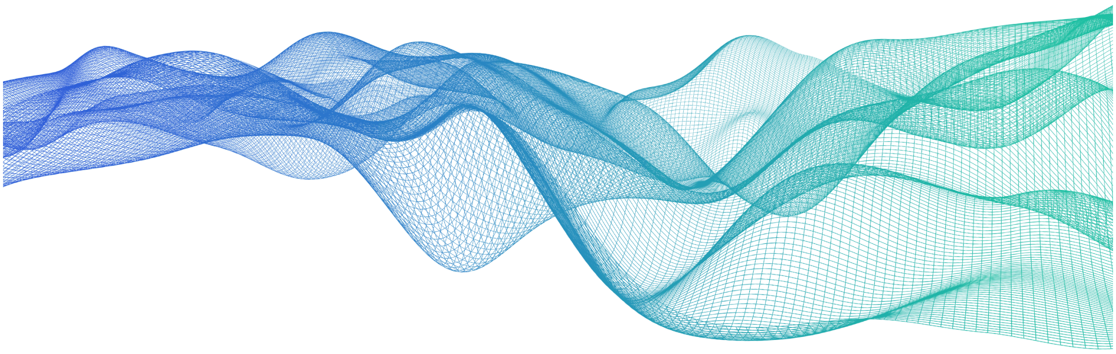
Book of Abstracts

FGP'22 - French German Portuguese

Conference on Optimization 2022

May 3-6, 2022 Porto, Portugal



Venue:

Faculdade de Economia da Universidade do Porto

Rua Dr. Roberto Frias

4200-464 Porto

Portugal

Editors/Editores: Maria do Rosário de Pinho, Fernando A.C.C. Fontes, Dalila B.M.M. Fontes

Contents

Preface

The French German Portuguese Conference on Optimization 2022 - FGP'22 – is the 20th edition of the series of French-German conferences which initiated in Oberwolfach, Germany, in 1980. Since 1998, the conference has been organized under the participation of a third invited European country. In this edition, it is jointly organized with Portugal and will take place at the University of Porto. Initially planned to 2021, the French German Portuguese conference in optimization was postponed to 2022.

Since the very beginning, the purpose of this conference series has been to bring together researchers working in all different areas of Optimization and particularly to encourage young researchers to present their work. Theoretical aspects of Optimization, in addition to applications and algorithms, are covered. The conference is held, typically, every two years and usually brings together up to 150 mathematicians. Communications are in the form of invited plenary talks, minisymposia, and contributed talks.

The conference joined 168 participants from 12 countries. This Book of Abstracts collects the summary of 141 talks presented at FGP'22, including 11 plenary talks by renowned keynote speakers, 15 minisymposia gathering a group of talks in a specialized area, and 23 contributed talks.

We thank all authors that contributed to this conference.

The Editors

Maria do Rosário de Pinho
Fernando A.C.C. Fontes
Dalila B.M.M. Fontes

Scientific Committee

The scientific committee is composed of optimization and optimal control eminent scholars from France, Germany, and Portugal

Samir Adly, Univ. de Limoges
Frédéric Bonnans, INRIA/Polytechnique
Claudia D'Ambrosio, CNRS/Polytechnique
Mounir Haddou, INSA Rennes
Vianney Perchet, ENSAE & Criteo

Helmut Maurer, Universität Münster
Alexander Martin, FAU Univ.
Sabine Pickenhain, Brandenburg Univ. of Technology
Michael Ulbrich, Technical Univ. of Munich
Andrea Walther, HU Berlin

Maria do Rosário de Pinho, Univ. Porto (Chair)
Agostinho Agra, Univ. Aveiro
Marcos Raydan, Univ. Nova Lisboa
Gueorgui Smirnov, Univ. Minho
João Xavier, IST-Univ. Lisboa

Organizing Committee

Maria do Rosário de Pinho, Univ. Porto (Chair)
Ana Luisa Custódio, Univ. Nova Lisboa
Dalila Martins Fontes, Inesc-Tec, Univ. Porto
Fernando ACC Fontes, Univ. Porto
João Gouveia, Univ. Coimbra
Ana Maria AC Rocha, Algoritmi, Univ. Minho
Cláudia Soares, Univ. Nova Lisboa

Local Team
Amélia Caldeira, Instituto Politécnico do Porto
Maria Margarida Ferreira, Univ. Porto
Sofia O. Lopes, Univ. Minho
Luís Tiago Paiva, Univ. Porto
Cristiana Silva, Univ. Aveiro

Logistic & Technical Support
Gustavo Andrade, Univ. Porto
Paulo Lopes, Univ. Porto
Sara Costa, Univ. Porto

Keynote Speakers

Fernando Lobo Pereira is a Full Professor at FEUP, Director of ISR-Porto, head of SYSTEC Research Unit and ARISE Associated Lab and coordinator of the Systems and Control area of DEEC. PhD in Control Theory from Imperial College of Science and Technology, Univ. of London, 1986. Research interests: control, dynamic optimization, impulsive systems, coordination and control of multiple dynamic systems, autonomous systems as well as control applications in mobile robotics, power, and manufacturing systems.

Participation in 50 R&D projects funded by FCT and other PT funding agencies, EU, NATO, and other agencies. These projects address a wide range of issues in decision and control, from theoretical – optimal and control theories, hybrid systems, and coordinated control – to applied – design, development, of advanced autonomous, and coordinated control systems for unmanned robotic vehicles, manufacturing systems-levels. Author of a book and of more than 400 publications in highly reputed peer-reviewed int. journals, conferences, and book chapters.

Francisco Saldanha da Gama is professor of Operations Research in the Department of Statistics and Operations Research at the Faculty of Science, University of Lisbon, where he received his PhD in 2002. Has taught undergraduate and post-graduate courses focusing on Operations Research, Mathematical Programming, Discrete Optimization, Stochastic Optimization, and Logistics. Has extensively published papers in scientific international journals mostly in the areas of location analysis, supply chain management, logistics and combinatorial optimization. Recently, together with Gilbert Laporte and Stefan Nickel co- edited the second edition of the volume "Location Science" published by Springer International Publishing (26 chapters, 767 pages). Has presented more than 100 contributed talks in scientific events being invited to innumerable scientific events as a plenary/semi-plenary/keynote speaker. Has been awarded several prizes and honors. Among those, the EURO prize for the best EJOR review paper (2012) and the Elsevier prize for the EJOR top cited article 2007–2011 (2012), both with the paper entitled "Facility location and supply chain management—A review" written together with Stefan Nickel and Teresa Melo. Has been member of innumerable scientific committees of international conferences and other scientific events. Is member of various international scientific organizations such as INFORMS—Institute for the Operations Research and Management Science, USA, CMAFcIO—Centro de Matemática Aplicações Fundamentais e Investigação Operacional da Fundação da Faculdade de Ciências, University of Lisbon, ECCO—European Chapter on Combinatorial Optimization, EWGSO—Working Group on Stochastic Optimization, SOLA—INFORMS Section on Location Analysis, and EWGLA—EURO Working Group on Locational Analysis, of which is one of the past coordinators. Currently is Editor-in-Chief of Computers & Operations Research as well as member of the Editorial Advisory Board of the Journal of the Operational Research Society (UK), Operations Research Perspectives, and Journal of Algorithms. The research interests include stochastic mixed-integer optimization, location theory and project scheduling.

Gerd Wachsmuth received his PhD from the TU Chemnitz in 2011. He was appointed as professor at the BTU Cottbus-Senftenberg in 2018. His research interest belong to the area of infinite-dimensional optimization. In articular, he is interested in optimal control problems with PDEs and non-smooth structures.

Helena Ramalhinho Lourenço is a Full Professor at the Economics and Business Department at the University Pompeu Fabra, Barcelona, Spain. She has a B.A. and Master degree in Statistics and Operations Research from the University of Lisbon, Portugal, and a Ph.D. in Operations Research from Cornell University, New York, USA. She has been involved in different research projects and consulting for business firms in the area of Operations Research and Logistics. Helena has published many articles in prestigious international scientific journals and has presented her work at international congresses and conferences. Helena teaches at various undergraduate, master's and PhD's programs at UPF and other universities. She is currently the director of the Business Analytics Research Group. Her research interests include Operations Research, Scheduling, Combinatorial Optimization, Metaheuristics, Iterated Local Search, Heuristic Search Optimization, Vehicle Routing, and applications on Supply Chain Management, Logistics, Operations Management and Health Care.

Jérôme Bolte is an applied mathematician at the University of Toulouse Capitole where he works on large scale optimization, gradient dynamical systems and on the geometrical aspects of optimization.

Luce Brotcorne is Senior researcher (Directeur de Recherche) at INRIA and currently leads the INOCS (Integrated Optimization with Complex Structure) project team of INRIA Lille-Nord Europe. The team includes about 15 French and foreign researchers. She holds a master degree in mathematics from the University of Namur (1992) and a Ph.D. in Operations Research, obtained at the ULB (Université Libre de Bruxelles) in 1998. She then worked as postdoctoral researcher at Cirrelt Montréal. Her research interests include bi-level pricing problems, equilibrium problems and stochastic optimization. She has been working on the optimization of transportation systems for more than 25 years and on demand response in the energy sector for more than 10 years. She is the coordinator of the Euro Working Group on Pricing and Revenue Management.

Mário Figueiredo received his PhD (1994) in Electrical and Computer Engineering from Instituto Superior Técnico, University of Lisbon, where he is an IST Distinguished Professor. He is a senior researcher and group leader at Instituto de Telecomunicações. His research areas include machine learning, signal processing, and optimization. He received several honors and awards, namely: Fellow of the Institute of Electrical and Electronics Engineers (IEEE), Fellow of the International Association for Pattern Recognition (IAPR), Fellow of the European Association for Signal Processing (EURASIP), W. R. G. Baker Award (IEEE), member of the Portuguese Academy of Engineering, member of the Lisbon Academy of Science. From 2014 to 2018 he was included in the annual list "Highly Cited Researchers".

Matthias Gerdts studied Mathematics with minor Computer Science at the University of Technology Clausthal, Germany, and graduated in 1997. He received his doctoral degree in 2001 and his Habilitation in 2006 from the University of Bayreuth, Germany. In 2003 he was a visiting professor atthe University of California, San Diego. From 2004 to 2007 he held a junior professorship for numerical optimal controlat the Department of Mathematics of the University ofHamburg, Germany, and moved to a lecturer position for mathematical optimization at the University of Birmingham, U.K., from 2007 to 2009. From 2009 to 2010 he was an associate professor for optimal control at the University of Würzburg, Germany. Since 2010 he is a full professor for engineering mathematics at the Department of Aerospace Engineering of the Bundeswehr University Munich, Germany. His primary research interests are optimal control, optimization techniques, model-predictive control, differential-algebraic equations, and sensitivity analysis with applications in automotive systems, robotics, and aerospace engineering.

Nelly Pustelnik (Member, IEEE) received the Ph.D. degree in signal and image processing from the Université ParisEst, Marne-la-Vallée, Paris, France, in 2010. From 2010 to 2011, she was a Postdoctoral Research Associate with Laboratoire IMS, Université de Bordeaux, Bordeaux, France, working on the topic of tomographic reconstruction from a limited number of projections. Since 2012, she has been a CNRS Researcher with the Signal Processing Team, Laboratoire de Physique de l'ENS de Lyon. Her research interests include optimization, wavelets, inverse problems, interface detection, and texture analysis. Between 2016 and 2020, she was an Associate Editor for the IEEE Signal Processing Letters (SPL), and since 2021, she has been a Senior Associate Editor for the IEEE SPL. Since 2016, she has been working with the IEEE MLSP Technical Committee.

Radu Ioan Bot is Professor for Applied Mathematics with Emphasis on Optimization at the Faculty of Mathematics of the University of Vienna and Member of the Research Platform \Data Science@Uni Vienna". Currently, he is the Dean of the Faculty of Mathematics and the Speaker of the Vienna Graduate Doctoral School on Computational Optimization. He received a Diploma and a M.Sc. degree in Mathematics from the Babe s-Bolyai University Cluj-Napoca, and a Ph.D. degree and the Habilitation in Mathematics from Chemnitz University of Technology. Radu Ioan Bot 's research interests include nonsmooth analysis, numerical algorithms for convex and nonconvex optimization and minimax problems, monotone operator theory, dynamical systems, and optimization methods for data science. His research is funded by the Austrian Science Fund, the German Research Foundation, the Romanian National Research Council, the Australian Research Council, and by industrial partners. He is (co-) author of the books Duality in Vector Optimization and Conjugate Duality in Convex Optimization published by Springer. Radu Ioan Bot is member of the Editorial Board of the journals Applied Mathematics and Optimization, Fixed Point Theory and Algorithms for Sciences and Engineering, Numerical Functional Analysis and Optimization, Computational Optimization and Applications, Minimax Theory and its Applications, Optimization Methods and Software, Optimization Letters, Journal of Optimization Theory and Applications, and SIAM Journal on Optimization.

Yurii Nesterov was born in 1956, Moscow. Master degree 1977, Moscow State University. Doctor degree 1984. Professor at Center for Operations Research and Econometrics, UCLouvain, Belgium. Author of 5 monographs and more than 120 refereed papers in the leading optimization journals. International recognition: Dantzig Prize 2000, John von Neumann Theory Prize 2009, Charles Broyden prize 2010, Francqui Chair (Liege University 2011-2012), SIAM Outstanding paper award (2014), EURO Gold Medal 2016.
Main research direction is the development of efficient numerical methods for convex and nonconvex optimization problems supported by the global complexity analysis: general interior-point methods (theory of self-concordant functions), fast gradient methods (smoothing technique), global complexity analysis of second-order and tensor schemes (cubic regularization of the Newton's method), accelerated proximal-point methods.

Minisymposia

MS1 Optimal control of ODE's: Theory and applications
Organizers: Sabine Pickenhain and Valeriya Lykina

MS2 Optimization for gas networks
Organizers: Falk Hante and Andrea Walther

MS3 Optimization in Medicine: contributions for radiotherapy treatments
Organizers: Joana Dias and Humberto Rocha

MS4 Geometric control, optimal feedback stabilization and applications
Organizers: Jérémy Rouot and Cristiana J. Silva

MS5 Recent trends in analytical and computational aspects on non-smooth optimization
Organizers: Kathrin Welker and Olga Weiß

MS6 Recent developments and new challenges in Derivative-Free Optimization
Organizers: Ana Luísa Custódio and Evelin Krulikovski

MS7 Internet of Manufacturing Things
Organizers: Dalila B.M.M. Fontes and S. Mahdi Homayouni

MS8 Optimization in distributed settings
Organizers: João Manuel de Freitas Xavier

MS9 New trends in optimal control and sweeping process with lack of regularity
Organizers: Alain Rapaport and Térence Bayen

MS10 Optimization in forest management and wildfire
Organizers: Ana Maria Rocha, Senhorinha Teixeira, and Filipe Alvelos

MS11 Optimal control of fluid flows
Organizers: Ana Leonor Silvestre and Jorge Tiago

MS12 The interplay between geometry and optimization
Organizers: Margarida Camarinha and Leonardo Colombo

MS13 Convergence rates of convex optimization methods
Organizers: Geovani Nunes Grapiglia and Masoud Ahookhosh

MS14 Optimization and data analytics in sustainable manufacturing and beyond
Organizers: Dalila B.M.M. Fontes and Sergiy Butenko

MS15 Mind the gap: new results on the Lavrentiev phenomenon
Organizers: Pierre Bousquet, Carlo Mariconda and Giulia Treu

Abstracts

Keynote Talks

Nonsmooth differentiation in deep learning: a mathematical approach

Jérôme Bolte

Toulouse School of Economics, Toulouse, France

jbolte@ut-capitole.fr

Modern problems in AI rely crucially on nonsmooth algorithmic differentiation (e.g., backpropagation). We will recall the importance of this approach in the training of neural networks, and we will then show how nonsmooth backpropagation can be modeled mathematically. Our approach leads to introducing a new-old regularity property called path-differentiability and a new sort of "weak gradients" called conservative fields. Many Lipschitz continuous functions are path differentiable: convex, concave, or semi-algebraic functions. Path-differentiability and conservativity reveal surprising relationships between the famous backpropagation algorithm and Whitney stratification. Our results are applied to establish the convergence of the training phase of neural networks - *as implemented in practice* - by "stochastic gradient descent".

Fast Augmented Lagrangian Method in continuous and discrete time

Radu Ioan Boţ*, Ernö Robert Csetnek, and Dang-Khoa Nguyen

Faculty of Mathematics, University of Vienna

radu.bot@univie.ac.at (*corresponding author), robert.csetnek@univie.ac.at, dang-khoa.nguyen@univie.ac.at

In this talk, we address the minimization of a continuously differentiable convex function under linear equality constraints.

First, we consider a second-order dynamical system with asymptotically vanishing damping term formulated in terms of the Augmented Lagrangian associated with the minimization problem ([1]). We show fast convergence of the primal-dual gap, the feasibility measure, and the objective function value along the generated trajectories, and also asymptotic weak convergence of the primal-dual trajectory to a primal-dual optimal solution of the underlying minimization problem.

By appropriate time discretization of the dynamical system we derive an inertial algorithm ([2]) with a general rule for the inertial parameters that covers the three classical rules by Nesterov, Chambolle-Dossal and Attouch-Cabot used in the literature in the context of fast gradient methods. We show that the algorithm exhibits in the convex regime convergence rates of order $O(1/k^2)$ for the primal-dual gap, the feasibility measure, and the objective function value. In addition, for the Chambolle-Dossal and Attouch-Cabot rules, the generated sequence of primal-dual iterates converges weakly to a primal-dual optimal solution.

For the unconstrained minimization of a convex differentiable function, we rediscover all convergence statements obtained in the literature for Nesterov's accelerated gradient method.

References

[1] R.I. Boţ, D.-K. Nguyen. (2021). *Improved convergence rates and trajectory convergence for primal-dual dynamical systems with vanishing damping*, Journal of Differential Equations 303, 369–406.

[2] R.I. Boţ, E.R. Csetnek, D.-K. Nguyen. (2021). *Fast Augmented Lagrangian Method in the convex regime with convergence guarantees for the iterates*, arXiv:2111.09370.

How to integrate and influence customers' behavior within pricing?

Luce Brotcorne

INRIA, France

Increased competition in a globalized economy, real-time access to a wealth of transparent information, the rise of a more knowledgeable and pragmatic generation of consumers is currently changing the perception and nature of optimal pricing. Nowadays, modeling the customer reaction to price is a central issue for most companies to generate revenue or define incentives to influence customer behavior.

Bilevel models are well suited to capture such situations where two decision-makers act non cooperatively and in a sequential way. More precisely, bilevel programs allow modeling situations where a main agent, the leader, strives to optimize a given quantity but controls only a subset of the decision variables. The remaining variables fall under the control of a second agent, the follower, who solves its own problem by taking into account the decisions taken by the leader. In other words, bilevel programs can be considered as demand-offer equilibrium models where the demand is the result of another mathematical problem.

We first present a brief theoretical study of bilevel problems in this talk. We illustrate the relevance of bilevel models for pricing problems in the energy field. We sketch the main lines of the solution approach and discuss numerical results.

Finally, we introduce the concept of near-optimality robustness for bilevel optimization problems, protecting the upper-level solution feasibility from limited deviations at the lower level. General properties and necessary conditions for the existence of solutions are derived for near-optimal robust versions of general bilevel optimization problems.

Alternating Direction Method of Multipliers in Imaging Inverse Problems: Overview of a Line of Work

Mário A. T. Figueiredo[1]

[1]Instituto de Telecomunicaes and Instituto Superior Tcnico,

Universidade de Lisboa, Portugal

mario.figueiredo@tecnico.ulisboa.pt

In this talk, I will review a line of work on the use of the ADMM (*alternating direction method of multipliers*, a member of the augmented Lagrangian family of methods) to address a wide variety of imaging inverse problems. At the core of this line of work is a way of using ADMM to tackle optimization problems where the objective function is the sum of two or more convex functions, each of which having a proximity operator that can be efficiently computed, and each possibly composed with a linear operator. The approach is illustrated on a variety of well-known problems, namely: image restoration and reconstruction with linear observations (for example, compressive imaging, image deblurring, image inpainting), which may be contaminated with Gaussian or Poisson noise, using synthesis, analysis, or hybrid regularization, and unconstrained or constrained regularization/variational formulations.

In the second part of the presentation, I will address the so-called plug-and-play (PnP) approach, wherein a formal regularizer is replaced with a black-box denoiser, aiming at leveraging state-of-the-art denoisers in more general inverse problems. Since these denoisers usually lack an optimization formulation, classical results on the convergence of ADMM cannot be directly invoked. A class of denoisers that we have proposed, while achieving excellent performance, also allow guaranteeing convergence of the resulting PnP-ADMM algorithm. These denoisers are particularly well suited to certain data fusion problems in imaging, which we will describe and which will be used to illustrate the proposed approach.

Dedication: this talk will be dedicated to the memory of my late colleague and friend, José Bioucas-Dias (1960-2020), in collaboration with whom most of this work was carried out.

Coordination of interacting systems using optimal control techniques

Matthias Gerdts[1]*

[1]Universität der Bundeswehr München, Department of Aerospace Engineering

matthias.gerdts@unibw.de (*corresponding author)

Automation and autonomy becomes more and more important in many robotics applications, especially for mobile robots, which move automatically in a production site, for automated cars, which move in the traffic, or in space robotics. These mobile robots often do not just follow a precomputed reference path, but they need to be able to update their trajectories in order to react on changing environments. This typically requires a feedback control strategy, which takes into account the current state of the robot and the environment. To this end we employ a model-predictive control (MPC) strategy, which requires to solve (discretized) optimal control problems repeatedly.

A particular focus of the talk is on methods for the coordination of interacting systems, which are not necessarily cooperative. We investigate suitable solution concepts and embed them into the MPC framework. The first approach uses generalized Nash equilibrium problems, which allow to model the coordination of automated agents without using pre-defined priorities. The second approach couples scheduling tasks with optimal control and leads to a bi-level optimization formulation.

Numerical experiments and case studies will be presented to illustrate the methods.

Acknowledgments: This research is funded by dtec.bw – Digitalization and Technology Research Center of the Bundeswehr which we gratefully acknowledge [projects EMERGENCY-VRD,MORE,SeRANIS,MissionLab].

A Maximum Principle for a Time-Optimal Bilevel Sweeping Control Problem

Fernando Lobo Pereira, and Nathalie Khalil

Faculdade de Engenharia da Universidade do Porto, Portugal

flp@fe.up.pt (*corresponding author), nathalie@fe.up.pt

This talk concerns a time-optimal bilevel optimal control problem. This problem instance arises in optimizing the motion of a structured crowd in a confined space. The dynamic constraints of the problem consist of a lower-level featuring drift term and a sweeping control process involving a truncated normal cone, and of a upper level depicting the overall crowd behaviour subject to state constraints. We establish necessary conditions of optimality in the form of a Maximum Principle as provided by Gamkrelidze for control problems with state constraints. The approach relies on the smooth approximation of the lower level sweeping control system, thereby dealing with the resulting lack of Lipschitzianity with respect to the state variable inherent to the sweeping process, and on the flattening of the bilevel structure via an exact penalization technique. Necessary conditions of optimality in the Gamkrelidze's form are applied to the resulting standard approximating penalized state-constrained single-level problem, being the main result obtained by passing to the limit.

Primal subgradient methods with predefined stepsizes

Yu. Nesterov

CORE/INMA, UCLouvain, Belgium

yurii.nesterov@uclouvain.be

In this talk, we present a new framework for analyzing primal subgradient methods for nonsmooth convex optimization problems. We show that the classical step-size rules, based on normalization of subgradient, or on the knowledge of optimal value of the objective function, need corrections when they are applied to optimization problems with constraints. Their proper modifications allow a significant acceleration of these schemes when the objective function has favorable properties (smoothness, strong convexity). We show how the new methods can be used for solving optmization problems with functional constraints with possibility to approximate the optimal Lagrange multipliers.

On strong convexity to the understanding and design of unsupervised and supervised deep image analysis

Nelly Pustelnik

CNRS, ENS de Lyon, France

nelly.pustelnik@ens-lyon.fr

Being fast, being flexible, handling with large dimensionality, and relying on a simple architecture are key issues for algorithms to be largely used in applicative fields such as signal and image processing. Among the last twenty years, a huge number of proximal algorithms satisfying these constraints have been developed, but identifying the most appropriate for a specific problem stays a challenging task. One of the simplest tools to compare algorithmic schemes is the convergence rate but at the price of assumptions such as the strong convexity. In this talk, we provide two data processing problems that turn out to be strongly convex: signal/image denoising and texture segmentation. We then establish a regime diagram with respect to Lipschitz and strong convexity constants allowing tight theoretical comparisons between standard first-order proximal schemes (forward-backward, Douglas-Rachford, Peaceman-Rachford). Numerical experiments in the context of unsupervised signal and image processing illustrate the tightness of these bounds. In the second part, we will detail how to take benefit of the strong convexity assumption in the design of supervised deep architecture. We especially highlight how faster unfolded schemes impact positively network architecture.

References

[1] B. Pascal, N. Pustelnik, and P. Abry, *Strongly Convex Optimization for Joint Fractal Feature Estimation and Texture Segmentation*, Applied and Computational Harmonic Analysis, vol. 54, pp 303-322, 2021.

[2] L. M. Briceño-Arias and N. Pustelnik, *Proximal or gradient steps for cocoercive operators*, submitted, 2022.

[3] H. T.V. Le, N. Pustelni, M. Foare, *The faster proximal algorithm, the better unfolded deep learning architecture ? The study case of image denoising*, submitted, 2022.

Optimization for Societal Impact

Helena Ramalhinho[1,2]*
[1]Universitat Pompeu Fabra
[2]Barcelona School of Management
helena.ramalhinho@upf.edu (*corresponding author)

The planet and the population are constantly changing and these changes have been more pronounced during the last decades. High pollution, increasing aging and risk population, frequent health issues and pandemics, depopulation of rural areas and overpopulation in cities, food security, conflicts in many areas with need of assistance, etc. are some of the main actual issues that need an efficient response by the society. Around these ideas there are many decisions to be considered on a day-to-day and strategic basis, which can bring many benefits to the planet and the population if they are made properly with the help of advances in research. We can express this benefit as the "Societal Impact", which can be defined as "the effect of research in the real world – a change or benefit beyond academia to the economy, society, culture, public policy or services, health, and the environment or quality of life"[1]. Analytics and Optimization should be part of the general response of the governments and the society to these problems, helping making better decisions and leading to a high positive impact on the society.

Analytics focuses on transforming data into insights by applying advanced analytical methods, based on mathematics, statistics, operations research and artificial intelligent models and algorithms, with the objective to improve the performance of an organization. One of the main tools in Analytics is Optimization that has been applied to improve different processes and systems in several industries, as for example in Retailing, Transportation, Manufacturing, etc. Recently there is a growing interest in applying Analytics and Optimization to non-profit or non-governmental organizations related to healthcare, social care, education, sustainable transport and humanitarian organizations.

In this talk, we present the Optimization methodologies, models, algorithms and tools applied to Non-Profit Organizations (NPO) and Non-Governmental Organizations (NGO) that had led to a positive impact on the wellbeing of the People and Planet. We will describe applications of Mathematical Programming Models and Metaheuristics Algorithms to Social Care, Healthcare, Public Services, Humanitarian Logistics and Environmental organizations. Examples of applications of optimization in these organizations are: home health and social care logistics and personnel scheduling (SUARA); applications of circular economy of Assistive Technology (Band de Moviments); location of the primary health care centers or schools in high growing population areas; food security and distribution (Food Bank); planning the humanitarian and health aid distribution and assistance; planning a sustainable transportation, scheduling special transportation for persons with reduced mobility (Barcelona City Council), etc.

Following these applications, we will discuss the main aspects and characteristics of the models and algorithms proposed, and the main differences to others solutions in business applications. One of the main differences are the objectives, which is not profit making centered, but focused on quality of life, sustainability, and wellbeing, which in summary we call the social value. Nevertheless, costs are an important barrier,

[1]https://www.purdue.edu/engagement/research-impacts/

and this makes the problems of multi-objective type. Additionally, the scenarios of these types of problems are surrounded by unexpected situations, as changes in the demand, chances in the resources available, etc. that are due mainly to the human component involved. This means uncertainty should be considered in the solutions proposed. In this case, it is important to have robust solutions to these problems, capable of adapting to different situations in real-time and meeting not only the "normal" but also disrupted situations. Therefore, the proposed models and algorithms must have adaptability and resilience characteristics incorporated so they can be applied in case of unexpected disruptions. Finally, the tools develop must be simple enough to be able to be used by the organizations which frequently have a different type of employees and users, as for example a large number of volunteers.

In conclusion, it can be noted that the problems related with the wellbeing of the people and planet require a new set of models and algorithms to bring an important societal impact. We describe the main characteristics of these models, algorithms and tools and present some real applications with a positive societal impact in different areas and organizations.

Districting Problems: Dealing with Uncertainty

Francisco Saldanha-da-Gama[1,2]

[1]Departamento de Estatística e Investigação Operacional, Faculdade de Ciências, Universidade de Lisboa, Portugal

[2]Centro de Matemática, Aplicações Fundamentais e Investigação Operacional, Faculdade de Ciências, Universidade de Lisboa, Portugal

fsgama@ciencias.ulisboa.pt

In a districting problems (DP) one aims at partitioning a set of basic geographic areas, called territorial units (TUs), into a set of larger clusters, called districts. These are problems with many applications that include the design of commercial areas to assign sales forces, school systems, design of police districts, waste collection, design of commercial areas, distribution logistics, et cetera. Compactness, contiguity and balancing are three major features sought when designing the districts. One aspect of practical relevance in DPs is the need to cope with demand that cannot be forecasted accurately. In this presentation demand is assumed to be represented by a random vector with a given joint cumulative distribution function. The resulting problem can be modeled as a two-stage mixed-integer stochastic programming problem. In the first stage a decision about an initial territory design is made maximizing the compactness of the districts. In the second stage, i.e. after demand is revealed, balancing requirements are to be met. This is ensured by means of two recourse actions: outsourcing and re-districting. The objective function accounts for the total expected cost that includes the cost for the first-stage territory design plus the expected cost incurred by outsourcing and reassignment in the second stage. The modeling framework discussed was assessed computationally using real geographical data. The obtained results are presented and discussed.

The role of curvature in second-order conditions

Gerd Wachsmuth[1]*, Constantin Christof[2], and Daniel Wachsmuth[3]

[1]BTU Cottbus-Senftenberg, [2]TU Mnchen, [3]Universitt Wrzburg

gerd.wachsmuth@b-tu.de (*corresponding author), christof@ma.tum.de,
daniel.wachsmuth@mathematik.uni-wuerzburg.de

We consider optimization problems of the form

$$\text{minimize} \quad F(x) + G(x) \quad \text{w.r.t. } x \in X.$$

Here, X is a Banach space and F is assumed to be twice differentiable. We are interested in (necessary and sufficient) second-order conditions for general functionals G. In particular, we are interested in the cases that G is the indicator function of a closed set C or that G is a convex but non-smooth functional.

It is clear that some kind of second-order derivative of G should appear in second-order conditions. In the case that G is the indicator function of C, these second-order derivatives can be interpreted as curvature of the boundary of C. In the finite-dimensional case, one can utilize the "second subderivative" (Rockafellar) and this concept has been generalized to reflexive spaces in the work of Do ("second-order epiderivative").

We transfer these ideas also to the non-reflexive situation. By posing a certain regularity condition, we are able to provide no-gap optimality conditions of second order. The theory is applicable to a broad spectrum of possible applications.

As an example, we address optimal control problems of the form

$$\begin{aligned} &\text{minimize} && \int_\Omega L(\cdot, y)\,\mathrm{d}x + \int_\Omega g(u)\,\mathrm{d}x \\ &\text{subject to} && -\Delta y + b(\cdot, y) = u \quad \text{in } \Omega, \qquad y = 0 \quad \text{on } \partial\Omega. \end{aligned}$$

Here, $g\colon \mathbb{R} \to \mathbb{R} \cup \{\infty\}$ is assumed to be convex. We are interested in the situation that g is not strongly convex, e.g.,

- $g = \delta_{[u_a,u_b]}$ is the indicator function of a closed interval; the control is typically bang-bang
- $g = |\cdot| + \delta_{[u_a,u_b]}$; which leads to sparse controls.

Although these problems are posed in Lebesgue spaces (w.r.t. the control u), we will see that we have to discuss these problems in the space of measures $X = \mathcal{M}(\Omega)$ to derive second-order conditions. Under a mild assumption on the adjoint state, we prove no-gap second-order conditions involving a generalized second-order derivative of the functional $u \mapsto \int_\Omega g(u)\,\mathrm{d}x$.

Acknowledgments: This work has been supported by the German research foundation (DFG) under grant numbers ME 3281/7-1, WA 3626/3-2, WA 3636/4-1, WA 3636/4-2.

Minisymposia

Minisymposium MS01

MS01 - Optimal control of ODE's: Theory and applications

Nonlinear metric subregularity in optimal control

Alberto Domínguez Corella[1]

[1] Vienna University of Technology

alberto.corella@tuwien.ac.at

Regularity is one of the fundamental concepts in modern analysis. The particular concept of nonlinear metric regularity, introduced in [1], aims at estimating (in a nonlinear way) the distance between preimages of set-valued mappings in terms of the residuals.

We will consider optimal control problems constrained by evolution equations. The talk will begin with a preliminary discussion on how the concept of nonlinear metric regularity applied to the optimality mapping associated with an optimal control problem relates with the local stability of the solutions of the problem. Later on, we will discuss some results concerning sufficient conditions for the optimality mapping of a problem to have the nonlinear regularity property. Finally, we will discuss some consequences of the results in optimal control as well as applications.

Acknowledgments: This research is supported by the Austrian Science Foundation (FWF) under grant No I4571.

References

[1] Ioffe, Alexander D. (2013) *Nonlinear regularity models*, Math. Program., 139 **1-2, Ser. B**, 223–242.

Different Relaxations in Infinite Horizon Optimal Control - Comparison and Existence

I. Dikariev[1]*, V. Lykina[1]**, S. Pickenhain[1]***,

[1] Brandenburg University of Technology at Cottbus-Senftenberg, Cottbus, Germany

dikarill@b-tu.de(*), valeriya.lykina@b-tu.de (**), sabine.pickenhain@b-tu.de (***)

In this paper we consider a Lagrange problem of optimal control on an unbounded time interval, in which convexity hypotheses are not assumed. Well known relaxation methods, introduced for the finite horizon case, like Γ-regularization, as in [3], convex combinations of Dirac-measures, as in [2] and Young-measures, also known as Gamkrelidzes relaxed controls, [4], are extended and compared in the infinite horizon case.

We succeed in proving equivalence results for these relaxations. Under mild assumptions existence results follow for the relaxed problems, in which Weighted Sobolev-spaces are used as state spaces.

