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Cartel Stability and Profits under Different Reactions to Entry in Markets with Growing Demand

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Cartel stability and profits under different reactions to entry in markets with growing demand

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Abstract. We study sustainability of collusion with optimal penal codes, in markets where demand growth may trigger the entry of a new firm. In contrast with grim trigger strategies, optimal penal codes make collusion easier to sustain before entry than after. We compare different reactions of the incumbents to entry in terms of: sustainability of collusion, incumbent's profits, entrant's profits, consumer surplus and social welfare. Surprisingly, the incumbent firms may prefer competition to collusion.

Keywords: Collusion; Demand growth; Optimal penal codes; Reactions to entry.

JEL Classification Numbers: K21, L11, L13.

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1 Introduction

Demand growth is usually understood as facilitating collusion, by reducing the short-term gains of deviating relatively to the costs of future retaliation (Ivaldi et al., 2007). However, demand growth may attract new firms to the market, which hinders collusion. Vasconcelos (2008) is innovative in studying this trade-off in a model where demand growth induces the entry of a new firm. Comparing the sustainability of collusion before and after the entry, he concludes that collusion is more difficult to sustain before the entry (i.e., when there are less firms in the market). The rationale for this result lies in the possibility that incumbents have of delaying entry by disrupting the collusive agreement.

In dynamic models of collusion, the standard continuation strategies after a deviation are: *grim trigger* strategies (Friedman, 1971) and *stick-and-carrot* strategies (Abreu, 1986). Vasconcelos (2008) considers that, after a deviation, firms revert to the non-cooperative equilibrium forever (i.e., he assumes grim trigger strategies). We intend to study whether his conclusions would remain valid with stick-and-carrot strategies. In particular, we consider that, folowing a deviation: there is a single period of punishment, and the collusive agreement is recovered thereupon. This punishment is more severe than that considered by Vasconcelos (2008), since it implies a zero continuation value after a deviation (optimal penal code).

Another direction in which we complement the results of Vasconcelos (2008) concerns the reaction of the incumbents to entry. Vasconcelos (2008) assumes that the incumbent firms immediately accommodate the entrant in a more inclusive agreement.[1] However, as explained by Harrington (1989), Friedman and Thisse (1994) and Vasconcelos (2004), this is just one of the possible cartel reactions to the entry of a new firm. Thus, we analyze three alternative scenarios: (i) the collusive agreement is discontinued when entry occurs; (ii) the entry is treated as a deviation from the collusive agreement, triggering the punishment strategy; (iii) the entrant is gradually accommodated in the collusive agreement (Friedman and Thisse, 1994).

The structure of our model is the same as in Vasconcelos (2008). We consider a market with two incumbent firms and one potential entrant that compete *à la* Cournot in an infinitely repeated game. Market demand is linear and grows at a constant rate.

[1] Vasconcelos (2008) also studies the sustainability of partial collusion. More precisely, he considers the possibility of the incumbents not including the entrant, behaving as a Stackelberg leader. He finds, however, that such an agreement is not sustainable.

Production costs are null. To become active, the potential entrant must support an entry cost. When choosing its entry period, the entrant faces the following trade-off: on the one hand, it wants to enter as soon as possible to produce and make profits; on the other hand, postponing entry decreases (the discounted value of) the entry cost. Entry occurs when the present value of the entrant's profits is maximum.

At the beginning of the game, the incumbent firms agree to produce quantities that maximize their joint profit in each period. Unilateral deviations trigger one period of punishment, after which the collusive agreement is restored. The punishment is as harsh as possible, inducing a zero continuation value after a deviation. As firms have positive profits when collusion is restored, they must make losses in the punishment period. For losses to be feasible (recall that there are no production costs), we assume that firms may incur in some form of observable dissipative costs (such as unproductive advertising or solidarity actions).[2]

We start by following Vasconcelos (2008) in assuming that the incumbents accommodate the entrant in their collusive agreement. In contrast with Vasconcelos (2008), we conclude that collusion is more difficult to sustain after entry than before entry. The result of Vasconcelos (2008) does not hold with stick-and-carrot punishment strategies, because a deviation before the entry ceases to be an effective way of delaying entry and enjoying additional pre-entry profits. A deviation can delay entry by one period, as the entrant would never enter in the punishment period. However, since the continuation value after the deviation is zero, the punishment absorbs all future profits (which include the gains from delaying entry).

The idea that firms benefit from establishing collusive agreements is widely accepted. Interestingly, we conclude that, in growing markets, the incumbent firms may prefer competition to collusion. Two conditions are necessary for this finding: (i) entry occurs later under competition (because the expected profits of the entrant are lower); and (ii) individual profits are greater under competition with two firms than under collusion with three firms. This implies that, under competition, the incumbents enjoy a longer pre-entry period, in which, although competing, they obtain higher profits than if they were colluding but entry had already occurred.

We compare our base case of immediate accommodation with three alternative reac-

[2]Costs of this kind were considered, in different contexts, by Milgrom and Roberts (1986), Bagwell and Ramey (1994), Lariviere and Padmanabhan (1997), Hertzendorf and Overgaard (2001) and Linnemer (2002).

tions to entry: cartel breakdown, predation and gradual accommodation. In the scenario of cartel breakdown, the collusive agreement is discontinued when entry occurs. As a result, entry occurs later than in the case of immediate accommodation. We find that, for this reason, the incumbents may prefer to break the cartel than to accommodate entry. The collusive agreement is easier to sustain in this case, mainly because it only involves two firms.

Alternatively, the cartel may respond to entry by implementing the following predatory strategy: if entry occurs, incumbent firms start a punishment process (one period of punishment followed by return to collusion), as if one of them had deviated from the collusive agreement. As a result, if a new firm entered the market, its continuation value would be null. Since it would not be profitable to support the entry cost, the severity of this threat allows incumbents to effectively deter entry. This is the best scenario for the incumbents. Of course, this outcome requires that the incumbents are able to credibly commit to this extreme form of punishment.

Finally, we analyze the sustainability of a collusive agreement that gradually accommodates the entrant, as proposed by Friedman and Thisse (1994). More precisely, we consider that: in the first period of activity, the entrant receives the non-cooperative profit; in the second period, it receives the mean between the non-cooperative profit and the collusive profit (one third of the monopoly profit); from the third period onwards, the entrant is fully integrated into the collusive agreement and receives one third of the monopoly profit. When compared to immediate accommodation, gradual accommodation is preferred by the incumbents because: (i) each incumbent receives a greater share of the monopoly profit during the accommodation periods; and (ii) entry is delayed. For the same reasons, the entrant obviously prefers immediate accommodation. As a result, the entrant has more incentive to disrupt the collusive agreement than incumbents. This unbalance implies that collusion sustainability is lower than with immediate accommodation.

In terms of social welfare, collusion is always detrimental relatively to competition. Naturally, it is less so if it is discontinued following entry. The remaining collusive scenarios are equivalent in terms of output and prices, as firms produce the monopoly output in every period. Still, predation is slightly preferred to accommodation because the entry cost is not supported. For the same reason, gradual accommodation is socially preferred to immediate accommodation.

The remainder of the paper is organized as follows. Section 2 presents the key assumptions of the model and derives the optimal entry period. Section 3 describes the collusive agreement in the case of immediate accommodation of the entrant. Section 4 analyzes alternative reactions to entry that may delay or even deter the entry of a new firm. Section 5 summarizes the main results of the paper. The Appendix contains the proofs of most propositions and lemmas.

