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SYMMETRY CONCEPTS AS DESIGN APPROACH. AESTHETICS BETWEEN IRREGULARITY AND REGULARITY

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Abstract: *The symmetry concept in modern mathematics as a concept of transformations leads to a design methodology which had been implemented in design teaching at Ulm School of Design. The concept is based on defining levels of symmetry, where the similarity grade of figures is the result of the respective transformation. The levels of symmetry follow the transformations with increasing or decreasing invariants. The aim of this research is to bring together the mathematical understanding of symmetry as transformations with a more associative approach in design disciplines following the systematics by Kurd Alsleben on the basis of the work of Wolf and Wolff. The concept will be related to the theory of information aesthetics, where the interplay between redundancy and innovation gets important for an aesthetic state, defined as a relation between an ordered to a not-ordered state according to Max Bense. Transformations with many invariants are related to repetitive redundancy whereas those with few or no invariants represent innovative creations. But innovation becomes only recognizable on the basis of redundancy. An example of the creation process of patterns in a student work illustrates the application of the theoretical concept as design methodology.*

Keywords: Symmetry Levels; Aesthetics; Design; Transformations; Ulm School of Design.

INTRODUCTION

Symmetry concepts as design approaches in architecture going back to the early theories of Vitruv. In this early understanding symmetry describes the combination of parts in a general way, not in a geometrical meaning. Further on, symmetry had been worked out in mathematics by the concept of transformations, where symmetry is then understood as the invariance under a certain kind of transformation group in frame of Felix Klein's *Erlangen Program.* In the design teaching at *Ulm School of Design*, existing between 1953 and 1968, symmetry concepts with the integration of different levels of symmetries according to such an understanding of symmetry as transformation had been one of the fundaments. The publication on symmetries by the German chemists Karl Lothar Wolf and Robert Wolff provided a useful background for the application in design disciplines. Levels of symmetry were brought in relationship to order and disorder according to the aesthetic theory. Kurd Alsleben, lecturer at Ulm School of Design, made a profound analysis of Wolf's symmetry classifications in his book about aesthetic redundancy. The book is related to information theory and aesthetics by referring to the work of the philosopher Max Bense and Abraham A. Moles, pioneer in information science with important inputs at Ulm School of Design. The symmetry levels are related to *information aesthetics*, where the interplay between redundancy and innovation gets important. As an artistic device, symmetry belongs to syntactically effective tools for Alsleben. This background of symmetry concepts for design disciplines with the distinction of levels of symmetry will be described and related to the theory of information aesthetics.

LEVELS OF SYMMETRIES

In accordance with the mathematical concept of symmetries introduced by Felix Klein's Erlangen Program of 1872, typologies of symmetries had been introduced as transformations. The invariants of figures characterise the respective transformation. If now we analyse the transformations in relation to the respective invariants, we receive a table of transformations with decreasing invariants. They correspond with the levels of symmetries, from high degree of symmetry to lower orders of symmetry. The Euclidean geometry in plane and space can be also called congruence geometry and is characterised via a transformation group of isometric and conformal transformations. Similarity transformations are not isometric and allow uniform scaling. The affine transformations are non-conformal transformations that maintain parallelism, but no lengths and angles. Then the perspective transformation or projective transformation only maintains collinearity and cross-ratio. Finally, only neighbourhoods remain in the topological transformation. March and Steadman (1974) gave an overview of the invariants of the respective transformation, called mapping by March and Steadman, but mentioned that they should be called more precisely transformations (March, Steadman, 1974, pp. 24-25).

Before, the German chemists Karl Lothar Wolf and Robert Wolff (1956), who were working on molecular structures, analysed symmetries as a foundation for morphology but also for applications to design. Their book *Symmetrie* has the subtitle *Versuch einer Anweisung zu gestalthaftem Sehen und sinnvollem Gestalten, systematisch dargestellt und an zahlreichen Beispielen erläutert*, which could be translated to *Attempt of an instruction for gestalt-like seeing and reasonable designing, systematically presented and explained with numerous examples*. Their characterization of symmetries is more determined by the view of analyzing and designing formations or objects, how the elementary parts, which are no longer symmetrical structures in themselves, are repeated, what remains invariant in the repetition of the elementary part. They also refer to Felix Klein's Erlangen Program (Wolf, Wolff 1956, p. 6).

