Viana, V., Nagy, D., Xavier, J., Neiva, A., Ginoulhiac, M., Mateus, L. & Varela, P. (Eds.). (2022). Symmetry: Art and Science | 12th SIS-Symmetry Congress [Special Issue]. *Symmetry: Art and Science*. International Society for the Interdisciplinary Study of Symmetry. 218-225. https://doi.org/10.24840/1447-607X/2022/12-27-218

POLYHEDRONS, ART, AND ARCHITECTURE IN RENAISSANCE VENICE

COSIMO MONTELEONE

Name: Cosimo Monteleone (b. Carosino, 1974)
Profession: Associate Professor
Fields of interest: History of Representation
Affiliation: Università degli Studi di Padova
E-mail: cosimo.monteleone@unipd.it
Homepage: www.research.unipd.it
Major publications and/or exhibitions:
Monteleone, C., Williams, K. (2021). *Daniele Barbaro's Perspective of 1568*. Springer.
Monteleone, C. (2020). *La prospettiva di Daniele Barbaro. Note critiche e trascrizione del manoscritto It. IV, 36=5446*. Aracne.
Monteleone, C. (2013). *Fran Lloyd Wright. Geometria e Astrazione nel Guggenheim Museum*. Aracne.

Abstract: *Scholars have long studied the influence of perspective in Italian Renaissance. The aim of this essay is to focus on a specific place and period of time, dealing with artists, mathematicians and scholars that lived and worked in Venice from the beginning to the second half of the of 16th century. In particular, I would like to explain why proportion and polyhedrons were welcomed into the artistic expressions of the Venetian Renaissance society. For this purpose, I would like to reconstruct the cultural environment that animated Venice, focusing on the role played by the mathematicians as well as analyzing the virtuous perspectives performed by the artists in the city. I would like to reconnect also the few but significant clues that art, mathematics, and perspective have handed down to us, starting from a sort of a cultural manifesto proposed by Luca Pacioli: the mathematization of the world.*

Keywords : Luca Pacioli; Daniele Barbaro; Polyhedrons; Renaissance Venice; Proportions.

INTRODUCTION

When in 1508 Luca Pacioli held a famous lecture on proportions in the church of St. Bartholomew in Venice, it was clear to the audience that God had organized the universe basing on precise mathematical rules (Black, 2013, pp. 87-104). The following year Pacioli (1509) published, also in Venice, a treatise entitled *Divina proportione*. His aim was to expand the contents of his lecture on proportions and discuss of plane and solid geometry. Pacioli's proportionate universe was based on polyhedrons, the same 'objects' that his friend and teacher Piero della Francesca had studied from a mathematical point of view in *Libellus de quinque corporibus regularibus* and had shown how to represent in *De prospectiva pingendi*. In essence: if polyhedrons embody the mathematics of the universe, then art and architecture are able to reveal through them their secret harmonies. This link between art and mathematics was justified by the fact that, also if the supreme polyphonic perfection of celestial world remained unattainable, artistic and architectural works could and should imitate the divine rules, revealing a mathematical-proportional order (Field, 1997, pp. 457-467). This is why for artists and architects it became a matter of primary importance knowing how to represent solids, because exercises on polyhedrons showed the way to manage the space, to investigate the universe as well as its immutable order. In this articulated and stimulating context, in which mathematics, art and architecture found a common exegetical horizon, we will see that an important role was played by a Renaissance Venetian scholar, Daniele Barbaro, whose works favored the study of polyhedrons.

