

MODEL IDENTIFICATION OF A FOUR WHEELED OMNI-DIRECTIONAL MOBILE ROBOT

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Abstract: This paper presents the experimental dynamic parameters identification of an omni-directional mobile robot with four wheels. Three methods of parameters identification related to dynamic equations are described, the parameters are the viscous frictions, the coulomb frictions and the inertia moment of the robot. A simulation environment, simulation results and real results are presented.
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1. INTRODUCTION

Dynamic modelling of mobile robots is very important to design of controllers, mainly when the robots need travel at higher velocity and perform heavy works. For example, in (Liu *et al.*, 2003) and (Watanabe, 1998), control strategies for omni-directional robots using the dynamic model are discussed. Omni-directional mobile robots have the ability to move simultaneously and independently in translation and rotation (Pin and Kilough, 1994). However, non-linearities, like motor dynamic constraints can greatly affect the robot behaviour, especially when the robot is accelerated and decelerated. This paper presents a robot model identification that could find the non-linear saturation elements. Three methods are described to identification of the model's parameters. These methods can be used in dynamic modelling where there are varying terrain topography or dynamic variations (Albagul and Wahyudi, 2004).

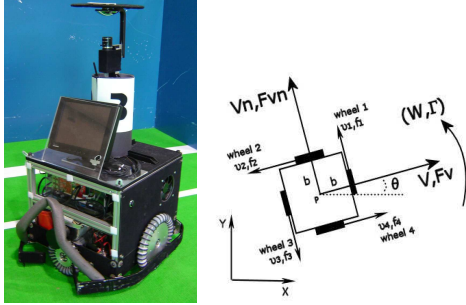
We focus attention on a omni-directional mobile robot with four motors, as shown in Fig.1(a), built for the 5dpo Robotic Soccer team from the Department of Electrical and Computer Engineering at the University of Porto at Porto, Portugal(Costa *et al.*, 1999). For this application (Robotic Soccer) the mobile robot needs to execute trajectories quickly and with a perfect position to the objective, for example, positioning to the ball, or to the goal, or to avoid dynamic obstacles. So, the dynamic characteristics of the motion are essential to follows the path correctly.

In section 2, the omni-directional mobile robot model is develop. The experimental methods of identification to model's parameters is presented in section 3. In section 4, simulation environment, simulation and real results of the identified model are presented. Finally, the conclusion and future works are drawn in section 5.

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2. THE OMNI-DIRECTIONAL MOBILE ROBOT MODEL

The omni-directional mobile robot model is developed based on the dynamics, kinematics and DC motors of the robot.



(a) Mobile robot. (b) Geometric parameters and coordinate frames.

Fig. 1. Omni-Directional robot

The World frame (X, Y, θ) , the robot's body frame and the geometric parameters is shown in Fig. 1(b). The following symbols, in SI unit system, are used to modelling:

- b [m] \rightarrow distance between the point P(center of chassis) and robot's wheels
- M [kg] \rightarrow robot mass
- r [m] \rightarrow wheel radius
- l \rightarrow motor reduction
- J [kg.m²] \rightarrow robot inertia moment
- B_v, B_{vn} [N/(m/s)] \rightarrow viscous friction related to V and V_n velocities
- B_w [N/(rad/s)] \rightarrow viscous friction related to W velocity
- C_v, C_{vn} [N] \rightarrow coulomb friction related to V and V_n velocities
- C_w [N.m] \rightarrow coulomb friction related to W velocity
- V, V_n [m/s] \rightarrow linear velocities of the robot
- W [rad/s] \rightarrow angular velocity of the robot
- θ [rad] \rightarrow orientation angle of the robot
- F_v, F_{vn} [N] \rightarrow traction forces of the robot
- Γ [N.m] \rightarrow rotation torque of the robot
- v_1, v_2, v_3, v_4 [m/s] \rightarrow wheels linear velocities
- f_1, f_2, f_3, f_4 [N] \rightarrow wheels traction forces
- T_1, T_2, T_3, T_4 [N.m] \rightarrow wheels rotation torque

