

# An extended invariant approach to laminate failure of fibre-reinforced polymer structures

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## ABSTRACT

This paper presents the extension and validation of omni-failure envelopes for first-ply failure (FPF) and last-ply failure (LPF) analysis of advanced composite materials under general three-dimensional (3D) stress states. Phenomenological failure criteria based on invariant structural tensors are implemented to address failure events in multidirectional laminates using the “omni strain failure envelope” concept. This concept enables the generation of safe predictions of FPF and LPF of composite laminates, providing reliable and fast laminate failure indications that can be particularly useful as a design tool for conceptual and preliminary design of composite structures. The proposed extended omni strain failure envelopes allow not only identification of the controlling plies for FPF and LPF, but also of the controlling failure modes. FPF/LPF surfaces for general 3D stress states can be obtained using only the material properties extracted from the unidirectional (UD) material, and can predict membrane FPF or LPF of any laminate independently of lay-up, while considering the effect of out-of-plane stresses. The predictions of the LPF envelopes and surfaces are compared with experimental data on multidirectional laminates from the first and second World-Wide Failure Exercise (WWFE), showing a satisfactory agreement and validating the conservative character of omni-failure envelopes also in the presence of high levels of triaxiality.

**Keywords:** Failure Criteria; Laminates; Polymer-matrix composites; Strength; Analytical modelling; Laminate mechanics

## NOMENCLATURE

<b>a</b>	Preferred direction
<b>A</b>	Structural tensor
CFRP	Carbon fibre reinforced polymer
$E_i$	Elastic modulus in the $i$ -direction
$F_{12}^*$	Interaction term from the Tsai-Wu failure theory
$FI_F$	Failure index for fibre tensile failure onset
$FI_K$	Failure index for fibre kinking
$FI_M$	Failure index for matrix failure onset
FPF	First-ply failure
FRP	Fibre reinforced polymer
$G_{12}, G_{23}$	In-plane and transverse shear moduli
GFRP	Glass fibre reinforced polymer
$I_i$	$i^{th}$ invariant of the stress tensor
LPF	Last-ply failure
$S_L, S_T$	In-plane and transverse shear strength
UD	Unidirectional
WWFE	World-Wide Failure Exercise
$X_c$	Compressive strength in the fibre direction
$X_t$	Tensile strength in the fibre direction
$Y_{bc}$	Transverse biaxial compressive strength
$Y_{bt}$	Transverse biaxial tensile strength

$Y_c$	Compressive strength in the transverse direction
$Y_t$	Tensile strength in the transverse direction

### Greek Symbols

$\alpha_{ij}$	Failure parameters
$\varepsilon$	Strain tensor
$\varepsilon_{xc}, \varepsilon_{xt}$	Compressive and tensile strain-to-failure
$\nu_{12}, \nu_{23}$	In-plane and transverse Poisson's ratio
$\sigma$	Stress tensor
$\sigma^p$	Crack inducing components of the stress tensor
$\sigma^r$	Reaction components of the stress tensor
$\varphi$	Kinking angle
$\varphi_c$	Kinking angle under pure longitudinal compression
$\chi$	Micro-mechanical parameter
$\psi$	Angle of the kinking plane

## 1.0 INTRODUCTION

Failure criteria are defined to set the limit values in stress (or strain) space beyond which the material experiences a certain level of structural degradation. Therefore, they provide a robust tool for predicting the loss of integrity that could lead to the structural collapse of the studied component<sup>(1)</sup>. For instance, failure criteria are crucial at early design stages, especially when providing failure envelopes in the stress (or strain) space from relatively simple experimental data. This enables the calculation of safety factors of composite structures subjected to complex loading and boundary conditions.

Over the last five decades, the development of failure criteria for composite materials has found continuous efforts worldwide that have led to the proposal of several failure criteria. As a direct consequence, many reviews of failure theories were published so far. One of the proposed classifications of these theories was made by distinguishing theories that do not account for different failure modes, denoted as *non-phenomenological failure criteria*, and failure theories that are able to identify the different failure modes, denoted as *phenomenological failure criteria*<sup>(2,3)</sup>. The first group comprises criteria in which a failure envelope is defined by using a mathematical expression, usually a polynomial form, which predicts failure by interpolating between a few experimental points. No attempt is made in order to predict which failure mode is taking place, and the criterion itself does not integrate any physical consideration. Tsai-Wu and Tsai-Hill are two common examples of non-phenomenological failure theories. Failure criteria of the second family predict failure based on physical considerations for the specific failure modes.

The need for failure criteria based on failure mechanisms dates back to 1973, when Hashin used his experimental studies to establish two different formulations in order to identify fibre failure and matrix failure mechanisms in a independent manner. According to some reviews, however, the first model distinguishing failure modes was proposed by Puck in 1969<sup>(4)</sup>, although his theory has found more resonance in the composites community with the work published in 1998<sup>(5)</sup>. Among the available phenomenological failure criteria, Hashin, Puck and LaRC failure theories can be highlighted<sup>(6,7,8)</sup>.

More recently, a new set of failure criteria based on structural invariants was proposed, commonly referred to as 3D invariant-based failure criteria<sup>(9)</sup>, which have provided satisfactory agreements with respect to experimental data and therefore they can be considered as potential predictive tools for composite designs in engineering practice.

These criteria can be implemented to perform ply-by-ply failure analysis with the aim of aiding the design of composite laminates. However, when the number of layers in the laminate is large or for large-scale simulations of composite structures, the computational cost of ply-by-ply or layerwise representations becomes huge, making the meso-scale analysis unsuitable for the mentioned applications. For these reasons, alternative approaches at the macro-scale, i.e., laminate level, must be considered in the numerical failure predictions<sup>(10,11)</sup>.

In this framework, to meet the industrial need for fast and practical tools at an early design stage and optimisation of large-scale composite structures (such as fuselage, wings or empennage), simplified and reliable approaches for the prediction of LPF (last-ply failure) of any composite laminates are particularly useful. Following this idea, Tsai and Melo proposed a novel invariant approach for the description of the mechanical response of composite laminates<sup>(12)</sup>. They introduced the “omni strain” failure envelope, also denoted as minimum FPF (first-ply failure) envelope, as the inner failure envelope in strain space, obtained by finding the controlling ply that would fail first for unit loading strain vectors from 0 to  $2\pi$ . This concept was then extended to address LPF, following the same approach but applying a matrix degradation factor<sup>(13)</sup>. Omni strain envelopes can be considered a material property independent of the laminate layup composition, thanks to their capability of describing the failure of any laminate by covering all possible fibre orientations.

However, since the “omni failure” concept was initially proposed using the Tsai-Wu failure criterion, the resulting method was able to highlight the controlling plies, but not the critical failure modes. Moreover, the Tsai-Wu criterion is mostly used for plane stress conditions, inhibiting the prediction of failure under complex triaxial stress states (as observed in mechanically fastened connections, or in thick composite structures, such as at the wing-to-fuselage assembly wingbox or at the bulkhead-stringer run-outs connection). Hence, to extend the applicability of the omni strain failure envelopes concept, the invariant-based failure theory is proposed for generation of phenomenological 3D omni strain failure surfaces, enabling the application of omni criteria in sizing of composite structures subjected not only to plane stress conditions, but also general 3D stress states.

With this aim, this paper is organized as follows: Section 2 presents the background on the 3D invariant-based failure criteria and omni strain failure envelopes; Section 3, departing from these theories, presents an extension of the omni strain failure envelopes to address laminate failure under general 3D stress states and predict the governing failure modes; Section 4 presents a series of validation case studies considering failure of multidirectional laminates under 2D and 3D stress states; Section 5 summarizes the main benefits and limitations of the proposed approach; finally, Section 6 includes the overall conclusions of the described work.

## 2.0 BACKGROUND

### 2.1 3D invariant-based failure criteria

Among the most recent phenomenological failure criteria proposed for fibre-reinforced polymers (FRPs), a set of advanced phenomenological failure theories was selected for the present work due to its unique 3D character. These new criteria are based on the transversely isotropic yield function developed by Vogler et al.<sup>(14)</sup>. They have an invariant quadratic formulation involving structural tensors that accounts for the preferred material directions of the anisotropic material. With this formulation, anisotropy is derived using structural tensors and not symmetry conditions based on a reference coordinate system. These advantageous features enable a simpler and elegant description of failure of composites<sup>(9)</sup>.

The invariant-based failure criterion for transverse failure of unidirectional composites is given as:

$$FI_M = \begin{cases} \alpha_1 I_1 + \alpha_2 I_2 + \alpha'_3 I_3 + \alpha'_{32} I_3^2 & \text{for } I_3 > 0 \\ \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3^c I_3 + \alpha_{32}^c I_3^2 & \text{for } I_3 \leq 0 \end{cases} \quad (1)$$

where the stress invariants ( $I_1, I_2, I_3$ ) for matrix failure are defined as:

$$I_1 = \frac{1}{4}\sigma_{22}^2 - \frac{1}{2}\sigma_{22}\sigma_{33} + \frac{1}{4}\sigma_{33}^2 + \sigma_{23}^2, \quad I_2 = \sigma_{12}^2 + \sigma_{13}^2, \quad I_3 = \sigma_{22} + \sigma_{33} \quad (2)$$

and the failure parameters  $\alpha$  are given by:

$$\begin{aligned} \alpha_1 &= \frac{1}{S_T^2}, \quad \alpha'_{32} = \frac{1 - \frac{Y_t}{2Y_{bt}} - \alpha_1 \frac{Y_t^2}{4}}{Y_t^2 - 2Y_{bt}Y_t}, \quad \alpha'_3 = \frac{1}{2Y_{bt}} - 2\alpha'_{32}Y_{bt}, \\ \alpha_2 &= \frac{1}{S_L^2}, \quad \alpha_{32}^c = \frac{1 - \frac{Y_c}{2Y_{bc}} - \alpha_1 \frac{Y_c^2}{4}}{Y_c^2 - 2Y_{bc}Y_c}, \quad \alpha_3^c = \frac{1}{2Y_{bc}} - 2\alpha_{32}^cY_{bc}. \end{aligned} \quad (3)$$

$S_T$  and  $S_L$  are respectively the transverse and in-plane shear strengths,  $Y_c$  and  $Y_{bc}$  are respectively the transverse uniaxial and biaxial compressive strengths, and  $Y_t$  and  $Y_{bt}$  are respectively the transverse uniaxial and biaxial tensile strengths. Through the sign of the third invariant  $I_3$ , this criterion is able to address failure under biaxial stress states.

