

n. 490 April 2013

ISSN: 0870-8541

Impossibility of market division  
with two-sided private information  
about production costs

João Correia-da-Silva <sup>1,2</sup>

<sup>1</sup> FEP-UP, School of Economics and Management, University of Porto

<sup>2</sup> CEF.UP, Research Center in Economics and Finance, University of Porto

# Impossibility of market division with two-sided private information about production costs

João Correia-da-Silva\*

CEF.UP and Faculdade de Economia, Universidade do Porto.

April 8<sup>th</sup>, 2013.

**Abstract.** In a market with several independent cities, two firms with private information about their production costs decide whether to open a store in each city or restrict their activity to some cities. In cities where a single firm opens a store, this firm is a monopolist. In cities where both firms open stores, there is price competition with full revelation of private information. In equilibrium, both firms open stores in all the cities. Tacit collusion to divide the market is impeded because, by restraining from opening additional stores, a firm reveals its inefficiency, which triggers an attack from its rival.

**Keywords:** Collusion, Market division, Two-sided private information, Adverse selection, Compromise game.

**JEL Classification Numbers:** C72, D82, L41.

---

I am grateful to Joana Pinho for her useful comments and suggestions. Financial support from FEDER and Fundação para a Ciência e Tecnologia (research grant PTDC/IIM-ECO/5294/2012) is gratefully acknowledged.

\*joao@fep.up.pt

# 1 Introduction

The aim of this paper is to build a bridge between results on the impossibility of cooperation in games with two-sided adverse selection (Milgrom and Stokey, 1982; Tirole, 1982; Carrillo and Palfrey, 2009, 2011) and the theory of collusion with two-sided private information (Roberts, 1985; Cramton and Palfrey, 1990; Kihlstrom and Vives, 1992; Athey and Bagwell, 2001, 2008; Chakrabarti, 2010; Miller, 2012).

When there is two-sided adverse selection, cooperation may not resist the fact that restraining from competing is interpreted as a sign of weakness. Consider a candidate equilibrium in which firms agree to cooperate if and only if their privately observed strenghts are below a certain threshold. Observing that the rival is willing to cooperate, a firm will know that the strenght of the opponent is below that threshold. As a result, if its strenght is sufficiently close to the threshold, the firm will prefer to compete.<sup>1</sup>

A similar phenomenon may occur when firms have the opportunity to divide the market, having private information about their costs. Consider a finite number of cities where a homogeneous good is demanded. To sell in a city, a firm needs to open a store there. After firms decide in which cities to open stores, there is price competition in the cities where both firms have stores and monopolies in the cities where there is a single store. In this setting, market division consists in firms not opening stores in all the cities.

Under perfect information, if firms have relatively similar production costs, there are mutually acceptable ways to divide the market. Full competition would imply that the low-cost firm captures all the market at a price that is equal to the marginal cost of the high-cost firm (or at the monopoly price, if it is lower). Therefore, both firms would be better off under any market sharing agreement that yields a higher payoff to the low-cost firm and a strictly positive payoff to the high-cost firm. For example: an agreement in which the low-cost firm would be a monopolist in all cities except one, and the high-cost firm would be a monopolist in that single city.

But, if firms have private information about their production costs, a firm's willingness

---

<sup>1</sup>This mechanism, explained in a simple setup by Carrillo and Palfrey (2009, 2011) is related to well-known *no trade* results. See Akerlof (1970), Aumann (1976), Milgrom and Stokey (1982), Tirole (1982), Myerson and Satterthwaite (1983), Chatterjee and Samuelson (1983), Cramton, Gibbons and Klemperer (1987) and Morris (1994).

to divide the market partially reveals its inefficiency. This may lead the rival to be competitive. If the rival remains willing to cooperate, this reveals, to an even greater extent, the inefficiency of the rival. And so on. Until a point is reached at which some firm finds it profitable to trigger a fully competitive scenario. It is this failure to cooperate that is addressed in this paper.