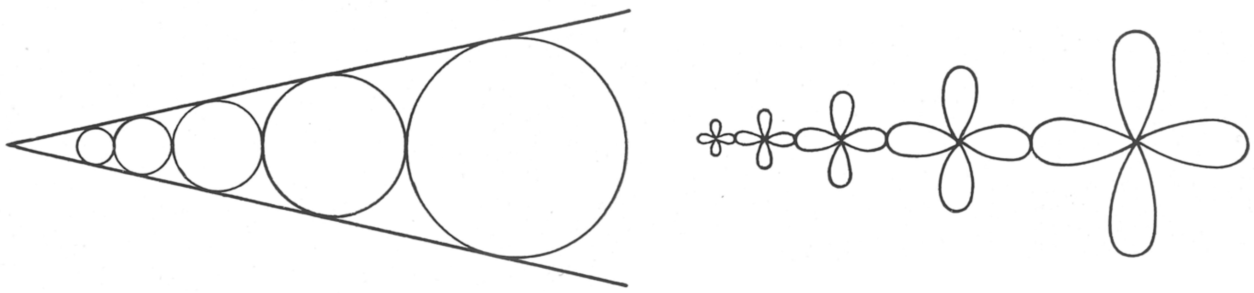

Figure 1 Homoeometric strips. (Wolf and Wolff, 1956, Tafelband, p. 4).

Identity and *isometry* are the same as in the table by March and Steadman. *Homeometric* formations are characterized as symmetric formations, where similar elements are repeated rulefully in size, position, or behaviour. There is a repetition of the same changes. As an example, they take a set of circles located between the legs of an angle, each touching its two neighbours and the legs of the angle, or corresponding arrangement of representational figures (Figure 1). The syngenomorphic formations are relationships in the sense of affine or projective geometry, like the conic section sequence. Therefore, *syngenometry* comprises affinity as well as perspectivity what corresponds to the geometric concept that affinity is a special case of perspectivity. A very low level of symmetry is already *katametry*. It is described that there is a rule by which the elements are connected in their sequence, like for example a sequence of regular polygons ordered by increasing number of vertices (Figure 2).

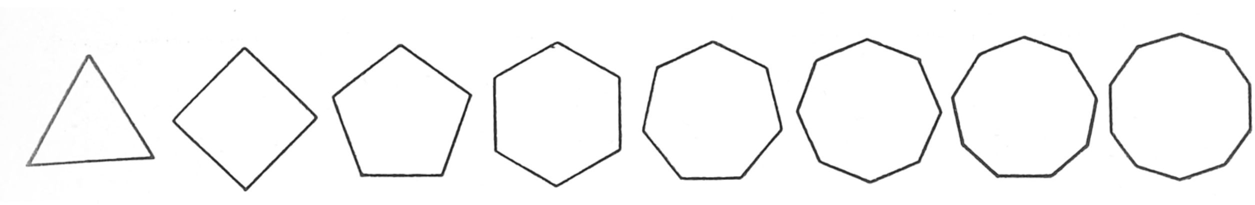

Figure 2 Katametric strip: sequence of regular polygons ordered by increasing number of vertices. (Wolf and Wolff, 1956, Tafelband, p. 5).

In *heterometry* an individual shape is defined and recognizable but there is no shape relation between the elements of the sequence. Finally, *ametry* is seen as a more theoretical case, existing only as idea, as an amorph formation where now shape is recognizable already in the element. It gets clear that these symmetry notions refer also to the designing rules of repetition not only to the comparison of the elements like in the mathematical understanding.

Table 1. Levels of symmetries according to Wolf and Wolff (1956, p. 5-6).

	position	size	angle	shape	rule	shape recognizable
identity	•	•	•	•	•	•
isometry		•	•	•	•	•
homoeometry			•	•	•	•
syngenometry				•	•	•
katametry					•	•
heterometry						•
ametry						

Huff (2000, p. 43) made a slightly different table according to his interpretation of Wolf and Wolff (Table 1) and introduced additionally hypometry between katametry and heterometry.

