

SYMMETRY IN LITERATURE, THE PERSPECTIVE OF A MATHEMATICIAN

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Abstract: *Mathematics and poetry are emanations of the human intellect to be valued in it and for itself. As the mathematician Hardy says, a mathematician, like a poet, is a maker of patterns. Their patterns should be beautiful, and symmetry is a source of beauty. The subjects here focused aim to emphasise interesting relationships between pattern makers from different areas of creativity, like poetry and mathematics. We live in a universe of patterns, where symmetry plays a central role. Symmetry is not only beauty to be admired, but it is also a useful instrument to discover the clues and rules of natural phenomena. In human creations, like science, arts and literature, symmetry may be easily recognized or not, so it deserves our special attention. This paper has two main goals: first, the public understanding of the wonderful world of fertile imagination of mathematicians and poets, second, the importance of symmetry in a deeper vision of life and of the universe.*

Keywords: Symmetry; Sestina; Poetry, Visual Poetry, Prose

INTRODUCTION

Symmetry is a fundamental concept in science, as well as in several different fields of creativity. Symmetry may be recognized in Arts in different civilizations and ages. It has such a strong presence in Nature, that it is usually seen as a decisive instrument to the understanding of life and phenomena. Many organisms exhibit a wide variety of symmetrical patterns. Sea stars have five-fold symmetry and flowers display a wide variety of symmetries. Symmetry at different levels is also found in literary work.

A first question naturally arises: what is symmetry? In a visual artistic work, symmetry intuitively means beauty and harmony of form. The word was born in Art, as it is due to the Greek sculptor Polykleitos in the 5th century B.C. A somewhat more rigorous definition is as follows: symmetry of an object is the group of all possible one-to-one transformations that preserve its original structure. Indeed, the circle has rotational symmetry, because when rotated in turn of the centre its form remains

invariant. The rectangle has bilateral symmetry since it remains invariant under a reflection in each of its diagonals. Humans are bilaterally symmetric; daisies have rotational and reflection symmetries.

Mathematics appears in literature, as according to Galileo, mathematics is the language of the Universe. Literary prose and poetry elect its terms and ideas as an inspired way to express feelings about the world, life and humanity. Literary aesthetic of prose style is influenced by the metaphorical use of mathematics, and its underlying rules and regularities assume an important role in poetic stylistic analysis. The presence of the mathematical glossary in literary texts, fusing the common sense of the terms with their technical meaning, is a valuable resource in semantics. Mathematical concepts in Portuguese poetry are present in many authors, e.g. in Ana Hatherly's *Tisanas*, from which work we mention the poem *Serpente infinita* (Infinite snake). The infinite, the so-called «miraculous jar of mathematics», alludes to the fact that infinite sets behave in a very special way: the whole may be equal to one of its parts.

Writers and mathematicians have their own universes, but despite their peculiarities, a cross fertilization exists between them. The challenge is in recognizing at what extent in mathematical and literary creations a subtle mutual attraction and inspiration arise. Many formal concepts are understood within the background of literary metaphor. Chaos, a scientific subject that has an important mathematical theory, is one, but a more fundamental one is symmetry itself. The use of formal logic is used by writers in the construction of consistent discourses. The entanglement of logical ideas in a novel is a most valuable instrument in the development of action. A mathematical proof is a chain of logical reasoning arguments, where inconsistency is forbidden. So, logical chains of arguments are significant at structural level in both creativity worlds and an inspiration source to get deepest insights.

Poets and prose-writers explore the use of symmetry in their works, as symmetry plays a central role in the universe of ideas and in the world around us. In this paper we revisit symmetry in a short story by Jorge Luis Borges and in the celebrate poem “The tiger”, by William Blake. In the same perspective, we recall a sestina by Camões, a poem with a complex style ruled by repetition of end-words in six stanzas and an envoi. Interestingly, a fundamental concept appears in a 16th century poem well before being formalized in differential calculus. Namely, the idea of “second derivative” in a poem of the sixteenth poet Camões, and of fractality in Álvaro de Campos, one of the heteronyms of Fernando Pessoa, in the last century. The role of symmetry in visual poetry is also touched.

SYMMETRY IN BLAKE AND BORGES

Metrics in poetry is clearly related to symmetry. Symmetry is present in literary glossary, both in poetry and prose. A paradigmatic example is the famous William Blake's poem *The Tyger*, published in "Songs of Experience" in 1794.