The impossibility result that is obtained crucially depends on the assumption that the marginal cost of a firm can be as high as the market reservation price. This implies that, when both firms have costs that are as high as possible, profits with market division are null (as with full competition). Without this assumption, two-sided private information does not completely rule out the possibility of market sharing agreements. These will still take place whenever both firms have very high production costs.<sup>2</sup>

## 2 The model

Consider a market with two firms,  $i \in \{A, B\}$ , that potentially sell homogeneous goods in two cities,  $j \in \{1, 2\}$ . In each city, demand is  $q_j = 1 - p_j$ . At  $\tau = 0$ , nature draws the marginal costs of the firms,  $c_A$  and  $c_B$ , which are i.i.d. uniformly in the interval  $[0, 1]$ . The actual values of these parameters are private information of each firm. Then, at any  $\tau \in (0, \frac{1}{2})$ , each firm may open a store in one or two cities. These choices are observable. Once a store is open, it cannot be closed. A firm may start by opening a store in one city, and another store later (possibly as a response to the store-openings of the rival). At  $\tau \in [\frac{1}{2}, 1)$ , firms publicly post prices in each of their stores. Firms can always decrease, but never increase, their selling price. In cities with a single store, the firm that owns the store sets the monopoly price.

The consequence of considering an open interval of time during which firms act (together with the irreversibility of store openings and price decreases) implies that firms are always able to respond to the rival's actions. A firm is not able to deviate unilaterally

---

<sup>2</sup>Another crucial assumption is that the probability density over marginal costs is not too decreasing. Otherwise, the posterior probability distribution over the rival's cost would place a low weight on extreme inefficiency, and this would lead sufficiently inefficient firms to prefer market division relatively to full competition.

at the last moment, because there isn't a last moment. Firms' choices, besides being optimal responses, are common knowledge.

In the price-setting stage, firms sequentially undercut the prices set by the other firm until the high-cost firm is not able to decrease its price further. The low-cost firm will, then, satisfy all the demand at a price equal to the marginal cost of the high-cost firm or at the monopoly price.

Since opening stores is costless, the only reason why a firm may not open stores in both cities is to sustain a tacit agreement to divide the market. To open zero stores is a dominated action. We can suppose that firms either open one or two stores. There are two kinds of possibly optimal courses of action: (i) open a single store as long as the rival also opens a single store, and open two stores if the rival opens two stores; (ii) open two stores.

We wish to investigate the impacts of two-sided private information about production costs on the incentives of the firms to tacitly collude by locating in a single city.

We start the analysis by calculating the payoffs of the firms under market division (each firm is a monopolist in one city) and under full competition (both firms open stores in the two cities). The profit of firm  $i$  when it is a monopolist in one city is:

$$\pi_i^m(c_i) = \frac{1}{4}(1 - c_i)^2.$$

When there is competition in the two cities, the profit of firm  $i$  is:

$$\pi_i^c(c_i, c_j) = \begin{cases} \frac{1}{2}(1 - c_i)^2, & \text{if } c_j > \frac{1+c_i}{2} \\ 2(c_j - c_i)(1 - c_j), & \text{if } c_j \in [c_i, \frac{1+c_i}{2}] \\ 0, & \text{if } c_j < c_i. \end{cases}$$

Under complete information, firms agree to divide the market if and only if the low-cost firm has higher profits by being a monopolist in a single city than by competing in both cities. The high-cost firm surely prefers to divide the market.

**Proposition 1.** *Firm  $i$  is better off with market division than with full competition if and only if:  $c_j < \frac{1}{4}[(2 - \sqrt{2}) + (2 + \sqrt{2})c_i]$ .*

*Proof.* See Appendix. □

This means that, with complete information, firms will divide the market whenever their costs are sufficiently similar. But, when firms have private information about their own costs, is it still possible that they refrain from opening stores in the two cities? The answer is no.

**Proposition 2.** *With two-sided private information, firms always open stores in both cities.*

*Proof.* See Appendix. □

This negative result regarding the possibility of market division can be extended to the case in which there is an arbitrary number of cities.