The symmetry concept by Wolf and Wolff shows an expansion for applications in design disciplines, which led to fruitful approaches, used especially in the design concepts by Maldonado, Alsleben, and Huff at Ulm School of Design. Tomás Maldonado, professor at Ulm School of Design between 1955 and 1967, used especially katametric design approaches in his courses. He brought the early computer artist Kurd Alsleben as a lecturer to Ulm School of Design, where he taught Structure Theory and Boolean Algebra between 1965 and 1968. In his book *Ästhetische Redundanz*, published already in 1962, Alsleben made a profound analysis of the symmetry theory by Wolf and Wolff and applied it to designing methodologies and aesthetics. He described it as important to give rules for image creations, even if the particularly productive low symmetry levels still elude a clear systematics. The different levels of the used symmetry notions had been explained by him as:

Automentry: Identical
Isometry: Equality of elements and their uniform repetition
Homoeometry: Similar, repetition in the same change in size, position or behavior
Syngenometry: Shape related
Katametry: Design related
Heterometry: Shape different
Ametry: Unformed, given only in the idea

According to these descriptions of the levels of symmetries, Alsleben showed examples of grades of similarities between two-dimensional figures (Figure 3). They demonstrate at the same time how elements can be transformed according to the respective rule. Autometry and ametry are not shown in the examples, because they only complete the system of symmetry levels on the two extremes.

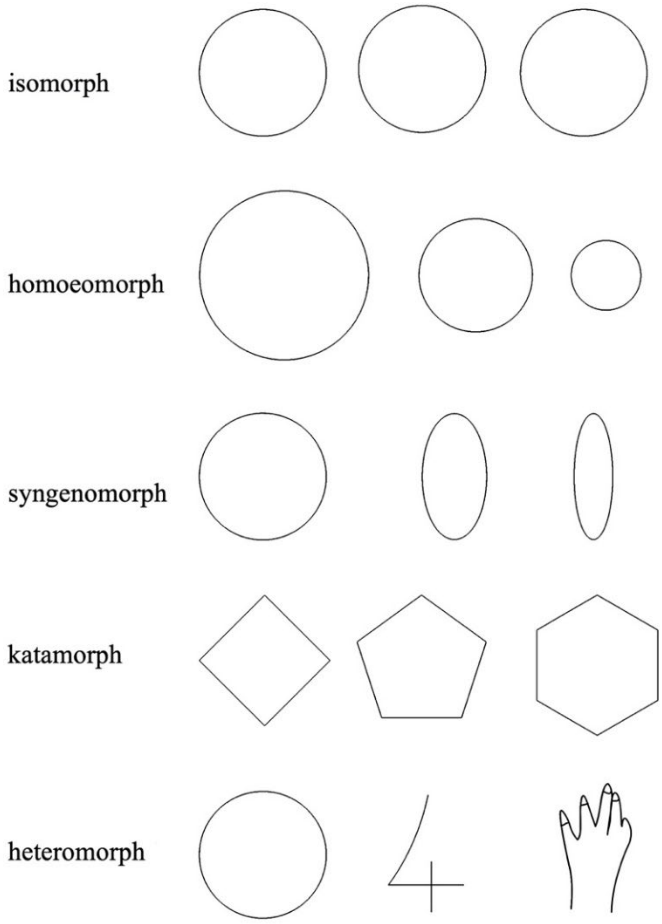

Figure 3 Grades of similarities between figures, according to Alsleben (1962, p. 61)

Alsleben characterized symmetry as corresponding to repetitive redundancy, which could be used as an artistic device, as a syntactically effective tool.

