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Joao Correia-da-Silva*

Carlos Hervés-Beloso

* CEMPRE - Centro de Estudos Macroeconómicos e Previsão

U.PORTO

FEP FACULDADE DE ECONOMIA UNIVERSIDADE DO PORTO

Faculdade de Economia da Universidade do Porto
Rua Dr. Roberto Frias, 4200-464 Porto | Tel. 225 571 100
Tel. 225571100 | www.fep.up.pt

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João Correia-da-Silva

Faculdade de Economia. Universidade do Porto. PORTUGAL.

Carlos Hervés-Beloso

RGEA. Facultad de Económicas. Universidad de Vigo. SPAIN.

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Abstract. In economies with uncertain delivery, objects of choice are lists of bundles instead of bundles. Agents trade their endowments for lists, and it is the market that selects one of the bundles in the list for actual delivery. Knowledge of the selection mechanism allows agents to predict the bundle that is to be selected in each state of nature. A small but informed agent is introduced in the economy in order to guarantee existence of a *rational expectations equilibrium.*

Keywords: General equilibrium, Private information, Uncertain delivery, Rational expectations, Options, Default.

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1 Introduction

In economies with uncertain delivery, objects of choice are lists of bundles instead of bundles. Agents buy lists, and it is the market that selects one of the bundles in the list for delivery. The market may be seen as consisting of a group of competitive brokers, who offer lists in exchange for the agent's endowments. Taking the prices of lists as given, agents choose the list that they prefer in their budget set. The brokers, in order to obtain one of the bundles that they promised to deliver, trade among themselves in conditions of perfect information. In this internal market, and an Arrow-Debreu equilibrium is generated.

This internal equilibrium consists of an allocation together with equilibrium prices for the contingent goods. The allocation must give to each broker one of the alternatives defined in the list selected by the agent that the broker deals with. The (internal) equilibrium allocation defines, then, which bundle is delivered to the agent. This bundle should be the cheapest at the equilibrium prices. Otherwise, there would be an opportunity for arbitrage, as the broker could deliver the same list at a lower cost (that is, with profit).

In sum, the agents trade their endowments for a list of bundles. The market delivers to them one of the bundles, the cheapest at the equilibrium prices. We suggest an interpretation, based on the existence of a group of competitive brokers, who trade the endowments of the agents among themselves under perfect information (the internal market)[1]. Naturally, each broker buys the cheapest bundle that allows to keep the contract for uncertain delivery, that is, the cheapest bundle among the alternatives in the list.

In previous papers, we have studied prudent expectations equilibrium (2005) and subjective expectations equilibrium (2006). The meaning of prudent expectations is that agents expected to receive the worst possible bundle in the list. Agents with subjective expectations were more sophisticated. Their beliefs on the prob-

[1]The meaning of perfect information is that in this internal market a state of nature is publicly announced to the brokers, as in Debreu (1959, chapter 7).

abilities of delivery of the different alternatives depended on the prices that they observed, and on the alternatives specified in the list. In this paper, we study the existence of rational expectations equilibrium.

We assume that agents observe prices in the internal market and that, knowing the selection mechanism, they can predict which bundle is to be selected for delivery (the cheapest). That is, agents know that they will receive the cheapest bundle, and, since they can observe prices in the internal market, a simple calculation allows them to predict the market's selection. Another way to justify their ability to predict which alternative is delivered is to consider that the brokers are friendly, and communicate the future selection to the agents. In this setting, the market appears as a friendly, truthful, mechanism. Prudent expectations were associated to the opposite paradigm, as the market was expected to deliver the worst alternative bundle to the agent.

Since, by observing prices, a rational agent can find out what will be the delivered bundle (the cheapest), the agent can find which consumption bundle results from each of the lists. Therefore, instead of choosing among lists, the agent can choose directly among these resulting consumption bundles. The market delivers one of the cheapest alternatives, thus the consumption bundles that result from a list are those that are the cheapest among the alternatives specified in the list. Note that this correspondence depends on the equilibrium prices.

The set of resulting consumption bundles is a subset of the usual finite dimensional euclidian space, that depends on the agent's information structure, and on the equilibrium prices. An informational restriction is, therefore, still present. More precisely, there are endogenous incentive compatibility restrictions that must be satisfied. For a bundle to result from a list, an agent who does not distinguish between states s and t should select x^s and x^t such that $p^s \cdot x^s \leq p^s \cdot x^t$ and $p^t \cdot x^t \leq p^t \cdot x^s$. If these incentive compatibility conditions are not satisfied, the agent will not receive x^s in state s and x^t in state t. An agent with rational expectations chooses among bundles which are incentive compatible in this sense.

2 The model

An economy with uncertain delivery is essentially a differential information economy in which objects of choice are lists of bundles instead of bundles.[2] If an agent buys a list with more than one bundle, the market then selects one of the alternative bundles of the list to be delivered to the agent.

We consider a finite number of agents ($i = 1, ..., n$), a finite number of possible states of nature ($s = 1, ..., \Omega$), a finite number of commodities, ($j = 1, ..., l$). The private information of agent i is represented by a partition of the set of states of nature such that agent i can distinguish states that belong to different sets of the partition P_i. The set of states that agent i does not distinguish from s is denoted $P_i(\omega_s)$.[3] A function that is constant across elements of P_i is said to be P_i-measurable. Consumption of agent i in state s is $x_i^s \in \mathbb{R}_+^l$, and the contingent consumption plan of agent i is $x_i = (x_i^s)_{s\in\Omega} \in \mathbb{R}_+^{\Omega l}$.

