Eng. Opt., 1991, Vol. 17, pp. 229–235
Reprints available directly from the publisher
Photocopying permitted by license only

PROBABILISTIC DESIGN AND OPTIMIZATION OF REINFORCED CONCRETE FRAMES

MARC HOIT

Department of Civil Engineering, University of Florida, Gainesville, Florida 32611, USA.

ALFREDO SOEIRO

Departamento de Engenharia Civil, FEUP, University of Porto, 4099 Porto Codex, Portugal.

and

FERNANDO FAGUNDO

Department of Civil Engineering, University of Florida, Gainesville, Florida 32611, USA.

A structural optimization algorithm which includes global displacements as decision variables is presented. The formulation addresses the possibility of using a universal procedure for obtaining optimal solutions independently of local code restrictions. A comparison of current ACI code safety requirements and reliability constraints with examples of optimal limit design techniques is presented. The flexural performance of the elements was evaluated as a function of the actual stress-strain diagrams of the materials. For the non-linear case, formation of fictitious rotational hinges was allowed and the equilibrium constraints were updated accordingly. The adequacy of the frames was guaranteed by imposing constraints, representing the maximum probability of failure of the members and the global displacements allowed, combined with a prescribed limited system probability of failure.

KEY WORDS: Structural optimization, integrated analysis, frames, reinforced concrete, reliability.

BACKGROUND

Standard classical structural frame optimization problems usually consist of cycling between two distinct phases defined as analysis and optimal design. This work presents a formulation that combines both phases by adding the global displacements to the set of design variables. This option has been implemented in several previous studies[1,2,3]. This option was selected considering the future application of this approach to frames with material non-linear behavior.

The first research stage was to optimize linear elastic plane frames subjected to monotonic loading. Only rectangular sections were considered. The objective function was the volume of the structure. Constraints of the optimization problem were expressed in terms of equalities representing structural equilibrium equations and of inequalities assuring serviceability and safety requirements.

The initial optimization strategy adopted consisted of using the method of augmented Lagrangian multipliers[4]. The results with this formulation were encouraging. The convergence rate for the optimal solution was dependent on the initial design, scaling, penalty parameters and Lagrangian multiplier values. The computational effort was considerable when compared with other explored techniques based on optimality criteria and mathematical programming methods. The unconstrained minimization was difficult to accomplish since the augmented Lagrangian function was generally very steep, with great sensitivity to any small variation of the displacement variables.

To improve the optimization procedure another non-linear programming technique was used. The generalized reduced gradient method was chosen because the method is based on the iterative solution of a system of equations, involving the active design variables, to find a feasible solution while using the gradient to optimize the problem over a reduced set of variables[5].

ELEMENT DEFINITION

A typical reinforced concrete frame element consists of a rectangular cross section, doubly reinforced with equal areas on both faces. Non-linear material behavior may be incorporated by modelling the reinforced concrete element as a one-component model. In this model rotational springs are added to the ends of the elastic element to simulate the formation of plastic hinges. The stiffnesses of the linear elastic element and of the springs were condensed using the flexibility formulation.

The determination of the flexural characteristics of each reinforced concrete section was based on the stress-strain diagrams of concrete and of reinforcing steel. The yielding and ultimate moments for each cross-section were used to determine the characteristics of the springs for each element. The spring stiffness was considered infinite whenever the element moment was below the yielding moment. When the moment was above the yielding value, the spring stiffness was updated accordingly. The secant stiffness approach was adopted for the evaluation of spring stiffness, thus incremental loading or unbalanced iteration were not taken into consideration[6].

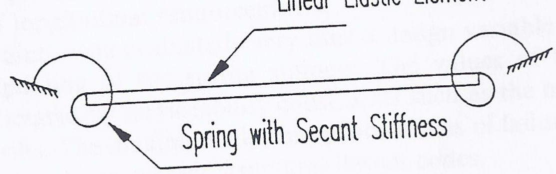

Figure 1 One-component element model.