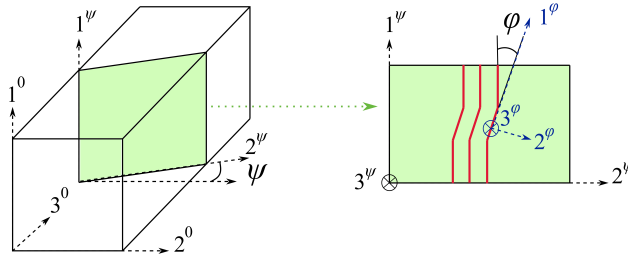
Fibre failure under tension is predicted using the non-interactive maximum allowable strain criterion, following the LaRC03 criteria<sup>(6)</sup>. Since the failure mechanism under longitudinal compression of carbon fibre reinforced polymers (CFRPs) involves mainly the formation of a kink band, resulting from micro-buckled fibres and local matrix cracking, Camanho et al.<sup>(9)</sup> proposed a 3D kinking model by recalling the invariant-based failure criterion for transverse failure, but in the misalignment frame of the kinked fibres. In fact, by determining the angle of the kinking plane  $\psi$  and the kinking angle  $\varphi$  based on the kinematics of fibre kinking, it is possible to formulate the invariant-based failure criterion for kinking failure prediction in the fibres misalignment frame.

Figure 1 shows the kinking plane and the 3 coordinate systems: the initial coordinate system ( $1^0 2^0 3^0$ ), associated with the global preferred material directions of the composite, the

coordinate system related with the kinking plane ( $1^\psi 2^\psi 3^\psi$ ) and the coordinate system related with the misaligned fibres ( $1^\varphi 2^\varphi 3^\varphi$ ). The invariant-based failure criterion for fibre kinking is formulated in the latter coordinate system. In a compact form, the failure criteria for fibre failure can be presented in the following way:

$$FI_F = \frac{\varepsilon_1}{\varepsilon_{Xt}} \quad \text{for } \sigma_{11} \geq 0 \quad (4)$$

$$FI_K = \begin{cases} \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3^t I_3 + \alpha_{32}^t I_3^2 & \text{for } \sigma_{11} < 0 \quad \text{and} \quad I_3 > 0 \\ \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3^c I_3 + \alpha_{32}^c I_3^2 & \text{for } \sigma_{11} < 0 \quad \text{and} \quad I_3 \leq 0 \end{cases} \quad (5)$$



**Fig. 1.** 3D kinking model with the definition of the kinking angle  $\varphi$  and the angle of the kinking plane  $\psi$ .

where  $\varepsilon_{Xt}$  is the ultimate tensile strain. In order to formulate the stress tensors, the definition of the preferred direction  $\mathbf{a}$ , or principal direction, of the transversely isotropic material, is required. This direction, which, for UD composites, corresponds to the fibre direction, in the frame of the misaligned fibres is obtained by performing two consecutive transformations of coordinate system to reach the initial ( $1^0 2^0 3^0$ ) system:

$$\mathbf{a}^{(0)} = \begin{bmatrix} \cos \varphi \\ \cos \psi \sin \varphi \\ \sin \psi \sin \varphi \end{bmatrix} \quad (6)$$

The structural tensor  $\mathbf{A}$  that represents the transversely isotropic properties of the material is defined as:

$$\mathbf{A} = \mathbf{a} \otimes \mathbf{a} \quad (7)$$

The stress invariants needed for the coordinate system-free formulation of the invariant-based criterion are now formulated as a function of the preferred direction  $\mathbf{a}$ , the structural tensor  $\mathbf{A}$ , and the stress tensor  $\boldsymbol{\sigma}$  based on its crack inducing components  $\boldsymbol{\sigma}^p$  and the reaction components  $\boldsymbol{\sigma}^r$ :

$$I_1 = \frac{1}{2} \text{tr} (\boldsymbol{\sigma}^p)^2 - \mathbf{a} (\boldsymbol{\sigma}^p)^2 \mathbf{a}, \quad I_2 = \mathbf{a} (\boldsymbol{\sigma}^p)^2 \mathbf{a}, \quad I_3 = \text{tr} \boldsymbol{\sigma} - \mathbf{a} \boldsymbol{\sigma} \mathbf{a} \quad (8)$$

with:

$$\boldsymbol{\sigma}^r = \frac{1}{2} (\text{tr} \boldsymbol{\sigma} - \mathbf{a} \boldsymbol{\sigma} \mathbf{a}) \mathbf{1} - \frac{1}{2} (\text{tr} \boldsymbol{\sigma} - 3 \mathbf{a} \boldsymbol{\sigma} \mathbf{a}) \mathbf{A}, \quad \boldsymbol{\sigma}^p = \boldsymbol{\sigma} - \boldsymbol{\sigma}^r \quad (9)$$

The angle of the kinking plane  $\psi$  is determined using a pragmatic approach considering the shear stresses acting on the transversely isotropic plane (if these components are not zero)<sup>(7,8)</sup>:

$$\psi = \arctan\left(\frac{\sigma_{13}}{\sigma_{12}}\right) \quad (10)$$

Otherwise the angle of the kinking plane is calculated by the maximum principal stress that acts on the transversely isotropic plane:

$$\psi = \frac{1}{2} \arctan\left(\frac{2\sigma_{23}}{\sigma_{22} - \sigma_{33}}\right) \quad (11)$$

The kinking angle  $\varphi$  is determined as proposed by Catalanotti et al.<sup>(8)</sup>, in which a micro-mechanical parameter  $\chi$  is introduced to account for the micro-structural effects that control the development of fibre kinking:

$$\chi = -\frac{\sin 2\varphi_c X_c}{2\varphi_c} \quad (12)$$

where  $\varphi_c$  is the kinking angle under pure longitudinal compression and  $X_c$  is the longitudinal compressive strength. As suggested by the authors of this formulation<sup>(8)</sup>, the kinking angle  $\varphi$  can be found imposing the stress equilibrium in the frame of the kink band and solving the nonlinear equation, applying the bisection method. The determination of the kinking angle  $\varphi_c$  is made by solving the invariant-based failure criteria for a pure longitudinal compressive stress state and knowing that, at failure, the criterion in Eq. (5) yields 1, with  $\sigma_{11} = X_c$  and  $\varphi = \varphi_c$ . Solving for  $\varphi_c$ :

$$\varphi_c = \frac{1}{2} \arccos \left\{ \left[ 4 \sqrt{\alpha_1 - 4\alpha_2 + \alpha_2^2 X_c^2 + (\alpha_3^c)^2 + 2\alpha_2 \alpha_3^c X_c + 4\alpha_{32}^c} + (\alpha_1 + 4\alpha_{32}^c) X_c + 4\alpha_3^c \right] \cdot \left[ (\alpha_1 - 4\alpha_2 + 4\alpha_{32}^c) X_c \right]^{-1} \right\} \quad (13)$$

It is important to emphasize that this set of invariant-based failure criteria is completely formulated in a 3D setting, unlike other phenomenological failure criteria that were initially formulated in a 2D setting and then extended to the 3D case. Additional details, such as a pragmatic approach for the determination of the fracture plane and the definition of the *in situ* properties, can be found in Ref.<sup>(15)</sup>.

Several validation studies of these criteria were performed, comparing the predictions with experimental data obtained for different material systems under various scenarios of multiaxial loading. In particular, these criteria are capable of predicting the evolution of the shear stress with hydrostatic pressure, which no previous failure criteria have taken into account. Furthermore, a good agreement between the failure envelopes predicted by computational micro-mechanics and the 3D invariant-based failure criteria were reported. While Hashin's criteria show an open failure envelope for the biaxial transverse compression quadrant, thus making it unsuitable for representing the fracture of composites subjected to high hydrostatic stresses, the 3D invariant-based failure criteria accurately represented the failure envelopes under both tension-tension and compression-compression biaxial stress states<sup>(9)</sup>.

The validation studies performed by Camanho et al.<sup>(9)</sup> include experimental results from open literature (such as the first WWFE, where only biaxial tests are discussed) and additional data obtained using computational micro-mechanics to complement the available database, without requiring very expensive test setups.

To further challenge the 3D invariant-based theory in the prediction of failure under triaxial stress states, experimental results from the WWFE-II<sup>(16)</sup> are considered here. Figs. 2-5 show the correlation of the predicted envelopes against experimental results from the WWFE-II for UD S-glass/epoxy, carbon/epoxy, E-glass/MY750 and T300/PR319 respectively. The predictions of the 3D invariant-based theory are also compared with those obtained using the Tsai-Wu failure surface for the same test cases.

The mechanical properties of the tested materials are found in Ref.<sup>(16)</sup>; however, since biaxial strengths are not provided, they have been assumed after assessing their effect on the predicted envelopes. Furthermore, the transverse shear strength  $S_T$  is assumed equal to the transverse tensile strength  $Y_T$ , following the observations from Refs.<sup>(17,5)</sup>. The influence of  $Y_{bc}$  on the strength predictions of the invariant-based theory was investigated for each case and it is included in the respective figures. With the same scope, the effect of the interaction term  $F_{12}^*$  from the Tsai-Wu theory was studied and it is also shown. All the test cases highlight that the 3D invariant-based theory offers a remarkable flexibility in fitting complex results with  $Y_{bc}$ , while the interaction term from Tsai-Wu cannot always help the correlation with experimental data. Indeed, except for the third case (Fig. 4) where the strength predictions are very sensitive to the value of the interaction term, a non-zero  $F_{12}^*$  results in open envelopes for the first 2 cases (Figs. 2-3) and in almost coincident predictions in the last case (Fig. 5).

In general, the failure predictions based on Tsai-Wu criterion exhibited some deviations with respect to the experimental data when dealing with triaxial failure stress states. In spite of the large scatter of the test results and the complexity of the considered test cases, a generally good fit is observed when using the 3D invariant-based failure criteria.