The poem focuses on God and Creation, and the following challenging question inevitably occurs. Is it the same God, the one who created the sweet lamb and the wild tiger, the gentle creatures and the terrible ones? The duality of contrasting forces, like tenderness and ferocity, is focused. The first and second stanzas end with the rhyming words "bright" and "night" (AA), while the fifth and fourth end rhymed (BB), and this regular rhyme structure informs all the poem. The bilateral symmetry of the tiger, the main poem actor, is omnipresent, his two brain hemispheres, his almost symmetric heart, his longitudinal stripes. Despite this harmony of forms, his wild nature prevails.

Symmetry in Jorge Luís Borges work is a demanding issue. In his short story «The death and a compass», the detective Eric Lönnrot is a pure reasoner with a strong logical thinking. Symmetry plays a major role in the discovery of a crime, in particular the mirror symmetry that gives for an object a reflected image. The difficulty of the case prevents us from finding a simple solution. Solutions with obvious symmetry are rejected in favour of those ones with more complex symmetry. Three deaths occurred in the afternoons of the third day of three consecutive months in places defining the "perfect" vertices of an equilateral and "mystical" triangle. An anonymous letter appeared suggesting that the symmetry is complete and so are the crime episodes. The detective Eric rejects this hypothesis: at the place of the three murders there was a reference to the tetragrammaton, the four-letter name of God: JHVH. In fact, the deaths had occurred at day "four" of each month, because the Jewish calendar records the day from sunset to sunset, and so Eric joins the missing point to the previously considered triangle, thus determining a perfect diamond. The diamond, the mirror reflection, the labyrinths that surround men and which it is impossible to escape from, populate this beautiful creation. Lönnrot considered for the last time the problem of symmetrical and periodic deaths, and using mathematical reasoning, brightly solved the intricate case.

SYMMETRY IN A SESTINA BY CAMÕES

There are cases in which poetry voluntarily submits to rigorous mathematical rules. In this context, a sestina by the great poet Luís Vaz de Camões is considered. The sestina has 6 stanzas, each one with 6 verses, and an envoy of three lines, usually unrhymed. The final words of the stanzas, represented by the natural numbers 1,2,3,4,5,6, obey a strict rule stated below. At the middle point of the three lines of the envoy appear either the final words 1,3, 6 or 4, 1,2 .

We briefly analyse the sestina pattern in the following poem by Luís de Camões from which we reproduce stanzas 1 and 2, as well as the envoy.

*Foge-me, pouco a pouco, a curta vida,
Se por acaso é verdade que inda vivo;
Vai-se-me o breve tempo de entre os olhos;
Choro pelo passado; e, enquanto falo,
Se me passam os dias passo a passo.
Vai-se-me, enfim, a idade e fica a pena.*

*Que maneira tão áspera de pena!
Pois nunca uma hora viu tão longa vida
Em que posso do mal mover-se um passo.
Que mais me monta ser morto que vivo?
Pera que choro, enfim? Pera que falo,
Se lograr-me não pude de meus olhos?*

*Morrendo estou na vida, e em morte vivo;
Vejo sem olhos, e sem língua falo;
E juntamente passo glória e pena.*

Jumpy words can be observed at a first glance in the poem, but beyond this we can notice two obvious things: (1) there is no particular rhyme pattern (2) the final words of the six lines of the first stanza occur again as final words in the second stanza, however in a different order.

Considering only the final words of the first stanza, the sequence of words (*vida, vivo, olhos, falo, passo, pena*) gives rise to the sequence (*pena, vida, passo, vivo, falo, olhos*), or symbolically, the sequence 1, 2, 3, 4, 5, 6 gives rise to 6, 1, 5, 2, 4, 3.

These two sequences formed by the first six natural numbers (with no repetitions) are called permutations, which are elements of the so-called symmetric group of degree 6. The permutation rule for the final words of each stanza must be successively followed in each stanza. More concretely, in every stanza of the poem, the same permutation must be used, when passing from the stanza 2 to 3, the stanza 3 to 4, the stanza 4 to 5, and the stanza 5 to 6. Applying the permutation to the final words of the sixth stanza, one will necessarily arrive at the original sequence of final words of stanza 1. And this is one relevant key of the pattern.

ORIGINAL MATHEMATICS IN PORTUGUESE POETRY

The use of the mathematical glossary in poetic texts is far from being a rare issue. The next poem by Álvaro de Campos is perhaps the most popular poem in Portuguese poetry that refers to mathematics. The author states that “Newton's binomial is as beautiful as the Venus of Milo/ There are few people to notice this”.

