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Multidimensional Screening with Complementary Activities: Regulating a Monopolist with Unknown Cost and Unknown Preference for Empire-Building

Ana Pinto Borges [1]
Didier Laussel [2]
João Correia-da-Silva [3,4]

[1] **NIDISAG, Núcleo de Investigação do Instituto Superior de Administração e Gestão**
[2] **Aix-Marseille Université (AMSE), GREQAM, CNRS and EHESS**
[3] **FEP-UP, School of Economics and Management, University of Porto**
[4] **CEF.UP, Research Center in Economics and Finance, University of Porto**

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Ana Pinto Borges

NIDISAG - Núcleo de Investigação do Instituto Superior de Administração e Gestão

Didier Laussel

Aix-Marseille Université (AMSE), GREQAM, CNRS and EHESS.

João Correia-da-Silva

CEF.UP and Faculdade de Economia, Universidade do Porto.

Abstract. We study optimal regulation of a monopolist when intrinsic efficiency (intrinsic cost) and empire-building tendency (marginal utility of output) are private information but actual cost (difference between intrinsic cost and effort level) is observable. This is a problem of multidimensional screening with complementary activities. Results are mainly driven by two elements: the correlations between types; and the relative magnitude of the uncertainty along the two dimensions of private information. If the marginal utility of output varies much more (resp. less) across managers than the intrinsic marginal cost, then we have empire-building (resp. efficiency) dominance. In that case, an inefficient empire-builder produces more (resp. less) and at lower (resp. higher) marginal cost than an efficient money-seeker. It is only when variabilities are similar that we obtain the natural ranking of activities (empire-builders produce more while efficient managers produce at a lower cost).

Keywords: Multidimensional screening, regulation, procurement, empire-building, adverse selection.

JEL Classification Numbers: D82, H42, L51.

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1 Introduction

Armstrong and Rochet (1999) have provided a "user's guide" for studying multidimensional screening problems. They studied a model with two activities, focusing on the case in which the utility functions of the agent and of the principal are additively separable in the levels of the two activities (independent activities). Furthermore, they considered that the agent's types are defined by two parameters coming from a binary distribution, with each parameter corresponding to one of the activities, in the sense that it only influences the utility of the agent and of the principal that is associated with that activity. They provided a full solution for this setup, and concluded that the qualitative properties of the solution are determined by the correlation between types and by the amount of "symmetry" between the two activities.[1]

The methodology proposed by Armstrong and Rochet (1999) is the following: (1) start by considering a relaxed problem where only the downward incentive-compatibility constraints are accounted for; (2) solve this relaxed problem; (3) find conditions which ensure that the solution of the relaxed problem is the solution of the fully constrained one. They noted, however, that it may be the case that upward or diagonal constraints bind, and outlined the resulting equilibria (in that case, activities may be distorted upward and not only downward).

We consider here a somewhat different problem. We start from the well-known model of Laffont and Tirole (1986), which deals with the regulation of a monopolist that has private information about his/her intrinsic marginal cost. In this model, the manager of the firm chooses a level of effort, which decreases the marginal cost of production but is costly to the manager. The effort level is also private information of the manager, but the regulator observes the resulting production cost. Borges and Correia-da-Silva (2011) modified this framework by assuming that the manager may have a preference for empire-building, i.e., may have a positive marginal utility for output (or employment, if we assume that employment determines output via a deterministic production function).[2] They showed that the regulator's welfare is increasing with the manager's tendency for empire-building: the more the manager is interested in a non-monetary reward, the lower is the monetary informational rent he/she requires. In a subsequent paper, Borges, Correia-da-Silva and Laussel (2012) studied the case in which the magnitude of the tendency for empire-building is private information of the manager, while the intrinsic marginal cost is

[1] Their definition of symmetry cannot be used in a model with non-separable utility functions.

[2] The tendency of managers for empire-building has been studied, among others, by Niskanen (1971) and documented by Donaldson (1984). Jensen (1986, 1993) has emphasized it as an origin of excess investment and output: "Managers have incentives to cause their firms to grow beyond the optimal size. Growth increases managers' power by increasing the resources under their control. It is also associated with increases in managers' compensation, because changes in compensation are positively related to the growth in sales."

observable. Here, we study the case in which the private information of the manager bears simultaneously on the value of the intrinsic marginal cost and on the value of the marginal utility of output. This leads to a two-dimensional screening model with complementary activities.

We suppose that both the level of efficiency and the tendency for empire-building can be either high or low (the intrinsic marginal cost and the marginal utility of output are drawn from a binary distribution). There are, therefore, four possible manager types: the efficient money-seeker, the efficient empire-builder, the inneficient money-seeker and the innefficient empire-builder. The resulting problem differs from the one considered by Armstrong and Rochet (1999) because the utility function of the regulator is not separable in the two activities - output and effort are complementary. More precisely, since effort reduces the marginal cost of output, more effort yields a larger optimal output level. In turn, a larger output level increases the returns from any given effort level and thus leads to a larger optimal effort level.

Our purpose is then twofold. First, it is a substantive one: we aim at analyzing the characteristics of optimal contracts between regulator and manager in the two-dimensional case where the manager's preference for high output is private information as well as his/her intrinsic efficiency. Second, it is a methodological one: we want to see how the conclusions of Armstrong and Rochet (1999) are modified in the case of complementary activities.

When analyzing our model, we realized that the approach of Armstrong and Rochet (1999) did not provide a complete picture of the possible kinds of solutions. In fact, the solutions of the relaxed problem obtained by considering only the downward incentive constraints (and ignoring the upward and diagonal incentive constraints)[3] rarely solve the fully constrained problem, and finding general conditions for that seems to be very hard. More precisely, with complementary activities, the diagonal constraints are frequently binding. This is why we analyze a less relaxed problem, where only the upward incentive compatibility constraints are discarded. Inclusion of the diagonal incentive constraints increases the number of *a priori* possible combinations of binding and non-binding incentive constraints to 63, which makes the analysis much more difficult and tedious.

One of our main findings is that an important determinant of the kind of solution that is obtained is the ratio between the variability (across managers) of marginal utility of output and the variability of intrinsic efficiency. When these variabilities are very different, the model becomes similar to a one-dimensional model where the relevant private information concerns the dimen-

[3]The downward (resp. upward) constraints are those which require that a worse (resp. better) type should not benefit from mimicking a better (resp. worse) type. One speaks of of a diagonal constraint when the two types cannot be ranked: each type is better in one dimension and worse in the other.

sion in which managers differ in a greater degree. Since Armstrong and Rochet (1999) showed that the correlation between types is the main driver of the kind of solution that is obtained when activities are independent, our results suggest that, when activities are complementary, there is another element that significantly drives the results: the relative magnitude of the uncertainty along each dimension of private information.

When intrinsic efficiency varies much more than marginal utility of output, there is "efficiency dominance": more efficient managers have lower marginal cost and larger output levels than the less efficient ones (an efficient money-seeker produces more than an inefficient manager). When it is large, there is "empire-building dominance": manager types with a stronger tendency for empire-building types have larger output and lower marginal cost levels than managers with a weaker tendency for empire-building (an inefficient empire-builder exerts more effort than an efficient money-seeker).

It is only when these varibilities are similar that we get output bunching or marginal cost bunching between the intermediate types (inefficient empire-builder and efficient money-seeker), or even, if empire-building tendency and efficiency are strongly positively correlated, the natural ranking of activities (more efficient types producing at lower marginal cost, types with stronger tendency for empire-building producing more output).

The rest of the paper is organized as follows. Section 2 introduces the model and the multi-dimensional screening problem. Section 3 focuses on a relaxed problem. Section 4 presents the solutions to several cases that differ qualitatively. Section 5 concludes the paper with some remarks. The systems of equations that characterize each case and the proofs of the formal results are presented in the Appendix.

2 The model

The firm produces an observable quantity of a good, $q \geq 0$, with a total observable cost $C = (\beta - e)q$, where β is the intrinsic marginal cost of the manager and e is the level of effort that is exerted by the manager. Neither the intrinsic marginal cost, β, nor the effort level, e, are directly observable, but the marginal cost can be inferred: $c = \beta - e = C/q$.

The regulator pays the observed production cost plus a net transfer t to the manager. The manager atributes utility to this monetary reward and also to the output in itself. The utility of the manager is:

$$U \ = \ t - \psi(e) + \delta q \ = \ t - \psi(\beta - c) + \delta q,$$

where $\psi(e)$ is the disutility of effort, assumed to be a convex function, and δ is the marginal utility of output.

The marginal utility of output is private information of the manager (as well as the intrinsic marginal cost). It measures the importance of the empire-building component of the manager's utility. A positive value of δ means that the manager likes to produce a higher output, or, equivalently, to have authority over more employees.

The manager requires a minimum utility level (which we set to zero for convenience) to accept the contract. The participation constraint is: $U \geq 0$.

A level of output equal to q generates a consumer surplus that is given by $S(q)$. Social welfare is measured as the difference between the total surplus (consumer surplus plus firm surplus) and the cost of raising funds to compensate the firm, $(1+\lambda)(C+t)$, with $\lambda > 0$:

$$\begin{aligned} W &= S(q) - (1+\lambda)(C+t) + U, \\ &= S(q) - (1+\lambda)\left[cq + \psi(\beta - c) - \delta q\right] - \lambda U. \end{aligned}$$

Notice that the regulator's welfare is increasing with the manager's marginal utility of output because, when the manager enjoys more a given level of output, this reduces the money transfer that is necessary to compensate him/her. It obviously follows that, other things equal (and specifically the intrinsic cost β), the regulator prefers an empire-builder to a pure money-seeker.

There are two possible values of β, namely $\beta_E < \beta_I$, and two possible values of δ, namely $\delta_M < \delta_B$. There are, then, four possible types of managers: the efficient money-seeker, (β_E, δ_M); the efficient empire-builder, (β_E, δ_B); the inefficient money-seeker, (β_I, δ_M); and the inefficient empire-builder, (β_I, δ_B). The prior probabilities associated with each of these types are $\alpha_{EB}, \alpha_{EM}, \alpha_{IB}$ and α_{IM}, all assumed to be strictly positive.

Obviously, the efficient empire-builder (EB) is the best type for the principal and the inefficient money-seeker (IM) is the worst type. It is also clear that EB is a better type than IB and EM; and that IB and EM are better types than IM. It is not possible to rank *a priori* the two intermediate types, i.e., the inefficient empire-builder (IB) and the efficient money-seeker (EM).

Finally, let $\omega \equiv \frac{\Delta\delta}{\Delta\beta} = \frac{\delta_B - \delta_M}{\beta_I - \beta_E}$ be the relative variability of empire-building tendency and efficiency (among the different types). We will see that, in this model with complementary activities, the results are mainly driven by the value of this parameter.

The regulator maximizes:

$$\sum_{i=\{E,I\}} \sum_{j=\{M,B\}} \alpha_{ij} W_{ij},$$

where $W_{ij} = S(q_{ij}) - (1+\lambda)\left[c_{ij}q_{ij} + \psi(\beta_i - c_{ij}) - \delta_j q_{ij}\right] - \lambda U_{ij}$.

It is very important to notice that, from the regulator's point of view, the two activities, output and efficiency, are complements, i.e., $\frac{\partial^2 W_{ij}}{\partial q_{ij}\partial c_{ij}} = -(1+\lambda) < 0$. Higher efficiency makes a larger output level more desirable, and vice versa. This is a substantial difference with respect to the setup of Armstrong and Rochet (1999), where both the agent and the principal have additively separable utility functions.