AESTHETICS BETWEEN REDUNDANCY AND INNOVATION

Alsleben's analysis and concept is embedded in the information aesthetics, which had been developed by the philosopher Max Bense (1965) and communication theorist Abraham A. Moles (1966), both professors at Ulm School of Design. Especially Max Bense influenced the philosophical and methodological background of the design school (Leopold, 2022). Information aesthetics developed criteria for aesthetic measures and evaluations. The aesthetic state had been defined by Bense as the relation of an ordered to a not-ordered state. Order relations correspond to redundancy in finding order relationships like symmetries. Innovations become only recognizable in relation to redundant order characteristics. Therefore, the interplay of redundancy and innovation or information – order and chaos – should be in an optimal relation to achieve an aesthetic state. Alsleben formulated: "Redundancy makes the style of a work of art, information its originality." The artist or creator moves between originality with perfect irregularity and banality with perfect regularity as shown in Figure 4.

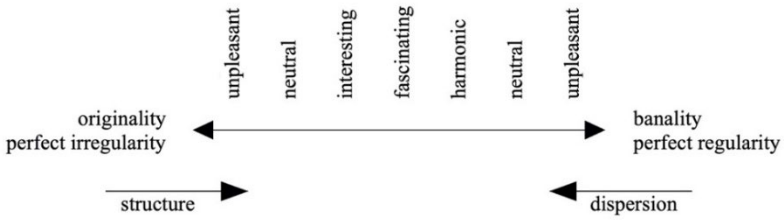

Figure 4 Originality – banality – scale, according to Alsleben (1962, p. 22) and Moles (1966)

APPLICATION EXAMPLE AS DESIGN METHODOLOGY

Using transformations of figures according to the levels of symmetry, grades of similarities between figures, offer a design methodology for creations between originality and banality. The design process can follow a methodology, starting with an element and applying different geometric transformations. This methodological concept based on geometric transformations, interpreted as symmetry levels, is explained more in detail by the author (Leopold, 2020). An example of the creation of patterns according such a way between redundancy and innovation is shown in Figure 5, where our student Carolin Dönneweg followed systematic affine transformations of an initial element, creating syngenomorph figures according to Alsleben's systematics, with amazing result. The selected basic figure was transformed by systematically changing the inclination angle of the figure. These sequential transformations of the figure created the 2D pattern with transitions in Figure 6. The 2D pattern had been interpreted as packings of 3D solids. The 3D pattern had been derived using spatially suitable packings of solids and their interspaces (Figure 7).

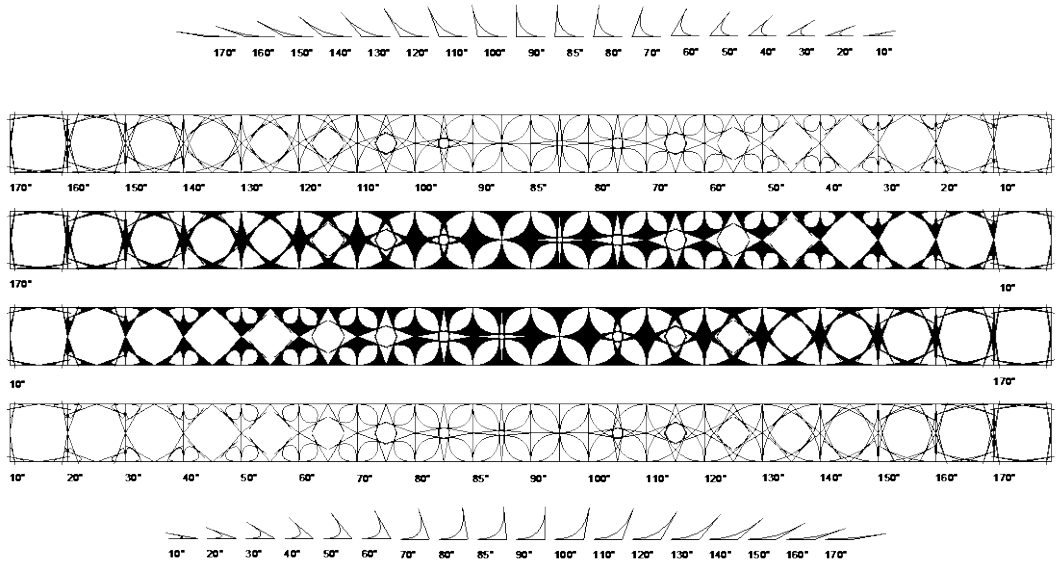

Figure 5 Sequential transformations of the initial element by student Carolin Dönneweg, fatuk, 2021.