Suppose, now, that there is an arbitrary number of cities,  $n \in \mathbb{N}$ . May there exist an equilibrium in which, when  $c_A \in [c_A^*, 1]$  and  $c_B \in [c_B^*, 1]$ , firm  $A$  opens stores in  $n_A < n$  cities and firm  $B$  opens stores in  $n_B < n$  cities? The answer is, again, no.

**Proposition 3.** *If there is a finite number of cities, with two-sided private information, firms always open stores in all the cities.*

*Proof.* See Appendix. □

If the upper bound of the marginal costs is strictly lower than the reservation price, then market division occurs whenever both firms are sufficiently inefficient. To understand why cooperation becomes possible if  $c_H < 1$ , notice that, when both firms have the maximal marginal costs ( $c_A = c_B = c_H$ ), competitive payoffs are null while the cooperation payoffs are now strictly positive.

**Proposition 4.** *If  $c_H < 1$ , there exists a threshold,  $c^* = 3c_H - 2$ , such that firms divide the market if and only if  $c_A \in [c^*, c_H]$  and  $c_B \in [c^*, c_H]$ .*

*Proof.* See Appendix. □

### 3 Concluding remarks

There is a striking similarity between the mechanisms that generate the collapse of the market for lemons (Akerlof, 1970), the impossibility of agreeing to disagree (Aumann, 1976), the absence of trade based on private information alone (Milgrom and Stokey, 1982), the inefficiency of trade with two-sided private information (Myerson and Satterthwaite, 1983), the inability to cooperate in the compromise game (Carrillo and Palfrey, 2009), and, in the setting of this paper, the non-sustainability of tacit collusive agreements to divide the market.

All these theoretical results are related to the fact that the willingness to accept some kind of agreement reveals information that induces the other party to reject that agreement. To study how mechanisms of this kind operate when firms with two-sided private information are incapable of reaching a collusive agreement, it was considered that firms' actions take place in an open interval of time. This uncommon structure for the strategic interaction rules out unilateral deviations, as there is always time for the rival to respond. Relatively to standard models with instantaneous and simultaneous decisions, this setting seems to favor cooperation. In spite of that, in the model presented in this paper, firms still deviate, being unable to settle on mutually beneficial market-sharing arrangements.

### References

- Akerlof, G.A. (1970): The market for 'lemons': Quality uncertainty and the market mechanism. *Quarterly Journal of Economics*, 84(3), 488-500.
- Athey, S. and K. Bagwell (2001): Optimal collusion with private information. *RAND Journal of Economics*, 32(3), 428-465.
- Athey, S. and K. Bagwell (2008): Collusion with Persistent Cost Shocks. *Econometrica*, 76(3), 493-540.
- Aumann, R.J. (1976): Agreeing to Disagree. *Annals of Statistics*, 4(6), 1236-1239.
- Carrillo, J.D. and T.R. Palfrey (2009): The Compromise Game: Two-Sided Adverse

- Selection in the Laboratory. *American Economic Journal: Microeconomics*, 1(1), 151-81.
- Carrillo, J.D. and T.R. Palfrey (2011): No trade. *Games and Economic Behavior*, 71(1), 66-87.
- Chakrabarti, S.K. (2010): Collusive equilibrium in Cournot oligopolies with unknown costs. *International Economic Review*, 51(4), 1209-1238.
- Chatterjee, K. and W. Samuelson (1983): Bargaining under incomplete information. *Operations Research*, 31(5), 835-851.
- Cramton, P., R. Gibbons and P. Klemperer (1987): Dissolving a partnership efficiently. *Econometrica*, 55(3), 615-632.
- Cramton, P. and T.R. Palfrey (1990): Cartel enforcement with uncertainty about costs. *International Economic Review*, 31(1), 17-47.
- Kihlstrom, R. and X. Vives (1992): Collusion by Asymmetrically Informed Firms. *Journal of Economics & Management Strategy*, 1(2), 371-396.
- Milgrom, P. and N. Stokey (1982): Information, trade and common knowledge. *Journal of Economic Theory*, 26(1), 17-27.
- Miller, D.A. (2012): Robust collusion with private information. *Review of Economic Studies*, 79(2), 778-811.
- Morris, S. (1994): Trade with heterogeneous prior beliefs and asymmetric information. *Econometrica*, 62(6), 1327-1347.
- Myerson, R.B. and M.A. Satterthwaite (1983): Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 29(2), 265-281.
- Roberts, K. (1985): Cartel behaviour and adverse selection. *Journal of Industrial Economics*, 33(4), 401-413.
- Tirole, J. (1982): On the possibility of speculation under rational expectations. *Econometrica*, 50(5), 1163-1181.