References

[1] Lykina, V.; Pickenhain, S. (2017). Weighted functional spaces approach to infinite horizon optimal control problems: a systematic analysis of hidden opportunities and advantages. *Journal of Mathematical Analysis and Applications* **454**, Issue 1, pp. 195 – 218.

[2] Carlson, D.A. Nonconvex and relaxed infinite – horizon optimal control problems. *Journal of optimization theory and applications*, 78(3):465–491, 1993.

[3] Ekeland, I.; Temam, R. *Convex Analysis and Variational Problems*. SIAM, 1999.

[4] Gamkrelidze, R.V. *Principles of Optimal Control Theory*. Plenum Press, New York and London, 1978.

OPTIMAL INVESTMENT WITH VINTAGE CAPITAL: EQUILIBRIUM DISTRIBUTIONS

SILVIA FAGGIAN [1], FAUSTO GOZZI [2], AND PETER M. KORT [3]

ABSTRACT. The paper concerns the study of equilibrium points, or steady states, of economic systems arising in modeling optimal investment with *vintage capital*, namely, systems where all key variables (capitals, investments, prices) are indexed not only by time τ but also by age s. Capital accumulation is hence described as a partial differential equation (briefly, PDE), and equilibrium points are in fact equilibrium distributions in the variable s of ages. Investments in frontier as well as non-frontier vintages are possible. Firstly a general method is developed to compute and study equilibrium points of a wide range of infinite dimensional, infinite horizon boundary control problems for linear PDEs with convex criterion, possibly applying to a wide variety of economic problems. Sufficient and necessary conditions for existence of equilibrium points are derived in this general context. In particular, for optimal investment with vintage capital, existence and uniqueness of a long run equilibrium distribution is proved for general concave revenues and convex investment costs, and analytic formulas are obtained for optimal controls and trajectories in the long run, definitely showing how effective the theoretical machinery of optimal control in infinite dimension is in computing explicitly equilibrium distributions, and suggesting that the same method can be applied in examples yielding the same abstract structure. To this extent, the results of this work constitutes a first crucial step towards a thorough understanding of the behavior of optimal controls and trajectories in the long run.

Key words: Equilibrium Points; Equilibrium Distributions; Vintage Capital Stock; Age-structured systems; Maximum Principle in Hilbert spaces; Boundary control; Optimal Investment.

Journal of Economic Literature: C61, C62, E22

[1]S. FAGGIAN, DEPARTMENT OF ECONOMICS, UNIVERSITÁ "*Ca' Foscari*" VENEZIA, ITALY. faggian@unive.it.

[2] DIPARTIMENTO DI ECONOMIA E FINANZA, UNIVERSITÁ LUISS - "GUIDO CARLI, I-00162, ROMA, ITALY. fgozzi@luiss.it.

[3] CENTER, DEPARTMENT OF ECONOMETRICS & OPERATIONS RESEARCH, TILBURG UNIVERSITY, P.O. BOX 90153, 5000 LE TILBURG, THE NETHERLANDS; DEPARTMENT OF ECONOMICS, UNIVERSITY OF ANTWERP, PRINSSTRAAT 13, 2000 ANTWERP 1, BELGIUM, kort@uvt.nl.

Value Function in Optimal Control on Wasserstein Spaces

B. Bonnet[1,2] and H. Frankowska [1]*

[1] CNRS IMJ-PRG and Sorbonne Université, Paris, France

[2] CNRS, LAAS, Toulouse, France

Benoit.BONNET@imj-prg.fr , helene.frankowska@imj-prg.fr (*corresponding author)

Mean field control problems refer broadly to situations in which a centralised policy-maker emits a control signal at the macroscopic level, in order to stir a microscopic multi-agent system towards a desired goal. Such models have led to optimal control problems stated on the so-called Wasserstein spaces. Various results of the classical control theory of ODEs adapt well to the Wasserstein spaces framework because flows of solutions to controlled ODEs generate solutions to the controlled continuity equation

$$\partial_t \mu(t) + \mathrm{div}_x\big(v(t,\mu(t),u(t))\mu(t)\big) = 0, \quad \mu(0) = \mu^0, \tag{1}$$

where $\mu^0 \in \mathcal{P}_c(\mathbb{R}^d)$ and $\mathcal{P}_c(\mathbb{R}^d)$ denotes the set of Borel probability measures on $\mathbb{R}^d$ having a compact support. In particular, the depiction of optimality conditions in the form of variants of the Pontryagin Maximum Principle (PMP) has attracted a lot of attention.

In this article – which is a continuation of our previous works [1] – we discuss various properties and applications of the value function $\mathcal{V} : [0,T] \times \mathcal{P}_c(\mathbb{R}^d) \to \mathbb{R}$ associated to a Mayer mean-field optimal control problem: Let U be a given compact metric space and consider the set of admissible controls $\mathcal{U} := \{u : [0,T] \to U \text{ is Lebesgue-measurable}\}$. Let the time-evolution of the control system be generated by the *controlled non-local velocity field* $v : [0,T] \times \mathcal{P}_c(\mathbb{R}^d) \times U \times \mathbb{R}^d \to \mathbb{R}^d$ and $\varphi : \mathcal{P}_c(\mathbb{R}^d) \to \mathbb{R}$ be a final cost. The value function is defined by:

$$\mathcal{V}(\tau,\mu_\tau) := \inf_{u(\cdot)\in\mathcal{U}} \{\varphi(\mu(T)) \mid \mu \text{ solves the controlled continuity equation (1) with } \mu(\tau) = \mu_\tau\}$$

for any $(\tau,\mu_\tau) \in [0,T] \times \mathcal{P}_c(\mathbb{R}^d)$. In particular, we show the following sensitivity relations: for every local minimiser $(\mu^*(\cdot), u^*(\cdot))$ there exists a the state-costate curve ν^* satisfying (PMP) such that

$$\Big(H(t,\nu^*(t),u^*(t))\,,\, -\bar{\nu}^*(t)\Big) \in \partial^+\mathcal{V}(t,\mu^*(t)) \text{ a.e. in } [0,T], \quad -\bar{\nu}^*(t) \in \partial^+_\mu \mathcal{V}(t,\mu^*(t)) \text{ for all } t \in [0,T],$$

where $\bar{\nu}^*(t) \in L^\infty(\mathbb{R}^d,\mathbb{R}^d;\mu^*(t))$ denotes the barycentric projection of the state-costate curve $\nu^*(t)$ onto $\mu^*(t)$ and ∂^+ stands for the Dini superdifferential.

Acknowledgments: This work has been supported by the Air Force Office of Scientific Research under award number FA9550-18-1-0254.

References

[1] BONNET B. & FRANKOWSKA H. (2021) *Necessary optimality conditions for optimal control problems in Wasserstein spaces*, Applied Mathematics and Optimization, **84**, 1281-1330

Optimal control problem on Riemannian manifolds under probability knowledge of the initial condition

Frédéric Jean[1], Othmane Jerhaoui[1,2]*, and Hasnaa Zidani[2]

[1]UMA, ENSTA Paris, Institut Polytechnique de Paris, 91120 Palaiseau, France

[2]Normandie Univ, INSA Rouen Normandie, Laboratoire de Mathématiques Appliquées (LMI), 76000 Rouen, France

Frederic.Jean@ensta-paris.fr, Othmane.Jerhaoui@ensta-paris.fr (*corresponding author), Hasnaa.Zidani@insa-rouen.fr

We consider an optimal control problem on a compact Riemannian manifold M with imperfect information on the initial state of the system. The lack of information is modeled by a Borel probability measure along which the initial state is distributed. This problem is of fundamental importance both in terms of real world applications and mathematical theory.

Indeed, this layout appears in the modelling of many optimal control problems related to mechanics, robotics or quantum systems. The initial condition is not perfectly known either due to the lack of measurements, errors of measurements, or even due to the nature of the system itself, meaning that the uncertainties are inevitable. As for the theoretical interest, since the optimal control problem with partial information is defined in the 2-Wasserstein space over the Riemannian manifold $\mathcal{P}_2(M)$, we need to define proper tools in this space to describe the problem.

Similar to optimal control problems with perfect information, we introduce the value function of this problem and an associated Hamilton Jacobi Bellman (HJB) equation defined on $\mathcal{P}_2(M)$. We propose to extend the same techniques commonly used in viscosity theory to the space $\mathcal{P}_2(M)$ in order to prove that the value function is the unique viscosity solution to the HJB equation. In particular, we want to define a notion of "smooth" solutions to the HJB equation, define the set of test functions for viscosity super and subolutions, prove a local comparison principle that guarantees uniqueness of the viscosity solution and finally prove that the value function verifies a dynamic programming principle that will guarantee existence of the solution.

The main result is that the value function of the problem is the unique viscosity solution to an HJB equation. The notion of viscosity is defined by exploiting the Riemannian-like structure on $\mathcal{P}_2(M)$.

Strong metric subregularity of optimal control problems constrained by semilinear elliptic PDE's

Alberto D. Corella[1], Nicolai Jork[1]*, and Vladimir Veliov[1]

[1]TU Vienna

alberto.corella@tuwien.ac.at , nicolai.jork@tuwien.ac.at*, vladimir.veliov@tuwien.ac.at

We investigate Lipschitz stability of solutions in affine optimal control problems constrained by semilinear elliptic partial differential equations. This is done by studying the optimality mapping associated to the system of necessary conditions of the control problem. The property of stability of set-valued mappings that we study is known in the literature as strong subregularity. We establish some preliminary results concerning the differentiability of the functions involved, especially the so-called switching mapping. With those results at hand, we then prove stability properties of the optimality mapping. We illustrate the applicability of our results obtaining estimates for the Tikhonov regularization.

Acknowledgments: This research is supported by the Austrian Science Foundation (FWF) under grant No I4571.

FGP2022 on Optimization

Solving Infinite Horizon Optimal Control Problems in the Framework of Neural Network Approach

Valeriya Lykina[1]*, Sabine Pickenhain[1]

[1]Brandenburg University of Technology at Cottbus-Senftenberg, Germany

Valeriya.Lykina@b-tu.de (*corresponding author), Sabine.Pickenhain@b-tu.de

In this talk a class (P) of primal infinite horizon control-constrained optimal control problems in the weighted functional spaces setting is considered. The dynamics is supposed to be affine in control. The goal is to establish the connection between the duality concept of Klötzler [1] which is based on minimizing the defect of the HJ inequality and suitable for the solution of nonlinear and nonconvex optimal control problems and the neural network approach used in [2] to solve control-constrained infinite horizon optimal control problems via the corresponding HJB equation. For the dual problem (D), a nonlinear ansatz for the dual variable $S(t,\xi) = \sum_{k=0}^{N} w_k(t)\sigma_k(\xi)$ with linear independent functions $\sigma_k(\cdot)$ is used. The time dependent weights $w_k(\cdot)$ are chosen from the accordingly weighted Sobolev spaces. The solution algorithm is interpreted in terms of neural networks framework.

References

[1] Klötzler, R. (1979). On a general conception of duality in optimal control. In: *Equadiff IV. Proceedings of the Czechoslovak Conference on Differential Equations and their Applications held in Prague, August 22 – 26, 1977.* (Fábera, J. (Ed.)), 189 – 196, Springer, Berlin (Lecture Notes in Mathematics **703**).

[2] Abu-Khalaf, M.; Lewis, F.L. (2005) *Nearly optimal control laws for nonlinear systems with saturating actuators using a neural network HJB approach*, Automatica, **41**, 779–791.

Optimal Control of Cancer Chemotherapy with Delays and State Constraints

Poh Lin Tan [1], Helmut Maurer [2]*, Jeevan Kanesan [2] and Joon Huang Chuah [2]

[1]University of Malaya, Kuala Lumpur, Malaysia

[2]Westfl̈ische Wilhelms-Universität Münster, Münster, Germany

paulinetan0916@gmail.com;, maurer@math.uni-muenster.de (*), jievan@um.edu.my, jhchuah@um.edu.my

We consider an optimal control model for cancer chemotherapy with the objective of minimizing a weighted sum of tumor cells and drug dosage [3]. The control process is subject to three state constraints involving an upper bound on drug toxicity, a lower bound on the white blood cell (WBC) population and a constraint to prevent the WBC count from staying too long below a fixed upper level. The state constraints are imposed for safeguarding the health of patients during treatment. The dynamics of the WBC population involves delays due to the delay chain of granulocyte development. The control problem is based on a similar model presented in Iliadis and Barbolosi [2]. However, the authors do not treat necessary conditions, which only recently have been presented in the literature, and could not use appropriate numerical methods.

We discuss the necessary optimality conditions in Vinter [4] and succeed in finding analytic expressions of the multipliers associated with the state constraints. We use the discretization methods developed in Gl̈lmann and Maurer [1] to compute solutions of the time-delayed optimal control problems. In all cases we obtain *extremal solutions* satisfying precisely the necessary conditions. In some cases we are able to solve the Induced Optimization Problem of directly optimizing the switching times. This allows to verify second-order sufficient conditions. Since the structure of optimal controls is rather complex, we propose approximate controls with fewer control arcs which can be administered in practice more easily.

Acknowledgments: This work is supported by Research grant under Faculty of Engineering, University of Malaya, (Grant No: GPF055A-2020).

References

[1] L. Gllmann and H. Maurer (2014). *Theory and applications of optimal control problems with multiple time-delays*, Journal of Industrial and Management Optimization **10**, 413–441.

[2] A. Iliadis and D. Barbolosi (2000). *Optimizing drug regimens in cancer chemotherapy by an efficacytoxicity mathematical model*, Computers and Biomedical Research **33(3)**, 211–226.

[3] Poh Ling Tan, Helmut Maurer, Jeevan Kanesan and Joon Huang Chuah (2021). *Optimal control of cancer chemotherapy with delays and state constraints*, to appear

[4] R.B. Vinter (2018). *State constrained optimal control problems with time delays*, Journal of Mathematical Analysis and Applications, **457**, 1696–1712.

Optimal control model for COVID-19: Economic collapse vs health system overcrowding

E. Paparelli[1]*, R. Giambò[2,3], H. Maurer[4]

[1]Department of Mathematical Sciences "G. L. Lagrange", Dipartimento di Eccellenza 2018-2022, Politecnico di Torino, Italy

[2]Mathematics Division, School of Science and Technology, University of Camerino, Camerino, Italy

[3]INFN, Sezione di Perugia, Perugia, 06123, Italy

[4]Institute for Analysis and Numerics, University of Münster, Münster, Germany

elisa.paparelli@polito.it (*), roberto.giambo@unicam.it, helmut.maurer@uni-muenster.de

The Covid-19 outbreak leads governments to impose restrictions on people freedom for containing the infectivity and mortality. On the other hand, lockdown policies make the production and consumption to decrease, leading to an economic damage. In this frame, the main issue arises: which policy is the best to contain both fatalities and economic losses?

The answer is investigated though the study of an optimal control problem constrained to a SEAIRD epidemic model [1], with fixed initial condition and fixed time horizon. One challenge is to find a continuous control function that minimizes the total cost due to economic losses and human deaths. After showing the existence of that function, the necessary conditions of optimality provided by Pontryagin Maximum Principle are applied. Also the uniqueness of the control function is guaranteed for a sufficiently small time interval. The second challenge is to face the same problem with an additional state constraint on infected people. Its presence is justified by the need of maintaining the infectious level under the health-facilities capacity threshold in order to avoid health system overcrowding. Also in this case, we discuss the necessary optimality conditions given in Hartl et al. [2]. To solve the optimal control problems we use the forward-backward sweep method [3], discretization and nonlinear programming methods [4] and the DIDO solver [5].

References

[1] A. Aspri, E. Beretta, A. Gandolfi, E. Wasmer (2021) *Mortality containment vs. Economics Opening: Optimal policies in a SEIARD model*, Journal of Mathematical Economics, **93**, 102490.

[2] R.F. Hartl, S.P. Sethi and R.G. Vickson (1995) *A survey of the maximum principles for optimal control problems with state constraints*, SIAM Review, **37(2)**, 181–218.

[3] M. McAsey, L. Mou, W. Han (2012) *Convergence of the Forward-Backward Sweep Method in Optimal Control*, Comput Optim Appl, **53**, 207–226

[4] A. Wächter, L. T. Biegler (2006) *On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming*, Mathematical Programming **106(1)**, 25-57

[5] I. M. Ross (2015), *A Primer on Pontryagin's Principle in Optimal Control*, Second Edition, Collegiate Publishers, San Francisco

Lyapunov-exponential Asymptotic Stabilization of Control Constrained Dynamical Systems

K. Kolo[2]*, V. Lykina[1]**, S. Pickenhain[1]***,

[1] Brandenburg University of Technology at Cottbus-Senftenberg, Cottbus, Germany

[2] Titus Research GmbH, Wildau, Germany

katharina.kolo@titus-research.eu (*), valeriya.lykina@b-tu.de (**), sabine.pickenhain@b-tu.de (***)

The approach we follow here is based on optimal control techniques, see [1], [3], [4]. More precisely, we consider an infinite horizon optimal control problem, realizing the Lyapunov-exponential stability by the choice of corresponding state spaces as Weighted Sobolev spaces. We consider the following infinite horizon optimal control problem:

$$
\begin{aligned}
(P)_\infty : \quad J_\infty(x,u) &= \int_0^\infty \frac{1}{2}\left(x^T(t)Wx(t) + u^T(t)Ru(t)\right) e^{\beta t} dt \to \min ! \\
(x,u) &\in W_2^{1,n}(\mathbb{R}^+, e^{\beta t}) \times \mathbb{U}, \quad \beta > 0 \\
\dot{x}(t) &= f(t, x(t), u(t)) \text{ a. e. on } \mathbb{R}^+, \\
x(0) &= x_0, \\
u(t) &\in U \text{ a. e. on } \mathbb{R}^+
\end{aligned}
$$

The presented work focuses on a dual-based treatment of the problem $(P)_\infty$. This dual problem, obtained by a Lagrange-type duality, leads to an infinite horizon variational problem, where the dual variables belong again to a weighted Sobolev space $W_2^{1,n}(\mathbb{R}^+, e^{-\beta t})$ with a density function $e^{-\beta t}$. We succeeded to prove saddle point conditions as well as necessary optimality conditions for the dual problem. For the numerical treatment of the dual problem we present a direct pseudo-spectral method based on generalized Laguerre polynomials. The application of the proposed method is illustrated by a SEIR-model for the covid-19 viral disease, where the parameter of this model were adopted to the statistical data of the actual pandemic, [2].

References

[1] Kalman, R. E. (1960). Contribution to the Theory of Optimal Control. *Boletin Sociedad Matemática Mexicana* **5**, 102 – 119.

[2] Kolo, K. (2021). A dual-based approach to the treatment of optimal control problems with an a priori infinite horizon. *Dissertation thesis* (in German), BTU Cottbus-Senftenberg, Germany.

[3] Letov, A. M. (1955). *Stability of Nonlinear Control Systems* (in Russian), Moscow, Gostekhizdat.

[4] Lykina, V.; Pickenhain, S. (2017). Weighted functional spaces approach to infinite horizon optimal control problems: a systematic analysis of hidden opportunities and advantages. *Journal of Mathematical Analysis and Applications* **454**, Issue 1, pp. 195 – 218.

A Stochastic Gradient Method for Nonlinear PDEs-Constrained Optimal Control Problems under Uncertainty

T. Scarinci[1]*, C. Geiersbach[2]

[1]DISIM, University of l'Aquila (Italy)

[2]Weierstrass Institute, Berlin (Germany)

teresa.scarinci@univaq.it (*corresponding author), caroline.geiersbach@wias-berlin.de

In this talk, we will describe a class of optimal control problems for semilinear elliptic partial differential equations with random data. Moreover, we will discuss some convergence results of a stochastic optimization method utilized to solve these model problems and, moreover, that can be applied also to a more general class of non-convex stochastic optimization problems. Namely, one might consider

$$\min_{u \in H} \{ j(u) = \mathbb{E}[J(u, \xi)] \},$$

where H is a Hilbert space, ξ is defined on some sample space Ξ, $u \in H$ is a deterministic control, and $\mathbb{E}[\cdot]$ is the expectation. Furthermore, $J : H \times \Xi \to \mathbb{R}$ is a cost that is smooth but might be nonconvex. The application of this method to the aforementioned class of PDE-constrained optimization problems will be addressed in detail. Some numerical experiments will conclude the analysis. This talk is based on results from [1].

References

[1] C. Geiersbach, T. Scarinci (2021). A stochastic gradient method for a class of nonlinear PDE-constrained optimal control problems under uncertainty. Preprint, https://arxiv.org/abs/2108.11782.

On the accuracy of the model predictive control method for optimal control

Vladimir M. Veliov[1]

[1] Vienna University of Technology

vladimir.veliov@tuwien.at.ac

The talk will present new results about the accuracy of the Model Predictive Control (MPC) method for finding on-line approximate optimal feedback control for Lagrange type problems on a fixed finite horizon. The predictions for the dynamics, the state measurements, and the solution of the auxiliary open-loop control problems that appear at every step of the MPC method may be inaccurate. The main result provides an error estimate of the MPC-generated solution compared with the optimal open-loop solution of the "ideal" problem, where all predictions and measurements are exact. The technique of proving the estimate involves an extension of the notion of strong metric sub-regularity of set-valued maps and utilization of a specific new metric in the control space, which makes the proof non-standard. The result is specialized for two problem classes: coercive problems, and affine problems.

The talk is based on joint research with A. Domínguez Corella.

Acknowledgments: This work has been supported by the Austrian Science Foundation (FWF) under grant No I4571.

Sensitivity relations for minimax control problems

Cristopher Hermosilla* Hasnaa Zidani †

Abstract

We consider a class of optimal control problems governed by ordinary differential equations. We analyse the sensitivity relations satisfied by the co-state arc of the Pontryagin maximum principle and the value function that associates the optimal value of the control problem to the initial time and state. Such a relationship is very well-known in the case of control problems in Mayer or Bolza form. We first investigate the case of minimax problems and derive new sensitivity relations that hold in this context. Then we use this result to investigate some control problems with pointwise state constraints. Such problems have been studied only under some controllability assumptions to guarantee Lipschitz regularity property of the value function. Here, we consider the case without any controllability assumption on the system, and without Lipschitz regularity of the value function. Because of this lack of regularity, the sensitivity relations cannot be expressed with the subdifferentials of the value function. We will show that the constrained problem can be reformulated with an auxiliary value function which is more regular and suitable to express the sensitivity of the adjoint arc of the original state-constrained control problem along an optimal trajectory. Furthermore, we will discuss the sensitivity analysis for normal and abnormal optimal solutions.

*Universidad Técnica Federico Santa María, 2390123 Valparaíso, Chile, Cristopher.Hermosill@usm.cl

†Université de Normandie, INSA-Rouen, F-76000 Rouen, France, Hasnaa.Zidani@insa-rouen.fr

Spatial growth models over networks

Fausto Gozzi (1),

(1) Luiss University, Roma, Italia

In this talk we present some recent results on a spatial growth model where the space variable is given by the nodes of an oriented graph. Technically speaking the model is an Optimal Control problem of an ODE, or in the stochastic case, of a SDE. Here we present some first results on existence, uniqueness and regularity of the solutions of the HJB equation and, consequently, on the existence of optimal feedback paths. We also show a special case where explicit solutions can be obtained. This is based on ongoing papers with: A. Calvia, M.Leocata, A. Xepapadeas, A. Yannacopoulos.

On Integer Optimal Control Problems with Total Variation Regularization

Jonas Marko[1] and Gerd Wachsmuth[1]

[1]BTU Cottbus-Senftenberg, Chair of Optimal Control

markojo1@b-tu.de, wachsmuth@b-tu.de

We investigate the integer optimal control problem

$$\begin{aligned} \text{Minimize} \quad & F(u) + \beta \mathrm{TV}(u) \\ \text{such that} \quad & u(t) \in \{\nu_1, \ldots, \nu_d\} \subset \mathbb{Z} \text{ for a.a. } t \in (0, T), \end{aligned} \tag{P}$$

with $\beta > 0$, where $\mathrm{TV}(u)$ denotes the total variation of u, which penalizes jumps of the control. The contribution F is assumed to be differentiable, e.g. it could realize the tracking of the state given by an ODE dependent on u. The presence of the TV-term can be used to prove existence of a minimizer. Moreover, it averts rapid switching between multiple control levels in a short amount of time, which is desirable from an application point of view.

We show local optimality conditions of first and second order via a finite-dimensional switching point problem. Additionally, a non-local optimality condition treating back-and-forth switches will be formulated.

For a numerical solution, we propose a proximal gradient method. The emerging discretized subproblems will be solved by an algorithm which is polynomial in the mesh size and in the admissible control levels. An adaption of this algorithm can be used to handle subproblems of the trust-region method proposed in [1]. We show properties of the proximal gradient method and of the generated sequence of iterates. Finally, we apply both methods to a control problem governed by the Lotka-Volterra equations.

References

[1] Sven Leyffer, Paul Manns (2021) *Sequential Linear Integer Programming for Integer Optimal Control with Total Variation Regularization*, `https://arxiv.org/abs/2106.13453`.

Minisymposium MS02

MS02 - Optimization for gas networks

FGP2022 on Optimization
Towards Sequential Action Control for Gas Network Problems

Yan Brodskyi[1]* and Falk M. Hante[1]

[1]Department of Mathematics, Humboldt-Universitt zu Berlin,
Unter den Linden 6, 10099 Berlin, Germany

yan.brodskyi@hu-berlin.de (*corresponding author), falk.hante@hu-berlin.de

In this talk, we discuss two extensions of the framework of sequential action control (SAC) principle. The main idea of SAC is to use the so-called moving horizon strategy. Instead of finding in each step of a receding horizon the full time dependent optimal control solution of the optimization problem, SAC finds a constant optimal control value and time to act that maximally improves the performance. Our two extensions concern firstly systems of partial differential equations which can be posed as abstract semilinear control problems in a Hilbert space and secondly proof of the corresponding concept for transient control of gas networks. Both extensions are motivated by the properties of gas network problems. We present theoretical results and a numerical study.

Acknowledgments: This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) [Project-ID - 239904186] TRR 154 - subproject A03.

References

[1] Y. Brodskyi, F. M. Hante and A. Seidel (2021) *Stabilization of PDEs by sequential action control*, Preprint. arXiv:1905.11717 [math.OC].

Nonconvex Equilibrium Models for Energy Markets: Exploiting Price Information to Determine the Existence of an Equilibrium

Julia Grübel[1]*, Olivier Huber[2], Lukas Hümbs[3], Max Klimm[4], Martin Schmidt[5], and Alexandra Schwartz[6]

[1]Friedrich-Alexander-Universität Erlangen-Nürnberg, Chair of Economic Theory, Lange Gasse 20, 90403 Nürnberg

[2]Humboldt-Universität zu Berlin, Department of Mathematics, Unter den Linden 6, 10099 Berlin

[3]Friedrich-Alexander-Universität Erlangen-Nürnberg, Department of Data Science and Department of Mathematics, Chair of EDOM, Cauerstraße 11, 91058 Erlangen

[4]Technische Universität Berlin, Institute of Mathematics, Straße des 17. Juni 136, 10623 Berlin

[5]Universität Trier, Department of Mathematics, Universitätsring 15, 54296 Trier

[6]Technische Universität Dresden, Faculty of Mathematics, Zellescher Weg 12-14, 01069 Dresden

julia.gruebel@fau.de (*corresponding author), olivier.huber@hu-berlin.de, lukas.huembs@fau.de, klimm@tu-berlin.de, martin.schmidt@uni-trier.de, alexandra.schwartz@tu-dresden.de

Motivated by examples from the energy sector, we consider a special type of market equilibrium problems (MEPs) involving players with nonconvex strategy spaces or objective functions, where the latter are assumed to be linear in the market prices. Such problems naturally occur in settings in which energy trading is combined with aspects of the actual energy transport through networks. We propose an algorithm that determines if an equilibrium of such a MEP exists and that computes a market equilibrium in the case of existence. Three key prerequisites have to be met. First, necessary optimality conditions of some players, e.g., of those facing convex optimization problems, have to yield enough information to appropriately bound market prices. Second, a technical assumption is required for those prices that are not uniquely determined by the derived price bounds. In particular, for all players, the part of their objective functions affected by the non-unique prices has to be minimal or maximal in the welfare-optimal solution. Third and finally, the presented algorithm relies on solving nonconvex optimization problems to global optimality, which cannot be avoided in our setting since it already includes nonconvex player problems. We show that these three prerequisites are met for relevant energy market applications. Furthermore, to demonstrate the performance of our proposed algorithm, we test it on well-known instances from the power and gas literature. There, nonconvexities—either due to integrality conditions or nonlinearities—mainly arise from considering the transmission system operator as an additional player who, e.g., switches lines or faces nonlinear physical laws. Our numerical results indicate that an equilibrium often exists, especially for the case of continuous nonconvexities in the context of gas market problems.

Acknowledgments: All authors thank the Deutsche Forschungsgemeinschaft for their support within projects A05, A07, B02, B07, B08, and B09 in the Sonderforschungsbereich/Transregio 154 "Mathematical Modelling, Simulation and Optimization using the Example of Gas Networks". Julia Grübel gratefully acknowledges the use of the services and facilities of the Energie Campus Nürnberg and financial support of the state of Bavaria.

On a New Solver for Constrained Piecewise Linear Optimization

Timo Kreimeier[1]* and Andrea Walther[1]

[1]Humboldt-Universität zu Berlin

timo.kreimeier@hu-berlin.de (*corresponding author), andrea.walther@math.hu-berlin.de

The optimization of parameters in networks, such as those encountered in gas pipeline distribution in the energy market, constitutes a current area of research in mathematical optimization. In this talk, we present an optimization algorithm developed for solving constrained piecewise linear optimization problems [1]. The presented algorithm - called CASM, for Constrained Active Signature Method - explicitly exploits the non-smoothness of the function by rendering it in a matrix-vector representation called abs-linear form. One highlight is that the special structure allows to check the optimality of a point with only polynomial effort.

Subsequently, results regarding an application of CASM in the context of gas network optimization are discussed. Here, CASM acts as a solver for subproblems generated in a bundle method for robust optimization [2]. After a relaxation, these subproblems can be formulated either as mixed integer problems (MIP) or as constrained piecewise linear problems. The latter can be solved with CASM and additionally allow a warm start in case of a refinement of the relaxation [3]. In contrast, simplex based branch-and-bound approaches require a solution of the refined problem from scratch.

Acknowledgments: The authors thank the Deutsche Forschungsgemeinschaft for their support within Project B10 in the Sonderforschungsbereich / Transregio 154 Mathematical Modelling, Simulation and Optimization using the Example of Gas Networks.

References

[1] Timo Kreimeier, Andrea Walther and Andreas Griewank (2021) *An active signature method for constrained abs-linear minimization*, submitted at Computational Optimization and Applications, available at Optimization Online.

[2] Martina Kuchlbauer, Frauke Liers and Michael Stingl (2022) *Outer approximation for mixed-integer nonlinear robust optimization*, url: https://opus4.kobv.de/opus4-trr154/frontdoor/index/index/docId/414

[3] Timo Kreimeier, Martina Kuchlbauer, Frauke Liers, Michael Stingl and Andrea Walther (2022) *Towards the Solution of Robust Gas Network Optimization Problems Using the Constrained Active Signature Method*, accepted for proceedings of INOC 2022.

A Decomposition Method for MINLPs with Multivariate Lipschitz Continuous Nonlinearities

Julia Grübel[1], Richard Krug[1]*, Martin Schmidt[2], and Winnifried Wollner[3]

[1]Friedrich-Alexander-Universität Erlangen-Nürnberg

[2]Trier University

[3]Technische Universität Darmstadt

richard.krug@fau.de (*corresponding author)

We continue the work of [1] where a decomposition method for mixed-integer optimization problems with Lipschitz continuous nonlinearities is presented. Now, we extend this method by considering multivariate Lipschitz continuous functions. Due to this, the number of problems that fit in our framework is increased significantly. This includes, e.g., problems where some constraints cannot be evaluated exactly which is the case for many control problems governed by partial differential equations or bilevel problems with non-convex lower level. We develop an algorithm that solves these mixed-integer optimization problems with multivariate Lipschitz continuous nonlinearities by iteratively approximating the nonlinear functions using their Lipschitz constant. We prove convergence to an approximate global solution and give a worst-case estimate for the number of iterations. In case studies, we derive the necessary Lipschitz constants and apply the method to bilevel problems and gas transport problems.

Acknowledgments: The authors thank the DFG for their support within projects A05 and B08 in CRC TRR 154.

References

[1] M. Schmidt, M. Sirvent, W. Wollner (2019) *A Decomposition Method for MINLPs with Lipschitz Continuous Nonlinearities*, Mathematical Programming, 178 **1**, 449–483.

Control Strategies for Transport Networks under Demand Uncertainty

Thomas Schillinger[1]*, Simone Göttlich[1]

[1]University of Mannheim, Department of Mathematics, 68131 Mannheim, Germany

schillinger@uni-mannheim.de (*corresponding author), goettlich@uni-mannheim.de

We consider transport networks with uncertain demands. Network dynamics are given by hyperbolic partial differential equations with linear flux functions and non-linear damping terms. At the network junctions suitable coupling conditions are discussed. Demands are incorporated as solutions to stochastic differential equations for which we compare a Jacobi and an Ornstein-Uhlenbeck process. For the demand satisfaction, we solve a constrained optimal control problem and investigate different demand update strategies. Controls in terms of network inputs are then calculated explicitly for different assumptions. Numerical simulations are performed to underline the theoretical results.

References

[1] Göttlich, Simone, Korn, Ralf, Lux, Kerstin (2019) *Optimal control of electricity input given an uncertain demand*, Mathematical Methods of Operations Research, Volume **90**, 1–28.

[2] Göttlich, Simone, Schillinger, Thomas (2021) *Control Strategies for Transport Networks under Demand Uncertainty*, preprint arXiv2111.09674.

[3] Coskun, Sema, Korn, Ralf (2021) *Modeling the Intraday Electricity Demand in Germany*, Mathematical Modeling, Simulation and Optimization for Power Engineering and Management, Springer International Publishing, 3–23.

FGP2022 on Optimization
Probabilistic Constrained Optimization on Gas Networks

Michael Schuster[1]*, Elisa Strauch[2], Martin Gugat[1] and Jens Lang[2]

[1]FAU Erlangen-Nürnberg

[2]TU Darmstadt

michi.schuster@fau.de (*corresponding author),
strauch@mathematik.tu-darmstadt.de,
gugat@math.fau.de,
lang@mathematik.tu-darmstadt.de

Uncertainty often plays an important role in gas transport and probabilistic constraints are an excellent modeling tool to obtain controls and other quantities that are robust against perturbations in e.g., the boundary data. We consider both, a stationary and a dynamic gas transport model with uncertain boundary data on networks. We provide an efficient way to compute the probability that random boundary data is feasible. In this context feasible means that the pressure corresponding to the random boundary data meets some box constraints at the network junctions.

