

Flow liquefaction susceptibility of iron ore mining tailings from CPTu data: State parameter and yield-stress-ratio approaches

Sensibilité à la liquéfaction en flux des résidus miniers de minerai de fer à partir des données du CPTu: approches des paramètres d'état et du rapport de contrainte-rendement

António Viana da Fonseca*, Fausto Molina-Gómez*, António Fonseca* & Davide Besençon*⁺

*CONSTRUCT-GEO/IC, Faculdade de Engenharia da Universidade do Porto, Portugal (FEUP), viana@fe.up.pt

⁺Engineering Faculty in Earth Sciences, Escuela Superior Politécnica del Litoral (ESPOL), Ecuador

Marcos Arroyo

Department of Civil Engineering, Geosciences Division, Universitat Politècnica de Catalunya, Spain

ABSTRACT: Flow liquefaction is a geotechnical hazard in mine tailings storage facilities. Susceptibility to liquefaction is typically evaluated with the CPTu by estimating the state parameter (ψ). This parameter allows estimating the contractive or dilative behaviour combining density and stress level under critical state soil mechanics (CSSM) framework. Soils with soil behaviour index (I_c) less than 2.60 and contractive behaviour are more susceptible to trigger liquefaction. Contractive/dilative behaviour can be also evaluated by alternative approaches, such as yield-stress-ratio (YSR). YSR values under a threshold value also given by CSSM indicate a contractive behaviour, while values above indicate dilative behaviour. This paper addresses the interpretation of CPTu data from a Brazilian mine tailings dams, categorising their states in terms of contractive/dilative behaviour to assess the liquefaction susceptibility of these complex materials. Although the two procedures applied have similar basis, the results obtained have similar basis, but the results obtained were not always coincident. The reasons for that are discussed in this paper, highlighting the justifications for such differences.

RÉSUMÉ: La liquéfaction en flux est un danger géotechnique dans les installations de stockage des résidus miniers. La sensibilité à la liquéfaction est généralement évaluée avec le CPTu en estimant le paramètre d'état (ψ). Ce paramètre permet d'estimer le comportement contractile ou dilatant combinant densité et état de contrainte de confinement sous le cadre de la mécanique des sols à l'état critique (CSSM). Les sols avec un indice de comportement du sol (I_c) inférieur à 2,60 et un comportement contractile sont plus susceptibles de déclencher liquéfaction. Le comportement de contraction / dilatation peut également être évalué par des approches alternatives, telles que le rapport limite d'élasticité (YSR). Les valeurs de YSR en dessous d'une certaine limite basée sur le CSSM indiquent un comportement de contraction, tandis que les valeurs ci-dessus indiquent un comportement dilatant. Cet article traite de l'interprétation des données CPTu provenant de barrages de résidus miniers brésiliens, en catégorisant leurs états en termes de comportement contractile/dilatant pour évaluer la susceptibilité à la liquéfaction de ces matériaux complexes. Bien que les deux procédures appliquées aient une base similaire, mais ils ne sont pas toujours coïncidents. Les raisons en sont discutées dans cet article, en mettre en évidence les justifications de ces différences.

KEYWORDS: site investigation, critical state soil mechanics, tailing storage facilities, liquefaction.

1 INTRODUCTION

Static liquefaction in Tailings Storage Facilities (TSF) has resulted in two recent well-documented case-histories of catastrophic collapse in Brazil: Fundão dam in Mariana, on November 5, 2015, and B1 dam in Brumadinho, on January 25, 2019. The public reports from review panels (Morgenstern et al., 2016; Robertson et al., 2019), which investigated the cause of the failures suggest that loose saturated silty and sandy tailings liquefaction was somehow triggered, causing deformation-induced undrained stress-paths in those sensitive granular materials. To reach this conclusion, piezocone penetration tests (CPTu) data –performed prior to the TSF failures– were interpreted under a critical state approach, together with laboratory tests results with remoulded representative samples that allowed the definition of the Critical State Locus (CSL).