2.1 Robot Dynamics

By Newton's law of motion and the robot's body frame, in Fig. 1(b), we have

$$F_v(t) = M \frac{dV(t)}{dt} + B_v V(t) + C_v \text{sgn}(V(t)) \quad (1)$$

$$F_{vn}(t) = M \frac{dV_n(t)}{dt} + B_{vn} V_n(t) + C_{vn} \text{sgn}(V_n(t)) \quad (2)$$

$$\Gamma(t) = J \frac{dW(t)}{dt} + B_w W(t) + C_w \text{sgn}(W(t)) \quad (3)$$

where,

$$\text{sgn}(\alpha) = \begin{cases} 1, & \alpha > 0, \\ 0, & \alpha = 0, \\ -1, & \alpha < 0. \end{cases}$$

The relationships between the robot's traction forces and the wheel's traction forces are,

$$F_v(t) = f_4(t) - f_2(t) \quad (4)$$

$$F_{vn}(t) = f_1(t) - f_3(t) \quad (5)$$

$$\Gamma(t) = (f_1(t) + f_2(t) + f_3(t) + f_4(t))b \quad (6)$$

The wheel's traction force(f) and the wheel's torque(T), for of each DC motor, is as follow:

$$f(t) = \frac{T(t)}{r} \quad (7)$$

$$T(t) = l \cdot K_t \cdot i_a(t) \quad (8)$$

where $i_a(t)$ is the armature current and K_t is motor torque constant. The dynamics of each DC motor can be described using the following equations,

$$u(t) = L_a \frac{di_a(t)}{dt} + R_a i_a(t) + K_v w_m(t) \quad (9)$$

$$T(t) = K_t i_a(t) \quad (10)$$

where L_a is the armature inductance, R_a is the armature resistance, $u(t)$ is the applied armature voltage, $w_m(t)$ is the rotor angular velocity in rad/sec, k_v is the emf constant. In SI unit system, the values of K_t and K_v are identical, see (Kuo, 1995): $K_t[N.m/A] = K_v[Volts/(rad/sec)]$. With the DC motor working in steady state ($L_a \frac{di_a}{dt} = 0$), we can simplify the equation 9, as

$$u(t) = R_a i_a(t) + K_v w_m(t) \quad (11)$$

Rearranging the equation 11, we get

$$\frac{u(t)}{i_a(t)} = K_v \frac{w_m(t)}{i_a(t)} + R_a. \quad (12)$$

To measure the constants K_v and R_a , we can use equation 12. For a given applied voltage $u(t)$, we measured the armature current $i_a(t)$ and the angular velocity $w_m(t)$ in steady state.

2.2 Robot Kinematics

By geometric parameters of the robot and the robot's body frame, in Fig. 1(b), is possible to derive the motion equations,

$$\begin{aligned}\frac{dx(t)}{dt} &= V(t)\cos(\theta(t)) - V_n(t)\sin(\theta(t)) \\ \frac{dy(t)}{dt} &= V(t)\sin(\theta(t)) + V_n(t)\cos(\theta(t)) \\ \frac{d\theta(t)}{dt} &= W(t)\end{aligned}\quad (13)$$

The relationships between wheel's linear velocities (v_1, v_2, v_3 and v_4) and robot velocities (V, V_n and W) are,

$$\begin{aligned}v_1(t) &= V_n(t) + bW(t) \\ v_2(t) &= -V(t) + bW(t) \\ v_3(t) &= -V_n(t) + bW(t) \\ v_4(t) &= V(t) + bW(t)\end{aligned}\quad (14)$$

Where $x(t)$ and $y(t)$ is the localization of the point P , and $\theta(t)$ the orientation angle of the robot.

3. MODEL'S PARAMETERS IDENTIFICATION

The parameters related to dynamic equations of the mobile robot were experimentally identified. The parameters are the viscous frictions (B_v, B_{vn}, B_w), the coulomb frictions (C_v, C_{vn}, C_w) and the inertia moment (J) of the mobile robot. The robot mass was balanced ($M = 35kg$). Three methods were used to identify the parameters, which are detailed in the next subsections. The coulomb and viscous frictions were identified with the methods 1 and 2. The inertia moment was identified by two ways: combining the method 1 with the method 2, and with the method 3. Thus, we used two methods for identify the frictions (1 and 2) and two methods (1+2 and 3) for identify the inertia moment.