A list is a finite set of bundles, indexed by $k = 1, ..., K$.[4] The list selected by agent i for delivery in state s is denoted $\tilde{x}_i^s \in \mathbb{R}_+^{Kl}$, with the k^{th} alternative being $\tilde{x}_i^{sk} \in \mathbb{R}_+^{\Omega l}$. The contingent list plan of agent i is $\tilde{x}_i \in \mathbb{R}_+^{\Omega K l}$.

The economy extends over two time periods. In the first, taking prices as given, agents trade their state-contingent endowments for P_i-measurable vectors of state-contingent lists, $\tilde{x}_i = (\tilde{x}_i^1, \tilde{x}_i^2, ..., \tilde{x}_i^\Omega)$, specifying the bundles that the market may deliver in each state of nature. In the second period, agents receive their information, and consume one of the bundles in the list that corresponds to the state of nature that occurs. If the state of nature is s, agent i receives one of the bundles, $\tilde{x}_i^{sk}$, in the list $\tilde{x}_i^s$.

[2]A comprehensive volume on differential information economies was recently edited by Glycopantis and Yannelis (2005).

[3]This kind of information setting corresponds to what Laffont (1986) calls *fixed information structures without noise*.

[4]If the agents wants to guarantee delivery of a precise bundle, all the alternatives must be equal.

With the delivery of a bundle $x^s \in \mathbb{R}^l_+$ in state s, the market keeps the promise of delivery of any list in $\tilde{X}(x^s)$, defined as:

$$\tilde{X}(x^s) = \{\tilde{x}^s = (\tilde{x}^{s1}, ..., \tilde{x}^{sK}) \in \mathbb{R}^{Kl}_+ : \exists k \ s.t. \ \tilde{x}^{sk} \leq x^s\}.$$

Each agent chooses a P_i-measurable vector of contingent lists, $\tilde{x}_i = (\tilde{x}_i^1, ..., \tilde{x}_i^K)$, so it makes sense to extend this correspondence to the whole set of states of nature. Delivery of $x_i = (x_i^1, ..., x_i^\Omega) \in \mathbb{R}^{\Omega l}_+$ keeps the contract for delivery of any list in $\tilde{X}^\Omega(x)$, defined as:

$$\tilde{X}^\Omega(x_i) = \tilde{X}(x_i^1) \times \tilde{X}(x_i^2) \times ... \times \tilde{X}(x_i^\Omega).$$

Observe that the lists offered for delivery in state s only depend on the primitives commodities that correspond to this state, x^s. A more explicit definition of the same correspondence is:

$$\tilde{X}(x_i^s) = \cup_{k=1}^K \{(\mathbb{R}^l_+)^{k-1} \times [0, x_i^s] \times (\mathbb{R}^l_+)^{K-k}\}.^5$$

The price of list $\tilde{x}$ is assumed to be equal to the price of the cheapest bundle, x, that keeps the contract for the delivery of $\tilde{x}$. In the internal market, the broker must trade the agent's endowments for a bundle, x, such that $\tilde{x} \in \tilde{X}^\Omega(x)$. If $\tilde{p}(\tilde{x}) > p \cdot x$, another broker could offer the list at a lower price. If $\tilde{p}(\tilde{x}) < p \cdot x$, no broker would be willing to make the trade. Therefore, it is enough to determine the prices of the contingent goods (primitives). The prices of lists (derivatives) follow as a consequence.

As usual, prices of the contingent commodities are normalized to the simplex:

$$p \in \Delta^{\Omega l}_+ = \left\{ p \in \mathbb{R}^{\Omega l}_+ : \sum_{s=1}^{\Omega} \sum_{j=1}^{l} p^{sj} = 1 \right\}.$$

The price of a list, $\tilde{p}(\tilde{x}_i)$, is:

$$\tilde{p}^s(\tilde{x}_i^s) = \min_k \{p^s \cdot \tilde{x}_i^{sk}\};$$

[5] In this definition, $[0, x^s]$ denotes the set of bundles y^s such that $0 \leq y^s \leq x^s$. For example, with two alternatives and a single commodity: $x = 1$ implies $\tilde{X}(x) = \{[0,1] \times \mathbb{R}_+\} \cup \{\mathbb{R}_+ \times [0,1]\}$. This formulation makes it clear that $\tilde{X}$ is a continuous correspondence, because it is a finite union of a finite product of continuous correspondences.

$$\tilde{p}(\tilde{x}_i) = \sum_{s=1}^{\Omega} \tilde{p}^s(\tilde{x}_i^s) = \sum_{s=1}^{\Omega} \min_k \{p^s \cdot \tilde{x}_i^{sk}\}.$$

Therefore, the budget restriction faced by agent i is:

$$\tilde{B}_i(e_i, p) = \{\tilde{x}_i \in \mathbb{R}_+^{\Omega K l} : \sum_{s=1}^{\Omega} \min_k \{p^s \cdot \tilde{x}_i^{sk}\} \leq p \cdot e_i\}.$$