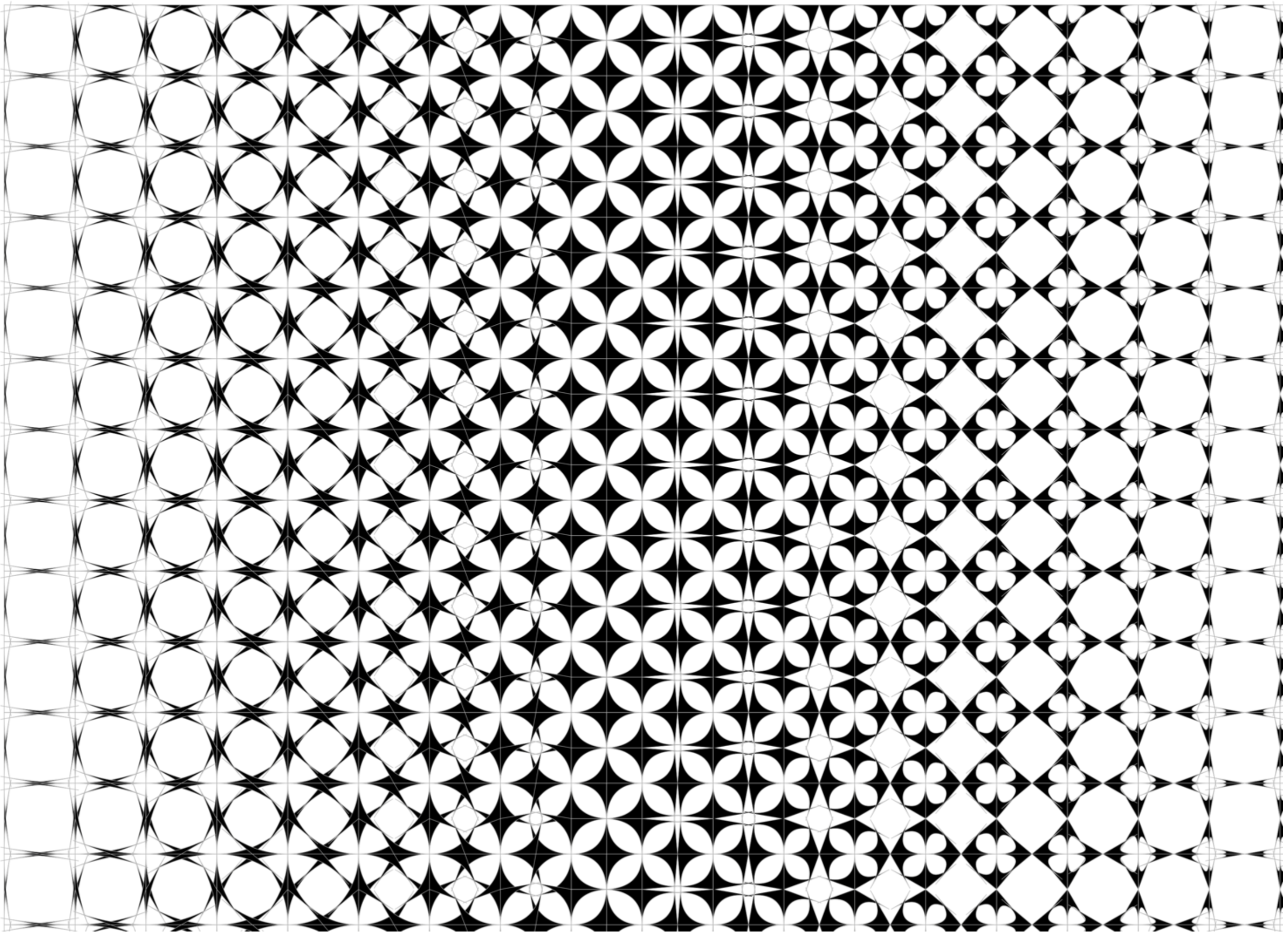

Figure 6 Designed 2D pattern with transitions between the elements by student Carolin Dönneweg, fatuk, 2021.

