

# Motion coordination of Autonomous Underwater Vehicles under acoustic communications

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**Abstract:** The problem of coordinating the motion of Autonomous Underwater Vehicles under constrained acoustic communications is formulated and solved in the Model Predictive Control framework. The impact of acoustic communications and perturbations on the motion performance and robustness is discussed along with several coordination schemes. The discussion is complemented with the presentation of simulation results. This is done in the context of an evaluation framework aimed at exercising key aspects of performance.

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*Keywords:* Model Predictive Control, Autonomous Underwater Vehicles, Formation Control

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## 1. INTRODUCTION

Motion coordination for autonomous underwater vehicles (AUV) is a challenging problem. This is because of the non-linear dynamics and of communication constraints. Non-linear dynamics arise naturally from the application of the laws of physics to AUV modeling. Communication constraints arise because radio waves are severely attenuated underwater thus making acoustics the typical choice for underwater communications. Acoustic communications are severely constrained in terms of bandwidth and reliability, Riksfjord et al. (2009).

The motivation for AUV motion coordination, namely formation control, comes mainly from oceanographic field studies, Paley et al. (2008); Zhang et al. (2007); Fiorelli et al. (2004), as well as from military applications de Sousa et al. (2009); de Sousa and Martins (2010).

Several approaches have been proposed to address the problem of vehicle formation control under communication constraints, Franco et al. (2004, 2008); Keviczky et al. (2006, 2008); Fax and Murray (2004); Olfati-Saber and Murray (2004); Semsar-Kazerouni and Khorasani (2008); Goodwin et al. (2004); Fontes et al. (2009); Gruene et al. (2009); Allen et al. (2002); Liu et al. (2001).

The problem of cooperative control of a team of distributed agents with decoupled nonlinear discrete-time dynamics and exchanging delayed information is addressed in Franco et al. (2004, 2008). Each agent is assumed to evolve in discrete-time, based on locally computed control laws, which are computed by exchanging delayed state information with a subset of neighboring agents. The cooperative control problem is formulated in a receding-horizon framework, where the control laws depend on the local state

variables (feedback action) and on delayed information from cooperating neighboring agents (feedforward action). A rigorous stability analysis exploiting the input-to-state stability properties of the receding-horizon local control laws is carried out. The stability of the team of agents is then proved by utilizing small-gain theorem results. Building on the work reported in Keviczky et al. (2006), a decentralized scheme for the coordinated control of formations of autonomous vehicles is presented in Keviczky et al. (2008). A high level receding horizon control and coordination strategy is obtained for each vehicle by solving a linear quadratic optimization problem featuring control saturation constraints, linear dynamics constraints, and formation constraints with neighboring vehicles defined by a graph. An appropriate graph structure describes the underlying communication topology between the vehicles. On each vehicle, information about neighbors is used to predict their behavior and plan conflict-free trajectories that maintain the coordination and achieve the team objectives. When feasibility of the decentralized control is lost, collision avoidance is ensured by invoking emergency maneuvers that are computed via invariant set theory. A stabilization analysis is also discussed in Keviczky et al. (2006).

Information exchange strategies that improve the formation stability and performance and, at the same time, are robust to changes in the communication topology are considered in Fax and Murray (2004) to address the problem of cooperative control of vehicle formations. The sensed and communicated information flow is modeled by a graph whose topology has implications in the control stability. By exploiting the interplay between communications and control, necessary and sufficient conditions for the stability of an interconnected system of identical vehicles can be derived. Stated in terms of the Popov criterium for networked control systems, these conditions involve the eigenvalues of the graph Laplacian and reveal how to

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shape the information flow in order to ensure stability and achieve superior performance.

The problem of unreliable communication channel between the MPC controller output and the actuator input, has been addressed in, among others, Gruene et al. (2009). Here, a mechanism for compensation of packet dropouts has been incorporated in the MPC scheme for discrete time problems. The basic idea consists in extending the control horizon until the next successful communication event happens and, in the meantime, use the best available control estimate, namely the one that has already been computed for the longer time interval. This article also includes some stability and sub-optimality analysis under an asymptotic controllability assumption. In order to show stability, the authors prove that, under the considered assumptions, the value function associated with the optimal control problem also exhibits properties of a Lyapunov function.