In an economy with uncertain delivery, $\mathcal{E} \equiv (e_i, u_i, P_i, q_i)_{i=1}^n$:

- A partition of Ω, P_i, represents the private information of agent i. The set of states that agent i does not distinguish from state s is denoted $P_i(s)$.
- Agents assign subjective probabilities to the different states of nature. To each state s, corresponds a prior probability q_i^s, with $\sum_{s=1}^{\Omega} q_i^s = 1$.
- For each state s, a rational expectations function, $R_i^s(\tilde{x}_i^s, p) : [0, T]^{Kl} \times \Delta^{\Omega l} \rightarrow [0, T]^l$, returns the bundle that agent i expects to receive. This bundle is the cheapest among the alternatives in the list.[6]
- Preferences are the same in undistinguished states, represented by a vector of Von Neumann-Morgenstern (1944) utility functions $u_i^s : \mathbb{R}_+^l \rightarrow \mathbb{R}_+$, which are assumed to be continuous, weakly monotone and strictly concave. The objective function combines beliefs with preferences for consumption: $\tilde{U}_i(\tilde{x}_i, p) = \sum_{s=1}^{\Omega} q_i^s u_i^s(R_i^s(\tilde{x}_i^s, p))$.
- The initial endowments are constant across undistinguished states, and strictly positive: $e_i^s \gg 0$ for all $s = \{1, ..., \Omega\}$.

The problem of agent i is to maximize the expected utility function, restricted to the budget set.

$$\max_{\tilde{x}_i \in \tilde{B}_i(e_i, p)} \tilde{U}_i(\tilde{x}_i, p) = \max_{\tilde{x}_i \in \tilde{B}_i(e_i, p)} \sum_{s=1}^{\Omega} q_i^s \; u_i^s(R_i^s(\tilde{x}_i^s, p)).$$

[6] In case of a tie, an agent can expect to receive the bundle with the highest utility among the cheapest. As a result, it is enough to satisfy the endogenous incentive compatibility in equality.

When several alternatives have the same price, the agent can agree with the broker on which alternative is delivered. Since the agent can calculate which alternative is delivered in each state of nature, the problem of the consumer can be written as a decision among bundles instead of lists, under a set of incentive compatibility conditions:

$$\max_{x_i \in \Phi_i(p) \cap B_i(e_i,p)} U_i(x_i) = \max_{x_i \in \Phi_i(p) \cap B_i(e_i,p)} \sum_{s=1}^{\Omega} q_i^s \ u_i^s(x_i^s).^7$$

A rational expectations equilibrium of the economy with uncertain delivery is a pair, (x^*, p^*), composed by a price system p^* and an allocation $x^* = (x_1^*, ..., x_n^*)$. These are such that, for every agent i:

(1) The bundle x_i^* maximizes expected utility, $U_i(x_i^*)$, in the choice set, $\Phi_i(p^*) \cap B_i(e_i, p^*)$.

(2) The bundles selected for delivery, x_i^*, do not violate the endogenous incentive compatibility restrictions. That is, for all s and $t \in P_i(s)$, $p^{*s} \cdot x_i^{*s} \leq p^{*s} \cdot x_i^{*t}$.

(3) The allocation, x^*, is feasible. That is, $\sum_i x_i^* \leq \sum_i e_i = e_T$.

Taking prices as given, each agent trades its initial endowments, e_i, for a vector of bundles that maximizes expected utility, $U_i(x_i)$, belonging to the endogenous incentive compatible set, $\Phi_i(e_i, p^*)$, and to the budget set.[8]

In the interpretation that we suggested, the selection mechanism is based on an internal market in which a group of perfectly informed brokers trade among themselves. The brokers take the endowments of the agents to an internal market for contingent goods, where they trade among themselves, seeking to buy bundles that satisfy the requirements of the lists that they promised to deliver to the

[7]Remember that the price of the list and the price of the delivered bundle must coincide.

[8]The measurability restriction placed on the vector of state-contingent lists is an informational restriction. The agent's information is such that, if state s occurs, they can only claim the right to receive one of the bundles x_i^t with $t \in P_i(s)$. Possible deliveries from a vector of state-contingent lists, $\tilde{x}_i$, are, then, given by a P_i-measurable vector of lists, with the set of alternatives in each state s being $\tilde{x}_i^s = \cup_{t \in P_i(s)} \{\tilde{x}_i^t\}$.

agents. Brokers should buy the cheapest of the bundles that keep their promises, and, in this case, the price that they pay for these bundles is equal to the price that they charged for the list. These are the bundles that the agents actually receive for consumption, and obviously must constitute a feasible allocation.

3 Non-existence of equilibrium - an example

Consider an economy with two agents, a single good, and three states of nature. Endowments are:

$e_1 = (100, 100, 1)$ and $e_2 = (1, 100, 100)$.

Agents observe only their endowments.

$P_1 = \{\{s_1, s_2\}; \{s_3\}\}$ and $P_2 = \{\{s_1\}; \{s_2, s_3\}\}$.

Consumption must be positive, and a significant level of risk aversion leads agents to trade ex-ante.

$U_i : \mathbb{R}^{\Omega l}_+ \rightarrow \mathbb{R}$;

$U_i(x_i^s) = \sum_s q_i^s \sqrt{x_i^s}$.