The regulator offers a menu of contracts to the manager, such that the type ij manager produces q_{ij} at marginal cost c_{ij} and receives a net transfer t_{ij}, implying a utility level U_{ij}. In this problem, there are four participation constraints and twelve incentive constraints.

The only binding participation constraint is $U_{IM} \geq 0$, because it implies that all the other types are able to attain a strictly positive utility level. The inefficient money-seeker (worst type) obtains its reservation utility.

The incentive constraints may be downward, upward or diagonal. The downward (resp. upward) constraints are those in which the constrained type is better (resp. worse) than the constraining type in both dimensions. In the diagonal constraints, each of the types is better in one dimension and worse in the other.

The incentive constraint which imposes that the constrained type ij cannot be better off by mimicking the constraining type $i'j'$ will be denoted constraint $ij/i'j'$. There are 5 downward constraints (EB/IM, EB/EM, EB/IB, EM/IM and IB/IM), 5 upward constraints (IM/EB, EM/EB, IB/EB, IM/EM and IM/IB) and 2 diagonal constraints (EM/IB and IB/EM).

A manager of type ij that claims to be of type $i'j'$ obtains the utility level of type $i'j'$, plus the difference in the empire-building component of utility, $(\delta_j - \delta_{j'})q_{i'j'}$, and minus the difference in the disutility of effort component, $\psi(\beta_i - c_{i'j'}) - \psi(\beta_{i'} - c_{i'j'})$. The corresponding incentive compatibility constraint $(ij/i'j')$ is:

$$U_{ij} \;\geq\; U_{i'j'} + (\delta_j - \delta_{j'})q_{i'j'} + \psi(\beta_{i'} - c_{i'j'}) - \psi(\beta_i - c_{i'j'}). \tag{1}$$

The following monotonicity property is a direct consequence of the incentive constraints.

Remark 1. *Ceteris paribus, an empire-builder produces more output than a money-seeker and an efficient manager produces with a lower cost than an inefficient manager:*

$$q_{iB} \geq q_{iM}, \ \forall i \in \{E, I\}, \tag{2a}$$

$$c_{Ej} \leq c_{Ij}, \ \forall j \in \{M, B\}. \tag{2b}$$

Proof. Adding the two incentive constraints between types ij and $i'j'$, we obtain: $0 \geq (\delta_j - \delta_{j'})(q_{i'j'} - q_{ij}) + \psi(\beta_{i'} - c_{i'j'}) - \psi(\beta_i - c_{i'j'}) + \psi(\beta_i - c_{ij}) - \psi(\beta_{i'} - c_{ij})$. Considering types iB and iM, we obtain $0 \geq (\delta_B - \delta_M)(q_{iM} - q_{iB})$, which implies that $q_{iB} \geq q_{iM}$. Considering types Ej and Ij, we obtain $0 \geq \psi(\beta_I - c_{Ij}) - \psi(\beta_E - c_{Ij}) + \psi(\beta_E - c_{Ej}) - \psi(\beta_I - c_{Ej})$. Since ψ is a convex function, this implies that $c_{Ej} \leq c_{Ij}$. □

We will focus on the case in which $\psi(e) = \frac{e^2}{2}$ and $S(q) = 2q - q^2$. To ensure that the problem is concave, we also assume that $\lambda < 1$.[4] In this case, the incentive constraints can be written as:

$$U_{ij} \ \geq \ U_{i'j'} + (\delta_j - \delta_{j'})q_{i'j'} + \frac{1}{2}(\beta_{i'}^2 - \beta_i^2) + c_{i'j'}(\beta_i - \beta_{i'}), \tag{3}$$

for all pairs ij and $i'j'$.

Denote by q_{ij}^* and c_{ij}^* the perfect information output and marginal cost for each manager type. The first-order conditions are:

$$\begin{aligned} S'(q_{ij}^*) &= (1+\lambda)\left(c_{ij}^* - \delta_i\right), \\ \psi'(\beta_i - c_{ij}^*) &= q_{ij}^*. \end{aligned}$$

Since $\psi(e) = \frac{e^2}{2}$ and $S(q) = 2q - q^2$, the first-best solution (perfect information benchmark) is:

$$\begin{aligned} q_{ij}^* &= \frac{1}{1-\lambda}\left[2 - (\beta_i - \delta_j)(1+\lambda)\right], \\ c_{ij}^* &= \frac{1}{1-\lambda}\left[-2 + 2\beta_i - (1+\lambda)\delta_j\right]. \end{aligned}$$

The above expressions illustrate the complementarity between output and efficiency, which is the main characteristic of the model. For instance, a high value of the marginal utility of output translates not only into a high first-best output level level but as well into a high first-best efficiency level. Reciprocally, a low value of the intrinsic marginal cost translates not only into a

[4] It is usual to assume that $-S''(q)\psi''(e) > (1+\lambda)$. See, for example, Laffont and Tirole (1986). For the specific functions on which we focus, this is equivalent to $\lambda < 1$.

high level of efficiency but as well into a high output level. It is not true, contrary to a model with separable utility functions, that intrinsically efficient managers always exhibit first-best efficiency levels and more output-oriented managers produce their first-best output levels. Due to the complementarity property, downward distortions along one dimension result in downward distortions along the other dimension.

The complementarity of activities seriously complicates the analysis of the model. Armstrong and Rochet (1999), when analyzing the case of independent activities, considered first a "relaxed" problem obtained by considering only the downward incentive constraints, i.e., by neglecting the upward incentive constraints and the two diagonal incentive constraints (those between the two intermediate types). In a second stage, they checked that the neglected constraints were indeed satisfied by the solutions of the relaxed problem (or found conditions that ensured that they were satisfied). When activities are complementary, the solutions of such a relaxed problem almost never satisfy the diagonal constraints. This is why we consider a less relaxed problem, where only the upward constraints are discarded.

3 The relaxed problem

We define a relaxed problem in which only the downward and the diagonal incentive constraints are considered, together with the participation constraint for the worst type (the only one that is binding):

$$\max_{q,c,U} \sum_{(i,j)} \alpha_{ij} \left\{ S(q_{ij}) - (1+\lambda) \left[(c_{ij} - \delta_j)\, q_{ij} + \psi(\beta_i - c_{ij}) \right] - \lambda U_{ij} \right\}$$

subject to:

$$\begin{aligned}
U_{IM} &= 0, && \text{(4a)}\\
U_{EB} &\geq U_{EM} + \Delta\delta q_{EM}, && \text{(4b)}\\
U_{EB} &\geq U_{IB} - \Delta\beta c_{IB} + k, && \text{(4c)}\\
U_{EB} &\geq \Delta\delta q_{IM} - \Delta\beta c_{IM} + k, && \text{(4d)}\\
U_{IB} &\geq \Delta\delta q_{IM}, && \text{(4e)}\\
U_{EM} &\geq -\Delta\beta c_{IM} + k, && \text{(4f)}\\
U_{IB} &\geq U_{EM} + \Delta\delta q_{EM} + \Delta\beta c_{EM} - k, && \text{(4g)}\\
U_{EM} &\geq U_{IB} - \Delta\beta c_{IB} + k - \Delta\delta q_{IB}, && \text{(4h)}
\end{aligned}$$

where $\Delta\beta \equiv \beta_I - \beta_E$, $k \equiv \frac{1}{2}\left(\beta_I^2 - \beta_E^2\right)$ and $\Delta\delta \equiv \delta_B - \delta_M$.

The solution of the relaxed problem must be such that:

$$U_{IB} = \max\left\{\Delta\delta q_{IM} \ ; \ U_{EM} + \Delta\delta q_{EM} + \Delta\beta c_{EM} - k\right\}, \tag{5a}$$

$$U_{EM} = \max\left\{-\Delta\beta c_{IM} + k \ ; \ U_{IB} - \Delta\beta c_{IB} + k - \Delta\delta q_{IB}\right\}, \tag{5b}$$

$$U_{EB} = \max\left\{U_{EM} + \Delta\delta q_{EM} \ ; \ U_{IB} - \Delta\beta c_{IB} + k \ ; \ \Delta\delta q_{IM} - \Delta\beta c_{IM} + k\right\}. \tag{5c}$$

Equations (5a) and (5b) are the incentive constraints of the intermediate types (IB and EM). For each of them, the constraining type that binds may be the other intermediate type, the worst type, or both. Equation (5c) is the best type's incentive constraint: EB may indeed be constrained by IB, EM, IM, or by two or three of them (7 possibilities). Combining the possible solutions of (5), there are up to 63 possible patterns of binding incentive constraints.

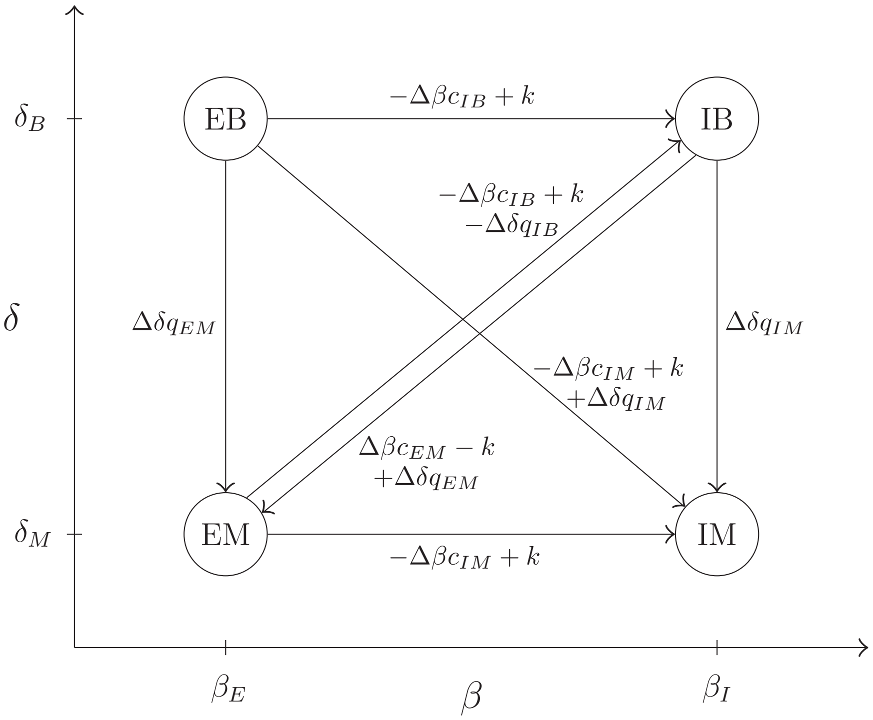

Figure 1: Downward and diagonal incentive constraints.

Figure 1 pictures all the possibly binding downward and diagonal incentive constraints. Notice that types on the left are intrinsically more efficient and types above are more output-oriented. It should be read as follows. The arrow starting from EB and going to IB represents the downward constraint EB/IB: the difference between U_{EB} and U_{IB} must be at least equal to $-\Delta\beta c_{IB} + k$.

Consider the arrow from EM to IB: the difference between U_{EM} and U_{IB} must be at least $-\Delta\beta c_{IB} + k - \Delta\delta q_{IB}$.

