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A COMBINED RESEARCH PLATFORM OF STRUCTURAL MORPHOLOGY, DEALING WITH THE ORDERED 3D SPACE

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Abstract: *The architecture of the human habitat and the whole domain of conceptual and physical space structures are mostly concerned with few basic 3D spatial features**:** 3D Space Networks-Lattices, Polyhedral (finite or infinite) close packing solids and space dividing surfaces. The interrelation of these prime 3D features provides for the Quintuplet Phenomenology of 3D Space: 3D Networks are associated with close-packing arrays of polyhedra and every dual network pair is associated with continuous surface partition, subdividing the space between the two. When combined with specific 'symmetry regimes', controlled by 3D Symmetry Space Groups, the focus is turned on Uniform 3D Space Networks-Lattices, with symmetrically identical vertex figures; on self-close-packing 3D polyhedra, and on highly symmetrical continuous surfaces, subdividing space between the dual, symmetrically identical 3D networks. Not even one of the a.m. features was exhaustively researched and enumerated in the past and that, mostly because no combined research platform was ever suggested. It is the author's belief and conviction that such a platform might be generated and operated, when based on manipulative scanning of the fundamental domains that are associated with the overall 3D Symmetry Space groups, provided that their exhaustive enumeration is complete.*

Keywords: Structural Morphology; Ordered Space; Symmetry Regime; Exhaustive Enumeration; Geometrical Uniformity.

INTRODUCTION

The architecture of the human habitat and the whole domain of conceptual and physical space structures and associated features is mostly concerned with few basic three-dimensional spatial features and their diverse interrelations:

- Networks (space lattices), arrays of interconnected vertices, representing physical or imaginary- virtual entities that might stretch to infinity and even to occupy n-dimensional space.
- Polyhedral finite or infinite volumetric solids, with plane or curved hyperbolical facets, that might be associated with compact close packing 0f 3D space.
- Space dividing surfaces, finite, or infinite, physical or virtual and imaginary, between any conceivable spatial entities, providing for their boundaries.

The structure and morphology of any configuration in 3D space might be represented by association of these features. Throughout history many attempts were performed to control intellectually this structural morphology domain and to accomplish the task of 'exhaustive enumeration' of its principal features; but all with a very limited success. Some failures are of a fundamental nature. No sound categorization (and no associated notation systems) of space networks, space packings and space dividing partitions is in existence. Even the claimed 'exhaustive enumeration of 3D symmetry space groups' by Fedorov, Schoenflies and Barlow, (1885/94), is raising in the authors mind serious doubts whether it is complete or not (!). It is high time to suggest a general research policy and program that carries the promise to confront the issue in its totality and promote a staged completion of the task.

BINDING MORPHOLOGICAL RELATIONS

The structure and morphology of any physical configuration in 3D space might be represented by the following morphological relations between the accounted features.

1. Every 3D network is associated with a specific polyhedral close packing.
2. All 3D networks come in dual network pairs.
3. Every dual network pair is associated with a hyperbolically extending 3D partition surface, subdividing the space between the two.
   The combination of the two dual networks, the two associated polyhedral compact-close packing modes and the associated 3D space partition surface between the dual networks pair constitute five spatial features, deeply associated, and interconnected with one another and provide for a critical characteristic of 3D space only. It is suggested to term this phenomenology as *The Quintuplet Phenomenology of 3D Space*.

The number of these morpho-topological spatial *Quintuplet* compounds is reaching to infinity and their order and periodicity characteristics stretch from the most accidental to the most ordered, symmetrically uniform. It should be stressed that:

- Determination of one feature of the five provides for the determination of the other complementary four.