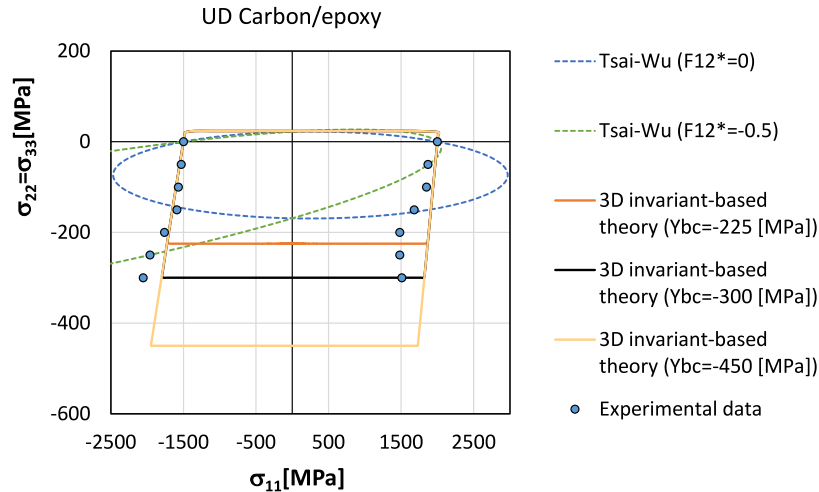
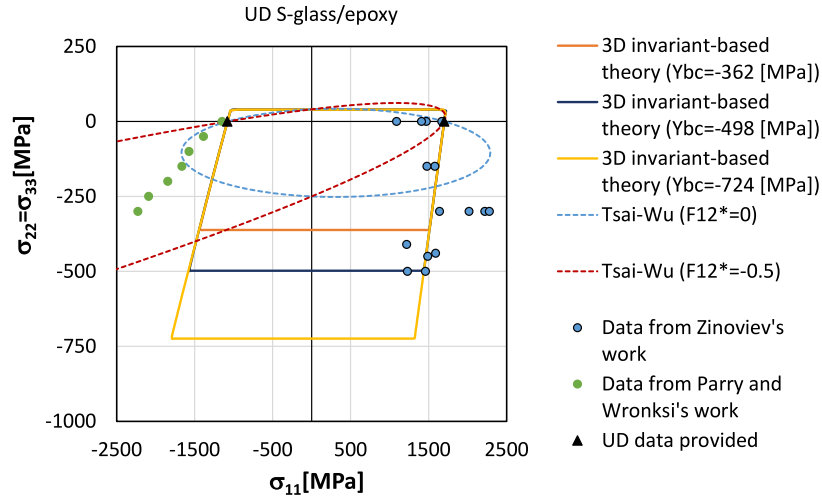
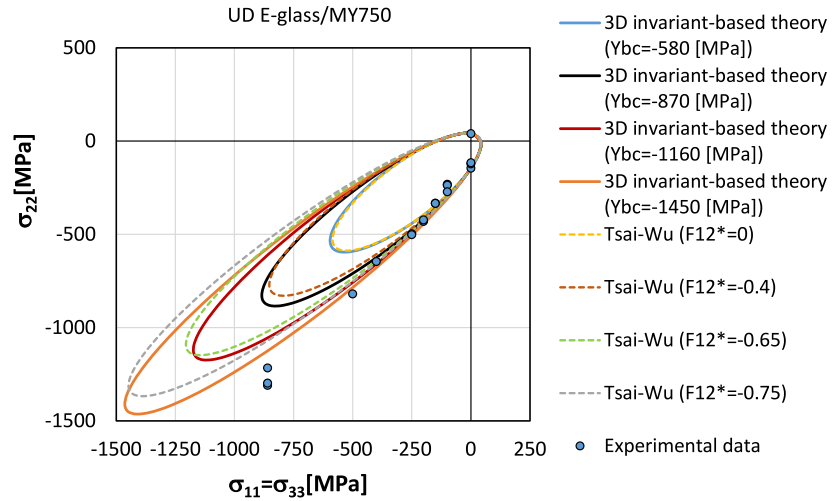


Fig. 2. Triaxial failure stress states:  $\sigma_{22}=\sigma_{33}$  vs. longitudinal stress  $\sigma_{11}$  for a UD carbon/epoxy.

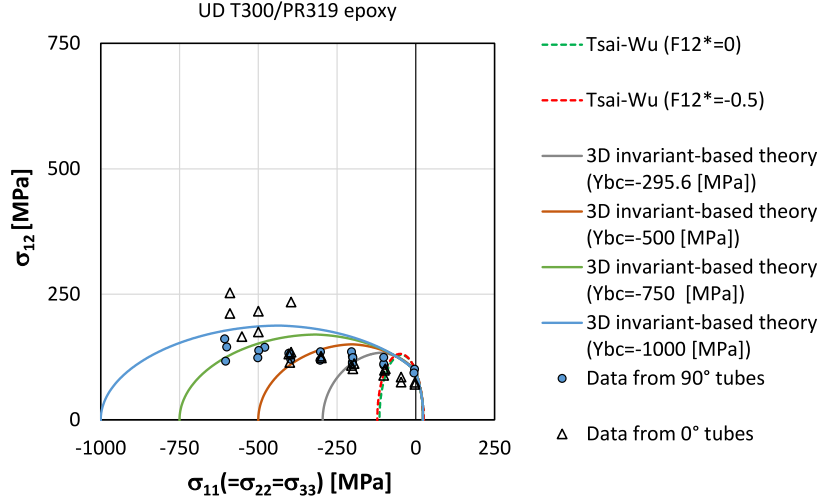




**Fig. 3.** Triaxial failure stress states:  $\sigma_{22}=\sigma_{33}$  vs. longitudinal stress  $\sigma_{11}$  for a UD S-glass/epoxy cube.



**Fig. 4.** Triaxial failure stress states:  $\sigma_{22}$  vs. longitudinal stress  $\sigma_{11}=\sigma_{33}$  for a UD E-glass/MY750 epoxy.



**Fig. 5.** Triaxial failure stress states: shear stress  $\sigma_{12}$  vs. hydrostatic pressure ( $\sigma_{11}=\sigma_{22}=\sigma_{33}$ ) for a UD T300/PR319 epoxy.

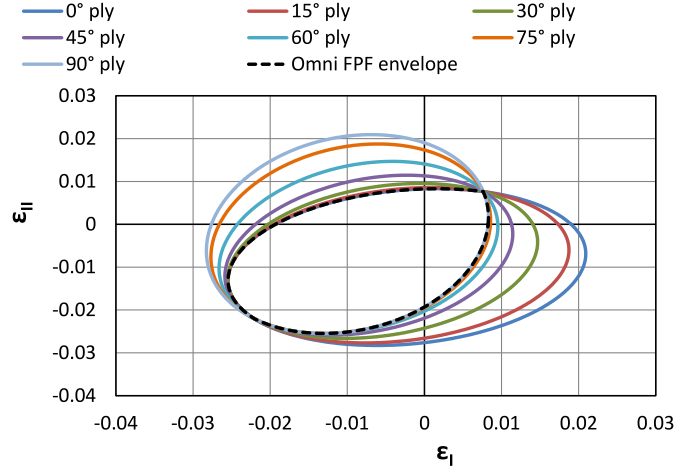
## 2.2 Omni strain failure envelopes

Omni strain envelopes are presented in strain space, since the shape of the envelope remains independent of adding other plies. In strain space it is possible to superimpose the failure envelopes for the different ply orientations and compute a laminate failure envelope. Fig. 6 shows the omni strain failure envelopes based on the Tsai-Wu failure criterion for two different materials. It can be noted that, with this approach, all laminate data can be displayed on one graph in strain space, realizing a very concise display of the strength of a given composite material. Furthermore, it is a very practical tool, enabling a fast selection of the stacking sequence according to the required mechanical properties, since it covers all the possible ply orientations.

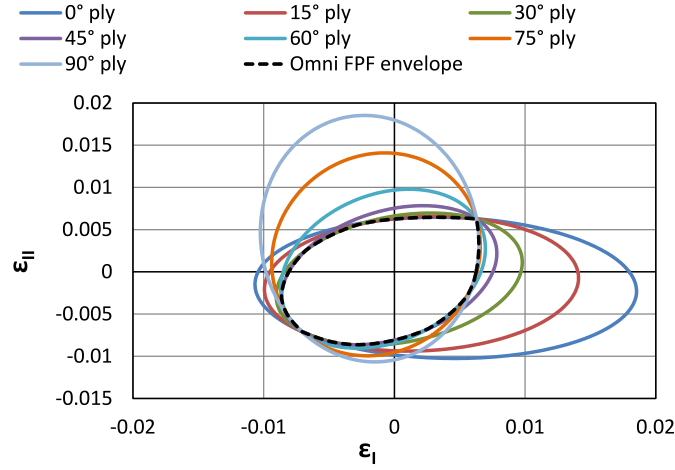
For the implementation of omni strain FPF envelopes, the use of polynomial tensor-based failure criteria is interesting, as there are established transformation relations that enable the reformulation of the criteria from stress space to strain space. These transformation relations are described in Ref. <sup>(12)</sup>, where the Tsai-Wu failure criterion is reformulated in strain space:

$$G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i = 1 \quad (14)$$

where  $G_{ij}$  and  $G_i$  are the strength parameters in strain space, which can be expressed as a function of the strength parameters in stress space  $F_{ij}$  and  $F_i$  as follows:  $G_{ij} = F_{kl}Q_{ki}Q_{lj}$ ,  $G_j = F_jQ_{ij}$ , where  $Q_{ij}$  is the in-plane stiffness matrix. However, any failure theory can be applied for the generation of omni FPF envelopes.



(a) T700/C-Ply 55.



