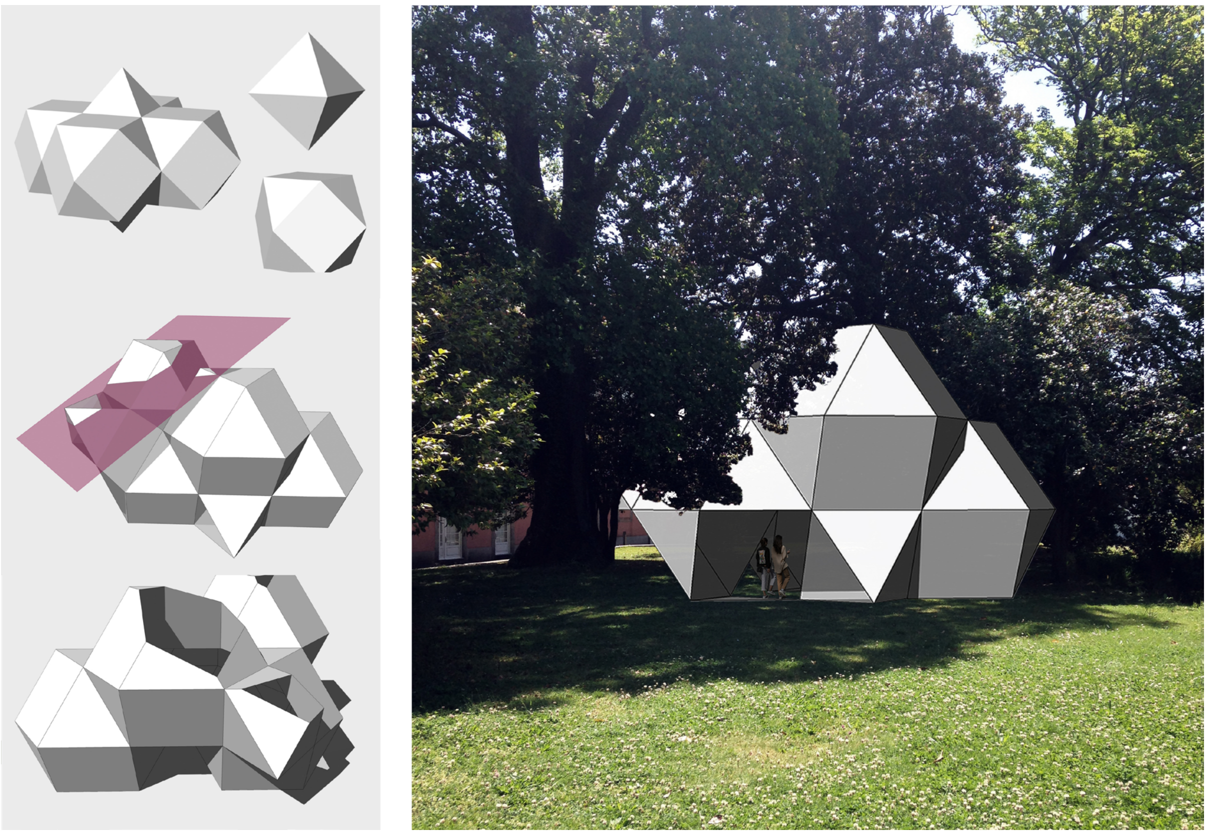

Fig. 3 - "Ponto de Paragem", Eliana Santos, Inês Mateus, Joana Gonçalves, Matheus Aliseda, 2016-17.

ISBN: 978-989-98926-8-2 | https://doi.org/10.24840/978-989-98926-8-2

REFERENCES

[01] Coxeter, H. S. M. (1973). *Regular Polytopes*. New York: Dover Publications.

[02] Conway, J. H., Burgiel, H., & Goodman-Strauss, C. (2008). *The symmetries of things.* Wellesley, MA: A.K. Peters.

[03] Pedersen, J. (1988). Why Study Polyhedra? In Senechal, M., & Fleck, G. M. (Eds.). *Shaping Space: A polyhedral approach*. Boston: Birkhäuser (pp. 133-147).

[04] Jakobsen, I. T., & Matthiasen, J. (2014). Descriptive Geometry and/or Computer Technology? What Mathematics is Required for Doing and Understanding Architecture? *Nexus Network Journal*, 16(2), 505-516. doi:10.1007/s00004-014-0199-3

[05] Grünbaum, B., & Shephard, G. C. (1980). Tilings with congruent tiles. *Bulletin of the American Mathematical Society* 3(3) 951-974. doi:10.1090/s0273-0979-1980-14827-2

[06] Grünbaum, B. (1994). Uniform tilings of 3-space. *Geombinatorics* 4: 49-56

[07] Viana, V. (2018). From Solid to Plane Tessellations, and Back. *Nexus Network Journal*, 20(3), 741-768. doi:10.1007/s00004-018-0389-5