2 Model

Consider an industry with two incumbents (firms 1 and 2) and a potential entrant (firm 3) that sell homogeneous products and compete by simultaneously choosing the quantities to produce, in an infinitely repeated setting. Production costs are null. Market demand in period $t \in \{0, 1, 2, ...\}$ is given by:

$$Q_t = (1 - p_t)\,\mu^t, \tag{1}$$

where $\mu > 1$ is the demand growth rate and p_t is the price in period t. We assume "free disposal". Therefore, the inverse demand function is:

$$p_t = \begin{cases} 1 - Q_t\mu^{-t} & \text{if } \ Q_t \leq \mu^t \\ 0 & \text{if } \ Q_t > \mu^t. \end{cases} \tag{2}$$

To enter the market, firm 3 must support a setup cost, $K > 0$. Before the entry of firm 3, the timing of each period's stage game is the following:

1st: Firm 3 decides whether or not to enter the market in the current period.

2nd: Active firms simultaneously choose the quantities to produce.

After the entry of firm 3, the single-period game is reduced to the second stage.

The objective of each firm is to maximize the discounted sum of its profits, $\sum_{t=0}^{+\infty} \delta^t \pi_t$, where $\delta \in (0, 1)$ denotes the common discount factor. For this sum to be finite, we make the following assumption.

Assumption 1. *Demand does not grow too fast:* $\mu\delta < 1$.

In period t, the profit function of each firm is given by:

$$\pi_t(q_t) = \left(1 - q_t\mu^{-t} - q_{-i,t}\mu^{-t}\right) q_t,$$

where $q_{-i,t}$ is the sum of the quantities produced by its competitors in period t. Profit-maximization yields the following best-reply function:

$$q_t(q_{-i,t}) = \frac{1}{2}\mu^t - \frac{q_{-i,t}}{2}. \tag{3}$$

When there are $n \in \{2,3\}$ firms engaging in Cournot competition, the output and profit of each firm, in period t, are given by:

$$q_t^{cn} = \frac{1}{1+n}\mu^t \quad \text{and} \quad \pi_t^{cn} = \frac{1}{(1+n)^2}\mu^t. \tag{4}$$

If firms collude and maximize their joint profit, the individual output and profit are:

$$q_t^{mn} = \frac{1}{2n}\mu^t \quad \text{and} \quad \pi_t^{mn} = \frac{1}{4n}\mu^t. \tag{5}$$

If firm 3 enters the market in period T and obtains a profit $\pi_t = \Pi\mu^t$ in all the following periods, $t \geq T$, the present value of its profits is:

$$V_3(T) = \sum_{t=T}^{+\infty} \delta^t \Pi \mu^t - \delta^T K = \frac{(\mu\delta)^T}{1-\mu\delta}\Pi - \delta^T K.$$

When deciding when to enter the market, firm 3 faces a trade-off: on the one hand, an earlier entry means start receiving profits sooner; on the other hand, by delaying entry, firm 3 decreases the (discounted) entry cost. Entry occurs when $V_3(T)$ is maximal.

Lemma 1. *If firm 3 expects its profits in each period to be given by $\pi_t = \Pi\mu^t$, it enters the market at:*

$$T = \begin{cases} int(\tau) & \text{if } V_3(int(\tau)) \geq V_3(int(\tau)+1) \\ int(\tau)+1 & \text{if } V_3(int(\tau)) < V_3(int(\tau)+1), \end{cases}$$

where $int(\tau)$ denotes the integer part of τ, the real number that maximizes $V_3(\cdot)$:

$$\tau = \frac{1}{ln(\mu)} ln\left[\frac{(1-\mu\delta)ln(\delta)K}{ln(\mu\delta)\Pi}\right]. \tag{6}$$

Proof. See Appendix. □

To calculate the optimal entry period for the case in which firm 3 expects to collude with the incumbents, T^m, simply replace $\Pi = \frac{1}{12}$ in (6) and in the expression for $V_3(T)$. Similarly, to calculate the optimal entry period for the scenario in which firm 3 expects to engage in Cournot competition, T^c, replace $\Pi = \frac{1}{16}$.

The following assumption guarantees that $T^m \geq 1$, which means that the incumbents are alone in the market at $t = 0$ (even if the entrant expects to collude after entry).[3]

Assumption 2. *The entry cost is not too small:* $K > \frac{1}{12(1-\delta)}$.

3 Collusion with entry accommodation

In this section, we consider a perfect collusive agreement with an optimal penal code, assuming that the entrant is included in the cartel after entering the market. Under this agreement, firms maximize their joint profits and inflict the severest possible punishment on deviators.

3.1 Collusive agreement with an optimal penal code

At the beginning of period $t = 0$, firms 1 and 2 make a collusive agreement (which extends to firm 3 when it enters the market) establishing that, in each period:

- Firms produce the quantities that maximize the industry profit, if the agreement was honored in the previous period.
- Firms carry out a stick-and-carrot punishment, if the agreement was violated in the previous period.

The stick-and-carrot punishment strategy (Abreu, 1986) consists in inflicting a single period of losses and reverting to the collusive agreement afterwards. If any of the firms avoids the single-period loss, the punishment is restarted.

[3]It is straightforward to check that $V_3(1) > V_3(0)$ if and only if $K > \frac{\Pi}{1-\delta}$. Since $\pi_t^{m3} = \frac{1}{12}\mu^t$, replace $\Pi = \frac{1}{12}$ to obtain $K > \frac{1}{12(1-\delta)}$.

Incentives for complying with the collusive agreement naturally depend on the strength of the punishment. The harshest possible punishment gives zero continuation value (to all firms) after a deviation.[4] It requires that firms have strictly negative profits in the punishment period.

For losses to be feasible (recall that production costs are null), we assume that firms can make observable unproductive expenditures, such as dissipative advertising. This possibility has been considered before by Milgrom and Roberts (1986), Bagwell and Ramey (1994), Lariviere and Padmanabhan (1997), Hertzendorf and Overgaard (2001) and Linnemer (2002), among others.

We focus on the following penal code, which gives zero continuation value to a deviator. If a deviation occurs in period t, and there are $n \in \{2, 3\}$ active firms:

- In period $t+1$, firms produce quantities that completely satiate the market (implying that the price is zero, even if one of the firms deviates and produces nothing):

$$q_{t+1}^{pn} = \frac{1}{n-1} \mu^{t+1},$$

  and spend an amount in dissipative advertising which implies that the continuation value is null (this amount is calculated in the Appendix).

- If no firm deviates in the punishment period, the collusive agreement is reinstated in period $t+2$. Otherwise, the punishment is restarted and the return to collusion is delayed by one period.

Given the severity of this punishment, firms might deviate in the punishment phase. Firms adhere to the punishment if and only if the following incentive compatibility constraint is satisfied:

$$V_{t+1}^{pn} \geq \pi_{t+1}^{dpn} + \delta V_{t+2}^{pn} \ , \quad \forall t \geq 0, \tag{7}$$

where π_{t+1}^{dpn} is the maximum profit that a firm can earn by deviating in the punishment period, while the remainder $n-1$ firms play the punishment strategy; and V_{t+1}^{pn} is the discounted value of profits along the punishment path (i.e., if firms cooperate in the punishment at $t+1$ and revert to collusion at $t+2$).