Each one of the approaches considered above is interesting and has its own merits. However, these are not designed to meet the requirements arising in the coordinated control of AUVs that feature not only very strict power, communication, and computational constraints, but also high unreliable, low data rate communications, as well as significant motion uncertainty.

Here, we present a different approach to the problem of AUV formation control. This approach is based on Model Predictive Control (MPC) techniques. In this approach vehicles exchange information over acoustic communication channels. Limited bandwidth precludes closing low-level (fast) feedback loops over acoustic communications. We introduce a distributed layered control framework to address this problem. The two layers are distributed over the AUVs in formation. The AUVs have the same layered control structure which is amenable to decentralization. The lower layer deals with the fast low-level control for each vehicle. The upper layer deals with acoustic communications and control corrections to the lower layer. Each vehicle has a fast low-level formation controller. This is a feedback controller for the whole formation. We use a model-based approach to close the control loop around state estimates from the vehicle and from models of the other vehicles. This is done without communications with the other AUVs. We use MPC for the high-level controller which runs in each vehicle. The model is reset when a message with the true of other AUVs is received. The MPC is run with the model updates to generate a sequence of control inputs for the AUVs in the formation. These control inputs are sent to the other AUVs for coordination. The MPC cost function is targeted at balancing the quadratic error to a reference trajectory for the formation and the control effort. Control and state constraints are also considered to reflect control saturations, as well as to prevent collisions with obstacles.

Our approach is targeted at a field demonstration with AUVs from Porto University. This demonstration will take place in 2011 at the Porto Harbor during the final review meeting of the Control for Coordination FP7 project. We are tasked to use the NAUV and one LAUV vehicles from Porto University for this purpose. The LAUV SeaCon AUV is based on evolutions of the award winning Light



Fig. 1. LAUV Vehicle on the top and NAUV on the bottom

Autonomous Vehicle (LAUV) developed by Porto University. The LAUV is a torpedo shaped vehicle made of composite materials with one propeller and four control fins. It has an advanced miniaturized computer system running modular controllers on a real-time Linux kernel. It is easily configurable for multiple operation profiles and sensor configurations to facilitate test and evaluation of new technologies. The LAUV is an open system which lends itself to the integration of new systems and technologies.

The NAUV vehicle is an extended version (180x20cm) of LAUV vehicle providing more accurate positioning, underwater communications and more sensors. The NAUV is equipped with Benthos acoustic underwater modem (ATM-885PCB) with a very low communication rate (up to 1000bps). For example, the transmission time of  $n$  bits takes  $t = 1.75 + n/v$  where  $v$  is the data rate velocity (in bps) and 1.75 s is the setup time. The NAUV also provides underwater imaging through a sidescan sonar and has a more accurate positioning by using an ADCP (Acoustic Doppler Current Profiler) together with IMU (inertial measuring unit).

The paper is organized as follows. We present the problem formulation and assumptions in section 2. Section 3 presents background material on Model Predictive Control. Section 4 describes our approach and discusses its properties. We discuss the evaluation the approach and simulation results in section 5. The conclusions and future work are discussed in the last section.

## 2. AUV FORMATION CONTROL PROBLEM

Here, we formulate the AUV formation control problem. This basically consists in controlling a set of AUVs to track a trajectory while maintaining a formation under constraints on the state (safety requirements), control (saturations) and communications.

