A General Attainable-Set Model Predictive Control Scheme. Application to AUV Operations ⋆

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Abstract: In this article, a Model Predictive Control (MPC) scheme that, by taking advantage of the control problem time invariant ingredients, replaces as much as possible the on-line computational burden of the conventional schemes, by off-line computation, is presented and its asymptotic stability shown. The generated data is stored onboard in look-up tables and recruited and adapted on-line with small computation effort according to the real-time context specified by communicated or sensed data. This scheme is particularly important to the increasing range of applications exhibiting severe real-time constraints. The approach presented here provides a better re-conciliation of onboard resources optimization with state feedback control - to deal with the typical a priori high uncertainty - while managing the formation with a low computational budget which otherwise might have a significant impact in power consumption.

Keywords: Model predictive control, Attainable set, Value function, Stability, AUV formation control, Obstacle avoidance

1. INTRODUCTION

The goal of this article is to discuss a new Model Predictive Control (MPC) scheme that, by taking advantage of the time invariant objects of the control problem, replaces as much as possible the on-line computational burden of the conventional schemes, by off-line computation. The class of systems envisioned in this research effort are those for which optimization, and robustness are key requirements, and, at the same time, scarcity of computational, and power resources, and time severe. A typical, and increasingly important class of systems are those involving Autonomous Underwater Vehicles (AUVs), either in stand-alone configuration, or possibly networked and articulated with other devices. The data generated by off-line computation - notably parameterized Attainable Sets and Value Functions - is stored onboard in look-up tables and recruited and adapted on-line with little computation effort according to the real-time context specified by communicated or sensed data. We call this scheme of Attainable Set MPC (AS-MPC) which departs significantly from previous ones as it is clear from state-of-the-art overview in 3. The AS-MPC (and its robust version, RAS-MPC) were presented and some of its properties discussed in Gomes and Pereira (2018) for the management and control of the AUV formations, which, in turn, follow from Gomes and Pereira (2017a), and Gomes and Pereira (2017b). In these references, a control architecture is also presented in order to also handle discrete events. Besides revisiting some of these results, now, we extend the investigation to encompass the asymptotic stability of the AS-MPC scheme. Moreover, we expand the considerations, accompanied by an illustrative example, on the hybrid control automaton in order to ensure the successful termination of a given mission. These developments are in Gomes (2017).

In section 2, we state the problem of controlling an AUV formation to track a planned trajectory along which payload data is gathered. Then, a selected brief state-of-the-art overview is given in 3. The presentation of the AS-MPC scheme for AUV motion control and the discussion of its properties is given in 4. Then, section 5 provides one of the main paradigmatic applications which requires the extension of the AS-MPC to the hybrid control systems. Some conclusions and prospective future work are provided in the last section.

2. THE AUV FORMATION TRACKING PROBLEM

The AUV formation control problem is formulated in order to define strategies for the relative behaviors of given set the vehicles, regarded as a single reconfigurable generalized vehicle. The operation in formation is required to fulfill the activities requirements to achieve the mission goal. Here, we focus in the mission of gathering payload data in a given water column volume along a pre-planned path. The AUVs motion control should be such that the integral error with respect to a given reference trajectory and the total control effort during a certain period of time is minimized subject to constraints as indicated below. A general formulation of the Optimal Control Problem (OCP) associated with this mission is:

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$$(P_T) \text{ Minimize } g(x(t_0{+}T)) + \int_{t_0}^{t_0+T} f_0(t, x(t), x_r(t), u(t))dt$$

$$\text{subject to } \dot{x}(t){=}f(t, x(t), x_r(t), u(t)) \quad \mathcal{L}-a.e.$$