To determine which of the constraints are binding, one has to compare the "lengths" of the paths from a point, ij, to another, $i'j'$. Constraints are non-binding if there is a longer path between the two points. For instance, to go from EB to IM there are three possible paths: EB/IM, $EB/IB + IB/IM$ and $EB/EM + EM/IM$. To determine which is the longest, one has to compare $\Delta\delta q_{IM} - \Delta\beta c_{IM} + k$, $\Delta\delta q_{IM} - \Delta\beta c_{IB} + k$ and $\Delta\delta q_{EM} - \Delta\beta c_{IM} + k$. To go from EM to IM there are two possible paths: EM/IM and $EM/IB + IB/IM$, and so on.

Let γ_1 to γ_5 be the non-negative Lagrange multipliers associated with the five downward constraints (EB/EM, EB/IB, EB/IM, IB/IM and EM/IM, respectively) and γ_6 and γ_7 the multipliers associated with the diagonal constraints (IB/EM and EM/IB, respectively).

The first-order conditions with respect to U_{EB}, U_{EM} and U_{IB} are:

$$-\lambda\alpha_{EB} + \gamma_1 + \gamma_2 + \gamma_3 = 0, \tag{6a}$$

$$-\lambda\alpha_{EM} - \gamma_1 + \gamma_5 - \gamma_6 + \gamma_7 = 0, \tag{6b}$$

$$-\lambda\alpha_{IB} - \gamma_2 + \gamma_4 + \gamma_6 - \gamma_7 = 0. \tag{6c}$$

For $S(q) = 2q - q^2$ and $\psi(e) = \frac{e^2}{2}$, the first-order conditions with respect to q_{ij} and c_{ij} yield:[5]

$$
\begin{aligned}
q_{EB} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_E-\delta_B)\right], & \text{(7a)}\\
q_{EM} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_E-\delta_M)-\frac{(\gamma_1+\gamma_6)\Delta\delta-\gamma_6\Delta\beta}{\alpha_{EM}}\right], & \text{(7b)}\\
q_{IB} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_I-\delta_B)-\frac{(\gamma_2+\gamma_7)\Delta\beta-\gamma_7\Delta\delta}{\alpha_{IB}}\right], & \text{(7c)}\\
q_{IM} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_I-\delta_M)-\frac{(\gamma_3+\gamma_4)\,\Delta\delta+(\gamma_3+\gamma_5)\Delta\beta}{\alpha_{IM}}\right], & \text{(7d)}\\
c_{EB} &= \frac{1}{1-\lambda}\left[-2+2\beta_E-(1+\lambda)\delta_B\right], & \text{(7e)}\\
c_{EM} &= \frac{1}{1-\lambda}\left[-2+2\beta_E-(1+\lambda)\delta_M+\frac{(\gamma_1+\gamma_6)\Delta\delta-\gamma_6\frac{2\Delta\beta}{1+\lambda}}{\alpha_{EM}}\right], & \text{(7f)}\\
c_{IB} &= \frac{1}{1-\lambda}\left[-2+2\beta_I-(1+\lambda)\delta_B+\frac{(\gamma_2+\gamma_7)\frac{2\Delta\beta}{1+\lambda}-\gamma_7\Delta\delta}{\alpha_{IB}}\right], & \text{(7g)}\\
c_{IM} &= \frac{1}{1-\lambda}\left[-2+2\beta_I-(1+\lambda)\delta_M+\frac{(\gamma_3+\gamma_4)\,\Delta\delta+(\gamma_3+\gamma_5)\frac{2\Delta\beta}{1+\lambda}}{\alpha_{IM}}\right]. & \text{(7h)}
\end{aligned}
$$

Besides the classical "no distortion at the top" (i.e., for the efficient empire-builder), what we mainly observe in these results is that the distortion of the activities of type ij manager's activities decreases with the probability α_{ij} associated with his/her type. On the other hand, it increases with the probability of type $i'j'$ if the incentive constraint $i'j'/ij$ is binding. Finally, a distortion along the intrinsic efficiency dimension effects not only the cost level but also the output level and reciprocally for a distortion along the empire-building dimension.

It is not surprising that the efficient empire-builder (EB) must produce more and at a lower marginal cost than the inneficient money-seeker (IM).

Remark 2. *In any solution of the relaxed problem, we have:*

$$q_{EB} > q_{IM}, \quad c_{EB} < c_{IM} \quad \textit{and} \quad e_{EB} < e_{IM}.$$

Proof. Follows from (7a), (7e), (7d) and (7h), given the non-negativity of the multipliers. □

After finding the solution of the relaxed problem, we will be interested in checking that the upward incentive constraints are satisfied. The next result is helpful for that purpose.

[5] See Appendix 6.1 for further details.

Remark 3. *If a downward incentive constraint is binding, the corresponding upward incentive constraint is surely satisfied if the activity levels satisfy the monotonicity property (2).*

Proof. See Appendix 6.1. □

4 Several possible scenarios

In this Section, we analyze possible solutions of the relaxed problem and of the original problem. We always use the same methodology. Each Case is defined by a list of binding and non-binding incentive constraints. Then, the solution candidate associated with each Case (output, cost and utility for each type) must be computed from the first-order conditions, (6) and (7), and from the incentive constraints, (4). While the binding incentive constraints provide additional equations, the non-binding incentive constraints allow us to set the corresponding multipliers to zero. Finally, we study the conditions under which the solution candidate that corresponds to each Case is an actual solution of the relaxed problem and of the original, fully constrained, problem.

The number of possible cases is *a priori* very large, so it is almost impossible to study all of them. Only few of them lead to activity levels that solve the principal's problem for some set of parameter values. We present some of these cases, focusing on the importance of $\omega = \frac{\Delta\delta}{\Delta\beta}$ in determining the nature of the solutions.

4.1 Case A: Strong positive correlation

We start with the first case that was presented by Armstrong and Rochet (1999). The solution of the fully constrained problem is of this kind when efficiency and tendency for empire-building are strongly positively correlated. All the downward constraints are binding, while the diagonal constraints are not binding.

In this case, there is bunching of output levels of the money-seekers ($q_{EM} = q_{IM}$) and bunching of cost levels of the inneficient managers ($c_{IB} = c_{IM}$). Besides that, the ranking of activity levels is "natural", in the sense that: the ranking of output is primarily determined by preference for output, while the ranking of observed efficiency is primarily determined by intrinsic efficiency.

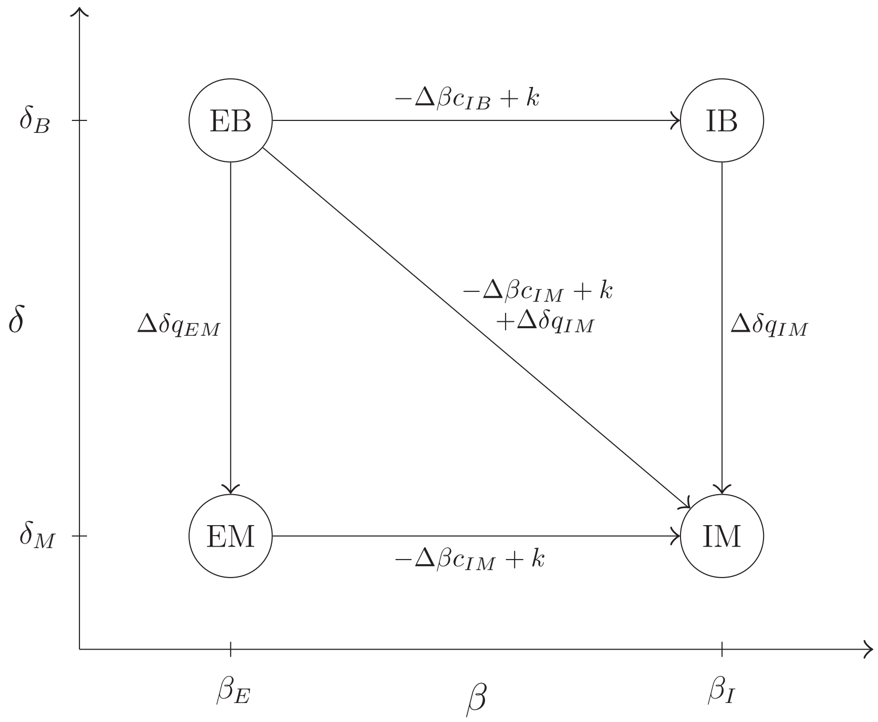

Figure 2: All the downward constraints are binding in Case A.

Remark 4. *When Case A is optimal in the relaxed problem, output and marginal cost levels are ranked in the natural way, with bunching of the worse types in each activity:*

$$q_{EB} > q_{IB} \geq q_{EM} = q_{IM},$$
$$c_{EB} < c_{EM} \leq c_{IB} = c_{IM}.$$

Proof. See Appendix 6.2. □

The solution of the relaxed problem that is obtained in Case A is also the solution of the fully constrained problem if the correlation between efficiency and empire-building is strong enough.

Proposition 1. *If α_{EM} and α_{IB} are sufficiently small, then Case A is optimal in the relaxed problem and in the fully constrained problem.*

Proof. See Appendix 6.2. □

The precise meaning of Proposition 1 is that, for given values of the remaining parameters, there exist threshold values of the probabilities of the intermediate types (α_{EM} and α_{IB}) below which Case A provides the solution of the original problem.

4.2 Cases B and C: Similar variabilities of β and δ

Several cases can only occur if ω is close enough to 1. In these cases, the ranking of types is not primarily determined by their ranking along a single dimension (there is neither "empire-building dominance" nor "efficiency dominance").

In Case B, which occurs when the correlation between empire-building and efficiency is weak, the ranking of managers according to their preference for output determines the ranking of their output levels, while the ranking of managers according to their intrinsic efficiency determines the ranking of their marginal cost levels. In this case, we are close to a model with independent activities. In fact, it coincides with the second of the cases that were analyzed by Armstrong and Rochet (1999).

In Case C, which occurs when empire-building and efficiency are negatively or weakly positively correlated, the output levels of the inefficient empire-builder and the efficient money-seeker are identical (partial bunching). This case did not appear in the work of Armstrong and Rochet (1999).

4.2.1 Case B: Natural ranking of activity levels

In Case B, we suppose that the diagonal incentive constraints are not binding, while all the downward constraints, except EB/IM, are binding. This case holds when efficiency and tendency for empire-building are weakly correlated and ω is close to 1.

There is a natural ranking of activity levels. The ranking of output levels is primarily determined by the tendency for empire-building, while the ranking of marginal cost levels is primarily determined by intrinsic efficiency. Types that have a stronger preference for high output produce more, and types that are intrinsically more efficient produce at a lower cost.