3.1 Method 1 - Robot on steady state velocity

This method was used to identify the viscous frictions (B_v, B_{vn}, B_w) and the coulomb frictions (C_v, C_{vn}, C_w) of the mobile robot. This method consists in apply velocities V, V_n and W in robot, and measure the traction forces (F_v, F_{vn}) and the torque (Γ), with the robot on steady state velocity. Due to steady state velocity (null derivatives) and for positive velocities, we can simplify the equations 1, 2 and 3:

$$F_v(t) = B_v V(t) + C_v \quad (15)$$

$$F_{vn}(t) = B_{vn} V_n(t) + C_{vn} \quad (16)$$

$$\Gamma(t) = B_w W(t) + C_w \quad (17)$$

Three experiments had been made, for V, V_n and W separately. The experiment consists in apply four velocities and measure the traction forces

and the torque. The table 1 shows the applied velocities and the resulting forces and torque. The forces and torque were calculated based in the motor's currents, using the set of equations 4...8. The velocity and currents of the first experiment with $V = 1(m/s)$ are shown in Fig. 2.

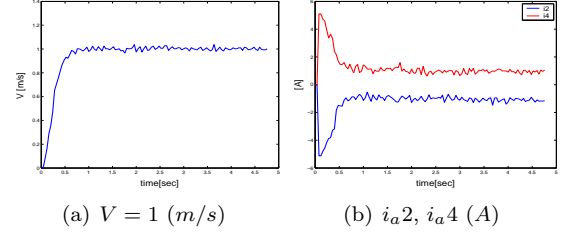


Fig. 2. Velocity and currents - first experiment.

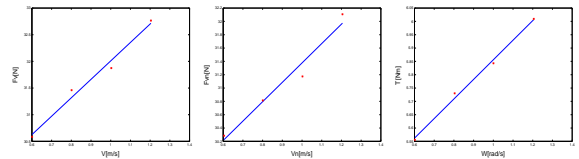
V (m/s)	F_v (N)	V_n (m/s)	F_{vn} (N)	W (rad/s)	Γ (N.m)
0.6	30.585	0.6	30.285	0.6	5.556
0.8	31.459	0.8	30.814	0.8	5.730
1	31.874	1	31.174	1	5.842
1.2	32.765	1.2	32.106	1.2	6.009

Table 1. Applied velocities and resulting forces and torque.

Since we have the values for velocities and forces, we can estimate the viscous and coulomb frictions. The least-squares line method was used to approximate the set of data to a linear model ($y = ax + b$). The Fig. 3 shows the applied velocities, resulting forces and the best fitting line. The resulting equations and the frictions are presented in the table 2.

	Equations	B_v, B_{vn} B_w	C_v, C_{vn} C_w
V	$F_v(t) = 3.45V(t) + 28.55$	3.45	28.55
V_n	$F_{vn}(t) = 2.90V_n(t) + 28.46$	2.90	28.46
W	$\Gamma(t) = 0.73W(t) + 5.12$	0.73	5.12

Table 2. Resulting equations and frictions - method 1.



(a) $F_v = f(V)$ (b) $F_{vn} = f(V_n)$ (c) $\Gamma = f(W)$

Fig. 3. Forces and torque vs. velocities.

3.2 Method 2 - Robot with null traction forces

In this method, the velocity variation with null traction forces was used to estimate the viscous frictions (B_v and B_{vn}), the coulomb frictions (C_v and C_{vn}) and the inertia moment J of the robot. Firstly, a constant velocity was applied on the robot, and then the robot motors were turned off, which resulted in null currents and null forces, consequently, provoking a velocity decrease until the robot stopped.

From the equations 1, 2 and 3 with null forces and null torque, we have the following equations for positive velocities:

$$\frac{dV(t)}{dt} = -\frac{B_v}{M}V(t) - \frac{C_v}{M} \quad (18)$$

$$\frac{dV_n(t)}{dt} = -\frac{B_{vn}}{M}V_n(t) - \frac{C_{vn}}{M} \quad (19)$$

$$\frac{dW(t)}{dt} = -\frac{B_w}{J}W(t) - \frac{C_w}{J} \quad (20)$$

Three experiments had been made, for V , V_n and W separately, see Fig. 4. The accelerations ($\frac{dV}{dt}$, $\frac{dV_n}{dt}$ and $\frac{dW}{dt}$) were calculated from the velocities (V , V_n and W), using the Euler's method (Franklin *et al.*, 1997)). We used the range of data, where the forces and torque were null to estimate the parameters, as in Fig.5.