Further we consider and analyze optimization problems with probabilistic constraints in the stationary and the dynamic setting where the probabilistic constraints are approximated by the kernel density estimator approach. Last we compare the solutions of the probabilistic constrained optimization problems with the solutions of the corresponding deterministic problems.

Acknowledgments: The authors are supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) within the collaborative research center TRR154 "Mathematical modeling, simulation and optimisation using the example of gas networks" (Project-ID 239904186, TRR154/2-2018, TP B01 (Lang, Strauch), TP C03 (Gugat, Schuster) and TP C05 (Giesselmann, Gugat)).

References

[1] M. Schuster, E. Strauch, M. Gugat and J. Lang (2021) *Probabilistic Constrained Optimization on Flow Networks*, Optim. Eng. (2021), *https://doi.org/10.1007/s11081-021-09619-x*.

[2] M. Schuster (2021) *Nodal Control and Probabilistic Constrained Optimization using the Example of Gas Networks*, Dissertation, FAU Erlangen-Nürnberg, Germany, 2021, *https://opus4.kobv.de/opus4-trr154/frontdoor/index/index/docId/410*.

Multiscale Control of Stackelberg Games

Michael Herty[1], Sonja Steffensen[1], and Anna Thünen[2]*

[1] RWTH Aachen University

[2] Technical University Clausthal

{herty,steffensen}@igpm.rwth-aachen.de, anna.thuenen@tu-clausthal.de (*corresponding author)

Energy markets have an important role to play in the transition to the renewable energy era, as they offer the possibility of demand side management to avoid peaks in consumption and resulting blackouts. We present a model motivated by the bilevel structure in energy markets. The difference to classical models is that we zoom into the consumer population. We use interacting agent models and compute their mean-field limit. Specifically, we investigate the interplay between bilevel optimization and the limit process. In particular, we establish conditions for consistency and propose a numerical method based on the derived models and present numerical examples.

Acknowledgments: This work has been supported by the DFG under Grant STE2063/2-1 and HE5386/19-1.

References

[1] Michael Herty, Sonja Steffensen, and Anna Thünen. (2020). *Multiscale Control of Stackelberg Games*, arXiv:2011.03405.

[2] Michael Herty and Anna Thünen. (2021) *Consistent Control of a Stackelberg Game with Infinitely many Followers*, IEEE CDC 2021.

Stochastic Gradient Descent for Optimal Control Problems using Domain Decomposition

Lukas Wolff[1]*, Falk Hante[2], and Enrique Zuazua[1]

[1]Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU), Germany

[2]Humboldt-Universität zu Berlin (HUB), Germany

lukas.wolff@fau.de (*corresponding author), falk.hante@hu-berlin.de, enrique.zuazua@fau.de

Solving optimal control problems on large domains can be computationally very expensive. We present an approach to reduce the numerical complexity of these problems by employing stochastic gradient descent (SGD) methods, tailored to the optimal control of PDE's by using domain decomposition methods (DDM).

The main idea of DDM as introduced in [3] is to decompose the PDE system on a domain Ω into subsystems on a set of non-overlapping subdomains $(\Omega_i)_{i=1}^M$ with interfaces $\Gamma_{i,j} = \overline{\Omega}_i \cap \overline{\Omega}_j$. This is achieved by coupling the solutions y_i on Ω_i to their neighbours with Robin boundary conditions of the form

$$\partial_{\nu_i} y_i^{(n+1)} + y_i^{(n+1)} = -\partial_{\nu_j} y_j^{(n)} + y_j^{(n)} \quad \text{on } \Gamma_{i,j}, i = 1, ..., M, j \neq i$$

in an iterative process over $n \in \mathbb{N}$ and with ν_i being the outward pointing normal of subdomain Ω_i. This procedure has been succesfully extended to the optimality systems of optimal control problems [1, 2].

We give a brief overview over these results. We then present a novel approach to solve optimal control problems in an iterative stochastic descent procedure using DDM, allowing us to compute the descent direction in each step based only on the dynamics on a randomly selected subgraph. This offers a significant reduction in computational time, as we will show in numerical results for the control of gas flow on large networks. This is also compared to the promising results recently obtained for a similar procedure in a finite dimensional setting [4]. We furthermore discuss the stability and consistency of our proposed method.

Acknowledgments: This work has been supported by the Deutsche Forschungsgemeinschaft (DFG) within project C07 of Transregio 154 *Mathematical Modelling, Simulation and Optimization using the Example of Gas Networks*.

References

[1] J.-D. Benamou and B. Desprès (1997) *A Domain Decomposition Method for the Helmholtz Equation and Related Optimal Control Problems*, Journal of Computational Physics, 136(1): 68–82.

[2] J. Lagnese and G. Leugering. (2004). *Domain decomposition methods in optimal control of partial differential equations* Birkhäuser Verlag, Basel, Switzerland.

[3] P.-L. Lions (1990) *On the Schwarz alternating method. III: A variant for nonoverlapping subdomains*, Third international symposium on domain decomposition methods for partial differential equations, volume 6, pages 202–223.

[4] D. Veldman and E. Zuazua (2021) *A framework for randomized time-splitting in linear-quadratic optimal control*, Preprint, TRR 154, URL: https://opus4.kobv.de/opus4-trr154/frontdoor/index/index/docId/443.

Minisymposium MS03

MS03 - Optimization in Medicine: contributions for radiotherapy treatments

An automated bi-level optimization approach for IMRT

Pedro Carrasqueira[1], Maria João Alves[2,3], Humberto Rocha [2,3] and Joana Dias[1,3]

[1]University of Coimbra, Inesc Coimbra, Portugal

[2]University of Coimbra, CeBer, Portugal

[3]University of Coimbra, Faculty of Economics, Portugal

pedro.carrasqueira@deec.uc.pt (*corresponding author), mjalves@fe.uc.pt, hrocha@fe.uc.pt joana@fe.uc.pt

Intensity-modulated radiation therapy (IMRT) is used worldwide to treat cancer patients. The objective of this treatment is to deliver a prescribed radiation dose to the tumor while sparing, as much as possible, all the healthy tissues, especially organs at risk (OAR). This means that the planning of a radiotherapy treatment should take into consideration conflicting objectives: to be able to spare as much as possible the OAR guaranteeing, at the same time, that the desired radiation is delivered to the volumes to treat. While the volumes to treat can be adequately irradiated from almost any set of directions, the radiation directions that are chosen have a determinant impact on the OAR. This means that those directions that provide an improved OAR sparing should be selected. The choice of radiation directions (beam angles) can thus be interpreted as being fundamentally determined by the OAR, with the radiation intensities associated with each of these directions being determined by the needed radiation to be delivered to the volumes to treat. In this presentation we will interpret the radiotherapy treatment planning problem as a bi-level optimization problem. At the upper level, OAR control the choice of the beam angles, which are selected aiming at OAR sparing. At the lower level, the optimal radiation intensities are decided by the volumes to treat, considering the beam angle ensemble obtained at the upper level. A pool of 10 clinical nasopharyngeal tumor cases already treated at IPO Coimbra was considered to assess the quality of the treatment plans obtained by the bi-level optimization approach for coplanar and noncoplanar beam ensembles. The results were then compared to the coplanar equidistant seven-beam solution, considered as a benchmark, which was significantly outperformed in terms of both upper and lower level objective functions' values. Treatment plans obtained by the bi-level algorithm, either for coplanar or noncoplanar cases, improved significantly OAR sparing and simultaneously maintain or increase PTV irradiation levels. These results show that the bi-level approach for the BAO problem is an effective tool for achieving higher quality treatment plans.

Acknowledgments: This work was supported by project grant POCI-01-0145-FEDER-028030 and by the Fundação para a Ciência e a Tecnologia (FCT) under project grants UIDB/05037/2020 and UIDB/00308/2020.

References

[1] Carrasqueira, P., Alves, M. J., Dias, J. M., Rocha, H., Ventura, T., Ferreira, B. C., and Lopes, M. C. (2021) *An automated bi-level optimization approach for IMRT*, International Transactions in Operational Research.accepted for publication.

A robust fluence map optimization for proton therapy treatment planning

Joana Dias[1,3], Humberto Rocha[2,3], Joana Neves [1,4] and Brígida Ferreira[1,5]

[1]University of Coimbra, Inesc Coimbra, Portugal

[2]University of Coimbra, CeBer, Portugal

[3]University of Coimbra, Faculty of Economics, Portugal

[4]University of Coimbra, Faculty of Science and Technology, Portugal

[5]IBEB, Faculty of Sciences, University of Lisbon, Portugal

joana@fe.uc.pt (*corresponding author), hrocha@fe.uc.pt, joana.g.neves23@gmail.com, bcferreira@fc.ul.pt

Radiotherapy is one of the possible treatments for cancer, and it is indeed delivered to more than half of all cancer patients. Proton therapy (PT) has specific features that make it highly appropriate for the treatment of cancer as compared with photon radiotherapy. Intensity Modulated Proton Therapy (IMPT), the most advanced form of PT, is delivered using electromagnetically scanned narrow beams ("beamlets") of protons. The process of treatment planning consists of deciding the number and directions of beams and the optimization of intensities and energies of beamlets comprising each of the beams so that a homogeneous dose distribution is obtained in the target volume and the normal tissues are maximally spared. Proton beams have unique depth-dose characteristics, since protons slow down as they penetrate matter, their rate of energy transfer increases with depth, coming to an abrupt stop just beyond where energy deposition is maximum producing the so-called Bragg peak. By positioning several Bragg peaks inside the target volume, excellent tumor conformity is obtained, and adjacent organs at risk are spared. However, PT is much more vulnerable to uncertainties due to this very steep Bragg peak. These uncertainties should be explicitly taken into account when planning the treatment, otherwise it is not possible to guarantee that the treatment will achieve its objectives, guaranteeing proper tumor coverage and sparing of the organs at risk. In this presentation a new fluence map optimization (FMO) approach for PT is presented, based on [1], where both positioning uncertainties and range uncertainties are taken into consideration. The initial existing structures are cloned, and FMO guarantees the satisfaction of all the existing dose constraints (defined by the medical prescription) in all these auxiliary structures. The obtained treatment plans are then assessed using Monte Carlo simulation, that takes explicitly into account the effect of treatment fractionation.

Acknowledgments: This work has been supported by the Fundação para a Ciência e a Tecnologia (FCT) under project grants UTA-EXPL/FMT/0079/2019, PTDC/CCI-INF/28030/2017, UIDB/05037/2020 and UIDB/00308/2020

References

[1] Dias, J., Rocha, H., Ventura, T., Ferreira, B., and Lopes, M. D. C. (2016) *Automated fluence map optimization based on fuzzy inference systems*, Medical physics, 43 **3**, 1083-1095.

Proton and Photon radiotherapy treatments: a comparison considering the impact of uncertainties

Joana Neves[1,4], Brígida Ferreira[1,5], Humberto Rocha[2,3] and Joana Dias [1,3]

[1]University of Coimbra, Inesc Coimbra, Portugal

[2]University of Coimbra, CeBer, Portugal

[3]University of Coimbra, Faculty of Economics, Portugal

[4]University of Coimbra, Faculty of Science and Technology, Portugal

[5]IBEB, Faculty of Sciences, University of Lisbon, Portugal

joana.g.neves23@gmail.com (*corresponding author), bcferreira@fc.ul.pt, hrocha@fe.uc.pt, joana@fe.uc.pt

The objective of radiotherapy treatments is to deliver the prescribed radiation dose to the volumes to treat and, at the same time, spare as much as possible all healthy structures. Photons and protons beams are both used as radiation sources in radiotherapy. When compared, protons are usually able to produce dose distributions that better conform with the target volume while simultaneously achieving a better sparing of healthy tissues. This can be explained by the typical depth dose curve of proton beams compared to the photon beam profile (Figure 1) that, due to the steep Bragg peak, makes proton beam radiotherapy much more vulnerable to uncertainties (intra and inter positional errors, dose computation, range, among others). In this presentation, the impact of uncertainties in photon and proton treatments are compared by using Monte Carlo simulation and the role of Operations research in this field is discussed.

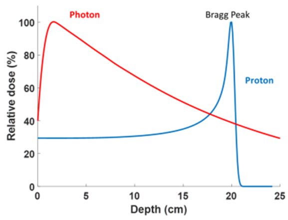


Figure 1: Depth-dose curves, normalized to 100 at maximum dose (based on [1]).

Acknowledgments: This work has been supported by the Fundação para a Ciência e a Tecnologia (FCT) under project grants UTA-EXPL/FMT/0079/2019, PTDC/CCI-INF/28030/2017, UIDB/05037/2020 and UIDB/00308/2020

References

[1] R. Mohan and D. Grosshans (2017) *Proton therapy – Present and future*, Advanced Drug Delivery Reviews, 109 **3**, 26-44.

Beam Angle Optimization in Intensity-Modulated Proton Therapy using a Probabilistic Descent Direct-Search Approach

Humberto Rocha[1,2]* and Joana Dias[1,2]

[1] FEUC, CeBER, University of Coimbra, Coimbra, Portugal

[2] INESC-Coimbra, University of Coimbra, Coimbra, Portugal

hrocha@fe.uc.pt (*corresponding author), joana@fe.uc.pt

Radiation therapy is a cancer treatment that aims to eradicate the cancer cells by irradiating the tumor with a prescribed dose while sparing, as much as possible, the surrounding organs and tissues. Intensity-modulated proton therapy (IMPT) is a promising treatment modality due to the unique depth-dose characteristics of protons that allow a better compromise between the irradiation of the tumor and the inevitable radiation of adjacent structures. Nevertheless, obtaining high-quality treatment plans taking the most possible advantage of the unique characteristics of IMPT requires the optimization of different parameters including the optimal selection of proton beam directions. Differentiating characteristics between protons and photons makes the beam angle optimization (BAO) problem, i.e., the optimal selection of irradiation directions, more complex in IMPT: (i) the number of beams in IMPT is typically lower than in intensity-modulated radiation therapy for photons (IMRT), being the selection of the beam directions even more critical; (ii) different levels of energy are available with protons, increasing the degrees of freedom; (iii) different sources of uncertainty require robustness to be considered. Thus, obtaining optimal beam irradiation directions in a clinically acceptable time becomes even more important considering the existence of more degrees of freedom and different scenarios required for robust plans. In previous works, direct-search methods proved to be suited for BAO in IMRT [1]. Recently, almost-sure probabilistic convergent direct-search approaches showed quality results in practice with faster computational time [2]. In this study, we extend our preliminary work for IMPT BAO [3] and compare the results of a probabilistic descent direct-search approach that considers a tailored set of polling directions with the results obtained by a deterministic counterpart.

Acknowledgments: This work has been supported by the Fundação para a Ciência e a Tecnologia (FCT) under project grants UTA-EXPL/FMT/0079/2019, PTDC/CCI-INF/28030/2017, UIDB/05037/2020 and UIDB/00308/2020.

References

[1] Rocha, H., Dias, J., Ferreira, B.C., Lopes, M.C. (2013) *Selection of intensity modulated radiation therapy treatment beam directions using radial basis functions within a pattern search methods framework*, J. Glob. Optim., **57**, 1065–1089.

[2] Gratton, S., Royer, C. W., Vicente, L. N., Zhang, Z. (2015) *Direct search based on probabilistic descent*, SIAM J. Optim., **25**, 1515–1541.

[3] Rocha, H., Dias, J. (2021) *A randomized direct-search approach for beam angle optimization in intensity-modulated proton therapy*. In: Gervasi, O., *et al.* (eds.) ICCSA 2021. LNCS, vol. 11621, pp. 199–214.

Minisymposium MS04

MS04 - Geometric control, optimal feedback stabilization and applications

Optimal Control of the Generalized Lotka-Volterra Equation with Application to the Intestinal Microbiota

Bernard J. M. Bonnard[1] and Cristiana J. Silva[2]*

[1] Institut de Mathématiques de Bourgogne, Université de Bourgogne, Franche-Compté, France

[2]Center for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal

bernard.bonnard@u-bourgogne.fr, cjoaosilva@ua.pt (*corresponding author)

This work is inspired by [1] based on a model described in [2], where the authors proposed to cure the microbiota instestine infected by the *C. difficile* bacteria, based on a sequence of antibiotic treatment (prior to infection) and fecal injection (post to infection). This treatment uses a $2d$-reduced model of an original Generalized Lotka-Volterra (GLV) model with dynamics in dimension $N = 11$. While the reduced model codes only $2^2 = 4$ interacting equilibria, the original model codes up to $2^{11} = 2048$ interacting equilibria.

In this talk, we propose a theoretical control approach to compute robust optimized policies based on the combination of two approaches. First of all, the problem can be set in the frame of geometric optimal control where the free coordinates Lie algebraic computations, based on Pontryagin Maximum Principle and developed in [3] to analyze complex interacting chemical networks, can be applied to GLV-dynamics and analyze up to 9 entangled equilibria (using the dictionary given in [3]). In this case, the antibiotic treatment is assumed to be permanent, but the problem can be set in the frame of optimal sampled-data control taking into account the logistic constraints about changing the dose the antibiotic treatment. In this frame, the problem boils down to a finite dimensional optimization problem which can be efficiently analyzed with a Nonlinear Model Predictive Control (NMPC) method, where convergence of the algorithm in related to the analysis of the permanent case.

Acknowledgments: B. Bonnard was supported by FMJH Program PGMO and by EDF Thales, Orange. C. J. Silva was supported by Portuguese funds through CIDMA, The Center for Research and Development in Mathematics and Applications of University of Aveiro, and the Portuguese Foundation for Science and Technology (FCT – Fundação para a Ciência e a Tecnologia), within project UIDB/04106/2020 and by the FCT Researcher Program CEEC Individual 2018 with reference CEECIND/00564/2018.

References

[1] E. W. Jones, P. Shankin-Clarke, J. M. Carlson. (2020). *Navigation and control of outcomes in a generalized Lotka-Volterra model of the microbiome*, in Advances in Nonlinear Biological Systems: Modeling and Optimal Control, AIMS Series on Applied Mathematics, **11**, 97–120.

[2] R. R. Stein, V. Bucci, N. C. Toussaint, C. G. Buffie, G. Rätsch, E. G. Pamer, et al. (2013). *Ecological Modeling from Time-Series Inference: Insight into Dynamics and Stability of Intestinal Microbiota*, PLoS Comput Biol **9(12)**, e1003388.

[3] B. Bonnard, J. Rouot. (2020). *Towards Geometric Time Minimal Control without Legendre Condition and with Multiple Singular Extremals for Chemical Networks*, in in Advances in Nonlinear Biological Systems: Modeling and Optimal Control, AIMS Series on Applied Mathematics, **11**, 1–34.

[4] B. Bonnard, C. J. Silva. *Geometric optimal control of the generalized Lotka-Volterra model of the intestinal microbiome*, in preparation.

Learning an optimal feedback operator semiglobally stabilizing semilinear parabolic equations

Karl Kunisch[1,2], Sérgio S. Rodrigues[1,*], and Daniel Walter[1]

[1]Johann Radon Institute for Computational and Applied Mathematics, ÖAW, Altenbergerstr. 69, 4040 Linz, Austria.

[2]Institute for Mathematics and Scientific Computing, 621, Heinrichstrasse 36, 8010 Graz, Austria.

karl.kunisch@uni-graz.at, sergio.rodrigues@ricam.oeaw.ac.at (*corr. author), daniel.walter@ricam.oeaw.ac.at

Stabilizing feedback operators are presented which depend only on the orthogonal projection of the state onto the finite-dimensional control space. A class of mono- tone feedback operators mapping the finite-dimensional control space into itself is considered. The special case of the scaled identity operator is included. Conditions are given on the set of actuators and on the magnitude of the monotonicity, which guarantee the semiglobal stabilizing property of the feedback for a class of semilinear parabolic-like equations. Subsequently an optimal feedback control minimizing the quadratic energy cost is computed by a deep neural network, exploiting the fact that the feedback depends only on a finite dimensional component of the state. Numerical simulations demonstrate the stabilizing performance of explicitly scaled orthogonal projection feedbacks, and of deep neural network feedbacks.

Acknowledgments: S. Rodrigues acknowledges partial support from the Austrian Science Fund (FWF): P 33432-NBL.

References

[1] K. Kunisch and S. S. Rodrigues and D. Walter (2021) *Learning an Optimal Feedback Operator Semiglobally Stabilizing Semilinear Parabolic Equations*, Appl. Math. Optim., Volume 84 **1**, 277–318. doi:10.1007/s00245-021-09769-5

Time Minimal Syntheses using Geometric and Computational Techniques for Chemical Reactors

Bernard Bonnard[1], Jérémy Rouot[2]
jeremy.rouot@univ-brest.fr

[1]INRIA Sophia Antipolis, Équipe McTAO
[2]Université de Bretagne Occidentale, LMBA, Brest

Abstract

The topic of the talk is the optimization of the production of one chemical species for the McKeithan network using an approach based on geometric optimal control and singularity theory. The objective is to classify the generic (local) syntheses for analytic systems of the form $\dot{q}(t) = X(q(t)) + u(t)Y(q(t))$, where the aim is to reach in minimum time a terminal manifold of codimension one. Computations techniques and the use of semi-normal forms illustrate the role of singularity theory in geometric optimal control, in relation with abnormal geodesics and regularity of the value function solution of Hamilton-Jacobi-Bellman equation.

References

[1] B. Bonnard, J. Rouot. *Towards Geometric Time Minimal Control without Legendre Condition and with Multiple Singular Extremals for Chemical Networks.* Advances in Nonlinear Biological Systems: Modeling and Optimal Control, American Institute of Mathematical Sciences, pp.1-34, 2020, Applied Mathematics, 978-1-60133-025-3.

[2] B. Bonnard, J. Rouot, B. Wembe. *Accessibility Properties of Abnormal Geodesics in Optimal Control Illustrated by Two Case Studies.* Submitted, 2022.

Acknowledgments J. Rouot was supported by the project PEPS "Jeunes chercheurs et jeunes chercheuses de l'Insmi 2022".

Optimal control of predator-prey systems complex network

Cristiana J. Silva[1]* and Guillaume Cantin[2]

[1]Center for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal

[2]Laboratoire des Sciences du Numérique, LS2N UMR CNRS 6004, Université de Nantes, France

cjoaosilva@ua.pt (*corresponding author), guillaume.cantin@univ-nantes.fr

In this work, we consider a biological environment in which (at least) two species p and q interact. We assume that the geographical habitat of the species is perturbed by the anthropic extension, so that it is fragmented in several patches. This fragmentation is likely to alter the equilibrium of the biological ecosystem. More precisely, p and q are linked by a predator-prey type interaction, which can be determined by a system of ordinary differential equations of Lotka-Volterra type:

$$\begin{cases} \dot{p} = p(1-p) - \frac{Qpq}{p+A}\,, \\ \dot{q} = Sq\left(1 - \frac{q}{p+C}\right)\,, \end{cases} \tag{PP}$$

where p and q denote the prey density and the predator density, respectively. Depending on the values of the parameters A, C, S, Q, the solutions of system (PP) can be attracted to a coexistence equilibrium or to an extinction equilibrium. We construct a complex network of predator-prey systems which models the evolution of the species in the fragmented environment, in which the coupling operator models the migrations of the species from one patch of the fragmented environment to another. The main goal of this work is to reach a coexistence equilibrium state for the whole complex network, applying optimal control strategies on the migrations of the species.

Acknowledgments: C. J. Silva was supported by Portuguese funds through CIDMA, The Center for Research and Development in Mathematics and Applications of University of Aveiro, and the Portuguese Foundation for Science and Technology (FCT – Fundação para a Ciência e a Tecnologia), within project UIDB/04106/2020 and by the FCT Researcher Program CEEC Individual 2018 with reference CEECIND/00564/2018.

Optimal control and cardiovascular modeling

Jorge Tiago[1]*

[1]CEMAT, Instituto Superior Técnico

jorge.tiago@tecnico.ulisboa.pt (*)

The mathematical investigation of the cardiovascular system and, in particular, of blood flow in major arteries, has seen vast and clear progress in the last three decades. Such progress allows, for the first time, to obtain computational solutions of mathematical models which can be regarded as being sufficiently general to be considered realistic. However, a big gap between the state of the art of such modeling strategies and its use by the medical community remains open. This is due to the fact that, for diagnosis and prognosis purposes, numerical simulations should be sufficiently accurate and reliable to be considered patient-specific. Optimal control techniques have a role in overcoming such a gap. In this talk, we will discuss some of these techniques, present numerical results, as well as some lines of research within the ongoing project HemoControl.

Acknowledgments: This work has been supported by the FCT Project PTDC/MAT-APL/7076/2020.

References

[1] Author. (year). *Title of the book*, publisher, City, initial page–final page.

[2] Author (year) *Title of the paper*, Journal, Volume **issue**, initial page–final page.

FGP2022

Zermelo Navigation Problems on Surfaces of Revolution

B. Bonnard[1] and B. Wembe[2*]

[1]INRIA Sophia Antipolis, McTAO team, Valbonne, France, bbonnard@u-bourgogne.fr.

[2]ENSEEIHT, IRIT, Toulouse, France, boris.wembe@irit.fr (*corresponding author).

This work is motivated by one of the founding problem of calculus of variations set by Zermelo-Carathéodory [1] which consists to minimize the transfer time for a ship navigating at constant speed on a river and aiming to reach the opposite shore. The control being the heading angle of the ship. This historical problem can be set in a more general frame, taking a surface of revolution with the induced Riemannian metric, the current being a vector field on the surface, invariant by rotation. The surface decomposes into domains of weak and strong current, the second one being when the speed of the current cannot be compensated by the speed of the ship. The problem is investigated using Pontryagin Maximum Principle to select geodesic curves, candidates as minimizers. We analyze the geodesic flow to determine the conjugate and cut points, using geometric methods and numerical computations to calculate the accessibility set and its boundary, in relation with regularity properties of the time minimal value function.

We present fine results describing the conjugate and cut loci. In particular one important phenomenon, already observed in the original example, is a cusp singularity of the so-called abnormal or limit curves when meeting the boundary between strong and weak current domain, which we analyze in our more general context [3]. We present shortly two cases studies analyzed in details in [2] related to the navigation of a ship near a vortex in hydrodynamics and toy model in space mechanics connected to orbital transfer with low propulsion.

Keywords: Zermelo navigation problem on surfaces of revolution, Geometric optimal control, Time minimal value function, Conjugate and cut loci, abnormal geodesics, cusp points, (non) isochronous separating points.

References

[1] C. Carathéodory, *Calculus of Variations and Partial Differential Equations of the First Order, Part 1, Part 2.* Holden-Day, San Francisco, California, 1965-1967; Reprint: 2nd AMS printing, AMS Chelsea Publishing, Providence, RI, USA (2001), 412 pages.

[2] B. Wembe, *Méthodes Géométriques et Numériques en Contrôle Optimal et Problèmes de Zermelo sur les Surfaces de Révolution - Applications*, Phd thesis, Université Toulouse Paul Sabatier, Toulouse, 2021.

[3] B. Bonnard, O. Cots, J. Gergaud, B. Wembe, *Abnormal Geodesics in 2D-Zermelo Navigation Problems in the Case of Revolution and the Fan Shape of the Small Time Balls.* To appear at System Control & Letters, hal-02437507 (2021).

Minisymposium MS05

MS05 - Recent trends in analytical and computational aspects on non-smooth optimization

Convexification of branched transport

Julius Lohmann

University of Münster, Germany

juliuslohmann@uni-muenster.de

The *branched transport problem* is a non-convex and non-smooth variational problem on Radon measures. In this one tries to find an optimal mass flux $\mathcal{F}$ (in the Eulerian formulation a vector-valued measure) between two given probability measures μ_+ and μ_-, which may for example describe the initial and final distribution of some material. The optimality here is with respect to a subadditive *transportation cost* describing the effort $\tau(m)$ to move an amount of material m per unit distance. The subadditivity of the transportation cost leads to a branched structure of the network on which the transport described by $\mathcal{F}$ takes place. We study a convexification of this problem to a *multimaterial transport problem* [1], which is also of its own interest. The cost function h appearing in this relaxed problem is nonnegative homogeneous and describes the effort to move combinations of certain artificially generated materials. With regard to the branched transport problem, for some cases one can design the function h such that the relaxation is tight. Using Fenchel's duality theorem in combination with a result on the conjugation of integral functionals [3], we view the multimaterial transport problem as the dual problem to a variant of the Kantorovich–Rubinstein formula for the Wasserstein distance (which in its classical form is used to solve the transportation problem with $\tau(m) = m$). The primal-dual optimality conditions then naturally lead to our definition of a calibration, a certificate for optimality of a minimizing candidate of the (dual) multimaterial transport problem. Further, we relax the function space of the primal problem to ensure existence of solutions and simultaneously derive the notion of a weak calibration using a suitable pairing between a (in our case) curl-free L^∞-matrixfield and a divergence measure vectorfield [2]. We give conditions under which a weak calibration can be represented by a (strong) calibration and provide a procedure of how to construct a calibration from a weak calibration. Further, we give examples of calibrations and will use them to prove properties of branched transport networks.

Joint work with: Bernhard Schmitzer, Benedikt Wirth.

Acknowledgments: This work is supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under the priority program SPP 1962, grant WI 4654/1-1, and under Germany's Excellence Strategy EXC 2044 – 390685587, Mathematics Münster: Dynamics–Geometry–Structure. B.W.s and J.L.'s research is supported by the Alfried Krupp Prize for Young University Teachers awarded by the Alfried Krupp von Bohlen und Halbach-Stiftung.

References

[1] A. Marchese, A. Massaccesi, R. Tione (2019) *A multi-material transport problem and its convex relaxation via rectifiable G-currents*, SIAM Journal on Mathematical Analysis, 51 **3**, 1965–1998.

[2] M. Šilhavý (2008) *Divergence measure vectorfields: their structure and the divergence theorem*, Mathematical Modelling of Bodies with Complicated Bulk and Boundary Behavior, 20, 217–237.

[3] M. Valadier (1975) *Convex integrands on Suslin locally convex spaces*, Pacific Journal of Mathematics, 20 **3**, 267–276.

Asymptotics of domain decomposition for optimal transport

Mauro Bonafini[1], **Ismael Medina**[1]*, Bernhard Schmitzer[1]

[1] Institute of Computer Science, University of Göttingen

[bonafini, ismael.medina (*corresponding author), schmitzer]@cs.uni-goettingen.de

Domain decomposition addresses big (entropy-regularized) optimal transport problems by dividing them into small subproblems and solving these in parallel. By alternating between overlapping subdomains, information is allowed to flow until convergence to an optimal transport plan. In [1] the authors proved linear convergence to the global minimizer, as well as excellent computational performance in practice. However, the worst-case convergence rate is rather pessimistic, very far from the fast convergence observed in regular problems (i.e. Wasserstein-2 on a convex domain). Providing theoretical guarantees for this fast convergence remains one of the significant challenges for the wide adoption of the domain decomposition method.

In [2] we work towards such theoretical guarantees. With the purpose of studying the algorithm for very fine resolutions, we derive the Γ-limit of the domain decomposition algorithm in the limit of increasingly small subdomains. In this limit, the sequence of iterates becomes a trajectory in the space of couplings, which follows a horizontal continuity equation. We will show that the velocity field of this continuity equation can be extracted from the limit of the flow of mass between neighboring subdomains in the finite-scale algorithm. The study of the velocity field reveals that the limit trajectory converges to a Monge map as $t \to \infty$; this in turn suggests that the number of iterations to convergence for the finite-scale domain decomposition grows linearly with the problem size.

Our work thus provides theoretical support for the fast experimental convergence observed in the domain decomposition algorithm. Heuristically it also explains its good performance when in combination with a multi-scale scheme, as done in [1].

References

[1] M. Bonafini, B. Schmitzer (2021). *Domain decomposition for entropy regularized optimal transport*, Numerische Mathematik, Volume **149**, 819–870.

[2] M. Bonafini, I. Medina, B. Schmitzer (2021). *Asymptotic analysis of domain decomposition for optimal transport*, arXiv:2106.08084.

Solving Semi-Linear Elliptic Optimal Control Problems with L^1-Cost via Regularization and RAS-Preconditioned Newton

Gabriele Ciaramella[1], Michael Kartmann[2], and Georg Müller[3]*

[1]Polytechnic University of Milan

[2]University of Konstanz

[3]University of Heidelberg

gabriele.ciaramella@polimi.it, michael.kartmann@uni-konstanz.de, georg.mueller@uni-heidelberg.de(*corresponding author)

This presentation is concerned with the numerical solution of a class of optimal control problems governed by semi-linear elliptic partial differential equations (PDEs) and characterized by an L^2/L^1-regularized cost functional. To tackle the non-linearity and non-smoothness of the problem, we propose an efficient computational framework by combining regularization of the non-differentiable projection operator appearing in the optimality system and the application of preconditioned Newton methods. Two preconditioning approaches based on the Restricted Additive Schwarz method (RAS) are considered. In the first approach, RAS is used as a preconditioner for the Newton linear system. In the second approach, a RAS Preconditioned Newton method (RASPEN) is applied directly to the (regularized) nonlinear optimality system.

FGP2022 on Optimization: Inexact Proximal Newton methods in Hilbert Spaces

Bastian Pötzl[1]*, Anton Schiela[1], and Patrick Jaap[2]

[1]Universität Bayreuth

[2]TU Dresden

bastian.poetzl@uni-bayreuth.de (*corresponding author), anton.schiela@uni-bayreuth.de, patrick.jaap@tu-dresden.de

In the present work we extend the idea of second order semi-smooth Proximal Newton methods as presented in [1] to admit an inexact computation of update steps by solving the respective subproblem only up to prescribed accuracy. We consider the composite minimization problem

$$\min_{x\in X} F(x) := f(x) + g(x)$$

on some real Hilbert space $(X, \langle\cdot,\cdot\rangle_X)$ where $f : X \to \mathbb{R}$ is assumed to be smooth in some adequate sense and $g : X \to \mathbb{R}$ is possibly not.

Update steps for iteratively solving such optimization problems can be computed by solving the regularized second order subproblem

$$\Delta x(\omega) := \operatorname*{argmin}_{\delta x\in X} f'(x)\delta x + \frac{1}{2}H_x(\delta x, \delta x) + \frac{\omega}{2}\|\delta x\|_X^2 + g(x+\delta x) - g(x)$$

constituting a Proximal Newton method. Exactly computing such update steps can become very costly for real world applications like elastoplasticity. Thus, we develop inexactness criteria for update step computation that are sufficiently easy to evaluate, help us preserve convergence properties of the exact case and reduce the computational effort significantly.

In existing literature, a gradient-based inexactness criterion for update step computation is used. In infinite dimensional space, the evaluation of the so-called gradient mapping for such criteria is as costly as computing the exact update step itself which is why we resort to a relative error criterion.