Remark 5. *When Case B is optimal in the relaxed problem, output and marginal cost levels are*

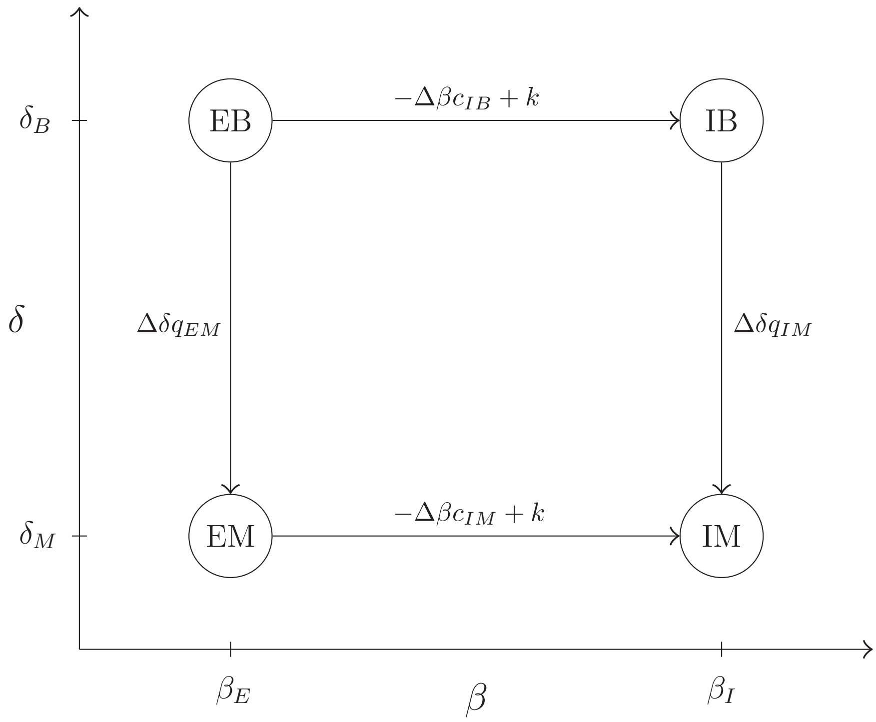

Figure 3: Incentive constraints that are binding in Case B.

ranked in the natural way:

$$q_{EB} > q_{IB} \geq q_{EM} \geq q_{IM},$$
$$c_{EB} < c_{EM} \leq c_{IB} \leq c_{IM}.$$

Proof. See Appendix 6.3. □

Case B provides the solution to the fully constrained problem when ω is close to 1 and the correlation between efficiency and empire-building is not too strong.

Proposition 2. *If* $\frac{\alpha_{EM}\alpha_{IB}}{\alpha_{EB}\alpha_{IM}} \leq \frac{2}{1-\lambda}$ *and* $\alpha_{IM} \in \left[\frac{\alpha_{EM}}{\alpha_{EM}+\alpha_{EB}}, 1 - \frac{1+3\lambda}{1+\lambda}\alpha_{EB} + \frac{1-\lambda}{1+\lambda}\alpha_{EM}\right]$*, then Case D is optimal in the relaxed problem and in the fully constrained problem when* $\omega = 1$*.*

Proof. See Appendix 6.3. □

4.2.2 Case C: Bunching of intermediate output levels

In Case C, we consider the case in which all the (downward and diagonal) incentive constraints are binding except EB/IM and IB/EM.

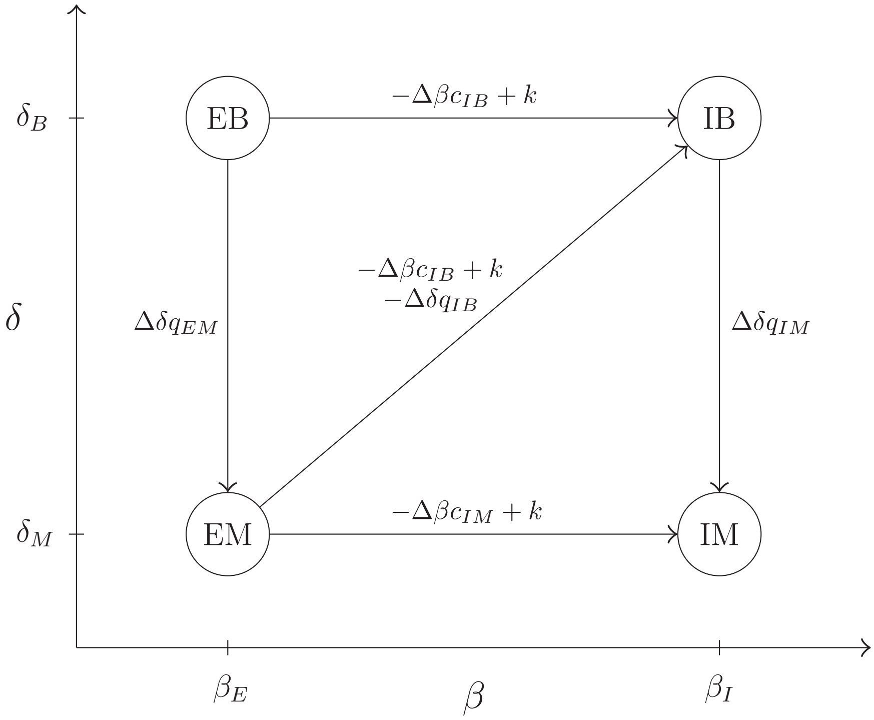

Figure 4: Incentive constraints that are binding in Case C.

Remark 6. *When Case C is optimal in the relaxed problem, output and marginal cost levels are ranked as follows:*

$$q_{EB} > q_{EM} = q_{IB} \geq q_{IM}$$
$$c_{EB} < c_{EM} < c_{IB} \leq c_{IM}.$$

Proof. See Appendix 6.4. □

There is bunching of the output levels of the two intermediate types ($q_{EM} = q_{IB}$). On the other hand, there is no bunching of the marginal cost levels ($c_{EM} < c_{IB}$). The efficient money-seeker produces at a lower marginal cost than the inefficient empire-builder.

The solution of the original problem is as in Case C if the correlation between efficiency and money-seeking is relatively high. More precisely, if the proportion of money-seekers among the efficient managers is higher than the proportion of inneficient money-seekers among all managers.

Proposition 3. *Case C is optimal in the relaxed problem and in the fully constrained problem when $\omega = 1$ if and only if $\alpha_{IM} \geq \frac{\alpha_{EM}}{\alpha_{EM}+\alpha_{EB}}$ and $\alpha_{EB} \leq (1-\alpha_{IM})\frac{\lambda+\alpha_{IM}}{\lambda+\alpha_{IM}+\lambda\alpha_{IM}}$.*

Proof. See Appendix 6.4. □

4.3 Case D: Empire-building dominance ($\Delta\delta \gg \Delta\beta$)

If the variability of the empire-building tendency parameter ($\Delta\delta$) is significantly larger than that of the intrinsic marginal cost parameter ($\Delta\beta$), then the inefficient empire-builder should be a better type than the efficient money-seeker (because IB has a much stronger empire-building tendency and is only slightly less efficient than EM). The resulting ordering of types (from the best to the worst) is, then: EB, IB, EM, IM.

In Case D, the constraints between types that are adjacent according to the ordering mentioned above are binding: the efficient empire-builder has to be prevented from mimicking the inefficient empire-builder (EB/IB), the inefficient empire-builder from mimicking the efficient money-seeker (IB/EM) and the efficient money-seeker from mimicking the inefficient money-seeker (EM/IM). In addition, the constraint that prevents the inefficient empire-builder from mimicking the inefficient money-seeker (IB/IM) is also binding.

We will show that this corresponds to the solution of the fully constrained problem when ω is sufficiently large.

Output and marginal cost levels are ranked in the same way and primarily according to the tendency of the manager for empire-building. An inefficient empire-builder produces at lower cost than an efficient money-seeker. This is the intuitive consequence of the complementarity between effort and output. When the variability of the marginal utility of output becomes very large relative to the variability of the intrinsic marginal cost, the greater effort provided by the inefficient empire-builder compensates the lower intrinsic efficiency with respect to the efficient money-seeker.

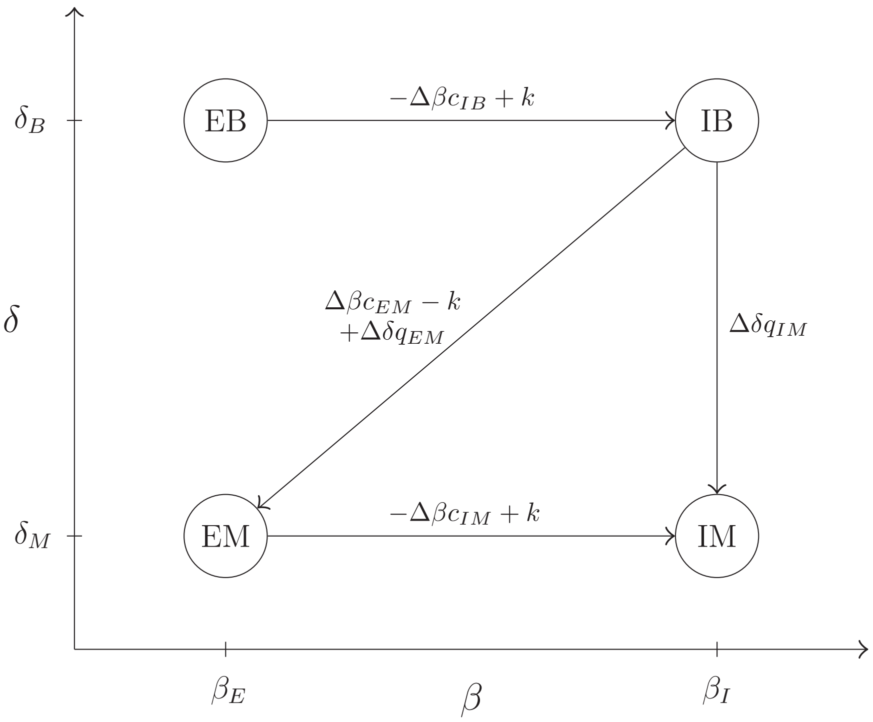

Figure 5: Incentive constraints that are binding in Case D.

Remark 7. *When Case D is optimal in the fully constrained problem, we must have* $\omega > 1$ *and the following ranking of activity levels:*

$$q_{EB} > q_{IB} \geq q_{EM} > q_{IM},$$
$$c_{EB} < c_{IB} \leq c_{EM} < c_{IM}.$$

Proof. See Appendix 6.5. □

There always exists a threshold value for ω, above which Case D provides the solution for the original problem.

Proposition 4. *If* ω *is sufficiently large, then Case D is optimal in the relaxed problem and in the fully constrained problem.*

Proof. See Appendix 6.5. □

More precisely, Proposition 4 should be read as follows: given the values of the remaining parameters, there exists a threshold value of the variability of intrinsic efficiency ($\Delta\beta$) below which

Case D provides the solution of the original problem.

4.4 Case E: Efficiency dominance ($\Delta\beta \gg \Delta\delta$)

An opposite situation occurs when the variability of the intrinsic marginal cost ($\Delta\beta$) is significantly larger than that of the marginal utility of output ($\Delta\delta$). In that case, EM is a better type than IB, because EM is much more efficient and only slightly less empire-builder than IB. The intuitive ordering of types, from the best to the worst, is: EB, EM, IB, IM.

In Case E, we assume that the binding constraints are those between adjacent types in the above sense (EB/EM, EM/IB, IB/IM), and the one that prevents the efficient money-seeker from mimicking the inefficient money-seeker (EM/IM). We will show that this kind of solution is optimal when ω is sufficiently small.