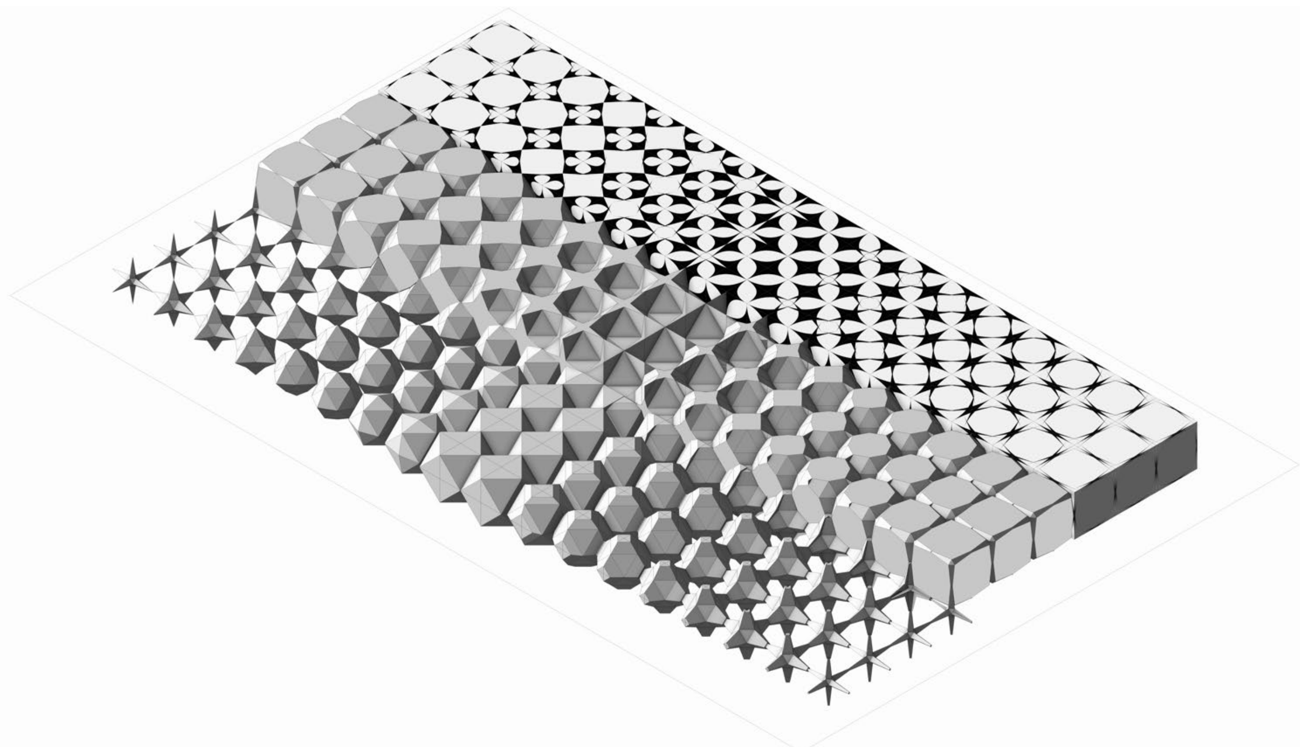

Figure 7 Derived 3D pattern using spatially suitable packings of solids and their interspaces by student Carolin Dönneweg, fatuk, 2021.

CONCLUSION

The research brings the mathematical background of symmetries as transformations together with applied concepts in design disciplines, based on information aesthetics. Different levels of symmetry follow the transformations with decreasing invariants. Rules by which the elements are connected in their sequence are related to the respective level of symmetry. Symmetry is characterized as corresponding to repetitive redundancy and proposed as an artistic device. According to information aesthetics the interplay of redundancy and innovation should be in an optimal relation to achieve an aesthetic state. Innovative creations become recognizable on the basis of redundancy. Therefore, aesthetic creations are moving between originality with perfect irregularity and banality with perfect regularity. Applications show the concept as an effective design tool.

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