$$u{\in}\mathcal{U}, \quad x(t_0{+}T){\in}C_f$$

$$h(t, x(t)){\leq}0, \quad g(t, x(t), u(t)){\leq}0$$

where $g{:}\boldsymbol{R}^n{\to}\boldsymbol{R}$ is the endpoint cost functional, $f_0{:}\boldsymbol{R}{\times}\boldsymbol{R}^n{\times}\boldsymbol{R}^m{\to}\boldsymbol{R}$ is the running cost integrand, $f{:}\boldsymbol{R}{\times}\boldsymbol{R}^n{\times}\boldsymbol{R}^m{\to}\boldsymbol{R}^n$, $h : \boldsymbol{R}\times\boldsymbol{R}^n \to \boldsymbol{R}^q$, and $g : \boldsymbol{R}\times\boldsymbol{R}^n\times\boldsymbol{R}^m \to \boldsymbol{R}^k$ represent, respectively, the vehicle dynamics, the state constraints, and the mixed constraints, $\mathcal{U}{:=}\{u{:}[t_0, T]{\to}\boldsymbol{R}^m{:}u(t){\in}\Omega\}$, $\Omega \subset \boldsymbol{R}^m$ is the set of measurable controls, $C_f \subset \boldsymbol{R}^n$ is a target set specified in order to either represent application domain features or to ensure stability, and x_r is the trajectory to be tracked. The mappings g, f, f_0, g and h, and the sets C_f and Ω satisfy the following assumptions (S): $\forall(t, u) \in \boldsymbol{R}{\times}\Omega$, are Lipschitz in x with constant K_m where m represents the respective map, and continuous in (t, u) for all $x{\in}\boldsymbol{R}^n$, $\exists K{\geq}0$ such that $\|f(t, x, \Omega)\|{\leq}K$, $\forall(t, x){\in}\boldsymbol{R}^{n+1}$, and C_f and Ω are compact.

To instance (P_T) in the context of AUV formation of N vehicles tracking a reference trajectory x_r, consider, for the AUV i:

- $x{=}col(\eta^i, \nu^i)$, and $u{=}\tau^i$, $i{=}1, \ldots, N$.
- $f_0(\cdot, \cdot, \cdot){=}(\eta^i(t){-}\eta_r^i(t))^T Q(\eta^i(t){-}\eta_r^i(t)){+}\tau^{iT}(t)R\tau^i(t)$, $g(\cdot){=}0$, being $\eta_r^i(\cdot)$ the i^{th} vehicle reference trajectory, and τ^i its control (denoted by u in (P_T)).
- By considering, for each vehicle (we drop the index i), the state and the controls given by $\eta = [x, y, \psi]^T$ and $\nu = [u, v, r]^T$ and $\tau = [\tau_u, \tau_r]^T$, respectively, the dynamics are given in Fossen (1994). For details, Gomes and Pereira (2018).
- Other constraint types include: (i) endpoint state constraints, $\eta^i(t + T){\in}C_{t+T}$, (ii) control constraints, $\tau^i(s) \in \mathcal{U}^i$, (iii) state constraints, $(\eta^i(s), \nu^i(s)) \in \mathcal{S}^i$, (iv) communication constraints $g^c_{i,j}(\eta^i(s), \eta^j(s)) \in C^c_{i,j}, \forall j{\in}\mathcal{G}^c(i)$, and (v) formation constraints $g^f_{i,j}(\eta^i(s), \eta^j(s)){\in}C^f_{i,j}, \forall j{\in}\mathcal{G}^f(i)$.

The OCP formulation can be either centralized or decentralized. In the later case, each vehicle has its own controller requiring two components: one underlying its own motion and other activities, and another concerning the cohesion of the required formation pattern. Each vehicle communicates acoustically with its neighbors, and their relative positions have to be such that full connectivity of the formation bidirectional communication graph is guaranteed. Each vehicle is a node of this graph whose arcs are the communication links. Communicating vehicles navigate sufficiently close to one another to mitigate the packets loss. Modes of operation include data gathering, obstacle collision avoidance, communication, and loitering. Each mode operation mode has its own formation pattern.