CPTu methods provide a rapid screening method for liquefaction susceptibility using well-calibrated procedures for determining the in situ state parameter (ψ). CPTu represents an inverse boundary value problem since the tip resistance is itself a result of the intrinsic material properties and the state of the soil. Although methods based on full-field test simulation are now becoming available (Monforte et al. 2021). Current approaches to estimate ψ , using normalised CPTu results, rely on empirical approximations considering the compressibility (λ), inspired by cavity expansion theory and critical state soils mechanics (SCE-

CSSM), like recently published method for analysis of cone penetration in brittle liquefiable soils (Monforte et al., 2021), or use empirical approximations to calculate λ .

However, to obtain reliable results, CPTu data must be complemented with information obtained by laboratory tests, namely triaxial testing that allow defining the CSL. In engineering practice, the most applied method to estimate ψ uses stress normalised tip resistance and soil behaviour type index (I_c), which indirectly incorporates compressibility and other soil behaviour and CSL parameters.

Several models –but not may– have been used to analyse catastrophic cases where the trigger mechanism was clearly shown to be of static origin conditions, like the recent failures in Brazil (Fundão, in 2015, and Brumadinho, in 2019) and Australia, Cadia (2018). The public reports (Morgenstern et al. 2016, Robertson et al. 2019, and Jefferies et al. 2019) included numerical analyses with such models, in particular, the state parameter ψ (Been and Jefferies 1985) –based constitutive model NorSand (Jefferies 1993). More recently, a new report from CIMNE and UPorto worked with an alternative model, Clay and Sand Model (CASM), originally developed by Yu (1998) and its improvements that allow flow instability (Gens, 2019).

This paper addresses the interpretation of CPTu data from two TSF historical cases, Fundão and Brumadinho, which collapsed by liquefaction triggering of very fragile iron tailings. Such an interpretation involves the categorisation of these

geomaterials in terms of contractive/dilative behaviour by estimating the ψ profiles –using the Robertson (2010) and Plewes, et al. (1992) methods– and the yield-stress-ratio (YSR) profiles, originally introduced by Mayne (2014) and further developed for flow liquefaction assessment by Mayne (2017). Both ψ and YSR approaches are based on the CSSM framework. Therefore, both approaches can be used analogously to detect the contractive/dilative behaviour of geomaterials, providing relevant insights about how liquefaction susceptibility in TSF can be successfully assessed.

2 INTERPRETATION OF CPTU DATA BY THE CSSM

2.1 State parameter

One of the main advantages of CSSM framework is the combination of initial stress-state and void ratio to identify contractive or dilative soil behaviour. Been & Jefferies (1985) introduced the concept of state parameter (ψ), defined as:

$$\psi = e_0 - e_{cs} \quad (1)$$

where e_0 and e_{cs} are void ratios in situ (at rest) and at critical state, respectively, for the same mean effective stress (p'). Such stress has to be computed from the overburden vertical effective stress (σ'_{v0}) and a reliable estimation of the rest coefficient of earth pressure ($K_0 = \sigma'_{ho}/\sigma'_{v0}$). Soils with a state denser than the critical state (i.e. $\psi < 0$) will be dilative and, in contrast, soils with a state looser than the critical state (i.e. $\psi > 0$) will be contractive. Soils with contractive behaviour are particularly more susceptible to liquefaction. To account for measurement and method uncertainty a threshold $\psi = -0.05$ is adopted as a liquefaction indicator limit for practical purposes (Jefferies & Been, 2015).

To estimate ψ from CPTu results, different methods are available. Robertson (2010) proposed the following relation:

$$\psi = 0.56 - 0.33 \log(Q_{tn,cs}) \quad (2)$$

being $Q_{tn,cs}$ the normalised cone resistance in granular soils referred to an equivalent clean sand defined by (Robertson & Wride, 1998):

$$Q_{tn,cs} = K_c Q_{cs} \quad (3)$$

where K_c is a correction factor, which considers grain characteristics, fines content, mineralogy and plasticity of the soil; K_c can be estimated using soil behaviour index (I_c), proposed by Robertson (2009), as follows:

$$K_c = 1 \quad \text{if } I_c \leq 1.64 \text{ or } I_c < 2.36 \text{ \& } Fr < 0.05 \quad (4)$$

$$K_c = -0.403I_c^4 + 5.58I_c^3 - 21.63I_c^2 + 33.75I_c - 17.88 \quad \text{if } I_c > 1.64 \quad (5)$$