[4] Note that firms can always guarantee a payoff of zero by producing zero in every period.

By construction, the optimal penal code gives zero continuation value after a deviation ($V^{pn}_{t+1} = V^{pn}_{t+2} = 0$). This allows us to write the ICC (7) as:

$$\pi^{dpn}_{t+1} \leq 0 \ , \quad \forall t \geq 0.$$

This condition is satisfied because, as mentioned before, if $n-1$ firms adhere to the punishment in period $t+1$, they produce an output which ensures that the resulting price is zero, regardless of the output of the deviator. Therefore, it is impossible for a deviatior to attain a strictly positive profit. The best it can do is have a null (instead of negative) profit by not spending on dissipative advertising.

We will now study whether firms, knowing that the penal code is credible, have incentives to deviate from the collusive agreement.

3.2 Sustainability of collusion

The collusive agreement is sustainable if firms do not have incentives to deviate neither before nor after the entry of firm 3, which occurs at T^m.

Collusion is sustainable after entry if and only if the following incentive compatibility constraint holds for all $t \geq T^m$:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi^{m3}_s \geq \pi^{d3}_t,$$

where π^{d3}_t is the deviation profit. The continuation value after the deviation is omitted from the ICC because, under the optimal penal code, it is null.

Proposition 1. *Collusion is sustainable after entry if and only if:* $\mu\delta \geq \frac{1}{4}$.

Proof. See Appendix. □

Before the entry of firm 3, the collusive agreement is sustainable if and only if the following incentive compatibility constraint holds for all $t \leq T^m - 1$:

$$\sum_{s=t}^{T^m-1} \delta^{s-t} \pi^{m2}_s + \sum_{s=T_m}^{+\infty} \delta^{s-t} \pi^{m3}_s \geq \pi^{d2}_t, \tag{8}$$

where π_t^{d2} is the deviation profit before the entry (the continuation value after a deviation is, again, omitted because it is null).

Lemma 2. *Let $\mu\delta \geq \frac{1}{9}$. If the ICC (8) is satisfied in the period that immediately preceeds entry, $t = T^m - 1$, it is satisfied in all previous periods, $t \in \{0, 1, ..., T^m - 2\}$.*

Proof. See Appendix. □

Considering the ICC at $t = T^m - 1$, we obtain the critical discount factor above which collusion is sustainable before the entry of firm 3.

Proposition 2. *Collusion is sustainable before entry if and only if: $\mu\delta \geq \frac{3}{19}$.*

Proof. See Appendix. □

Combining Propositions 1 and 2, we conclude that collusion is less sustainable after entry than before. The reason is related to the standard result that collusion is less sustainable if the number of firms is higher (Ivaldi *et al.*, 2003).

Our conclusion is the opposite of the one obtained by Vasconcelos (2008) and by Brandão *et al.* (2012), who have assumed that if one firm deviates from the collusive agrement, firms revert to Cournot competition forever (grim trigger strategies). In their models, incumbents have an additional incentive to deviate before the entry of firm 3, because a deviation effectively delays its entry. The delay results from the reduction of the profits that firm 3 expects to receive, which decrease from the perfect collusion level to the Cournot competition level. In the meantime, the incumbents benefit because Cournot competition between the two is more profitable than perfect collusion with three firms. In our model, firms use stick-and-carrot strategies. Thus, the best that incumbents can do is to delay entry by one single period (by deviating in the period that immediately preceeds the entry of firm 3, i.e., at $T^m - 1$). However, since the optimal penal code absorbs all the continuation value, delaying entry is irrelevant. Firms deviate if and only if the single-period deviation profit exceeds the value of colluding forever.

Corollary 1. *Collusion is sustainable (before and after entry) if and only if: $\mu\delta \geq \frac{1}{4}$.*

We have concluded that, if $\mu\delta \geq \frac{1}{4}$, the collusive agreement is sustainable and, therefore, entry occurs at T^m.

In the next section, we consider alternative reactions of the cartel to entry and compare them with this benchmark case. We also analyze the case in which the incumbents do not establish any collusive agreement.

4 Can the incumbents profitably delay entry?

In this section, we study four alternative scenarios:

(i) no collusion;

(ii) collusion before entry and competition after entry;

(iii) collusion before entry and punishment if and when entry occurs;

(iv) gradual accommodation of the entrant in the collusive agreement.

The motivation for studying the case in which there is no collusion is obvious, as we are interested in understanding the implications of collusion. It is also the scenario that appears if the discount factor is not sufficiently high for collusion to be sustainable. The remaining alternative scenarios correspond to relevant variations regarding the reaction of the incumbents to the entry of firm 3, previously considered and compared by Harrington (1989) and Friedman and Thisse (1994).[5]

4.1 No collusion

If there is no collusion, the present value of the profits of each incumbent is:

$$V_i^c = \sum_{s=0}^{T^c-1} \delta^s \pi_s^{c2} + \sum_{s=T^c}^{+\infty} \delta^s \pi_s^{c3}, \tag{9}$$

where T^c is the period at which firm 3 enters the market. The next Proposition shows that collusion is not always beneficial to the incumbents.

[5] See also the discussion by Harrington (1991).

Proposition 3. *The incumbents are better off under competition than under collusion with entry accommodation if and only if:* $2 - 6(\mu\delta)^{T^m} + 7(\mu\delta)^{T^c} < 0$.

Proof. See Appendix. □

The parameter values for which the incumbents prefer competition to collusion with accommodation of the entrant are represented in Figure 1. In the painted area, the collusive agreement with entry accommodation would be sustainable, since $\mu\delta \in \left(\frac{1}{4}, 1\right)$, but the incumbents prefer the competitive scenario.

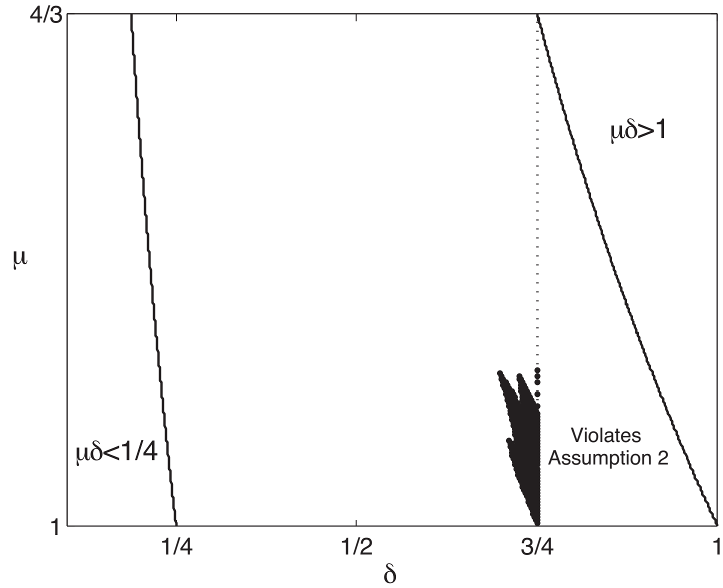

Figure 1: Competition versus entry accommodation (for $K = \frac{1}{3}$).

It may seem counterintuitive that the incumbent firms prefer not to collude. Notice, however, that: (i) a collusive agreement with accommodation of the entrant induces an earlier entry (at T^m rather than at T^c); and (ii) incumbents benefit from higher profits when there is competition between two firms than when there is collusion involving three firms. Of course, the entrant surely prefers the collusive scenario.