The different states occur with objective and publicly known probabilities:

$q = (q^1, q^2, q^3) = (0.45, 0.1, 0.45)$.

With strictly positive prices, agents select equal consumption in states that they do not distinguish.

$x_1 = (x_1^{12}, x_1^{12}, x_1^3)$ and $x_2 = (x_2^1, x_2^{23}, x_2^{23})$.

Since they are at the frontier of their budget sets:

$(p^1 + p^2)x_1^{12} + p^3 x_1^3 = 100(p^1 + p^2) + p^3$;

$$p^1 x_2^1 + (p^2 + p^3) x_2^{23} = p^1 + 100(p^2 + p^3).$$

Adding the two:

$$p^1(x_1^{12} + x_2^1) + p^2(x_1^{12} + x_2^{23}) + p^3(x_1^3 + x_2^{23}) = 101p^1 + 200p^2 + 101p^3.$$

For this to be an equilibrium, the allocation must be feasible:

$$x_1^{12} + x_2^1 \leq 101;$$

$$x_1^{12} + x_2^{23} \leq 200;$$

$$x_1^3 + x_2^{23} \leq 101.$$

The previous relation implies that these conditions are verified in equality:

$$x_1^{12} + x_2^1 = 101;$$

$$x_1^{12} + x_2^{23} = 200;$$

$$x_1^3 + x_2^{23} = 101.$$

Subtracting the first from the second, and the third from the second:

$$x_2^{23} - x_2^1 = 99;$$

$$x_1^{12} - x_1^3 = 99.$$

This implies that consumption are of the form:

$$x_1 = (x_1^{12}, x_1^{12}, x_1^3) = (x_1^3 + 99, x_1^3 + 99, x_1^3);$$

$$x_2 = (x_2^1, x_2^{23}, x_2^{23}) = (x_2^1, x_2^1 + 99, x_2^1 + 99).$$

The only individually rational allocation corresponds to the initial endowments.

$$x_1 = (x_1^{12}, x_1^{12}, x_1^3) = (100, 100, 1);$$

$$x_2 = (x_2^1, x_2^{23}, x_2^{23}) = (1, 100, 100).$$

But are agents maximizing their utility levels?

$$U(x_1) = 0.45 * 10 + 0.1 * 10 + 0.45 * 1 = 5.95;$$

$$U(x_2) = 0.45 * 1 + 0.1 * 10 + 0.45 * 10 = 5.95.$$

Suppose that $p^1 = p^3$. Agent 1 can trade consumption in s_1 for consumption in s_3. But consuming less in s_1 implies that delivery in s_2 will also be of this lower quantity. In any case, the agent can select:

$$x'_1 = (x'^{12}_1, x'^{12}_1, x'^{3}_1) = (81, 81, 20).$$

The corresponding utility level is:

$$U(x'_1) = 0.45 * 9 + 0.1 * 9 + 0.45 * 4.47 = 6.96.$$

In the case with asymmetric prices ($p^1 \neq p^3$), the same trade is even more favorable for one of the agents. We reached a contradiction, implying that there is no equilibrium with strictly positive prices.

With zero prices, an alternative bundle can be big enough to violate feasibility and still be incentive compatible. There aren't, in fact, any incentive compatibility restrictions because there are of the form $0 \cdot x^s \leq 0 \cdot x^t$. The cheapest alternative can violate feasibility and agents are better off increasing further and further their consumption in this state with zero prices. This means that there cannot be a rational expectations equilibrium when all prices are zero in one of the states.

4 A catalyzer

To avoid the existence of zero prices, suppose that there is a perfectly informed agent with a linear utility function. This agent will force the existence of a lower bound in prices. Below a certain price level, the agent selects a consumption level that violates feasibility for the whole economy.

Let the information partition of agent $n+1$ be:

$$P_{n+1} = \{\{s_1\}, \{s_2\}, ..., \{s_\Omega\}\}.$$

The utility function of the speculator is:

$U_{n+1}(x^s_{n+1}) = \sum_{s=1}^{\Omega} q^s_{n+1} v^s_{n+1} \cdot x^s_{n+1} = \sum_{s=1}^{\Omega} q^s_{n+1} \sum_{j=1}^{l} v^s_{n+1,j} x^s_{n+1,j}.$

Its endowments can be arbitrarily small.

This agent will use all its endowments to buy a single contingent good, the one with the highest ratio between utility and cost: $q^s_{n+1} v^s_{n+1,j}/p^s_j$. The quantity that the agent will buy is:

$x^s_{n+1,j} = \frac{p \cdot e_{n+1}}{p^s_j}.$

It is obvious that a sufficiently small p^s_j will induce $x^s_{n+1,j} > e^s_{T,j}$, violating feasibility. The demand of this agent will exceed the endowments of all the agents taken together. The actual effect of this agent is to impose a strictly positive lower bound on equilibrium prices.