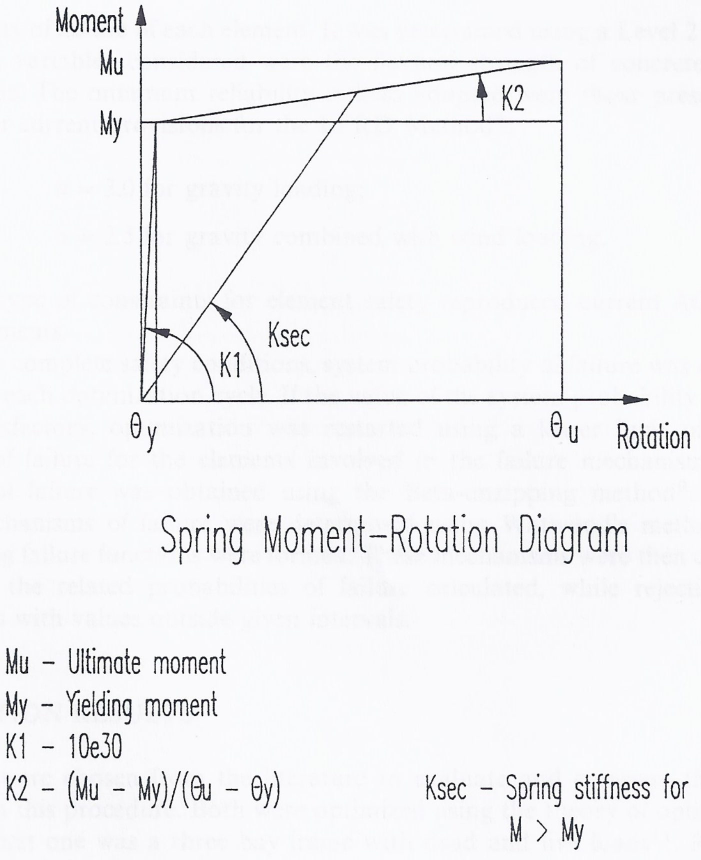

Figure 2 Spring stiffness.

PROCEDURE IMPLEMENTATION

The objective function was the total cost of the materials, in this case concrete and steel. The final cost of the optimized structure is compared with previous results and the adequacy of the design is also compared with current code requirements. The constraints were comprised of the equilibrium equalities, imposed maximum global displacements and required element reliabilities. Design variables were the section sizes and the areas of longitudinal reinforcements.

Equilibrium constraints were evaluated every time a design variable changed with the corresponding updating of the spring stiffness. The values of the maximum displacements were dictated by serviceability constraints such as the maximum joint rotations or storey drifts. The maximum element probabilities of failure were chosen considering practices involving current structural design codes.

Element safety was tested using two types of constraints. The first one considered the probability of failure of each element. It was determined using a Level 2 method[7]. The random variables considered were the flexural strength of concrete and the external loads. The minimum reliability indices adopted were those prescribed by AISC in their current provisions for the LFRD Method[8]:

$a = 3.0$ for gravity loading;

$a = 2.5$ for gravity combined with wind loading.

The second type of constraints for element safety reproduced current ACI 318-83 code requirements.

In order to complete safety conditions, system probability of failure was evaluated at the end of each optimization cycle. If the value of the system probability of failure was not satisfactory, optimization was restarted using a lower limit of element probability of failure for the elements involved in the failure mechanism. System probability of failure was obtained using the Beta-unzipping method[9]. The elementary mechanisms of failure were determined using Watwood's method[10] and corresponding failure functions were formed. These mechanisms were then combined linearly and the related probabilities of failure calculated, while rejecting those combinations with values outside given intervals.

OPTIMIZATION RESULTS

Two frames were chosen from the literature to evaluate and compare the results obtained with this procedure. Both were optimized using the theory of optimal limit design. The first one was a three bay frame with dead and live loads[11]. Results of reinforcement optimization are shown in Table 1. The second example was a more detailed study of a two bay frame and the characteristics are presented in Ref. [12]. Optimization results are shown in Table 2. In both cases the values obtained were close to those corresponding to the expected optimal values from other examples. Reliability constraints were satisfied, displacements were within the limits and equality constraints were satisfied.

Table 1 Three bay frame.

Optimized Reinforcement Areas (in^2)			
Section	*OLD*	*Reliability*	*ACI*
1	2.63	1.97	2.37
2	2.61	2.26	2.84
3	3.53	2.30	2.90
4	0.71	0.67	0.57
5	3.12	1.95	2.42
Steel Cost	59,940	44,856	52,596

Table 2 Two bay frame

Optimized Reinforcement Areas (in^2)			
Section	*OLD*	*Reliability*	*ACI*
1	2.70	3.05	4.10
2	4.80	3.88	5.25
3	4.80	4.94	6.73
4	3.32	2.33	3.11
Total Cost	65,952	65,700	83,844

The algorithm proved to be almost insensitive to the initial design points. Although these frames allowed only for optimization of the steel reinforcement area, the initial design points had section dimensions superior to the assigned values in the referenced frames. Other examples tested proved that, since the cost of reinforcement steel is low in the USA, minimal dimensions were always obtained unless element reliability or code constraints were active.