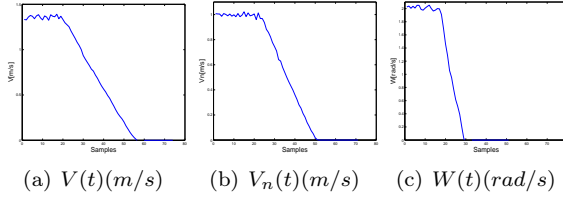


Fig. 4. Velocities behaviour.

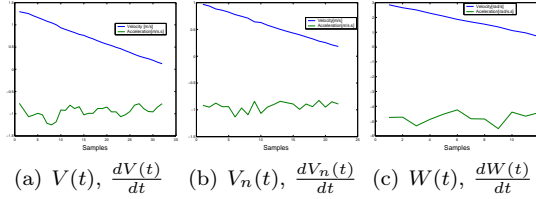


Fig. 5. Velocities and accelerations.

In order to estimate the viscous and the coulomb frictions, the mass M and the inertia moment J values are necessary. As there is no inertia moment J value, an estimation of J was firstly obtained. We used the equation 20, the velocity curve $W(t)$ and the acceleration curve $\frac{dW(t)}{dt}$ shown in Fig. 5(c). The viscous (B_w) and the coulomb (C_w) frictions, estimated with method 1, were used on equation 20. The estimated robot inertia moment was $J = 1.358[kg.m^2]$.

Since we have the values of robot mass, velocity and acceleration, we can apply on equations 18 and 19, to obtain viscous frictions (B_v and B_{vn}) and coulomb frictions (C_v and C_{vn}). The table 3 shows the equations obtained through least-squares method and the friction values.

3.3 Method 3 - Inertia Moment Identification

A practical method for estimating the moment of inertia J of the robot, to compare with the

	Equations	$B_v,$ B_{vn}	$C_v,$ C_{vn}
V	$\frac{dV(t)}{dt} = -0.11V(t) - 0.86$	4.01	30.35
V_n	$\frac{dV_n(t)}{dt} = -0.10V_n(t) - 0.87$	3.77	30.76

Table 3. Resulting equations and frictions - method 2.

estimation of the methods 1+2, is presented. The robot was hanged from the ceiling by wire, see figure 6, to eliminate frictions between the robot and the floor. The mass (m_o) was hanged at the disc attached to the robot, by a wire.

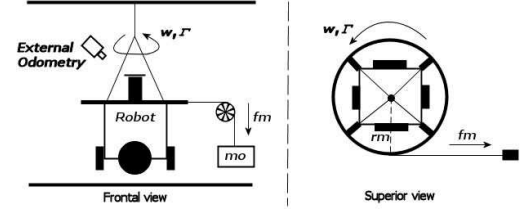


Fig. 6. Schematic of the experience.

By applying a known torque ($\Gamma(t)$) to robot body, and measuring the resultant angular position ($\theta(t)$), by a external odometry system based on vision, we can compute the moment of inertia of the robot. Based in rotational equation of motion, see equation 3, and considering null friction values (B_w e B_{cw}), we get:

$$\Gamma(t) = J \frac{dW(t)}{dt} \quad (21)$$

The torque $\Gamma(t)$ and the applied force $f_m(t)$ are

$$\Gamma(t) = f_m(t)r_m \quad (22)$$

$$f_m(t) = m_o g \quad (23)$$

where g is the acceleration of gravity ($\approx 9.8[m/s^2]$), m_o is the mass and r_m is the radius disc ($0.25[m]$).

Four experiments had been made to identify the value of J . The experiments 1, 2 and 3 were performed with a object with mass equal $0.510 Kg$, and the experiment 4 with a object with mass equal $1 Kg$. Our objective was to verify the repeatability of the experiments.

The figure 7(a) shows the angular position ($\theta(t)$) curve for the first experiment. The angular velocities ($W(t)$) were calculated using the derivative of the angular position. The figure 7(b) shows the angular velocity and the best fitting line to the angular velocity, calculated by least-squares line method, that give us the value of the angular accelerations.