While this relative error criterion in particular ensures local fast convergence of our method, global convergence is secured by a Cauchy decrease criterion which uses cheaply computable so-called Cauchy decrease steps in order to measure functional value descent for the actual inexact update steps.

We tested these modifications of Proximal Newton methods for a simple model in function space.

Acknowledgments: This work was funded by the DFG SPP 1962: Non-smooth and Complementarity-based Distributed Parameter Systems – Simulation and Hierarchical Optimization; Project number: SCHI 1379/6-1

References

[1] BP, Anton Schiela and Patrick Jaap. (2021). *Second order semi-smooth Proximal Newton methods in Hilbert Spaces*, arxiv.org, arXiv:2103.14344v2.

Dimension reduction of dynamic superresolution

Martin Holler[1], **Alexander Schlüter**[2]*, and Benedikt Wirth[2]

[1]Institute of Mathematics and Scientific Computing, University of Graz

[2]Institute for Applied Mathematics, University of Münster

martin.holler@uni-graz.at, alex.schlueter@wwu.de (*corresponding author), benedikt.wirth@uni-muenster.de

Stars in the sky or cells in the blood stream: many imaging problems consist of moving point sources imaged over multiple frames. The central question is how to resolve point locations and velocities from images where fine scale information is lost, e.g. due to the diffraction of light at the aperture of an optical instrument, and how to efficiently combine information from multiple frames.

We build on a model by Alberti et al. [1], who proposed to solve a convex optimization problem over the space of Radon measures, where point sources are represented by linear combinations of Dirac measures. To incorporate dynamic information, these measures live in phase space, the space combining positions and velocities, which results in a high problem dimensionality and makes numerical solutions challenging. We introduce a novel dimension reduction technique based on projections of phase space onto lower-dimensional subspaces, which reduces the problem dimension from $2d$ to $d+1$, where d is the space dimension. Indeed, we prove that known exact reconstruction results stay true after dimension reduction, and we additionally prove new error estimates for reconstructions from noisy data in optimal transport metrics which are of the same quality as one would obtain in the non-dimension-reduced case.

Acknowledgments: The work is supported by the European Fund for Regional Development (ERDF) within the project TRACAR: Non-invasive Imaging of Pharmacokinetics and Optimization of CAR T-Cell Therapies for Solid Tumors. It was further supported by the Deutsche Forschungsgemeinschaft (DFG) under Germany's Excellence Strategy EXC 2044 – 390685587, Mathematics Münster: Dynamics-Geometry-Structure, and by the Alfried Krupp Prize for Young University Teachers awarded by the Alfried Krupp von Bohlen und Halbach-Stiftung. MH acknowledges support by the Austrian Science Fund (FWF) (Grant J 4112).

References

[1] G. S. Alberti, H. Ammari, F. Romero, and T. Wintz (2019) *Dynamic spike superresolution and applications to ultrafast ultrasound imaging*, SIAM Journal on Imaging Sciences, 12(3):1501– 1527.

On simultaneous mesh and shape optimization in VI

Volker Schulz[1]* and Daniel Luft[1]

[1]Trier University, Germany

volker.schulz@uni-trier.de (*corresponding author), luft@uni-trier.de

Computational meshes arising from shape optimization routines commonly suffer from decrease of mesh quality or even destruction of the mesh. In this talk, we provide an approach to regularize shape optimization problems constrained by PDE and VI, in order to increase both shape and volume mesh quality. For this, we employ pre-shape calculus. Existence of regularized solutions is guaranteed. Further, consistency of modified pre-shape gradient systems is established. We present pre-shape gradient system modifications, which permit simultaneous shape optimization with mesh quality improvement. Optimal shapes to the original problem are left invariant under regularization. The computational burden of our approach is limited, since additional solution of possibly larger (non-)linear systems for regularized shape gradients is not necessary. The efficiency and efficacy of the techniques are illustrated in numerical results.

Acknowledgments: This work has been supported by BMBF (Bundesminis- terium für Bildung und Forschung) within the collaborative project GIVEN (FKZ: 05M18UTA). Further, the authors acknowledge the support of the DFG research training group 2126 on algorithmic optimization.

References

[1] Author. (year). *Title of the book*, publisher, City, initial page–final page.

[2] Author (year) *Title of the paper*, Journal, Volume **issue**, initial page–final page.

On a semismooth conjugate gradient method

Andrea Walther and Franz Bethke

Institut für Mathematik, Humboldt-Universität zu Berlin

andrea.walther@math.hu-berlin.de (*corresponding author), bethke@math.hu-berlin.de

In machine learning and other large scale applications, nowadays deterministic and stochastic variants of the steepest descent method are widely used for the minimization of objectives that are only piecewise smooth. As alternative, in this talk we present a deterministic descent method based on the generalization of rescaled conjugate gradients proposed by Phil Wolfe in 1975 for objectives that are convex. Without this assumption the new method exploits semismoothness to obtain conjugate pairs of generalized gradients such that it can only converge to Clarke stationary points. In addition to the theoretical analysis, we present preliminary numerical results.

Acknowledgments: This work is dedicated to the memory of Andreas Griewank

FGP2022 on Optimization: Geometry Segmentation with Total Variation Regularization

Manuel Weiß[1]*, Prof. Dr. Roland Herzog[1], Prof. Dr. Ronny Bergmann[2], Dr. Stephan Schmidt[3], Lukas Baumgärtner [3] and Dr. José Vidal Núñez [4]

[1] Interdisciplinary Center for Scientific Computing (Heidelberg)

[2] Norges teknisk-naturvitenskapelige universitet (NTNU Trondheim)

[3] Humboldt-Universität zu Berlin

[4] Universidad de Alcalá de Henares

manuel.weiss@iwr.uni-heidelberg.de (*), roland.herzog@iwr.uni-heidelberg.de, ronny.bergmann@ntnu.no, s.schmidt@hu-berlin.de, lukas.baumgaertner@hu-berlin.de, j.vidal@uah.es

The total variation has proven as a useful regularizer for various applications in Inverse imaging and Shape optimization problems. For the task of shape segmentation, we propose a model that combines normal vector data of a discrete surface and a total variation penalty and present suitable solvers.

Acknowledgments: This work has been funded by DFG SPP 1962, project P06: A Calculus for Non-Smooth Shape Optimization with Applications to Geometric Inverse Problems.

Control in the coefficients of the obstacle problem

Ira Neitzel[1]*, Nicolai Simon[2], and Winnifried Wollner[2]*

[1]Universität Bonn

[2]Technische Universität Darmstadt

neitzel@ins.uni-bonn.de, nsimon@mathematik.tu-darmstadt.de, wollner@mathematik.tu-darmstadt.de (*corresponding author)

Within this talk, we will discuss the optimal control of an obstacle problem by the choice of the coefficients.

$$\min J(q,u) = j(u) + \frac{\alpha}{2}\|q\|^2$$
$$\text{s.t.} \begin{cases} (q\nabla u, \nabla(v-u)) \geq (f, v-u) & \forall v \in K, \\ u \in K, & q \in Q^{\text{ad}}, \end{cases}$$

where $K \subset H_0^1(\Omega)$ and $Q^{\text{ad}} \subset L^2(\Omega;\mathbb{R}^{d\times d}_{\text{sym}})$ are suitable closed and convex subsets To cope with the product in the main part of the operator, tools from H-convergence, see, e.g., [1]. Further, we discuss first order optimality conditions obtained by extending a limiting approach for the control of the obstacle problem by the load f, see, e.g., [2, 3].

Acknowledgments: This work has been supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Projektnummer 314067056

References

[1] F. Murat, L. Tartar (1995) *H-convergence*, in *Topics in the mathematical modelling of composite materials*, 31, Birkhäuser, 21–43.

[2] K. Kunisch, D. Wachsmuth (2012), *Sufficient optimality conditions and semi-smooth Newton methods for optimal control of stationary variational inequalities*, ESAIM Control Optim. Calc. Var., 18 **2**, 520–547.

[3] C. Meyer, A. Rademacher, W. Wollner (2015) *Adaptive optimal control of the obstacle problem*, SIAM J. Sci. Comput., 37 **2**, A918–A945.

Minisymposium MS06

MS06 - Recent developments and new challenges in Derivative-Free Optimization

Derivative-free separable quadratic modeling and cubic regularization for unconstrained optimization

R. Garmanjani[1]*, A. L. Custódio[1], and M. Raydan[1]

[1] NOVA School of Science and Technology – CMA

r.garmanjani@fct.unl.pt *, alcustodio@fct.unl.pt, m.raydan@fct.unl.pt

Abstract

We present a derivative-free separable quadratic modeling and cubic regularization technique for solving smooth unconstrained minimization problems. The derivative-free approach is mainly concerned with building a quadratic model that could be generated by numerical interpolation or using a minimum Frobenious norm approach, when the number of points available does not allow to build a complete quadratic model. This model plays a key role to generate an approximated gradient vector and Hessian matrix of the objective function at every iteration. We add a specialized cubic regularization strategy to minimize the quadratic model at each iteration, that makes use of separability. We discuss convergence results, including worst case complexity, of the proposed schemes to first-order stationary points. Some preliminary numerical results are presented to illustrate the robustness of the specialized separable cubic algorithm.

Key words. Derivative-free optimization, fully-linear models, fully-quadratic models, cubic regularization, worst-case complexity.

Acknowledgments: This research was financially supported by Fundação para a Ciência e a Tecnologia (FCT) (Portuguese Foundation for Science and Technology) through the projects UIDB/MAT/00297/2020 (Centro de Matemática e Aplicações), PTDC/MAT-APL/28400/2017, and CEECIND/02211/2017.

A derivative-free method for convex constrained minimization

E. H. M. Krulikovski[1]*, A. L. Custódio[1], and M. Raydan[1]

[1] NOVA School of Science and Technology – CMA

e.krulikovski@fct.unl.pt *, alcustodio@fct.unl.pt, m.raydan@fct.unl.pt

Abstract

We present a new derivative-free optimization (DFO) approach for solving convex constrained minimization problems. We assume that the feasible set is the intersection of a finite collection of convex sets and, in order to comply with a wide variety of applications, we also assume that the projection onto each of the individual convex sets is relatively simple and inexpensive to compute, like is the case with boxes, spheres, half-spaces, or hyper-planes. Our iterative approach makes use of a Directional Direct Search (DDS) scheme, considering adequate positive basis as poll directions, until some local conditions hold, guaranteeing the quality of simplex gradient vectors as approximations to the real gradient vectors (which are not required). When those conditions hold, a Spectral Projected Gradient (SPG) scheme is activated in which the computed simplex gradient is used, instead of the real gradient, to maintain a DFO approach. Concerning the SPG iterations, if the convex feasible set is a simple set, then a direct projection is computed. If the feasible set is the intersection of finitely many convex sets, then Dykstra's alternating scheme is applied to obtain the required projection. Under standard assumptions, usually associated to derivative-free optimization methods, we prove the convergence of our combined scheme towards constrained stationary points.

Preliminary numerical results, on some well-known test functions, are presented to illustrate the performance of our combined scheme, when compared to the one that only uses the DDS method. Our results indicate that combining the method DDS with the methods SPG and Dykstra is a robust and effective approach for derivative-free convex constrained optimization.

Key words. Derivative-free Optimization · Convex constrained optimization · Projected gradient · Directional Direct Search · Simplex gradient · Dykstra's algorithm

Acknowledgments: This research was financially supported by Fundação para a Ciência e a Tecnologia (FCT) (Portuguese Foundation for Science and Technology) through the projects UIDB/MAT/00297/2020 (Centro de Matemática e Aplicações), PTDC/MAT-APL/28400/2017, and CEECIND/02211/2017. The first author is a postdoctoral researcher financially supported by FCT under the contract UI/297/2020 - 5/2021.

A Trust-region Approach for Computing Pareto Fronts in Multiobjective Optimization

Aboozar Mohammadi [1]*, Ana Luísa Custódio [1]

[1]NOVA School of Science and Technology – CMA

a.mohammadi@campus.fct.unl.pt*, alcustodio@fct.unl.pt

Multiobjective optimization is a challenging optimization area due to the existence of conflicting objectives that need to be simultaneously optimized. Rarely, the solution of a multiobjective optimization problem is a single point. Instead, when solving a multiobjective optimization problem, the goal is to compute the set of nondominated points, commonly known as the Pareto front.

However, the majority of the algorithms proposed in the literature, with a well supported convergence analysis, generate a sequence of iterates converging to a single Pareto point. Different initializations can be considered, in an attempt of capturing the complete Pareto front, but there are no guarantees of success.

In this work, we propose an algorithm based on a trust-region approach for computing an approximation to the complete Pareto front of a given multiobjective optimization problem. The algorithm alternates between two main steps, the extreme point step and the scalarization step, until appropriate stopping criteria are satisfied. The goal of the extreme point step is to expand the approximation to the Pareto front, by moving towards the extreme points of it, corresponding to the individual minimization of each of the objective function components. The scalarization step attempts to reduce the gaps on the Pareto front, by solving an adequate scalarization problem. In any of these two steps, models are built to replace the different components of the objective function, and minimized under a trust-region framework. Different approaches can be taken to build these models, depending of being on a derivative-based or a derivative-free setting.

A detailed description of the algorithmic structure will be provided, as well as some related theoretical results. Numerical experiments will illustrate the good performance of the method on some academic test problems.

Acknowledgments: This work has been supported by national funds through FCT, Fundação para a Ciência e a Tecnologia I. P., under the scope of projects UIDB/MAT/00297/2020, PTDC/MAT-APL/28400/2017, and 2020.08249.BD.

A Direct Multisearch Filter Method for Biobjective Optimization

Everton J. Silva[1]* and Ana L. Custódio[1]

[1] NOVA School of Science and Technology – CMA

ejo.silva@campus.fct.unl.pt *, alcustodio@fct.unl.pt

Abstract

In practical applications, it is common to have several conflicting objective functions to optimize. Frequently, these functions are of black-box type, preventing the use of multiobjective derivative-based optimization techniques. Direct Multisearch (DMS) is a multiobjective derivative-free optimization class of methods, with a well-established convergence analysis and competitive computational implementations, often successfully used for benchmark of new algorithms and in practical applications.

From the theoretical point of view, DMS is developed for continuous optimization with general constraints, making use of an extreme barrier approach, only evaluating feasible points. In this work, we propose the integration of a filter approach in DMS, to address biobjective optimization problems with linear and nonlinear constraints. The linear constraints are explicitly treated by the algorithm, by adapting the positive generating sets considered at each iteration to the geometry of the nearby constraints. The violations of the nonlinear constraints are aggregated in a third objective function and are treated as an additional objective to be minimized.

We will describe the proposed algorithmic structure in detail, provide results on the theoretical properties of the method, and report numerical experiments that state the good performance of this approach to address nonlinear constraints, both in an academic test set and in a real application.

Key words. Derivative-free Optimization · Multiobjective Optimization · Constrained Optimization · Direct Multisearch · Filter methods

Acknowledgments: This research was financially supported by Fundação para a Ciência e a Tecnologia (FCT) (Portuguese Foundation for Science and Technology) through projects UIDB/MAT/00297/2020 (Centro de Matemática e Aplicações), UI/BD/151246/2021 (CMA), and PTDC/MAT-APL/28400/2017 (CMA).

Minisymposium MS07

MS07 - Internet of Manufacturing Things

A bi-objective mixed integer linear programming model to optimize demand response to time-of-use prices in the electricity retail market

Carlos Henggeler Antunes [1,3]*, Maria João Alves[2,3]

[1]Department of Electrical and Computer Engineering, University of Coimbra, Portugal

[2]CeBER and Faculty of Economics, University of Coimbra, Portugal

[3]INESC Coimbra, Portugal

ch@deec.uc.pt (*corresponding author), mjalves@fe.uc.pt

Time-of-use energy prices are expected to become a common tariff scheme in smart grids, which can bring benefits for grid operators (contributing to alleviate congestion in distribution networks), retailers (enabling to manage wholesale buying and retail selling prices) and consumers (engaging in demand response actions to reduce the energy bill).

Consumers' energy behaviors adapt to these time-differentiated tariff schemes by resetting temperature setpoints or rescheduling appliance operation to profit from lower priced periods, thus balancing the cost and comfort evaluation dimensions. This work presents a bi-objective mixed integer linear programming model to optimize demand response in face of time-of-use energy prices, considering the minimization of energy costs and the minimization of the discomfort associated with changes regarding most preferred operation settings or time slots. This type of models and methods to compute nondominated solutions displaying different trade-offs between the economic and comfort dimensions will be at the core of energy management systems running 24/7 on computers with mild processing requirements, which should be parameterized with the consumer's preferences.

The physical accuracy of appliance operation and control, as well as fine grain time discretization, impose a significant computational burden due to the model combinatorial nature. Results obtained with a commercial solver are presented and the difficulties of developing customized meta-heuristic approaches to obtain good quality solutions in an acceptable computational time are discussed.

Acknowledgments: This work has been supported by FCT – Fundação para a Ciência e a Tecnologia, I.P., Projects MAnAGER (POCI-01-0145-FEDER-028040), UIBD/00308/2020 and UIDB/05037/2020.

Optimization of Joint Scheduling Speed Adjustable Machines and Vehicles in Job Shops

João M.R.C. Fernandes[1,2], S. Mahdi Homayouni[2], and Dalila B.M.M. Fontes[2,3]*

[1] Faculdade de Engenharia da Universidade do Porto

[2] LIAAD, INESC TEC, 4200-465 Porto

[3] Faculdade de Economia do Universidade do Porto

joaomiguelrcf@gmail.com, smh@inesctec.pt, fontes@fep.up.pt(*corresponding author)

This work addresses the joint scheduling of speed adjustable machines where the jobs are processed on and vehicles that transport the jobs around the shop floor. It extends the Job shop Scheduling Problem (JSP) [1] and the JSP with Transport resources (JSPT) [2] since in addition to determining, simultaneously, the sequence of the production operations for each machine, the allocation of the transport tasks to vehicles, and the sequence of the transport tasks for each vehicle, one also needs to decide on the processing speed of each production operation and the traveling speed of each task for the empty and for the loaded legs. The most common objective function for the JSP and the JSPT used to be makespan. However, due to extreme weather conditions, stricter environmental regulations, and volatile energy prices energy-efficient scheduling has become a new trend in industry and academia. Particularly, speed adjusting of machines has been extensively practiced to achieve energy-efficient scheduling. The possibility of adjusting the resources working speed is an immediate result of the recent advancements in the Internet of Manufacturing Things (IoMT) in the context of Industry 4.0. At a higher working speed the processing time decreases at the cost of a higher power consumption and vice versa. By adjusting the working speed of some resources one may reduce resources idle time as well as energy consumption without impacting productivity. Recently the energy-efficient JSPT (EEJSPT) under speed adjustment strategy for both machines and vehicles has been considered for the first time in [3] since it attempts to provide the same level of service while consuming less energy. Here, we extend it by proposing a non-dominated sorting genetic algorithm (NSGA-II) that finds effective solutions to the problem efficiently. We also provide a Pareto front analysis to infer on the advantages of considering speed adjusting machines and vehicles.

Acknowledgments: This work has been supported by FEDER/COMPETE2020/POCI/ MCTES/FCT through grants PTDC/EEI-AUT/2933/2014, POCI-01-0145-FEDER-031821, and PTDC/EEI-AUT/31447/2017.

References

[1] M. R. Garey, D. S. Johnson, and R. Sethi, "The complexity of flowshop and jobshop scheduling," *Mathematics of Operations Research*, vol. 1, no. 2, pp. 117–129, 1976.

[2] D. B. M. M. Fontes and S. M. Homayouni, "Joint production and transportation scheduling in flexible manufacturing systems," *Journal of Global Optimization*, vol. 74, no. 4, pp. 879–908, 2019.

[3] S. M. Homayouni and D. B. M. M. Fontes, "A milp model for energy-efficient job shop scheduling problem and transport resources," in *IFIP International Conference on Advances in Production Management Systems*, pp. 378–386, Springer, 2021.

Sustainable Scheduling of Single Machine and Preventive Maintenance Activities

S. Mahdi Homayouni[1] and Dalila B.M.M. Fontes[1,2]*

[1] LIAAD, INESC TEC, 4200-465 Porto

[2] Faculdade de Economia do Universidade do Porto

smh@inesctec.pt, fontes@fep.up.pt(*corresponding author)

This work addresses sustainable scheduling of manufacturing operations and preventive maintenance (PM) activities in a multi-state speed adjustable machine which requires a setup in between processing any pair of jobs/PM activities that its time depends on both preceding and subsequent jobs (i.e., sequence-dependent-setup-times are required). The higher the speed, the lower the processing time though, the higher the energy consumption and thus higher produced emissions. Decisions involve sequencing and timing of production processes jointly with PM activities, determining the processing speed of the machine for each operation, and allowing the machine to switch off when it is not needed for a relatively long time. We consider a tri-objective optimization problem; i.e. minimization of total weighted earliness/ tardiness (economic pillar), total energy consumption by machine (environmental pillar), and the number of undesired activities (social pillar). Social objectives are scarce in scheduling problems since they are believed to be hardly improved via scheduling decisions [1]. Here, we propose a novel social objective since PM activities and setups are considered as undesired activities which are preferred to be performed in morning and afternoon shifts (since they require substantial attention of the collaborators in night shifts). It is easy to see the link between early/tardy scheduling problems and speed adjustment strategy since by controlling the processing speed earliness and tardiness could be decreased. Although a few works researched joint scheduling of jobs/PM activities in single machine environments [2], sustainability objectives are yet to be considered. Here, we propose a sequence-based mathematical formulation of the problem where the PM activities are scheduled in predetermined time windows and the machine is turned "off" during the PM activities and also during machine idle time to save energy; though, turning it "on" needs large energy consumption. The decision variables are defined to determine the sequence of jobs/PM activities on the machine, the processing speed of each job, and whether the machine is switched "off" after processing a job. Constraints ensure that PM activities are performed in their allowed time window and identify whether jobs/PM activities are undesired. The model can be solved to optimality for small-sized problem instances.

Acknowledgments: This work has been supported by FEDER/COMPETE2020/POCI/ MCTES/FCT through grants PTDC/EEI-AUT/2933/2014, POCI-01-0145-FEDER-031821, and PTDC/EEI-AUT/31447/2017.

References

[1] M. Akbar and T. Irohara, "Scheduling for sustainable manufacturing: A review," *J Clean Prod*, vol. 205, pp. 866–883, 2018.

[2] I. H. Sin and B. Do Chung, "Bi-objective optimization approach for energy aware scheduling considering electricity cost and preventive maintenance using genetic algorithm," *Journal of Cleaner Production*, vol. 244, p. 118869, 2020.

Minisymposium MS08

MS08 - Optimization in distributed settings

Distributed real-time learning with safety guarantees in multi-agent systems

A. Pedro Aguiar

Faculty of Engineering, University of Porto

pedro.aguiar@fe.up.pt

In this talk I will address the distributed real-time learning using mobile multi-agent systems with safety guarantees. The presentation will focus on a recent methodology that combines distributed optimization, control barrier functions, and nonlinear Lyapunov based tools yielding powerful frameworks with formal guarantees of stability, optimality, performance, and safety. Illustrative examples in the area of motion control of multi-agent systems will be presented.

Acknowledgments: This work has been supported in part by projects: IMPROVE (POCI-01-0145-FEDER-031823), funded by FEDER funds through COMPETE2020 - POCI and by the Portuguese national FCT/MCTES (PIDDAC); RELIABLE (PTDC/EEI-AUT/3522/2020) funded by FCT/MCTES; and DynamiCITY (NORTE-01-0145-FEDER-000073), funded by NORTE2020/ PORTUGAL2020, through the European Regional Development Fund.

Distributed Banach-Picard Iteration for Locally Contractive Maps

Francisco Andrade[1,2], Mário A. T. Figueiredo[1,2], and João Xavier[1,3]

[1] *Instituto Superior Técnico, Lisboa, Portugal*
[2] *Instituto de Telecomunicações, Lisboa, Portugal*
[3] *Institute for Systems and Robotics*

The Banach-Picard iteration is widely used to find fixed points of locally contractive (LC) maps. In this presentation, we show how to extend the Banach-Picard iteration to distributed settings; specifically, we assume the map of which the fixed point is sought to be the average of individual (not necessarily LC) maps held by a set of agents linked by a communication network. An additional difficulty is that the LC map is not assumed to come from an underlying optimization problem, which prevents exploiting strong global properties such as convexity or Lipschitzianity. Yet, we present a distributed algorithm (DBPI- distributed Banach-Picard iteration) that maintains the linear rate of the standard Banach-Picard iteration for the average LC map. As an application we derive two distributed algorithms - distributed EM and Distributed PCA - whose local linear convergence guarantees follow from those of the DBPI.

Robust Target Localization via Linear-Fractional Representations

João Domingos[1]*, Cláudia Soares[2], and João Xavier[1]

[1] ISR- Instituto Superior Técnico

[2]NOVA LINCS, Computer Science Department, NOVA School of Science and Technology, Universidade NOVA de Lisboa

oliveira.domingos@tecnico.ulisboa.pt (*corresponding author), claudia.soares@fct.unl.pt, jxavier@isr.tecnico.ulisboa.pt

In this presentation we consider the problem of delimiting the region of possible target locations, given correlated range measurements with bounded error. This region is of primary interest in robust statistics since it is a tight majorizer of the set of Maximum-Likelihood (ML) estimates with arbitrary noise densities. We construct a superset of the aforementioned region through convex relaxations that use Linear Fractional Representations (LFRs), a well-known technique in robust control. We compare our method with a standard semidefinite relaxation and verify that the supersets created by our method are tighter than the benchmark ones, being about 20% smaller in size (area). Furthermore, our method tends to be actually tight since, empirically, the size of the supersets is, on median terms, within a 3% margin of a lower bound computed via a naive grid search.

Acknowledgments: This work has been supported by FCT, Fundação para a Ciência e a Tecnologia, under the projects 1801P.00763.1.01 HARMONY, PD/BD/150631/2020 and NOVA LINCS (UIDB/04516/2020).

Distributed Inverse Optimal Control

João M. Lemos[1,2]*, João P. Belfo[1], and A. Pedro Aguiar[2]

[1]INESC-ID/IST/Univ. Lisboa

[2]SYSTEC/FEUP

jlml@inesc-id.pt (*corresponding author), pedro.belfo@gmail.com, pedro.aguiar@fe.up.pt

Game theory provides a way of designing distributed optimal controllers for interacting agents (plants) in a network, either by using indirect or direct (model predictive control) methods [1]. For the linear quadratic problem, it is possible to obtain convergence results for the coordination procedure among agents based on a non-cooperative game [2]. In this presentation, it is shown how to generalize these results to nonlinear plants using techniques from inverse optimal control. Using arguments based on Lyapunov stability and Input-to-State Stability (ISS) theories, it is shown how to obtain conditions under which the proposed game converges to a fixed point and the overall multi-agent system is ISS with respect to the disturbance signals. Further details for the nonlinear case are given in [3]. The potential applications are wide including the control of water delivery canals [2], coordinated control of steam flow and pressure in large-scale boilers, coordinated joint control in biped locomotion, or coupled mechanical systems motion coordination.

Acknowledgments: This work has been supported by national funds through FCT, Fundação para a Ciência e a Tecnologia, under project UIDB/50021/2020

References

[1] J. M. Igreja, J. M. Lemos, F. M. Cadete, L. M. Rato and M. Rijo (2012). *Control of a water delivery canal with cooperative distributed MPC*. American Control Conference, Fairmont Queen Elizabeth, Montréal, Canada, June 27-June 29, 2012, 3346-3351.

[2] J. M. Lemos and L. F. Pinto (2012) *Distributed Linear-Quadratic Control of Serially Chained Systems – Application to a Water Delivery canal*, IEEE Control Systems Mag., Volume **32(6)**, 2096-2101.

[3] J. P. Belfo, A. Pedro Aguiar, J. M. Lemos (2021) *Distributed Inverse Optimal Control for Discrete-Time Nonlinear Multi-Agent Systems*, IEEE Control Systems Letters, Volume **5(6)**, 5(6): 2096-2101.

Minisymposium MS09

MS09 - New trends in optimal control and sweeping process with lack of regularity

On the asymptotic behaviour of the value function in optimal control problems

Veljko Ašković[1]*, Emmanuel Trélat [1], Hasnaa Zidani [2]

[1] Jacques Louis Lions Laboratory, Sorbonne University France

[2] INSA Rouen Normandie, LMI laboratory

veljkoaskovic@hotmail.com, emmanuel.trelat@sorbonne-universite.fr, hasnaa.zidani@insa-rouen.fr

The turnpike phenomenon stipulates that the solution of an optimal control problem in large time, remains essentially close to a steady-state of the dynamics, itself being the optimal solution of an associated static optimal control problem (see [2]). We use this property to propose an asymptotic expansion of the value function as the horizon T is large. Firstly, we prove the result in the linear quadratic case, using the Hamiltonian structure of the Pontryagin Maximum Principle extremal equations and some basic results of LQ theory. Then, based on some results obtained in [3], the result is generalized to a large class of nonlinear systems provided they satisfy the (strict) dissipativity property.

References

[1] Anderson, Kokotović (1987) *Optimal control problems over large time intervals*, Automatica, J.IFAC **23**, 355-363.

[2] E. Trélat E. Zuazua, (2015) *The turnpike property in finite-dimensional nonlinear optimal control*, Journal of Differential Equations **258**, 81–114.

[3] E. Trélat (2020) *Linear turnpike theorem*, Preprint HAL, 25 pages

Hybrid maximum principle with regional switching parameter

Térence Bayen[1]*, Anas Bouali[1], and Loïc Bourdin[2]

[1]Avignon Université, Laboratoire de Mathéematiques d'Avignon (EA 2151) F-84018

[2]Institut de recherche XLIM. UMR CNRS 7252. Université de Limoges

terence.bayen@univ-avignon.fr (*corresponding author), anas.bouali@univ-avignon.fr, loic.bourdin@unilim.fr

In this presentation, we consider a Mayer optimal control problem where the controlled system is defined over a partition of the euclidean space, and we assume that the dynamics depends on some additional regional switching parameter. This means that the parameter should remain constant over every stratum (but not necessarily constant over the time period). This framework is motivated by several applications arising in the context of aerospace engineering or in epidemiology (typically when a loss of control occurs). Our objective is to derive the necessary optimality conditions, in the form of a Pontryagin' principle. We shall see in this presentation how to obtain such conditions thanks to regional needle perturbations and to a careful sensitivity analysis in this hybrid setting. Furthermore, we present an application of this framework in the case of non permanent control (typically when a control loss occurs on certain regions).

References

[1] T. Bayen, A. Bouali, and L. Bourdin. Hybrid maximum principle with regional switching parameter. *preprint HAL*, 2022.

[2] Térence Bayen, Anas Bouali, and Loïc Bourdin. Optimal control problems with non-control regions: necessary optimality conditions. 2022.

LIE BRACKETS, STATE CONSTRAINTS, AND UNIQUENESS OF VISCOSITY SOLUTIONS OF HAMILTON-JACOBI PDE

GIOVANNI COLOMBO, NATHALIE T. KHALIL, AND FRANCO RAMPAZZO

ABSTRACT. The classical *inward pointing condition* (IPC) for a control system whose state x is constrained in the closure $C := \bar{\Omega}$ of an open set Ω prescribes that at each point of the boundary $x \in \partial\Omega$ the intersection between the dynamics and the interior of the tangent cone of $\bar{\Omega}$ at x be nonempty. Under this assumption, for every system trajectory $x(.)$ on a time-interval $[0,T]$, that possibly violates the constraint, one can construct a new system trajectory $\hat{x}(.)$ that satisfies the constraint and whose distance from $x(.)$ is bounded by a quantity proportional to the maximal deviation $d := \text{dist}(\Omega, x([0,T]))$. When (IPC) is violated, the construction of such a neighboring feasible trajectory is not possible in general, as simple counterexamples show. However, we prove that a "higher order" inward pointing condition involving Lie brackets of the dynamics' vector fields (together with a non-positiveness curvature-like assumption) and the implementation of a suitable "rotating" control strategy allows for a novel construction of a trajectory $\hat{x}(.)$ that satisfies the constraint and whose distance from the reference trajectory $x(.)$ is bounded by a quantity that is proportional to $\sqrt{d}$. As an application, we establish the continuity *up to the boundary* of the value function V of an optimal control problem subject to state constraints. This property allows to regard V as the unique *constrained* viscosity solution of the corresponding Bellman equation. This work generalizes a previous result by Monica Motta (SICON, 1995), that was the first to employ higher order inward pointing conditions.

(Giovanni Colombo) Università di Padova, Dipartimento di Matematica "Tullio Levi-Civita", via Trieste 63, 35121 Padova, Italy, and Istituto Nazionale di Alta Matematica "F. Severi", Unità di ricerca presso l'Università di Padova
Email address: colombo@math.unipd.it

(Nathalie T. Khalil) Research Center for Systems and Technologies SYSTEC, Faculty of Engineering, University of Porto, Rua Roberto Frias s/n 4200-465 Porto, Portugal
Email address: khalil.t.nathalie@gmail.com

(Franco Rampazzo) Università di Padova, Dipartimento di Matematica "Tullio Levi-Civita", via Trieste 63, 35121 Padova, Italy
Email address: rampazzo@math.unipd.it

Date: January 21, 2022.

Singular arc and non-smooth dynamics in optimal control: an example

O. Cots[1]*

[1]Université Toulouse, CNRS, ENSEEIHT-IRIT

olivier.cots@irit.fr (*corresponding author)

We consider an optimal control problem from irrigation application excerpted from [1] which has a non-smooth dynamics. The problem may be written in Mayer form with a control-system affine with respect to the control and with points of non differentiability with respect to the state. The peculiar feature is that the optimal solution contains a singular arc which appears at a non differentiable point (called corner point) of the dynamics. We present a direct shooting formulation to solve this optimal control problem when the structure is known *a priori* and present a regularization process combined with an homotopy technique to recover the fact that the singular arc is located at a corner point. The optimal control problem has several difficulties to consider:

1. The control appears linearly in the (extended) control-system so the optimal solution may be composed of bang arcs (such that $u(t) \in \{0, 1\}$) and singular arcs (such that $u(t) \in (0, 1)$).
2. The problem has a state constraint and so the optimal solution may also contain boundary arcs.
3. The problem dynamics has points of non differentiability and so we need a non-smooth version of the Pontryagin Maximum Principle [2] or a Maximum Principle for hybrid optimal control problems [4, 3].

The state space denoted M may be stratified with two strata of dimension 3 and one stratum at the interface, of dimension 2, denoted M^0. The main difficulty is due to the fact that the optimal trajectory stays on M^0 on a time interval of non-empty interior, where the dynamics is non differentiable. The optimal trajectory has a semi-transverse contact when reaching the stratum M^0 and so the associated time of contact is not a regular crossing time (as assumed in [3]).