3. BRIEF STATE-OF-THE-ART

A good reference in AUV motion control problems is Fossen (1994). Extended versions of these control systems for very diverse robot craft have been considered for single and multiple vehicles. Non-linear control theory and geometric control provide tools led to very popular design techniques, Kristiansen and Nicklasson (2009); Ren and Beard (2002); Lv et al. (2011). Early on, it became clear that, along with feedback control, resources optimization plays a key role in contexts of scarce resources. Thus, MPC became a design approach of choice for many applications. Conventional MPC schemes generate feedback control syntheses conciliating near optimization with feedback control. For details, see Gomes (2017). Moreover, MPC schemes inherit from optimal control a huge flexibility enabling the handling of very complex dynamics subject to very diverse types of constraints, encompassing those in vehicle formations. A wide variety of MPC controllers for formations of robotic vehicles were designed to deal with key challenges such as communications failures and delays in continuum and discrete times, centralized and decentralized contexts, linear and nonlinear dynamics, leader-follower and leaderless schemes, collision-free motion, cooperative and competitive strategies, single and multiple objectives. Moreover, a wide range of applications (surveillance, exploration, tracking paths and trajectories), have been considered in a vast literature of which we single out Mayne et al. (2000); Franco et al. (2004, 2008); Keviczky et al. (2006, 2008); Consolini et al. (2008); Chao et al. (2011); Quintero et al. (2015); Bertrand et al. (2014); Andrade et al. (2016); Shen et al. (2016). Many of these MPC approaches are weakly suitable for AUVs: (i) intense on-line computational burden, and (ii) design issues to ensure the required performances do note consider onboard resource constraints. This work considers some of these issues, and, in line of the one proposed in Gomes and Pereira (2017a) and is an extension of Gomes and Pereira (2018),concerns a substantially different approach to AUV formation control.

4. ATTAINABLE SET MPC

Now, we present the AS-MPC introduced in Gomes and Pereira (2017a, 2018) including some additional results. The key ideas are: (i) Replace the infinite dimensional optimization problems by simpler finite dimensional ones; (ii) Take advantage of time-invariant data such as vehicle dynamics, and environment features to pre-compute offline and store on-board approximations to reference, short term, Attainable Set (AS), and Value Function (VF) in an appropriate grid of points to be recruited on-line depending on real-time data.

Let $T_f > 0$ be a large number, with possibly $T_f = \infty$ and consider the short term "equivalent" cost functional, and the attainable set for the dynamic control system on the time horizon $[0, T_f]$, and let (P_{T_f}) be (P_T) with $T{=}T_f$ and (only for the sake of simplicity) without state and mixed constraints. Let Δ be the control horizon length and t_0 the current time. By defining

a) <u>Value Function</u>. $V(t, z) := \min_{u\in\mathcal{U}, \xi\in C_f} \{J(\xi, u)\}$, where $\xi = x(T_f)$, $\dot{x}(\tau) = f(\tau, x(\tau), u(\tau))$, $\mathcal{L}$-a.e., $x(t) = z$, $u{\in}\mathcal{U}$, and $J(\xi, u){=}g(\xi){+}\int_t^{T_f} f_0(\tau, x(\tau), u(\tau))d\tau$, and

b) <u>Attainable Set</u>. (Kurzhanski and Varaiya (2001))

$$\mathcal{A}_f(t; t_0, x_0){=}\{x(t){:}\dot{x}{=}f(t, x, u), u{\in}\mathcal{U}, x(t_0){=}x_0\}, \ t{>}t_0.$$

we have, by the Principle of Optimality, that (P_{T_f}) and $(P^a_{T_f})$ are equivalent, being

($P^a_{T_f}$) Minimize $V(t{+}\Delta, z))$ subject to $z{\in}\mathcal{A}_f(t{+}\Delta; t, x(t))$.

Note that the integrand was removed by a standard change of variable (no relabelling), Gomes and Pereira (2018). The principle of optimality states that, for $\Delta < T_f - t_0$, the solution to (P_{T_f}) restricted to $[t_0, t_0{+}\Delta]$ is also a solution to (P_{T_f}).

The simplicity of $(P^a_{T_f})$ is merely apparent due to the complexity of the AS and VF computation. The computational burden associated with $\mathcal{A}_f$ leads to select a suitable approximation. Polyhedral of either inner or outer type, Baturin et al. (2006); Graettinger and Krogh (1991), ellipsoidal, Kurzhanski and Vályi (1997), and "cloud of points" as endpoints of trajectories generated by piecewise constant controls have been proposed. A trade-off analysis between the on-line complexity and extent of the precision led us to opt for the latter. For positional systems, Krasovskii and Subbotin (1988), the VF may be computed by solving the Hamilton-Jacobi-Bellman equation (HJBE) which are known to be very computationally intensive, in spite of, the enormous research effort that lead to a number of efficient software packages, Sethian (1999); Michel et al. (2005). Generally, a set of VFs for given typified situations is defined. In real-time "mission" execution, the relevant VF is identified via on-line data and invoked to compute the optimal control at any (t, x). The time invariance of the dynamics allows the off-line pre-computation of an approximation to $\mathcal{A}_f(t_0 + \Delta; t_0, x_0)$ and of $V(t_0{+}\Delta, z)$ depicted in figure 1.