5 A sequence of economies

In order to establish existence of equilibrium, we construct a sequence of economies in which the utility functions are the only thing varying along the sequence. The consumption set is $\mathbb{R}^{\Omega l}_+$, bounded above for convenience. Choose T to be greater than the total endowments in the economy of any contingent good, and use it to bound the consumption set:

$X_i = \{x_i \in \mathbb{R}^{\Omega l}_+ : \; x_i << (T, T, ..., T)\} = [0, T]^{\Omega l}.$

Utility penalties are imposed on consumption bundles which are not incentive compatible. The penalty depends on the violation, in value, of the inequalities that guarantee incentive compatibility. In the economy $\mathcal{E}^k$, the utility penalty is equal to:

$U^k_{pen}(x_i, p) = k \max_{t \in P_i(s)} \{p^s \cdot x^s_i - p^s \cdot x^t_i\}.$

Notice that these penalties are never negative (from $s \in P_i(s)$ it follows that the maximum is at least zero), and that they increase towards infinity along the sequence of economies. For sufficiently high k, the penalty will be big enough for a bundle to have less utility than the initial endowments. With this trick we induce incentive compatibility in the limit economy.

In the economy $\mathcal{E}^k$, the utility functions of the agents are:

$$U_i^k(x_i, p) = U_i(x_i) - k \sum_{s=1}^{\Omega} q_i^s \max_{t \in P_i(s)} \{p^s \cdot x_i^s - p^s \cdot x_i^t\} =$$

$$= \sum_{s=1}^{\Omega} \{q_i^s \; u_i^s(x_i^s) - k \max_{t \in P_i(s)} \{p^s \cdot x_i^s - p^s \cdot x_i^t\}\}.$$

It is clear that, for any $k \in \mathbb{N}$, the utility functions are continuous in prices and bundles. The maximum of linear functions is a convex function, and multiplying a convex function by a negative constant $(-k)$ yields a concave function. Thus, the objective function $U_i^k(x_i, p)$ is also concave, and this implies existence of equilibrium. The fact that preferences depend, continuously, on prices is not a problem.[9]

The sequence of economies has a sequence of equilibria, $(x^k, p^k)_{k=1}^{\infty}$, in the compact set $[0, T]^{n\Omega l} \times \Delta^{\Omega l}$.[10] Therefore, there exists a subsequence that converges. For this limit, (x^*, p^*), to be a "Rational Expectations Equilibrium", the following conditions must be satisfied:

(1) Feasibility: $\sum_i x_i^* \leq \sum_i e_i = e_T$;

(2) Budget restriction: $\forall i: \; p^* \cdot x_i^* \leq p^* \cdot e_i$;

(3) Incentive compatibility: $\forall i: \; x_i^* \in \Phi_i(p^*)$;

(4) Maximality: $\forall i: \; x_i \in B(p^*, e_i) \cap \Phi_i(p^*) \Rightarrow U_i(x_i^*) \geq U_i(x_i)$.

[9] With price dependent preferences, it is known that equilibrium exists (Arrow and Hahn, 1971). Economies with price-dependent preferences were recently studied by Balasko (2003). See also our previous paper (2006).

[10] The equilibrium allocation is always in $[0, T]^{n\Omega l}$ because it is feasible.

5.1 Establishing existence of equilibrium

Conditions (1) and (2) are easy consequences of the fact that (x^*, p^*) is the limit of a sequence of equilibria.

(1) The limit allocation, x_i^*, is the limit of a sequence of feasible allocations, therefore it is feasible.[11]

(2) The limit allocation, x_i^*, is the limit of a sequence of allocations in the budget set, therefore it also belongs to the limit budget set.[12]

The limit allocation, x^*, satisfies the incentive compatibility restrictions (3) because if x^* violated one of the restrictions by some value $\mu > 0$, then, for a sufficiently high k, x^k would also violate the same restriction by at least $\mu/2$. That is, for $t \in P^s$:

$p^{s*} \cdot x^{s*} > p^{s*} \cdot x^{t*} + \delta \Rightarrow$

$Rightarrow p^{sk} \cdot x^{sk} > p^{sk} \cdot x^{tk} + \delta/2$, for all $k > k_0$.

Utility among feasible allocations is bounded by $U_i(e_T)$, so we can consider a k that is sufficiently high for $k\mu/2 > U_i(e_T) - U_i(e_i)$. It follows that $U_i^k(x^k) <$

[11] Suppose that feasibility (1) is not satisfied. Let $\sum_i e_i = e_T$, $\sum_i x_i^* = x_T^*$, and for a commodity l, $x_{Tl}^* - e_{Tl} = \epsilon > 0$. For a sufficiently large k, we know that $d(x^*, x^k) < \epsilon$, therefore, $x_{Tl}^k > x_{Tl}^* - \epsilon$. We know that the equilibrium allocations in the sequence are feasible, thus this leads to a contradiction: $x_{Tl}^k - e_{Tl} < 0$.

[12] Suppose that x_i^* does not satisfy agent i's budget restriction (2). Let $\alpha = 3\|e_T\| + 1$, and select $\epsilon > 0$ such that $p^* \cdot x_i^* - p^* \cdot e_i = \alpha\epsilon$. Choosing a sufficiently high k, we can guarantee that $d(x^*, x^k) < \epsilon$ and $d(p^*, p^k) < \epsilon$. With $p^k = p^* + dp$, $x^k = x_i^* + dx_i$, and manipulating:

$(p^* + dp) \cdot (x_i^* + dx_i) - (p^* + dp) \cdot e_i =$

$= p^* \cdot x_i^* - p^* \cdot e_i + p^* \cdot dx_i + dp \cdot x_i^* + dp \cdot dx_i - dp \cdot e_i =$

$= \alpha\epsilon + (p^* + dp) \cdot dx_i + dp \cdot (x_i^* - e_i) >$

$> \alpha\epsilon - \epsilon - \epsilon \cdot 3\|e_T\| = 0.$

This means that x^k does not satisfy the budget restriction with prices p_k, which is a contradiction.