# Appendix

## Proof of Proposition 1

It is clear that firm  $i$  prefers to compete if  $c_j > \frac{1+c_i}{2}$  (it becomes a monopolist in both cities instead of a single one) and that it prefers to divide the market if  $c_j < c_i$  (otherwise it has zero profits). When  $c_j \in [c_i, \frac{1+c_i}{2}]$ , there is a threshold,  $c^*$ , such that firm  $i$  prefers to divide the market if and only if  $c_j > c^*$ . This threshold can be calculated as follows:

$$\frac{1}{4}(1 - c_i)^2 > 2(c_j - c_i)(1 - c_j) \Leftrightarrow 8c_j^2 - 8(1 + c_i)c_j + 1 + 6c_i + c_i^2 > 0.$$

Using  $c_j \in [c_i, \frac{1+c_i}{2}]$  to select the relevant root, we obtain:

$$\begin{aligned} c_j &> \frac{1 + c_i}{2} - \frac{1}{2}\sqrt{(1 + c_i)^2 - \frac{1}{2}(1 + 6c_i + c_i^2)} \\ \Leftrightarrow c_j &> \frac{1 + c_i}{2} - \frac{1 - c_i}{2\sqrt{2}} \\ \Leftrightarrow c_j &> \frac{1}{4} \left[ (2 - \sqrt{2}) + (2 + \sqrt{2}) c_i \right]. \end{aligned} \quad \square$$

## Proof of Proposition 2

Consider that firms  $i \in \{A, B\}$  use threshold strategies: firm  $i$  opens a single store (the precise moment at which it does so is irrelevant) if  $c_i \geq c^*$  and if its rival opens a single store or no stores; firm  $i$  opens stores in both cities if  $c_i < c^*$  or if its rival opens stores in both cities.

For this to be an equilibrium, it is necessary that, when  $c_i \geq c^*$ , firm  $i$  perceives a higher expected value (conditionally on  $c_j \geq c^*$ ) with market division relatively to full competition. Formally:

$$\pi_i^m(c_i) \geq \frac{1}{1 - c^*} \int_{c^*}^1 \pi_i^c(c_i, c_j) dc_j, \quad \forall c_i \geq c^*.$$

Replacing the expressions for profits and considering (the critical case)  $c_i = c^*$ , we obtain:

$$\begin{aligned} \frac{(1 - c^*)^2}{4} &\geq \frac{1}{1 - c^*} \int_{c^*}^{\frac{1+c^*}{2}} 2(c_j - c^*)(1 - c_j) dc_j + \frac{1}{1 - c^*} \int_{\frac{1+c^*}{2}}^1 \frac{(1 - c^*)^2}{2} dc_j \Leftrightarrow \\ \frac{(1 - c^*)^2}{4} &\geq \frac{1}{1 - c^*} \int_{c^*}^{\frac{1+c^*}{2}} 2(c_j - c^*)(1 - c_j) dc_j + \frac{(1 - c^*)^2}{4} \Leftrightarrow \\ 0 &\geq \frac{1}{1 - c^*} \int_{c^*}^{\frac{1+c^*}{2}} (c_j - c^*)(1 - c_j) dc_j, \end{aligned}$$

which clearly cannot hold for  $c^* < 1$ . □

### Proof of Proposition 3

Consider threshold strategies as in the proof of Proposition 2, and suppose, w.l.o.g., that  $c_A^* \leq c_B^*$ . If  $c_B^* \geq \frac{1+c_A^*}{2}$ , then firm  $A$  surely deviates in order to become a monopolist in  $n$  cities instead of  $n_A$ . We can consider, therefore, that  $c_B^* < \frac{1+c_A^*}{2}$ .