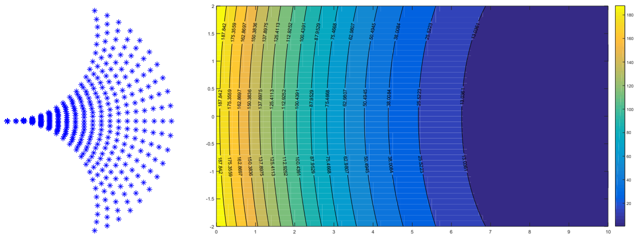

Fig. 1. i) Unicycle AS and (ii) Position control VF

Let Δ be the control horizon, and t_0 the current time. The AS-MPC scheme is follows:

1. Initialization: t_0, $x(t_0)$, other MPC parameters.
2. Solve $(P^a_{T_f})$ to obtain z^* and compute u^* on $[t_0, t_0{+}\Delta]$ to steer the system from x_0 at t_0 to z^* at $t_0{+}\Delta$.
3. Apply u^* during $[t_0, t_0{+}\Delta]$
4. Sample x at $t_0{+}\Delta$ to obtain $\bar{x} = x(t_0{+}\Delta)$
5. Slide time, i.e., $t_0{=}t_0{+}\Delta$, update the AS with the new $x(t_0)$ by appropriate translation and rotation, update the VF at the new $t_0{+}\Delta$, and goto 2.

The only on-line computational burden is due to the need on adapting the AS and VF for the current AUV configuration and the environment context.

In Gomes (2017), the properties considered in the Propositions of this section are stated and proved. Denote by $(x^*_{T,\Delta}, u^*_{T,\Delta})$ the AS-MPC scheme with (P_T) as optimal control process [1]. Let $J(x, u)$ be the cost functional, with the state endpoint term replaced by the running time equivalent, evaluated at (x, u) over $[0, \infty)$, and by $J(x, u)_{|[\alpha,\beta]}$ and $J_k(x, u)$, its restriction to $[\alpha, \beta]$, and to $[k\Delta, (k + 1)\Delta]$.

Proposition 1. *Consider (x^*, u^*) to be an optimal control process for the infinite horizon OPC such that $\lim_{t\to\infty} x^*(t) = \xi^*$, where ξ^* is an equilibrium in C_∞. Let $(x^*_{T,\Delta}, u^*_{T,\Delta})$ be s.t. $\lim_{t\to\infty} x^*_{T,\Delta}(t) = \xi^*$. Then,*

(i) $\displaystyle\lim_{\Delta\downarrow 0, T\uparrow\infty} \sum_{k=1}^{\infty} J_k(x^*_{T,\Delta}, u^*_{T,\Delta}) = J(x^*, u^*)$

(ii) $\displaystyle\lim_{k\to\infty} \left| J_k(x^*_{T,\Delta}, u^*_{T,\Delta}) - J(x^*, u^*)_{|[k\Delta,(k+1)\Delta]} \right| = 0.$

Since we are using the cloud of points as approximation to the AS, an estimate of the Hausdorff distance between these sets is required to determine the worst case of suboptimality. Let $\Omega_\varepsilon = \{u_i \in \Omega : i = 1, \ldots, N_\varepsilon\}$ such that: (i) $\Omega \subset \bigcup_{i=1}^{N_\varepsilon}(u_i {+} \varepsilon B)$, and (ii) $\forall i\, \exists j$ s.t. $\|f(t, x, u_i) - f(t, x, u_j)\| {<} \varepsilon$. Denote by $\mathcal{A}_f(t_1; t_0, x)$ and $\mathcal{A}^\varepsilon_f(t_1; t_0, x)$ the points attainable at $t_1 {>} t_0$ from x at t_0, by the dynamic system with controls, respectively, in L^∞ with values in Ω, and piecewise constant with values in Ω_ε.

Proposition 2. *Let $\Delta > 0$. Under the considered assumptions on the dynamics, we have,* $\forall (t, x) {\in} \boldsymbol{R} {\times} \boldsymbol{R}^n$,

$$d_H\left(\mathcal{A}_f(t{+}\Delta; t, x), \mathcal{A}^\varepsilon_f(t{+}\Delta; t, x)\right) {\le} \Delta(\varepsilon {+} \Delta K_f K).$$

The fact that $\bar{x} \in \boldsymbol{R}^n$ to which the system is steered at a given time is likely not listed in the VF look-up table, prompts the following VF interpolation estimate.