POLYHEDRONS, ART AND ARCHITECTURE

During the 16th century Venice was a city where the studies on Euclid multiplied. Scholars connected mathematics and aesthetics, while artists and architects realized works of art basing on proportional reasoning. This cultural context was favored by the *Renovatio Urbis*, a policy carried out by the *doge* Andrea Gritti who had the idea to modernize and renew the image of Venice in the eyes of the world. Manfredo Tafuri (1985, pp.163-166) has acutely emphasized that the changes promoted in Venice by Gritti can be traced back to a unitary politics that involved also artists and architects, who were invited to actively participate in the re-evaluation of the image of the city with their works. The case of San Francesco della Vigna is emblematic to explain the mathematical fervor that animated Venice in the 16th century. The discussion revolved around the renovation of this church and the proportions that had to be applied to its facade and choir. The dissertation was inaugurated by the words of the guardian of the church, Francesco Giorgio Veneto (known as Zorzi), who on 1 April 1535, after an explicit request of Andrea Gritti, listed some suggestions on the proportions to respect in architecture. This note was addressed to the official architect of the *Serenissima*, Jacopo Sansovino (Onda, 2008, pp. 55-88; Tafuri, 1983, pp. 48-69). Francesco Zorzi had the right titles to support his theories, being the author of a treatise entitled *Harmonia mundi totius cantica tria*, published in Venice in 1525. This work describes the science of numbers and musical proportions, intended as tools for deciphering the mysteries of God and universe. Zorzi's references were the physics of Plato's *Timaeus*, the harmonic and proportional speculations of Vitruvius, and the mathematics of Luca Pacioli (Lorenzin, 2013, p.23). Proportions, discovered in nature and mathematics, were embodied in many aspects by polyhedrons. This is why regular and semi-regular solids became in Venice a means to investigate and interpret the immutable order of the universe, as well as to reclaim the cultural role of city in international politics.

Given these premises, it is no coincidence that Luca Pacioli listed in his book the inlayers among the experts of mathematics, indeed, their figurative repertoire was notoriously based on complex solids. In particular, Pacioli (1509a, p. 23r) mentions the authors of the choir of San Francesco della Vigna, Giovan Marco and Lorenzo Canozi da Lendinara (Bagatin, 1990, p. 35), whose work was unfortunately destroyed in 1530 by Jacopo Sansovino, that designed and rebuilt the new choir. However, we can have an idea of the appearance that the old choir of San Francesco della Vigna must have had, considering the work carried out in the same period by friar Giovanni da Verona for the church of Santa Maria in Organo (Rognini, 2007). Giovanni da Verona realized wooden inlays that feature shelves with the trompe-l'oeil technique. The false doors let us see books on which wireframe Platonic and Archimedean solids lie. The appearance of these solids is very similar to that depicted by Leonardo da Vinci for Luca Pacioli's *Divina Proportione* (Figure 1). Putting such complex bodies into perspective is not a simple operation. It requires some specific geometric and mathematical knowledge. I will try to describe the cultural world of Venice in the Renaissance, focusing on the deep relationship between geometry, mathematics, and art.

Figure 1 Giovanni da Verona (1490-1500). Wireframe polyhedrons, inlays of Santa Maria dell'Organo, Verona.

The investigation of symmetrical and proportional properties of solids is part of mathematics as attested by Euclid's Book XIII of *Elements* and, at the beginning of 16th century, Venice witnessed a dispute on the correct interpretation of this text. This dispute was philological and Paul Lawrence Rose (1975, pp. 143-149) explained what pushed Bartolomeo Zamberti (1505) and Luca Pacioli (1509b) to ideally face on printed paper. Two Greek terms, *ἀναλογία* and *λόγος*, fall into this dispute. Zamberti translated them with Latin words *proportio* and *ratio*, i.e., distinguishing between proportion and proportionality, while Pacioli included the meaning of both Greek terms in the Latin word *proportio*, following the medieval tradition (Ciocci, 2017, p. 295; Malet, 2006, pp. 63-81). Beyond the reasons given by the two parties to plead their causes, it was clear to scholars that this discussion on *proportio* and *ratio* involved all mathematical disciplines that found their speculative reasoning on vision as painting and architecture. It is emblematic that in this period many specialized books on the link between mathematics and arts were published in Venice; among their authors there were artists, engineers, architects, and scholars such as Andrea Palladio, Silvio Belli, Cosimo Bartoli, and Daniele Barbaro (Williams, 2019, pp. 271-292). We should not be surprised then if in Venice, a city strongly affected by Euclid's thought, some artistic masterpieces were created in relation to mathematics and geometry as in the case of Vittore Carpaccio's star *mazzocchio*. This solid testifies the ability reached by painters under the guidance of mathematicians. On this topic it is worthwhile to consider the conclusions of Margaret Daly Davis (Daly Davis, 1980, p. 183-200), who linked Carpaccio's *Arrival of English Ambassadors* with the perspectival teachings of Geronimo Malatini, connecting the training of the Venetian painter to Luca Pacioli's teachings. Indeed, Pacioli (1494, p. 2r) mentioned Malatini among mathematicians, experts in perspective. An engraving by Giovanni Maria De Pian, dating back to 1785, which reproduces the Carpaccio's painting before it was resized, seems to confirm this hypothesis, showing another (lost today) starred *mazzocchio* on the left. Ludovico Zorzi (1988, p. 37) suggests that the man dressed in red in the foreground, whose index finger points to the star *mazzocchio* on the left could represent Geronimo Malatini who, with his gesture, underlines the difficulty of representing this solid (Figure 2).