The ICC condition for firm  $A$  when  $c_A = c_A^*$  is:

$$\begin{aligned} n_A \frac{(1 - c_A^*)^2}{4} &\geq \frac{n}{1 - c_B^*} \int_{c_B^*}^{\frac{1+c_A^*}{2}} (c_B - c_A^*)(1 - c_B) dc_B + \frac{n}{1 - c_B^*} \int_{\frac{1+c_A^*}{2}}^1 \frac{(1 - c_A^*)^2}{4} dc_B \Leftrightarrow \\ \left[ n_A - \frac{n(1 - c_A^*)}{2(1 - c_B^*)} \right] \frac{(1 - c_A^*)^2}{4} &\geq \frac{n}{1 - c_B^*} \int_{c_B^*}^{\frac{1+c_A^*}{2}} (c_B - c_A^*)(1 - c_B) dc_B, \end{aligned}$$

which implies that  $\frac{1-c_A^*}{1-c_B^*} < \frac{2n_A}{n}$ .

Similarly, the ICC condition for firm  $B$  when  $c_B = c_B^*$  is:

$$\begin{aligned} n_B \frac{(1 - c_B^*)^2}{4} &\geq \frac{n}{1 - c_A^*} \int_{c_B^*}^{\frac{1+c_B^*}{2}} (c_A - c_B^*)(1 - c_A) dc_A + \frac{n}{1 - c_A^*} \int_{\frac{1+c_B^*}{2}}^1 \frac{(1 - c_B^*)^2}{4} dc_A \Leftrightarrow \\ \left[ n_B - \frac{n(1 - c_B^*)}{2(1 - c_A^*)} \right] \frac{(1 - c_B^*)^2}{4} &\geq \frac{n}{1 - c_A^*} \int_{c_B^*}^{\frac{1+c_B^*}{2}} (c_A - c_B^*)(1 - c_A) dc_A, \end{aligned}$$

which implies that  $\frac{1-c_A^*}{1-c_B^*} > \frac{n}{2n_B}$ .

The two ICCs imply, therefore, that  $\frac{2n_A}{n} > \frac{n}{2n_B}$ . But this is impossible, as:

$$\frac{2n_A}{n} > \frac{n}{2n_B} \Rightarrow n_A^2 + 2n_A n_B + n_B^2 < 4n_A n_B \Leftrightarrow (n_A - n_B)^2 < 0. \quad \square$$

### Proof of Proposition 4

Suppose that  $c_H < 1$  and that firms choose market division when their costs are above  $c^* = c_H - \epsilon$ , for some  $\epsilon$  that is small enough for  $c_H \leq \frac{1+c^*}{2}$ .

This choice is optimal for firm  $i$ , when  $c_i = c^*$ , if and only if:

$$\begin{aligned} \frac{(1-c^*)^2}{4} &\geq \frac{1}{c_H - c^*} \int_{c^*}^{c_H} (c_j - c^*)(1 - c_j) dc_j \Leftrightarrow \\ (1-c^*)^2 &\geq \frac{4}{c_H - c^*} \int_{c^*}^{c_H} -c_j^2 + (1+c^*)c_j - c^* dc_j \Leftrightarrow \\ 1 + 2c^* + c^{*2} &\geq \frac{4}{3(c_H - c^*)}(c^{*3} - c_H^3) + \frac{2}{(c_H - c^*)}(1+c^*)(c_H^2 - c^{*2}) \Leftrightarrow \\ (1+c_H - \epsilon)^2 &\geq \frac{4}{3}(-3c_H^2 + 3\epsilon c_H - \epsilon^2) + 2(1+c_H - \epsilon)(2c_H - \epsilon) \Leftrightarrow \\ 1 - 2c_H + c_H^2 + \frac{\epsilon^2}{3} &\geq 0. \end{aligned}$$

The above condition is always true. Firms always prefer to cooperate if it is common knowledge that their costs are above any given  $c^*$  such that  $c_H \leq \frac{1+c^*}{2}$ .