Proposition 3. *Assume that V is not known at $\bar{x}$, and consider a grid of points G_δ in $\boldsymbol{R}^n$ s. t. the maximum distance between neighboring points in G_δ is less than $\delta{>}0$. Then, $\exists$ a simplex* [2] *$S_{\bar{x}} {=} \{x_i {:} i{=}1, \ldots n{+}1\} {\subset} G_\delta$ whose points are the closest to $\bar{x}$ s. t. the estimate $\tilde{V}$ of V at $\bar{x}$ is*

$$\tilde{V}(\bar{x}) {=} \frac{\sum_{i=1}^{n+1} V_i \|\bar{x} {-} x_i\|^{-1}}{\sum_{i=1}^{n+1} \|\bar{x} {-} x_i\|^{-1}}$$

where, for $i = 1, \ldots, n{+}1$, $V_i = V(x_i) {+} \nabla V(x_i) \cdot \bar{v}_i$, with $\bar{v}_i = \bar{x} {-} x_i$ and the $n {\times} (n{+}1)$ unknowns of the vectors $\nabla V(x_i)$, are given as a solution of the $n{+}1$ equations $\nabla V(x_i) {\cdot} (\bar{v}_i {-} \bar{v}_k) {=} \frac{V(x_k) - V(x_i)}{\|x_i - x_k\|}$. Moreover, $\exists c{>}0$, such that $\|V(\bar{x}) {-} \tilde{V}(\bar{x})\| {\le} \max_{x_i, x_j \in S_{\bar{x}}} \{|V(x_i) {-} V(x_j)|\} {+} c\delta$.

In order to discuss the stability of the AS-MPC, we will consider first the corresponding conventional MPC scheme with the optimal control problem (P_T) with finite T. Let us denote by $(\bar{x}^*, \bar{u}^*)$ the control process generated by this MPC scheme. A given equilibrium $\xi^* \in C_f$ is asymptotically stable if, under the above assumptions $\forall \varepsilon {>} 0$, $\exists N {\in} \boldsymbol{N}$ s.t. $\|\bar{x}^*(k\Delta) {-} \xi^*\| {\le} \varepsilon\ \forall k {>} N$. There is a vast literature on the stability for MPC schemes, in which some of the MPC ingredients - terminal, and running cost functionals, optimization, and control horizons, and terminal constraints - are chosen and/or endowed with properties to ensure the asymptotic stability of the control process generated by the MPC is asymptotically stable. Although not exhaustive, in Jadbabaie and Hauser (2005), various stability results are presented and the trade-off of the design intrusiveness is clearly discussed.

[1] This differs from the stated AS-MPC, where $T = \infty$

[2] A simplex in R^n is any set of $n{+}1$ points whose n vectors given by the difference to one of them form a linearly independent set.

We choose a result akin to the one in Jadbabaie and Hauser (2005) or in chapter 6 of Gruene and Pannek (2017) (albeit in discrete time), which are natural in the sense that they dispense with the need to shape C_f for this purpose and, in order to cover a wider as possible for the AS-MPC design, we might either require the terminal cost to be Control Lyapunov Function (guaranteeing exponential stability) in the attraction region, or, for more general terminal costs (including the null case), a certain asymptotic controllability assumption on the cost-to-go functional.

To facilitate the exposition, we consider only autonomous systems, $g_0 \equiv 0$, $x_{ref} = 0$, and $t_0 = 0$. Since, due to its infinite horizon VF, the stability of the AS-MPC follows in a straightforward manner from that for the conventional MPC in which the OPC is solved on a finite optimization horizon $[0, T]$, we will start by considering this context.

Denote by $\{P_T^l\}_{l=0}^{\infty}$ the sequence of OPCs of the MPC scheme on the interval $[l\Delta, l\Delta + T]$, and, in what follows, we have $T = N\Delta$ for some $N \in \boldsymbol{N}$.