4.2 Discontinuance of collusion

Suppose now that the collusive agreement is terminated when firm 3 enters the market. More precisely, the collusive agreement established by the incumbents is the following:

- Before the entry of firm 3, maximize joint profit, producing $q_t^{m2} = \frac{1}{4}\mu^t$, if the agreement was honored in the previous period.

- If there is a defection in period t, engage in one period of punishment, producing $q_{t+1}^{p2} = \mu^{t+1}$ and spending an amount in dissipative advertising that makes the continuation value null (optimal penal code).[6]

- After the entry of firm 3, compete in quantities.

In this scenario, the optimal entry period is the same as when there is no collusion.

The collusive agreement is obviously sustainable after entry, since firms are playing their best-response to the rivals' actions. Before the entry of firm 3, the incumbents abide by the collusive agreement if and only if:

$$\sum_{s=t}^{T^c-1} \delta^{s-t}\pi_s^{m2} + \sum_{s=T^c}^{+\infty} \delta^{s-t}\pi_s^{c3} \geq \pi_t^{d2} \ , \quad \forall t < T^c. \tag{10}$$

Lemma 3. *Let $\mu\delta \geq \frac{1}{9}$. If the ICC (10) is satisfied in the period that immediately preceeds entry, $t = T^c - 1$, it is satisfied in all previous periods, $t \in \{0, 1, ..., T^c - 2\}$.*

Proof. Follow exactly the steps of the proof of Lemma 2. □

Substituting $t = T^c - 1$ and the expressions for profits in the ICC (10), we obtain the following result.

Proposition 4. *The collusive agreement with cartel breakdown following entry is sustainable if and only if: $\mu\delta \geq \frac{1}{5}$.*

Proof. See Appendix. □

To investigate whether or not the incumbents prefer discontinuance of collusion to immediate accommodation, we compare the present value of profits in the two scenarios.

[6] The amount of disspative advertising is given by the same expression as in the base-case collusive agreement (calculated in the Appendix), except that T^c must be considered instead of T^m.

Proposition 5. *The incumbents are better off discontinuing the collusive agreement when entry occurs than accommodating the entrant if and only if:* $(\mu\delta)^{T^c - T^m} < \frac{2}{3}$.

Proof. See Appendix. □

The parameter values for which the incumbents prefer discontinuance of collusion to accommodation of the entrant are represented in Figure 2. In the painted area, the incumbents prefer reversion to competition to entry accommodation (collusion is sustainable in both scenarios).

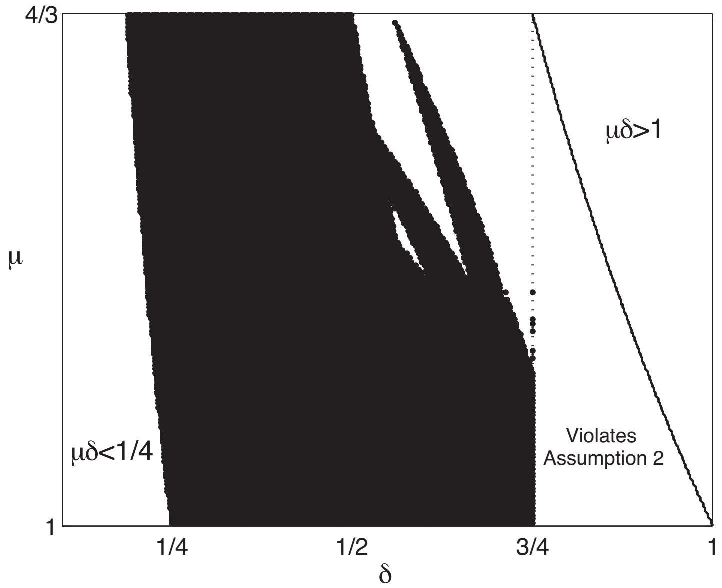

Figure 2: Discontinuance of collusion versus entry accommodation (for $K = \frac{1}{3}$).

4.3 Predation

Consider now that the incumbents are able to commit to the following agreement:

- Produce quantities that maximize the industry profit, $q_t^{m2} = \frac{1}{4}\mu^t$, if the agreement was honored in the previous period, and if firm 3 did not enter the market.
- If one of the incumbents deviates in period t, engage in punishment in period $t+1$, producing $q_{t+1}^{p2} = \mu^t$ and spending an amount in dissipative advertising that absorbs all the continuation value.[7]

[7] This amount can be calculated using the same method as in the case of collusion with entry accommodation.

- If firm 3 enters in period t, engage in punishment immediately: produce $q_t^{p3} = \mu^t$ and make an expenditure in dissipative advertising that is enough for the continuation value to be null.

Since the continuation value after the entry is zero, entering the market is not profitable (due to the entry cost, $K > 0$). Therefore, in this scenario, the incumbents effectively deter the entry of firm 3.

Proposition 6. *The collusive agreement with entry deterrence is sustainable if and only if:* $\mu\delta \geq \frac{1}{9}$.

Proof. See Appendix. □

If feasible, this is the best scenario for the incumbents. Each of them receives half of the monopoly profit in all periods. On the other hand, this is the worst scenario for consumers and for the entrant.

Remark 1. *The incumbents are always better off in this scenario (in which entry is deterred) that in the scenario of collusion with entry accommodation.*

4.4 Gradual accommodation

Following Friedman and Thisse (1994), we now assume that the accommodation of the entrant in the collusive agreement is not immediate. Firms still maximize the industry profit (before and after the entry), but there is an adjustment phase during which the entrant receives a smaller share of the industry profit than the incumbents.

We consider a version of the gradual accommodation process proposed by Friedman and Thisse (1994) that is simplified in two aspects: (i) in the first period of activity, the entrant receives exactly (instead of slightly more than) the non-cooperative profit; and (ii) the adjustment takes two (instead of n) periods. These assumptions simplify the analysis without qualitatively affecting the results.

In this scenario, the profit of firm 3 if it enters the market in period T is equal to:

- The Cournot competition profit (with 3 firms), in the entry period: $\pi_T^g = \frac{1}{16}\mu^T$.

- The arithmetic mean between the Cournot competition profit and the collusive profit, in the period that follows the entry: $\pi^g_{T+1} = \frac{7}{96}\mu^{T+1}$.
- The collusive profit, from period $T+2$ onwards: $\pi^g_t = \frac{1}{12}\mu^t$, $\forall t \geq T+2$.

The discounted value of the entrant's profits is, therefore:

$$V_3^g(T) = \frac{1}{16}(\mu\delta)^T + \frac{7}{96}(\mu\delta)^{T+1} + \frac{(\mu\delta)^{T+2}}{12(1-\mu\delta)} - \delta^T K.$$

Firm 3 enters the market when $V_3^g(T)$ is maximum, that is, in period:

$$T^g = \begin{cases} int(\tau^g) & \text{if} \quad V_3^g(int(\tau^g)) \geq V_3^g(int(\tau^g)+1) \\ int(\tau^g)+1 & \text{if} \quad V_3^g(int(\tau^g)) < V_3^g(int(\tau^g)+1), \end{cases}$$

where:

$$\tau^g = \frac{1}{ln(\mu)} ln\left[\frac{96(1-\mu\delta)ln(\delta)K}{ln(\mu\delta)\left[6+\mu\delta+(\mu\delta)^2\right]}\right].$$

It is straightforward that the incumbents are able to delay the entry of firm 3 by accommodating its entry gradually instead of immediately: $T^m \leq T^g \leq T^c$.