The threshold at which firms become indifferent between cooperation and competition must be low enough so that  $c_H > \frac{1+c^*}{2}$ , i.e.,  $c^* < 2c_H - 1$ . It is implicitly defined by:

$$\frac{(1-c^*)^2}{4} \geq \frac{1}{c_H - c^*} \int_{c^*}^{\frac{1+c^*}{2}} (c_j - c^*)(1 - c_j) dc_j + \frac{1}{c_H - c^*} \int_{\frac{1+c^*}{2}}^{c_H} \frac{(1-c^*)^2}{2} dc_j.$$

Simplifying, we obtain:

$$\begin{aligned} (1-c_H) \frac{(1-c^*)^2}{4} &\geq \int_{c^*}^{\frac{1+c^*}{2}} (c_j - c^*)(1 - c_j) dc_j \Leftrightarrow \\ (1-c_H) \frac{(1-c^*)^2}{4} &\geq \frac{1}{12}(1-c^*)^3 \Leftrightarrow \\ c^* &\geq 3c_H - 2. \end{aligned} \quad \square$$

## *Recent FEP Working Papers*

489	Meena Rambocas and João Gama, <i>Marketing Research: The Role of Sentiment Analysis</i> , April 2013
488	Liliana Araújo, Sandra Silva and Aurora A.C. Teixeira, <i>Knowledge Spillovers and Economic Performance of Firms Located in Depressed Areas: Does Geographical Proximity Matter?</i> , March 2013
487	João Correia-da-Silva, Joana Pinho and Hélder Vasconcelos, <i>Cartel Stability and Profits under Different Reactions to Entry in Markets with Growing Demand</i> , March 2013
486	Ana Pinto Borges, Didier Laussel and João Correia-da-Silva, <i>Multidimensional Screening with Complementary Activities: Regulating a Monopolist with Unknown Cost and Unknown Preference for Empire-Building</i> , February 2013
485	Carlos Seixas, António Brandão and Manuel Luís Costa, <i>Policy Choices by an Incumbent: A Case with Down-Up Problem, Bias Beliefs and Retrospective Voting</i> , February 2013
484	Pedro Mazedo Gil, Oscar Afonso and Paulo B. Vasconcelos, <i>Industry Dynamics and Aggregate Stability over Transition</i> , February 2013
483	Márcia Oliveira, Dalila B. M. M. Fontes and Teresa Pereira, <i>Multicriteria Decision Making: A Case Study in the Automobile Industry</i> , February 2013
482	Dina Cunha, Sandra Silva and Aurora A.C. Teixeira, <i>Are Academic Spin-Offs necessarily New Technology-Based firms?</i> , January 2013
481	Hélder Ferreira and Aurora A.C. Teixeira, <i>'Welcome to the experience economy': assessing the influence of customer experience literature through bibliometric analysis</i> , January 2013
480	Ana Carina Araújo and Aurora A.C. Teixeira, <i>Determinants of International Technology Transfer: an Empirical Analysis of the Enterprise Europe Network</i> , January 2013

*Editorial Board* (wps@fep.up.pt)

Download available at: <http://wps.fep.up.pt/wplist.php>

also in <http://ideas.repec.org/PaperSeries.html>

[www.fep.up.pt](http://www.fep.up.pt)

UNDERGRADUATE, MASTER AND PHD PROGRAMMES



School of Economics and Management, University of Porto  
Rua Dr. Roberto Frias | 4200-464 Porto | Portugal  
Telephone: +351 225 571 100