Results show that there are differences in the optimized solutions using the three different approaches studied. These differences may be attributed to the fact that

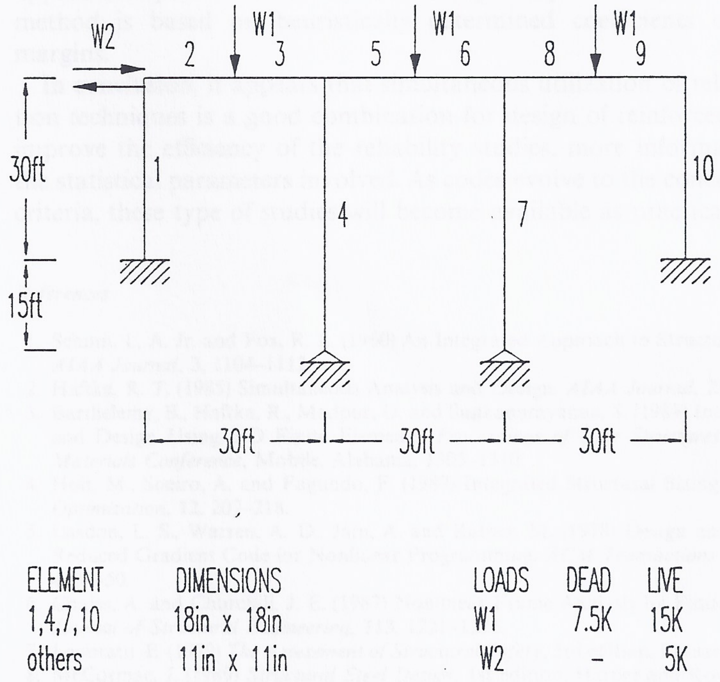

Figure 3 Three bay frame.

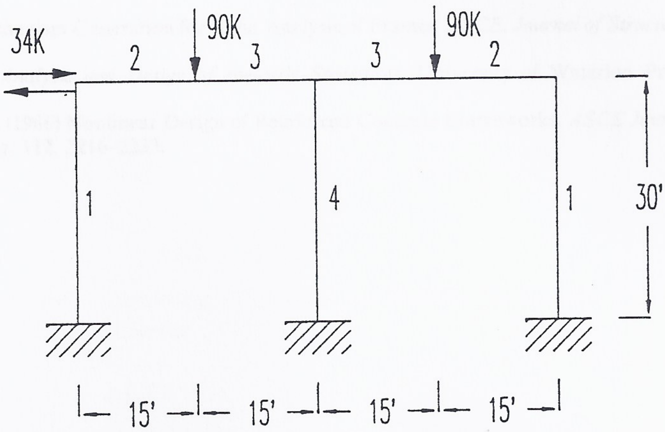

Figure 4 Two bay frame.

redistribution of the moments in the optimal limit design (OLD) examples is limited. Another possible reason may be the fact that safety requirements are different in the three approaches. OLD demands an ultimate global load factor, while the reliability approach requires a certain element and system probability of failure. The ACI design method is based on heuristically determined coefficients creating larger safety margins.

In conclusion, it appears that simultaneous utilization of reliability and optimization techniques is a good combination for design of reinforced concrete frames. To improve the efficiency of the reliability studies, more information is needed about the statistical parameters involved. As codes evolve to the concept of reliability based criteria, these type of studies will become available as practical design procedures.

References

1. Schmit, L. A. Jr. and Fox, R. L. (1960) An Integrated Approach to Structural Synthesis and Analysis. *AIAA Journal*, 3, 1104–1112.
2. Haftka, R. T. (1985) Simultaneous Analysis and Design. *AIAA Journal*, **23**, 1099–1103.
3. Barthelemy, B., Haftka, R., Madpur, U. and Sankanarayanan, S. (1989) Integrated Structural Analysis and Design Using 3-D Finite Elements. *Proceedings of 30th Structures, Structural Dynamics and Materials Conference*, Mobile, Alabama, 1305–1310.
4. Hoit, M., Soeiro, A. and Fagundo, F. (1987) Integrated Structural Sizing Optimization. *Engineering Optimization*, **12**, 207–218.
5. Lasdon, L. S., Warren, A. D., Jain, A. and Ratner, M. (1978) Design and Testing of a Generalized Reduced Gradient Code for Nonlinear Programming. *ACM Transactions on Mathematical Software*, **4**, 34–50.
6. Chajes, A. and Churchill, J. E. (1987) Nonlinear Frame Analysis by Finite Element Methods. *ASCE Journal of Structural Engineering*, **113**, 1221–1235.
7. Leporatti, E. (1979) *The Assessment of Structural Safety*, 3rd edition, Research Studies Press, Oregon.
8. McCormac, J. (1989) *Structural Steel Design*, 1st edition, Harper and Row, New York.
9. Thoft-Christensen, P. and Morotsu, Y. (1986) *Application of Structural Systems Reliability Theory*, 1st edition, Springer-Verlag, Berlin.

10. Watwood, V. (1979) Mechanism Generation for Limit Analysis of Frames. *ASCE, Journal of Structural Division*, **109**, 1–15.
11. Cohn, M. (Ed.) (1972) *Analysis and Design of Inelastic Structures*, University of Waterloo Press, Waterloo.
12. Ali, M. and Grierson, D. (1986) Nonlinear Design of Reinforced Concrete Frameworks. *ASCE Journal of Structural Engineering*, **112**, 2216–2233.