Models of AUVs are quite complex because of the nonlinear dynamics arising from hydrodynamics and actuation. In our developments we consider a simpler model with

coefficients based on the results from Prestero (2001) and from our own field experiments.

$$\dot{\eta} = \begin{bmatrix} u \cos(\psi) - v \sin(\psi) \\ u \sin(\psi) + v \cos(\psi) \\ r \end{bmatrix}, \quad (1)$$

$$\dot{\nu} = \begin{bmatrix} \frac{\tau_u - (m - Y_{\dot{v}})vr - X_{u|u}|u|u|}{m - X_{\dot{u}}ur - Y_{v|v}|v|v|} \\ \frac{m - Y_{\dot{v}}}{\tau_r - (Y_{\dot{v}} - X_{\dot{u}})uv - N_{r|r}|r|r|} \\ \frac{I_{zz} - N_{\dot{r}}}{I_{zz} - N_{\dot{r}}} \end{bmatrix}, \quad (2)$$

where  $\eta = [x, y, \psi]^T$  (from here onwards, a “ $T$ ” in upper script will denote transposed),  $\nu = [u, v, r]^T$ ,  $\tau = [\tau_u, \tau_r]$ , the coefficients  $X_{\dot{u}}, Y_{\dot{v}}, N_{\dot{r}}$  represents hydrodynamic added mass,  $X_{u|u}, Y_{v|v}, N_{r|r}$  the hydrodynamic drag and  $m$  the vehicle mass.

From the above, we are interested in control strategies which, for each AUV  $i$ ,  $i = 1, \dots, n_v$ , minimize, over a given time interval, a cost functional with two terms, one that penalizes the trajectory tracking error forcing vehicles to follow the desired path,  $\eta_{ref}^i$ , and another that penalizes the control effort, thus saving the limited energy on board of vehicles, i.e.,

$$\int_t^{t+T} [(\eta^i(s) - \eta_{ref}^i(s))^T Q (\eta^i(s) - \eta_{ref}^i(s)) + \tau^{iT}(s) R \tau^i(s)] ds, \quad (3)$$

and, at the same time, satisfies the following:

- (i) Kinematic and dynamic constraints (vehicle dynamics) given by (1) and (2);
- (ii) Endpoint state constraints,  $\eta^i(t+T) \in C_{t+T}$ ;
- (iii) Control constraints,  $\tau^i(s) \in \mathcal{U}^i$ ;
- (iv) State constraints,  $(\eta^i(s), \nu^i(s)) \in \mathcal{S}^i$ ;
- (v) Communication constraints  
 $g_{i,j}^c(\eta^i(s), \eta^j(s)) \in C_{i,j}^c, \forall j \in \mathcal{G}^c(i)$ ; and
- (vi) Formation constraints  
 $g_{i,j}^f(\eta^i(s), \eta^j(s)) \in C_{i,j}^f, \forall j \in \mathcal{G}^f(i)$ .

While the control constraints (iii) include, for example, saturations, the state constraints (iv) are specified to keep each vehicle in a specified set in order to satisfy safety or some other requirement. For example, to avoid collision with obstacles – known a priori or detected on the fly – or to prevent some variables to take on values that may damage components.

The satisfaction of the acoustic communication constraints (v) ensure that the motion of the vehicles is such that the required connectivity is preserved. The fact that closer the vehicles are, the lower the power consumption and packets loss, makes a strong case for each AUV to communicate with its neighbors and, hence, for decentralized control structure. The communications structure may be described by the triple  $(g^c, C^c, \mathcal{G}^c)$ , where  $g^c : \mathbf{R}^n \times \mathbf{R}^n \rightarrow \mathbf{R}^M$ ,  $C^c \in \mathbf{R}^M$  (here,  $M \leq n(n_v - 1)n_v$ , being  $n$  the dimension of the state space component of interest of each vehicle), and  $\mathcal{G}^c$  a graph where each node corresponds to each vehicle and an edge to a communication link. We point out that the communications graph is, in general, quite different from the formation or control graphs that we will introduce next.