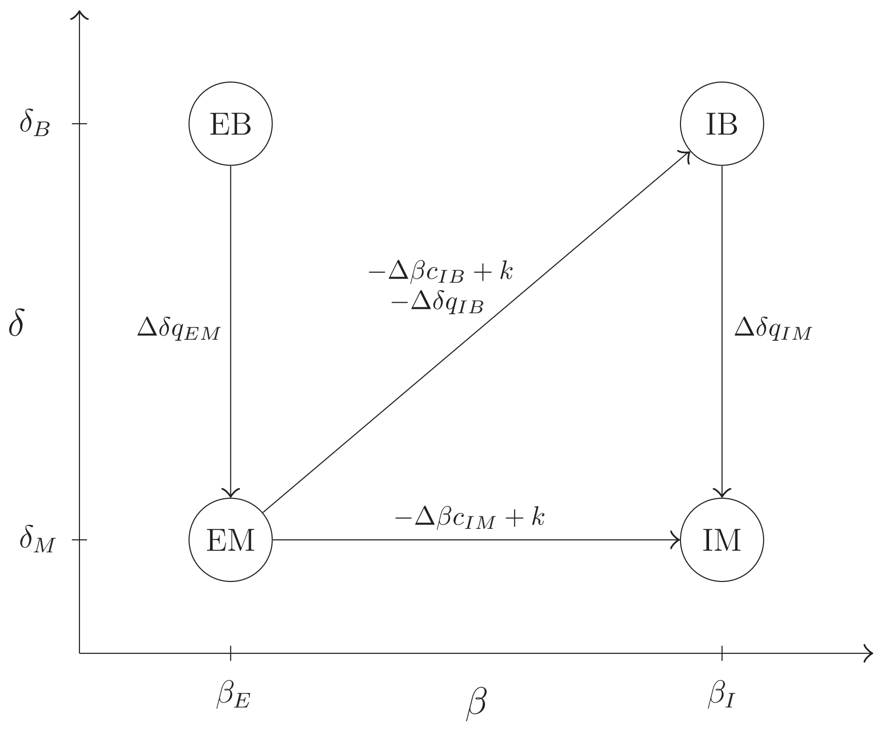

Figure 6: Incentive constraints that are binding in Case E.

In this case, the ranking of activity levels of the four types of managers is primarily determined by their ranking along the efficiency axis, i.e., more efficient managers not only produce at lower marginal cost but also produce larger outputs. The four types are unambiguously ranked, first according to their efficiency, and then according to their tendency for empire-building ("efficiency dominance").

Remark 8. *When Case E is optimal in the relaxed problem, output and marginal cost levels are ranked as follows:*

$$q_{EB} > q_{EM} \geq q_{IB} > q_{IM}$$
$$c_{EB} < c_{EM} \leq c_{IB} < c_{IM}.$$

Proof. See Appendix 6.6. □

Efficiency dominance is the result of the complementarity of effort and output levels. When managers differ much more in their intrinsic marginal cost than in their marginal utility of output, the optimal contract ranks their productivity and their output according to the value of this parameter. For instance, an efficient money-seeker produces more output than an inefficient empire-builder, though it has a lower marginal utility of output. This holds even of the probability of a manager being a money-seeker is small.

For given values of all the remaining parameters, if we decrease the variability of the marginal utility of output ($\Delta\delta$), then, below some threshold value, Case E is optimal.

Proposition 5. *If ω is sufficiently small, Case E is optimal in the relaxed problem and in the fully constrained problem.*

Proof. See Appendix 6.6. □

5 Concluding remarks

We analyzed a model of two-dimensional screening with complementary activities and types drawn from a binary distribution. The results show that one of the main determinants of the characteristics of the optimal contract is the relative variability of the characteristics of managers in the two dimensions. When the ratio $\frac{\Delta\delta}{\Delta\beta}$ differs enough from 1, the model becomes closer to a one-dimensional model. When it is low, our setup becomes close to the traditional model of Laffont and Tirole (1986), where the only piece of private information is the intrinsic cost: more efficient managers have both larger output and lower marginal cost levels. When it is large, our setup becomes close to a model where the only piece of private information is the manager's tendency for empire-building. In this case, an empire-builder produces more and at a lower marginal cost. These results are the obvious consequence of the complementarity of effort and output.

It is only when $\frac{\Delta\delta}{\Delta\beta}$ is close to 1, that the results are more mitigate. The "natural ranking" result where the ranking of observed efficiency levels is determined by intrinsic efficiencies and the ranking of output levels is determined by empire-building tendencies is only obtained under an additional condition, namely a large positive correlation between empire-building and efficiency.

6 Appendix

6.1 Relaxed Problem

The first-order conditions with respect to the q_{ij} are:

$$\begin{aligned}
S'(q_{EB}) - (1+\lambda)(c_{EB} - \delta_B) &= 0, \\
S'(q_{IB}) - (1+\lambda)(c_{IB} - \delta_B) &= \frac{-\gamma_7 \Delta\delta}{\alpha_{IB}}, \\
S'(q_{EM}) - (1+\lambda)(c_{EM} - \delta_M) &= \frac{(\gamma_1 + \gamma_6)\Delta\delta}{\alpha_{EM}}, \\
S'(q_{IM}) - (1+\lambda)(c_{IM} - \delta_M) &= \frac{(\gamma_3 + \gamma_4)\Delta\delta}{\alpha_{IM}}.
\end{aligned}$$

Those with respect to the c_{ij} are:

$$\begin{aligned}
q_{EB} - \psi'(\beta_E - c_{EB}) &= 0, \\
q_{IB} - \psi'(\beta_I - c_{IB}) &= \frac{(\gamma_2 + \gamma_7)\Delta\beta}{\alpha_{IB}(1+\lambda)}, \\
q_{EM} - \psi'(\beta_E - c_{EM}) &= \frac{-\gamma_6 \Delta\beta}{\alpha_{EM}(1+\lambda)}, \\
q_{IM} - \psi'(\beta_I - c_{IM}) &= \frac{(\gamma_3 + \gamma_5)\Delta\beta}{\alpha_{IM}(1+\lambda)}.
\end{aligned}$$

With $S(q) = 2q - q^2$ and $\psi(e) = \frac{e^2}{2}$, the activity levels are given by equations (7a)-(7h).

Proof of Remark 3

Suppose that $ij/i'j'$ is an upward incentive constraint and that $i'j'/ij$ is binding. From (3):

$$\begin{aligned}
U_{ij} &\geq U_{i'j'} + (\delta_j - \delta_{j'})q_{i'j'} + \frac{1}{2}(\beta_{i'}^2 - \beta_i^2) + c_{i'j'}(\beta_i - \beta_{i'}), \\
U_{i'j'} &= U_{ij} + (\delta_{j'} - \delta_j)q_{ij} + \frac{1}{2}(\beta_i^2 - \beta_{i'}^2) + c_{ij}(\beta_{i'} - \beta_i).
\end{aligned}$$

Adding the two, we obtain:

$$0 \geq (q_{i'j'} - q_{ij})(\delta_j - \delta_{j'}) + (c_{i'j'} - c_{ij})(\beta_i - \beta_{i'}). \tag{10}$$

Since $ij/i'j'$ is an upward incentive constraint: $\delta_j - \delta_{j'} \leq 0$ and $\beta_i - \beta_{i'} \geq 0$. With the ranking of activities being natural: $q_{i'j'} - q_{ij} \geq 0$ and $c_{i'j'} - c_{ij} \leq 0$. Hence, (10) holds. □

6.2 Case A

In Case A, the incentive compatibility constraints (4) can be written as:

$$U_{IM} = 0, \tag{11a}$$
$$U_{EB} = -\Delta\beta c_{IM} + k + \Delta\delta q_{EM}, \tag{11b}$$
$$c_{IB} - c_{IM} = 0, \tag{11c}$$
$$q_{EM} - q_{IM} = 0, \tag{11d}$$
$$U_{IB} = \Delta\delta q_{IM}, \tag{11e}$$
$$U_{EM} = -\Delta\beta c_{IM} + k, \tag{11f}$$
$$c_{IM} - c_{EM} \geq 0, \tag{11g}$$
$$q_{IB} - q_{IM} \geq 0. \tag{11h}$$

The first-order conditions (7) are:

$$q_{EB} = \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_E - \delta_B)\right], \tag{12a}$$
$$q_{EM} = \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_E - \delta_M) - \frac{\gamma_1 \Delta\delta}{\alpha_{EM}}\right], \tag{12b}$$
$$q_{IB} = \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_I - \delta_B) - \frac{\gamma_2 \Delta\beta}{\alpha_{IB}}\right], \tag{12c}$$
$$q_{IM} = \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_I - \delta_M) - \frac{(\gamma_3+\gamma_4)\,\Delta\delta + (\gamma_3+\gamma_5)\Delta\beta}{\alpha_{IM}}\right], \tag{12d}$$
$$c_{EB} = \frac{1}{1-\lambda}\left[-2 + 2\beta_E - (1+\lambda)\delta_B\right], \tag{12e}$$
$$c_{EM} = \frac{1}{1-\lambda}\left[-2 + 2\beta_E - (1+\lambda)\delta_M + \frac{\gamma_1 \Delta\delta}{\alpha_{EM}}\right], \tag{12f}$$
$$c_{IB} = \frac{1}{1-\lambda}\left[-2 + 2\beta_I - (1+\lambda)\delta_B + \frac{\gamma_2 \frac{2\Delta\beta}{1+\lambda}}{\alpha_{IB}}\right], \tag{12g}$$
$$c_{IM} = \frac{1}{1-\lambda}\left[-2 + 2\beta_I - (1+\lambda)\delta_M + \frac{(\gamma_3+\gamma_4)\,\Delta\delta + (\gamma_3+\gamma_5)\frac{2\Delta\beta}{1+\lambda}}{\alpha_{IM}}\right]. \tag{12h}$$

Proof of Remark 4

From the binding incentive constraints (11c) and (11d), we must have $c_{IB} = c_{IM}$ and $q_{EM} = q_{IM}$. From (11g) and (11h), $c_{IM} \geq c_{EM}$ and $q_{IB} \geq q_{IM}$. It is clear from the expressions (12) that $q_{EB} > q_{IB}$ and $c_{EB} < c_{EM}$. $\square$

Proof of Proposition 1

By Remark 3, the upward constraints are satisfied. Therefore, for this to be the solution of the relaxed problem and of the fully constrained problem, we only have to check that the multipliers are non-negative.

Since $\gamma_6 = \gamma_7 = 0$, from (6), we find that:

$$\begin{aligned} \gamma_3 &= -\gamma_1 - \gamma_2 + \lambda\alpha_{EB}, \\ \gamma_5 &= \gamma_1 + \lambda\alpha_{EM}, \\ \gamma_4 &= \gamma_2 + \lambda\alpha_{IB}. \end{aligned} \tag{13}$$

Using these relations between the multipliers and the first-order conditions (12), $q_{EM} = q_{IM}$ and $c_{IB} = c_{IM}$ imply that:

$$\gamma_1 = \left(\frac{\alpha_{EM}}{\omega}\right) \frac{2\alpha_{IM}\left[\lambda + \alpha_{IM} + \lambda\omega(\alpha_{EB} + \alpha_{IB})\right] + \lambda(1-\lambda)\omega\alpha_{IB}\left(1 - \alpha_{EM}\right) + \alpha_{IB}(2 - \omega - 3\lambda\omega)}{\alpha_{EM}\left[(1-\lambda)\alpha_{IB} + 2\alpha_{IM}\right] + 2\alpha_{IM}(\alpha_{IB} + \alpha_{IM})}.$$

Replacing $\alpha_{IB} = 0$ and $\alpha_{EM} = 0$, we obtain $\gamma_1 = 0$. To verify that $\gamma_1 > 0$ for small but positive α_{IB} and α_{EM}, notice that: the denominator is always positive; the term $\frac{\alpha_{EM}}{\omega}$ is also, obviously, positive; and the numerator converges to $2\alpha_{IM}\left(\lambda + \alpha_{IM} + \lambda\omega\alpha_{EB}\right) > 0$ when $(\alpha_{IB}, \alpha_{EM}) \to (0, 0)$. Therefore, $\exists \epsilon > 0 : \ (\alpha_{IB}, \alpha_{EM}) < (\epsilon, \epsilon) \Rightarrow \gamma_1 > 0$.