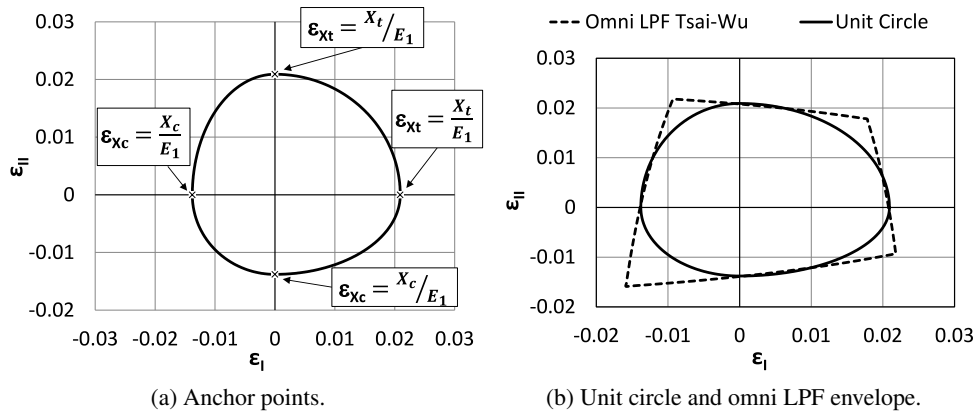
(b) IM7/977-3.

**Fig. 6.** Omni FPF envelopes in principal strain space for T700/C-Ply 55 (a) and IM7/977-3 (b) laminates, according to the Tsai-Wu failure criterion.

Because the omni FPF envelopes represent the most conservative design solution, where all the plies remain undamaged, Tsai and Melo proposed an extended version of this criterion, to define and predict the continued load-carrying capability of any laminate, after damage initiation. They introduced the omni last-ply failure (LPF) envelope<sup>(13)</sup>, which is an extension of the concept of omni FPF envelope to ultimate failure. The construction of these envelopes follows the same procedure as described before, but with degraded ply properties, based on a matrix degradation factor ( $E_m^*$ ) and micro-mechanics relations. Moreover, Tsai and Melo observed that, for all CFRP laminates, the inner LPF envelope is controlled by the 0° and 90° plies loaded along the respective fibre direction.

Following these observations, a further simplification of the failure analysis was performed

introducing the unit circle failure envelopes for CFRPs, which can be easily represented by defining the “anchor points” as shown in Fig. 7a. Comparing the omni strain LPF envelope and the unit circle failure envelope of the same material, as shown in Fig. 7b, the unit circle envelope is inscribed in the omni LPF envelope. Although the failure predictions related with this criteria are intentionally conservative, this theory is extremely useful. In particular, the great advantage of using the unit circle failure envelope is that it only requires the strains-to-failure of a  $0^\circ$  coupon measured in tension and in compression, instead of complete characterization of the ply properties required by the omni strain LPF envelope.

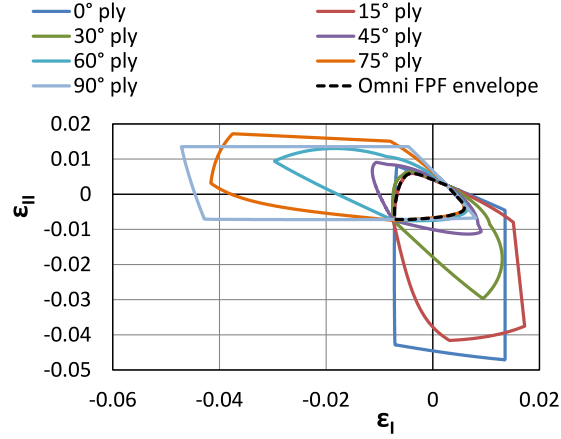


**Fig. 7.** (a) Definition of anchor points for a unit circle failure envelope in principal strain space; (b) unit circle (solid line) and omni strain LPF (dashed line) envelopes for T700/C-Ply 55 carbon/epoxy.

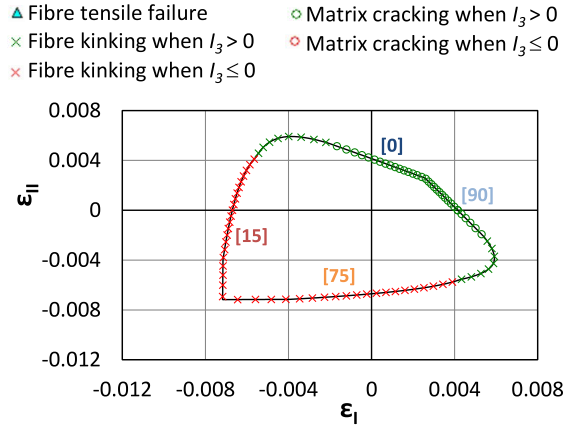
### 3.0 EXTENDED OMNI STRAIN FAILURE ENVELOPES

Exploiting the fully 3D description of failure provided by the invariant-based theory outlined in Section 2.1, omni strain failure envelopes can be extended by finding the controlling plies in the 3D principal strain space. Indeed, with this extension, the resulting design space can predict laminate failure under complex 3D stress states and address, for instance, the design of bolted joints or thick composite laminates, where through-thickness stress states cannot be neglected. Furthermore, in this case, the envelopes allow the identification of the critical failure modes for each controlling ply, which cannot be investigated with the Tsai-Wu based omni strain envelopes.

An example of omni FPF envelope, obtained using the invariant-based failure model for IM7/8552, is shown in Fig. 8. On the left-hand side, several ply failure envelopes are represented with different colours, from which the omni strain FPF envelope (outlined with a black dotted line) can be obtained. Fig. 8b provides a detailed view of the omni FPF envelope only, where the failure loci are represented using different markers in order to identify the critical failure modes for each controlling ply ( $[0]$ ,  $[90]$ ,  $[75]$  and  $[15]$ ). In this way, it is possible to identify fibre failure as the most prominent FPF mode shaping the omni FPF envelope of IM7/8552.



(a) Omni FPF envelope.

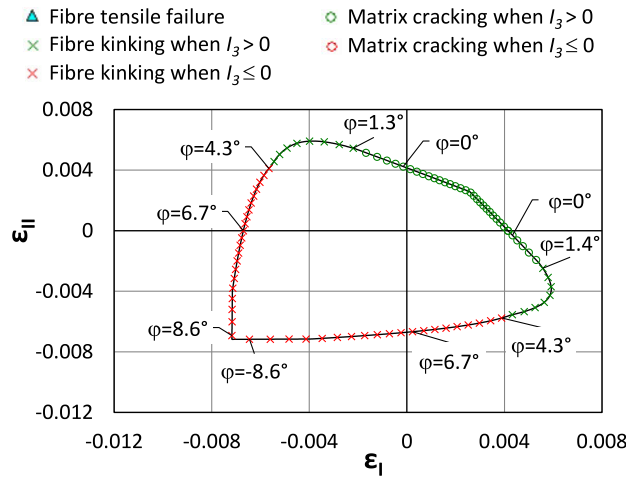


(b) Controlling plies and failure modes.

**Fig. 8.** Omni FPF envelope in strain space for IM7/8552, according to the 3D invariant-based failure criteria, with a detailed view on the critical failure modes.

Furthermore, when fibre kinking occurs, the kinking angle  $\varphi$  and the angle of the kinking plane  $\psi$ , computed as described in the failure model, can be recorded and added to the plot. To give an example, Fig. 9 shows the evolution of the kinking angle on the omni FPF envelope for IM7/8552. It can be observed that the predicted kinking angle in the minimum FPF envelope ranges, in absolute values, from  $0^\circ$  to  $8.6^\circ$ , reaching the maximum absolute value in the compression-compression quadrant, under the highest value of applied biaxial compressive strain.

Following Ref. <sup>(13)</sup>, the degraded ply properties of IM7/8552 are computed using micromechanics relations and a matrix degradation factor equal to 0.15. The omni LPF envelope can be finally generated, as shown in Fig. 10. In a similar illustrative scheme as for the omni FPF envelope, the controlling plies and the critical failure modes are highlighted in the figure. In particular, Fig. 10b shows that LPF is dominated by fibre failure, in agreement with the

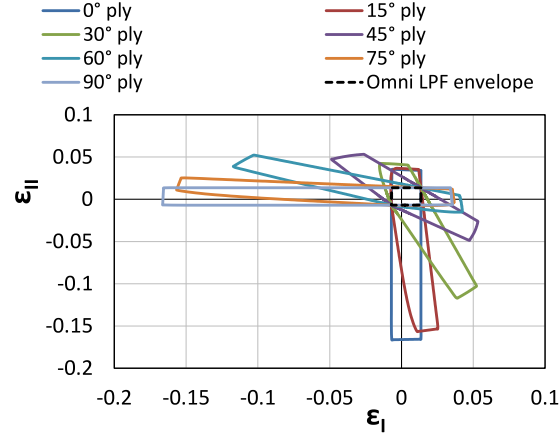


**Fig. 9.** Detailed view on the values of kinking angle  $\varphi$  on the omni FPF envelope in principal strain space for IM7/8552, according to the 3D invariant-based failure model.

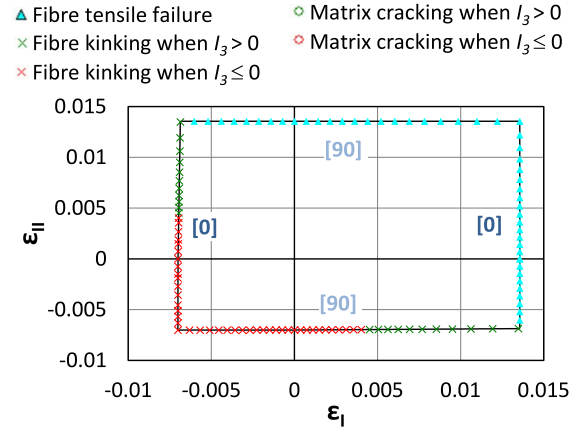
observations of Tsai and Melo<sup>(13)</sup>.

This invariant-based description of failure can be represented in 3D, as the set of invariant-based failure criteria captures well the effect of the out-of-plane direction. For IM7/8552, the failure surface in stress space is shown in Fig. 11, while the omni FPF and LPF surfaces are represented in Figs. 12a and 12b, respectively. As a remark, these surfaces predict membrane FPF or LPF of any laminate, independently of lay-up or stacking sequence, while considering the effect of out-of-plane stresses, and can be obtained using only the material properties of the UD material required by the failure model. For this reason, this approach can be very effective in guiding the conceptual design of composite structures subjected to any stress state.