- When periodic, all five constituents of the *Quintuplet* share the same symmetry regime.
- The *Level of the Quintuplet Constituent*'s periodicity-symmetry plays a defining role in their categorization as 3D spatial phenomena, ranging from the most accidental to the most symmetric-uniform periodicity level (Figures 1 and 2).
- Certain level of periodicity and spatial expansion nature leads to the evolution of 'Translation Networks' (Bravais -1848), controlled by translation symmetries that determine the shape, spatially defined, of their 'Translation Units'. Some of the translation networks may be associated with uniform (equi-vertex-edge) networks.
- A principal feature of the Bravais's Lattice translation units is the attached 'Order" characteristic, representing the number of the unit's fundamental domains, when associated with its 3D Symmetry Space Groups. This property characteristic defines the number of the equi-vertex figures, that might be associated with a related 'uniform network'.
- According to the prevailing crystallographic literature, maximal Order number associated with the 'Cubic Symmetry System' is reaching to 48, implying that no 'uniform network' related to Cubic Symmetry System might have more than 48 symmetrically identical vertex figures within one of its cubic translation Unit spaces.
- A conspicuous result of networks uniformity is the emergence of 'self-close packing' (identical) polyhedral solids.

One of the most mind-provoking 3D space networks categories is the one termed as self-dual**,** referring to a pair of two identical dual networks (Figures 2 and 3). Self-duality of networks results in a hyperbolical 3D partition surface that subdivides the entire space into two topologically-symmetrically identical sub-spaces. These self-dual networks partition surfaces are endowed with 2-fold symmetry characteristics, bearing a set of 2-fold rotation symmetry axes and or a set of inversion symmetry points, taking one network into coincidence with its complementary dual net, thus doubling the translation unit's 'fundamental domains. These symmetry elements are part of the associated partition surface's skeletal-generative structure. The number of this self-dual networks category members is very limited, justifying considering it as a very exclusive phenomenon.

Another worth mentioning *Quintuplet* related issue is that of the closely packing identical polyhedra: the phenomenon of the 'self-packs**'**. As a fact, every vertex of a uniform 3D network is a centroid of a self-packing volumetric polyhedral solid. Most of these polyhedral solids have hyperbolical surface facets, sometimes in combination with plane facets as well. Consequently, the number of self-packing polyhedra solutions corresponds to the number of possible uniform 3D network types, and they're both exhaustive enumerations are one and the same research issue. To be precise: Exhaustive enumeration of the self -packs in 3D space will be resolved following the completion of the exhaustive enumeration of uniform 3D networks.

STRUCTURAL MORPHOLOGY AND CRYSTALLOGRAPHY INTERFACE

In crystallographic (inorganic chemistry) science domain, uniform translation networks and the ordered polyhedral close packing configurations invoke the atomic-molecular-ionic patterns and the associated crystal structures. Many have contributed, over the years, to their research and development. Fedorov, around 1885, Schoenflies by 1889 and Barlow in 1894, must be especially regarded for their (considered to be) exhaustive enumeration (1885÷94) of the '230 3D Symmetry Space Groups' that are consistent with the 14 Bravais (translation) Lattices. Crystallographic literature, dealing with the space dividing surfaces between dual network pairs is invoking the 'Fermi surfaces' (1962). The shape of the 'Fermi surface is derived from the periodicity and symmetry of the crystalline lattice and from the occupation of electronic energy bands" ("Crystal_system", 2020).

It was stipulated by the author (ca. 40 years ago) that the only promising method of solving and exhaustively enumerating uniform 3D space networks is by exploring the interior of the 'symmetry space groups' related 'fundamental domains' for vertex locations that might generate uniform equi-vertex-edge network structures. It is manageable provided all spatial constructs of the fundamental domains, corresponding to all possible '3D symmetry space groups', are accounted, morphologically determined, and geometrically solved. Following this line of exploration, it was discovered that in cubic lattice's self-duality associated translation unit, the maximal number of 'fundamental domains' is doubled and reaching to 96 (Figure 3). It should be accepted as a proof that self-duality was not considered by Fedorov and others. Limiting research inquiries to 3D space and to the most periodic, symmetrically uniform characteristics of the a.m. *Quintuplet features*, it was realized that exhaustive enumeration of '3D symmetry space groups' should come first. The most critical observation and statement of this presentation is that the exhaustive enumeration of 3D Symmetry Space Groups, claimed by Fedorov & al, is not complete and reaching far beyond the claimed 230 items (!), and that because the self-duality feature of the Bravais lattices (except of the Hexagonal Lattice) and the associated additional symmetry elements, were overlooked and not considered as relevant.