Similarly, we obtain:

$$\begin{aligned} \gamma_2 = & \frac{\alpha_{IB}}{\alpha_{EM}\left[(1-\lambda)\alpha_{IB} + 2\alpha_{IM}\right] + 2\alpha_{IM}(\alpha_{IB} + \alpha_{IM})} \left[\lambda\alpha_{EM}(1 - \alpha_{IB})\right. \\ & \left. -\alpha_{EM}\alpha_{IM}(1 + 3\lambda - \omega - \lambda\omega) + \lambda\alpha_{IM}(2 + \omega + \lambda\omega - 2\alpha_{IB}) + \alpha_{IM}^2(-2\lambda + \lambda\omega + \omega)\right]. \end{aligned}$$

Again, replacing $\alpha_{IB} = 0$ and $\alpha_{EM} = 0$, we obtain $\gamma_2 = 0$. Following the same reasoning as for γ_1, notice that: the denominator is always positive; the probability α_{IB} is also, obviously, positive; and the term inside square brackets converges to $(2\lambda + \omega\lambda + \lambda^2\omega)\,\alpha_{IM} + (\lambda\omega - 2\lambda + \omega)\alpha_{IM}^2 > 0$

when $(\alpha_{IB}, \alpha_{EM}) \to (0,0)$. Therefore, $\exists \epsilon > 0 : \ (\alpha_{IB}, \alpha_{EM}) < (\epsilon, \epsilon) \Rightarrow \gamma_2 > 0$.

Since $\gamma_1 \to 0$ and $\gamma_2 \to 0$, from (13):

$$\lim_{(\alpha_{IB}, \alpha_{EM}) \to (0,0)} \gamma_3 = \lambda \alpha_{EB}.$$

We conclude that, for sufficiently small α_{IB} and α_{EM}, all the multipliers are non-negative.

We still need to check that $q_{IB} \geq q_{IM}$ and $c_{IM} \geq c_{EM}$. Replacing the limit values of the multipliers in (12), we obtain:

$$\begin{aligned}
q_{IB} &= \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_I - \delta_B)\right], \\
q_{IM} &= \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_I - \delta_M) - \frac{(\lambda\alpha_{EB} + \lambda\alpha_{IB})\,\Delta\delta + (\lambda\alpha_{EB} + \lambda\alpha_{EM})\Delta\beta}{\alpha_{IM}}\right], \\
c_{EM} &= \frac{1}{1-\lambda}\left[-2 + 2\beta_E - (1+\lambda)\delta_M\right], \\
c_{IM} &= \frac{1}{1-\lambda}\left[-2 + 2\beta_I - (1+\lambda)\delta_M + \frac{(\lambda\alpha_{EB} + \lambda\alpha_{IB})\,\Delta\delta + (\lambda\alpha_{EB} + \lambda\alpha_{EM})\frac{2\Delta\beta}{1+\lambda}}{\alpha_{IM}}\right].
\end{aligned}$$

It is clear from the expressions above that $q_{IB} \geq q_{IM}$ and $c_{IM} \geq c_{EM}$. □

6.3 Case B

In Case B, the incentive compatibility constraints (4) can be written as:

$$\begin{aligned}
U_{IM} &= 0, && \text{(14a)} \\
\omega(q_{IM} - q_{EM}) + c_{IM} - c_{IB} &= 0, && \text{(14b)} \\
U_{EB} &= \Delta\delta q_{IM} - \Delta\beta c_{IB} + k, && \text{(14c)} \\
c_{IM} - c_{IB} &\geq 0, && \text{(14d)} \\
U_{IB} &= \Delta\delta q_{IM}, && \text{(14e)} \\
U_{EM} &= -\Delta\beta c_{IM} + k, && \text{(14f)} \\
\omega(q_{IM} - q_{EM}) + c_{IM} - c_{EM} &\geq 0, && \text{(14g)} \\
\omega(q_{IB} - q_{IM}) + c_{IB} - c_{IM} &\geq 0, && \text{(14h)}
\end{aligned}$$

Since $\gamma_3 = \gamma_6 = \gamma_7 = 0$, from (7), the solution is of the form:

$$\begin{aligned}
q_{EB} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_E-\delta_B)\right], && (15a)\\
q_{EM} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_E-\delta_M)-\frac{\gamma_1\Delta\delta}{\alpha_{EM}}\right], && (15b)\\
q_{IB} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_I-\delta_B)-\frac{\gamma_2\Delta\beta}{\alpha_{IB}}\right], && (15c)\\
q_{IM} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_I-\delta_M)-\frac{\gamma_4\Delta\delta+\gamma_5\Delta\beta}{\alpha_{IM}}\right], && (15d)\\
c_{EB} &= \frac{1}{1-\lambda}\left[-2+2\beta_E-(1+\lambda)\delta_B\right], && (15e)\\
c_{EM} &= \frac{1}{1-\lambda}\left[-2+2\beta_E-(1+\lambda)\delta_M+\frac{\gamma_1\Delta\delta}{\alpha_{EM}}\right], && (15f)\\
c_{IB} &= \frac{1}{1-\lambda}\left[-2+2\beta_I-(1+\lambda)\delta_B+\frac{\gamma_2\frac{2\Delta\beta}{1+\lambda}}{\alpha_{IB}}\right], && (15g)\\
c_{IM} &= \frac{1}{1-\lambda}\left[-2+2\beta_I-(1+\lambda)\delta_M+\frac{\gamma_4\Delta\delta+\gamma_5\frac{2\Delta\beta}{1+\lambda}}{\alpha_{IM}}\right]. && (15h)
\end{aligned}$$

Proof of Remark 5

From equations (15), $q_{EB} > q_{IB}$ and $c_{EB} < c_{EM}$. Adding (14b) and (14h), we obtain $q_{IB} \geq q_{EM}$. Subtracting (14b) from (14h) yields $c_{EM} \leq c_{IB}$. From (14d), $c_{IB} \leq c_{IM}$. Then, from (14b), $q_{EM} \geq q_{IM}$. □

Proof of Proposition 2

We have to check that the multipliers are positive and that the discarded constraints are satisfied.

(i) When $\omega = 1$, we obtain:

$$\gamma_1 = \lambda\alpha_{EM}\frac{2\alpha_{EB}\alpha_{IM}-(1-\lambda)\alpha_{EM}\alpha_{IB}}{(1+\lambda)\alpha_{IM}\alpha_{IB}+(1-\lambda)\alpha_{EM}\alpha_{IB}+2\alpha_{EM}\alpha_{IM}},$$

which is positive when $\frac{\alpha_{EM}\alpha_{IB}}{\alpha_{EB}\alpha_{IM}} \leq \frac{2}{1-\lambda}$;

$$\gamma_2 = \lambda\alpha_{IB}\frac{(1+\lambda)\alpha_{EB}\alpha_{IM}+(1-\lambda)\alpha_{EM}(\alpha_{EM}+\alpha_{EB})}{2\alpha_{IM}\alpha_{EM}+\alpha_{IB}\left[(1+\lambda)\alpha_{IM}+(1-\lambda)\alpha_{EM}\right]} > 0.$$

From (6), γ_4 and γ_5 are always positive when γ_1 and γ_2 are positive.

(ii) To check that (14g) holds when $\omega = 1$, notice that:

$$\omega(q_{IM} - q_{EM}) + c_{IM} - c_{EM} = \Delta\beta + \Delta\beta \frac{\lambda\alpha_{EM}\left[\alpha_{IB} + \frac{2}{1+\lambda}(\alpha_{EM} + \alpha_{EB})\right]}{2\alpha_{IM}\alpha_{EM} + \alpha_{IB}\left[(1+\lambda)\alpha_{IM} + (1-\lambda)\alpha_{EM}\right]} > 0;$$

(iii) To check that (14h) holds when $\omega = 1$, notice that:

$$\omega(q_{IB} - q_{IM}) + c_{IB} - c_{IM} = \lambda\Delta\beta \frac{\alpha_{EB}\alpha_{IM} - \alpha_{EM}(1 - \alpha_{IM})}{(1-\lambda)\alpha_{EM}\alpha_{IB} + 2\alpha_{EM}\alpha_{IM} + (1+\lambda)\alpha_{IB}\alpha_{IM}}$$

which is positive if and only if $\alpha_{IM} \leq \frac{\alpha_{EM}}{\alpha_{EM}+\alpha_{EB}}$;

(iv) From (14b), condition (14d) is equivalent to positivity of $q_{EM} \geq q_{IM}$. This is equivalent to:

$$\begin{aligned}
&\Delta\beta(1+\lambda) - \gamma_1\frac{\Delta\delta}{\alpha_{EM}} + \gamma_4\frac{\Delta\delta}{\alpha_{IM}} + \gamma_5\frac{\Delta\beta}{\alpha_{IM}} \geq 0 \;\Leftrightarrow \\
&\Delta\beta(1+\lambda) - \gamma_1\frac{\Delta\delta}{\alpha_{EM}} + (\lambda\alpha_{IB} + \lambda\alpha_{EB} - \gamma_1)\frac{\Delta\delta}{\alpha_{IM}} + (\lambda\alpha_{EM} + \gamma_1)\frac{\Delta\beta}{\alpha_{IM}} \geq 0.
\end{aligned}$$

With $\omega = 1$:

$$\begin{aligned}
&(1+\lambda)\alpha_{IM} - \gamma_1\frac{\alpha_{IM}}{\alpha_{EM}} + \lambda\alpha_{IB} + \lambda\alpha_{EB} - \gamma_1 + \lambda\alpha_{EM} + \gamma_1 \geq 0 \;\Leftrightarrow \\
&\alpha_{IM} + \lambda - \gamma_1\frac{\alpha_{IM}}{\alpha_{EM}} \geq 0.
\end{aligned}$$

Replacing the expression of γ_1, we obtain:

$$\begin{aligned}
&\alpha_{IM} + \lambda - \lambda\alpha_{IM}\frac{2\alpha_{EB}\alpha_{IM} - (1-\lambda)\alpha_{EM}\alpha_{IB}}{(1+\lambda)\alpha_{IM}\alpha_{IB} + (1-\lambda)\alpha_{EM}\alpha_{IB} + 2\alpha_{EM}\alpha_{IM}} \geq 0 \;\Leftrightarrow \\
&(\alpha_{IM} + \lambda)\left[(1+\lambda)\alpha_{IB} + (1-\lambda)\frac{\alpha_{EM}\alpha_{IB}}{\alpha_{IM}} + 2\alpha_{EM}\right] - 2\lambda\alpha_{EB}\alpha_{IM} + \lambda(1-\lambda)\alpha_{EM}\alpha_{IB} \geq 0 \;\Leftrightarrow \\
&\frac{1+\lambda}{2}\alpha_{IB} + \frac{1-\lambda^2}{2}\frac{\alpha_{EM}\alpha_{IB}}{\alpha_{IM}} + \alpha_{EM} + \frac{\lambda(1+\lambda)}{2}\frac{\alpha_{IB}}{\alpha_{IM}} + \frac{\lambda(1-\lambda)}{2}\frac{\alpha_{EM}\alpha_{IB}}{\alpha_{IM}^2} + \lambda\frac{\alpha_{EM}}{\alpha_{IM}} \geq \lambda\alpha_{EB}.
\end{aligned}$$

This clearly holds if $\lambda\alpha_{EB} \leq \alpha_{EM} + \frac{1+\lambda}{2}\alpha_{IB}$, which is equivalent to $\alpha_{IM} \leq 1 - \frac{1+3\lambda}{1+\lambda}\alpha_{EB} + \frac{1-\lambda}{1+\lambda}\alpha_{EM}$.