Finally, the formation constraints (vi) specify the relations between data (typically, relative positions) of AUVs which have to be maintained with the help of appropriate control activity. These relative positions are specified in order to ensure the desired requirements for the AUVs formation. The formation structure may be described by triple  $(g^f, C^f, \mathcal{G}^f)$  where  $g^f : \mathbf{R}^n \times \mathbf{R}^n \rightarrow \mathbf{R}^M$ ,  $C^f \in \mathbf{R}^M$  (here,  $M \leq n(n_v - 1)n_v$ , being  $n$  the dimension of the state space component of interest of each vehicle), and  $\mathcal{G}^f$  a graph whose  $i^{th}$  component defines the vehicles with which the  $i^{th}$  vehicle has a formation relation.

### 3. MPC BACKGROUND

There is a vast body of literature on MPC – also designated by Receding Horizon Control (RHC) (see, for example, Mayne et al. (2000)). MPC is a control scheme in which the control action for the current time subinterval – control horizon – is obtained, at each sampling time, by solving on-line an optimal control problem over a certain large time horizon – the prediction horizon – with the state variable initialized at the current best estimate updated with the latest sampled value. Once the optimization yields an optimal control sequence, this is applied to the plant during the control horizon. Then, once this time interval elapses, the process is re-iterated. The MPC scheme involves the following steps:

1. Initialization. Let  $t_0$  be the current time, and set up the initial parameters or conditions specifying  $x_0$ ,  $T$ ,  $\Delta$ , initial filter parameters (in case the sampled data requires filtering, initial control for the recursive control optimization procedure, etc.
2. Sample the state variable at time  $t_0$ .
3. Compute the optimal control strategy,  $u^*$ , in the prediction horizon, i.e.,  $[t_0, t_0 + T]$ , by solving the optimal control problem ( $P$ ).
4. Apply the obtained optimal control during the current control horizon,  $[t_0, t_0 + \Delta]$ .
5. Slide time by  $\Delta$ , i.e.,  $t_0 = t_0 + \Delta$ , and adapt parameters and models as needed.
6. Go to step 2.

where  $x_0$  is the initial state,  $T$  is the prediction horizon for control optimization, and  $\Delta$  is the control horizon. A number of variants to this scheme have been considered by enriching some of the steps with additional processing:

- For the networked systems implementation, the data obtained in step 4. might be a composition of locally sampled data and data communicated from other vehicles or subsystems. For this class of systems, it might be of interest to replace data that failed to be transmitted by simulated data.
- Filtering the sampled state variable is usually required, being the Kalman filter widely used.
- For situations in which models are significantly uncertain or may vary over time, it might be of interest to use the sampled data to identify or refine the value of model parameters.
- Likewise, if external perturbations acting on the vehicles/systems are sensed or estimated, then they can be used to improve the models entering in the optimization procedure, and to change the MPC parameters.

- Communication may introduce delays and data packets might fail to arrive with serious consequences to the controller performance. To address this, true data may be replaced by simulated data or MPC parameters may be adjusted.

A typical general formulation of the optimal control problem (P) may be as follows:

$$\begin{aligned}
(P) \text{ Minimize } & g(x(t_0 + T)) + \int_{t_0}^{t_0+T} f_0(t, x(t), u(t)) dt \\
\text{subject to } & \dot{x}(t) = f(t, x(t), u(t)) \quad \mathcal{L} - a.e. \\
& u(t) \in \Omega \quad \mathcal{L} - a.e. \\
& h(t, x(t)) \leq 0 \\
& g(t, x(t), u(t)) \leq 0 \\
& x(t_0 + T) \in C_f
\end{aligned}$$

where  $g$  is the endpoint cost functional,  $f_0$  is the running cost integrand,  $f$ ,  $h$ , and  $g$  represent, respectively, the vehicle dynamics, the state constraints, and the mixed constraints,  $C$  is a target that may also be specified in order to ensure stability. If one wants to take into account the uncertainty with respect to the initial state, then one may consider an initial state constraint, i.e.,  $x(t_0) \in C_i$  where  $C_i$  is an estimate of the uncertainty set, being the minimization taken over the worst case of the initial state.

Now, we discuss stability and robustness, Mayne et al. (2000); Langson et al. (2004); Mayne et al. (2009).