$U_i(x_k) - k\mu/2 < U_i(e_T) - k\mu/2 < U_i(e_i) = U_i^k(e_i)$, which is a contradiction.

The most difficult part of the proof is to verify that the limit, (x^*, p^*), maximizes the utility (4) of the agents in the incentive compatible budget set, $B(p^*, e_i) \cap \Phi_i(p^*)$. The fact that $\Phi_i(p)$ is not lower hemicontinuous could prevent (x^*, p^*) from being optimal. There could be an incentive compatible bundle $x' \in B(p^*, e_i) \cap \Phi_i(p^*)$ that is not even nearly incentive compatible in any of the economies in the sequence. In spite of its low utility level in the sequence, this bundle may be optimal in the original economy, and, therefore, (x^*, p^*) may not be an equilibrium.

The strategy to prove that (4) holds is to pick a point, x' in $B(p^*, e_i) \cap \Phi_i(p^*)$ that is a candidate for an optimum (being preferred to x^*) and find that a neighbor of x' also belongs to $B(p^k, e_i) \cap \Phi_i(p^k)$, for large k. This would contradict that (x^k, p^k) is an equilibrium, because the neighbor of x' would also be preferred to x^k in the economy $\mathcal{E}^k$. It is not possible to do this for any candidate for an optimum. The first part of the proof is to show that, without loss of generality, we pick can candidates of a particular kind, described below.

Observe that if there are parallel prices for delivery in different states, $p^{*s} = ap^{*t}$, we can choose x'_i such that $x'^s_i = x'^t_i$ whenever $t \in P_i(s)$. To see this, notice that, in this case, two of the incentive compatibility inequalities originate an equality:

$$\begin{cases} p^{*s} \cdot x'^s \leq p^{*s} \cdot x'^t \\ p^{*t} \cdot x'^t \leq p^{*t} \cdot x'^s \end{cases} \Leftrightarrow p^{*s} \cdot x'^s = p^{*s} \cdot x'^t.$$

The two consumption vectors cost the same in both states. Preferences are also the same in both states, because they belong to the same element of the agent's partition of information. If $u^s(x'^s) > u^s(x'^t)$, the agent would be better off selecting x'^s for consumption in both states. Thus, we must have $u^s(x'^s) = u^s(x'^t)$. Since preferences are convex, the agent is not worse off consuming the average bundle in both states.

We want to take this a bit further. Whenever there are two symmetric incentive compatibility conditions (as above) satisfied in equality, then an agent can choose a consumption vector only among those which give the same consumption in the two states. Suppose that the following inequalities (note the symmetry) are not strict at x'_i:

$$\begin{cases} p^{*s} \cdot x'^s = p^{*s} \cdot x'^t \\ p^{*t} \cdot x'^t = p^{*t} \cdot x'^s \end{cases}$$

The reasoning is the same as for the case of parallel prices. We must have $u(x'^s) = u(x'^t)$, and a convex combination that gives the same average bundle for consumption in both states is better or at least indifferent.

This means that we can choose $x'^s = x'^t$ whenever two symmetric incentive compatibility conditions are satisfied in equality. This eliminates some inequalities from the original system of incentive compatibility conditions. But we must be careful when choosing a neighbor of x'. The displacements from x' to this neighbor must forcefully be such that $dx'^s = dx'^t$, in order to preserve the redundance of the eliminated inequalities.

We suppose, then, that there exists a $x''_i \in B(p^*, e_i) \cap \Phi_i(p^*)$ such that $U_i(x''_i) > U_i(x^*_i)$, and that it delivers the same in undistinguished states whenever two symmetric incentive compatibility conditions are satisfied in equality. The neighbor $x'_i = (1 - \epsilon_1)x''_i$ also belongs to $B(p^*, e_i) \cap \Phi_i(p^*)$ and has the advantage of belonging to the interior to the budget set.

$U_i(x'_i) = U_i(x^*_i) + v$, with $v > 0$.

From continuity of the utility functions, there exists $\epsilon_2 > 0$ such that:

$x_i \in B(x'_i, \epsilon_2) \Rightarrow U_i(x_i) > U_i(x^k)$, for sufficiently high k.

