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ANTISYMMETRY IN PORTUGUESE CERAMIC TILE FACADES

JOÃO NUNES, ANDREIA HALL and PAOLO VETTORI

Name: João Nunes (b. Coimbra, 1969), Andreia Hall (b. London, 1967), Paolo Vettori (b. Merano, Italy, 1969)
Profession: High School teacher, Associate professor, Assistant professor.
Affiliation: CIDMA - Centre for Research and Development in Mathematics and Applications, University of Aveiro
E-mail: nunes.joaol@gmail.com, andreia.hall@ua.pt, pvettori@ua.pt
Major publications and/or exhibitions:
Hall, A. Almeida, P. & Teixeira, R. (2019). Exploring Symmetry in Rosettes of Truchet Tiles, *Journal of Mathematics and the Arts*, 13(4), 308-335. https://doi:10.1080/17513472.2019.1581963

Abstract: *In this work we explore the presence of antisymmetry in Portuguese ceramic tile facades, showing examples from different styles and different antisymmetry groups. We further present some experiments with school children who explored symmetry and antisymmetry in a creative way. Children used a particular tile design, the Truchet tile, to create specific types of symmetry and antisymmetry. Truchet tiles can be found in several Portuguese facades from north to south.*

Keywords: Symmetry; Antisymmetry; Two-colour Symmetry; Truchet tile, GeoGebra.

INTRODUCTION

Symmetry is a very important feature in visual perception of images and has been a recurring element in art, architecture, and other artifacts of human construction for millenniums (Westphal-Fitch *et al.*, 2012, pp. 2007-2008). Symmetry in itself embeds the notion of repetition, regularity, or congruence. As Wade (2006, p. 1) says, symmetry is a universal principal and "it is as much interest to mathematicians as it is to artists and is as relevant to physics as it is to architecture". However, too much repetition and regularity become tedious and uninteresting and therefore it is not surprising that any notion of symmetry is intrinsically entangled with that of asymmetry or broken symmetry. One way to disturb symmetry without destroying it completely is using antisymmetry. Like symmetry, antisymmetry can be found in human productions since pre-historic times (Radovic & Jablan, 2001, p. 58). Both features can be useful when creating art works and interlacing mathematics with the arts in the classroom can be a successful way to promote the interest in mathematics.

ANTISYMMETRY

In geometry, a symmetry of a figure is an isometry that leaves it invariant. The set of symmetries of a figure F together with the operation composition forms a group which is known as the symmetry group of F. A symmetry group can either be discrete or continuous. Most figures we are interested in have discrete groups. In the plane there are only three categories of discrete symmetry groups:

rosette groups (they have a finite number of symmetries which can only be rotations or reflections); frieze groups (they have translation symmetry in only one direction); and wallpaper groups (they have translation symmetry in two directions and spread over the plane). There are two types of rosette symmetry groups, seven types of frieze groups and 17 types of wallpaper groups. Different notations may be found in the literature for these groups. In this paper, we denote by C_n the rosette cyclic groups of n rotations by multiples of $360°/n$ (being C_1 the identity) and by D_n the rosette dihedral groups, consisting of the rotations of C_n and of n distinct reflections (for example, D_4 is the symmetry group of a square). Figure 1 shows some examples of rosettes.

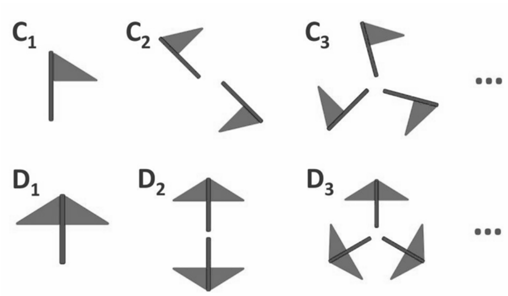


Figure 1 Examples of rosettes with symmetry groups C_n and D_n.

As for the wallpaper groups, in this paper we shall use the crystallographic notation: *p1, p2, pm, pg, pmm, pmg, pgg, cm, cmm, p4, p4m, p4g, p3, p3m1, p31m, p6, p6m*. The numbers stand for the greater order of the rotational symmetries in the pattern. The letters m and g indicate the presence of certain types of symmetry: m stands for mirror (reflection symmetry) and g stands for glide-reflection. The letters p and c have to do with the cell type (primitive or cantered). The classification of wallpaper patterns can be done using the flowchart proposed by Washburn and Crowe (1988). More details on symmetry groups may be found in Martin (1982) or in Veloso (2012).

Antisymmetry (also known as two-color symmetry) is closely connected to the concept of symmetry and may occur whenever each point of a figure or object has associated a dichotomous characteristic such as one of two colors, one of two electric charges, etc. An antisymmetry may be defined as a symmetry coupled to an exchange of colours (or exchange of the value of the dichotomous variable) that leaves the figure or object invariant. Since there are four possible types of symmetry on the plane, there are also four possible types of antisymmetry on the plane. The set of all antisymmetries and symmetries of a figure also forms a group. Antisymmetry groups can be derived from the symmetry groups by coupling a permutation group with only two elements, the colour-change transformation (see Radovic and Jablan (2001) for more details on antisymmetry). Classification and designation of antisymmetry groups can be obtained by analysing the symmetry group of the uncoloured figure (only with contours), G_u, and the symmetry group of the colored figure, G_c. The name of the anti-symmetry group is $G_u|G_c$. Figure 2 illustrates this process through the well-known yin-yang symbol which is an example of an antisymmetric figure. The yin-yang symbol exhibits some sort of symmetrical appeal (given by the rotational antisymmetry) despite having no symmetry. Without colouring, the symmetry group is C_2 while with coloring the symmetry group is C_1. Therefore, the antisymmetry group is $C_2|C_1$.