**Stability.** Two major MPC approaches have been considered to stability:

- Direct method using the fixed horizon value function as a Lyapunov function; and
- Indirect approach employing the monotonicity property of a sequence of value functions.

Regardless of the approach, a number of formulations involving either a certain terminal state constraint set  $C$ , or terminal cost  $f_0$ , or both, have been considered. In order to ensure the asymptotic stability of the obtained feedback control law, say  $u = k(x)$ , the required typical assumptions are:

- $0 \in C$  with  $C$  closed;
- $k(x) \in \Omega$  the control constraint set;
- $C$  is positively invariant under  $k(\cdot)$ ; and
- $f_0$  is locally a Lyapunov function.

**Robustness.** Robustness concerns the ability of the system in preserving a certain property - e.g., stability or performance - in the presence of uncertainties. For stability, this can be checked by concluding that the Lyapunov function for the nominal closed-loop system keeps the descent property for sufficiently small disturbances. While this is not very difficult to show for unconstrained problems, the consideration of constraints on states and controls raises substantial challenges as it is required to ensure that the constraints remain satisfied. Inherent robustness, min-max open loop control and feedback control are the general contexts considered to investigate robustness of MPC schemes.

The requirements for formation control of unmanned vehicles are easily encoded with optimal control formulations. However, the question of how to solve these optimal control problems for practical applications is not a trivial one. This is the reason why MPC has been one of the tools of choice to solve these problems. This can be done in two stages: the planning phase - solved off-line to provide the formation reference trajectory -, and the execution phase - solved on-line with the help of locally formulated control problems.

#### 4. APPROACH

In this section we describe our implementation of a decentralized version of a discrete time MPC system to control a formation of AUVs. The main features are:

- The decentralized character of the overall MPC controller is since each vehicle runs its own MPC scheme (which also encompasses the models of its neighboring AUVs) and communicates only with its neighbors;
- Computational efficiency is achieved by replacing the optimal control problem by a LQ optimization problem (for which an efficient MATLAB solver is used) and, for this, we consider (i) quadratic cost functionals, (ii) approximation of each AUV dynamics by a linear model, and (iii) state and control constraints (saturation) given by inequalities;
- Communication delays and packet dropouts can easily be incorporated; and
- Noise and disturbances can be easily considered in the vehicles simulated motion.

Each AUV runs the same type of controller in our decentralized MPC framework. This is discussed next.

Let  $N_p$ ,  $n_v$ , and  $T$  be, respectively, the prediction horizon, the number of vehicles, and the sampling period. Then, according to the previous considerations, the discrete time linear model of vehicle  $i = 1, \dots, n_v$ , is, for  $k = 0, \dots, N_p - 1$ , given by:

$$x_{k+1}^i = \Phi^i(T)x_k^i + \Psi^i(T)u_k^i, \quad y_k^i = C^i x_k^i, \quad (4)$$

where  $\Phi^i(T) = e^{A^i T}$ ,  $\Psi^i(T) = \int_0^T e^{A^i(T-s)} ds B^i$ , and

$x_k^i \in \mathcal{R}^{n_s}$ ,  $u_k \in \mathcal{R}^{n_c}$ , and  $y_k \in \mathcal{R}^{n_o}$  are, respectively, the system state, input and output variables, and  $n_s$ ,  $n_c$  and  $n_o$  are the associated space dimensions.