Asymptotically Controllable (AC) Assumption. The system is AC with respect to f_0 with rate $\beta \in \mathcal{KL}_0$ [3] if and only if, $\forall x_0 \in \boldsymbol{R}^n$ and $T > 0$, $\exists$ an admissible control sequence $\{u^l\}$, $\forall u^l \in \mathcal{U}^l(x_0)$, satisfying $f_0(x_{x_0}(t), u^l(t)) \leq \beta(\bar{f}_0(x_0), t)$, $\forall t \in [0, T]$.

Here, $\mathcal{U}^l(x_0)$ is the subset of feasible controls from the l^{th} iteration onwards with $\bar{f}_0(x_0) = \min_{u \in \Omega}\{f_0(x_{x_0,u}, u)\}$, $x(0) = x_0$, $x_{x_0,u}(\cdot)$ is a trajectory starting at x_0 driven by u (either sub-index might be omitted, whenever obvious), $\mathcal{U}_T(x_0)$ is the set of feasible controls on $[0, T]$ for $x(0) = x_0$.

In Gruene and Pannek (2017), it is shown that, from (CA) follows that, if for a given $x_0 \in \boldsymbol{R}^n$, $\exists \bar{u}^*$, optimal control for (P_T) with the specificities adopted for this result s. t. $\lambda_k = \int_{k\Delta}^{(k+1)\Delta} f_0(x_{x_0,\bar{u}^*}, \bar{u}^*(t))dt$ are nonnegative for $k = 0, \ldots, N-1$, then $\sum_{j=k}^{N-1} \lambda_j \geq \int_{k\Delta}^{T} \beta(\lambda_k, t)dt$ for $k = 0, \ldots, N-2$. Moreover, by letting $\nu = V_T(x_{x_0,\bar{u}^*}(1))$, then, for $k = 0, \ldots, N-2$, we have $\nu \leq \sum_{j=0}^{k-1} \lambda_{j+1} + \int_{k\Delta}^{T} \beta(\lambda_{k+1}, t)dt$, being $\beta \in \mathcal{KL}_0$. Then, it is shown that, if $\exists\, \alpha \in (0, 1]$ s.t. $\sum_{k=0}^{N-1} \lambda_k - \nu \geq \alpha \lambda_0$, we have that for such α and (P_T) satisfying (AC), the relaxed dynamic programming inequality

$$V_T(x) \geq \alpha \int_0^{\Delta} f_0(x_{x,\bar{u}^*}(t), \bar{u}^*(t))dt + V_T(x_{x,\bar{u}^*}(\Delta))$$

holds for all $x \in \boldsymbol{R}^n$, and u^* is the feedback control law generated by the MPC. A suitable value of α is the smallest one satisfying all the conditions, being a procedure to compute it presented in Gruene and Pannek (2017).

In the sequel, we will assume the additional assumptions (AS): (i) (P_T) with $T = \infty$ has a feasible solution, and (ii) $\exists\, \alpha_1, \alpha_2 \in \mathcal{K}^{\infty}$ [4] s. t. $\alpha_1(|x|) \leq \bar{f}_0(x) \leq \alpha_2(|x|)$, $\forall x \in \boldsymbol{R}^n$.

Proposition 4 (Theorem 6.18 of Gruene and Pannek (2017)). *Consider that the data of (P_T) with $T = \infty$ satisfying (S), (AC) and (AS) and that $\exists\, \alpha \in (0, 1]$ as above. Then, in the absence of disturbances, the conventional MPC scheme with horizon T generates a feedback control process (x^*, u^*) which is asymptotically stable. Moreover, $\forall x \in \boldsymbol{R}^n$, $J_{\infty}(x, u^*) \leq \frac{V_T(x)}{\alpha} \leq \frac{V_{\infty}(x)}{\alpha}$.*

It is important to note that the main interest of this result lies in the fact that, at the price of more elaborated arguments, the global scope and constructive nature to support the design of asymptotically stable MPC are obtained. The proof of the discrete version of this result appears in Gruene and Pannek (2017) and, it is straightforward in the light of previous results. Its continuum time version involves some additional technicalities but the underlying ideas remain. We recall that this was shown to the conventional MPC scheme. Since the AS-MPC scheme is equivalent to the conventional MPC with $T = \infty$ it turns out that the asymptotic stability is even easier to hold since the larger the value of T, the closer to a Lyapunov function $J_T(x, u^*)$ is in the attraction domain, Jadbabaie and Hauser (2005).