The bundle x'_i satisfies the incentive compatibility conditions for the equilibrium prices p^*. Those that correspond to an element of the agent's information partition, $A^j_i = \{j1, ..., jJ\}$, are (omitting the subscripts $_i$):

$$\begin{cases} p^{*j1} \cdot x'^{j1} \leq p^{*j1} \cdot x'^{j2}; \\ ... \\ p^{*j1} \cdot x'^{j1} \leq p^{*j1} \cdot x'^{jJ}; \\ p^{*j2} \cdot x'^{j2} \leq p^{*j2} \cdot x'^{j1}; \\ ... \\ p^{*j2} \cdot x'^{j2} \leq p^{*j2} \cdot x'^{jJ}; \\ ... \\ ... \\ p^{*jJ} \cdot x'^{jJ} \leq p^{*jJ} \cdot x'^{j1}; \\ ... \\ p^{*jJ} \cdot x'^{jJ} \leq p^{*jJ} \cdot x'^{jJ-1}. \end{cases} \Leftrightarrow \begin{cases} p^{*j1} \cdot x'^{j2} - p^{*j1} \cdot x'^{j1} = k_{12} \geq 0; \\ ... \\ p^{*j1} \cdot x'^{jJ} - p^{*j1} \cdot x'^{j1} = k_{1J} \geq 0; \\ p^{*j2} \cdot x'^{j1} - p^{*j2} \cdot x'^{j2} = k_{21} \geq 0; \\ ... \\ p^{*j2} \cdot x'^{jJ} - p^{*j2} \cdot x'^{j2} = k_{2J} \geq 0; \\ ... \\ ... \\ p^{*jJ} \cdot x'^{j1} - p^{*jJ} \cdot x'^{jJ} = k_{J1} \geq 0; \\ ... \\ p^{*jJ} \cdot x'^{jJ-1} - p^{*jJ} \cdot x'^{jJ} = k_{J,J-1} \geq 0. \end{cases}$$

Let $d(x, x') < \epsilon_2$. For sufficiently high k, we know that $U_i(x_i) > U_i(x^k)$ and also that $d(p^k, p^*) < \epsilon_2$. Let $dx = x - x'$ and $dp = p^k - p^*$. Manipulating the first condition:

$$p^{*j1} \cdot x'^{j2} - p^{*j1} \cdot x'^{j1} = (p^{kj1} - dp^{j1}) \cdot (x^{j2} - dx^{j2}) - (p^{kj1} - dp^{j1}) \cdot (x^{j1} - dx^{j1}) = k_{12} \Leftrightarrow$$

$\Leftrightarrow p^{kj1} \cdot x^{j2} - p^{kj1} \cdot x^{j1} = k_{12} + p^{kj1} \cdot dx^{j2} + dp^{j1} \cdot (x^{j2} - dx^{j2}) - p^{kj1} \cdot dx^{j1} - dp^{j1} \cdot (x^{j1} - dx^{j1}) \Leftrightarrow$

$$\Leftrightarrow p^{kj1} \cdot x^{j2} - p^{kj1} \cdot x^{j1} > k_{12} - \epsilon_2 - \epsilon_2(\|e_T\| + \epsilon_2) - \epsilon_2 - \epsilon_2(\|e_T\| + \epsilon_2 \Leftrightarrow$$

$$\Leftrightarrow p^{kj1} \cdot x^{j2} - p^{kj1} \cdot x^{j1} > k_{12} - 2\epsilon_2 - 2\epsilon_2(\|e_T\| + \epsilon_2).$$

Let $\epsilon_3 = 2\epsilon_2 + 2\epsilon_2(\|e_T\| + \epsilon_2) > 0$. We have:

$$p^{kj1} \cdot x^{j2} - p^{kj1} \cdot x^{j1} > k_{12} - \epsilon_3.$$

Let k_{min} be minimum over the set of strictly positive k_{ab}. Choosing ϵ_2 small enough to makes $\epsilon_3 < k_{min}$ guarantees that the strict inequalities for $d(x, x') < \epsilon_2$ remain strict under p^k, for large k.

If all inequalities still held, then we would have a contradiction (the equilibrium x^k would not be a maximizer of U_i^k, because x_i would be preferred) and thus there could not exist such x'_i preferred to x^*_i. Our problem reduces to guaranteeing that, with a sufficiently high k, the inequalities which are not strict at (x', p^*) are also satisfied at least in a point of $B(x'_i, \epsilon_2)$, with prices p^k.

Suppose that one of the inequalities is not strict (for example, $k_{12} = 0$). Consider a small neighborhood, in which $d(p^k, p^*) < \frac{\epsilon_2}{5\|e_T\|}$:

$$p^{kj1} \cdot x^{j2} - p^{kj1} \cdot x^{j1} =$$

$$= p^{*j1} \cdot (x'^{j2} + dx^{j2}) + dp^{j1} \cdot (x'^{j2} + dx^{j2}) - p^{*j1} \cdot (x'^{j1} + dx^{j1}) - dp^{j1} \cdot (x'^{j1} + dx^{j1}) =$$

$$= p^{*j1} \cdot dx^{j2} + dp^{j1} \cdot (x'^{j2} + dx^{j2}) - p^{*j1} \cdot dx^{j1} - dp^{j1} \cdot (x'^{j1} + dx^{j1}) >$$

$$> p^{*j1} \cdot dx^{j2} - \tfrac{\epsilon_2}{5\|e_T\|}(2\|e_T\|) - p^{*j1} \cdot dx^{j1} - \tfrac{\epsilon_2}{5\|e_T\|}(2\|e_T\|) >$$

$$> p^{*j1} \cdot dx^{j2} - p^{*j1} \cdot dx^{j1} - \tfrac{4\epsilon_2}{5}.$$

To preserve this single inequality, it would be enough to choose $dx^{j2} = 0$ and $dx^{j1} = -\frac{4\epsilon_2}{5}\frac{p^{*j1}}{\|p^{*j1}\|^2}$. The previous relation would become:

$$p^{kj1} \cdot x^{j2} - p^{kj1} \cdot x^{j1} > 0 + \tfrac{4\epsilon_2}{5} - \tfrac{4\epsilon_2}{5} = 0.$$