Q_{tn} is a normalised cone parameter to evaluate soil liquefaction calculated by the following relation:

$$Q_{tn} = \left(\frac{q_t - \sigma'_{v0}}{Pa} \right) \left(\frac{Pa}{\sigma'_{v0}} \right)^n \quad (6)$$

where q_t is the net tip resistance, σ'_{v0} and σ'_{v0} are the total and effective vertical stresses, respectively, Pa is the atmospheric pressure, and n is the variable stress exponent depending on I_c , which is estimated by an iterative procedure using:

$$n = 0.381I_c + \left(\frac{\sigma'_{v0}}{Pa} \right) - 0.15 \quad (7)$$

The state parameter may also be obtained using an empirical method proposed by Plewes et al. (1992) –hereafter referred as Plewes method– described by equations 8-15.

$$\psi = - \frac{\ln \left(\frac{Q_p}{\bar{k}} \right)}{\bar{m}} \quad (7)$$

$$\bar{Q}_p = Q_p \cdot (1 - B_q) + 1 \quad (8)$$

$$Q_p = \frac{(q_t - p)}{p'} = \frac{3Q_t}{(1 + 2K_0)} \quad (9)$$

$$Q_t = \frac{(q_t - \sigma'_{v0})}{\sigma'_{v0}} \quad (10)$$

$$Fr = \frac{fs}{q_t - \sigma'_{v0}} \quad (11)$$

$$B_q = \frac{u - u_0}{q_t - \sigma'_{v0}} \quad (12)$$

$$\bar{k} = \left(3 + \frac{0.85}{\lambda_{10}} \right) \cdot M_{tc} \quad (13)$$

$$\bar{m} = 11.9 - 13.3\lambda_{10} \quad (14)$$

where Q_p is the tip net resistance normalised by the mean effective stress, Q_t is the tip normalised cone resistance, Fr is the normalised friction ratio, B_q is the normalised excess pore pressure, M_{tc} is the critical state friction ratio, and \bar{k} and \bar{m} are soil-specific coefficients.

Therefore, Plewes method needs CSL values, like the angle of friction, or the M_{tc} (in compression stress path) and the inclination of the CSL in $e:\log p'$ space, λ_{10} . This last, although ideally being obtained in the laboratory by triaxial testing, can be derived by default, when laboratory data are not available, by correlations with the friction ratio ($R_f = (q/f_s)$ in CPTu:

$$\lambda_{10} = \max \left(0.01; \frac{\min[R_f; 7]}{10} \right) \quad (16)$$

This combination between CPTu measurements and key parameters defined through advanced laboratory tests provides a better approximation of ψ , due to the direct incorporation of soil compressibility –expressed by the slope of CSL (Shuttle & Cuning, 2007).

2.2 Yield stress ratio

The stress history of soils is a primary characteristic that relates to many fundamental aspects of soil behaviour (Mayne, 2017). The effective yield stress (σ'_p) provides a generalised form to define the boundaries between elastic and plastic behaviour of soils (Ku & Mayne, 2013). Yield stress ratio (YSR) is a normalised form of the stress history, which is equivalent to the overconsolidation ratio (OCR). Mayne (2014) proposed a method combining the cavity expansion theory and critical state soils mechanics (SCE-CSSM) to estimate the stress-history of soils from CPTu data. Agaiby & Mayne (2019) compiled a database of 93 field sites to formulate a general procedure to estimate the YSR profile as a function of net cone tip resistance. Such a database covered a wide variety of geomaterials, including, such as clays, silts, sands, and mixed soil types (see Equations 17-18).