5. THE CONTROL ARCHITECTURE

The control architecture emerges from the application of the AS-MPC scheme to an hybrid automata encompassing the desired modes of operations and the discrete events triggering the transitions between them. In this way, it organizes the motion control of the generalized vehicles in terms of simpler AUV formation control problems. These are required either due to the complexity of the mission - typically involving multiple phases - or to the variability of the environment due to the emergence of events with impact in the mission execution. This amounts to regard the overall formation model as a hybrid dynamic control system, i.e., a collection of dynamic control systems - one per mode of operation -, and a set of either controlled or uncontrolled discrete events associated each one of them. Thus, the implementation of the AS-MPC controller described in section 4 in the context of hybrid dynamics requires the need of an event-driven control strategy ensuring liveness and nonblocking properties, Cassandras and Lafortune (2008), to be embedding of the AS-MPC controller. The general approach consists in designing an hybrid supervisory controller that, once composed with the original system ensures the specified set of properties. This "closed loop" hybrid automaton will constitute the dynamics of the AS-MPC scheme. To eliminate the burden inherent to the explanation of general contexts, we focus in the motion control of a three AUVs formation in a plane carrying the required navigation and payload sensors whose mission consists in gathering data along a given path such that the observation requirements are satisfied. The automata representing the highest layer of the control architecture is shown in Figure 2. The considered

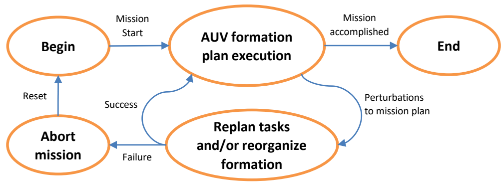

Fig. 2. Main System Automaton

tasks are: (i) Gather data along a given path while the

[3] $\mathcal{KL}_0$ is the class of functions $\beta : R^+ \times R^+ \to R^+$ for which $\beta(\cdot, t) \in \mathcal{K}^{\infty}$, i.e., it satisfies $\beta(0, \tau) = 0$ and $\lim_{r \to \infty} \beta(r, \tau) = \infty$, and $\forall r > 0$, $\lim_{\tau \to 0} \beta(r, \tau) = 0$

[4] This class of functions was defined in the previous footnote.

AUVs keep the triangle formation and the decentralized controller simultaneously ensures path tracking and formation pattern; (ii) Avoid collision with obstacles by detecting obstacles, characterizing them, collision-avoidance path replanning, and, possibly, formation reconfiguration; (iii) Communication required to transmit gathered data, enable mission follow-up, and receive new commands if necessary. The set of discrete modes and events triggering transitions are depicted in Figure 3. Once the mission

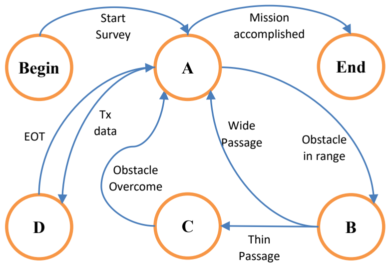

Fig. 3. Main operation modes and events

starts, the AUVs enters in the nominal mode **A** - data gathering while tracking the given path in a triangle formation. The *Mission Accomplished* event triggers the recovery operation. The mission follow-up requires monitoring the gathered - mode **D** - which involves the surfacing of an AUV from time to time (pre-planned or not) to transmit the gathered data and the AUVs health status, which, after a scrutiny, might entail mission changes. Once mode **D** is complete, the operation mode **A** is reactivated. If an *Obstacle in range* event is detected, then mode **B** - obstacle characterization and a collision avoidance path computation - is activated. Then, either a *Wide passage* is available and the formation is kept unchanged and the system returns to mode **A** tracking the original path, or a *Thin passage* is available and the system transits to mode **C** where the formation is reconfigured to overcome de obstacle, and, once the obstacle is overcame, the system returns to mode **A**.