It is also important to stress that, for typical CFRP laminates, such as aerospace industry-standard “quad” laminates characterized by different percentage of  $0^\circ$ ,  $\pm 45^\circ$  and  $90^\circ$  plies<sup>(18)</sup>, omni LPF and laminate LPF envelopes (the latter obtained from superposing in strain space only the envelopes of the ply orientations contained in the selected laminate) will lead to the same laminate failure predictions. This is justified by the presence of the  $[0]$  and  $[90]$  plies in these laminates, which will govern LPF according to both approaches. Therefore, for all CFRP quad laminates, the omni LPF envelopes ensure the same degree of conservatism as the laminate LPF envelopes, but without the need to recompute the failure envelope every time the layup changes. On the other hand, when tackling LPF of angle-ply laminates, omni LPF envelopes will have a certain degree of conservatism that will depend on the ply angles.

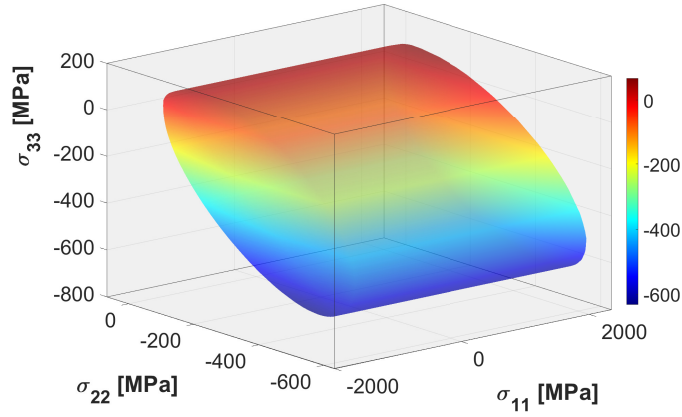


(a) Omni LFP envelope.

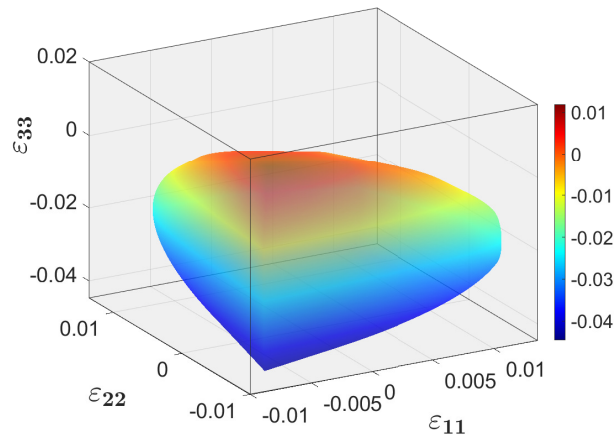


(b) Identification of controlling plies and failure modes.

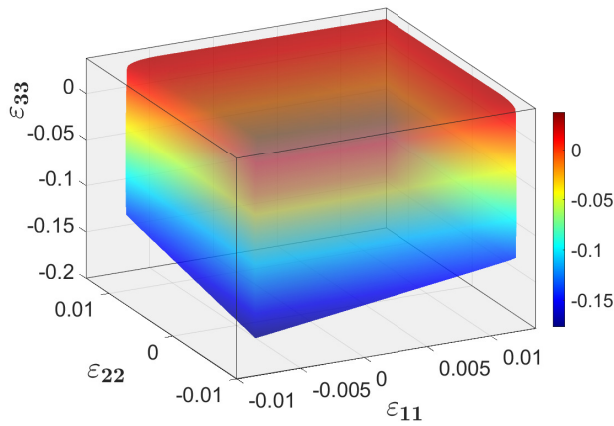
**Fig. 10.** Omni LFP envelope in principal strain space for IM7/8552, according to the 3D invariant-based failure criteria, with a detailed view of the controlling plies and critical failure modes.



**Fig. 11.** 3D failure surface in stress space for UD IM7/8552, obtained using the 3D invariant-based failure criteria.



(a) 3D omni strain FPF surface.



(b) 3D omni strain LPF surface.

**Fig. 12.** 3D omni strain FPF (a) and LPF (b) surfaces in strain space for IM7/8552, obtained using the 3D invariant-based failure criteria.



#### 4.0 VALIDATION OF THE EXTENDED OMNI ENVELOPES

In this section, the reliability and the overall performance of these “extended” omni strain LPF envelopes in predicting laminate failure is assessed using experimental data from the first and the second WWFE<sup>(16,19,20)</sup>. For the validation studies involving biaxial experimental data, omni LPF envelopes are generated using the Tsai-Wu and the 3D invariant-based failure criteria outlined in Section 2.1. For all the considered criteria, the envelopes are calculated using a matrix degradation factor of 0.15, which however lowers the differences in the failure predictions between the different criteria. In fact, when the matrix degradation factor approaches to zero, the predictions of the criteria in 2D become coincident. The degraded elastic properties used for omni LPF envelopes are:  $E_2(=E_3)$ ,  $G_{12}$  and  $\nu_{12}$ . The interaction term  $F_{12}^*$  of the Tsai-Wu failure theory is considered equal to -0.5 for all the considered test cases.

The first WWFE provides several test cases including biaxial failure stress envelopes for different laminates. In this validation study, the following laminates have been considered: AS4/3501-6 [90/±45/0]<sub>s</sub>, E-glass/LY556/HT907/DY063 [±30/90]<sub>s</sub> and E-glass/MY750/HY750/DY063 [±55]<sub>s</sub>. The material properties of their unidirectional plies are provided in Table 1. The additional mechanical properties required for the 3D invariant-based failure theory ( $Y_{bc}$ ,  $Y_{bt}$ ) are not provided in the WWFE-I, but they are scaled following the observations from Ref.<sup>(14)</sup>. Since the experimental data from literature are presented in stress space, laminate stress-strain relations have been used to represent omni LPF envelopes in stress space and compared with the available data.

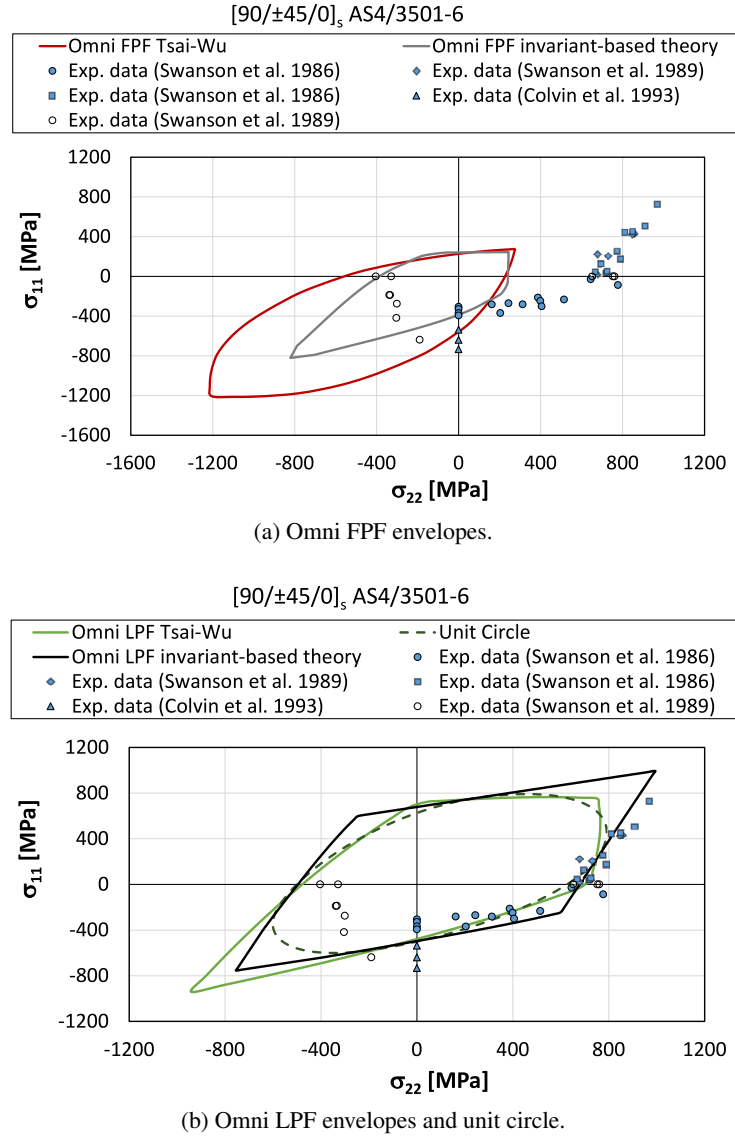
**Table 1**  
Material properties from WWFE-I<sup>(19)</sup>.

Material	$E_1$ [GPa]	$E_2$ [GPa]	$\nu_{12}$	$G_{12}$ [GPa]	$X_t$ - $X_c$ [MPa]	$Y_t$ - $Y_c$ [MPa]	$S_L$ [MPa]
AS4/3501-6	126.0	11.0	0.28	6.6	1950 - 1480	48 - 200	79
E-glass/LY556	53.48	17.7	0.28	5.83	1140 - 570	35 - 114	72
E-glass/MY750	45.6	16.2	0.28	5.83	1280 - 800	40 - 145	73

Fig. 13 presents the comparison between experimental results and the proposed envelopes for an AS4/3501-6 [90/±45/0]<sub>s</sub> laminate. On the left-hand side, Fig. 13a shows the omni FPF envelopes obtained with Tsai-Wu and the invariant-based theory, while in Fig. 13b the omni LPF envelopes based on the same criteria and the unit circle predictions are included. The conservatism of the omni FPF approach can be observed when using both theories and, in particular, in the strength predictions in the first and fourth quadrant. This can be explained by the critical failure mode under those stress states, which is matrix cracking for FPF.