We also mention the poem *Pitágoras* by Eugénio Lisboa, where the poet argues that “from the number everything was born/ other numbers, the truth/ The curve that the star followed/ The beauty, the life, the city”. According to the Pythagorean philosophy, everything is number, and nothing can exist in the world without number, so number is the bases of the universe.

Very interestingly, mathematical concepts appear in Portuguese poetry in a naïve form, even before their formal creation in mathematics. The Brazilian mathematician Elon Lages Lima (private communication) made a very interesting observation: he claimed that Luís de Camões, in the second tercet of his famous sonnet “*Mudam-se os tempos, mudam-se as vontades*” (Times change, desires change), introduces the notion of second derivative. In the aforementioned tercet, Camões develops the idea that everything is changing, always earning new qualities. The last 3 verses are particularly important to justify Elon’s point of view.

Everything in the world is made up of change / Always taking new qualities / And, apart from this every day change / Another change makes me astonished: / That nothing changes as it used to be.

Todo o mundo é composto de mudança/ mudam-se os tempos mudam-se as vontades/ Sempre ganhando novas qualidades/ E, afora esta mudança/ Outra mudança / Que nada muda já como soía

The change - already with new qualities, different from the previous ones—suggests the concept of second derivative. Mathematically speaking, the first derivative is the rate of change of a quantity, while the second derivative is the rate of change of the previous rate of change. For instance, the velocity of a body is a rate of change: the rate at which its distance from the starting point changes. The acceleration is a “second order” rate of change, i.e., a rate of change of a rate of change, different from the first one. Newton and Leibniz in the 17th century, independently, discovered these revolutionary concepts, and created an important new branch of mathematics, the calculus and the science of movement.

Álvaro de Campos touches the concept of fractal, as a structure that is repeated with various scales in an object. In the poem *Não, não é cansaço* (No, it's not fatigue), from which we reproduce the following fragment, the last verse is impressive in the sense that it suggests the fractal nature of the

world, a world not seen in the traditional Euclidean paradigm, which prevailed in culture for long centuries, but a more complex world with more complex patterns and symmetries.

The poet says that the world, with everything it contains, with everything that unfolds in it, is after all the same thing repeated in equal copies in different scales. This idea is in the heart of the notion of fractal. This notion is also present in the following statement by the poet, which actually is full of symmetrical qualities:

...I know myself fully, and through knowing myself fully, I know all of humanity fully.

SYMMETRY IN VISUAL POETRY

Poems are constructions with words, letters, phonetic, rhyme games, rhythm, ideograms, figures...Possible absence of verse, stanza, rhyme, are among characteristics of visual poetry. In this form of poetry, the diagram plays a principal role. Form and content, words, and shape, are inter-linked. The perceptions induced by expressive imagery resources and their relationships with the verbal text are crucial in visual poems. The chosen images used to express ideas, feelings, emotions, meet in symmetry an important way of expression. Symmetry matters, not only at an aesthetic and structural level, but also adding value and meaning to the poem.

Salette Tavares was one of the key figures of the Portuguese visual poetry, and her book *espelho cego* published in 1957 is a landmark in the area, from whom we reproduce the famous spider poem *A aranha*. The spider with its many legs has a motion involving a complex rhythmic pattern and subtle symmetries follow the sequence of these movements. The walk of a biped has only a glide reflection symmetry, well-illustrated by the leg's forms left in dry sand. This symmetry is the composition of a reflection and a translation. The spider with its many legs, that can be sought as mechanical systems, has a faster and much more complicated movement.

We also recall the poems by E.M. de Melo e Castro (1932-2022) *Pêndulo* and *Tontura*, which exhibit interesting symmetries. These visual poems illustrate this notorious fact: the forms induce by their own not only a vision of the thematic object, but other additional qualities. *Tontura* has perfect rotational symmetries. Other symmetries occur, like the reflection relatively to a vertical line through the centre of the circular disc. These symmetries combined with the lettering in different decreasing scales towards the centre provokes a certain mental confusion. The pendulum generates a rhythmic periodic oscillation, and *Pêndulo* is an inspired kinetic poem. Periodic oscillators possess a time reversal time-translational symmetry, that is, its law of motion is symmetric under time reversal.

The visual poem *Amazona* (Amazon) by our great artist Almada Negreiros deserves attention. The graphic representation involves a rhythmic pattern of movement, with a somewhat complex structure. Subtle symmetries follow the sequence of the horse movements. The Amazon, with her bilateral symmetry broken by the mobility of harms, strengthens the perception of rhythm, freedom and elegance.