The table 4 shows the torque values (Γ), angular accelerations ($\frac{dW(t)}{dt}$) and inertia moments (J) obtained by experiments. The inertia moments were similarly in all experiments.

	Exp. 1	Exp. 2	Exp. 3	Exp. 4
$\Gamma(N.m)$	1.2495	1.2495	1.2495	2.4500
$\frac{dW(t)}{dt}(rad/s^2)$	0.9126	0.8942	0.8947	1.7512
$J(kg.m^2)$	1.3691	1.3974	1.3965	1.3990

Table 4. Estimated inertia moments.

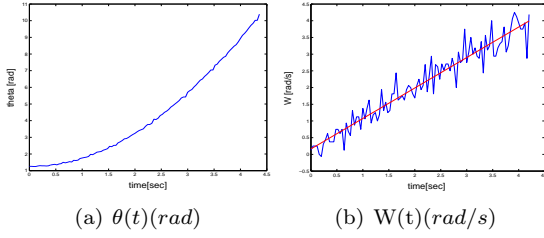


Fig. 7. First experiment.

3.4 Comparing methods

The identified parameters are presented in tables 5 and 6. We can see a acceptable difference between the estimated values, take into account that the experiments were made in hard conditions. For example, the measured values of currents and velocities have a considerable noise and any irregularity in the floor can cause alterations in robot parameters. The robot model is using the mean of estimated parameters, conform shown in tables.

Frictions	Viscous			Coulomb		
	B_v	B_{vn}	B_w	C_v	C_{vn}	C_w
Method 1	3.45	2.90	0.73	28.55	28.46	5.12
Method 2	4.01	3.77	-	30.35	30.76	-
Mean	3.73	3.34	0.73	29.45	29.61	5.12

Table 5. Estimated frictions.

	Inertia Moments
Methods 1+2	1.358
Method 3 (mean)	1.390
Mean	1.374

Table 6. Estimated inertia moments.

4. SIMULATION AND REAL RESULTS

In this section the model simulation, with the estimated parameters is presented. The simulation environment is implemented in the Matlab/Simulink (Mathworks, 2000) software, shown in Fig.8. The simulation environment is divided in three blocks:

- Kinematic model;
- Drivers-motors: has the discrete PID controllers, drives with PWM signals and the motors of the robot;
- Dynamic model.

The PID controllers calibration was discussed in (Conceicao *et al.*, 2005). Three simulations had been made (see Figs.9, 10 and 11), using the following reference velocities:

- (1) $V = 1[m/s]$, $V_n = 0[m/s]$, $W = 0[rad/s]$;
- (2) $V = 0[m/s]$, $V_n = 1[m/s]$, $W = 0[rad/s]$;

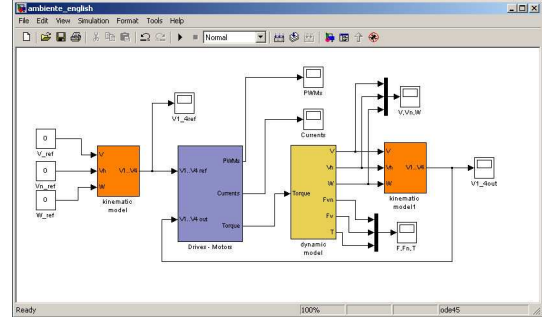


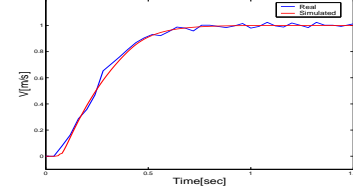
Fig. 8. Simulation environment.

- (3) $V = 0[m/s]$, $V_n = 0[m/s]$, $W = 1[rad/s]$.

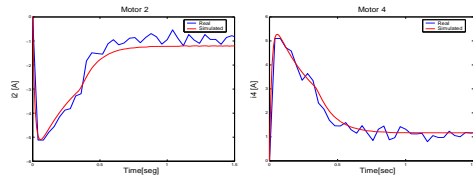
The Figs. 9(a), 10(a) and 11(a) show the results of the simulation and the real velocities of the robot. The simulation results are very similar to the real ones, in the transitory and the steady state too.

In Figs. 9(b), 9(c), 10(b) and 10(c) we can see some non-linearities of the robot, due to PWM saturation, shown in Figs. 9(d), 9(e), 10(d) and 10(e). The simulated currents had a good approximation to the real ones, because the model could find even some non-linearities of the system.