Small differences can be observed between the two omni LPF envelopes (obtained using the 3D invariant-based failure criteria in black and Tsai-Wu in green solid line), but in general, the predictions obtained with Tsai-Wu and the unit circle envelope are slightly more conservative compared with the invariant-based theory. The correlation between test data and predictions is excellent, except for the third quadrant where the predictions seem to overestimate the laminate strength under biaxial compression. However, those experimental results from Swanson and Colvin cannot be considered 100% reliable, as pointed out also by the organizers of the first WWFE<sup>(19)</sup>. In fact, those results are characterized by a large scatter

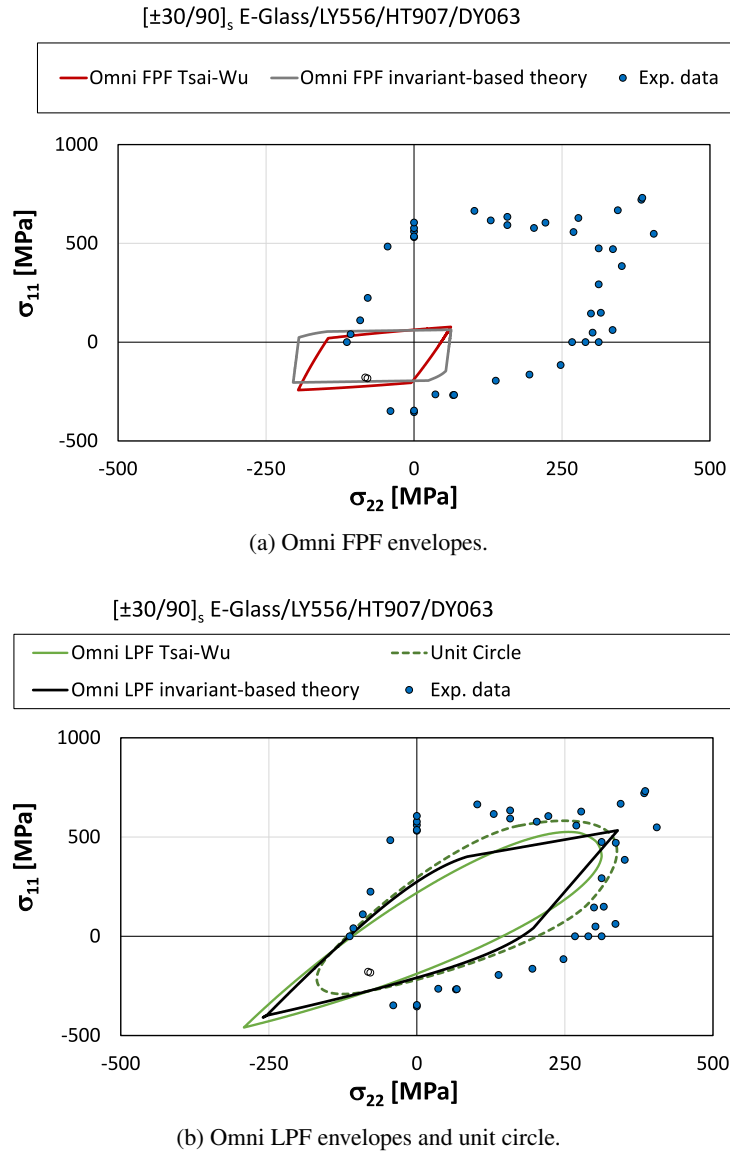
in the mean axial compressive strength, probably affected by buckling when testing longer specimens. Thus, the predictive capability of the considered approaches cannot be reliably assessed in the compression-compression quadrant.



**Fig. 13.**  $\sigma_{22}$  -  $\sigma_{11}$  failure envelopes versus experimental results from the WWFE-I for a [90/±45/0]<sub>s</sub> AS4/3501-6 carbon/epoxy laminate.

The correlation between the failure predictions using omni FPF envelopes and experimental data for a [±30/90]<sub>s</sub> laminate made of E-glass/LY556/HT907/DY063, shown in Fig. 14a, highlights the remarkable conservatism of the approach and the large difference when com-

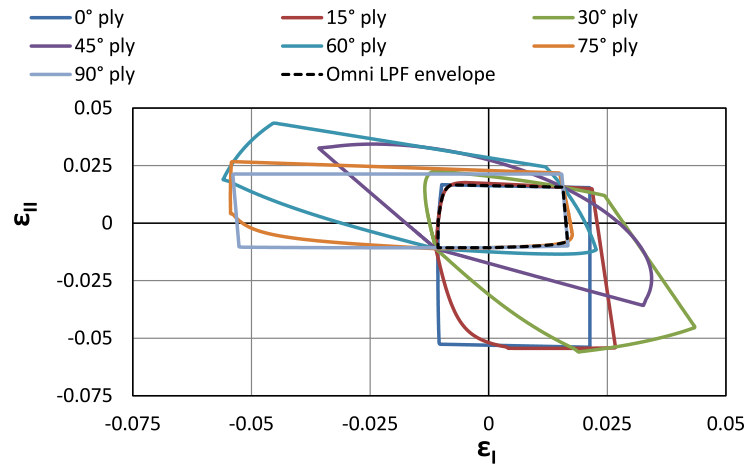
pared with omni LPF and unit circle envelopes shown in Fig. 14b. The failure predictions obtained with the two omni LPF envelopes are almost coincident, while the unit circle is less conservative than the omni LPF envelopes. Unlike CFRPs, fibre failure is not always the critical failure mode for glass-fibre reinforced plastics (GFRPs).



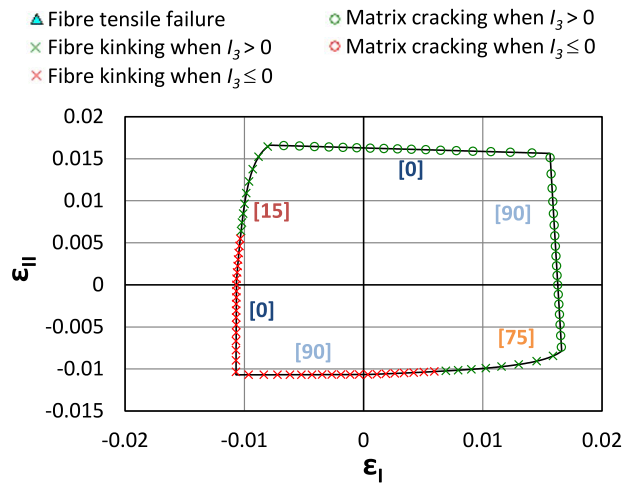
**Fig. 14.**  $\sigma_{22}$  -  $\sigma_{11}$  failure envelopes versus experimental results from the WWFE-I for a [ $\pm 30/90$ ]<sub>s</sub> E-glass/LY556/HT907/DY063 laminate.

To confirm this observation, Figs. 15 and 16 show the omni LPF envelopes in strain space for the E-glass composites herein (Table 1), highlighting that the controlling plies are not

always [0] and [90], and that matrix failure defines almost half of the envelopes for these materials. This confirms that the assumption of fibre failure for LPF is only true when studying CFRP laminates. As direct consequence of these remarks, the resulting unit circles for the same materials are no longer inscribed in the omni LPF envelopes, as represented in Fig. 17. Thus, for the considered GFRPs, with a lower degree of anisotropy, the tensile and compressive strains-to-failure of a [0] coupon cannot be considered the only anchor properties to define omni failure analysis. In other words, the simplification of the unit circle does not apply to GFRPs.

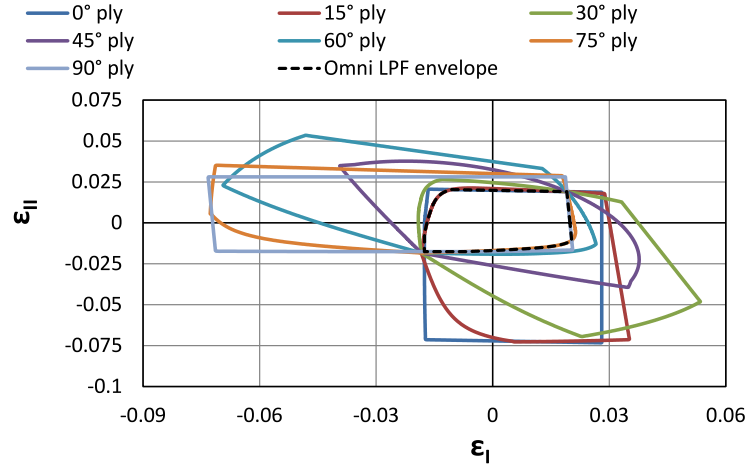


(a) Omni LPF envelope.

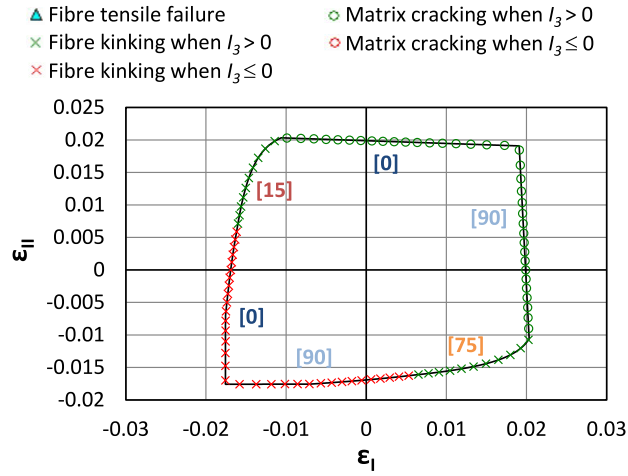


(b) Controlling plies and failure modes.

**Fig. 15.** Omni strain LPF envelope, controlling plies and critical failure modes obtained with the 3D invariant-based failure criteria for E-glass/LY556/HT907/DY063.

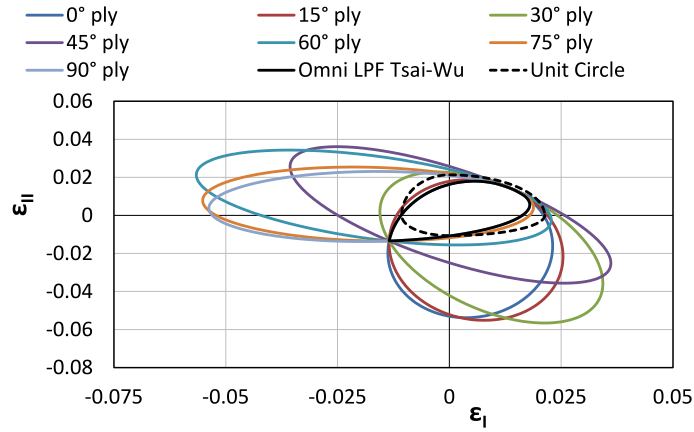


(a) Omni LPF envelope.