Figure 1 Aranha by the visual poet Sallete Tavares (1922-1994)¹

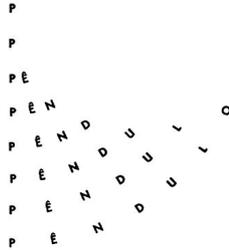


Figure 2 O Pêndulo²; Tontura, 1962 (*Ideogramas*, pág. 25)³

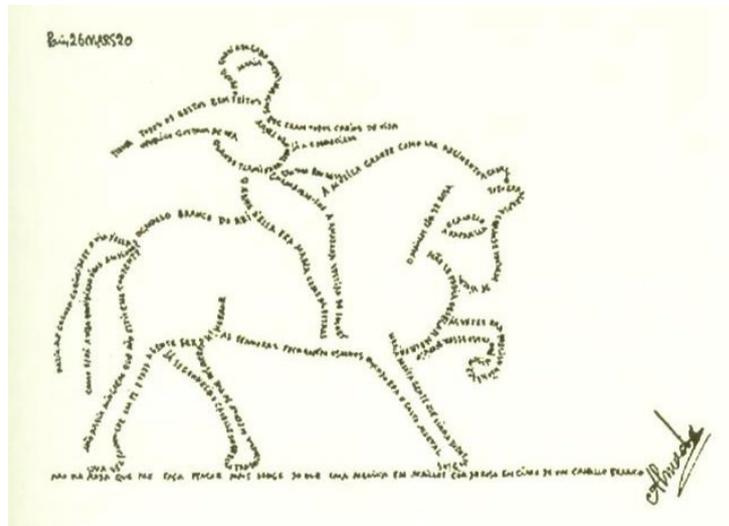


Figure 3 Amazona, calligramme by Almada Negreiros, 1920.⁴

In mathematics, a proof of a statement by a diagram evident in itself, without any explanatory reasoning, is a visual proof also called a proof without words. This kind of proofs can be considered more appealing than formal ones by their simplicity and elegance. Proofs without words as an exercise of visual thinking plays a special role in the art of problem solving.

The Pythagoras's theorem, stating that in a rectangle triangle the square of the hypotenuse equals the sum of the squares of the two other sides, has several visual proofs, one of them is shown in the following diagram. Determining the area of the large square by two different methods yields the relation between their sizes. One of these methods consists in decomposing the square as illustrated in the figure and summing these areas. The other method consists in directly evaluating the area of the large square. Finally, considering the equality between the obtained values by the two methods gives the stated result.

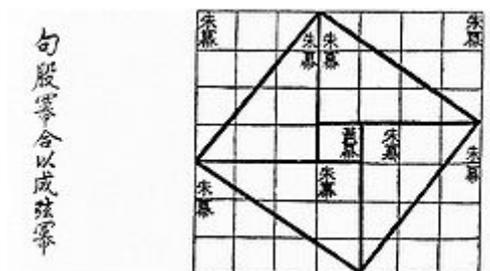


Figure 4 Diagram added by Zhao Shuang to the Zhoubi Suanjing that can be used to prove the Pythagorean Theorem⁵, retrieved from

CONCLUSION

Despite the intrinsic similarities of poetry and mathematics, they are often seen as unrelated expressions of human creativity. One of the goals of this paper is to point out some of their relationships. In this context a sestina by Camões is considered. This medieval form of poetry is difficult from a formal point of view, as it has a complex strophic structure. The complexity aimed at to impress the loved person to whom the poem was dedicated. The presence of mathematical concepts in poetry and prose, adding the common sense of the notions to the technical ones, is here noticed. The naive and intuitive appearance of mathematical concepts, like the second derivative and fractals, in poems by our great poets, before their formal creation in mathematics, is also pointed out. Symmetry is a most valuable instrument in literature, and particularly in visual poetry. In this framework, famous Portuguese visual poems are revisited.

¹ Source: <https://www.culturagenial.com/o-que-e-um-poema-ThThevisual-exemplos/>

² Source: <https://visual-poetry.tumblr.com/post/17944489142/p%C3%AAndulo-by-em-de-melo-e-castro-196162>.

³ Source: https://pt.wikipedia.org/wiki/E._M._de_Melo_e_Castro

⁴ Source: http://www.antonimiranda.com.br/poesia_visual/almada_negreiros.html

⁵ Source: Wikimedia. https://en.wikipedia.org/wiki/Zhoubi_Suanjing

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