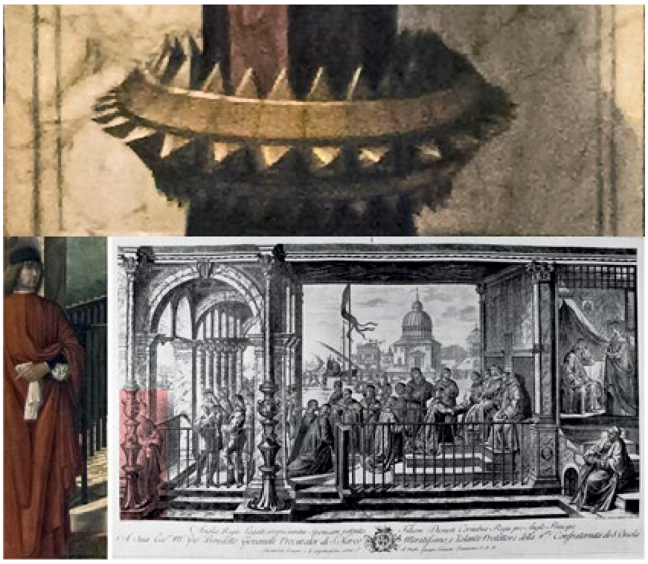

Figure 2 V. Carpaccio (1495), Star Mazzocchio and Man in Red (details of *The Arrival of the English Ambassadors*, Galleria dell'Accademia, Venice. G. M. De Pian (1785). Engraving of Carpaccio's painting before it was resized.

There is no doubt that there were in Venice painters able to represent polyhedrons as in the case of the *Portrait of Luca Pacioli and Giudubaldo da Montefeltro* at the Capodimonte Museum in Naples. It would take too long to retrace critical theses and interpretations on the authorship and meaning of this double portrait (Ciocci, 2017, pp. 14-33), here I would like to focus instead on some issues that summarize and confirm the outlined Venetian cultural context. Both, the *Elements* of Euclid, open to proposition VIII of Book XIII on polyhedrons, and Pacioli's *Summa* (1494), a fundamental text for the theory of proportions, were published in Venice when the friar moved in the city as tutor of Antonio Rompiasi's sons. So, two polyhedrons in the painting would represent practical applications of mathematics. In particular, the wooden dodecahedron, which is lying on the *Summa*, would be a reference to proportions; while the rhombicuboctahedron, that refracts through water and reflects on its surface, would refer to Optics (Figure 3).

Figure 3 J. de' Barbari (1495, attributed to), *Portrait of Luca Pacioli* with Guidubaldo da Montefeltro, Capodimonte Museum, Naples.

Looking carefully at the double portrait, it is relevant to emphasize a detail insofar neglected by critics: the rhombicuboctahedron, a solid discovered by Pacioli, is hanging on a thread that passes through its base that is an equilateral triangle. This position differs from the solid orientation assigned in *Divina Proportione*, where its base is a square. From this consideration we must deduce that not only the artists in Venice were able to represent the Platonic and Archimedean solids in perspective, but that they were even able to move away from the figurative repertoire proposed by Leonardo da Vinci in the *Divina Proportione*.

Therefore, the question that arises is: where did the artists in Venice get their mathematical knowledge from? The answer could come from the first theoretical work in which the rhombicuboctahedron appears in both positions, i.e., the one similar to *Divina Proportione* and the other rotated as in the double portrait, I am referring to the treatise on perspective written by the Venetian scholar Daniele Barbaro (1568, pp. 64-67) (Figure 4).