The industry profit is equal to the monopoly profit in every period. During the accommodation periods (T^g and T^g+1), each incumbent receives half of the difference between the monopoly profit and the entrant's profit:

$$\pi^g_{T^g} = \frac{3}{32}\mu^{T^g} \quad \text{and} \quad \pi^g_{T^g+1} = \frac{17}{192}\mu^{T^g+1}.$$

As in the previous sections, we consider that if one firm deviates from the collusive agreement, firms engage in one period of punishment and then return to collusion. In the punishment period, firms must support dissipative advertising expenses that exactly offset their future profits (the continuation value after a deviation is zero).

To study the sustainability of collusion with gradual accommodation, start by noticing that, after the accommodation phase, there is no difference between this scenario and the benchmark case. Therefore, from Proposition 1, firms honor the collusive agreement at any $t \geq T^g+2$ if and only if $\mu\delta \geq \frac{1}{4}$. However, this condition is not sufficient for the entrant to comply with the collusive agreement during the accommodation process.

Proposition 7. *The incentives for the entrant to deviate from the collusive agreement are the strongest at the entry period, T^g. The entrant does not deviate if and only if $\mu\delta \geq \frac{\sqrt{7753}-83}{16} \approx 0.316$.*

Proof. See Appendix. □

Contrarily to the entrant, the incumbents have more incentives to deviate after the accommodation phase.

Proposition 8. *The incentives for the incumbents to deviate from the collusive agreement are stronger after the accommodation phase (i.e., in periods $t \geq T^g + 2$) than during the accommodation phase. The incumbents do not deviate after entry if and only if $\mu\delta \geq \frac{1}{4}$.*

Proof. See Appendix. □

Finally, we need to analyze the incentives for incumbents to deviate before entry. The ICC for periods $t < T^g$ can be written as:

$$\frac{1}{8}\sum_{s=t}^{T^g-1}(\mu\delta)^s + \frac{3}{32}(\mu\delta)^{T^g} + \frac{17}{192}(\mu\delta)^{T^g+1} + \frac{1}{12}\sum_{s=T^g+2}^{+\infty}(\mu\delta)^s \geq \frac{9}{64}(\mu\delta)^t. \tag{11}$$

Lemma 4. *Let $\mu\delta \geq \frac{1}{9}$. If the ICC (11) is satisfied in the period that immediately preceeds entry, $t = T^g - 1$, it is satisfied in all previous periods, $t \in \{0, 1, ..., T^g - 2\}$.*

Proof. Follow exactly the steps of the proof of Lemma 2. □

Replacing $t = T^g - 1$ in the incentive constraint (11), we obtain the critical discount factor for the sustainability of collusion before entry.

Proposition 9. *Collusion is sustainable before entry if and only if $\mu\delta \geq 0.144$.*

Proof. See Appendix. □

The incentive compatibility constraint that binds is the one that guarantees that the entrant does not deviate in the entry period.

Corollary 2. *The collusive agreement with gradual accommodation is sustainable if and only if:* $\mu\delta \geq \frac{\sqrt{7753}-83}{16} \approx 0.316$.

It is straightforward that incumbents prefer to gradually accommodate the entrant, instead of treating it immediately as a full partner. There are two reasons for this: (i) gradual accommodation delays entry; and (ii) the share of incumbents in the industry profit is greater during the accommodation process. For the same reasons, the entrant is better off with immediate accommodation.

4.5 Welfare analysis

In this Section, we compare the different cartel reactions to entry in terms of the surplus of incumbents, entrant and consumers.

4.5.1 Incumbents surplus

Predation is clearly the scenario that gives the highest payoff to incumbents (since they receive half of the monopoly profit in every period).

It is also clear that the incumbents are better off by accommodating entry gradually rather than instantaneously ($V_i^g > V_i^a$), because entry is delayed and because they enjoy a greater profit during the acommodation process.

The comparison between the scenarios of competition and discontinuance of collusion is also immediate ($V_i^{dc} > V_i^c$), because: entry occurs at the same period; post-entry profits are the same; and pre-entry profits are lower under competition.

Remark 2. *The preferences of the incumbents regarding the different scenarios always satisfy the following partial order:*

$$V_i^a < V_i^g < V_i^p \quad \textit{and} \quad V_i^c < V_i^{dc} < V_i^p.$$

All the preference orderings that are compatible with the partial ordering in Remark 2 occur for some values of the parameters.

4.5.2 Entrant surplus

Obviously, the worst scenario for the potential entrant is the one in which the incumbents adopt a predatory behavior. In that case, entry does not even occur ($V_e^p = 0$).

It is irrelevant for the entrant whether the incumbents compete from the beginning or discontinue collusion when entry occurs ($V_e^c = V_e^{dc}$). Entry occurs at the same moment and the entrant's flow of profits is the same.

To verify that gradual accommodation is preferred to competition, suppose that entry under gradual accommodation occurred at the period in which it would be optimal to enter period under competition, T^c. Even in that case, gradual accommodation would be preferred to competition, because the discounted entry cost would be the same while the flow of profits would be greater under gradual accommodation. Having the possibility of entering at T^g instead of T^c only increases the value of the scenario of gradual accommodation. Therefore, $V_e^g > V_e^c$.

Following a similar reasoning, we conclude that the entrant is better off if it is immediately accommodated in the collusive agreement rather than gradually ($V_e^a > V_e^g$).

Remark 3. *The preferences of the entrant regarding the different scenarios always satisfy the following complete order:*

$$V_e^p < V_e^c = V_e^{dc} < V_e^g < V_e^a.$$

4.5.3 Consumer surplus

We measure consumer welfare as the discounted sum of each period's consumer surplus:

$$CS = \sum_{t=0}^{+\infty} \delta^t \frac{Q_t (1 - p_t)}{2} = \frac{1}{2} \sum_{t=0}^{+\infty} (\mu\delta)^t (1 - p_t)^2 .$$

Notice that consumers are not directly affected by the timing of entry nor by the way how incumbents react to entry. The only thing that interests consumers is total output and the resulting market price.

The non-cooperative scenario is the best for consumers, because the output is the highest in all periods. For the same reason, consumers prefer discontinuance of collusion to entry accommodation ($CS^a < CS^{dc}$), as there is competition from entry onwards. Predation and (instanteneous or gradual) accommodation are equivalent for consumers, because the output is at the monopoly level in all periods ($CS^p = CS^a = CS^g$).

The following remark summarizes these observations.

Corollary 3. *The preferences of consumers regarding the different scenarios always satisfy the following complete order:*

$$CS^p = CS^g = CS^a < CS^{dc} < CS^c.$$

4.5.4 Social welfare

As production costs are null, social welfare is increasing in total output. In addition, *ceteris paribus*, the later is the entry of firm 3 (i.e., the lower is the discounted value of the entry cost), the higher is social welfare.

It is clear that social welfare is higher if there is no collusion than if there is collusion with immediate accommodation, because the output is higher (in every period) and the entry occurs later. Likewise, discontinuance of collusion is socially better than entry accommodation, but worse than competition ($W^a < W^{dc} < W^c$).