Remark. The presented results are entirely reproducible online[1] without any installation, using the Binder[2] technology.

References

[1] K. Boumaza, N. Kalboussi, A. Rapaport, S. Roux and C. Sinfort. *Optimal control of a crop irrigation model under water scarcity*. 2020. hal-01991296.

[2] F. Clarke, *Functional Analysis*, Calculus of Variations and Optimal Control, Springer 2013.

[3] T. Haberkorn and E. Trélat, *Convergence results for smooth regularizations of hybrid nonlinear optimal control problems*, SIAM J. Control Optim., **49** (2011), no. 4, 1498-1522.

[4] H. J. Sussmann, *A nonsmooth hybrid maximum principle*, in Stability and Stabilization of Nonlinear Systems (Ghent, 1999), Lecture Notes in Control and Inform. Sci. 246, Springer, London, 1999, pp. 325–354.

[1]See the example (with same title) on the `ct` project gallery: `ct.gitlabpages.inria.fr/gallery`.
[2]`mybinder.org`

Optimal Control involving Sweeping Processes with End Point Constraints

Maria do Rosário de Pinho[1], M. Margarida A. Ferreira[1]*, and Georgi V. Smirnov[2]

[1]University of Porto, Faculty of Engineering, DEEC and SYSTEC, Porto, Portugal

[2]Universidade do Minho, Centre of Physics, Department of Mathematics, Braga, Portugal

mrpinho@fe.up.pt, mmf@fe.up.pt*, smirnov@math.uminho.pt

Introduced by J.J. Moreau in 1974 in the context of plasticity and friction theory, sweeping processes are evolution differential inclusions involving the normal cone to a set. From the beginning of this century, there has been a resurgence of interest in such systems due to their applications in mechanics, engineering, economics and crowd motion problems. Not surprisingly, attention to problems involving controlled sweeping systems have attracted the attention of many authors. These are systems defined by the differential inclusion

$$\begin{aligned} &\dot{x}(t) \in f(t, x(t), u(t)) - N_C(x(t)), \\ &x(0) \in C_0. \end{aligned} \tag{0.1}$$

where N_C is the normal cone to a set C and $u(t) \in U$ a.e.. Setting

$$F(t, x) = \{f(t, x, u) - N_C(x) : \ u \in U\},$$

the differential inclusion (0.1) can be rewritten as

$$\dot{x}(t) \in F(t, x(t)), \ x(0) \in C_0.$$

The remarkable feature of (0.1) is the presence of N_C since it destroys regularity conditions on F under which optimality conditions for optimal control problems are postulated.

We consider an optimal control problem involving a sweeping system and we generalize a previously obtained Maximum Principle for that problem ([2], [3]) to cover optimal control problems involving end point constraints and with $C = C(t)$. A new complementary condition is also added to the necessary conditions. This work is based on an ingenious smooth approximating family of problems.

Acknowledgments: This work was supported by the FEDER/COMPETE/NORTE2020/ POCI/FCT funds, through grants To CHAIR - POCI-01-0145-FEDER-028247, UPWIND-POCI-01-0145-FEDER-031447, UID/EEA/00147/2019-SYSTEC, and the Portuguese Foundation for Science and Technology in the framework of the Strategic Funding through grant UID/FIS/04650/2019.

References

[1] M. d. R. de Pinho, M. M. A. Ferreira and G. V. Smirnov (2019) *Optimal Control involving Sweeping Processes*, Set-Valued Var. Anal **27**, 523548.

[2] M. d. R. de Pinho, M. M. A. Ferreira and G. V. Smirnov (2019) *Correction to: Optimal Control Involving Sweeping Processes*, Set-Valued Var. Anal **27**, 1025–1027.

State-Constrained Sweeping Process Problems

Nathalie T. Khalil[1] and Fernando Lobo Pereira[1]

[1]SYSTEC, Department of Electrical and Computer Engineering, Porto University, and with the Institute for Systems and Robotics, 4200-465 Porto, Portugal

nathalie@fe.up.pt, flp@fe.up.pt

This talk concerns a Maximum Principle (MP) for a state-constrained optimal control problem with dynamics given by a sweeping process. The main novelty in this work is the additional ingredient, *a state constraint*, in the context of sweeping processes. More precisely, we are interested in the following problem:

$$\begin{aligned} &\text{Minimize} && g(x(1)) \\ &\text{subject to} && \dot{x}(t) \in f(t, x(t), u(t)) - N_C(x(t)) \;\; [0,1]\text{-a.e.} \\ &&& u(t) \in U, [0,1]\text{-a.e.} \\ &&& x(0) \in C_0 \subset C \\ &&& h(x(t)) \leq 0 \; \forall t \in [0,1]. \end{aligned} \tag{0.1}$$

Here, $N_A(z)$ is the limiting normal cone to the closed set A at point z in the sense of Mordukhovich. $U \subset \mathbb{R}^m$ is the control set, $C_0 \subset \mathbb{R}^n$ the endpoint constraint set, $g : \mathbb{R}^n \to \mathbb{R}$ the objective function, $f : [0,1] \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a vector-valued function, C is the sweeping set such that

$$C := \{x \in \mathbb{R}^n : \psi(x) \leq 0\} \tag{0.2}$$

where $\psi : \mathbb{R}^n \to \mathbb{R}$, such that $0 \in \text{int}\, C$ and $h : \mathbb{R}^n \to \mathbb{R}$ is the state constraint function.

To this problem, we derive the necessary optimality conditions in the Gamkrelidze's form which has the virtue of ensuring a much greater regularity of the measure multiplier. In fact, this multiplier is a monotonic function of bounded variation. We also show the relation with the MP in the Dubovistkii-Milyutin form.

Acknowledgments: The authors acknowledge the partial support of FCT R&D Unit SYSTEC - POCI-01-0145-FEDER-006933, and R& D Projects STRIDE - NORTE-01-0145-FEDER-000033, MAGIC - PTDC/POCI-01-0145-FEDER-032485, SNAP - NORTE-01-0145-FEDER-000085, HARMONY - NORTE-01-0145-FEDER-031411 funded by ERDF — COMPETE2020 — FCT/MEC — PT2020 — NORTE 2020, POCI and PIDDAC

References

[1] N.T. Khalil and F. Lobo Pereira (2022) *State-Constrained Sweeping Process Problems*, Submitted to IFAC-Paris 2022.

[2] MdR de Pinho, MMA Ferreira, and GV. Smirnov (2019) *Optimal control involving sweeping processes*, Set-Valued and Variational Analysis, 27 **2**, 523–548.

FGP2022 on Optimization: Goh and Legendre-Clebsch conditions for non-smooth optimal control problems

Franco Rampazzo*, Francesca Angrisani

Dipartimento di Matematica ”T.Levi-Civita”, Università di Padova

rampazzo@math.unipd.it, francesca.angrisani@unipd.it

For end-point constrained optimal control problems like

$$(P) \qquad \min_{u\in\mathcal{U}} \Psi(T, x(T)), \quad \begin{cases} \dfrac{dx}{dt} = f(x(t)) + \displaystyle\sum_{i=1}^{m} g_i(x(t))u^i(t), & \text{a.e. } t \in [0,T], \\ x(0) = \hat{x}, \qquad (T, x(T)) \in \mathfrak{T}, \end{cases}$$

where the cost Ψ and the vector fields $f, g_1, \ldots, g_m$ are Lipschitz continuous, various generalization of the original Maximum Principle (Pontryagin et al., 1962) have been produced in literature, possibly within different theoretical frameworks, starting from the pioneering works by F. Clarke in the Seventies up to recent papers. We present some ideas in the direction of adding some *higher order necessary conditions* to the the Maximum Principle. In particular, we will discuss how one can generalize the classical *Goh condition*

$$p(t) \cdot [g_i, g_j](x(t)) = 0, \qquad \forall t \in [0,T]$$

and the *Legendre-Clebsch condition* ($m = 1, g = g_1$),

$$p(t) \cdot [f, g](x(t)) = 0, \quad p(t) \cdot [g, [f, g]](x(t)) \leq 0 \qquad \forall t \in [0,T],$$

where $p(\cdot)$ is a solution of the adjoin tequation of the Maximum Principle. (For smooth vector fields X, Y, $[X, Y]$ is the Lie bracket, i.e. defined as $[X, Y] = DY \cdot X - DX \cdot Y$.)

I will describe some achievements from a work in progress with F. Angrisani (see[1]), where the data of the are nonsmooth. The main tools for the extensions of both Goh and Legendre-Clebsch condition consist in the notion of Quasi Differential Quotient ([3]) together with a set-valued generalization $[X, Y]_{set}$ of the Lie bracket introduced in [4] (see also [5][2]). In particular the Quasi Differential Quotient allows to treat within the same framework the two kinds of non-smothness affecting the problem ,namely the one concerning the adjoint inclusion and the one connected with the set-valued Lie brackets.

Acknowledgments: This work has been partially supported by the INdAM-GNAMPA Project 2020 “Extended control problems: gap, higher order conditions and Lyapunov functions” and by the Padua University grant SID 2018 “Controllability, stabilizability and infimum gaps for control systems”, BIRD 187147.

References

[1] Angrisani F., Rampazzo F. *Goh and Legendre-Clebsh conditions for nonsmooth control systems*, in preparation.

[2] Feleqi E., Rampazzo F., *Iterated Lie brackets for nonsmooth vector fields*, NoDEA Nonlinear Diff. Eq. Appl., vol. 24, no. 6, Art. 61, 2017.

[3] Palladino M., Rampazzo F.,*A geometrically based criterion to avoid infimum gaps in Optimal Control*, to appear in J.of Diff. Eqs.

[4] Rampazzo F., Sussmann H.J., *Set-valued differentials and a nonsmooth version of Chow-Rashevski's theorem*, Proceedings of the 40th IEEE Conference on Decision and Control, Orlando, Florida, (IEEE Publications, New York), vol. 3, pp. 2613-2618, 2001.

[5] Rampazzo F., Sussmann H.J., *Commutators of flow maps of nonsmooth vector fields*, J. Diff. Eq., vol. 232, no. 1, pp. 134-175, 2007.

Equivalent formulations of the min peak problem and applications

Emilio Molina[1,2,3], Alain Rapaport[1]*, and Hector Ramirez[2]

[1] MISTEA, Univ. Montpellier & INRAE, France

[2] DIM & CMM (AFB170001 - CNRS IRL2807), Santiago-de-Chile, Universidad de Chile,

[3] LJLL, Sorbonne Université & INRIA, France

emilio.molina@dim.uchile.cl, alain.rapaport@inrae.fr (*corresponding author), hramirez@dim.uchile.cl

We consider the optimal control problem which consists in minimizing the maximum over a time interval $[0,T]$ of a scalar function

$$\inf_{u(\cdot)} \max_{t\in[t_0,T]} y(t)$$

for a dynamics in $\mathbb{R}^{n+1}$ of the form

$$\left\{ \begin{array}{l} \dot{x} = f(x,y,u) \\ \dot{y} = g(x,y,u) \end{array} \right. \qquad u \in U \subset \mathbb{R}^p$$

This problem is not in the usual Mayer, Lagrange or Bolza forms of the optimal control theory, and thus does not allow to use directly numerical software based on direct or Hamilton-Jacobi Bellman methods.

We propose and discuss several reformulations of this problem in Mayer form

1. with state or mixed constraint , or
2. with upper semi-continuous differential inclusion without state constraint

We consider also a particular class of problems for which we are able to give an explicit optimal solution. This allows us to compare numerical solutions obtained for the reformulations that we propose on an example in this class of problems, and to show their potential merits as practical methods to determine optimal solution of L^∞ optimal control problems. This problem has been motivated by epidemiological questions, when one looks for minimizing the peak of an epidemic playing with restrictions as a control variable.

Acknowledgments: This work was partially supported by ANID-PFCHA/Doctorado Nacional/2018-21180348, FONDECYT grant 1201982 and Centro de Modelamiento Matemático (CMM), ACE210010 and FB210005, BASAL funds for center of excellence, three from ANID (Chile).

Necessary optimality conditions for multiprocesses with parameters

Abdallah Abdel Wahab[1] and Piernicola Bettiol[1]*

[1]Univ Brest, UMR CNRS 6205, Laboratoire de Mathématiques de Bretagne Atlantique, 6 Avenue Victor Le Gorgeu, Brest, 29200-F, France;

abed901@hotmail.com, piernicola.bettiol@univ-brest.fr(*corresponding author)

We consider a class of optimization problem involving multiprocesses (in the the sense of Clarke-Vinter SICON 1989), in which a parametrized family of control systems are linked with endpoint constraints and a minimax cost functional. For this class of problems we shall provide necessary optimality conditions.

References

[1] F. H. Clarke and R. B. Vinter. (1989). *Optimal Multiprocesses*, *Applications of Optimal Multiprocesses*, SIAM J. on Control and Optimization, Vol. 27, No. 5, pp. 1072–1091, 1048–1071.

[2] R. B. Vinter. (2005). *Minimax optimal control*, SIAM J. on Control and Optimization, Vol. 44, No. 3, pp. 939–968.

[3] F. H. Clarke. (2013), *Functional analysis, calculus of variations and optimal control*. Springer Science & Business Media, Vol. 264.

[4] R. B. Vinter. (2010). *Optimal control*. Birkhaüser, Boston.

Minisymposium MS10

MS10 - Optimization in forest management and wildfire

Optimizing drone base locations for assessing forest fires

Filipe Alvelos[1,2*], Eduardo Cunha[2], António Vieira[3], António Bento-Gonçalves[3], and Sarah Moura[3]

[1] Departamento de Produção e Sistemas, Universidade do Minho, Portugal

[2] Centro Algoritmi, Universidade do Minho, Portugal

[3] Centro de Estudos de Comunicação e Sociedade / Departamento de Geografia, Universidade do Minho, Portugal

falvelos@dps.uminho.pt (*corresponding author)

Forest fires are considered to be one of the main environmental problems in Portugal. Of particular concern is the high number of ignitions, which compromises the timely, adequate and informed response of the firefighting mechanism. The time of the first intervention is crucial for the success in extinguishing a fire outbreak, preventing it from turning into a major fire. To this end, it is essential to have credible and diversified data that can, in real time, provide fire fighting structures with the information they need to act with the greatest urgency, in a proportional and most effective way.

In order to solve some of the problems of validating information about the fire and providing updated information in real time to the fire-fighting structures, we propose the modelling and implementation of a network of suitably structured bases of drones (unmanned aerial vehicles) with the aim of responding to each emergency call and making an in-loco confirmation of the report made.

Besides being a very suitable mean of confirmation, the collection of images and sensor data will contribute to complement the information to be provided to the civil protection authorities. In the research hypothesis, there are topics such as the determination of a network of drone survey locations and its economic analysis in view of the benefit of reducing false alarms or obtaining additional data on the potential fire. The modelling of the network of drone bases will respond to the criteria defined for the identification of the areas with the highest risk of fire occurrence, the specific characteristics of the drones to be used in this structure and the conditions on the ground for the installation of the drone bases.

The problem's simplest version can be formulated by defining two sets: a set of potential location for drone bases (PLDB) and a set of potential ignition locations (PIL). The problem amounts to select the subset of PLDB such that the sum of the distance between each PIL and its nearest PLDB is minimum. Extensions include different drones with different autonomies and costs, other objectives and network design decisions (e.g. minimizing the distance between the selected bases). Mixed integer programming is at the core of the approaches to solve these optimization problems since it has been used successfully in many location [1] and location with network design [2] problems. We intend to validate the proposed approach with data from the municipality of Baião, Portugal.

Acknowledgments: This work is supported by FCT - Fundação para a Ciência e Tecnologia within the R&D Units Project Scope UIDB/00319/2020 and within project PCIF/GRF/0141/2019 "O3F - An Optimization Framework to reduce Forest Fire".

References

[1] Laporte, G., Nickel, S., & da Gama, F. S. (Eds.). (2019). Location science. Cham, Switzerland: Springer.

[2] Contreras, I., & Fernández, E. (2012). General network design: A unified view of combined location and network design problems. European Journal of Operational Research, 219(3), 680-697.

A Forest Fire Monitoring System Based on IoT Technology

Thadeu Brito[1,2,3]*, Beatriz Flamia Azevedo [1,4] João Mendes[1,4], Matheus Zorawski[1], Florbela Fernandes[1], Ana I. Pereira[1,4], and José Lima[1,2]

[1] Research Centre in Digitalization and Intelligent Robotics (CeDRI) - Instituto Politécnico de Bragança, Campus de Santa Apolónia, Bragança, Portugal
[2] INESC TEC - INESC Technology and Science, Porto, Portugal
[3] Faculty of Engineering of University of Porto, Porto, Portugal
4 Algoritmi Research Centre, University of Minho, Campus de Gualtar, Braga, Portugal
{brito(*corresponding author), beatrizflamia, matheuszorawski, joao.cmendes, fflor, apereira, jllima}@ipb.pt

Protecting forests is an emerging issue worldwide, given the enormous importance of these ecosystems to the planet. The possibilities and challenges in developing a robust surveillance system are innumerable. Besides, forest monitoring systems could be helpful to provide authorities support in the decision making. In this regard, the project Forest Alert Monitoring System (SAFe) proposes to develop innovative technologies to allow efficient forest monitoring based on Internet of Things (IoT) [1, 2]. The proposed system aims to improve nature monitoring systems through sensors in low-cost modules using Long Range (LoRa). The modules will be developed according to the necessities of each forest and scattered there to transmit data that will identify an early forest fire ignition. Indeed, the modules scattering needs to be optimized, otherwise this task will place many modules than is necessary (increasing the project's execution cost and the data acquisition). Therefore, soil recognition via satellite images can supply an algorithm that will determine the best position for individual sensor modules. In this way, the modules group will perform a Wireless Sensor Network (WSN) capable of acquiring critical ecological variables that are essential to avoid wildfires. The big data generated by WSN should be filtered individually inside the modules before the communication process, which implies the sample rate needs to be dynamic to taking advantage the whole bandwidth. After storing the data acquired in a cloud, the Artificial Intelligence (AI) system will predict regular situations to identify possible alerts based on the historical events and the last sensor reading (from the individual module, and his neighborhood). Only the combat fire authorities can access the data (raw and predicted by AI) through an online platform, which will display the geolocation with possible fire ignition.

Acknowledgments: This work has been supported by Fundação La Caixa and FCT — Fundação para a Ciência e Tecnologia within the Project Scope: UIDB/5757/2020. Thadeu Brito is supported by FCT Grant Reference SFRH/BD/08598/2020.

References

[1] Brito, T., Azevedo, B.F., Valente, A., Pereira, A.I., Lima, J., Costa, P. (2021) *Environment monitoring modules with fire detection capability based on iot methodology*, In: Paiva, S., Lopes, S.I., Zitouni, R., Gupta, N., Lopes, S.F., Yonezawa, T. (eds.) Science and Technologies for Smart Cities. pp. 211–227.

[2] Azevedo, B.F., Brito, T., Lima, J., Pereira, A.I. (2021) *Optimum sensors allocation for a forest fires monitoring system*, Forests, 12(4). https://doi.org/10.3390/f12040453, https://www.mdpi.com/1999-4907/12/4/453

A scheduling problem for fighting several fire fronts

Marina A. Matos[1]*, Ana Maria A. C. Rocha[1,2], Lino A. Costa[1,2] and Filipe Alvelos[1,2]

[1]ALGORITMI Center, University of Minho, 4710-057 Braga, Portugal

[2]Production and Systems Department, University of Minho, 4710-057 Braga, Portugal

mmatos@algoritmi.uminho.pt (*corresponding author), arocha@dps.uminho.pt, lac@dps.uminho.pt, falvelos@dps.uminho.pt

The number of forest fires has been increasing significantly causing great concern worldwide. Forest fires damage the ecosystem, leading to negative consequences for life, economy and the environment. A forest fire is a natural phenomenon that totally or partially destroys the ecosystem of a given area. Thus, its control in terms of time and space is crucial. In Portugal there was also an increase in forest fires. The year 2017 was considered a critical year, since there were several forest fires induced by human and natural causes.Firefighting is an important topic in optimization, since improvements in resources and quick actions are needed by professionals working in this area [1]. Firefighting depends not only on the resources involved, but also on the number, skills and level of preparedness of teams [2]. Resource management in forest firefighting can be modeled as a scheduling problem, where the machines correspond to resources and the jobs correspond to the fires [3, 4, 5].

In this paper, a scheduling problem to maximize the unburned area will be solved using the Genetic Algorithm, where only one resource is available for fighting several fire fronts. Thus, the aim of this work is to obtain the optimal sequence order, indicating when each of the fire fronts should be attacked. In this scheduling problem, the processing time (time to extinguish each fire front) and the travel time of the resource to access each fire front are considered. The processing time of each fire front depends on the time elapsed from the ignition, that is, the longer the resource takes to start the initial attack, the greater the fire spread. Preliminary results will be drawn showing the effectiveness of the proposed approach.

Acknowledgments: This work has been supported by PCIF/GRF/0141/2019: "O3F - An Optimization Framework to reduce Forest Fire" and the PhD grant with the reference UI/BD/150936/2021, both funded by FCT, Fundação para a Ciência e Tecnologia.

References

[1] F. Alvelos. Mixed integer programming models for fire fighting. In *International Conference on Computational Science and Its Applications*, pages 637–652. Springer, 2018.

[2] N. Khakzad. Optimal firefighting to prevent domino effects. In *Dynamic Risk Assessment and Management of Domino Effects and Cascading Events in the Process Industry*, pages 319–339. Elsevier, 2021.

[3] C. P. Pappis and N. P. Rachaniotis. Scheduling a single fire fighting resource with deteriorating fire suppression times and set-up times. *Operational Research*, 10(1):27–42, 2010.

[4] N. P. Rachaniotis and C. P. Pappis. Scheduling fire-fighting tasks using the concept of "deteriorating jobs". *Canadian Journal of Forest Research*, 36(3):652–658, 2006.

[5] N. P. Rachaniotis and C. P. Pappis. Minimizing the total weighted tardiness in wildfire suppression. *Operational Research*, 11(1):113–120, 2011.

Integrating remote sensing and wildfire simulation: a case study

Eduardo Cunha[1]*, Filipe Alvelos[1,2], António Vieira[3,4], António Bento-Gonçalves[3,4], and Sarah Moura[3,4]

[1] Centro Algoritmi, Universidade do Minho, Portugal

[2] Departamento de Produção e Sistemas, Universidade do Minho, Portugal

[3] Centro de Estudos de Comunicação e Sociedade, Universidade do Minho, Portugal

[4] Departamento de Geografia, Universidade do Minho, Portugal

eduardonunesdacunha@gmail.com(*corresponding author)

Portugal is one of the European countries most affected by wildfires, despite its smaller land area when compared to other countries - specifically those in the Mediterranean region. Consequently, it is considered one of the most prone countries to forest fires in the south of the European continent. In the year 2019 the municipality of Baião, located in Northern Portugal, was affected by the Great Forest Fire of Baião, which was considered the third largest fire that occurred in Portugal in that same year, and burned an area corresponding to 853 ha (ICNF data) in a single occurrence, corresponding to 356 ha of forest, 463 ha of bush and 34 ha of other classes.

For the area studied between the years 1975 to 2019, a maximum of 12 wildfires occurrences were identified with areas suffering 11 fire recurrences. In this sense, the development of studies that seek to understand the model of fire spread in this region is justified, which can contribute to an effective management of natural resources in the region.

Using Remote Sensing and Geographic Information Systems technologies, we first characterize the landscape in terms of the social (e.g. distance to roads), environmental (e.g. land use), and physical (e.g. slope, wind) factors involved in wildfire ignition and spread. Based on this characterization, we derive the information required by the wildfire models, such as fuel models [1].

We derive a map of ignition probabilities using results from the literature [2] and a fire spread simulator based on the minimum travel time principle [3]. The latter provides the fire arrival time at each cell of the landscape, given an ignition cell and assuming an estimate of the fire transmission times between adjacent cells – which is usually obtained by Rothermel's model or variants.

We intend to validate the proposed approach with real data from the Great Forest Fire of Baião, but keeping the approach as general as possible.

Acknowledgments: This work is supported by FCT - Fundação para a Ciência e Tecnologia within the R&D Units Project Scope UIDB/00319/2020 and within project PCIF/GRF/0141/2019 "O3F - An Optimization Framework to reduce Forest Fire".

References

[1] Fernandes, P., Gonçalves, H., Loureiro, C., Fernandes, M., Costa, T., Cruz, M. G., & Botelho, H. (2009, July). Modelos de combustível florestal para Portugal. In Actas do 6 Congresso Florestal Nacional, Ponta Delgada, Portugal. Sociedade Portuguesa de Ciências Florestais, Lisboa, Portugal (pp. 348-354).

[2] Catry, F. X., Rego, F. C., Bação, F. L., & Moreira, F. (2009). Modeling and mapping wildfire ignition risk in Portugal. International Journal of Wildland Fire, 18(8), 921-931.

[3] Finney, M. A. (2002). Fire growth using minimum travel time methods. Canadian Journal of Forest Research, 32(8), 1420-1424.

Reducing Forest Fires throughout Fuel Management in a Landscape-Level

Mariana Godoi Dias[1]*, Susete Marques[1,2], Isabel Martins[2,3],

[1] Instituto Superior de Agronomia,
Universidade de Lisboa

[2] Centro de Estudos Florestais e Laboratório
TERRA, Universidade de Lisboa

[3]Centro de Matemática, Aplicações
Fundamentais e Investigação Operacional,
Universidade de Lisboa

isa125735@isa.ulisboa.pt, smarques@isa.ulisboa.pt,
isabelinha@isa.ulisboa.pt

Forest fires are a major problem in Mediterranean countries, where they have increased substantially during the last decades. In Portugal the occurrence of fires has become a threat, [1], causing social, environmental and economic impact, and it is a key factor in ecosystems and landscape degradation. The emergence and spread of forest fires, include fuel characteristics [2], ignition sources, climate and topographic conditions. Fuel characteristics are a function of the structure and composition of the vegetation, in addition to anthropogenic factors [1]. The description of forest vegetation as input data for fire behavior simulators plays a relevant role for fire behavior, so management activities which reduce the fuel load have direct impact in minimizing the occurrence, intensity and spread of fires, as well as the consequent losses [3]. This work presents a methodology for minimizing the propagation of forest fires, through the selection of forest fuels management model, by combining forest stand dynamic models with optimization models, while taking into account the productivity preferences of landowners. We propose a perspective that allows us to estimate how the fire spreads through the landscape (using the FlamMap tool, Minimum Travel Time), according to the selection of the best distribution of species and management options (designed as an integer programming model) We apply this approach in Forest Intervention Zone of Paiva and Entre Douro e Sousa.

Acknowledgments: *This work has been supported by FCT, the Portuguese Foundation for Science and Technology within the Project Scope: PCIF/GRF/0141/2019 "O3F – An Optimization Framework to reduce Forest Fire.*

References

[1] Marques, S., Borges, J. G., Garcia-Gonzalo, J., Moreira, F., Carreiras, J. M. B., Oliveira, M. M., Cantarinha, A., Botequim, B., & Pereira, J. M. C. (2011). *Characterization of wildfires in Portugal*. European Journal of Forest Research, 130(**5**), 775–784.

[2] Rothermel, RC. (1972). *A mathematical model for predicting fire spread in wildland fuels*. USDA forest service research papers INT-115, Ogden, Utah, USA

[3] Fernandes, P., Loureiro, C., & Fernandes, M. (2009). *Modelos de Combustível Florestal para Portugal*

Modelling the effect of forestry using a Simplified CFD Model

Inês Gonçalves[1]*, Francisco Pinto[2], João Pedro Silva [1,2], Filipe Alvelos [1], José Carlos Teixeira [2], and Senhorinha Teixeira [1]

[1]ALGORITMI, Universidade do Minho

[2]MEtRICs, Universidade do Minho

b12799@algoritmi.uminho.pt*, a75138@alunos.uminho.pt, js@dem.uminho.pt, falvelos@dps.uminho.pt,jt@dem.uminho.pt, st@dps.uminho.pt

Wildland fires are considered extremely complex and destructive phenomena. The difference between being considered a natural disaster or a necessary perturbation of an ecosystem depends on the impact on the environment and economy, as well as on the management and the land use between natural and urban areas. Their behavior depends on the state of vegetation, meteorological conditions, and ground terrain, being, therefore, considered an expensive and challenging task which makes the development of robust and accurate models of wildfire behavior an extremely important activity [1] [2].

This work presents the development of atmospheric flow simulation in the presence of an obstruction, caused by the existence of trees in the domain, which modifies the velocity profile because of the drag produced by the forest presence. For the success of this project, the model must include the interaction of the forest with the surrounding atmosphere by modification of the environment, leading to the inclusion of key parameters such as ground roughness, turbulence, canopy height, and wind speed [3]. This type of CFD modeling is challenging by the fact that it must include source and sink terms in the Reynolds Averaged Navier Stokes (RANS) equations to account for forestry effects.

Here, an approach is proposed that will allow to study the effect of forestry on the atmospheric flow with the ultimate objective of analyzing its influence by modifying different parameters related to the vegetation and turbulence.

Acknowledgments: This work has been supported by FCT, the Portuguese Foundation for Science and Technology within the Project Scope: PCIF/GRF/0141/2019 "O3F – An Optimization Framework to reduce Forest Fire", and within the R&D Units Project Scope UIDB/00319/2020 (ALGORITMI) and UIDP/04077/2020 (MEtRICs). Furthermore, João Silva would like to also express his acknowledgments for the support given by the FCT through the PhD grant SFRH/BD/130588/2017.

References

[1] N. Frangieh, D. Morvan, S. Meradji, G. Accary and O. Bessonov (2018) *Numerical simulation of grassland fires behavior using an implicit physical multiphase model*, Fire Safety Journal, Volume **102**, 37-47.

[2] V. Agranat and V. Perminov (2020) *Mathematical modeling of wildland fire initiation and spread*, Journal of Environmental Modelling and Software, Volume **125**.

[3] C. Desmond, S. Watson and P. Hancock (2017) *Modelling the wind energy resources in complex terrain and atmospheres. Numerical simulation and wind tunnel investigation of non-neutral forest canopy flows*, Journal of Wind Engineering and Industrial Aerodynamics, Volume **166**, 48-60.

Modeling of a Wildfire Propagation using Surface and Crown Fire Models

João Marques[1]*, João Pedro Silva[1,2], Senhorinha Teixeira[1], José Carlos Teixeira[2], Susete Marques[3], Filipe Alvelos[1]

[1] ALGORITMI, Universidade do Minho

[2]MEtRICs, Universidade do Minho

[3]Centro de Estudos Florestais, Universidade, de Lisboa

b12802@algoritmi.uminho.pt, js@dem.uminho.pt, st@dps.uminho.pt, jt@dem.uminho.pt, smarques@isa.ulisboa.pt, falvelos@dps.uminho.pt

Forest fires, as phenomena of natural origin or human negligence, cause the destruction of extensive forest areas across the planet, causing material, as well as, human damage, due to their unpredictability, velocity, and difficulties in controlling its propagation. Dealing with this complex phenomenon represents a challenge for several people, from land managers to operation coordinators, given its undeniable social relevance. The wildfire behavior is closely associated to the environment where it occurs and is the result of the interaction of three components, namely topography, vegetation, and weather conditions, forming the so-called fire triangle [1]. This work presents the development of the simulation of the forest fire propagation combining mathematical models with the integration of empirical models for the prediction of, for instance, the intensity and the fire front propagation velocity in different types of vegetation [2]. The model includes the interaction of the forest fire with the surrounding atmosphere by modifying the fire environment, which leads to the inclusion of key parameters such as moisture, ambient temperature, wind speed and direction [3].

We propose an approach that allows us to estimate the speed of fire propagation (using surface and crown fire models) in homogeneous stands with the ultimate objective of, given an ignition, obtaining the fire paths and fire arrival times at all locations. This is accomplished, according to the Minimum Travel Time principle, through a shortest path tree algorithm (Dijsktra). We apply this approach in Forest Intervention Zone of Castelo de Paiva e Entre Douro e Sousa.

Acknowledgments: This work has been supported by FCT, the Portuguese Foundation for Science and Technology within the Project Scope: PCIF/GRF/0141/2019 "O3F – An Optimization Framework to reduce Forest Fire", and within the R&D Units Project Scope UIDB/00319 /2020 (ALGORITMI) and UIDP/04077/2020 (MEtRICs). Furthermore, João Silva would like to also express his acknowledgments for the support given by the FCT through the PhD grant SFRH/ BD/130588/2017.

References

[1] C. Countryman, "The fire environment concept," pp. 1–12, 1972.

[2] R. C. Rothermel, "A Mathematical Model for predicting fire spread in wildland fuels," Ogden, 1972.

[3] A. Bakhshaii and E. A. Johnson, "A review of a new generation of wildfire–atmosphere modeling," Can. J. For. Res., vol. 49, no. 6, pp. 565–574, Jun. 2019, doi: 10.1139/cjfr-2018-0138.

Bringing wildfire resistance into forest management planning

I. Martins[1,2]*, F. Alvelos[3,4], S. Marques[1,5], and M. Dias[1]

[1]Instituto Superior de Agronomia, Universidade de Lisboa

[2]Centro de Matemática, Aplicações Fundamentais e Investigação Operacional, Universidade de Lisboa

[3]Escola de Engenharia, Universidade do Minho

[4]Centro de Algoritmi, Universidade do Minho

[5]Centro de Estudos Florestais e Laboratório Terra, Universidade de Lisboa

isabelinha@isa.ulisboa.pt (*corresponding author), falvelos@dps.uminho.pt, smarques@isa.ulisboa.pt, marii.plant@gmail.com

The integration of fire damage in forest management planning is an important tool to bring wildfire resistance into plans. In this study, we propose a novel simulation-based optimization approach in which the forest management planning is iteratively guided by the feedback of a fire spread simulation.

The management problem consists of selecting one alternative prescription for each stand to maximize the net present value of timber production. The prescription selection is subject to restrictions on volume of timber harvested and environmental indices as biodiversity, carbon, and erosion. This problem is formulated in mixed integer program. In each iteration of the approach, the optimization problem is solved, and the fire travel time between stands is calculated, according to the selected prescriptions and other conditions (wind characteristics and slope of the stands). Then, an ignition stand is chosen and the fire spread is simulated based on the minimum travel time algorithm [1]. Paths with a fire travel speed greater than a given threshold (unacceptable paths) are identified and, to exclude them from further fire spread simulations, constraints on the prescription selection are added to the model. The model is re-optimized and the whole process is resumed until there are no unacceptable paths or the management problem is unfeasible. In this case, the most recent added constraints are removed from the model and stands of the unacceptable paths are selected to be barriers to fire spread. The forestry problem is re-optimized for the managed stands.

We tested the proposed approach with the forest of the Zona de Intervenção Florestal of Paiva and Entre Douro e Sousa, situated in the north of Portugal.