From the considerations of the formation control problem formulation and assumed simplifications, it follows that the underlying optimal control problem for AUV  $i$ , ( $LQP^i$ ), involves data from all its neighboring vehicles as specified by the formation graph, consisting in minimizing the quadratic cost functional

$$\begin{aligned}
& \sum_{k=1}^{N_p} \|y_{t+k}^{ref,i} - y_{t+k}^i\|_{Q^i}^2 + \sum_{k=0}^{N_p-1} \|u_{t+k}^i\|_{R^i}^2 \\
& + \sum_{k=1}^{N_p} \sum_{j \in \mathcal{G}(i)} \|D^{ij}(y_{t+k}^i - y_{t+k}^j) - d^{ij}\|_{L^{ij}}^2
\end{aligned} \quad (5)$$

$$\text{subject to: } x_{t+k+1}^j = \Phi^j(T)x_{t+k}^j + \Psi^j(T)u_{t+k}^j, \quad (6)$$

$$y_{t+k}^j = C^j x_{t+k}^j \quad (7)$$

$$x_{t+k}^j \in [x_{LB,t}^j, x_{UB,t}^j] \quad (8)$$

$$u_{t+k}^j \in [u_{LB}^j, u_{UB}^j] \quad (9)$$

$$x_t^j = x_0^j, \quad (10)$$

where constraints hold for  $j \in \{i\} \cup \mathcal{G}(i)$ , being, for each time  $k$ ,  $\mathcal{G}(i)$  the set of nodes of the graph specifying the vehicles linked to AUV  $i$ . Here,  $y_{t+k}^i$  and  $y_{t+k}^{ref,i}$  are, respectively, the vector of outputs of vehicle  $i$  and its reference,  $x_0^j$  is the initial state of vehicle  $j$  at the initial time  $t$ ,  $D^{ij}$  is a matrix reflecting the formation relation between vehicles  $i$  and  $j$ ,  $d^{ij}$  is a parameter vector specifying distances between vehicles  $i$  and  $j$ ,  $x_{LB,t}^j$ ,  $x_{UB,t}^j$ ,  $u_{LB}^j$ , and  $u_{UB}^j$  are bounds for state and control at time  $t$ , respectively.

Now, we describe the MPC scheme for the control of a formation of AUVs. This scheme runs in each vehicle and will be the same for all AUVs. Thus, if there is no loss of information in the communication, then, all the vehicles have the same data and the control strategy generated for each vehicle is known to all of them. In the event of packet dropouts or communication delays, the missing sampled data is replaced by simulated data, and there will be some differences between the control strategies computed by the various vehicles for a given vehicle.

The MPC scheme for AUV  $i$  is as follows:

1. Initialization: prediction and control horizons, other optimal control problem parameters that depend on specific mission requirements, such as, level of perturbations, existence of obstacles, relative importance of trajectory tracking and formation pattern errors.
2. Sample the state variable, compute its estimate, and communicate it to its neighbors via acoustic modem.
3. Obtain the state variable of its neighbors via acoustic modem.
  - (a) If data is available go to step 4.
  - (b) Otherwise, generate estimates of the neighbors' state by running their models.
4. Solve the linear quadratic optimization problem ( $LQP^i$ ) at the current time  $t$ , and for the current prediction horizon (of length  $N_p$ ) and the given reference output trajectory. This yields an optimal control sequence for vehicle  $i$ .
5. Apply the control  $u^{i*}$  for the current control horizon.
6. Slide time for the optimization problem and adjust parameters if needed.
7. Let time elapse until the end of the current control horizon, and go to step 2.

## 5. SIMULATION RESULTS

### 5.1 Evaluation

We developed an evaluation system targeted at exercising our framework under conditions representative of field operations. We introduce three metrics for performance evaluation: trajectory tracking, formation tracking and control effort. The trajectory tracking metric for vehicle

$i$  is given by  $TM_i = \sqrt{T \sum_{k=1}^{N_p} (y_k^{ref,i} - y_k^i)^2}$ . This metric