$$YSR = \frac{\sigma'_p}{\sigma'_v} = \frac{0.33(q_t - \sigma'_{v0})^{m'}}{\sigma'_{v0}} \quad (17)$$

$$m' = 1 - \frac{0.28}{1 + (I_c/2.65)^{25}} \quad (18)$$

YSR profile is used to estimate the contractive/dilative behaviour of geomaterials by comparing with the corresponding

value at critical state (YSR_{CSL}). This relative position is associated to the stress history on soil behaviour (Mayne, 2017):

$$YSR_{CSL} = \left(\frac{2}{\cos \phi'} \right)^{1/\Lambda} \quad (19)$$

$$\Lambda = 1 - \frac{\kappa}{\lambda} = 1 - \frac{Cs}{Cc} \quad (20)$$

where ϕ' is the critical friction angle, κ the is swelling index in the $e : \log p'$ space and λ is the slope of CSL. By default, this ratio can be assessed by the ration between dividing unidimensional swelling and compression indices (Cs/Cc), as terminated in oedometric tests. The value of Λ ranges 0.8 ± 0.1 for most soils, including clays and sands (Been et al., 1988; Mayne, 1988, 2017). On the other hand, considering a range of ϕ' between 30° to 45° and a representative value $\Lambda = 0.8$ (Agaiby & Mayne, 2019) the corresponding range of values YSR_{CSL} is typically close to 3 (Mayne, 2017). The YSR profile compared with its corresponding YSR_{CSL} threshold profile provides an assessment of contractive/dilative behaviour, which leads to a rational procedure for screening liquefaction susceptibility.

3 DESCRIPTION OF CASE STUDIES

3.1 Fundão TSF

On November 5, 2015, the Fundão tailings dams, constructed by upstream method, located in Mariana (Minas Gerais, Brazil) collapsed. The TSF released 32 million m^3 of iron ore tailings, consisting of a mixture of sandy/silty with a dam break volume that claimed the lives of 19 villagers in Bento Rodrigues town. A report issued in August 2016 by a panel of experts (Morgenstern et al., 2016) indicated the collapse was induced by flow liquefaction of sandy loose tailings below and behind the left abutment of this TSF. The panel identified that liquefaction was triggered by induced deformations in a compressible layer composed by slimes (very fine tailings).

Reid (2019) digitalised and made available the data of five CPTu carried out before the failure of Fundão TSF –from January to March 2015. These data were originally provided in the Panel report (Morgenstern et al., 2016). Moreover, Reid (2019) provided additional insights on the causes that can explain the liquefaction susceptibility interpreting the CPTu results to obtain the ψ profiles using three screening-level methods, including Robertson (2010) and Plewes methods. In this work, F-02 and F-05 CPT are studied. Figure 1 shows the location of the CPT using an aerial photograph taken on July of 2015 to illustrate the conditions at that time of Fundão TSF. Figure 2 presents the CPT data of F-02 and F-05.



Figure 1. Location of CPTu in Fundão TSF (map data from Google Earth®)

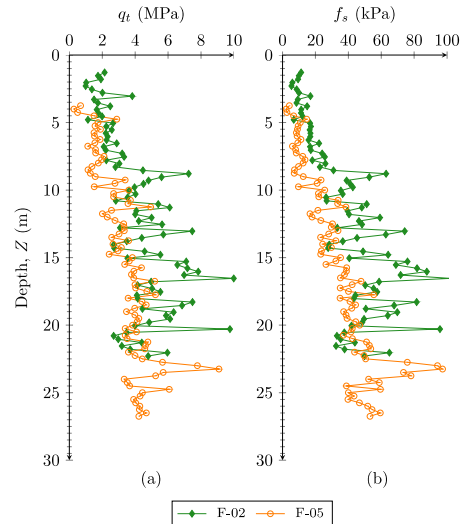


Figure 2. CPT results at Fundão TSF: (a) q_t profiles; (b) f_s profiles.