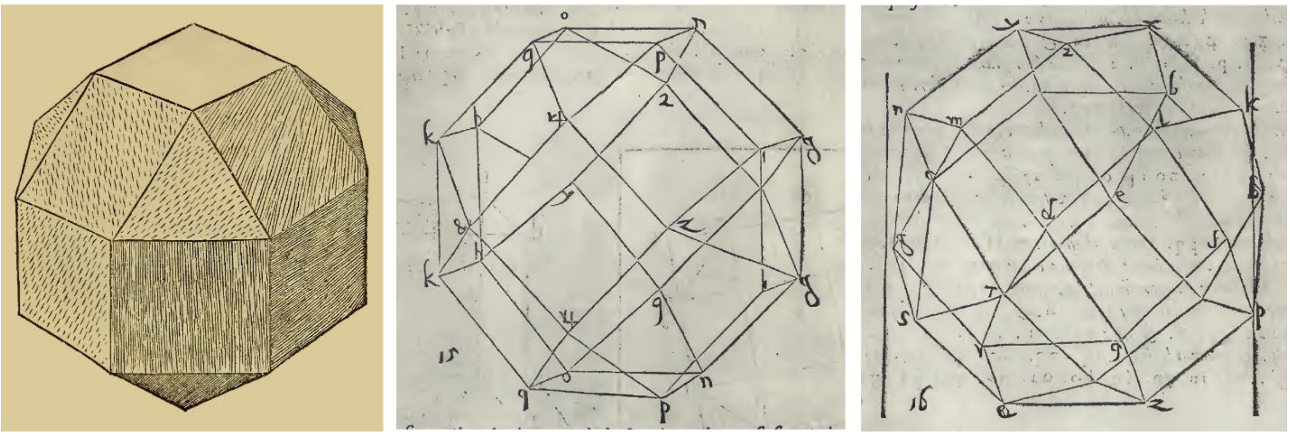

Figure 4 Left: L. Pacioli (1509a, XXXV), Rhombicuboctahedron lying on a square.
Centre and Right: D. Barbaro (1568, p. 66), Rhombicuboctahedron lying on a square and a triangle.

There are some clues that link Barbaro's *La pratica della perspettiva* to the artistic and mathematical context of Venice in the early 16th century. At the beginning of his treatise Daniele Barbaro indicates Giovanni Zamberti as his only practical teacher, specifying that he added the theory to the precepts explained with the rulers and the compass (Barbaro, 1568, pp. 2-3). It is not easy to outline an elusive profile like that of Giovanni Zamberti, of whom we have only indirect information. First of all, we know that Giovanni was the brother of Bartolomeo Zamberti, the scholar who discussed with Luca Pacioli about the correct translation of Euclid. His name also appears in a letter dated 1511 (Molmenti, 1907, p. 240), written by Vittore Carpaccio, confirming Giovanni Zamberti's relationship with the Venetian artistic world.

Basing on the dedicatory letter at the opening of his brother Bartolomeo's Latin translation of Euclid's *Optics*, critics have always maintained that Barbaro was trained by Giovanni Zamberti on perspective principles (Camerota, 2006, p. 149). I do not agree with this position because the first two chapters of *La pratica della perspettiva* are a copy of Piero della Francesca's work. It would be better to connect Giovanni Zamberti's help to the polyhedrons, as the words of Barbaro's Italian manuscript unequivocally confirm (Barbaro, n.d., p. 298r):

> *In the end I met an honorable citizen of our homeland, called Giovanni Zamberti, an old man who practices this science, from whom I confess, and I am delighted to have learned many beautiful things and, especially, those relating to regular and irregular bodies.*

If we trust the Barbaro's words, we must place the practical skills of Giovanni Zamberti in the teaching of the abacus schools, places where Euclid's treatises were studied in order to learn how to measure inaccessible distances visually (Field, 1997, p. 129). Then, how does Barbaro explain to artists to realize the perspectives of complex objects such as the Platonic and Archimedean solids? In Book III of his work Daniele Barbaro suggests to section a solid at different heights. The final perspective of a polyhedron could be depicted shortening each section and merging their vertices (Figure 5). If the perspective of plane figures is quite simple, identifying the real shape of each single section, i.e., drawing the plan of the solid, is not a trivial matter and it is properly in the composition of these preparatory drawings (plans and elevations) that the advice of mathematicians would have helped the artists (Williams, Monteleone, 2021, pp. 63-82) (Figure 6).

In this regard it is worth remembering that the final purpose of perspective is to represent what the eye sees, not only for artistic but also for scientific purposes. Perspective supports all mathematical disciplines, such as architecture, astronomy, gnomonics and geography, which need to represent on a plane the spatial problems that they face. Indeed, perspective is a measured representation of space, where each object occupies a precise spot in relation to its context (Camerota, 2006, pp. 22-

34). At this point, Girolamo Malatini and Giovanni Zamberti could be considered like mathematicians, who apply their skills to every field of knowledge, including art. At the end we could state that in Venice, at the beginning of 16th century, mathematicians were involved with artists and architects to solve proportional problems and issues related to the representation of space. The solution arrived from a mathematical and geometrical training in the abacus schools that, indeed, became part of the standard artistic *curriculum*.