Table 1. MPC Performance Table

Noise Level Situation	Mean Var.	0 0	0 0.05	0 0.1	0 0.25	0.1 0.02
Comms Disabled	TM	0.7	3.2	11.8	33.5	211.7
	FM	0.2	1.4	2.8	4.8	39.6
	CM	8.2	27.6	40.6	48.2	57.7
	C	34.4	206.6	524.9	1158.0	8197.0
Comms Enabled Delay=0 Sec	TM	0.7	0.8	0.8	1.0	1.1
	FM	0.2	0.2	0.3	0.4	0.4
	CM	8.2	10.6	14.7	25.9	17.6
	C	34.4	41.6	48.4	70.3	81.3
Comms Enabled Delay=0.1 Sec	TM	0.7	0.8	0.9	1.2	1.6
	FM	0.2	0.3	0.3	0.4	0.8
	CM	8.2	16.0	24.5	34.7	18.3
	C	34.4	44.2	52.5	74.9	105.5

accounts for the trajectory tracking errors. The formation

metric is given by  $FM = \sqrt{T \sum_{k=1}^{N_p} \sum_{j \in \mathcal{G}(i)} (y_k^i - y_k^j - d^{ij})^2}$ .

Similarly, this metric accounts for deviations with respect to the predefined formation pattern. Finally, the control metric is a measure of fuel consumption and is given by

$$CE = \sqrt{T \sum_{k=1}^{N_p} u_k^2}.$$

To assess the framework performance, three different scenarios were considered for a side-by-side formation of two vehicles moving along a sine trajectory with a nominal velocity of 1 m/sec: communications disabled, communications enabled without delays and communications enabled with a delay of 0.1 seconds (corresponding to a communication distance between vehicles of 150m). In this last scenario a prediction model is used to mitigate the impact of the delay. We considered also several levels of perturbations for each scenario going from no noise, to 0.1 mean and 0.02 variance.

Table 1, shows how our MPC controller performed. Here, the overall trajectory tracking metric  $TM$  is given by  $TM = \frac{TM_1 + TM_2}{2}$ . The entries in this table were obtained by averaging the performance of 10 runs with independent realizations of the input random variables.

From the data in the table some conclusions arise:

- Performance metrics degrades as the performance of the formation controller degrades with noise.
- The performance of the controller improves significantly with enabled communications relatively to running in open-loop (communications disabled).
- If we do not use prediction to mitigate de impact of delay, then the performance degrades significantly. On the other hand, if we do compensate the transmitted vehicle position, the performance becomes acceptable – as shown in table 1 – when compared with the scenario of no delay.

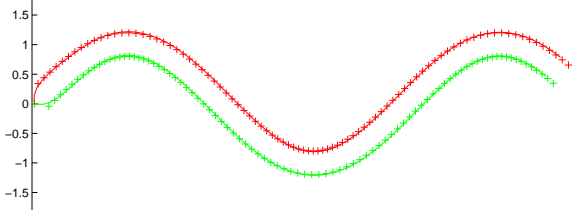


Fig. 2. Formation trajectories without noise or delay

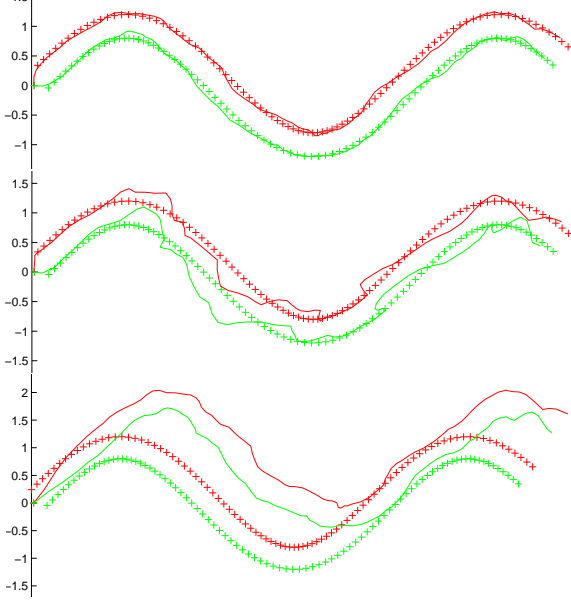


Fig. 3. Formation trajectories with AUV communications and noise: (Mean = 0, Variance=0.05), (Mean = 0, Variance=0.25), (Mean = 0.1, Variance=0.02)

## 5.2 Runs

In this section some simulation runs are shown and the results of our evaluation discussed.