The output is the same under immediate accommodation, gradual accommodation and predation. However, social welfare is higher when accommodation is gradual than when it is immediate, because the entry cost is supported later, and it is even higher under predation, because the entry cost is not even supported ($W^a < W^g < W^p$).

Remark 4. *In terms of social welfare, the different scenarios always satisfy the following partial order:*

$$W^a < W^{dc} < W^c \quad \text{and} \quad W^a < W^g < W^p.$$

The comparison between discontinuance of collusion and predation is not straightforward. On the one hand, total output is greater when collusion is discontinued. On the other hand, the entry cost is not supported under predation. We find that the additional output more than compensates the entry cost, which means that discontinuance of collusion is preferable to predation.

Proposition 10. *Social welfare is higher if incumbents discontinue collusion when entry occurs than if they adopt a predatory behavior:* $W^p < W^{dc}$.

Proof. See Appendix. □

Together with Remark 4, Proposition 10 allows us to complete the social welfare ordering of the different scenarios.

Corollary 4. *In terms of social welfare, the different scenarios always satisfy the following complete order:*

$$W^a < W^g < W^p < W^{dc} < W^c.$$

5 Conclusions

In this paper, we study the sustainability of collusion in markets where the demand growth may attract the entry of a new firm. In a similar model, Vasconcelos (2008) assumed that, after a deviation from the collusive agreement, firms revert to the Nash equilibrium forever (trigger strategies). However, it is well known that, under quantity competition, firms can improve the sustainability of the collusive agreement by adopting Abreu-type punishment strategies. Motivated by this idea, we modified the work of Vasconcelos (2008) by characterizing a security level penal code (i.e. a punishment strategy according to which all firms are driven down to a zero continuation value in case a deviation occurs). More precisely, we assume that a deviation triggers a period of severe losses, after which the collusive agreement is restored (stick-and-carrot strategies).

Following Vasconcelos (2008), we started by considering that the incumbents accommodate the entrant in a more inclusive agreement as soon as entry occurs. We found that, in contrast with the conclusions of Vasconcelos (2008), collusion is more difficult to sustain after entry than before. This finding conforms to the idea that: the higher is the number of firms in the market, the less sustainable is collusion. The origin of the difference between our result and that of Vasconcelos (2008) is in the ability of the incumbents to increase their profits by delaying entry (following a disruption of the collusive agreement). If firms use grim trigger strategies, the cartel breakdown effectively delays entry. This benefits the incumbents because Cournot competition between the two is more profitable than collusion among three firms. With optimal penal codes, breaking the agreement leads to a punishment that absorbs all future profits. As a result, an entry delay would be irrelevant.

One of our most surprising results is that incumbents may prefer to compete since the beginning of the game rather than to make a collusive agreement that accommodates the entrant. Once again, the explanation for this result lies in the possibility of delaying entry and in the fact that Cournot competition with two firms is more profitable than collusion with three firms.

We have also considered other reactions to entry, which may delay (or even prevent) the entry of a new firm. In particular: (i) discontinuance of collusion; (ii) predation; and (iii) gradual accommodation.

The best scenario for the incumbents is the one in which they are able to deter entry by credibly threatening to regard entry as a deviation (i.e., predation). In addition, the incumbents surely prefer to gradually accommodate the entrant than to have it as a full partner immediately after entry. Depending on the model parameters, the incumbents may be better off with discontinuance of collusion or with immediate accommodation. On the one hand, entry occurs later if the collusive agreement is discontinued when entry occurs; on the other hand, the incumbents profit more after the entry if they accommodate the entrant. Unsurprisingly, the worst scenario for the entrant is the one in which the incumbents adopt a predatory behaviour; while the best scenario is the one in which the incumbents immediately include it in the collusive agreement.

Consumer surplus is the same under predation and under immediate or gradual accommodation. The reason for this result is that, in all these scenarios, output and price are at their monopoly levels. As products are homogeneous, entry is irrelevant for consumers unless it impacts the output and price levels. Of course, consumers are better off if firms compete in all periods. As there are no production costs, the social welfare is also increasing in the output level. For this reason, competition is also the socially preferred scenario, followed by discontinuance of collusion. The timing of entry must also be taken into account, as it is socially beneficial to support the entry cost later and it is even better to avoid it altogether. This is why entry accommodation is more damaging to social welfare than predation.

6 Appendix

Proof of Lemma 1

Let us start by dealing with t as a continuous variable and denote it by τ. The first-order condition for the maximization of $V_3(\tau)$ is:

$$\frac{\Pi}{1-\mu\delta}(\mu\delta)^{\tau}ln(\mu\delta) - K\delta^{\tau}ln(\delta) = 0 \;\Leftrightarrow\; \tau = \frac{1}{ln(\mu)}ln\left[\frac{(1-\mu\delta)ln(\delta)K}{ln(\mu\delta)\Pi}\right].$$

As time is discrete, the optimal entry period will be one of the two integers that are closest to τ, depending on which of those yields the highest value for $V_3(t)$. □

Calculation of the amount of dissipative cost (A_{t+1})

Here, we obtain the value of the dissipative cost, A_{t+1}, for the collusive agreement defined in Section 3.1. This value must be such that the continuation value after a deviation is zero.

Recall that the output of each firm in the punishment period is high enough for the resulting market price to be null, even if there is a deviation. Therefore, as production costs are null, the profit of each firm in the punishment period $t+1$ is $\pi^p_{t+1} = -A_{t+1}$.

Keep in mind that the decision of firm 3 regarding its entry period only depends on the flow of profits that it expects to obtain after the entry. If firm 3 expects to get one third of the monopoly profit in all periods, entry occurs in period T^m. Firm 3 does not care about the existence of a deviation in any period $t \leq T^m - 2$, because the collusive agreement would be restored at T^m. The entry decision is only affected if one incumbent deviates at $T^m - 1$. In that case, the punishment takes place at T^m and firm 3 delays its entry to $T^m + 1$.

Suppose that the deviation takes place at $t \leq T^m - 3$. The punishment will be carried out at $t+1$, there will be collusion with two firms from $t+2$ until $T^m - 1$, entry will take place at T^m, and there will be collusion with three firms from T^m onwards. In this case, for the continuation value to be null:

$$\begin{aligned} A_{t+1} &= \sum_{s=t+2}^{T^m-1} \delta^{s-(t+1)} \pi_s^{m2} + \sum_{s=T^m}^{+\infty} \delta^{s-(t+1)} \pi_s^{m3} = \delta^{-(t+1)} \left[\sum_{s=t+2}^{T^m-1} \frac{(\mu\delta)^s}{8} + \sum_{s=T^m}^{+\infty} \frac{(\mu\delta)^s}{12} \right] \\ &= \frac{\mu\delta \left[3 - (\mu\delta)^{T^m-t-2} \right]}{24(1-\mu\delta)} \mu^{t+1}. \end{aligned}$$

If the deviation takes place at $t = T^m - 2$, entry will still take place at T^m, and there will be collusion with three firms from $t+2$ onwards. Therefore:

$$A_{t+1} = \sum_{s=t+2}^{+\infty} \delta^{s-(t+1)} \pi_s^{m3} = \frac{\mu\delta}{12(1-\mu\delta)} \mu^{t+1}.$$

If a firm deviates at $t = T^m - 1$, the punishment will be carried out in period T^m. In that case, firm 3 postpones its entry to period $T^m + 1$ (in which firms will be, again, colluding). After the punishment period, there will be collusion with three firms. Again, the amount of dissipative advertising must be exactly that which absorbs the value of collusion with three firms from $t+2$ onwards.