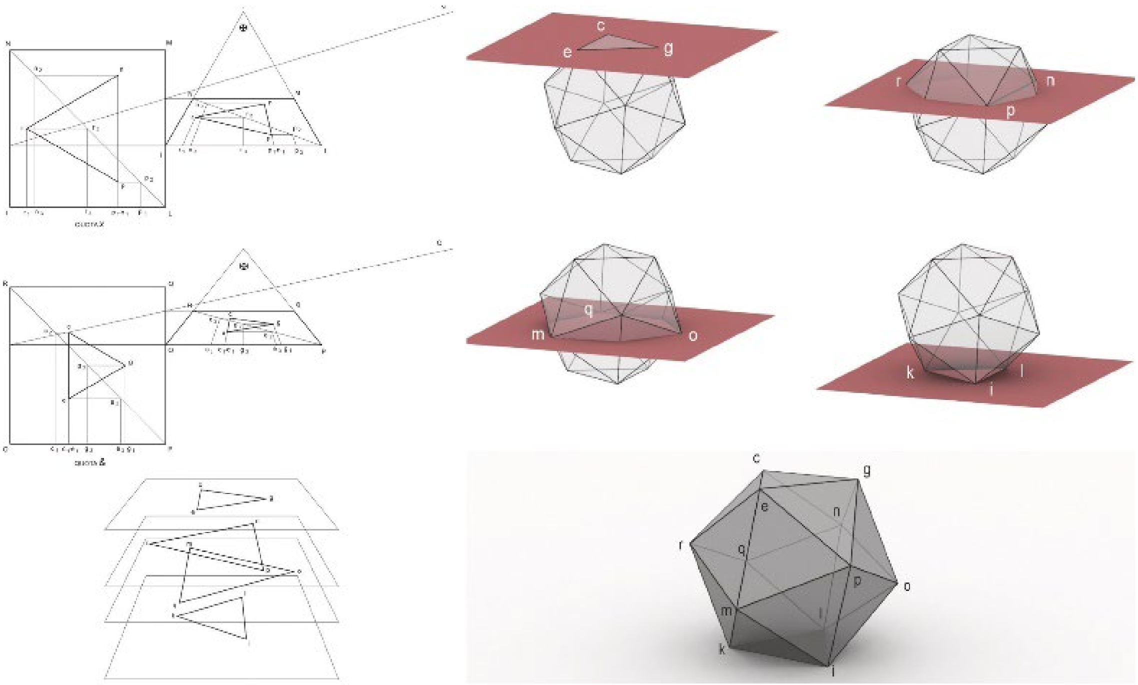

Figure 5 Perspective of an icosahedron by means of shortened parallel sections.

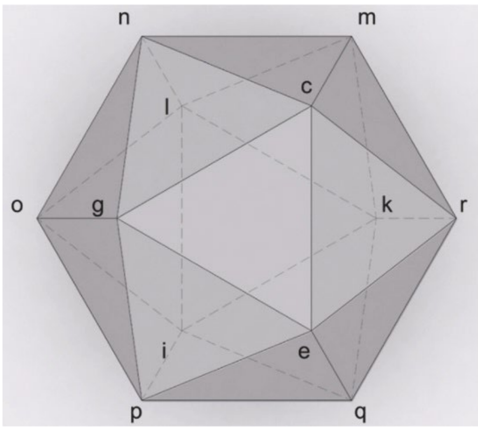

Figure 6 Plan of an icosahedron.

CONCLUSION

This essay started with Luca Pacioli's public lecture in the church of St. Bartholomew in Venice, an event that fuelled the interest of scholars in the study of Euclid's works. This new scientific fervour coincided with the policy of *doge* Andrea Gritti, who wished to affirm a new image of Venice in the world. To pursue this purpose Gritti asked for the collaboration of artists and architects. Thus, in the city, architectures were created following the proportional rules of celestial spheres as well as perspectival artworks that reproduced scientifically on canvas what men see. The geometric perfection of polyhedrons became symbols of beauty, thanks also to the work on perspective of Daniele Barbaro that widespread in the artistic world of Venice the way to represent Platonic and Archimedean solids in any position. So, on one hand, polyhedrons symbolized a new this scientific training and, on the other, they embodied the harmony and perfection that God imposed to the universe.