First, it is not surprising that, in the deterministic case (no noise and no delays), the performance is very good. Figure 2 shows exactly that for a 2-vehicle formation. Note that the reference trajectory for vehicle  $i$  is represented with a “+” symbol. Red and green colors refer to vehicle  $V_1$  and  $V_2$  respectively. Solid line refer to the formation trajectories.

On the other hand, if we now add some Gaussian noise to the process we can observe performance degradation. Figure 3 shows the performance degradation for three different noise profiles. The impact of the mean is more significant than that of the variance. The analysis of the trajectory realizations depicted in this figure reveals the controller robustness to perturbations and time delays.

To show the flexibility of the framework, Figure 4 shows the control of three vehicles in triangle formation avoiding an obstacle. If we regard the emergence of the obstacle as perturbation that causes vehicles’ states to deviate from their nominal values, we conclude that the designed MPC controller is strongly stable.

Figure 5 shows the effect of communication dropouts. These have a significant impact in the controller perfor-

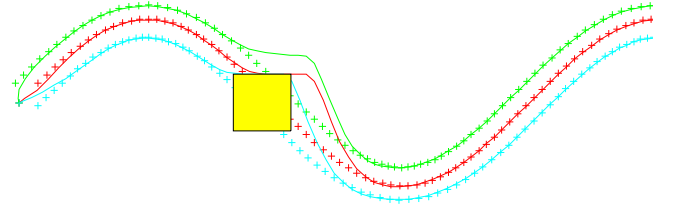


Fig. 4. Formation control of three AUVs in a triangle formation and obstacle avoidance

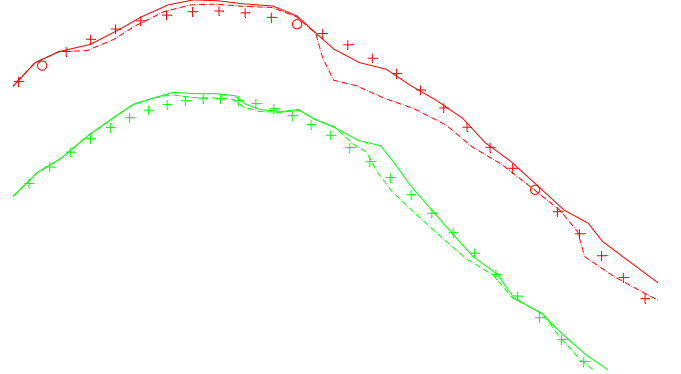


Fig. 5. Effect of communications dropouts in formation control of two AUVs

mance since communication messages carry the vehicles state between vehicles. Dropout times are modeled with a normal distribution, being the dropout of the message received on vehicle  $i$  at time  $k$  represented with an “o” in its reference trajectory. In this figure, red and green colors refer to vehicles  $V_1$  and  $V_2$ , respectively. While the reference trajectories are represented with “+”, the solid line refers to the vehicle trajectory for a well defined noise profile, and the dash-dot line shows what would happen if the dropouts shown in the figure occurred. The following comments are in order:

- When  $V_1$  fails to receive the position data of  $V_2$ , the MPC formation controller becomes open loop and the trajectory  $V_1$  degrades.
- If we focus on the trajectory of  $V_2$ , we observe that, every time  $V_1$  drives away from the expected trajectory (due to dropouts),  $V_2$  – which is kept receiving the data from  $V_1$  – adjusts its position in such a way that the formation constraint holds.

The inspection of the trajectory realizations in this figure reveals that the perturbations caused by the packets dropouts are reasonably well countered in a sustained way by the MPC controller.

## 6. CONCLUSIONS

A formation controller based on model predictive control has been presented. Multiple simulations runs revealed that the proposed framework produced the intended control strategies according to the requirements. Many research challenges remain in order to achieve the computational tractability for problems with more complex formations and larger number of vehicles. This will require new

ways of taking into account the decentralization character and are the subject of current research.

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