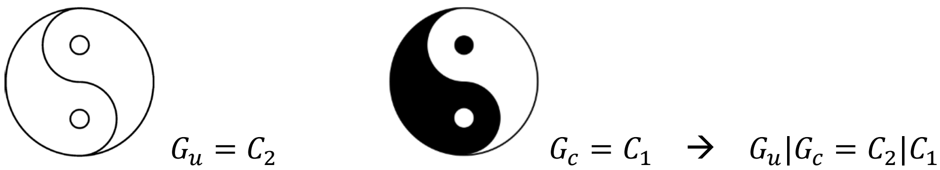


Figure 2 Antisymmetry group classification procedure with the yin-yang symbol:
left - uncoloured symbol; right - coloured symbol.

Grunbaum and Shephard (1987, pp. 402-413) provide a description of antisymmetry groups, with examples for each group. These groups are more diverse than the symmetry ones: there are 17 antisymmetry frieze groups and 46 antisymmetry wallpaper groups.

ANTISYMMETRY AND THE TRUCHET TILE IN PORTUGUESE FACADES

Portugal has a long tradition in tiles, not only in artistic panels (churches and other monuments) but also in house facades. Some of these tiles are antisymmetric and produce antisymmetric wallpaper patterns. One such tile is the Truchet tile, consisting of a square divided by one of its diagonals into two triangles of different colours, ◣. The name Truchet dates back to the 18th century in honour to a French priest, Sébastien Truchet. He explored several patterns made from this motive and did some counts of possible patterns. However, the motive appears in human ornamental art long before Truchet was born. For example, Radovic & Jablan, (2001, p. 60 and 64) give examples of friezes and wallpaper patterns from the Neolithic period.

Figure 3 shows some Portuguese artistic panels/facades using the Truchet tile. The two images on the top of the figure are from the facade of the rectory of the University of Lisbon (1961) by Fred Kradolfer (on the left) and from a panel in the Students Union of the University of Coimbra (1960) by Alberto José Pessoa and João Abel Manta (on the right). The bottom image is from the panel "The Sea" (1959) by Maria Keil in Av. Infante Santo, Lisbon. Note that in the two panels of Figure 3 the artists also use rectangular versions of the tile.

Traditional Truchet tiles used in house facades in Portugal are usually blue and white (in Spain they are green and white). The Truchet tile itself has antisymmetry group $D_2|D_1$. Depending on the disposition of the tiles, different wallpaper patterns, and hence different symmetry and antisymmetry groups, may be found. In Figure 4 we show pictures of facades with antisymmetric wallpaper patterns made from Truchet tiles.

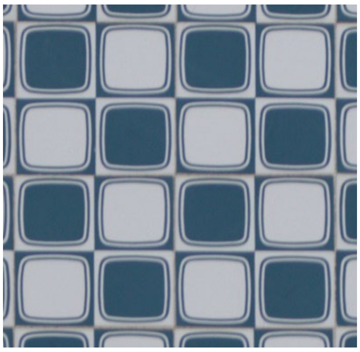
Figure 3 Examples of Truchet tiles from Portuguese panels/facades.

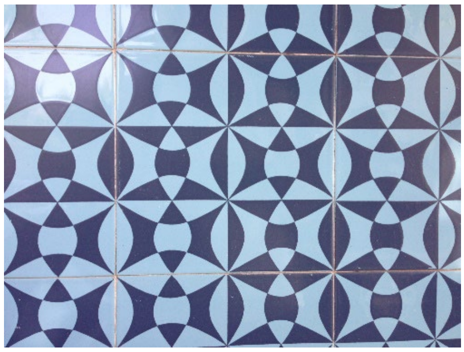
$cmm|cm$ $p4m|p4g$

$cmm|cm$ $p4m|p4m$

Figure 4 Four facades using ceramic Truchet tiles, three in Porto and one in Cabeceiras de Bastos (right-bottom), Portugal.

Other antisymmetric ceramic tile facades can be found in Portuguese facades. Figure 5 shows some examples from Aveiro.





$p2|p2$ $p4m|p4m$ $pmm|pmm$

Figure 5 Examples of antisymmetric tiles from Portuguese facades in Aveiro, Portugal.