If a firm deviates after the entry, at $t \geq T^m$, there will also be collusion with three firms from $t+2$ onwards. Therefore, the same expression for A_{t+1} applies.

We conclude that:

$$A_{t+1} = \begin{cases} \frac{\mu\delta\left[3-(\mu\delta)^{T^m-t-2}\right]}{24(1-\mu\delta)} \mu^{t+1}, & \text{if } t \leq T^m - 3 \\ \frac{\mu\delta}{12(1-\mu\delta)} \mu^{t+1}, & \text{if } t \geq T^m - 2. \end{cases}$$

Proof of Proposition 1

Substituting $q_{-i,t} = \frac{1}{3}\mu^t$ in (3), we obtain the deviation output, $q_t^{d3} = \frac{1}{3}\mu^t$, which yields $\pi_t^{d3} = \frac{1}{9}\mu^t$. Substituting the expressions for profits in the ICC, we obtain:

$$\frac{1}{12}\delta^{-t} \sum_{s=t}^{+\infty} (\mu\delta)^s \geq \frac{1}{9}\mu^t \Leftrightarrow \mu\delta \geq \frac{1}{4}. \qquad \square$$

Proof of Lemma 2

Substituting $q_{-i,t} = \frac{1}{4}\mu^t$ in the best-reply function (3), we obtain the deviation quantity, $q_t^{d2} = \frac{3}{8}\mu^t$, and the corresponding profit, $\pi_t^{d2} = \frac{9}{64}\mu^t$. Substituting the expressions for profits in the ICC (8), we obtain:

$$\frac{1}{8}\sum_{s=t}^{T^m-1}(\mu\delta)^s + \frac{1}{12}\sum_{s=T^m}^{+\infty}(\mu\delta)^s \geq \frac{9}{64}(\mu\delta)^t.$$

The ICC in period $t = T^m - \tau$, with $1 \leq \tau \leq T^m$, is given by:

$$\frac{1}{8}\sum_{s=T^m-\tau}^{T^m-1}(\mu\delta)^s + \frac{1}{12}\sum_{s=T^m}^{+\infty}(\mu\delta)^s \geq \frac{9}{64}(\mu\delta)^{T^m-\tau}. \tag{12}$$

We want to show that if (12) is satisfied for $\tau = k$, then it is also satisfied for $\tau = k+1$. Our hypothesis is, therefore, that:

$$\frac{1}{8}\sum_{s=T^m-k}^{T^m-1}(\mu\delta)^s + \frac{1}{12}\sum_{s=T^m}^{+\infty}(\mu\delta)^s - \frac{9}{64}(\mu\delta)^{T^m-k} \geq 0. \tag{13}$$

For $\tau = k+1$, the ICC (12) can be written as:

$$\frac{1}{8}\sum_{s=T^m-k-1}^{T^m-1}(\mu\delta)^s + \frac{1}{12}\sum_{s=T^m}^{+\infty}(\mu\delta)^s - \frac{9}{64}(\mu\delta)^{T^m-k-1} \geq 0. \tag{14}$$

Subtracting the left-hand side of (13) from that of (14), we obtain:

$$\frac{1}{8}(\mu\delta)^{T^m-k-1} - \frac{9}{64}\left[(\mu\delta)^{T^m-k-1} - (\mu\delta)^{T^m-k}\right] = \frac{9\mu\delta - 1}{64}(\mu\delta)^{T^m-k-1}.$$

This expression is positive, meaning that (13) implies (14), if and only if $\mu\delta \geq \frac{1}{9}$. □

Proof of Proposition 2

Collusion is sustainable at $T^m - 1$ (and, therefore, at all $t < T^m$) if and only if:

$$\frac{1}{8}(\mu\delta)^{T_m-1} + \frac{1}{12}\sum_{s=T_m}^{+\infty}(\mu\delta)^s \geq \frac{9}{64}(\mu\delta)^{T_m-1} \;\Leftrightarrow\; \frac{(\mu\delta)^{T_m}}{1-\mu\delta} \geq \frac{3}{16}(\mu\delta)^{T_m-1} \;\Leftrightarrow\; \mu\delta \geq \frac{3}{19}.$$

□

Proof of Proposition 3

The incumbents prefer no collusion to collusion with immediate accommodation if and only if:

$$\sum_{s=0}^{T^m-1} \delta^s \left(\pi_s^{c2} - \pi_s^{m2}\right) + \sum_{s=T^m}^{T^c-1} \delta^s \left(\pi_s^{c2} - \pi_s^{m3}\right) + \sum_{s=T^c}^{+\infty} \delta^s \left(\pi_s^{c3} - \pi_s^{m3}\right) \geq 0.$$

Manipulating this condition, we obtain:

$$\begin{aligned}
&-\frac{1}{72}\sum_{s=0}^{T^m-1}(\mu\delta)^s + \frac{1}{36}\sum_{s=T^m}^{T^c-1}(\mu\delta)^s - \frac{1}{48}\sum_{s=T^c}^{+\infty}(\mu\delta)^s \geq 0 \;\Leftrightarrow \\
&-2\left[1-(\mu\delta)^{T^m}\right] + 4\left[(\mu\delta)^{T^m} - (\mu\delta)^{T^c}\right] - 3(\mu\delta)^{T^c} \geq 0 \;\Leftrightarrow \\
&-2 + 6(\mu\delta)^{T^m} - 7(\mu\delta)^{T^c} \geq 0.
\end{aligned}$$

□

Proof of Proposition 4

Replacing the expressions for profits, we can write the ICC (10) as follows:

$$\frac{1}{8}\sum_{s=t}^{T^c-1}(\mu\delta)^s + \frac{1}{16}\sum_{s=T^c}^{+\infty}(\mu\delta)^s \geq \frac{9}{64}(\mu\delta)^t. \tag{15}$$

Lemma 3 allows us to consider $t = T^c - 1$ to obtain the critical discount factor:

$$\frac{1}{8}(\mu\delta)^{T^c-1} + \frac{1}{16}\sum_{s=T^c}^{+\infty}(\mu\delta)^s \geq \frac{9}{64}(\mu\delta)^{T^c-1} \;\Leftrightarrow\; \frac{(\mu\delta)^{T^c}}{16(1-\mu\delta)} \geq \frac{(\mu\delta)^{T^c-1}}{64} \;\Leftrightarrow\; \mu\delta \geq \frac{1}{5}.$$

□

Proof of Proposition 5

The present value of profits of an incumbent firm in the collusive agreement with immediate accommodation of the entrant is:

$$V^m = \sum_{t=0}^{T^m-1} \delta^t \pi_t^{m2} + \sum_{t=T^m}^{+\infty} \delta^t \pi_t^{m3},$$

while, in the case of reversion to competition, it is:

$$V^{rc} = \sum_{t=0}^{T^c-1} \delta^t \pi_t^{m2} + \sum_{t=T^c}^{+\infty} \delta^t \pi_t^{c3}.$$