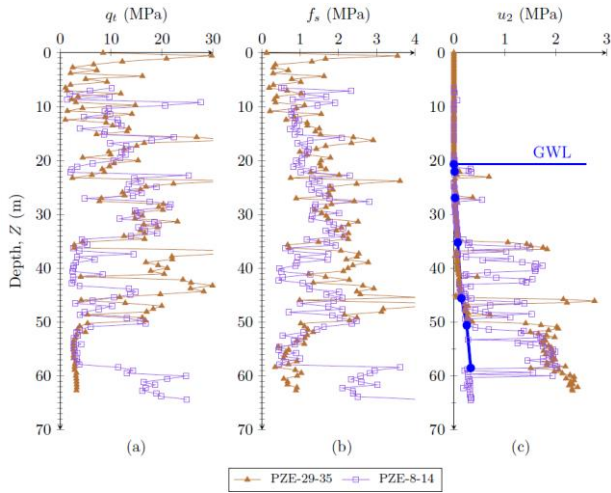
3.2 Brumadinho TSF

The Brumadinho disaster occurred due to the collapse of B1 dam at Vale Córrego do Feijão iron ore mine TSF (Minas Gerais, Brazil) on 25 January, 2019, when the 85 m high upstream dam suffered a catastrophic failure. The collapse occurred in approximately 10 seconds, releasing 12 million m^3 of iron ore tailings. The mudflow due to dam break advanced downstream through the offices of the mine –including a ‘cafeteria’ during lunchtime. Besides, the flow destroyed houses, farms, inns and roads of Brumadinho municipality. About 270 people died and 11 are still missing.

To identify the causes of the failure of B1 dam, in Brumadinho, an expert panel review was commissioned by Vale. A report issued in December 2016 by such a panel of experts (Robertson et al., 2019) concluded that the collapse was due to flow liquefaction of the tailings in the TSF, like in Fundão TSF. Several possible triggers were examined by the Panel and additional studies are still in development about this important aspect. Within the scope of such studies, two representative CPTu, namely PZE-29-35 and PZE-8-14, are interpreted to define the state of the soils before the TSF collapse. Both CPTu were carried out in 2018 in the central portion of the dam. Figure 3 shows the location of the CPTu located in the zone where the collapse occurred. Figure 4 presents the CPTu data of PZE-29-35 and PZE-8-14. This aerial photograph taken on July of 2018, illustrating the conditions of B1 dam before its collapse.



Figure 3. Location of CPTu in B1 dam in Brumadinho TSF (map data from Google Earth®)


 Figure 4. CPTu results at Brumadinho TSF: (a) q_t profiles; (b) f_s profiles.

4 APPLICATION OF ψ AND YSR APPROACHES

The three screening methods, introduced in Section 2, were applied for estimating the contractive/dilative behaviour of iron ore tailings for both case studies using CPTu data. Table 1 presents the soil parameters used in this study to outline ψ and

YSR profiles of these geomaterials. The parameters in Table 1 were inferred from Morgenstern et al. (2016) and Robertson et al. (2019) for Fundão and Brumadinho, respectively.

Table 1. Soil parameters for the computation of screening methods.

Parameter	Method	Fundão	Brumadinho
γ (kN/m ³)	YSR	22	25
K_0	Plewes	0.5	0.5
M_{ic}	Plewes	1.33	1.38
ϕ' (°)	YSR	33	34
κ	YSR	$0.1\lambda_{10}$	$0.1\lambda_{10}$
λ_{10}	YSR	0.055	0.039

Figures 5, 6 and 7 present the interpretation results of CPTu data in terms of soil behaviour type index (Robertson, 1990), state parameter (Plewes et al., 1992; Robertson, 2010) and yield stress ratio (Mayne, 2017). For the iron tailings from Fundão TSF, the soil profile drawn from the material index behaviour (I_c) revealed layers composed of tailings that can behave like ‘silty sand to sandy silt’ and ‘clayey silt to silty clay’. However, F-05 showed tailings, between 12 m to 20 m depth, which is in the limit of clays soil class, i.e. $I_c \approx 2.8$. This layer has been identified previously by Morgenstern et al. (2016) and Reid (2019), highlighting the presence of compressible soils that can induce pore pressure built-up during rapid loading. Moreover, I_c profiles revealed that the site investigation point F-02 has coarser materials –more granular– than the site investigation point F-05.