Acknowledgments: This research was supported by project PCIF/GRF/0141/2019 "An Optimization Framework to reduce Forest Fire", funded by FCT – Fundação para a Ciência e Tecnologia. This research was also supported by the Forest Research Center - Project UIDB/00239/2020, and the Center of Mathematics, Fundamental Applications and Operations Research - Project UIDB/04561/2020, both funded by FCT.

References

[1] Finney, M.A. (2002) *Fire growth using minimum travel time methods*, Can. J. For. Res., 32(8), 1420–1424.

A VNS metaheuristic for a robust fire suppression resource assignment problem

André Bergsten Mendes[1],Filipe Alvelos[2,3]

[1] Departamento de Engenharia Naval e Oceânica, Universidade de São Paulo, Brasil

[2] Centro Algoritmi, Universidade do Minho, Portugal

[3] Departamento de Produção e Sistemas, Universidade do Minho, Portugal

andbergs@usp.br (*corresponding author)

Combating wildfires is a major concern throughout the world. With the ever-increasing occurrence and intensity of forest fires, it is paramount to extinguish them as quickly as possible before they become uncontrollable.

To model the assignment and dispatching of suppression resources, several characteristics associated with a landscape must be considered, such as its topography, fuel level, and weather conditions (e.g., wind direction and speed, humidity, temperature). These aspects define the expected fire spreading rate throughout the landscape nodes represented in a raster form, starting from a known ignition node. Also, the expected number of resources to place at each node (to suppress the fire and block the fire propagation through it) can be estimated based on known fire propagation parameters.

The problem addressed in this research consists of defining where to position resources that become available at different time instants so that the burned area is minimized. We extend the work of [2], by considering that multiple resources can be required at each node (instead of one resource per node).

In this research, we consider that extra resources may be requested to protect a node in extreme fires and treat the worst-case resource requirement as a robust optimization problem. We propose a VNS metaheuristic to solve the robust resource assignment problem in grids with up to 400 nodes. Based on estimated fire propagation rates, our method dynamically calculates the fire propagation paths using the minimum travel principle [1], whereby the fire advances to the nearest nodes in its vicinity. The resource assignment decisions are taken with the purpose of minimizing the burned area by a given target instant.

The robust resource assignment is proposed for different protection levels, and risk assessments are conducted. The VNS results are compared with a MIP robust optimization model, solved by Gurobi.

Acknowledgments: This work is supported by FCT - Fundação para a Ciência e Tecnologia within the R&D Units Project Scope UIDB/00319/2020 and within project PCIF/GRF/0141/2019 "O3F - An Optimization Framework to reduce Forest Fire".

References

[1] Finney, M. A. (2002) *Fire growth using minimum travel time methods.*, Canadian Journal of Forest Research, 32 **(8)**, 1420—1424.

[2] Alvelos, F. (2018). Mixed integer programming models for fire fighting. In International Conference on Computational Science and Its Applications, Springer, 637–652.

Stochastic programming for resources positioning in wildfire fighting

David Neto[1], André Bergsten Mendes [2], Filipe Alvelos[1,3]

[1] Centro Algoritmi, Universidade do Minho, Portugal

[2] Departamento de Engenharia Naval e Oceânica, Universidade de São Paulo, Brasil

[3] Departamento de Produção e Sistemas, Universidade do Minho, Portugal

a83798@alunos.uminho.pt (*corresponding author)

Forest fires are a major problem that affects the entire world, destroying fauna and flora, and is getting worse due to climate change. In this context, the importance of approaching this phenomena with optimization models has been increasing.

In this work, we integrate fire spread in a stochastic programming model aiming at selecting the optimal positioning of fire fighting resources (e.g. firefighters teams).

The model is based on the representation of a forest landscape as a graph. The different locations are represented by nodes with different properties and the edges represent the adjacency between these locations. Considering an ignition on a node, the principle of minimum travel time (MTT) [1] can be applied, which states that the fire arrival times for each node will follow the quickest path, considering fire transmission times associated with the arcs.

As detailed in [2], a mixed integer programming models can be derived based on the MTT principle, integrating both fire spread (fire arrival times are decision variables) and the positioning decisions.

We consider uncertainty by defining a set of scenarios defined by different ignition locations, and wind speeds and directions. We obtain the fire transmission times of each scenario by an estimate of the rate of spread in the wind direction, the Huygens's principle and the standard the assumption that fire propagation has an ellipse shape.

A single mixed integer programming (the deterministic equivalent) integrates all the scenarios. An optimal solution provides the position of the resources that maximizes the average among all scenarios (each, weighted by its probability of occurrence) of the fire arrival times.

We present experimental results.

Acknowledgments: This work is supported by FCT - Fundação para a Ciência e Tecnologia within the R&D Units Project Scope UIDB/00319/2020 and within project PCIF/GRF/0141/2019 "O3F - An Optimization Framework to reduce Forest Fire".

References

[1] Finney, M. A. (2002) *Fire growth using minimum travel time methods.*, Canadian Journal of Forest Research, 32 **(8)**, 1420—1424.

[2] Alvelos, F. (2018). Mixed integer programming models for fire fighting. In International Conference on Computational Science and Its Applications, Springer, 637–652.

Discrete Event Simulation of Forest Fires

Catarina Santos[1,2]*, Ana Raquel Xambre[2,3], Helena Alvelos[2,3], Andreia Hall[1,3], and Filipe Alvelos[4,5]

[1]Department of Mathematics, University of Aveiro

[2]Department of Economics, Management, Industrial Engineering and Tourism, University of Aveiro

[3]Centre for Research and Development in Mathematics and Applications, University of Aveiro

[4]Department of Production and Systems, University of Minho

[5]Algoritmi, University of Minho

catarinassantos@ua.pt (*corresponding author), raquelx@ua.pt, helena.alvelos@ua.pt
andreia.hall@ua.pt, falvelos@dps.uminho.pt

In recent years, forest fires around the world have drastically affected entire communities. Moreover, the most significant drivers of forest fire ignition are fuel, weather and topography [1]. In Portugal, the dry summer climate, the environmental and socio-economic conditions, the growing abandonment of agricultural land, and the consequent accumulation of fuels, as well as the vegetation itself in the area, makes the country increasingly prone to fires [2].

In this work a discrete event simulation model is used to simulate the propagation of forest fire. The forest is represented by a graph where nodes correspond to stands and arcs to the adjacency between those stands. The weight of each arc is the spread probability between the two nodes. Another important aspect to simulate is where and how the fire starts so, with regard to fire ignitions, the following options were assumed: (i) one ignition point versus multiple ignitions points; (ii) node(s) selected randomly or through an ignition probability function; and (iii) using variable ignition point(s), for each run of the simuation, versus fixed ignition point(s). The model was tested using three different networks and the results were analyzed. The model was developed in order to obtain a greater insight into how fire can spread through a specific landscape, and how the factors considered can affect the behavior of the fire and the time it takes to burn the entire area. It should thus contribute to the understanding of a problem that highly impacts the environment and the life of the communities.

Acknowledgments: This work has been supported by Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia) through project PCIF/GRF/0141/2019 - *An Optimization Framework to reduce Forest Fire*.

References

[1] A. Ganteaume, A. Camia, M. Jappiot, J. San-Miguel-Ayanz, M. Long-Fournel and C. Lampin. (2013) *A Review of the Main Driving Factors of Forest Fire Ignition Over Europe*, Environmental Management, 51, 651–662.

[2] M. Carmo, F. Moreira, P. Casimiro and P. Vaz. (2011) *Land use and topography influences on wildifre occurence in northern Portugal*, Landscape and Urban Planning, 100, 169–176.

Minisymposium MS11

MS11 - Optimal control of fluid flows

Optimal Control of the 2D Evolutionary Navier-Stokes Equations with Measure Valued Controls

Eduardo Casas[1,2]* and Karl Kunisch[1]

[1]Departamento de Matemática Aplicada y Ciencias de la Computación, E.T.S.I. Industriales y de Telecomunicación, Universidad de Cantabria, 39005 Santander, Spain

[2]Institute for Mathematics and Scientific Computing, University of Graz, Heinrichstrasse 36, A-8010 Graz, Austria

eduardo.casas@unican.es (*corresponding author), karl.kunisch@uni-graz.at

In this talk, we consider an optimal control problem for the two-dimensional evolutionary Navier-Stokes system. Looking for sparsity, we take controls as functions of time taking values in a space of Borel measures. The cost functional does not involve directly the control but we assume some constraints on them. We prove the well-posedness of the control problem and derive necessary and sufficient conditions for local optimality of the controls.

Acknowledgments: The first author was supported by Spanish Ministerio de Economía, Industria y Competitividad under research project MTM2017-83185-P. The second was supported by the ERC advanced grant 668998 (OCLOC) under the EU's H2020 research program.

Injection-suction control for 2D stochastic Navier-Stokes equations with slippage

Fernanda Cipriano[1]*, Nikolai Chemetov[2]

[1]Departamento de Matemática, Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa and Centro de Matemática e Aplicações, Lisbon, Portugal

[2]Department of Computing and Mathematics, University of São Paulo, Brazil

cipriano@fct.unl.pt (*corresponding author), nvchemetov@ gmail.com

We consider a velocity tracking problem for stochastic Navier-Stokes equations in a 2D-bounded domain. The control acts on the boundary through an injection-suction device and the flow is allowed to slip against the surface wall. We study the well-posedness of the stochastic state equations, stochastic linearized state equations and stochastic adjoint equations. In addition, we show the existence of an optimal solution and establish the first order optimality condition.

Acknowledgments: This work has been supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID/MAT/00297/2019 (Centro de Matemática e Aplicações).

References

[1] N. V. Chemetov, F. Cipriano (2018) *Optimal control for two-dimensional stochastic second grade fluids*, Stochastic Processes Appl., Volume **128**, 2710–2749.

[2] Nikolai Chemetov and Fernanda Cipriano (2018) *Injection-suction control for 2D Navier-Stokes equations with slippage*, SIAM J. Control Optim., Volume **56**, 1253–1281.

FGP2022 on Optimization, Multiobjective optimal control problems. Navier-Stokes equations

Irene Marín Gayte[1]*,

[1] Departamento de Matemática, Instituto Técnico Superior, Universidade de Lisboa

irene.m.gayte@tecnico.ulisboa.pt (*corresponding author),

This talk deals with the solution of some multi-objective optimal control problems for stationary Navier-Stokes equations. More precisely, we look for Pareto and Nash equilibria associated to standard cost functionals. First, we prove the existence of equilibria and we deduce appropriate optimality systems. Then, we analyze the existence and characterization of Pareto and Nash equilibria for the Navier-Stokes equations. Here, we use the formalism of Dubovitskii and Milyoutin., see [3]. Finally, we also present a finite element approximation of the bi-objective problem and we illustrate the techniques with several numerical experiments. The work is based on [2] and [1].

Acknowledgments: This work has been done in collaboration with the professor Enrique Fernández Cara of the University of Seville.

References

[1] E. Fernández-Cara, I. Marín-Gayte, (2021) *Theoretical and numerical bi-objective optimal control: Nash equilibria*, ESAIM: Control, Optimisation and Calculus of Variations, **27**, 50.

[2] E. Fernández-Cara, I. Marín-Gayte, (2020) *Theoretical and numerical results for some bi-objective optimal control problems*, Communications on Pure and Applied Analysis, **19**, 4, 210–2126.

[3] I. V. Girsanov, (1972) *"Lectures on mathematical theory of extremum problems,"* Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, Berlin, **67**.

Controllability of low Reynolds numbers swimmers of ciliate type

Jérôme Lohéac[1]*, and Takéo Takahashi[2]

[1]Université de Lorraine, CNRS, CRAN, F-54000 Nancy, France

[2]Université de Lorraine, CNRS, Inria, IECL, F-54000 Nancy, France

jerome.loheac@univ-lorraine.fr (*corresponding author), takeo.takahashi@inria.fr

We study the locomotion of a ciliated microorganism in a viscous incompressible fluid. We use the Blake ciliated model: the swimmer is a rigid body with tangential displacements at its boundary that allow it to propel in a Stokes fluid, [1, 2]. This can be seen as a control problem: using periodical displacements, is it possible to reach a given position and a given orientation? We are interested in the minimal dimension d of the space of controls that allows the microorganism to swim. Our main result states the exact controllability with $d = 3$ generically with respect to the shape of the swimmer and with respect to the vector fields generating the tangential displacements. The proof is based on analyticity results and on the study of the particular case of a spheroidal swimmer.

The talk is based on the paper [3].

References

[1] J. R. Blake (1971) *A spherical envelope approach to ciliary propulsion*, J. Fluid Mech., 46, 199–208.

[2] J. R. Blake (1973) *A finite model for ciliated micro-organisms*, J. Biomechanics, 6, 133–140.

[3] J. Lohéac and T. Takahashi (2020) *Controllability of low Reynolds numbers swimmers of ciliate type*, ESAIM, Control Optim. Calc. Var., 26, 1–30.

Optimal boundary control problem for steady Navier-Stokes equations with directional do-nothing boundary condition

Pedro Nogueira[1]*, Ana L. Silvestre[2], and Jorge Tiago[2]

[1]CEMAT, Instituto Superior Técnico, Universidade de Lisboa, Portugal

[2] CEMAT and Department of Mathematics, Instituto Superior Técnico, Universidade de Lisboa, Portugal

pedro.mendes.nogueira@tecnico.ulisboa.pt, ana.silvestre@math.tecnico.ulisboa.pt, jorge.tiago@tecnico.ulisboa.pt

Inspired by the modelling of blood flow redirected by a bypass surgery in a tract of an artery [5], we consider the Navier-Stokes equations with mixed boundary conditions including a non-homogeneous Dirichlet condition in the inlets of the fluid domain and a directional do-nothing (DDN) outflow boundary condition [1, 2, 3, 4].

In an appropriate functional framework, associated with a saddle point approach [5], we begin by establishing the well-posedness of the direct problem. Then, aiming at flow regularization, we analyse the boundary control problem which consists in the minimization of quadratic cost functionals of the velocity field (tracking-type or vorticity) by means of the inflow velocity. We prove the existence of optimal solutions, justify the Gâteaux derivative of the control-to-state map and deduce the first order necessary conditions for optimality. The results are obtained under smallness restrictions on the inflow boundary controls.

Keywords: Navier-Stokes equations; mixed boundary conditions; directional do-nothing condition; optimal boundary control.

Acknowledgments: This work has been supported by Fundação para a Ciência e a Tecnologia, through the projects UIDB/04621/2020 and UIDP/04621/2020 of CEMAT/IST-ID and MAT-PTDC/MAT-APL/7076/2020.

References

[1] Braack M. and Mucha P.B. (2014) *Directional do-nothing condition for the Navier-Stokes equations*, Journal of Computational Mathematics, Vol. 32 , **5**, pp 507-521.

[2] Bruneau C.-H. (2000) *Boundary conditions on artificial frontiers for incompressible and compressible Navier-Stokes equations*, ESAIM Math. Model. Numer. Anal., Vol. 34, **2** , 303-314.

[3] Bruneau C.-H. and Fabrie P. (1994) *Effective downstream boundary conditions for incompressible Navier-Stokes equations*, *Int. J. Numer. Meth. Fluids*, Vol. 19, 693-705. doi:10.1002/fld.1650190805

[4] Bruneau C.-H. and Fabrie P. (1996) *New efficient boundary conditions for incompressible Navier-Stokes equations: a well-posedness result*, RAIRO-MMAN, Vol. 30, **7**, 815-840.

[5] Manzoni A., Quarteroni A. and Salsa S. (2019) *A saddle point approach to an optimal boundary control problem for steady Navier-Stokes equations*, Mathematics in Engineering Vol. 1 , **2**, 252–280.

Stabilization of fluid flows using reduced order model based on spectral projections

Jean-Pierre Raymond[1,2]*

[1]IMT - Université Paul Sabatier Toulouse III, 31062 Toulouse-Cedex 9 - France

raymond@math.univ-toulouse.fr

We will present results concerning the stabilization of fluid flows, or systems coupling fluid flow equations to convection-diffusion equations. We are interested in the local stabilization around unstable stationary solutions. We build feedback laws (in the case of total information), and feedback laws coupled to an observer (in the case of partial information), for reduced models defined by spectral projection. We prove, in some of the cases studied, that the feedback law defined from the reduced model also stabilizes the initial system with an exponential decay rate a priori fixed. We also establish convergence rates of feedback laws for models approximated by a Finite Element Method towards the feedback law of the original model.

This work is the result of multiple collaborations with (in alphabetical order): C. Airiau, M. Badra, J.-M. Buchot, K. Dubey, P. Chandrashekar, M. Fournié, M. Ndiaye, M. Ramaswamy, R. Sandilya, G. Tissot, J. Weller-Calvo (see [1], [2], [3], [4]).

Acknowledgments: This work has been supported by the ANR-Project IFSMACS (ANR 15-CE40.0010).

References

[1] C. Airiau, J.-M. Buchot, R. K. Dubey, M. Fournié, J.-P. Raymond, J. Weller-Calvo (2017) Stabilization and best actuator location for the Navier-Stokes equations. SIAM J. Sci. Comput. 39, no. 5, B993-B1020.

[2] P. Chandrashekar, M. Ramaswamy, J.-P. Raymond, R. Sandilya (2021), Numerical stabilization of the Boussinesq system using boundary feedback control. Comput. Math. Appl. 89, 163-183.

[3] M. Badra, J.-P. Raymond (2022), Numerical approximation of feedback gains stabilizing the Oseen system, preprint.

[4] M. Badra, J.-P. Raymond (2022), Approximation of feedback gains using spectral projections - Applications to the Navier-Stokes equations, preprint.

Feedback semiglobal stabilization to trajectories for the Kuramoto–Sivashinsky equations

Sérgio S. Rodrigues[1,*], and Dagmawi A. Seifu[1]

[1]Johann Radon Institute for Computational and Applied Mathematics, ÖAW, Altenbergerstr. 69, 4040 Linz, Austria.

sergio.rodrigues@ricam.oeaw.ac.at (*corr. author), dagmawi.seifu@ricam.oeaw.ac.at

It is shown that an oblique projection based feedback control is able to stabilize the state of the Kuramoto–Sivashinsky equation evolving in rectangular domains. The number of actuators is finite and consists of a finite number of indicator functions supported in small subdomains. Simulations are presented showing the stabilizing performance of the feedback control.

Acknowledgments: S. Rodrigues acknowledges partial support from the Austrian Science Fund (FWF): P 33432-NBL.

References

[1] S. S. Rodrigues and D. A. Seifu (2022) *Feedback semiglobal stabilization to trajectories for the Kuramoto–Sivashinsky equations*, ongoing work.

Stabilization of a rigid body moving in a compressible viscous fluid

Arnab Roy[1]

[1]BCAM, Basque Center for Applied Mathematics, Mazarredo 14, E48009 Bilbao, Bizkaia, Spain

royarnab244@gmail.com

In this talk, we discuss the stabilizability of a fluid-structure interaction system of a viscous, compressible fluid and rigid ball in a three dimension space with an external force acting on the ball produced by a spring and a damper connecting the centre of the ball to a fixed point h_1. We prove global-in-time existence result for this model under the condition that the initial velocities are small and the center of mass of the body is sufficiently near to h_1. In this scenario, we discuss the asymptotic behaviour of the whole system.

Acknowledgments: This work has been supported by the ANR research project IFSMACS (ANR-15-CE40-0010).

References

[1] Author. (year). *Title of the book*, publisher, City, initial page–final page.

[2] Author (year) *Title of the paper*, Journal, Volume **issue**, initial page–final page.

Minisymposium MS12

MS12 - The interplay between geometry and optimization

Dynamic interpolation on Riemannian manifolds

Margarida Camarinha[1]* and Fátima Silva Leite[2]

[1]University of Coimbra, CMUC, Department of Mathematics, Coimbra, Portugal

[2]University of Coimbra, Department of Mathematics, Coimbra, Portugal
and Institute of Systems and Robotics, DEEC-UC, Coimbra, Portugal

mmlsc@mat.uc.pt (*corresponding author), fleite@mat.uc.pt

The dynamic interpolation problem arose as an attempt to extend spline-based methods to Riemannian manifolds and led to the study of the so-called Riemannian cubic splines [2]. These curves are required to be $\mathcal{C}^2$-smooth, interpolate a given set of distinct points on a Riemannian manifold at prescribed instants of time, satisfy certain boundary conditions and minimize the average of the intrinsic acceleration. This optimization problem has been investigated in different research fields, from the calculus of variations to the optimal control, with a wide range of applications in engineering, physics and medicine, such as rigid body control in robotics, spacecraft control in aeronautics, quantum control in quantum information processing, 3D animation in computer graphics or regression schemes for computational anatomy in medical imaging.

Explicit solutions for the dynamic interpolation problem have rarely been reached. Recently, in [3], bi-Jacobi fields were used to compute approximations of these solutions. This issue is especially challenging when boundary conditions are imposed on the velocity beyond the interpolation points. In view of this limitation, the interest in studying the existence and uniqueness of Riemannian cubics and their relation to bi-Jacobi fields has gained ground.

In this talk, we will survey the main results on dynamic interpolation and report on some of its applications. Then we will discuss the recent advances on local existence and uniqueness of Riemannian cubics satisfying boundary conditions [1]. The proposed approach relies strongly on a generalization of some tools from the variational theory of geodesics: the biexponential map, bi-Jacobi fields and biconjugate points. This topic, beside being of interest per se, can be important for developing methods to approximate cubic splines.

Acknowledgments: The research of M. Camarinha was partially supported by the Centre for Mathematics of the University of Coimbra - UIDB/00324/2020, funded by the Portuguese Government through FCT/MCTES. F. Silva Leite acknowledges Fundação para a Ciência e Tecnologia (FCT-Portugal) and COMPETE 2020 program for the financial support to the project UIDB/00048/2020.

References

[1] M. Camarinha, F. Silva Leite and P. Crouch (2021) *Existence and uniqueness for Riemannian cubics with boundary conditions*, in Lecture Notes in Electrical Engineering, **695**, LNEE, 322–331.

[2] P. Crouch and F. Silva Leite (1995) *The dynamic interpolation problem on Riemannian manifolds, Lie groups and symmetric spaces*, J. Dynam. Control Systems, **1**, 177–202.

[3] L. Noakes and T. S. Ratiu (2016) *Bi-Jacobi fields and Riemannian cubics for left-invariant $SO(3)$*, Commun. Math. Sci., **14**, 55–68.

Title: **Accelerated optimization via discrete variational calculus**

Speaker: David Martín de Diego (ICMAT, Spain)

Abstract: Many of the new developments in machine learning are connected with gradient-based optimization methods. Recently, these methods have been studied using a variational perspective. This has opened up the possibility of introducing variational and symplectic methods using geometric integration. In particular, in this talk, we will introduce variational integrators which allow us to derive different methods for optimization. Using both, Hamilton's and Lagrange-d'Alembert's principle, we derive two families of optimization methods in one-to-one correspondence that generalize Polyak's heavy ball and the well known Nesterov's accelerated gradient method. We will additionally show how to design these methods for optimization in manifolds using retraction maps.

Joint work with Cédirc M. Campos and José Torrente.

References

[1] C. M. Campos, A. Mahillo and D. Martín de Diego. A Discrete Variational Derivation of Accelerated Methods in Optimization. arXiv:2106.02700.

[2] V. Duruisseaux, Schmitt J., and Leok M. Adaptative hamiltonian variational integrators and applications to symplectic accelerated optimization. arXiv: 1709.01975, 2020.

[3] Michael I. Jordan. Dynamical symplectic and stochastic perspectives on gradient-based optimization. In Proceedings of the International Congress of Mathematicians-Rio de Janeiro 2018. Vol. I. Plenary lectures, pages 523549. World Sci. Publ., Hackensack, NJ, 2018.

[4] J. E. Marsden and M. West. Discrete mechanics and variational integrators. Acta Numer., 10: 357514, 2001

[5] Yu. E. Nesterov. A method for solving the convex programming problem with convergence rate $O(1/k^2)$. Dokl. Akad. Nauk SSSR, 269(3): 543547, 1983.

[6] Andre Wibisono, Ashia C. Wilson, and Michael I. Jordan. A variational perspective onaccelerated methods in optimization.Proc. Natl. Acad. Sci. USA, 113(47): E7351E7358, 2016.

Title: Symmetries of deformable bodies
Speaker: M. Esmeralda Sousa Dias (IST-UL, Lisbon, PT)

Abstract: We review the geometry of rotating bodies which can stretch and shear. Although there is some similarity with rigid bodies, the symmetries of these models are richer, since internal observable qualities of the particles are allowed. Symmetry reduction, in particular Euler-Poincaré and Lie-Poisson reduction, will be briefly referred.

Towards an Algebraic Perspective on Optimal Control

Christopher Gadzinski (me@cgad.ski)[1]

[1]University of Coimbra

In the book "Control Theory From a Geometric Viewpoint" by Agrachev and Sachkov [1], it is frequently useful to view differential geometric objects from an *algebraic point of view*, i.e. as homomorphisms or augmented derivations between certain commutative algebras.

In our own work, we apply such an algebraic perspective to the Pontryagin maximum principle. By extending the usual Poisson bracket for functions on a symplectic manifold to a "sub-Poisson bracket" that operates on a certain class of non-differentiable functions, we are able to state the Pontryagin maximum principle for time-optimal control in a particularly elegant way. Our new algebraic perspective yields a one-line proof of a "Noether's theorem for optimal control" already known to Torres in [2] and opens some ongoing research questions.

References

[1] Andrei A. Agrachev and Yuri L. Sachkov. Control theory from the geometric viewpoint. ISBN: 9783662064047, OCLC: 1157341037.

[2] Delfim F. M. Torres. On the noether theorem for optimal control. pages 244–249, 09 2001.

Local Minimizers for Variational Obstacle Avoidance on Riemannian manifolds

Jacob R. Goodman

Instituto de Ciencias Matematicas (CSIC-UAM-UC3M-UCM),
Calle Nicolas Cabrera 13-15, 28049, Madrid, Spain

jacob.goodman@icmat.es

In this talk I will discuss a variational obstacle avoidance problem on complete Riemannian manifolds. That is, I minimize an action functional, among a set of admissible curves, which depends on an artificial potential function used to avoid obstacles. In particular, I extend the theory of bi-Jacobi fields and biconjugate points, and present necessary and sufficient conditions for optimality in this setting. Local minimizers of the action functional are divided into two categories—called Q-local minimizers and Ω-local minimizers—and subsequently classified, with local uniqueness results discussed in both cases.

Acknowledgments: The project that gave rise to these results received the support of a fellowship from "la Caixa" Foundation (ID 100010434). The fellowship code is LCF/BQ/DI19/11730028. Additionally, support has been given by the "Severo Ochoa Programme for Centres of Excellence" in R&D (SEV-2015-0554).

References

[1] Jacob R. Goodman. (2022). *Local Minimizers for Variational Obstacle Avoidance on Riemannian manifolds*. ArXiv preprint arXiv:2201.04395.

[2] Jacob R. Goodman, Leonardo J. Colombo. (2021). *Collision Avoidance of Multiagent Systems on Riemannian Manifolds*. SIAM Journal on Control and Optimization, 60(1), 168–188.

Retraction maps: a seed of geometric integrators for optimal control problems

María Barbero Liñán[1]* and David Martín de Diego[2]

[1] Departamento de Matemática Aplicada, Universidad Politécnica de Madrid, Spain

[2] Instituto de Ciencias Matemáticas (CSIC-UAM-UCM-UC3M), Spain

m.barbero@upm.es (*corresponding author), david.martin@icmat.es

The notion of retraction map was first introduced in 1931 by Borsuk [5] from a topological perspective. However, it did not become an interesting and resourceful tool in optimization theory and numerical analysis until the early XXIst century (Absil, Adler, Mahony, Sepulchre, Shub, et al. [1, 2]). Here we use the notion of retraction map to define a new map, called discretization map, that for every point and velocity in the tangent bundle of the configuration manifold assigns two points in the configuration manifold. After suitably lifting the new retraction map to the cotangent bundle [4] , the typical phase space for Hamiltonian mechanical systems, we will be able to define geometric integrators for optimal control problems [3].

Acknowledgments: The authors acknowledge financial support from the Spanish Ministry of Science and Innovation, under grants PID2019-106715GB-C21, the Spanish National Research Council, through the "Ayuda extraordinaria a Centros de Excelencia Severo Ochoa" R&D (CEX2019-000904-S) and I-Link Project (Ref: linkA20079) from CSIC (CEX2019-000904-S). MBL has been financially supported by "Programa propio de I+D+I de la Universidad Politécnica de Madrid: Ayudas dirigidas a jóvenes investigadores doctores para fortalecer sus planes de investigación".

References

[1] P.-A. Absil, R. Mahony, and R. Sepulchre (2008). *Optimization algorithms on matrix manifolds*, Princeton University Press, Princeton, NJ.

[2] R. L. Adler, J.-P. Dedieu, J. Y. Margulies, M. Martens, and M. Shub (2002). *Newton's method on Riemannian manifolds and a geometric model for the human spine*, IMA J. Numer. Anal., 22(3):359–39.

[3] M. Barbero Liñán, H. Cendra, E. García Toraño, and D. Martín de Diego (2019). *Morse families and Dirac systems*, J. Geom. Mech., 11(4):487–510.

[4] M. Barbero Liñán, D. Martín de Diego (2021) *Extended retraction maps: a seed of geometric integrators*, arXiv:2106.00607.

[5] K. Borsuk (1931) *Sur les retractes*, Fund.Math., 17.

A numerical optimization approach to solve the least square problem on curved spaces

Luís Machado[1,2]

[1]Institute of Systems and Robotics - University of Coimbra

DEEC - University of Coimbra - Polo II, 3030-290 Coimbra, Portugal

[2]University of Trás-os-Montes e Alto Douro (UTAD)

5000-801 Vila Real, Portugal

lmiguel@utad.pt

The absence of explicit expressions for polynomials on general curved spaces is the main drawback for obtaining a straightforward generalization for the classical least square problem. To overcome this challenge, we formulate the following variational problem on a locally connected Riemannian manifold M:

$$\min_{\gamma\in\Omega} J(\gamma) = \frac{1}{2}\sum_{i=0}^{N} d^2\big(q_i, \gamma(t_i)\big) + \frac{\lambda}{2}\int_0^T \left\langle \frac{D^m\gamma}{dt^m}, \frac{D^m\gamma}{dt^m} \right\rangle dt,$$

depending on the smoothing parameter λ. Due to the high nonlinearity of the Euler-Lagrange equations, we propose a numerical optimization approach to obtain solutions for the proposed variational problem. We will see that the geometric mean and the geodesic that best fits the given data set of points $q_0, \ldots, q_N$, will arise as limiting processes of the proposed problems. Numerical simulations will be provided.

Acknowledgments: This work has been supported by Fundação para a Ciência e a Tecnologia (FCT) under the project UIDB/00048/2020

References

[1] Machado, L.; Monteiro, M. Teresa T. (2017) *A numerical optimization approach to generate smoothing spherical splines*, Journal of Geometry and Physics, Volume 111, 71–81.

[2] Machado, L.; Silva Leite, F.; Krakowski, K. (2010) *Higher-order smoothing splines versus least squares problems on Riemannian manifolds*, Journal of Dynamical and Control Systems, Volume 16(1), 121–148.

Jacobi multipliers and optimization

Patrícia Santos [1,2]*

[1] Polytechnic Institute of Coimbra, ISEC, Portugal

[2] CMUC, University of Coimbra, Portugal

patricia@isec.pt (*corresponding author)

A Jacobi multiplier is a non-vanishing function on a manifold that verifies the generalised Liouville equation, along the integral curves of a vector field on the manifold. This is a geometrical tool that can provide constants of motion for ordinary differential equations and Lagrangian descriptions for second-order ordinary differential equations [1]. In this work the geometric theory behind the Jacobi multipliers is recalled and some relations between Jacobi multipliers and optimization problems are given [2].

Acknowledgments: This work has been supported by Centre for the Mathematics of the University of Coimbra–UIDB/00324/2020

References

[1] José F. Cariñena, Patrícia Santos (2021). *Jacobi multipliers and Hamel's formalism*, J. Phys. A: Math. Theor., **54**, 225203 (28pp).

[2] J. M. Maruskin and A. M. Bloch (2011). *The Boltazmann-Hamel equations for the optimal control of mechanics systems with nonholonomic constraints*, Int. Robust Nonliner Control, **21** 373-386.

Minisymposium MS13

MS13 - Convergence rates of convex optimization methods

High-Order Proximal-Point Methods Beyond Classical Settings

Masoud Ahookhosh[1]* and Yurii Nesterov[2]

[1]Department of Mathematics, University of Antwerp, Middelheimlaan 1, B-2020 Antwerp, Belgium.

[2]Center for Operations Research and Econometrics (CORE), Catholic University of Louvain (UCL).

masoud.ahookhosh@uantwerp.be (*corresponding author), Yurii.Neterov@uclouvain.be

We introduce a Bi-level OPTimization (BiOPT) framework for minimizing the sum of two convex functions, where both of them can be nonsmooth. The BiOPT framework involves two levels of methodologies. In the upper level of BiOPT, the objective function is regularized by a $(p+1)$th-order proximal term that we assume to be minimized exactly or inexactly at a reasonable cost. In this level, we are particularly interested in the generic inexact high-order proximal-point (with/without segment search) scheme and its acceleration using the estimation sequence technique. In the lower level of BiOPT, the corresponding pth-order proximal auxiliary problem needs to be solved inexactly. It is shown that can be done by a modified lower-order non-Euclidean composite gradient scheme (with/without bisection procedure). Ultimately, if the accelerated proximal-point method is applied in the upper level, and the auxiliary problem is handled by the non-Euclidean composite gradient scheme, then we end up with an efficient method with either an optimal or a superfast convergence rate.

Acknowledgments: This project has received funding from the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation programme (grant agreement No.788368).

References

[1] Masoud Ahookhosh and Yurii Nesterov. (2021) *High-order methods beyond the classical complexity bounds: inexact high-order proximal-point methods*, https://arxiv.org/pdf/2109.12303.pdf, 1–28.

[2] Masoud Ahookhosh and Yurii Nesterov. (2021) *High-order methods beyond the classical complexity bounds: inexact high-order proximal-point methods with segment search*, https://arxiv.org/pdf/2107.05958.pdf, 1–28.

Optimization Methods for Fully Composite Problems

Nikita Doikov[1]*, Yurii Nesterov[1]

[1]Institute of Information and Communication Technologies, Electronics and Applied Math. (ICTEAM) and Center for Operations Research and Econometrics (CORE), UCLouvain, Belgium

nikita.doikov@uclouvain.be (*corresponding author), yurii.nesterov@uclouvain.be

In this talk, we discuss a Fully Composite Formulation of convex optimization problems [1, 2, 3, 4, 5]:

$$\min_{x \in \mathrm{dom}\,\phi} \Big\{ \phi(x) \stackrel{\mathrm{def}}{=} F(x, f(x)) \Big\}$$

It includes, as a particular case, the problems with functional constraints, max-type minimization problems, and problems of Composite Minimization, where the objective can have simple nondifferentiable components.