(v) From Remark 3, we only need to check the upward constraints between types that exhibit non-binding downward constraints, i.e., between IM and EB:

$$\begin{aligned} 0 &\geq U_{EB} - \Delta\delta q_{EB} - k + c_{EB}\Delta\beta \iff \\ 0 &\geq (q_{IM} - q_{EB})\omega + c_{EB} - c_{IB}. \end{aligned}$$

From Remark 5, this condition is satisfied. □

6.4 Case C

In Case C, the incentive constraints (4) can be written as:

$$U_{IM} = 0, \tag{16a}$$

$$\omega(q_{IM} - q_{EM}) + c_{IM} - c_{IB} = 0, \tag{16b}$$

$$U_{EB} = \Delta\delta q_{IM} - \Delta\beta c_{IB} + k, \tag{16c}$$

$$c_{IM} - c_{IB} \geq 0, \tag{16d}$$

$$U_{IB} = \Delta\delta q_{IM}, \tag{16e}$$

$$U_{EM} = -\Delta\beta c_{IM} + k, \tag{16f}$$

$$\omega(q_{IM} - q_{EM}) + c_{IM} - c_{EM} \geq 0, \tag{16g}$$

$$\omega(q_{IB} - q_{IM}) + c_{IB} - c_{IM} = 0. \tag{16h}$$

With $\gamma_3 = \gamma_6 = 0$, from (7), the activity levels are given by:

$$\begin{aligned}
q_{EB} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_E-\delta_B)\right], &\text{(17a)}\\
q_{EM} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_E-\delta_M)-\frac{\gamma_1\Delta\delta}{\alpha_{EM}}\right], &\text{(17b)}\\
q_{IB} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_I-\delta_B)-\frac{(\gamma_2+\gamma_7)\Delta\beta-\gamma_7\Delta\delta}{\alpha_{IB}}\right], &\text{(17c)}\\
q_{IM} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_I-\delta_M)-\frac{\gamma_4\Delta\delta+\gamma_5\Delta\beta}{\alpha_{IM}}\right], &\text{(17d)}\\
c_{EB} &= \frac{1}{1-\lambda}\left[-2+2\beta_E-(1+\lambda)\delta_B\right], &\text{(17e)}\\
c_{EM} &= \frac{1}{1-\lambda}\left[-2+2\beta_E-(1+\lambda)\delta_M+\frac{\gamma_1\Delta\delta}{\alpha_{EM}}\right], &\text{(17f)}\\
c_{IB} &= \frac{1}{1-\lambda}\left[-2+2\beta_I-(1+\lambda)\delta_B+\frac{(\gamma_2+\gamma_7)\frac{2\Delta\beta}{1+\lambda}-\gamma_7\Delta\delta}{\alpha_{IB}}\right], &\text{(17g)}\\
c_{IM} &= \frac{1}{1-\lambda}\left[-2+2\beta_I-(1+\lambda)\delta_M+\frac{\gamma_4\Delta\delta+\gamma_5\frac{2\Delta\beta}{1+\lambda}}{\alpha_{IM}}\right]. &\text{(17h)}
\end{aligned}$$

Proof of Remark 6

(i) Since $\gamma_6 = 0$, from (17), $q_{EB} > q_{EM}$ and $c_{EB} < c_{EM}$.

(ii) Adding (16b) and (16h), we obtain $q_{IB} = q_{EM}$. From (17):

$$\begin{aligned}
q_{EM} + c_{EM} &= \beta_E,\\
q_{IB} + c_{IB} &= \frac{1}{1-\lambda}\left[(1-\lambda)\beta_I + \frac{\gamma_2\left(\frac{2}{1+\lambda}-1\right)\Delta\beta}{\alpha_{IB}}\right].
\end{aligned}$$

It is clear that $q_{IB} + c_{IB} > q_{EM} + c_{EM}$, which means that $c_{EM} < c_{IB}$.

(iii) From (16d), $c_{IB} \leq c_{IM}$ and, from (16h), $q_{IB} \geq q_{IM}$. □

Proof of Proposition 3

We must check that, around $\omega = 1$: the obtained multipliers (γ_1, γ_2, γ_4, γ_5 and γ_7) are positive; and the constraints (16d) and (16g) are satisfied.

(i) Using the relations between the multipliers, (6), and the expressions for the activity levels, (17), in the binding incentive constraints, (16b) and (16h), we obtain, with $\omega = 1$:

$$\begin{aligned}
\gamma_1 &= \lambda \frac{\alpha_{EB}\alpha_{EM}}{\alpha_{IB} + \alpha_{EM}} > 0, \\
\gamma_2 &= \lambda \frac{\alpha_{EB}\alpha_{IB}}{\alpha_{IB} + \alpha_{EM}} > 0, \\
\gamma_5 &= \lambda \frac{\alpha_{IM}\left(\alpha_{EB} + \alpha_{EM}\right)}{\alpha_{IB} + \alpha_{IM}} > 0, \\
\gamma_7 &= \lambda \frac{\alpha_{IB}[\alpha_{EM} - \alpha_{IM}(\alpha_{EM} + \alpha_{EB})]}{(\alpha_{IB} + \alpha_{EM})(\alpha_{IB} + \alpha_{IM})}.
\end{aligned}$$

The multiplier γ_7 is positive if and only if $\alpha_{IM} \geq \frac{\alpha_{EM}}{\alpha_{EM}+\alpha_{EB}}$. The multiplier γ_4 is surely positive, as γ_2 and γ_7 are positive.

(ii) The constraint (16d) holds if and only if $q_{EM} - q_{IM} \geq 0$, which, evaluated at $\omega = 1$, equals:

$$\frac{\Delta\beta}{1-\lambda}\left[\frac{-\lambda(1-\alpha_{IM})\alpha_{IM} + \left(\alpha_{EM} + \alpha_{IB}\right)\left(\lambda + \alpha_{IM} + \lambda\alpha_{IM}\right)}{(\alpha_{IB} + \alpha_{EM})\alpha_{IM}}\right].$$

The above expression is positive if and only if $(1-\alpha_{EB}-\alpha_{IM})(\lambda+\alpha_{IM}+\lambda\alpha_{IM}) \geq \lambda\alpha_{IM}(1-\alpha_{IM})$, and this is equivalent to:

$$\alpha_{EB} \leq (1-\alpha_{IM})\frac{\lambda + \alpha_{IM}}{\lambda + \alpha_{IM} + \lambda\alpha_{IM}}.$$

(iv) To check the constraint (16g), observe that, at $\omega = 1$:

$$\omega(q_{IM} - q_{EM}) + c_{IM} - c_{EM} = \Delta\beta\left[\frac{\alpha_{IB} + \alpha_{IM} + \lambda}{(1+\lambda)(\alpha_{IB} + \alpha_{IM})}\right] > 0.$$

(v) Since the activity levels are ranked in the natural way, from Remark 3, we only need to check the only upward constraint that corresponds to a non-binding downward constraint, i.e., IM/EB. This constraint can be written as:

$$c_{IB} - c_{EB} \quad \geq \quad 0,$$

which, by Remark 6, holds. □

6.5 Case D

In Case D, since $\gamma_1 = \gamma_3 = \gamma_7 = 0$, from (6) we obtain:

$$\gamma_2 = \lambda\alpha_{EB}, \tag{18a}$$

$$\gamma_5 - \gamma_6 = \lambda\alpha_{EM}, \tag{18b}$$

$$\gamma_4 + \gamma_6 = \lambda(\alpha_{IB} + \alpha_{EB}). \tag{18c}$$

From (7), the activity levels are given by:

$$q_{EB} = \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_E - \delta_B)\right], \tag{19a}$$

$$q_{EM} = \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_E - \delta_M) - \frac{\gamma_6\Delta\delta - \gamma_6\Delta\beta}{\alpha_{EM}}\right], \tag{19b}$$

$$q_{IB} = \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_I - \delta_B) - \frac{\lambda\alpha_{EB}\Delta\beta}{\alpha_{IB}}\right], \tag{19c}$$

$$q_{IM} = \frac{1}{1-\lambda}\left[2 - (1+\lambda)(\beta_I - \delta_M) - \frac{\gamma_4\Delta\delta + \gamma_5\Delta\beta}{\alpha_{IM}}\right], \tag{19d}$$

$$c_{EB} = \frac{1}{1-\lambda}\left[-2 + 2\beta_E - (1+\lambda)\delta_B\right], \tag{19e}$$

$$c_{EM} = \frac{1}{1-\lambda}\left[-2 + 2\beta_E - (1+\lambda)\delta_M + \frac{\gamma_6\Delta\delta - \gamma_6\frac{2\Delta\beta}{1+\lambda}}{\alpha_{EM}}\right], \tag{19f}$$

$$c_{IB} = \frac{1}{1-\lambda}\left[-2 + 2\beta_I - (1+\lambda)\delta_B + \frac{\lambda\alpha_{EB}\frac{2\Delta\beta}{1+\lambda}}{\alpha_{IB}}\right], \tag{19g}$$

$$c_{IM} = \frac{1}{1-\lambda}\left[-2 + 2\beta_I - (1+\lambda)\delta_M + \frac{\gamma_4\Delta\delta + \gamma_5\frac{2\Delta\beta}{1+\lambda}}{\alpha_{IM}}\right]. \tag{19h}$$

The incentive constraints of the relaxed problem (4) can be written as:

$$\begin{aligned}
U_{IM} &= 0, && \text{(20a)}\\
c_{EM} - c_{IB} &\geq 0, && \text{(20b)}\\
U_{EB} &= \Delta\delta q_{IM} - \Delta\beta c_{IB} + k, && \text{(20c)}\\
c_{IM} - c_{IB} &\geq 0, && \text{(20d)}\\
U_{IB} &= \Delta\delta q_{IM}, && \text{(20e)}\\
U_{EM} &= -\Delta\beta c_{IM} + k, && \text{(20f)}\\
\omega(q_{IM} - q_{EM}) + c_{IM} - c_{EM} &= 0, && \text{(20g)}\\
\omega(q_{IB} - q_{IM}) + c_{IB} - c_{IM} &\geq 0. && \text{(20h)}
\end{aligned}$$

The equality (20g) is the additional relation that, together with equations (18) and (19), allows us to determine the multipliers (γ_4, γ_5 and γ_6) and the activity levels (q_{ij} and e_{ij}).

Proof of Remark 7

From (19), $q_{EM} + c_{EM} - q_{IM} - c_{IM} < -\Delta\beta$. This implies that $\omega(q_{EM} - q_{IM}) + c_{EM} - c_{IM} < -\Delta\beta + (\omega - 1)(q_{EM} - q_{IM})$. From (20g), the left term is null. From Remark 1, a solution of the general problem must be such that $q_{EM} \geq q_{IM}$. Thus, for a solution of type A to be a solution of the general problem, we need $\omega > 1$ and $q_{EM} > q_{IM}$. Then, from (20g), $c_{EM} < c_{IM}$. Subtracting (20b) from (20g), we obtain $c_{IB} \leq c_{EM}$. Adding (20b) and (20h), we obtain $q_{IB} \geq q_{EM}$. From (19), $q_{EB} > q_{IB}$ and $c_{EB} < c_{IB}$. □

Proof of Proposition 4

Using (18), (19) and (20g), it is possible to obtain γ_4, γ_5 and γ_6 as a function of ω (among other parameters). The points below are based on the solution that is obtained.