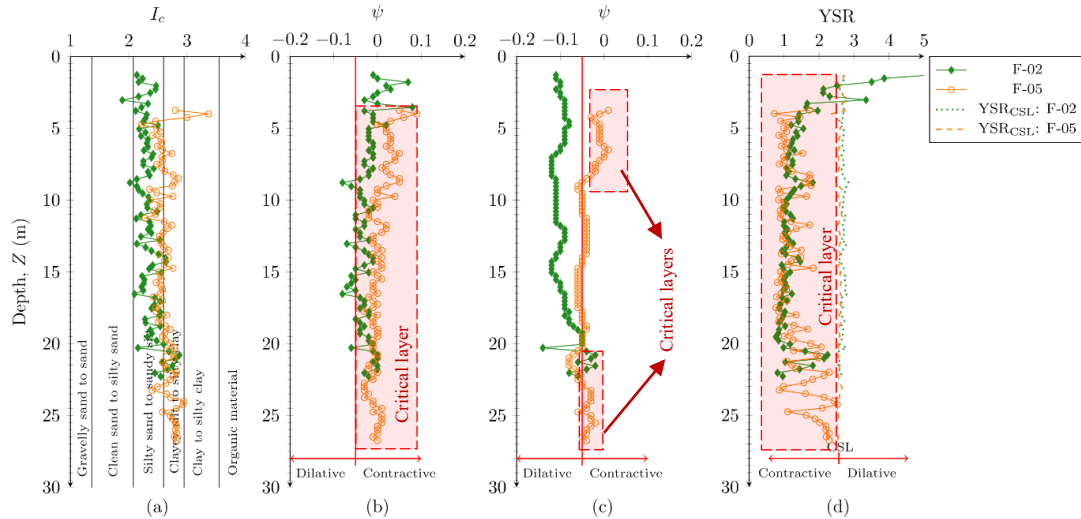


Figure 5. CPTu interpretation in Fundão TSF: (a) soil behaviour index profile; (b) Robertson method; (c) Plewes method; (d) Mayne method.

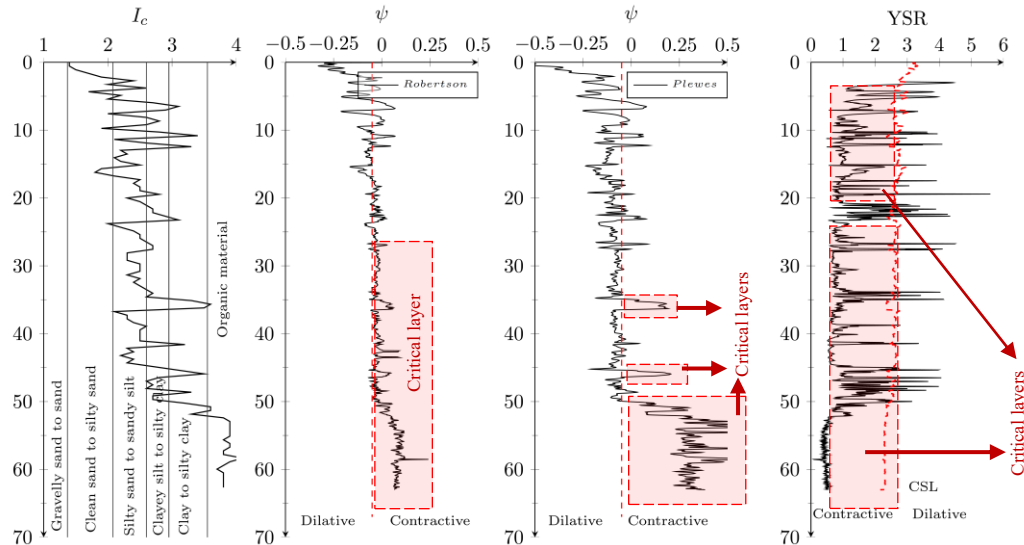


Figure 6. CPTu interpretation in Brumadinho TSF – PZE-29-35: (a) soil behaviour index; (b) Robertson method; (c) Plewes method; (d) Mayne method.