ANTISYMMETRY IN SCHOOLS: A DIDACTIC EXPERIMENT

In the past decade several professional development courses have taken place at the University of Aveiro, addressing the topic of symmetry and antisymmetry with mathematics teachers of all grades (including elementary school teachers), Some participants explored the topic of symmetry and antisymmetry with their students. They observed facades, pavements, and other real word objects in the streets, and back in the classroom students were challenged to create figures with specific types of symmetry/antisymmetry. Teachers reported that this type of approach helped their students get involved and motivated for the subject, and further contributed to a better understanding and assimilation of the concept of symmetry. Figures 6 and 7 show some of the results, obtained during the professional development course “Matemática de Muitas Maneiras” in 2018 (images reproduced with permission of the participants). Can you spot the flaw in the middle rosette of Figure 7?

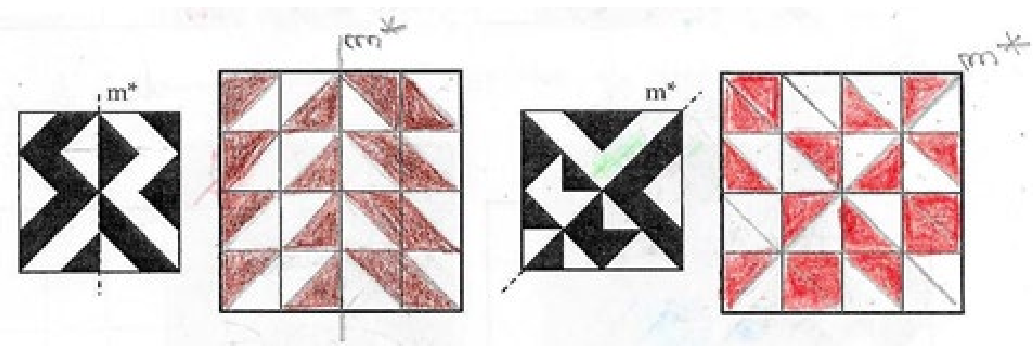


Figure 6 Examples of Truchet rosettes created by elementary school students, following particular models.

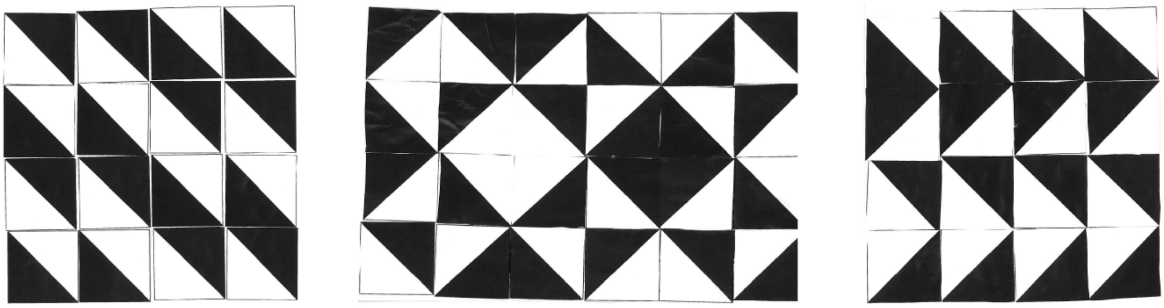


Figure 7 Examples of Truchet rosettes freely created by elementary school students.

CONCLUSION

Antisymmetry stems from symmetry by coupling to it a colour-exchange transformation. Antisymmetric figures, like symmetric ones, have an appealing effect but with an added richness caused by the colouring effect. Antisymmetry can be found in many human productions such as ceramic tiles facades. Exploring antisymmetry with school children through the analysis of real-world patterns and through the creation of figures from a given module, may improve students' motivation towards learning mathematics and contribute to a better understanding of symmetry.

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João NUNES

João Luís da Costa Nunes is a high school teacher since 1992 when he graduated in Mathematics at the University of Coimbra. Twice master's in educational mathematics: *The Mathematics of Identification Systems*, 2009, University of Coimbra; *Polynomial zeros: location, solving formulas and applications*, 2020, University of Aveiro. Since 2021, he is a student of the doctoral program in Educational Mathematics at the University of Aveiro. He is interested in geometry, algebra, arts and their applications in mathematical education. His current research includes *Truchet's tile*, rosettes, friezes and wallpapers.

Andreia HALL

Andreia Hall, PhD in Probability and Statistics, is Associate Professor at the Mathematics Department of the University of Aveiro since 2005. She started her academic career in extreme value theory and later worked in statistical data analysis. Presently she is mostly interested in mathematical education and mathematics and the arts. She teaches mathematics for pre-service primary teachers and for in-service teachers through professional development courses. She is a member of the *Mathematical Circus Project* which promotes the interest in mathematics through shows of mathematical magic. In 2020 she did an exhibition of mathematical quilts - *Between lines: mathematics through patchwork* – at the University of Aveiro.

Paolo VETTORI

Paolo Vettori, PhD in Mathematical Systems Theory, is Assistant Professor at the Mathematics Department of the University of Aveiro since 2008. In his research career he has been studying various aspects of Linear Control Systems and Coding Theory, in particular their symmetries and generalizations to quantum systems. He has been teaching linear algebra, geometry, calculus, informatics and control theory to bachelor and master students. As a member of the Mathematical Circus Project, he performs (supposedly) entertaining and challenging mathematical tricks to a wide variety of audiences (from first graders to elderly people with lung diseases)