REFERENCES

Bagatin, L. (1990). *L'arte dei Canozi Lendinaresi*. Lint Editoriale.

Barbaro, D. (1568). *La pratica della perspettiva*. Fratelli Borgominieri.

Barbaro, D. (n.d.). *Manuscript It. IV, 39=5446*. Biblioteca Nazionale Marciana

Black, E. M. (2013). La prolusione di Luca Pacioli del 1508 nella chiesa di S. Bartolomeo e il contesto intellettuale veneziano. In N. Bonazza et al. (ed. by). *La chiesa di S. Bartolomeo e la comunità tedesca a Venezia*. Venice, Marcianum Press.

Camerota, F. (2006). La prospettiva del Rinascimento. *Arte, architettura e scienza*. Electa.

Ciocci, A. (2017). *Luca Pacioli. La vita e le Opere*. Sansepolcro, Centro Studi Mario Pancrazi.

Daly Davis, M. (1980). Carpaccio and the Perspective of Regular Bodies. In M. Dalai Emiliani (ed. by). *La prospettiva rinascimentale. Codificazioni e trasgressioni*. Florence, Centro Di.

Field, J. V. (1977). *The invention of infinity. Mathematics and Art in the Renaissance*. Oxford University Press.

Lorenzin, R. (2013). Francesco Zorzi Veneto, De Harmonia Mundi Torius Cantica Tria. Teorie Musicali e Kabbalah, PhD Thesis in Hystory and Critics of Cultural Heritage, XXV cicle, Università degli Studi di Padova.

Malet, A. (2006). Renaissance notions of number and magnitude. In *Historia Mathematica*, 33, 2006, pp. 63-81.

Molmenti, P. (1907). *The Life and Works of Vittore Carpaccio*. John Murray.

Onda, S. (2008). *La chiesa di San Francesco della Vigna e il convento dei frati Minori*. Parrocchia di San Francesco della Vigna.

Pacioli, L. (1494). *Summa de Arithmetica geometria. Proportioni: et proportionalità*. Venice, Paganino de' Paganini.

Pacioli, L. (1509a). *Divina proportione*. Venice, Paganino de' Paganini.

Pacioli, L. (1509b). *Euclidis Megarensis* [...]. Venice, Paganino de' Paganini.

Rognini, L. (2007). *La sagrestia di Santa Maria in Organo. Le vicende storiche e artistiche della "più bella sagrastia che fusse in tutta Italia*". Cierre Edizioni.

Rose, P. L. (1975). *The Italian Renaissance of Mathematics. Studies on humanists and mathematicians from Petrarch to Galile*o. Géneve, Droz.

Tafuri, M. (1983). *L'armonia e i conflitti. La chiesa di San Francesco della Vigna nella Venezia del Cinquecento.* Einaudi.

Tafuri, M. (1985). *Venezia e il Rinascimento*. Einaudi.

Williams, K. (2019). Daniele Barbaro on Geometric Ratio. In *Nexus Network Journal*, 21, 2019, 271-292.

Williams, K & Monteleone, C. (2021). *Daniele Barbaro's Perspective of 1568*. Cham: Springer.

Zamberti, B. (1505). *Euclidis megarensis* [...]. Venice, Giovanni Tacuino

Zorzi, L. (1988). *Carpaccio e la rappresentazione di Sant'Orsola. Ricerche sulla visualità dello spettacolo nel Quattrocent*o. Einaudi.

Cosimo MONTELEONE

Cosimo Monteleone is Associate Professor of Descriptive Geometry and Digital Representation of Architecture at the Università degli Studi di Padova. In 2003 he obtained his degree in Architecture at the University IUAV of Venice, where he also earned his Ph.D. in 2010 with a study on the Guggenheim Museum by Frank Lloyd Wright. His research focuses on architectural, urban and landscape survey; 3D modeling of architecture and city; augmented and virtual reality; gnomonic; science and technique applied to art and architecture; history of representation with particular skills related to Renaissance perspective. On the topics of his research, he has published several essays, presented conferences, lectures, directed digital installations for national and international exhibitions. He published three books: C. Monteleone, K. Williams (2021). Daniele Barbaro's Perspective of 1568. Cham: Springer. C. Monteleone (2020). La prospettiva di Daniele Barbaro. Note critiche e trascrizione del manoscritto It. IV, 36=5446. Rome: Aracne. C. Monteleone (2013). Frank Lloyd Wright. Geometria e Astrazione nel Guggenheim Museum. Rome: Aracne.