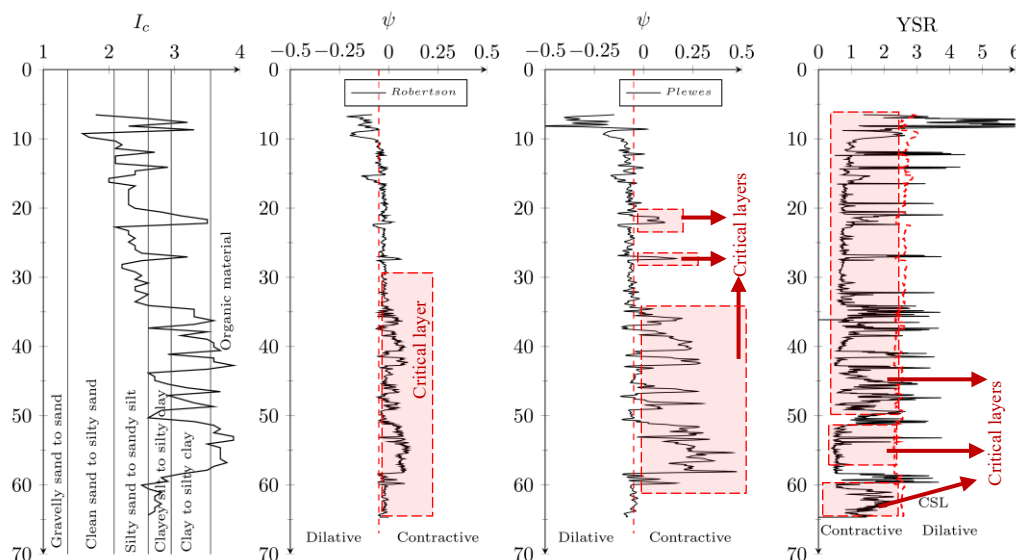


Figure 7. CPTu interpretation in Brumadinho TSF – PZE-8-14: (a) soil behaviour index; (b) Robertson method; (c) Plewes method; (d) Mayne method.

On the other hand, for the iron tailings from B1 dam in Brumadinho TSF, the soil profile from material index shows a complex layering with tailings that covers a wide variety of soil behaviour, including ‘clean sand to silty sand’, ‘silty sand to sandy silt’, ‘clayey silt to silty clay’ and ‘clay to silty clay’ behaviours. Both soil profiles revealed the alternating presence of granular materials ($I_c < 2.6$), with loose conditions which are favourable to trigger liquefaction. Although site investigation points PZE-29-35 and PZE-8-14 are only 75 m apart, they present relevant differences in terms of I_c profiles. Such differences are mainly at 35 m to 48 m depth, where the soil classes significantly changed, showing I_c values between 2.1 to 3.8, this last one characteristic of slimes.

In terms of liquefaction susceptibility, Figure 5 showed differences between interpretations of the three screening methods, clearly evidenced between 12 m to 20 m depth. The ψ profiles computed using the Robertson method and YSR profiles revealed a quasi-continuous critical layer –susceptible to trigger soil liquefaction– from 3 m to 27 m depth in Fundão TSF. On the other hand, ψ profiles estimated from Plewes method exposed a single 8 m high critical layer for the F-02 site investigation point and two critical layers of similar size for the F-05 site investigation point, which clearly differs from the results of Robertson and YSR methods.

For the case of B1 dam in Brumadinho TSF, Figures 6 and 7 revealed differences between all screening methods. The results from the YSR method are the most pessimistic, giving an almost continuous indication of potentially liquefiable material along the profile. The results from Robertson method are less critical in the upper 25 m, but then indicate 35-40 m of a fully liquefiable profile beneath that level. On the other hand, Plewes method showed significant differences. In the site investigation PZE-29-35, it was identified three critical layers with two embedded layers composed of apparent stable tailings in critical layer in about 38 m and 45 m depth; whereas in the site investigation PZE-8-14, it was observed an almost continuous critical layer at 35 m to 58 m depth.