We treat all these formulations in a unified way, highlighting the existence of very natural optimization schemes of different order. As an immediate consequence of our results, we get, in particular, new high-order methods with global linear rate of convergence for convex minimization with functional constraints. Our new first-, second-, and third-order methods can be implemented in practice using the existing polynomial-time technique [6].

We prove the global rates for our methods under the most general conditions. Assuming additionally that the upper-level component of our objective function is *subhomogeneous*, we develop efficient modification of the basic Fully Composite first-order and second-order methods, and propose their accelerated variants.

Acknowledgments: This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 788368).

References

[1] Arkadii Nemirovskii and David Yudin. Problem complexity and method efficiency in optimization. 1983.

[2] James Burke. Descent methods for composite nondifferentiable optimization problems. *Mathematical Programming*, 33(3):260–279, 1985.

[3] Yurii Nesterov. *Effective methods in nonlinear programming*. Moscow, Radio i Svyaz, 1989.

[4] Yurii Nesterov. Gradient methods for minimizing composite functions. *Mathematical Programming*, 140(1):125–161, 2013.

[5] Dmitriy Drusvyatskiy and Courtney Paquette. Efficiency of minimizing compositions of convex functions and smooth maps. *Mathematical Programming*, 178(1):503–558, 2019.

[6] Yurii Nesterov. Implementable tensor methods in unconstrained convex optimization. *Mathematical Programming*, pages 1–27, 2019.

First-order methods for some nonconvex quartic problems

Radu-Alexandru Dragomir[1]* Yurii Nesterov[1]

[1] Center for Operations Research and Econometrics (CORE), UCLouvain

radu.dragomir@uclouvain.be (*corresponding author), yurii.nesterov@uclouvain.be

We study optimization problems of the form

$$\min_{x \in \mathcal{K}} \rho(x) - g(x) \tag{P}$$

where $\mathcal{K}$ is a finite-dimensional convex cone, and ρ, g are two convex functions on $\mathcal{K}$. We assume that ρ is a quartic polynomial induced by a symmetric 4-linear form. Such structure arises in low-rank optimization, Euclidean distance matrix recovery and phase retrieval.

We propose a bilevel scheme for finding a stationary point of Problem (P). It involves solving successive smooth optimization problems where the squared inner objective satisfies a variant of the *uniform convexity* property. Using this structure, we design fast gradient methods for solving the subproblems.

Accelerated Alternating Minimization Methods

Sergey Guminov[1,2], Pavel Dvurechensky[3,2,*], Nazarii Tupitsa[1,2], and Alexander Gasnikov[1,2]

[1]Moscow Institute of Physics and Technology, Dolgoprudny, Russia

[2]Institute for Information Transmission Problems RAS,Moscow, Russia

[3]Weierstrass Institutefor Applied Analysis and Stochastics, Berlin, Germany

ergey.guminov@phystech.edu, pavel.dvurechensky@wias-berlin.de (*corresponding author), tupitsa@phystech.edu, gasnikov@phystec

We combine alternating minimization (AM) and Nesterov-type momentum acceleration and propose a generic accelerated alternating minimization method with a $1/k^2$ convergence rate in terms of the objective for convex problems and $1/k$ in terms of the squared gradient norm for non-convex problems, where k is the iteration counter. Our method does not require any knowledge of neither convexity of the problem nor function parameters such as smoothness constant, i.e. it is adaptive to convexity and smoothness. Further, we develop its primal-dual modification for convex problems with linear constraints. We consider two applications of our methods to highlight their properties. The first one is the non-convex collaborative filtering problem, and the second one is optimal transport, where our primal-dual method takes a form of accelerated Sinkhorn's algorithm or accelerated Iterative Bregman Projections algorithm. In both applications, we show numerically that our methods outperform the baselines. In the second application, we also obtain state-of-the-art complexity results for optimal transport problems.

Acknowledgments: This research was funded by Russian Science Foundation (project 18-71-10108).

Convergence rate of Langevin MCMC methods for non-smooth potential functions

Susan Ghaderi[2], Masoud Ahookhosh[2]*, Panos Patrinos[1], Yves Moreau[1]

[1]Department of Electrical Engineering (ESAT), KU Leuven.

[2]Department of Mathematics, University of Antwerp, Middelheimlaan 1, B-2020 Antwerp, Belgium.

[3]Luxembourg Centre for Systems Biomedicine (LCSB), University of Luxembourg.

susan.ghaderi@kuleuven.be,masoud.ahookhosh@uantwerp.be,
panos.patrinos@kuleuven.be,alexander.skupin@uni.lu,yves.moreau@kuleuven.be

Using gradient-based MCMC sampling methods shed the light to accelerate using Bayesian inference learning for high-dimensional problems. In particular, using Langevin and Hamiltonian dynamics in collaboration with canonical sampling methods make these methodologies very popular in machine learning and data science problems, socially when there is no analytical solution for these sampling methods. In this study, we consider the potential function as the summation of a smooth (non-convex) log-likelihood function and a (nonsmooth) convex log-prior function., in which the log-prior can be nonsmooth (e.g., Laplace distribution), that the potential function can consequently be nonsmooth. Using the Forward-Backward Envelope (FBE), which is considered as a continuously differentiable approximation of the potential function, we present a new FBE MCMC method for sampling from a nonsmooth potential function. Owing to Lipschitz smoothness and (strong) convexity of FBE under mild assumptions, it is possible to provide asymptotic and non-asymptotic convergence results for this algorithm. Our numerical experiments illustrate the efficiency of the proposed algorithm.

References

[1] Ghaderi, Susan. (2021) *Mathematical Approaches to Biological Complexity in Systems Biomedicine, Diss, Univeristy of Luxembourg, Esch-sure-Alzette, Luxembourg*.

[2] Susan Ghaderi, Masoud Ahookhosh, Adam Arny, Panos Patrinos, Alexander Skupin, and Yves Moreau.(2022) *Non-smooth Unadjusted Langevin Algorithms via Forward-Backward Envelope, (submited)*.

Adaptive Third-Order Methods for Composite Convex Optimization

Geovani Nunes Grapiglia[1]* and Yurii Nesterov[2]

[1] INMA, Université catholique de Louvain (UCLouvain), Belgium.

[2] CORE, Université catholique de Louvain (UCLouvain), Belgium.

geovani.grapiglia@uclouvain.be (*corresponding author), Yurii.Neterov@uclouvain.be

In this work we propose third-order methods for composite convex optimization problems in which the smooth part is a three-times continuously differentiable function with Lipschitz continuous third-order derivatives. The methods are adaptive in the sense that they do not require the knowledge of the Lipschitz constant. Trial points are computed by the inexact minimization of models of the objective function that consist in the nonsmooth part of the objective plus a quartic regularizations of third-order Taylor polynomial of the smooth part. Specifically, approximate solutions of the auxiliary problems are obtained by using a Bregman gradient method as inner solver. Different from existing adaptive approaches for high-order methods, in our new schemes the regularization parameters are tunned by checking the progress of the inner solver. With this technique, we show that the basic method finds an ϵ-approximate minimizer of the objective function performing at most $\mathcal{O}\left(|\log(\epsilon)|\epsilon^{-\frac{1}{3}}\right)$ iterations of the inner solver. An accelerated adaptive third-order method is also presented with total inner iteration complexity of $\mathcal{O}\left(|\log(\epsilon)|\epsilon^{-\frac{1}{4}}\right)$.

Acknowledgments: This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No.788368).

Escaping limit cycles: Global convergence for constrained nonconvex-nonconcave minimax problems

Thomas Pethick[2], Puya Latafat[1*], Panagiotis Patrinos[1], Olivier Fercoq[3], Volkan Cevher[2]

[1]Department of Electrical Engineering (ESAT-STADIUS), KU Leuven

[2]Laboratory for Information and Inference Systems (LIONS), EPFL

[3]Laboratoire Traitement et Communication d'Information, Télécom Paris, Institut Polytechnique de Paris

puya.latafat@kuleuven.be (*corresponding author)

We introduce a new extragradient-type algorithm for a class of nonconvex-nonconcave minimax problems. It is well-known that finding a local solution for general minimax problems is computationally intractable. This observation has recently motivated the study of structures sufficient for convergence of first order methods in the more general setting of variational inequalities when the so-called *weak Minty variational inequality* (MVI) holds. This problem class captures non-trivial structures as we demonstrate with examples, for which a large family of existing algorithms provably converge to limit cycles. Our results require a less restrictive parameter range in the weak MVI compared to what is previously known, thus extending the applicability of our scheme. The proposed algorithm is applicable to constrained and regularized problems, and involves an adaptive stepsize allowing for potentially larger stepsizes. Our scheme also converges globally even in settings where the underlying operator exhibits limit cycles. Moreover, a variant with stochastic oracles is proposed—making it directly relevant for training of generative adversarial networks. For the stochastic algorithm only one of the stepsizes is required to be diminishing while the other may remain constant, making it interesting even in the monotone setting.

Acknowledgments: This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement n° 725594 - time-data). This work was supported by the Swiss National Science Foundation (SNSF) under grant number 200021_205011. The work of the second and third author was supported by the Research Foundation Flanders (FWO) postdoctoral grant 12Y7622N and research projects G081222N, G0A0920N, G086518N, and G086318N; Research Council KU Leuven C1 project No. C14/18/068; Fonds de la Recherche Scientifique – FNRS and the Fonds Wetenschappelijk Onderzoek – Vlaanderen under EOS project no 30468160 (SeLMA); European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 953348. The work of Olivier Fercoq was supported by the Agence National de la Recherche grant ANR-20-CE40-0027, Optimal Primal-Dual Algorithms (APDO).

Subgradient Ellipsoid Method for Nonsmooth Convex Problems

Anton Rodomanov[1]*, Yurii Nesterov[2]

[1]Institute of Information and Communication Technologies, Electronics and Applied Mathematics (ICTEAM), Catholic University of Louvain (UCL), Louvain-la-Neuve, Belgium.

[2]Center for Operations Research and Econometrics (CORE), Catholic University of Louvain (UCL), Louvain-la-Neuve, Belgium

anton.rodomanov@uclouvain.be (*corresponding author), yurii.nesterov@uclouvain.be

We present a new ellipsoid-type algorithm for solving nonsmooth convex optimization problems. Our algorithm can be seen as a combination of the standard Subgradient and Ellipsoid methods. However, in contrast to the latter one, the proposed method has a reasonable convergence rate even when the dimensionality of the problem is sufficiently large. The talk is based on the recent work [1].

Acknowledgments: This work has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 788368).

References

[1] A. Rodomanov and Y. Nesterov (2021). *Subgradient Ellipsoid Method for Nonsmooth Convex Problems*, arXiv preprint arXiv:2106.13340.

Minisymposium MS14

MS14 - Optimization and data analytics in sustainable manufacturing and beyond

Cluster Detection via Polynomial Optimization

Sergiy Butenko, Mykyta Makovenko, and Akis Pardalos

Wm Michael Barnes '64 Department of Industrial & Systems Engineering
Texas A&M University, College Station, TX 77843-3131, USA

Various complex systems of diverse nature and origin can be conveniently modeled using graphs or networks. If the components of the complex system of interest are thought of as vertices, then the pairwise interactions between different components can be naturally described by edges, yielding a network representation of the complex system. A common problem arising in network models is to detect clusters that satisfy some required structural properties and optimize a given performance measure. Oftentimes, clusters are modeled using various graph-theoretic structures relaxing certain characteristics of the classical clique concept, referred to as clique relaxations. Due to their ubiquitous nature, clique relaxations have attracted a considerable attention of researchers working in diverse fields. This talk will focus on novel methods for computing cliques and clique relaxations in a graph based on polynomial programming formulations. The proposed methods can be thought of as extensions of the classical Motzkin-Straus formulation, expressing the clique number in terms of the maximal value of a standard quadratic program. Previously, this intriguing relationship has motivated significant developments in quadratic optimization, copositive programming, and complexity in nonlinear optimization. Inspired by such developments, we establish cubic formulations for several clique relaxations and introduce a hierarchy of polynomial programming formulations for the maximum clique problem. The hierarchical feature of the proposed approach is fundamentally different from other well-known hierarchies in optimization. Nalemy, instead of relying on tighter and tighter convexifications of the original problem, we seek to reduce the set of local optima that are not global with each next formulation in our hierarchy. In particular, every local maximum of the last formulation in the hierarchy is guaranteed to be global.

Artificial Intelligence, Data Sciences, and Smart/Green Manufacturing

Panos M. Pardalos
University of Florida, USA
www.ise.ufl.edu/pardalos, https://nnov.hse.ru/en/latna/

Smart/green manufacturing (Industry 4.0) is the fourth industrial revolution. With advances in artificial intelligence, information and telecommunication technologies and data-enabled decision-making, smart/green manufacturing can be an essential component of sustainable development. We are going to discuss some successes in smart/green manufacturing, and focus on artificial intelligence, data enabled decision making and optimization applications. In addition, we will discuss future research directions and new challenges to society.

Minisymposium MS15

MS15 - Mind the gap: new results on the Lavrentiev phenomenon

Analysis and numerics for (non)local models and differential forms with Lavrentiev gap: beyond regularity.

Lavrentiev gap is the key phenomenon in calculus of variations to study many properties of problems with general growth. It leads to many challenges in analysis and numerics such as non-density of smooth functions and disconvergence of numerical methods. We present new density results and examples on Lavrentiev gap for general classes of differential forms and non-local models using fractal contact sets. We also construct examples for the general classes of non-local problems and design the finite element scheme to study numerically so called W-minimizers for such kind of problems.

This talk is based on several joint works with Lars Diening, Moritz Kassmann (Bielefeld), Mikhail Surnachev (Keldysh Institute of Applied Mathematics, Moscow),Johannes Storn(Bielefeld) and Christoph Ortner(UBC, Vancouver).

Some Regularity Properties of minimizers and of the value function in the Calculus of Variations

Piernicola Bettiol[1]* and Carlo Mariconda[2]

[1]Univ Brest, UMR CNRS 6205, Laboratoire de Mathématiques de Bretagne Atlantique, 6 Avenue Victor Le Gorgeu, Brest, 29200-F, France;

[2]Università degli Studi di Padova, Dipartimento di Matematica "Tullio Levi-Civita", Via Trieste 63, 35121 Padova, Italy.

piernicola.bettiol@univ-brest.fr(*corresponding author), carlo.mariconda@unipd.it

We consider a nonautonomous Bolza type problem in the context of the Calculus of Variations, and assuming a slow growth condition on the Lagrangian we derive regularity properties of the minimizers. As a consequence we deduce the local Lipschitz continuity of the value function. These results can be obtained without necessarily imposing convexity or Lipschitz regularity of the Lagrangian with respect to the state variable or the velocity variable.

References

[1] A. Cellina, 2004, *The classical problem of the calculus of variations in the autonomous case: relaxation and Lipschitzianity of solutions*, Trans. Amer. Math. Soc., 356, 415–426.

[2] F.H. Clarke, 1993, *An indirect method in the calculus of variations*, Trans. Amer. Math. Soc., 336, 655–673.

[3] C. Mariconda, 2021, *Equi-lipschitz minimizing trajectories for non coercive, discontinuous, non convex Bolza controlled-linear optimal control problems*, Trans. Amer. Math. Soc.

[4] C. Mariconda and G. Treu, 2007, *Lipschitz regularity of the minimizers of autonomous integral functionals with discontinuous non-convex integ rands of slow growth*, Calc. Var. Partial Differential Equations, 29, 99–117.

FGP2022 on Optimization

Non Occurrence of the Lavrentiev Gap for Multidimensional Autonomous Problems

Pierre Bousquet

Université Toulouse 3, France

pierre.bousquet@math.univ-toulouse.fr

We establish that the Lavrentiev phenomenon does not occur for *autonomous* problems in the multiple integrals Calculus of Variations, provided that the integrand is convex with respect to the gradient variable. The main novelty is that no other (artificial) restriction is assumed on the integrand or on the domain. The core of the proof is based on a new approximation result for a parametric version of the variational problem.

No Lavrentiev gap for some double phase integrals

Filomena De Filippis[1,*] and Francesco Leonetti[1]

[1] *DISIM, University of L'Aquila, Via Vetoio snc, Coppito, 67100 L'Aquila, Italy*
*filomena.defilippis@graduate.univaq.it (*corresponding author),*
francesco.leonetti@univaq.it

In [1] we prove the absence of the Lavrentiev gap for non-autonomous functionals

$$\mathcal{F}(u) := \int_{\Omega} f(x, Du(x))\, dx,$$

where the density $f(x,z)$ is α-Hölder continuous with respect to $x \in \Omega \subset \mathbb{R}^n$, it satisfies the (p,q)-growth conditions

$$|z|^p \leqslant f(x,z) \leqslant L(1+|z|^q),$$

where $1 < p < q < p\left(\frac{n+\alpha}{n}\right)$, and it can be approximated from below by suitable densities f_k.

Acknowledgments: We thank UnivAQ, MIUR, GNAMPA, INdAM for the support.

References

[1] F. De Filippis, F. Leonetti (2021) *No Lavrentiev gap for some double phase integrals*, Preprint.

State-Constrained Average Cost Problems

Piernicola Bettiol[1] and Nathalie T. Khalil[2]

[1]Laboratoire de Mathématiques, Université de Bretagne Occidentale 6 Avenue Victor Le Gorgeu, 29200 Brest, France

[2]SYSTEC, Department of Electrical and Computer Engineering, Porto University, and with the Institute for Systems and Robotics, 4200-465 Porto, Portugal

piernicola.bettiol@univ-brest.fr, nathalie@fe.up.pt

In this talk, we study problems with an objective function given in terms of an average cost, where unknown parameters $\omega \in \Omega$ (Ω being a compact set) intervene in the dynamics, the cost function and the left and right end-point constraints. The novelty related to the presented work is the extra ingredient, *a state constraint*, added to its formulation, which will also depend on ω. More precisely, we are interested in the following problem:

$$\begin{cases} \text{minimize} \quad \displaystyle\int_\Omega g(x(S,\omega), x(T,\omega); \omega)\, d\mu(\omega) \\ \text{over measurable functions } u : [S,T] \to \mathbb{R}^m \text{ and } W^{1,1} \text{ arcs } \{x(\cdot,\omega) : [S,T] \to \mathbb{R}^n \mid \omega \in \Omega\} \\ \text{such that} \quad u(t) \in U(t) \quad \text{a.e. } t \in [S,T] \\ \text{and, for each } \omega \in \Omega, \\ \qquad \dot{x}(t,\omega) = f(t, x(t,\omega), u(t), \omega) \quad \text{a.e. } t \in [0,T], \\ \qquad \displaystyle\int_\Omega d_{C(\omega)}(x(S,\omega), x(T,\omega))\, d\mu(\omega) = 0 \\ \qquad h(t, x(t,\omega), \omega) \le 0 \quad \text{for all } t \in [S,T]. \end{cases} \tag{P}$$

(Here, $d_E(x)$ is the Euclidean distance of a point x from the set E.) The data for this problem comprise a time interval $[S,T]$, a probability measure μ defined on a metric space Ω, functions $g : \mathbb{R}^n \times \mathbb{R}^n \times \Omega \to \mathbb{R}$, $f : [S,T] \times \mathbb{R}^n \times \mathbb{R}^m \times \Omega \to \mathbb{R}^n$, and $h : [S,T] \times \mathbb{R}^n \times \Omega \to \mathbb{R}$, a nonempty multifunction $U : [S,T] \rightsquigarrow \mathbb{R}^m$, and a family of closed sets $\{C(\omega) \subset \mathbb{R}^n \times \mathbb{R}^n \mid \omega \in \Omega\}$.

The main goal of the talk is to provide necessary optimality conditions for (P), allowing the unknown parameters to belong to a compact metric space Ω. At the end of the talk, we shall discuss possible directions in the calculus of variations setting.

References

[1] P. Bettiol and N. Khalil (2019) *Necessary optimality conditions for average cost minimization problems*, Discrete and continuous dynamical systems Serie B, 24 **5**, 2093–2124.

[2] R.B. Vinter (2005) *Minimax optimal control*, SIAM Journal on Control and Optimization, 44, 939–968.

New results on Lavrentiev's gap in the one dimensional case

C. Mariconda [1]

We consider a positive one dimensional functional (the 'energy")

$$F(y) := \int_t^T L(s, y(s), y'(s))\, ds$$

of the calculus of variations, defined among the absolutely continuous functions $y : [t, T] \to \mathbb{R}^n$. Starting from the classical theorem by Alberti and Serra Cassano on the avoidance of the gap at a given function y with $F(y) < +\infty$ when the Lagrangian L is autonomous, we emphasize how different sets of conditions are needed, depending on the number of boundary conditions (one, or two prescribed end points) that are considered in the Lipschitz approximation in norm and in energy of y. We then formulate an extension of the results to the non autonomous case.

[1] Dipartimento di Matematica "Tullio Levi–Civita", Università di Padova, Italy

FGP2022 on Optimization: Unbounded Control, Infimum Gaps, and Higher Order Normality

Monica Motta[1]*, Michele Palladino[2], and Franco Rampazzo[1]

[1]Department of Mathematics “Tullio Levi-Civita”, Padua University, Italy

[2]Gran Sasso Science Institute - GSSI, L'Aquila, Italy

monica.motta@unipd.it (*corresponding author), michele.palladino@gssi.it, franco.rampazzo@unipd.it

In optimal control theory one sometimes extends the minimization domain of a given problem, with the aim of achieving the existence of an optimal control. However, this issue is naturally confronted with the possibility of a gap between the original infimum value and the extended one. Avoiding this phenomenon is not a trivial issue, especially when the trajectories are subject to endpoint constraints. Since the seminal works by J. Warga in 1970s [3, 4], some authors have recognized ‘normality’ of an extended minimizer as a condition guaranteeing the absence of an infimum gap. (Let us recall that an extremal is called *abnormal* provided the corresponding cost multiplier in the Maximum Principle can be chosen equal to zero, and *normal* otherwise.) In particular, in [2] a generalization of Warga's criterion to a vast class of endpoint-constrained minimum problems' extensions has been recently achieved through the combined use of the notion of *abundance* (see [1] and [5, 6]) and of a suitable set separation theorem.

Yet, normality is far from being necessary for this goal, a fact that makes the search for weaker assumptions a reasonable aim. In relation with a control-affine system with unbounded controls, we provide a sufficient no-gap condition based on a notion of *higher order normality*, which is less demanding than the standard normality and involves iterated Lie brackets of the vector fields defining the dynamics.

The proof is based on a geometric approach: one the one hand, we use a set separation result obtained in [2], where the occurrence of a gap is shown to imply the linear separability of an approximating cone to the original reachable set from any approximating cone to the endpoint constraint, for a suitable concept of *approximating cone*. This linear separability translates into the abnormality of the minimizer. On the other hand, we build the approximating cones to the reachable set as convex hulls of the so-called needle variations plus new *higher order* (in the time-scale) variations involving iterated Lie brackets.

Acknowledgments: This work has been partially supported by the INdAM-GNAMPA Project 2020 “Extended control problems: gap, higher order conditions and Lyapunov functions” and by the Padua University grant SID 2018 “Controllability, stabilizability and infimum gaps for control systems”, BIRD 187147.

References

[1] B. Kaskosz, (1999) *Extremality, controllability and abundant subsets of generalized control systems*, Journal of Optimization Theory and Applications, vol. 101, pp. 73-108.

[2] M. Palladino, F. Rampazzo, (2020), *A geometrically based criterion to avoid infimum gaps in optimal control*, Journal of Differential Equations, Vol. 269, Issue 11, pp. 10107-10142.

[3] J. Warga, (1971) *Normal Control Problems have no Minimizing Strictly Original Solutions*, Bulletin of the Amer. Math. Soc., 77, 4, pp. 625-628.

[4] J. Warga, (1972) *Optimal Control of Differential and Functional Equations*, Academic Press, New York.

[5] J. Warga, (1983), *Controllability, extremality, and abnormality in nonsmooth optimal control*, J. Optim. Theory and Applic., vol. 41, no. 1, pp. 239-260.

[6] J. Warga, (1983), *Optimization and Controllability Without Differentiability Assumptions*, SIAM J. Control and Optim., vol. 21, no. 6, pp. 837-855.

Contributed Talks

An Accelerated Spectral Method for Solving Large Scale Systems of Nonlinear Equations

Sani Aji[1,2]* and Poom Kumam[1]

[1]KMUTTFixed Point Research Laboratory, Room SCL 802 Fixed Point Laboratory, Science Laboratory Building, Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thrung Khru, Bangkok 10140, Thailand.

[2]NOVA School of Scince and Technology - CMA

sani.aji@mail.kmutt.ac.th*, poom.kum@kmutt.ac.th

Systems of nonlinear equations arise from many practical applications, such as in ℓ_1-norm regularized optimization problems from compressed sensing, variational inequalities and chemical equilibrium problems. Some of the popular iterative methods for solving these systems include Newton and quasi-Newton methods which have fast convergence from good initial points. However, these methods require computing the Jacobian matrix or an approximation to it at every iteration, which limits their adequacy to handle large scale problems.

In this work, we propose a novel iterative algorithm for solving large scale systems of nonlinear equations based on a combination of the inertial step, the spectral gradient and projection methods. We incorporate the inertial step in the algorithm specifically to accelerate its speed of convergence. We state the global convergence of the algorithm under suitable assumptions, and present numerical experiments on some test problems. From these experiments, our approach proves to be more efficient when compared with some existing algorithms in literature.

Acknowledgments: The authors acknowledge the financial support provided by the Center of Excellence in Theoretical and Computational Science (TaCS-CoE), KMUTT and Center under Computational and Applied Science for Smart Innovation research Cluster (CLASSIC), Faculty of Science, KMUTT. The first author was supported by the Petchra Pra Jom Klao Ph.D. research Scholarship from King Mongkut's University of Technology Thonburi (KMUTT) Thailand (Grant No. 19/2562). Also, this research project is supported by Thailand Science Research and Innovation (TSRI) Basic Research Fund: Fiscal year 2021 under project number 64A306000005.

A new exact method for linear bilevel problems with multiple objective functions at the lower level

Maria João Alves[1,3]*, Carlos Henggeler Antunes[2,3]

[1]CeBER and Faculty of Economics, University of Coimbra, Portugal

[2]Department of Electrical and Computer Engineering, University of Coimbra, Portugal

[3]INESC Coimbra, Portugal

mjalves@fe.uc.pt (*corresponding author), ch@deec.uc.pt

We propose a general-purpose exact method to solve the linear bilevel programming problems with multiple objective functions at the lower level (semivectorial linear bilevel problems). The method is based on the exhaustive search of efficient extreme solutions of an associated multiobjective linear programming (MOLP) problem with many objective functions. Therefore, its development required the design and implementation of an effective vector-maximum algorithm for MOLP problems.

The method relies on a proposition stating that an optimistic optimal solution to the semivectorial linear bilevel problem is an efficient extreme point of this MOLP problem for which the number of objective functions is equal to the number of lower-level objective functions plus the number of upper-level decision variables plus 1. Since the number of objective functions of the associated MOLP problem increases with the number of upper-level decision variables, and the number of efficient extreme points of a MOLP problem grows very quickly with the number of objective functions, this method is mainly adequate to bilevel problems with a small number of upper-level decision variables. This number is the dimension with the major impact on the computational effort required by the method.

Computational results using randomly generated instances are presented to compare the performance of this new method with an exact method proposed by other authors and with a local search heuristic we also developed based on the same principles, which has shown to be quite effective in problems where the global optimum is difficult to obtain within a reasonable timeframe.

Acknowledgments: This work has been supported by FCT – Fundação para a Ciência e a Tecnologia, I.P., Projects UIDB/05037/2020, UIBD/00308/2020 and MAnAGER (POCI-01-0145-FEDER-028040).

FGP2022 on Optimization

Handling infinitely many inequality constraints in function optimization problems using kernel methods

Pierre-Cyril Aubin-Frankowski[1]* and Alessandro Rudi [1]

[1] INRIA - Département d'Informatique de l'École Normale Supérieure, PSL Research University

pierre-cyril.aubin@inria.fr (*corresponding author), alessandro.rudi@inria.fr

Many optimization problems over function spaces (density estimation, optimal transport, state-constrained linear optimal control,. . .) involve an infinite number of pointwise inequality constraints. This side information can originate from both physical and theoretical constraints on the model such as "stay within boundaries" in path-planning or "be nonnegative and integrate to one" in density estimation. On the other hand, reproducing kernels are propitious for pointwise evaluations and some kernels encode very rich classes of functions, suitable to approximate the function spaces of interest.

However, representer theorems, which ensure the numerical applicability of kernels, cannot be applied for an infinite number of evaluations. Through constructive algebraic and geometric arguments, I will present how to tackle this question by perturbing the constraints, through coverings in infinite dimensions [1] and through kernel Sum-Of-Squares [3]. Both schemes entail an extra computational price, involving second-order conic or SDP constraints, but assessing the amount of perturbation enables to prove rates on the convergence of the schemes. Tightening guarantees the satisfaction of the constraints and was used in state-constrained optimal control [2] and density estimation [3]. Relaxation instead gives better approximation rates for global optimization [4].

Acknowledgments: This work has been supported by the European Research Council (grant REAL 947908).

References

[1] P.-C. Aubin-Frankowski, Z. Szabo (2020) *Hard Shape-Constrained Kernel Machines*, NeurIPS, Volume **33**, 384–395.

[2] P.-C. Aubin-Frankowski (2021) *Linearly-constrained Linear Quadratic Regulator from the viewpoint of kernel methods*, SIAM Journal on Control and Optimization, Volume **59(4)**, 2693–2716.

[3] U. Marteau-Ferey, F. Bach, A. Rudi (2020) *Non-parametric Models for Non-negative Functions*, NeurIPS, Volume **33**, 12816–12826.

[4] A. Rudi, U. Marteau-Ferey, F. Bach (2020) *Finding Global Minima via Kernel Approximations*, Under review, https://arxiv.org/abs/2012.11978.

Solving Nonconvex-Nonconcave Min-Max Problems exhibiting Weak Minty Solutions

Axel Böhm[1]

[1]University of Vienna

axel.boehm@univie.ac.at

We investigate a structured class of nonconvex-nonconcave min-max problems exhibiting so-called *weak Minty* solutions, a notion which was only recently introduced, but is able to simultaneously capture different generalizations of monotonicity. We prove novel convergence results for a generalized version of the optimistic gradient method (OGDA) in this setting matching the ones recently shown for the extragradient method (EG). In addition we propose an adaptive stepsize version of EG, which does not require knowledge of the problem parameters.

Acknowledgments: This work has been supported by Research supported by the doctoral programme *Vienna Graduate School on Computational Optimization (VGSCO)*, FWF (Austrian Science Fund), project W 1260.

Extremal functional determinants: progress in the Dirichlet and periodic cases

J.-B. Caillau[1]*, Y. Chitour[2], P. Freitas[3] and Y. Privat[4]

[1]Université Côte d'Azur, CNRS, Inria, LJAD

[2]Université Paris-Saclay, CNRS, L2S

[3]University of Lisbon

[2]Université de Strasbourg, CNRS, IRMA

jean-baptiste.caillau@univ-cotedazur.fr (*corresponding author)
yacine.chitour@centralesupelec.fr
psfreitas@fc.ul.pt
yannick.privat@unistra.fr

Maximisation of the functional determinant of Sturm-Liouville operators with respect to a potential has been recently studied in [1]. We continue this work by considering more generally extremisation of this determinant over different boundary conditions, still in one dimension. In particular, we address the question of the Dirichlet case for integrable potentials, and of periodic conditions [2, 3] for essentially bounded potentials. Notwithstanding the topological difference between the two situations, we show that extremal solutions share some common properties. The key ingredient of the analysis is the reduction to a suitable optimal control problem. In particular, we prove the following.

Theorem. For any positive M, the functional determinant of the differential operator $-\Delta + V$ has a unique maximiser and a unique minimiser over the set of essentially bounded functions V on the circle such that $\|V\|_\infty \leq M$.

Acknowledgments: This project is supported by the FMJH Program PGMO and EDF-Thales-Orange (extdet PGMO grant).

References

[1] Aldana, C.; Caillau, J.-B.; Freitas, P. Maximal determinants of Schrödinger operators. *J. Ec. polytech. Math.* **7** (2020), 803–829.

[2] Burghelea, D.; Friedlander, L.; Kappeler, T. On the Determinant of Elliptic Differential and Finite Difference Operators in Vector Bundles over S^1. *Commun. Math. Phys.* **138**, 1–18 (1991).

[3] Forman, R. Determinants, Finite-Difference Operators and Boundary Value Problems. *Commun. Math. Phys.* **147**, 485–526 (1992).

Optimal switching a formation of Dubins cars

Amélia C.D. Caldeira[1]*, Luís T. Paiva[2], Dalila B.M.M. Fontes[3] and Fernando A.C.C. Fontes[2]

[1] SYSTEC-ARISE, LEMA-ISEP, Politécnico do Porto, Porto, Portugal
[2]SYSTEC-ARISE, Faculdade de Engenharia, Universidade do Porto, Porto, Portugal
[3]LIADD- INESC TEC, Faculdade de Economia, Universidade do Porto, Porto, Portugal
acd@isep.ipp.pt(*corresponding author), ltpaiva@fe.up.pt, fontes@fep.up.pt, faf@fe.up.pt

We address the problem of optimally reconfigure the geometry of a formation of undistinguishable mobile robots with nonholonomic dynamics. In this problem, each robot of the formation moves from the current oriented position to a selected target oriented position using the shortest path. Here, we combine results from previous work on optimal formation switching when the agents are holonomic with results on the structure of the shortest path for nonholonomic agents. The problem of switching the shape of the formation, has been addressed previously (see e.g. [1] and references therein). However, here we are considering that the agents are nonholonomic systems and so, the initial and final headings of each agent have a major influence in the trajectories used. The problem of finding the path of minimum distance with limited curvature between two oriented points in the plane is known as the Dubin's problem. This is equivalent to the problem of the path of minimum length between two oriented points for a car system with minimum turning radius. The solution to this problem was characterized by Dubins [2] . The same result was later shown by [3, 4] using Pontryagin's Maximum Principle instead of geometric arguments The problem of determining how a structured formation of nonholonomic vehicles can be reorganized into another formation is studied in this work. Given a set of initial nodes (oriented positions) and a set of final target nodes (also oriented positions), the questions addressed are: a) which of the vehicles should go to which terminal position? b) which is the path between each pair of initial and final positions? — such that the total distance travelled by the agents of the formation is minimized.