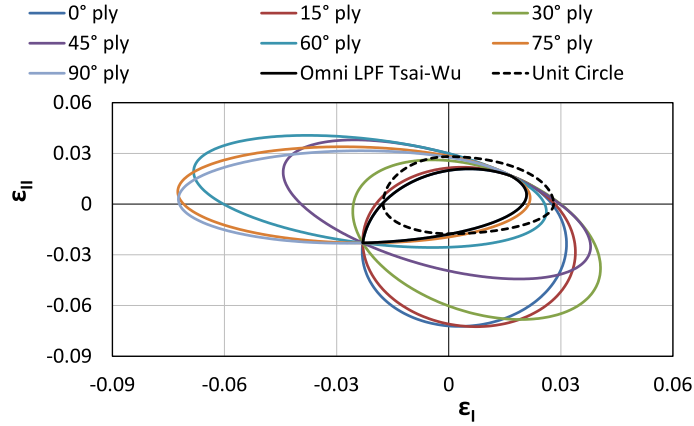


(b) Controlling plies and failure modes.

**Fig. 16.** Omni strain LPF envelope, controlling plies and critical failure modes obtained with the 3D invariant-based failure criteria for E-glass/MY750/HY750/DY063.



(a) E-glass/LY556/HT907/DY063.



(b) E-glass/MY750/HY750/DY063.

**Fig. 17.** Omni LPF envelopes, based on Tsai-Wu, and unit circle envelopes in principal strain space for (a) E-glass/LY556/HT907/DY063 and (b) E-glass/MY750/HY750/DY063.

Finally, it is noted that, as in the previous case (Fig. 13), the failure envelopes in Fig. 14 underpredict the experimental data in the compression–compression quadrant, which can be attributed to compression instability of the specimens under that loading condition.

The experimental results for a  $[\pm 55]_s$  E-glass/MY750/HY750/DY063 laminate are compared with the proposed omni LPF and unit circle envelopes in the  $\sigma_{22} - \sigma_{11}$  stress space, as illustrated in Fig. 18a. The conservatism of omni FPF envelopes is confirmed in this test case, as shown in Fig. 18b, while the predictive capability of omni LPF envelopes can be acknowledged also in this study.

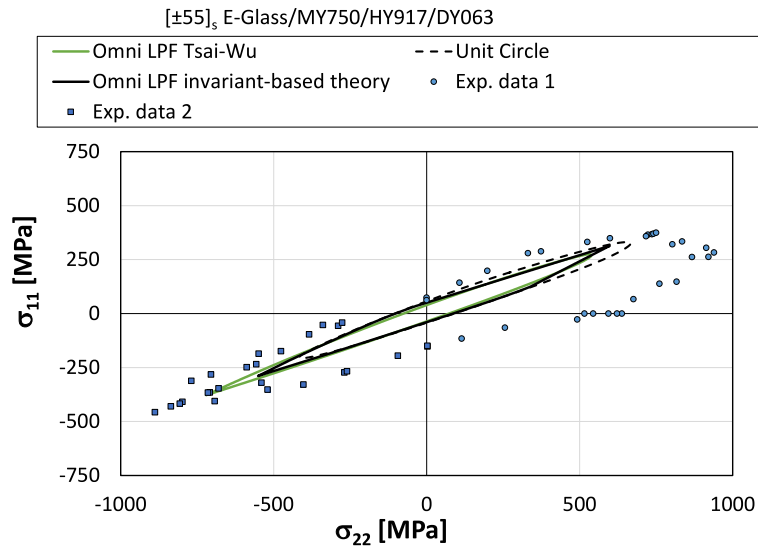
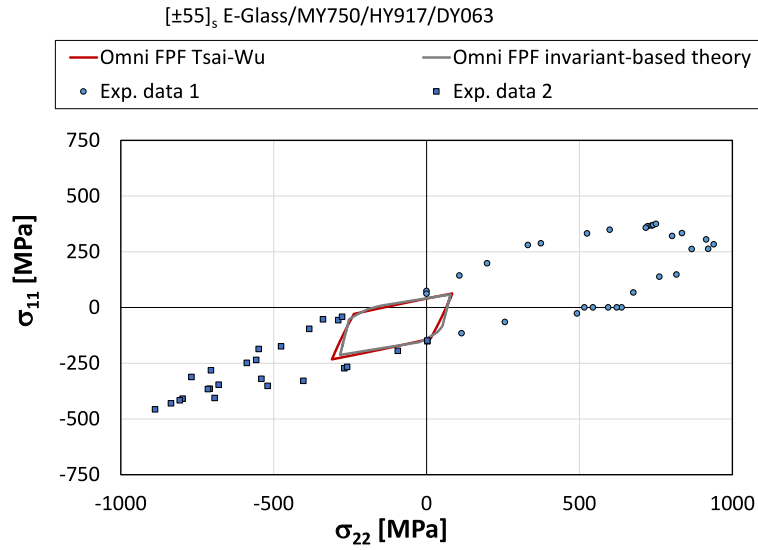
These three test cases provide clear indications on the huge benefits in using a LPF approach instead of FPF predictions. The larger domain when using LPF predictions allows to reduce conservatism in a remarkable way, without incurring additional computational time. These benefits can be exploited immediately from the conceptual design stage of composite

aerostructures, since the presented tool is invariant with respect to the laminate layup. The beneficial impact of this approach on the composites industry, where the consolidated practice in early design stage is to use FPF theories, such as maximum strain or Tsai-Wu, can be significant.

A triaxial test case for laminate failure is also available from the WWFE-II. Due to the complexity of imposing a triaxial stress state with suitable load introduction systems, there is lack of reliable experimental results involving triaxiality. For UD laminates, such validation studies have been supported in the past by computational micromechanics; however, for multidirectional laminates, computational micromechanics is still not suitable for similar studies due to the computational cost of running a micro-scale representative volume element of a multidirectional laminate. Herein, the aim is to assess the reliability of the proposed quick analytical approach to composite laminate failure, accepting a certain degree of conservatism. In the selected test case, glass/epoxy tubes were first subjected to an equal internal and external pressure, and then an axial compression load was incrementally applied up to failure, while keeping the pressure constant during the test<sup>(16)</sup>. The mechanical properties of E-glass/MY750/HY750/DY063 in the out-of-plane direction can be found in Ref.<sup>(16)</sup>.

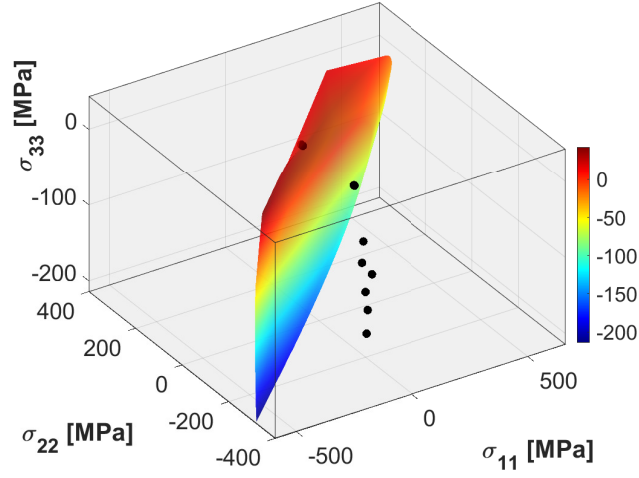
In order to predict the variation of the compressive strength  $\sigma_{22}$  with through-thickness stress  $\sigma_{33}$  (where  $\sigma_{11}=\sigma_{33}$ ), a fully 3D omni LPF surface in stress space was generated for this material, to extract the envelope in the relevant section. To assess the conservatism of the proposed 3D omni LPF surface, a laminate failure envelope obtained superposing only ply failure envelopes of the relevant orientations ( $\pm 35^\circ$ ) and the same failure model, was included in this study. Both surfaces and experimental data are shown in Fig. 19.

The predictions using the full 3D omni surface and the  $[\pm 35^\circ]_s$  LPF surface in the relevant section are illustrated with a blue solid line and a dotted red line, respectively, and compared with experimental results in Fig. 20. This comparison shows that the laminate LPF envelope allows to reduce the conservatism of 3D omni LPF surfaces, with more accurate predictions, in the case of angle-ply laminates. However, in spite of providing conservative predictions, the omni LPF envelopes define, in a physically-based setting, a safe approach for laminate failure prediction that is independent of the particular lay-up sequence, thus making its application straightforward for any laminate of a given material system. It can, therefore, be used for preliminary design, analysis and optimisation of composite structures without the need for recalculating the failure envelope. This allows simple generalization of ply-based criteria to laminate-based criteria, which is expected to contribute to significant time savings during the massive operations taking place, for instance, in multidisciplinary design optimisation of composite aerostructures.

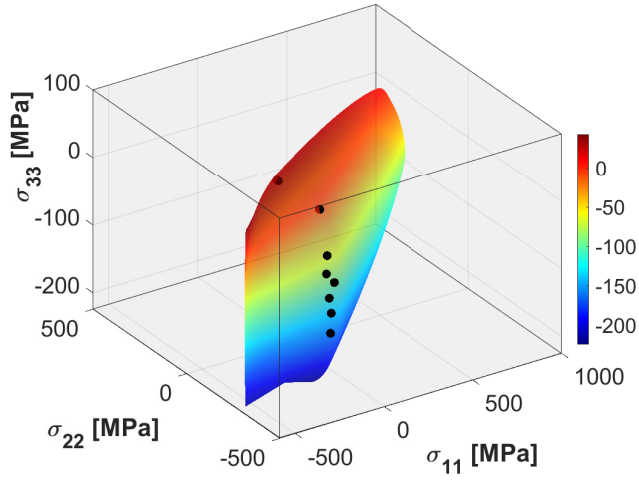


**Fig. 18.**  $\sigma_{22}$  -  $\sigma_{11}$  failure envelopes versus experimental results from the WWFE-I for a [ $\pm 55$ ]<sub>s</sub> E-glass/MY750/HY750/DY063 laminate.