While in Fundão the critical layers were identified in tailings with q_t lower than 5 MPa, in Brumadinho critical layers were identified much with higher values, especially in the site investigation PZE-29-35. Such layers are mainly composed of sand and silt mixtures, that is, I_c higher than 2.05, but bellow of 2.8. Experimental evidence obtained in the laboratory has shown that silty sized tailings are sensitive and unstable geomaterials (Fourie & Papageorgiou, 2001; Carrera et al. 2011; Bedin et al., 2012). Besides, tailings with more plastic fines content are less

susceptible to trigger liquefaction (Li & Coop, 2019; Torres-Cruz & Santamarina, 2020), which is note case in these non-plastic tailings. Still, interbedded layers composed of soil mixtures in all profiles, which were produced by the segregation of materials during the construction of both TSF delineate the variations between dilative or contractive behaviour.

The presence of such layers demands the collection of undisturbed or representative integral samples at these depths for physical and mechanical characterisation in the laboratory. This characterisation allows estimating the factors influencing liquefaction susceptibility of iron tailings, such as grading, fines content and critical state parameters. On the other hand, the definition of critical state parameters provides reliable input data for the application of Plewes and YSR methods.

Therefore, the uncertainties and variations of CSL parameters of the interbedded layers composed of soil mixtures tailings, in both Fundão and Brumadinho case studies, explained the differences between the profiles of Figures 5, 6 and 7. However, further laboratory studies ongoing are not the focus of this paper.

Nowadays, the authors are conducting further studies to assess, in the laboratory, the geomechanical behaviour and liquefaction susceptibility of iron tailings with the purpose of providing relevant insights on the connection between ψ obtained through in situ and laboratory testing.

Specialised practitioners have increasingly pursued the use of CSM models in numerical analyses to evaluate the stability in TSF structures engineering practice, but some uncertainties have to be still solved, especially the ones that can have implications to the evaluation of in situ state parameter (ψ). The CSM based techniques that are used depend highly on the accurate assessment of ψ , utmost importance for the overall distribution of the stress-strain relations towards the instability stress ratio (η_{IL}), since this depends highly on in situ stress ratio ($K_0 = \sigma'_{ho}/\sigma'_{vo}$), in each and all points in the soil mass. It is consensual that this factor plus the principal stress angle α –related to fabric anisotropy and resulting in an induced shear stress (τ_{xy}) acting on the horizontal plane (as a result of the slope geometry)– are decisive for different strength conditions (e.g. the maximum (peak) and consequently, the residual) to which the stress state evolves after the instability. This principal stress angle is dependent on K_0 , as it is also a function of the deviatoric stress. Therefore some investment has to be made in testing TSF to evaluate how K_0 varies in distinct zones if these structures (more or less distant to the face of dams –crest– or the slope of piles).

To deal with these uncertainties in evaluating if these slopes can develop static liquefaction triggering, numerical studies should be performed varying the state parameter inferred from methods that depend of K_0 and α . This is obviously the case of CSM models, which use the mean effective stress for the normalisation of stress state –the fundamental reference to evaluate the position of stress-voids-relation to CSL.

5 CONCLUSION

Different methodologies for the interpretation of piezocone tests, based on the critical state soil mechanics framework, have been explored in this study. These methodologies were described and applied to delineate in situ profiles to estimate the liquefaction susceptibility of two TSF case studies in Brazil. Soils with in-situ $\psi < -0.05$ or $YSR < 3$ revealed contractive behaviour and high susceptibility for trigger liquefaction. The results of this paper showed differences between the profiles delineating with the addressed methods. These differences were attributed to the intrinsic variability of soil profiles –evidenced through interbedded layers composed of soil mixtures. Besides, these differences were associated with uncertainties in the input parameters used in the computation of Plewes and YSR methods, which depend on laboratory data. Therefore, the authors recommended further studies in the laboratory to characterise the physical and critical state parameters of iron tailings, providing a reliable connection between ψ obtained through in situ and laboratory testing. Such a connection will allow defining the method explored in this paper to better delineating the contractive/dilative behaviour profile from CPTu data.

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