Due to the fact that it illustrates well the point concerning the interaction between mission planning and control, Obstacle Collision Avoidance is analyzed in detail. The following is considered: (i) Obstacle detection and characterization data is obtained by a range finder; (ii) Unmapped obstacles are relatively sparse; (iii) Obstacles are locally modelled by circles; (iv) The range finder sensor distance is much larger than that transversed by the AUV within Δ time units. The automaton 4 shows the various modes and associated transition events. Once an obstacle is detected, it is characterized to compute a path remaining as close as possible to the one to be tracked, while avoiding collision. The formation pattern can be adjusted to facilitate the obstacle characterization and overcoming. Whenever an AUV is close to an obstacle, then its AS-MPC is modified by adding penalization function to the VF to ensure a safety distance d_s to the obstacle. This procedure easily handles multiple obstacles, being the event "safety of a passage between obstacles" very relevant. Figure 5 depicts the logic generating this event. The passage is safe if $H_1 + H_2 - R_1 - R_2 - 2d_s > 0$ where d_s is given, R_1, R_2, C_1, C_2, and P_L are estimated with the range finder, $H_1 = \sqrt{R_1^2 + L_1^2}$, $H_2 = \sqrt{R_2^2 + L_2^2}$, $L_2 = |P_L - A|$, $L_1 = |A - P_V| - |\sqrt{(R_1 + d_s)^2 - R_1^2}|$,

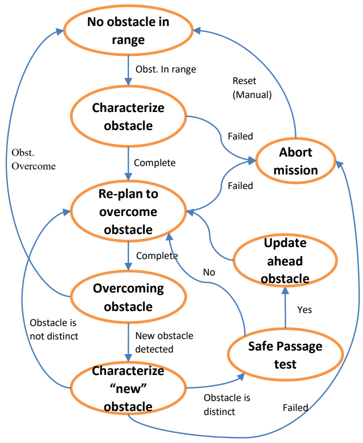

Fig. 4. Obstacle Collision Avoidance Automaton

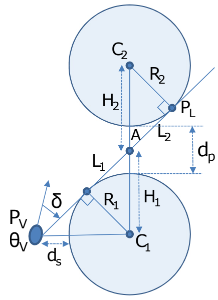

Fig. 5. Safe passage detection

P_V is the position of the AUV, and the point A is the intersection of the segments $\overline{C_1, C_2}$ and $\overline{P_V, P_L}$.

Simulation results for the considered mission in the presence of unexpected obstacles are shown in Figure 6. Af-

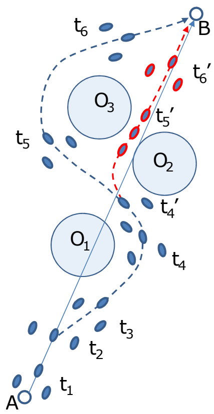

Fig. 6. Simulation results for obstacle avoidance

ter deployment, the AUVs are loitering in the triangle formation for the survey around the departure point A. Once the survey starts at time t_1, mode **A** is activated and the triangle formation tracks the given path to the final destination B. At t_2, obstacle O_1 is detected. Then, the formation switches to mode **B** to characterize the obstacle. In order to do this efficiently the AUVs may adapt the formation pattern and, once this is done, a path to overcome is generated by the AS-MPC by modifying the VF around O_1 by adding an adequate penalization function forcing the vehicle to overcome the obstacle by the right. Since there is plenty of space, the AUVs return to the triangle formation at t_3 while circumventing the obstacle. At t_4, O_2 is detected with O_1 still in range on the right. The VF modification due to O_1 is kept while a "safe passage between obstacles" event does not occur. Once this event occurs the VF is now modified around O_1 and O_2 to prevent any collision, and, moreover the triangle formation can be safely maintained. At t'_4, O_3 is detected and mode **B** determines two alternatives to pursue the mission:(i) Follow through a thin passage between O_2 and O_3 with longitudinal formation along a path closer to the original one and with loss of the gathered data quality; (ii) Circumvent O_3 by the left along a longer route far away from the original path, but preserving the triangle formation. A simple onboard optimization procedure determines that the first option is the best one. Once the obstacles are overcome at t'_6, the triangle formation is adopted until point B is reached and the mission mode changes to **D** to proceed with data transmission.

6. CONCLUSIONS

An AS-MPC scheme - conciliating limited onboard computational complexity with optimal feedback control - for a single AUV presented in Gomes and Pereira (2018) to the path tracking control of a formation of AUVs is considered and its properties, with emphasis to stability analysis are investigated. Moreover, by considering the dynamics given by an hybrid automata, the flexibility of the AS-MPC is revealed, via the simulation of a paradigmatic example, to be able to avoid collisions with unmapped obstacles as well as the various tasks of the mission and the management of the formation pattern. The obtained simulation results encourage the next step: migrate the developments to a multiple AUV based system for field testing.

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