(i) The expression of γ_6 is a ratio between two second-order polynomials in ω with positive coefficients in ω^2. Thus, γ_6 is strictly positive for ω greater than a critical value ω_{A6}. In fact, $\lim_{\omega\to\infty} \gamma_6 = \frac{\lambda\alpha_{EM}(\alpha_{EB}+\alpha_{IB})}{\alpha_{EM}+\alpha_{IM}}$.

(ii) The expression of γ_4 is also a ratio between two second-order polynomials in ω with positive coefficients in ω^2. Thus, γ_4 is strictly positive for ω greater than a critical value ω_{A4}. It can be computed that $\lim_{\omega\to\infty} \gamma_4 = \frac{\lambda\alpha_{IM}(\alpha_{EB}+\alpha_{IB})}{\alpha_{EM}+\alpha_{IM}}$.

(iii) From (18b), γ_5 is strictly positive when γ_6 is positive.

(iv) Observe that $\lim_{\Delta\beta\to 0}(c_{IB}-c_{EB})=0$. From (19), this implies that, in the limit, $c_{IB}<c_{IM}$ and $c_{IB}<c_{EM}$. The constraints (20b) and (20d) are satisfied.

(v) The constraint (20h) can be written as $\Delta\delta(q_{IB}-q_{IM})+\Delta\beta(c_{IB}-c_{IM})\geq 0$. When $\Delta\beta\to 0$, it is implied by $q_{IB}>q_{IM}$. This clearly holds, from (19), when $\Delta\beta\to 0$.

(vi) It remains to check that the upward constraints are satisfied. Writing, respectively, IM/EB, EM/EB, IB/EB, IM/EM and IM/IB:

$$\begin{aligned}
U_{IM} &\geq U_{EB}-\Delta\delta q_{EB}-k+\Delta\beta c_{EB},\\
U_{EM} &\geq U_{EB}-\Delta\delta q_{EB},\\
U_{IB} &\geq U_{EB}-k+\Delta\beta c_{EB},\\
U_{IM} &\geq U_{EM}-k+\Delta\beta c_{EM},\\
U_{IM} &\geq U_{IB}-\Delta\delta q_{IB}.
\end{aligned}$$

After some manipulation:

$$\begin{aligned}
\omega(q_{EB}-q_{IM})+c_{IB}-c_{EB} &\geq 0,\\
\omega(q_{EB}-q_{EM})+c_{IB}-c_{EM} &\geq 0,\\
c_{IB}-c_{EB} &\geq 0,\\
c_{IM}-c_{EM} &\geq 0,\\
q_{IB}-q_{IM} &\geq 0.
\end{aligned}$$

When $\Delta\beta\to 0$, the first and second of these conditions clearly hold, as they are implied by $q_{EB}>q_{IM}$ and $q_{EB}>q_{EM}$. It is also clear that $c_{IB}\geq c_{EB}$ and that, when $\Delta\beta\to 0$, $q_{IB}>q_{IM}$.

Only the fourth condition remains to be checked. Replacing the expressions of the multipliers in (19), we obtain:

$$c_{IM}-c_{EM}=\frac{\omega^2\Delta\beta(\lambda+\alpha_{IM})}{\alpha_{IM}\left[2+(1+\lambda)(\omega^2-2\omega)\right]},$$

which is positive. $\square$

6.6 Case E

Given that $\gamma_2 = \gamma_3 = \gamma_6 = 0$, the solution in Case E is of the form:

$$
\begin{aligned}
q_{EB} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_E-\delta_B)\right], && (21a)\\
q_{EM} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_E-\delta_M)-\frac{\gamma_1\Delta\delta}{\alpha_{EM}}\right], && (21b)\\
q_{IB} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_I-\delta_B)-\frac{\gamma_7\Delta\beta-\gamma_7\Delta\delta}{\alpha_{IB}}\right], && (21c)\\
q_{IM} &= \frac{1}{1-\lambda}\left[2-(1+\lambda)(\beta_I-\delta_M)-\frac{\gamma_4\Delta\delta+\gamma_5\Delta\beta}{\alpha_{IM}}\right], && (21d)\\
c_{EB} &= \frac{1}{1-\lambda}\left[-2+2\beta_E-(1+\lambda)\delta_B\right], && (21e)\\
c_{EM} &= \frac{1}{1-\lambda}\left[-2+2\beta_E-(1+\lambda)\delta_M+\frac{\gamma_1\Delta\delta}{\alpha_{EM}}\right], && (21f)\\
c_{IB} &= \frac{1}{1-\lambda}\left[-2+2\beta_I-(1+\lambda)\delta_B+\frac{\gamma_7\frac{2\Delta\beta}{1+\lambda}-\gamma_7\Delta\delta}{\alpha_{IB}}\right], && (21g)\\
c_{IM} &= \frac{1}{1-\lambda}\left[-2+2\beta_I-(1+\lambda)\delta_M+\frac{\gamma_4\Delta\delta+\gamma_5\frac{2\Delta\beta}{1+\lambda}}{\alpha_{IM}}\right], && (21h)
\end{aligned}
$$

where:

$$
\begin{aligned}
\gamma_1 &= \lambda\alpha_{EB}, && (22a)\\
\gamma_5+\gamma_7 &= \lambda(\alpha_{EM}+\alpha_{EB}), && (22b)\\
\gamma_4-\gamma_7 &= \lambda\alpha_{IB}. && (22c)
\end{aligned}
$$

In Case E, the incentive constraints can be written as:

$$
\begin{aligned}
U_{IM} &= 0, && \text{(23a)}\\
U_{EB} &= -\Delta\beta c_{IM} + k + \Delta\delta q_{EM}, && \text{(23b)}\\
q_{EM} - q_{IB} &\geq 0, && \text{(23c)}\\
q_{EM} - q_{IM} &\geq 0, && \text{(23d)}\\
U_{IB} &= \Delta\delta q_{IM}, && \text{(23e)}\\
U_{EM} &= -\Delta\beta c_{IM} + k, && \text{(23f)}\\
\omega(q_{IM} - q_{EM}) + c_{IM} - c_{EM} &\geq 0, && \text{(23g)}\\
\omega(q_{IB} - q_{IM}) + c_{IB} - c_{IM} &= 0, && \text{(23h)}
\end{aligned}
$$

Proof of Remark 8

From (21), $q_{EB} > q_{EM}$ and $c_{EB} < c_{EM}$. From (23c), $q_{EM} \geq q_{IB}$. Adding (23g) and (23h), we obtain $\omega(q_{IB} - q_{EM}) + c_{IB} - c_{EM} \geq 0$. From (23c), $q_{IB} - q_{EM} \leq 0$, which implies that $c_{EM} \leq c_{IB}$.

After solving the whole system to obtain the values of multipliers, we find that:

$$
c_{IM} - c_{IB} = \frac{\omega^2 \Delta\beta(\lambda + \alpha_{IM})}{\alpha_{IM}\left[2 + (1+\lambda)(\omega^2 - 2\omega)\right]},
$$

which is positive.

By (23h), $c_{IB} < c_{IM}$ implies that $q_{IB} > q_{IM}$. □

Proof of Proposition 5

Using (21), (22) and (23h), we can obtain the solution as a function of ω and the other parameters. After finding this solution, we observe the following.

(i) When ω is sufficiently small, γ_5 and γ_7 are positive, because:

$$
\lim_{\omega \to 0} \gamma_5 = \lambda \frac{\alpha_{IM}(\alpha_{EB} + \alpha_{EM})}{\alpha_{IB} + \alpha_{IM}} > 0,
$$
$$
\lim_{\omega \to 0} \gamma_7 = \lambda \frac{\alpha_{IB}(\alpha_{EB} + \alpha_{EM})}{\alpha_{IB} + \alpha_{IM}} > 0.
$$

The value of γ_4 is always positive when γ_7 is positive.

(ii) The constraint $q_{EM} - q_{IB} \geq 0$ is equivalent to the non-negativity of a ratio between two polynomials in ω that have positive constant terms. Therefore, for small ω, the ratio is positive. In fact:

$$\lim_{\omega \to 0} (q_{EM} - q_{IB}) = \frac{(\lambda + \alpha_{IB} + \alpha_{IM})\Delta\beta}{(1-\lambda)(\alpha_{IB} + \alpha_{IM})} > 0.$$

(iii) The constraint $q_{EM} - q_{IM} \geq 0$ is also equivalent to the positivity of a ratio between two polynomials in ω that have positive constant terms. Thus, it holds for sufficiently small ω. In fact, we also have:

$$\lim_{\omega \to 0} (q_{EM} - q_{IM}) = \frac{(\lambda + \alpha_{IB} + \alpha_{IM})\Delta\beta}{(1-\lambda)(\alpha_{IB} + \alpha_{IM})} > 0.$$

(iv) The constraint (23g) is equivalent to the positivity of a polynomial in ω that has a positive constant term. It is also satisfied for small ω. In fact:

$$\lim_{\omega \to 0} \left[\omega(q_{IM} - q_{EM}) + c_{IM} - c_{EM}\right] = \frac{2(\lambda + \alpha_{IB} + \alpha_{IM})\Delta\beta}{(1-\lambda^2)(\alpha_{IB} + \alpha_{IM})} > 0.$$

(v) From Remark 3, we only need to check that the upward incentive constraints IB/EB and IM/EB are satisfied. Respectively:

$$\begin{aligned} \omega(q_{IM} - q_{EM}) + c_{IM} - c_{EB} &\geq 0, \\ \omega(q_{EB} - q_{EM}) + c_{IM} - c_{EB} &\geq 0. \end{aligned}$$

From Remark 8, the second is satisfied. The first is implied by condition (23g). □

References

[1] Armstrong, M. and J.C. Rochet, 1999, Multidimensional screening: A user's guide, European Economic Review, 43, 959-979.

[2] Basov, S., 2005, Multidimensional screening, Berlin: Springer Verlag.

[3] Borges, A.P. and J. Correia-da-Silva, 2011, Using cost observation to regulate a manager who has a preference for empire building, The Manchester School, 79, 29-44.

[4] Borges, A.P., J. Correia-da-Silva and D. Laussel, forthcoming, Regulating a manager whose empire-building preferences are private information, Journal of Economics, doi: 10.1007/s00712-012-0325-1.

[5] Donaldson, G., 1984, Managing corporate wealth, New York: Praeger.

[6] Jensen, M.C., 1986, Agency costs of free cash flow, corporate finance and take-overs, American Economic Review, 76 (2), 3323-329.

[7] Jensen, M.C., 1993, The modern industrial revolution, exit, and the failure of internal control systems, Journal of Finance, 48, pp. 831-880.

[8] Laffont, J.J. and J. Tirole, 1986, Using cost observation to regulate firms, Journal of Political Economy 94, 614-641.

[9] Niskanen, W.A., 1971, Bureaucracy and the representative government, Chicago IL: Aldine Press.

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School of Economics and Management, University of Porto
Rua Dr. Roberto Frias | 4200-464 Porto | Portugal
Telephone: +351 225 571 100