Therefore, incumbents prefer reversion to competition relatively to accommodation if and only if:

$$\begin{aligned} V^m > V^{rc} \;&\Leftrightarrow\; \sum_{t=T^m}^{T^c-1} \delta^t (\pi_t^{m2} - \pi_t^{m3}) + \sum_{t=T^c}^{+\infty} \delta^t (\pi_t^{c3} - \pi_t^{m3}) > 0 \\ &\Leftrightarrow\; \frac{1}{24} \sum_{t=T^m}^{T^c-1} (\mu\delta)^t - \frac{1}{48} \sum_{t=T^c}^{+\infty} (\mu\delta)^t > 0 \;\Leftrightarrow\; (\mu\delta)^{T^c - T^m} < \frac{2}{3}. \end{aligned}$$

□

Proof of Proposition 6

Since the continuation value after a deviation is null, the incumbents are willing to sustain the collusive agreement in period $t \geq 0$ if:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi_s^{m2} \geq \pi_t^{d2} \;\Leftrightarrow\; \frac{1}{8} \sum_{s=t}^{+\infty} (\mu\delta)^s \geq \frac{9}{64} (\mu\delta)^t \Leftrightarrow \frac{(\mu\delta)^t}{1-\mu\delta} \geq \frac{9}{8} (\mu\delta)^t \;\Leftrightarrow\; \mu\delta \geq \frac{1}{9}.$$

□

Proof of Proposition 7

(i) We start by analyzing the incentives for the entrant to abide by the collusive agreement in period T^g+1. As the price is equal to $p_{T^g+1} = \frac{1}{2}$ and the profit of each incumbent is $\pi_{T^g+1} = \frac{17}{192}\mu^{T^g+1}$, we conclude that each incumbent produces $\frac{17}{96}\mu^{T^g+1}$. Substituting this in the best-reply function of the entrant, given in (3), we obtain the deviation output of the entrant and the corresponding profit:

$$q_{T^g+1}^{dg} = \frac{31}{96}\mu^{T^g+1} \quad \text{and} \quad \pi_{T^g+1}^{dg} = \frac{961}{9216}\mu^{T^g+1}.$$

Therefore, the entrant does not defect in period T^g+1 if the following ICC is satisfied:

$$\begin{aligned} &\frac{7}{96}\mu^{T_g+1} + \sum_{s=T_g+2}^{+\infty} \delta^{s-T_g-1} \frac{1}{12}\mu^s \geq \frac{9 61}{9216}\mu^{T_g+1} \;\Leftrightarrow \\ &\frac{1}{12}\frac{\mu\delta}{1-\mu\delta} \geq \frac{289}{9216} \;\Leftrightarrow\; \mu\delta \geq \frac{289}{1057} \approx 0.273. \end{aligned}$$

(ii) Now, we do the same for period T^g. In this period, the entrant should receive the Cournot competition profit, $\pi_{T^g} = \frac{1}{16}\mu^{T^g}$, but the price is equal to the monopoly price, $p_{T^g} = \frac{1}{2}$. According to the agreement, the entrant should produce $\frac{1}{8}\mu^t$, while the incumbents jointly produce $\frac{3}{8}\mu^t$. The best possible deviation by the entrant yields:

$$q_{T_g}^{d3} = \frac{5}{16}\mu^{T_g} \quad \text{and} \quad \pi_{T_g}^{d3} = \frac{25}{256}\mu^{T_g}.$$

As a result, the entrant abides by the collusive agreement if and only if:

$$\frac{1}{16}\mu^{T^g} + \delta\frac{7}{96}\mu^{T^g+1} + \sum_{s=T^g+2}^{+\infty} \delta^{s-T^g}\frac{1}{12}\mu^s \geq \frac{25}{256}\mu^{T^g} \quad \Leftrightarrow$$
$$8(\mu\delta)^2 + 83\mu\delta - 27 \geq 0 \;\Leftrightarrow\; \mu\delta \geq \frac{\sqrt{7753} - 83}{16} \approx 0.316. \qquad \square$$

Proof of Proposition 8

(i) We start by studying collusion sustainability in period $T_g + 1$. If the agreement was respected, each incumbent would produce $\frac{17}{96}\mu^{T_g+1}$ and the entrant would produce $\frac{7}{48}\mu^{T_g+1}$. Therefore, if an incumbent deviates, its outupt and profit are:

$$q_{T_g+1}^{d3} = \frac{65}{192}\mu^{T_g+1} \quad \text{and} \quad \Pi_{T_g+1}^{d3} = \frac{4225}{36864}\mu^{T_g+1}.$$

Therefore, each incumbent does not deviate in period $T_g + 1$ if and only if:

$$\frac{17}{192}\mu^{T_g+1} + \sum_{s=T_g+2}^{+\infty} \delta^{s-T_g-1}\frac{1}{12}\mu^s \geq \frac{4225}{36864}\mu^{T_g+1} \quad \Leftrightarrow \; \mu\delta \geq \frac{961}{4033} \approx 0.238.$$

(ii) Now, we analyze the incentives for incumbents to collude in period T_g. According to the agreement, each incumbent should produce $\frac{3}{16}\mu^{T_g}$, while the entrant should produce $\frac{1}{8}\mu^{T_g}$. Therefore, the output and the profit of a deviating incumbent is:

$$q_{T_g}^{d3} = \frac{11}{32}\mu^{T_g} \quad \text{and} \quad \Pi_{T_g+1}^{d3} = \frac{121}{1024}\mu^{T_g}.$$

As a result, each incumbent complies with the collusive agreement iff:

$$\frac{3}{32}\mu^{T_g} + \delta\frac{17}{192}\mu^{T_g+1} + \sum_{s=T_g+2}^{+\infty}\delta^{s-T_g}\frac{1}{12}\mu^{s} \geq \frac{121}{1024}\mu^{T_g} \Leftrightarrow$$

$$-16(\mu\delta)^2 + 247\mu\delta - 75 \ \geq \ 0.$$

The last inequality is satisfied if:

$$\mu\delta \geq \frac{347-\sqrt{115609}}{32} \approx 0.218.$$

□

Proof of Proposition 9

Manipulating condition (11), we obtain:

$$-\frac{1}{64} + \frac{3}{32}(\mu\delta) + \frac{17}{192}(\mu\delta)^2 + \frac{1}{12}\frac{(\mu\delta)^3}{1-\mu\delta} \geq 0 \Leftrightarrow$$

$$-\frac{1}{64} + \frac{7}{64}\mu\delta - \frac{1}{192}(\mu\delta)^2 - \frac{1}{192}(\mu\delta)^3 \geq 0,$$

which holds for $\mu\delta \geq 0.144$ (approximately). □

Proof of Proposition 10

In the scenario of discontinuance of collusion, the sum of the incumbents' profits with the consumer surplus is:

$$\sum_{t=0}^{T^c-1}\frac{1}{4}(\mu\delta)^t + \sum_{t=T^c}^{+\infty}\frac{1}{8}(\mu\delta)^t + \frac{1}{2}\sum_{t=0}^{T^c-1}\frac{1}{4}(\mu\delta)^t + \frac{1}{2}\sum_{t=T^c}^{+\infty}\frac{9}{16}(\mu\delta)^t \ = \ \frac{12+(\mu\delta)^{T^c}}{32(1-\mu\delta)}.$$

Under predation, it is lower:

$$\sum_{t=0}^{+\infty}\frac{1}{4}(\mu\delta)^t + \frac{1}{2}\sum_{t=0}^{+\infty}\frac{1}{4}(\mu\delta)^t \ = \ \frac{12}{32(1-\mu\delta)}.$$

This concludes the proof, since the entrant's surplus is surely positive (otherwise, it would not enter the market). □

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