COMBINED RESEARCH AND DEVELOPMENT PROGRAM

After considering the exhaustive enumeration of the 3D symmetry space groups, the author suggests a combined research and development platform and a program aiming at the exhaustive enumeration of the most symmetrically uniform features of the *Quintuplet Phenomenology*, concerning:

1. Uniform (equi-vertex-edge) 3D space networks-lattices.
2. Self-packing polyhedra of 3D space.
3. Hyperbolic continuous surfaces that subdivide between uniform dual network pairs.
4. Hyperbolic surfaces that subdivide between self-dual network pairs.

It is right to report here that few steps of the suggested research program are in progress, with fascinating and encouraging results. After completing the networks categorization and developing their notation system, according to the author's findings, the number of constructed- solved uniform networks is reaching, so far, beyond the 700 (!) and the number of the 3D symmetry space groups is reaching to 562 (!).

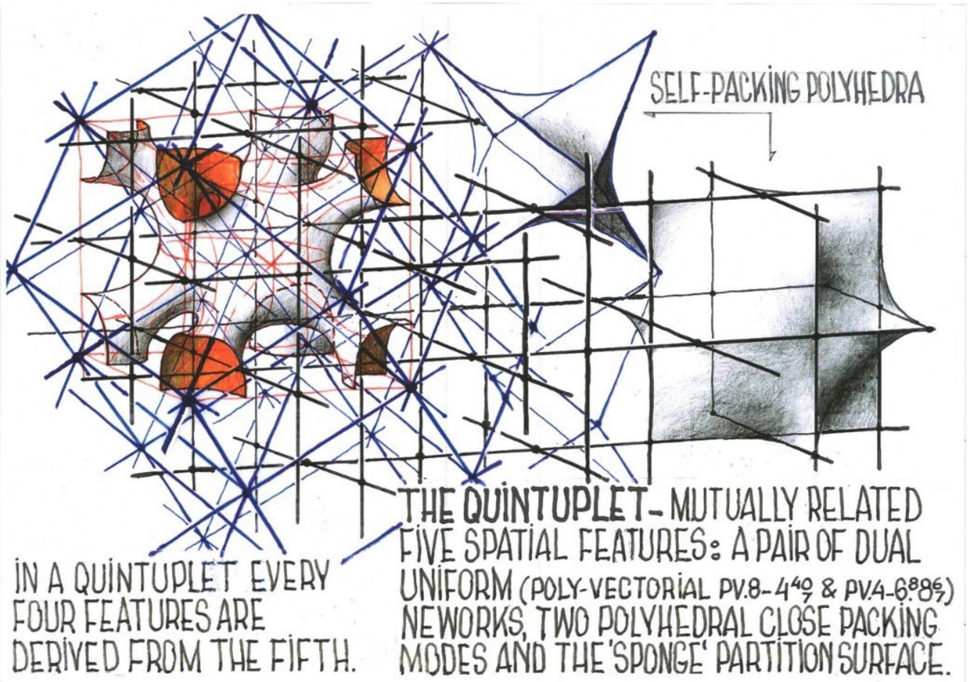

Figure 1 A *Quintuplet* relating to cubic symmetry system, with self-close packing polyhedral solids having hyperbolic facets.