In third simulation(see Fig.11), the motors do not saturate easily, because there are 4 motors simultaneously applying force. In this way, the transitory state become much more fast.

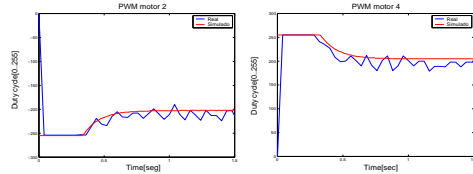


(a) Real and simulated $V(m/s)$.



(b) Current $i2(A)$

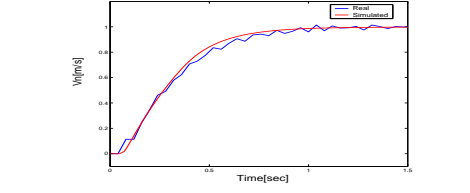
(c) Current $i4(A)$



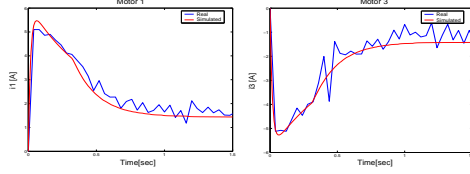
(d) PWM motor 2

(e) PWM motor 4

Fig. 9. Velocity $V = 1$, simulation 1.

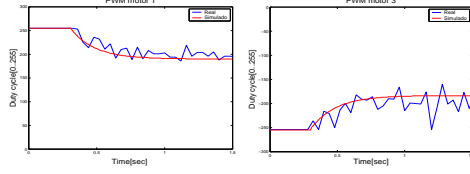


(a) Real and simulated $V_n(m/s)$.



(b) Current $i1(A)$

(c) Current $i3(A)$



(d) PWM motor 1

(e) PWM motor 3

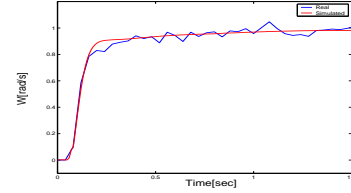
Fig. 10. Velocity $V_n = 1$, simulation 2.

5. CONCLUSION AND FUTURE WORKS

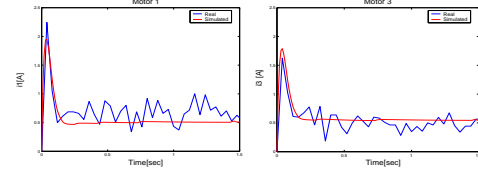
In this paper an experimental identification of the dynamic parameters of an omni-directional mobile robot has been developed. Three methods of the parameters identification that even can be used in dynamic modelling were described. A robot model that could find the non-linear elements is very important to design of controllers and trajectories, mainly in application where critical trajectories must be executed at higher velocity, for example in robotic soccer. In the near future, we will use the robot model to design of controllers.

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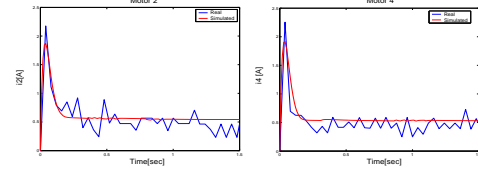


(a) Real and simulated $W(rad/s)$.



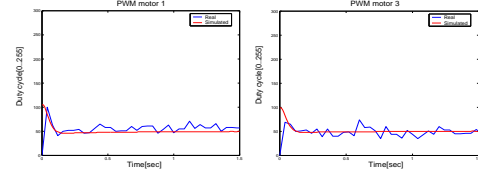
(b) Current $i1(A)$

(c) Current $i3(A)$



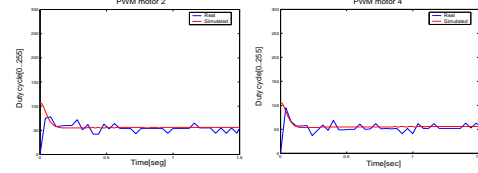
(d) Current $i2(A)$

(e) Current $i4(A)$



(f) PWM motor 1

(g) PWM motor 3



(h) PWM motor 2

(i) PWM motor 4

Fig. 11. Velocity $W = 1$, simulation 3.

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