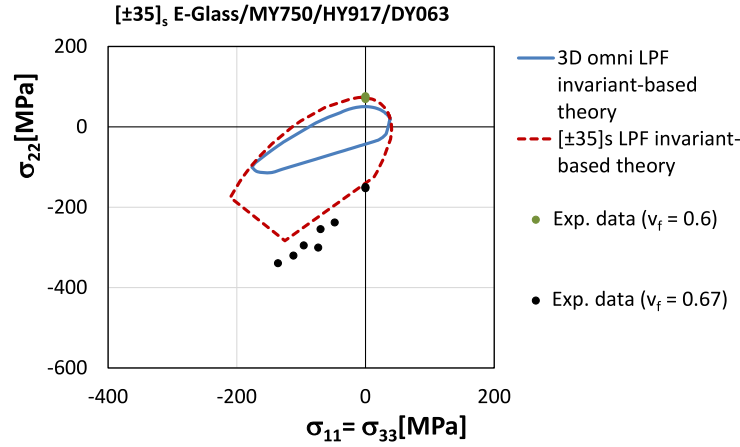




(a) 3D omni LPF surface.

(b) 3D  $[\pm 35]_s$  LPF surface.

**Fig. 19.** 3D omni LPF (a) and  $[\pm 35]_s$  laminate LPF (b) surfaces versus experimental results from WWFE-II for a  $[\pm 35]_s$  E-glass/MY750 epoxy laminate.

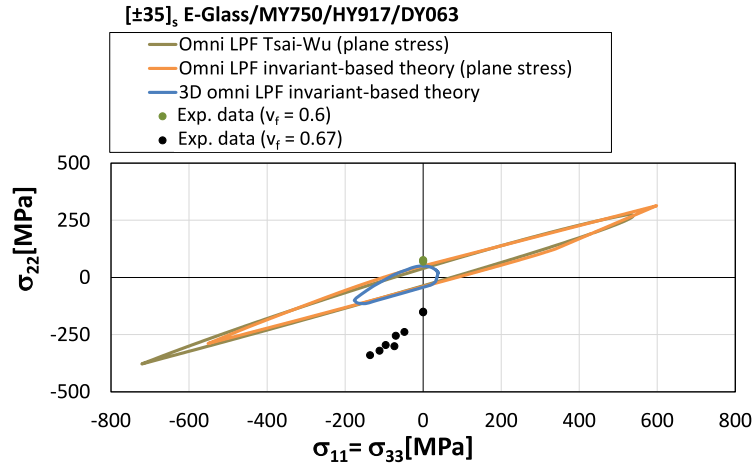


**Fig. 20.**  $\sigma_{22} - \sigma_{11} = \sigma_{33}$  failure envelopes versus experimental results from WWFE-II for a  $[\pm 35]_s$  E-glass/MY750 epoxy laminate. The experimental data is available for two different fibre volume fractions ( $v_f$ ).

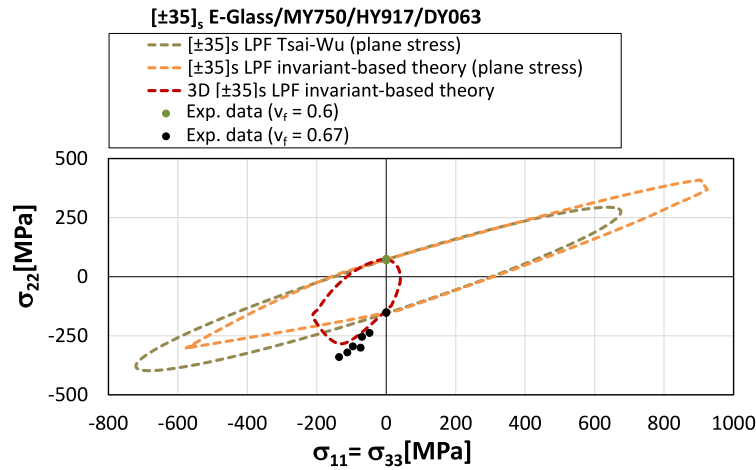
Finally, in order to assess the importance of including the out-of-plane stresses in the failure analysis, the triaxial test case for a  $[\pm 35]_s$  E-glass/MY750 epoxy laminate from WWFE-II is again considered. In this case, the omni LPF envelopes assuming plane stress conditions are compared with the ones under general 3D stress states as shown in Fig. 21a. In Fig. 21b, laminate LPF envelopes under 2D and 3D stress states are shown.

The omni LPF envelopes under plane stress provide similar predictions for both theories, as observed for the biaxial test cases from WWFE-I. When comparing these envelopes with the omni LPF surface, generated with the invariant-based theory, the design space is considerably reduced in the first and third quadrant, as a result of the effect of the out-of-plane stress. Although the available test data do not allow to assess with rigour how overestimated are the predictions of the plane stress models, the remarkable difference suggests that accounting for the effect of hydrostatic pressure can be very important in obtaining safe failure predictions for general laminates.

This becomes clear by analysing the laminate LPF envelopes shown in Fig. 21b. The failure envelope from the  $[\pm 35]_s$  LPF surface confirms that the influence of the out-of-plane stress allows to capture the increase of strength under hydrostatic pressure and the reduced strength under triaxial tension/compression, and therefore, to achieve an improved correlation with the experimental data.



(a) Omni LPF predictions.

(b) [ $\pm 35$ ]<sub>s</sub> LPF predictions.

**Fig. 21.** 3D omni LPF (a) and [ $\pm 35$ ]<sub>s</sub> laminate LPF (b) predictions under general stress states and plane stress conditions versus experimental results from WWFE-II for a [ $\pm 35$ ]<sub>s</sub> E-glass/MY750 epoxy laminate.

## 5.0 STRENGTHS AND LIMITATIONS OF THE EXTENDED OMNI FAILURE CONCEPT

Omni strain failure envelopes, extended with recourse to fully 3D phenomenological failure criteria, represent the original contribution introduced with this research work. The goal of this implementation was to address two unanswered questions arising from the omni strain failure envelope concept, firstly proposed by Tsai and Melo<sup>(12,13)</sup>: (i) whether this concept could be used to predict FPF and LPF of general laminates in the presence of high levels of triaxiality, and (ii) whether this concept could account for the influence of layup on the FPF and LPF modes, and predict them.

The extension of omni strain failure envelopes was realized by implementing the 3D invariant-based failure theory. The failure predictions from the first and second WWFE were selected as a suitable benchmark to validate this method, since they describe challenging biaxial and triaxial experimental data and the performance of competing failure theories involved in those WWFEs was available for comparison. In this way, the application of this concept to scenarios of general 3D stress states was analysed, while predicting the FPF and LPF modes, and assessing its conservatism compared with other approaches.

The validation study has shown that the extended omni LPF concept is a reliable method to address laminate failure under biaxial and triaxial stress states. The benefits of this implementation can be appreciated especially when predicting laminate failure under high values of hydrostatic pressure, where omni LPF envelopes based on non-phenomenological failure criteria give overconservative predictions, leading to reduced design spaces. Then, extended omni LPF envelopes under membrane loading generated similar predictions compared to the ones obtained with Tsai-Wu, but the extended approach provided also the critical failure modes.

An additional observation was made after assessing the effect of the stacking sequence on the prediction of omni LPF envelopes. This study has shown that LPF envelopes obtained superposing only ply failure envelopes contained in the considered laminate allow to reduce the conservatism of omni LPF envelopes, with more accurate predictions, in the angle-ply laminates. However, typical structural laminates are often characterized by the presence of [0] and [90] plies, and, consequently, omni LPF envelopes will typically provide satisfactory predictions in those cases.

Thus, the extended omni failure approach resulted to be a simple and generic framework that leads to safe prediction of the strength of general multidirectional laminates, independently of the stress state and level of triaxiality imposed on the composite. And despite its simplicity, the strength predictions are either equally good or better than other methods, for instance, considered in the first and second WWFE. Hence, the proposed extended method provides reliable and fast failure indications of composite laminates that can be particularly useful as a design tool for conceptual and preliminary design of composite structures.

## 6.0 CONCLUSIONS

In the present work, an extension of a recently introduced concept, called omni strain failure envelope, is proposed by implementing a set of 3D phenomenological failure theories, known as the 3D invariant-based failure criteria, in order to address laminate failure under general 3D stress states and to identify critical failure modes. This concept allows simple generalization of ply-based criteria to laminate-based criteria, overcoming two important constraints in multidisciplinary design optimisation of composite aerostructures: (i) computing time, by establishing laminate failure criteria independently of ply discretization, and (ii) excessive conservatism, by incorporating safe LPF criteria instead of overconservative FPF criteria.

A validation study of the predicting capability of omni LPF envelopes was performed using experimental results from the first and second WWFE. A good agreement was observed for the 2D cases, where the added value brought by the proposed envelopes was highlighted when analysing glass-fibre composites, whose LPF is governed by different failure modes; LPF of CFRP laminates, on the other hand, is always governed by fibre failure.

Finally, a triaxial test case was studied. The proposed 3D omni LPF surface presented some degree of conservatism with respect to experimental data on an angle-ply laminate, providing, nevertheless, a safe design space with minimal analysis cost due to its laminate invariant character. However, this conservatism will not be observed in industry-standard CFRP quad laminates, containing  $[0]$ ,  $[\pm 45]$  and  $[90]$  plies. Moreover, it is shown that the influence of the out-of-plane stress cannot be neglected to properly capture laminate failure under hydrostatic pressure and to obtain safe LPF predictions under general 3D stress states.

So, the described method proves to be a very robust tool, giving reliable and fast failure indications in either 2D or complex 3D stress states. Being a quick tool, it can be used for preliminary design and integrated in multidisciplinary design optimisation platforms, meeting the industrial need for computational efficiency, while simplifying the failure analysis and sizing of composite laminates.

## ACKNOWLEDGEMENTS

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