Acknowledgments: We acknowledge the support of FEDER/COMPETE2020/NORTE2020/POCI/PIDDAC/MCTES/FCT funds through grants PTDC/MAT-APL/28247/2017-ToChair, 02/SAICT/2017-31447|POCI-01-0145-FEDER-031447|FCT–UPWIND, and 02/SAICT/2017-31821|POCI-01-0145-FEDER-031821|FCT–FAST.

References

[1] A. C. D. Caldeira, D. B. M. M. Fontes, and F. A. C. C. Fontes, "Obstacle avoidance in optimal switching of a formation geometry," in *Proceedings of CONTROLO 2012*, 2012.

[2] L. E. Dubins, "On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and tangents," *American Journal of mathematics*, vol. 79, no. 3, pp. 497–516, 1957.

[3] J.-D. Boissonnat, A. Cérézo, and J. Leblond, "Shortest paths of bounded curvature in the plane," *Journal of Intelligent and Robotic Systems*, vol. 11, no. 1-2, pp. 5–20, 1994.

[4] H. J. Sussmann and G. Tang, "Shortest paths for the reeds-shepp car: a worked out example of the use of geometric techniques in nonlinear optimal control," Rutgers Center for Systems and Control, Tech. Rep. SYCON-91-10, September 1991.

A Distributionally Robust Approach for the Optimal Cable Design of Wind Farms

Adelaide Cerveira[1]* and Agostinho Agra[1]

[1]DMAT, University of Trás-os-Montes and Alto Douro and INESC-TEC

[2]DMat and CIDMA, University of Aveiro

cerveira@utad.pt (*corresponding author), aagra@ua.pt

We consider the problem of designing the cable network that interconnects the turbines to the substation in wind farms, aiming to minimize both the infrastructure cost and the cost of the energy losses in the cables. In [1], several integer linear programming models are proposed to solve the case where the losses are deterministic. However, in practical situations, energy losses depend on wind direction and speed, which is rarely known with certainty. Here we propose a distributionally robust two-stage mixed integer model that adapts a model proposed in [1], where the energy losses follow a probability distribution which is assumed to belong to an ambiguity set. Following [2], a decomposition algorithm is employed to solve the model. Computational results are reported.

Acknowledgments: This work is financed by National Funds through the Portuguese funding agency, FCT - Fundação para a Ciência e a Tecnologia, within project LA/P/0063/2020, and grants UIDB/04106/2020 and UIDP/04106/2020.

References

[1] Adelaide Cerveira, Amaro de Sousa, E. J. Solteiro Pires, and José Baptista (2016) *Optimal Cable Design of Wind Farms: The Infrastructure and Losses Cost Minimization Case*, IEEE TRANSACTIONS ON POWER SYSTEMS, 31, **6**, 4319–4329.

[2] Manish Bansal, Kuo-Ling Huang, and Sanjay Mehrotra *Decomposition algorithms for two-stage distributionally robust mixed binary programs*, SIAM Journal on Optimization, 28, **3**, 2360–2383. 2018.

Optimal control for the groundwater pollution: a game theory approach

Emmanuelle Augeraud-Véron[1], Catherine Choquet[2], Éloïse Comte[3]* and Moussa M. Diédhiou[2]

[1]GREThA, University of Bordeaux, Bordeaux, France

[2]MIA, La Rochelle University, La Rochelle, France

[3]LISC, INRAE, Aubière, France

emmanuelle.augeraud@u-bordeaux.fr, catherine.choquet@univ-lr.fr,
eloise.comte@inrae.fr (*corresponding author), moussa_mory.diedhiou@univ-lr.fr

The purpose of this talk is the study of a spatial differential game in the case of non cooperative consequence for the optimal control problem of groundwater pollution.

Two polluters with different spread policies are considered. Two economic objectives are defined, taking into account the polluter's private benefits and the environmental damage due to the pollution. These spatio-temporal objectives are constrained by the hydrogeological state equations, namely the spread of the pollutant in the underground and the velocity of the flow, respectively modeled by a convection-diffusion-reaction equation coupled with an elliptic partial differential equation ([1]).

We prove the existence result of a Nash equilibrium by using a fixed point strategy. Nonlinearities of the model are complex issues for proving the uniqueness. We characterize the Nash equilibrium by a system of partial differential equations derived from Pontryagin's approach and its corresponding optimality conditions ([2]). A uniqueness result follows, according to some additional assumptions on the objective functions. We conclude with numerical simulations in a realistic context of agricultural pollution due to fertilizer spreading.

References

[1] E. Augeraud-Véron, C. Choquet and É. Comte (2019) *Existence, uniqueness and asymptotic analysis of optimal control problems for a groundwater pollution*, ESAIM : Control, Optimisation and Calculus of Variations, 25, Article number 53.

[2] E. Augeraud-Véron, C. Choquet, É. Comte and M. M. Diédhiou, *A game theory approach for the groundwater pollution control*, Submitted.

Accurate identification of time-optimal solutions of fast-oscillating control systems

Lamberto Dell'Elce *, Jean-Baptiste Caillau, and Jean-Baptiste Pomet

Inria & Université Côte d'Azur CNRS

lamberto.dell-elce@inria.fr (*corresponding author)

For a control system with one fast periodic variable φ, with a small parameter measuring the ratio between time derivatives of fast and slow variables, we consider the Hamiltonian equation resulting from applying Pontryagin maximum principle (PMP) for the minimum time problem with fixed initial and final slow variables and free fast variable, namely

$$\begin{aligned} t_f \to \min \quad & \text{subject to} \\ & \dot{I} = \varepsilon \left[F_0(I, \varphi, \varepsilon) + \sum_{i=1}^{m} u_i F_i(I, \varphi, \varepsilon) \right], \\ & \dot{\varphi} = \omega(I) + \varepsilon \left[G_0(I, \varphi, \varepsilon) + \sum_{i=1}^{m} u_i G_i(I, \varphi, \varepsilon) \right], \\ & |u| = \sqrt{u_1^2 + \cdots + u_m^2} \leq 1, \\ & I(0) = I_0, \qquad I(t_f) = I_f. \end{aligned} \tag{0.1}$$

The slow state variables, I, live on some manifold M of dimension n, and there are fixed endpoint conditions I_0, I_f. There are no endpoint conditions neither on $\varphi \in \mathbb{R}$, nor on its angle class in $\mathbb{S}^1 \simeq \mathbb{R}/2\pi\mathbb{Z}$. We assume that the pulsation $\omega(I)$ is uniformly bounded on M by some positive constant.

One may perform averaging at least under normalization of the adjoint vectors and define a "limit" average system. The paper is devoted to the convergence properties of this problem as the small parameter tends to 0. We show that using the right transformations between boundary conditions of the "real" and average systems, and second-order corrections of the dynamics of I (that can be evaluated via a simple integration after solving the averaged shooting problem) leads to a reconstruction of the fast variable of order ε on interval of times of order $1/\varepsilon$. Relying on this, we propose a procedure to efficiently reconstruct the solution of the two point boundary problem for nonzero ε using only the solution of the average optimal control problem.

The practical interest of this contribution is the relationship that is established between the magnitude of the small parameter ε (read control) and the boundary phases, φ_0 and φ_f. Specifically, the number of possible solutions and their quantitative assessment for a specific ε can be estimated using only the information given by the average solution. In addition, the proposed methodology provides insight into how new combinations of φ_0 and φ_f (i.e., multiple potential local optima) are generated when ε is decreased. This qualitative behavior cannot be fully understood by inspecting solutions of the original problem owing to numerical difficulties related to the integration of trajectories for very low ϵ.

The methodology is illustrated by means of a low-thrust orbital transfer in space mechanics.

Sensitivity approach for the Simulation based Optimization of Multiphase Flow with a dynamic Contact Line

Elisabeth Diehl[1]*, and Stefan Ulbrich[1]

[1]Department of Mathematics
Technische Universitt Darmstadt
Dolivostrae 15, 64293 Darmstadt, Germany

diehl@mathematik.tu-darmstadt.de (*corresponding author), ulbrich@mathematik.tu-darmstadt.de

In this talk, we present a simulation based optimization approach for optimal control of multiphase flow in the context of wetting phenomena. In general, the mathematical model of multiphase flow is governed by the Navier-Stokes equations together with jump conditions to connect the flow of the different fluids or phases at their interfaces. Based on L_p-maximal regularity of the underlying linear twophase problem we proved the differentiability of the solution with respect to initial and distributed controls for appropriate spaces. To describe dynamic wetting or dewetting, a multiphase flow problem with a liquid-gas interface and a dynamic contact line, the Navier-Stokes Equations are complemented by a transport equation for flow advection. This transport equation originates from an algebraic Volume-of-Fluid approach, that leads to an One-Field-Formulation of the problem. For this purpose, we assume the whole domain to be filled with one single fluid which is not constant in density and viscosity. The model is completed by appropriate initial and boundary conditions, where for example the dynamic contact angle enters as a boundary condition.

We use the introduced model for the simulation and optimization of a wetting process, motivated by gravure printing. For good printing results, it is essential to remove excess ink from the printing plate, except for a thin film that remains. For this purpose, a steel strap is pulled over the surface, which is also called a doctor blade. Our aim is to develop a gradient-based multilevel optimization method for shape optimization of the doctor blade and parameter identification problems arising in wetting processes. To achieve this, we derive sensitivity equations for the continuous flow problem together with a suitable descretization procedure. Furthermore, we show numerical results with an adapted numerical solver, based on the well known interFoam solver from the OpenFOAM library.

Acknowledgments: This work has been supported by the DFG within CRC 1194.

References

[1] T. Bitsch, J. Schfer, E. Diehl, H. M. Sauer, E. Drsam and S. Ulbrich (2019) *An experimental and numerical cooperative research concept for doctor blading*, In: Proceedings of the 46th International Research Conference of iarigai, Stuttgart, Germany, September 15-18, 68-74.

[2] E. Diehl, J. Haubner, M. Ulbrich and S. Ulbrich (2020) *Differentiability results and sensitivity calculation for optimal control of incompressible two-phase Navier-Stokes equations with surface tension*, Technical Report.

Optimal Control meets Reinforcement Learning: Docking Maneuver in Space

Simon Gottschalk[1]*

[1]Universität der Bundeswehr München

simon.gottschalk@unibw.de (*corresponding author)

A docking maneuver of a satellite to a target object is a frequently needed maneuver in space. It is, for instance, of special interest for space shuttles transporting astronauts to a space station or for space debris removal, where a satellite tries to dock to a tumbling debris object. From a mathematical point of view, such a maneuver contains some challenges. Additional to a possibly complex dynamical system, the corresponding optimal control problem may have several demanding constraints. In particular, constraints in order to avoid a collision with the target or third parties objects are difficult to deal with. Additionally, we are interested in a framework, which is able to act online and to react to small disturbances.

For this purpose, we use Reinforcement Learning (RL), which trains a policy (controller) based on simulated data in an offline phase and is able to apply this controller very fast in the actual control phase. This concept has already been linked to classical optimal control theory in [1]. For an application of RL to a docking maneuver and a comparison to classical optimal control methods, we refer to [2]. Note that the collision constraints can be comparatively easy integrated. However, sometimes in the offline phase, the data based approach leads to inefficient training behavior for simple movements. In these cases, a classical optimization based control, which has system insights, would outperform this approach.

As a remedy, we will present a technique, which benefits from both worlds. We will discuss, how an optimization problem can be integrated into the RL framework in a hierarchical manner. Thereby, we will explicitly mention how the optimization problem can be efficiently solved without slowing down the trainings phase too much. Note, however, that we assume to have a model of the considered dynamical system, which is not needed in the classical RL framework. Finally, we will discuss this new approach by applying it to a docking maneuver of two satellites.

Keywords: Reinforcement Learning, optimal control with ODEs, docking maneuvers for satellites, space debris removal, collision avoidance

Acknowledgments: This research has been conducted within the project frame of SeRANIS Seamless Radio Access Networks in the Internet of Space. The project is funded by dtec.bw Digitalization and Technology Research Center of the Bundeswehr, grant number 150009910.

References

[1] S. Gottschalk, M. Burger and M. Gerdts (2020) *A projected primal-dual gradient optimal control method for deep reinforcement learning*, Journal of Mathematics in Industry, 10, 1 – 22.

[2] C. E. Oestreich, R. Linares and R. Gondhalekar (2021) *Autonomous six-degree-of-freedom spacecraft docking with rotating targets via reinforcement learning*, Journal of Aerospace Information Systems, 18 **7**, 417–428.

Optimal Control of Solar Sails for a Sun Occultation Mission

Alesia Herasimenka [1]*, Lamberto Dell'Elce[1], Nicolò Bernardini[2] and Nicola Baresi[2]

[1] Université Côte d'Azur, CNRS, Inria, LJAD

[2] Surrey Space Center, University of Surrey

alesia.herasimenka@univ-cotedazur.fr (*corresponding author)

Studying solar corona is fundamental to gain insight into a plethora of phenomena with major impact on satellite dynamics and operations. Due to the very-modest brightness of the corona (namely, roughly one million times weaker than the Sun), observations are achieved by means of coronagraphs, which are optical instruments provided with a mask to occult the Sun's disk. However, ground-based measurements suffer from scattering due to the atmosphere. A novel concept was recently proposed, which consists of using natural bodies as occulting disks.[1,2] The idea is to place a satellite in proximity of the tip of the umbra cone generated by a celestial body (*e.g.*, Earth or Moon). Assuming that electrical power is gathered via solar panels, the satellite is constrained to periodically leave the observation zone end expose itself to sunlight to recharge its batteries.

The possibility of using a solar sail to maneuver the satellite in a propellantless fashion was suggested.[1] This work further investigates this option by offering a detailed study of optimal solar-sail-actuated maneuvers for repeated observations. The circular restricted three-body problem (Earth-Sun system, where Earth serves as occulting body) is used to model the motion of the satellite. Trajectory design is formulated as a periodic optimal control problem aimed at maximizing the duty cycle of the observations, defined as the fraction of orbital period devoted to observations. Charge of the batteries is part of the state variables, and its rate of change is function of the position of the satellite and increases from zero to its maximum value across the penumbra region. Pontryagin maximum principle (PMP) is then applied to deduce necessary conditions of optimality for the aforementioned cost function (duty cycle) and dynamical model. PMP reveals the periodicity of boundary conditions of both state and adjoint variables. Numerical solution is achieved by using indirect techniques. To this purpose, a reliable initial guess is obtained by outcomes of reference[1] and differential continuation is used to degrade the reflectivity of the sail and to explore various capacities of the batteries and size of the sail. Stability of the periodic trajectory is also discussed, as well as efficiency and convergence of the numerical methods.

Acknowledgments: This work has been supported by ESA, contract 4000134950/21/NL/GLC/my

References

[1] Bernardini, Nicolo and Baresi, Nicola and Armellin, Roberto and Eckersley, Steve. (2020). *Feasibility Study of Sun Occultation Missions Using Natural Bodies*, International Astronautical Congress (IAC), IAF.

[2] Eckersley, Steve and Kemble, Stephen (2017) *Method of solar occultation*, Google Patents, US Patent 9,676,50.

A Recursive Multilevel Algorithm for Deep Learning

Stefan Ulbrich[1]*, Isabel Jacob[1],

[1]Technical University of Darmstadt, Department of Mathematics, 64291 Darmstadt, Germany

ulbrich@mathematik.tu-darmstadt.de (*corresponding author), jacob@mathematik.tu-darmstadt.de

Neural networks are one of the most popular approaches in machine learning. They have been applied in many different scenarios like image classification or voice recognition. As computing capacity advances in modern computers, the complexity of neural networks also increases leading to deep networks. This entails the challenge of finding more efficient learning algorithms.

In this talk, we formulate residual neural networks as discretisations of an Euler forward method. Motivated by this formulation, we explore measures to enhance the stability of residual neural networks. Moreover, we propose a recursive multilevel optimisation approach for training residual neural networks for image recognition by alternating training of networks on a coarse and on a fine dataset. We present numerical results to demonstrate the usefulness of our approach.

Sensitivity analysis of an irrigation optimal linear-control problem

Ana P. Lemos-Paião[1]*, Sofia O. Lopes[2,4], and M. d. R de Pinho[3]

[1]CFIS, Universidade do Minho, Portugal

[2]CFIS and Departamento de Matemática e Aplicações, Universidade do Minho, Portugal

[3]SYSTEC and ISR-Porto, Faculdade de Engenharia, Universidade do Porto, Portugal

[4]Collaborator with SYSTEC and ISR-Porto, Universidade do Porto, Portugal

anapaiao@fisica.uminho.pt (*corresponding author), sofialopes@math.uminho.pt, mrpinho@fe.up.pt

The aim of this work is to study a parametric optimal linear-control (POLC) problem associated with the one analysed in [3]. It consists in optimizing the water usage for the daily irrigation of a given farmland and guaranteeing that the field crop is kept in a good state of preservation, by considering that $x(t) \geq x_{\min}$ for all instant of time t under study, where $x(t)$ represents the soil moisture along time and $x_{\min}$ is the hydrological need of the crop. The percentage of water loss due to the run-off and deep infiltration, β, is an important parameter for the daily irrigation modelling. However, it is hard to estimate and it may be subject to perturbations. So, the proposed problem is seen as a parametric optimal control one in function of β, where the control appears linearly in the dynamic and cost. Thus, through sensitivity analysis, we intend to know how the optimal solution and optimal cost depends on β.

For POLC problems, the idea is to transcribe them into non-linear programming problems to which the sensitivity analysis developed in [2] can be applied. For this class of problems one can find, e.g., [1, 4]. Due to the control solution profile for the proposed irrigation problem, it make sense to follow [4], where the authors study POLC problems with a state constraint, assuming that the optimal control has finitely many bang-bang and boundary arcs and it is discontinuous at junction times.

Acknowledgments: This work was supported by the FEDER/COMPETE/NORTE2020/POCI/FCT funds, under project To CHAIR - POCI-01-0145-FEDER-028247.

References

[1] C. Büskens, H. J. Pesch and S. Winderl (2001) *Real-time solutions of bang-bang and singular optimal control problems*. In: Online optimization of large scale systems. Springer, Berlin, 129–142.

[2] C. Büskens and H. Maurer (2001) **Sensitivity analysis and real-time optimization of parametric nonlinear programming problems*. In: Online optimization of large scale systems. Springer, Berlin, 3–16.

[3] S. O. Lopes, F. A. C. C. Fontes, R. M. S. Pereira, MdR de Pinho and A. M. Gonçalves (2016) *Optimal control applied to an irrigation planning problem*, Math. Probl. Eng., 2016 **Article ID 5076879**, 10 pages.

[4] H. Maurer and G. Vossen (2007) *Sufficient conditions and sensitivity analysis for optimal bang-bang control problems with state constraints*. In: System Modeling and Optimization. Springer, Berlin, Heidelberg. 2016 **Article ID 5076879**, 82–99.

Large-scale nonconvex optimization: randomization, gap estimation, and numerical resolution

Frédéric Bonnans[1], Kang Liu[1,2], Nadia Oudjane[3], Laurent Pfeiffer[1] and Cheng Wan[3]

[1]Inria-Saclay and CentraleSupélec

[2]École Polytechnique

[3]EDF R&D

Frederic.Bonnans@inria.fr, kang.liu@polytechnique.edu, nadia.oudjane@edf.fr, laurent.pfeiffer@inria.fr, cheng.wan@edf.fr

We address a large-scale nonconvex optimization problem, involving an aggregative term. This term can be interpreted as the sum of the contributions of N agents to some common good, with N large. Our setting is similar to the one investigated by Wang in [1]. We introduce a randomized relaxation for this problem, and prove that the relaxation gap decreases with the number of agents N, independently of the dimension of the aggregative term. We give a stochastic method to construct an approximate minimizer of the original problem, given an approximate solution of the randomized problem. With the help of McDiarmid's concentration inequality [4], we quantify the probability of success of the method. The Frank-Wolfe (FW) algorithm [2] is used to solve the randomized problem, and it allows to decompose this large-scale problem to N individual sub-problems at each iteration. A sub-linear convergence rate is obtained for the FW algorithm. In order to handle the memory overflow problem possibly caused by the FW algorithm, we propose a stochastic Frank-Wolfe (SFW) algorithm, which ensures the convergence in both expectation and probability senses. Numerical experiments on a mixed-integer quadratic program and a multi-agent optimal control problem illustrate the efficiency of the method.

Acknowledgments: This work benifited from the FMJH program PGMO and from the support to this program from EDF.

References

[1] M. Wang. (2017). Vanishing price of decentralization in large coordinative nonconvex optimization. *SIAM Journal on Optimization*, 27:1977–2009.

[2] M. Jaggi. (2013). Revisiting Frank-Wolfe: Projection-free sparse convex optimization. *International Conference on Machine Learning*, 427–435.

[3] C. Alasseur, J. F. Bonnans, N. Oudjane, A. D. Paola, A. Seguret and V. Trovato. (2020). Decomposition of high dimensional aggregative stochastic control problems. *arXiv:2008.09827*.

[4] C. McDiarmid. (1989). On the method of bounded differences. *Surveys in combinatorics*, 141:148–188.

[5] B. Delyon. (2015). Exponential inequalities for dependent processes. *hal-01072019*.

Fast method for solving monotone equation

Radu Ioan Boţ[1]*, Ernö Robert Csetnek[1], and Dang-Khoa Nguyen[1]

[1]Faculty of Mathematics, University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria

radu.bot@univie.ac.at(*corresponding author), robert.csetnek@univie.ac.at, dang-khoa.nguyen@univie.ac.at

In the framework of a real Hilbert space, we consider the problem of finding zeros of a monotone and Lipschitz operator. We study as a starting point a second-order dynamical system that involves both viscous and Hessian-driven dampings associated with the problem. Our methods achieve fast convergence rates on the operator norm and variational inequality. In addition, the trajectory asymptotically weakly converges to a solution of the original problem. We then propose an inertial-type algorithm, which results from the time discretization of the second-order dynamical system. The generated iterate shares compatible features with those obtained in the continuous setting.

Acknowledgments: This work has been supported by FWF (Austrian Science Fund), projects W 1260, P 29809-N32, and P 34922-N.

Generalized conditional gradient method for potential mean field games

Pierre Lavigne[1], Laurent Pfeiffer[2*]

[1]Institut Louis Bachelier

[2]Inria and L2S, CentraleSupélec, Université Paris-Saclay

pierre.lavigne@institutlouisbachelier.org, laurent.pfeiffer@inria.fr (*corresponding author)

The starting point of our work is a general connexion, in the context of potential non-atomic games, between the Frank-Wolfe algorithm and the fictitious play, a best-response procedure (Ref. 2). We investigate this connexion in the context of potential mean-field games, in their formulation as a coupled system of two PDEs. More precisely, we show that in this framework, the fictitious play algorithm (as described in Ref. 1) is nothing but a particular implementation of the so-called generalized conditional gradient method. At a technical level, we investigate the rate of convergence of the variables of the model for various stepsize rules.

References

[1] P. Cardaliaguet and S. Hadikhanloo (2017) *Learning in mean field games: The fictitious play*, ESAIM: Control, Optimization, and Calculus of Variations, Volume **23**, Number **2**, 569 - 591.

[2] S. Sorin (2022) *Continuous Time Learning Algorithms in Optimization and Game Theory*, Dynamic Games and Applications, Volume **34**.

Nonlinear robust shape optimization using second-order approximations and model order reduction for electric motors

Björn Polenz[1]* and Stefan Ulbrich[1]

[1]Department of Mathematics
Technische Universität Darmstadt
Dolivostraße 15, 64293 Darmstadt, Germany

polenz@mathematik.tu-darmstadt.de (*corresponding author), ulbrich@mathematik.tu-darmstadt.de

We examine the optimization of an asynchronous electrical machine. Our goal is to optimize the width and height of conductive bars, located in the rotor of the machine, such that the average of the joule losses in these bars over a given time horizon is minimized while a fixed torque is preserved. To compute the state of the machine we solve a magnetoquasistatic approximation of Maxwell's equations coupled with circuit equations for the windings in the stator and the bar network in the rotor. We model the rotation of the rotor by the equation of motion and consider the transient phase as well as the steady state of the machine. This leads to a system of partial differential algebraic equations.

Since the motor is used in environments with different temperatures and the temperature has an influence on the electrical conductivity of the bars, we consider the conductivity as uncertain. We use a worst-case approach to treat this uncertainty and end up with a bi-level structured problem. Since these problems are difficult to solve computationally, we employ a strategy utilizing a quadratic approximation of the robust formulation as a surrogate model combined with an adaptive strategy to control the introduced error.

The problem formulation as well as the robustification of the optimization lead to high computational cost. To accelerate our computations, we apply model order reduction techniques in the form of the proper orthogonal decomposition. Numerical results are presented.

Acknowledgments: This work has been supported by BMBF Grant 05M2018RDA (PASIROM).

References

[1] O. Lass and S. Ulbrich. Model order reduction techniques with a posteriori error control for nonlinear robust optimization governed by partial differential equations, *SIAM J. Sci. Comput.*, 39(5), S112–S139, 2017.

[2] P. Kolvenbach, O. Lass and S. Ulbrich. An approach for robust PDE-constrained optimization with application to shape optimization of electrical engines and of dynamic elastic structures under uncertainty, *Optim. Eng.*, 19, 697–731, 2018.

Mixed-Integer Nonlinear Optimization of Heating Networks

Lea Rehlich[1]*, Stefan Ulbrich[1] and Marc Pfetsch[1]

[1]Department of Mathematics
Technische Universitt Darmstadt
Dolivostrae 15, 64293 Darmstadt, Germany

rehlich@mathematik.tu-darmstadt.de (*corresponding author), ulbrich@mathematik.tu-darmstadt.de,
pfetsch@mathematik.tu-darmstadt.de

The decarbonization of the heating sector plays a major role in reducing CO_2 emissions due to its big share on the total energy usage. Therefore district heating networks and their optimal operating strategies are of great importance as heating alternative in combination with renewable energy generation and waste heat. However, especially in future heating networks involving low temperatures and decentralized structures with flexible consumers and multiple suppliers, optimal operating strategies are challenging.

We investigate a global optimization approach of heating networks that aims at finding the cost-optimal operating strategy. The systems state is described by the underlying physical equations including operating parameters such as the mass flow rate, temperature and pressure. It is based on nonlinear equations together with binary variables, e.g., for determining the flow direction in the network pipes. Thus, we obtain a Mixed-Integer Nonlinear Optimization Problem which is solved with the solver SCIP.

For real-sized district heating networks the solving process leads to high computational costs, especially for networks with high flexibilities. We examine the difficulties in the solving process and consider methods to lower the computational costs. This includes deriving additional variable bounds and adding inequalities. Numerical results with example networks based on real district heating networks are presented.

Acknowledgments: This work has been supported by the German Federal Ministry for economic affairs and energy (BMWi) - project number 03EN3012A.

A preliminary approach to optimisation of data routing and schedule length in TSCH networks

António Pereira[1], and Cristina Requejo[1]*

[1]University of Aveiro

antoniop@ua.pt, crequejo@ua.pt (*corresponding author)

The data transmission process is of the utmost importance nowadays and consists of transferring data between digital devices. The process enables devices or components within devices to communicate between them and occurs via point-to-point data streams or channels. These channels may previously have been in the form of copper wires but are now much more likely to be part of a wireless network.

Time Slotted Channel Hopping (TSCH) is one of the medium access control modes that is defined in IEEE 802.15.4e standard, a recent protocol designed for the Industrial Internet of Things (IIoT) applications such as smart city, smart home, and smart factory. Data transmission in TSCH networks is performed according to a tight schedule. The higher the schedule length, the higher the energy consumption of the network nodes and the end-to-end transmission delay. The amount of data transferred within a given time period is the data transfer rate, which dictates whether or not a network supports tasks that require complex, data-intensive applications. A higher data throughput improves user experience and increases reliability.

In this work, we consider the problem of finding optimal routing topologies that minimize the schedule length. We study the problem, propose a mixed integer linear programming formulation, and evaluate the influence of the problem's characteristics on the optimal solution. A computational experience is carried out to assess the solutions obtained and the proposed solution strategies.

Acknowledgments: This research has been partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through projects UIDB/04106/2020 and UIDP/04106/2020.

FGP2022 on Optimization: A semismooth Newton stochastic proximal point algorithm with variance reduction

Andre Milzarek[2], Fabian Schaipp[1]*, and Michael Ulbrich[1]

[1]Chair of Mathematical Optimization, Department of Mathematics, Technical University of Munich

[2]School of Data Science (SDS), The Chinese University of Hong Kong, Shenzhen.

andremilzarek@cuhk.edu.cn, schaippf@ma.tum.de (*corresponding author), mulbrich@ma.tum.de

We develop an implementable stochastic proximal point (SPP) method for optimization problems that typically arise in statistical learning. The objective function is composed by a weakly convex, finite-sum loss function and a nonsmooth, convex regularization term. In general, the SPP method requires to solve an implicit equation and therefore it is often only analyzed from a theoretical perspective. We propose an efficient way of implementing the SPP update by a semismooth Newton method, using Fenchel duality and allowing for inexact solves in our convergence theory. Moreover, we then extend the SPP method by SVRG-type variance reduction techniques. We obtain convergence to a stationary point with a sublinear rate for constant step sizes for Lipschitz-smooth loss functions. In the strongly convex case we obtain linear convergence to the minimizer, similar to results for variance-reduced stochastic (proximal) gradient methods such as SVRG. Numerical experiments show that the proposed algorithm can compete with state-of-the art methods and achieves higher robustness with respect to step size selection.

A Relaxed Inertial Forward-Backward-Forward Algorithm for Solving Monotone Inclusions with Application to GANs

Radu Ioan Boţ[1,2], Michael Sedlmayer[2], and Phan Tu Vuong[3]

[1]Faculty of Mathematics, University of Vienna, Austria

[2]Research Network Data Science @ Uni Vienna, University of Vienna, Austria

[3]Mathematical Sciences, University of Southampton, United Kingdom

{radu.bot, michael.sedlmayer}@univie.ac.at, T.V.Phan@soton.ac.uk

We introduce a relaxed inertial forward-backward-forward (RIFBF) splitting algorithm for approaching the set of zeros of the sum of a maximally monotone operator and a single-valued monotone and Lipschitz continuous operator. This work aims to extend Tseng's forward-backward-forward method by both using inertial effects as well as relaxation parameters. We formulate first a second order dynamical system that approaches the solution set of the monotone inclusion problem to be solved and provide an asymptotic analysis for its trajectories. We provide for RIFBF, which follows by explicit time discretization, a convergence analysis in the general monotone case as well as when applied to the solving of pseudo-monotone variational inequalities. We illustrate the proposed method by applications to a bilinear saddle point problem, in the context of which we also emphasize the interplay between the inertial and the relaxation parameters, and to the training of Generative Adversarial Networks (GANs).

On nonconvex generalized Nash equilibrium problems in infinite-dimensional spaces: Existence of equilibria and solution methods

Michael Ulbrich[1] and Julia Wachter[1]*

[1]Technical University of Munich, Chair of Mathematical Optimization, Department of Mathematics, Boltzmannstr. 3, 85748 Garching, Germany

mulbrich@ma.tum.de, wachter@ma.tum.de (*corresponding author)

This talk considers generalized Nash equilibrium problems (GNEPs) in infinite-dimensional spaces, using Nikaido–Isoda functionals and augmented Lagrangian methods.

The theory of and numerical methods for (generalized) Nash equilibrium problems are well-investigated in the finite-dimensional setting under suitable convexity assumptions. Here, we investigate GNEPs in an infinite-dimensional nonconvex setting. In the case of convex constraints, we express the GNEP in terms of the regularized Nikaido–Isoda functional and apply a generalized version of the Kakutani fixed point theorem in order to prove the existence of variational and normalized equilibria. Further, we develop differentiability results for the Nikaido–Isoda merit function using a generalized version of Danskin's theorem, thus enabling the use of derivative-based solution methods. For handling general, possibly nonconvex, constraints, we also briefly present an augmented Lagrangian method for GNEPs in infinite-dimensional spaces. In contrast to previous studies, such as work by Kanzow and Steck, we require weaker assumptions in order to provide convergence results. In particular, we can work with multiplier-penalty terms in more general spaces than Hilbert spaces.

Acknowledgments: This work has been supported by DFG through the Priority Programme SPP 1962 "Non-smooth and Complementarity-based Distributed Parameter Systems: Simulation and Hierarchical Optimization."

– SPONSORS

FGP22 Conference organization would like to thank the support of the conference sponsors.

Faculty of Economics, University of Porto

Faculty of Engineering of the University of Porto

Research Center for Systems and Technologies

International Mathematics Center

Institute of Systems and Computer Engineering, Technology and Science

Advanced Production and Intelligent Systems Associated Laboratory

Centre for Mathematics - University of Coimbra

Foundation for Science and Technology

Center for Mathematics and Applications, Universidade Nova de Lisboa

Laboratory for Computer Science and Informatics, Universidade Nova de Lisboa

University of Porto | Banco Santander Totta, SA

Tourism in Porto and North of Portugal

Centro ALGORITMI – Universidade do Minho

Institute for Systems and Robotics

This Book of Abstracts and many works therein were financed by the ERDF - European Regional Development Fund through the Operational Programme for Competitiveness and Internationalisation - COMPETE 2020, INCO.2030, under the Portugal 2020 Partnership Agreement and by National Funds, Norte 2020, through CCDRN and FCT - Fundação para a Ciência e a Tecnologia, I.P. (Portuguese Foundation for Science and Technology) within projects:

FAST - Flexible And SusTainable manufacturing (PTDC/EGE-OGE/31821/2017 - POCI-01-0145-FEDER-031821);

To Chair - The Optimal Challenges in Irrigation (POCI-01-0145-FEDER-028247)

UPWIND - A Multi-Kite System to Harvest High Altitude Wind Power (PTDC/EEI-AUT/31447/2017 - POCI-01-0145-FEDER-031447);

MAGIC - Multi-Agent Control and Estimation for Multi-Horizon Goals Conciliation (POCI-01-0145-FEDER-032485);

SNAP - Sustainable Management and Control of Agro-Production Systems (NORTE-01-0145-FEDER-000085);

MLDLCOV - Impact of COVID-19 lockdown measures on mobility, air pollution, health and macroeconomic indicators in Portugal: a Machine Learning Approach (DSAIPA/CS/0086/2020);

DYNAMICITY - Fostering Dynamic Adaptation of Smart Cities to Cope with Crises and Disruptions (NORTE-01-0145-FEDER-000073);

and SYSTEC R&D unit – Research Center for Systems and Technologies (UIDB/00147/2020).

We are also grateful for the funding provided by the University of Porto and Banco Santander Totta, S.A., within the scope of the multi-annual patronage program contract established between the two institutions - "Santander Universidades".

Cofinanciado por:

COMPETE 2020 PORTUGAL 2020 UNIÃO EUROPEIA Fundo Europeu de Desenvolvimento Regional FCT Fundação para a Ciência e a Tecnologia