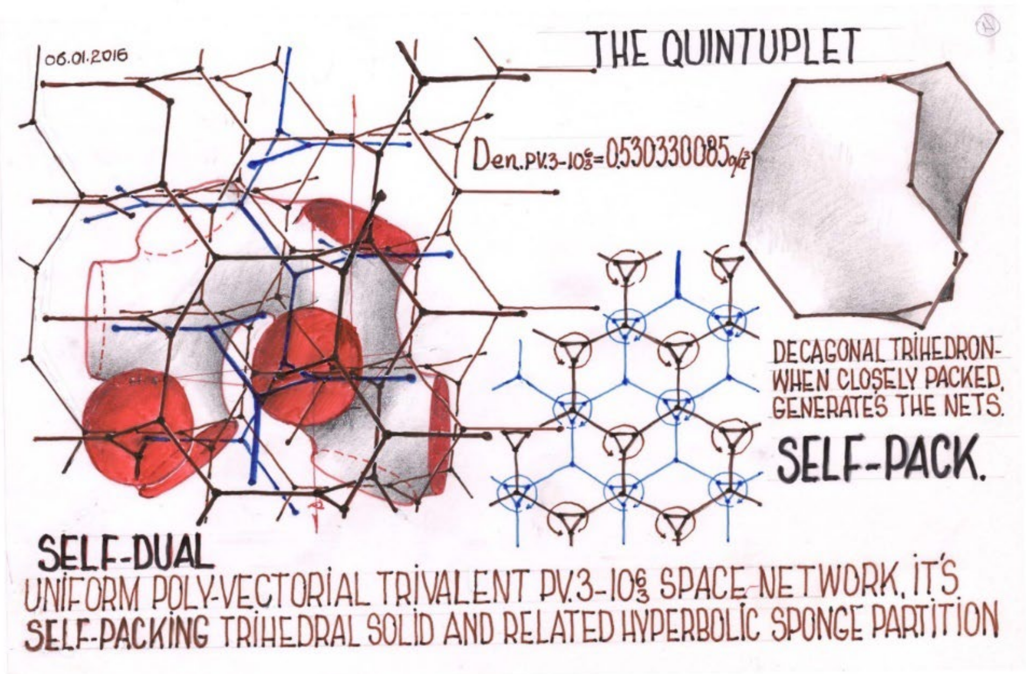

Figure 2 A self-dual tri-valent uniform network with self-packing trihedra.

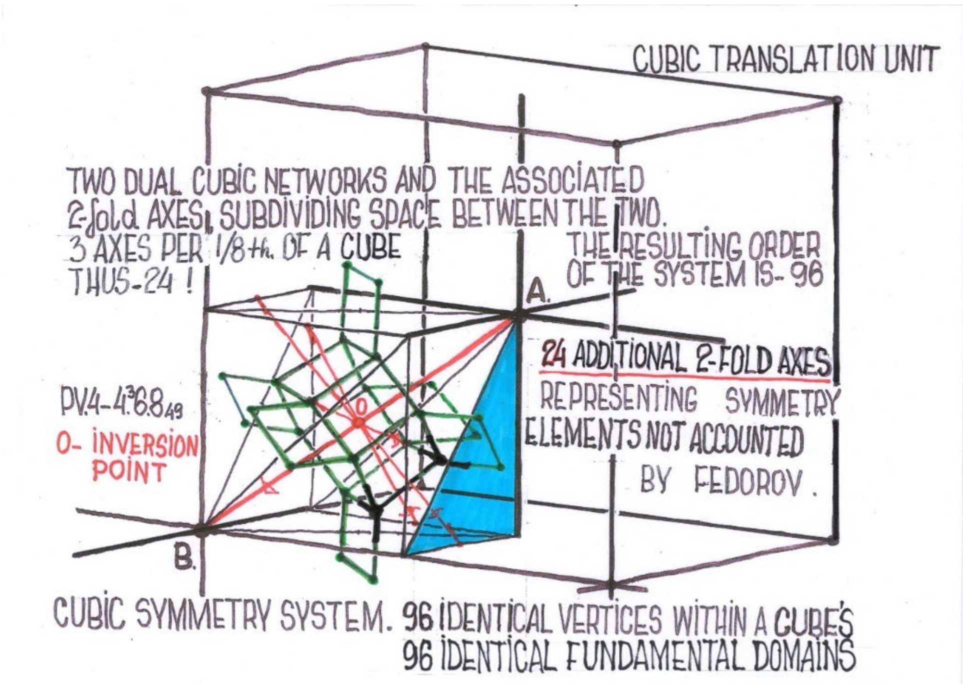

Figure 3 Self-dual cubic network and the generated symmetry axes,
raising the number of the system's fundamental domains from 48 to 96.

CONCLUSION

The expected research platform calls for a coordinated multifarious expertly team efforts of structural morphologists, geometers-mathematicians, and crystallographers. Geometrical-Topological solution and exhaustive enumeration of the above might contribute considerably to our present-day perception and understanding of 3D space related structural-spatial morphology at large. It might significantly promote many research issues within the domains of chemistry and crystallography, morphology of physical space structures and technology, concerned with close packing and associated physical space dividing partition surfaces.

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