This procedure is possible only if $\|p^{*j1}\| > 0$. If limit prices in this state were zero, with $d(p^k, p^*) < \frac{\epsilon_4}{4\|e_T\|}$ the condition would become:

$$p^{kj1} \cdot x^{j2} - p^{kj1} \cdot x^{j1} =$$

$$= p^{*j1} \cdot (x'^{j2} + dx^{j2}) + dp^{j1} \cdot (x'^{j2} + dx^{j2}) - p^{*j1} \cdot (x'^{j1} + dx^{j1}) - dp^{j1} \cdot (x'^{j1} + dx^{j1}) =$$

$$= dp^{j1} \cdot (x'^{j2} + dx^{j2}) - dp^{j1} \cdot (x'^{j1} + dx^{j1}) <$$

$$< -\tfrac{\epsilon_4}{4\|e_T\|}(2\|e_T\|) - \tfrac{\epsilon_4}{4\|e_T\|}(2\|e_T\|) = \epsilon_4.$$

When limit prices are zero, we can only guarantee that the incentive compatibility condition is violated by a small ϵ_4. Unfortunately, this small violation is amplified by a large k. To avoid zero prices in the limit, we included a small informed agent (the catalyst) that induces a lower bound on equilibrium prices in the sequence. Since we have avoided this problem of zero prices, the difficulty now lies in preserving all inequalities simultaneously.

Suppose that the following inequalities (note the symmetry, $k_{12} = k_{21} = 0$) are not strict at x'_i:

$$\begin{cases} p^{*j1} \cdot x'^{j1} = p^{*j1} \cdot x'^{j2} \\ p^{*j2} \cdot x'^{j2} = p^{*j2} \cdot x'^{j1} \end{cases}$$

As explained above, in this case, we have $x'^{j1} = x'^{j2}$, and some inequations become redundant. All inequalities that correspond to prices p^{*j2} parallel to p^{*j1} can be eliminated because they are equivalent to those with prices p^{*j1}. Remember that when choosing a neighbor of x', the displacements must be such that $dx'^{j1} = dx'^{j2}$, preserving the redundance of the eliminated inequalities.

With this procedure we make sure that the relevant system of inequalities does not have any $k_{ab} = k_{ba} = 0$.

$$\begin{cases} p^{*j1} \cdot x'^{j2} - p^{*j1} \cdot x'^{j1} = k_{12}; \\ ... \\ p^{*j1} \cdot x'^{jJ} - p^{*j1} \cdot x'^{jJ} = k_{1J}; \\ p^{*j2} \cdot x'^{j1} - p^{*j2} \cdot x'^{j2} = k_{21}; \\ ... \\ ... \\ p^{*jJ} \cdot x'^{jJ-1} - p^{*jJ} \cdot x'^{jJ} = k_{JJ-1}. \end{cases}$$

Denote $\min_{a,b}(1 - \frac{p^{ja} \cdot p^{jb}}{\|p^{ja}\| \|p^{jb}\|}) \|p^{ja}\| = \gamma > 0$. Since we have a lower bound on prices, this minimum is strictly positive, unless prices were parallel in two states. Keep x sufficiently close to x' in order to preserve the strict inequalities, and maintain the strategy of selecting displacements parallel to prices: $dx^{ja} = -\frac{\epsilon_2}{2} \frac{p^{kja}}{\|p^{kja}\|}$. Consider also a small deviation in prices, such that $d(p^k, p^*) < \epsilon_5 = \frac{\epsilon_2 \gamma}{8\|e_T\|}$.

Given an inequality that is not strict, $k_{12} = 0$:

$$p^{kj1} \cdot x^{j2} - p^{kj1} \cdot x^{j1} =$$

$$= p^{*j1} \cdot (x'^{j2} + dx^{j2}) + dp^{j1} \cdot (x'^{j2} + dx^{j2}) - p^{*j1} \cdot (x'^{j1} + dx^{j1}) - dp^{j1} \cdot (x'^{j1} + dx^{j1}) =$$

$$= p^{*j1} \cdot dx^{j2} + dp^{j1} \cdot (x'^{j2} + dx^{j2}) - p^{*j1} \cdot dx^{j1} - dp^{j1} \cdot (x'^{j1} + dx^{j1}) >$$

$$> p^{*j1} \cdot dx^{j2} - \epsilon_5(\|e_T\| + \epsilon_2) - p^{*j1} \cdot dx^{j1} - \epsilon_5(\|e_T\| + \epsilon_2) >$$

$$> p^{*j1} \cdot dx^{j2} - p^{*j1} \cdot dx^{j1} - 2\epsilon_5(\|e_T\| + \epsilon_2) =$$

$$= -p^{*j1} \cdot \tfrac{\epsilon_2}{2} \tfrac{p^{*j2}}{\|p^{*j2}\|} + p^{*j1} \cdot \tfrac{\epsilon_2}{2} \tfrac{p^{*j1}}{\|p^{*j1}\|} - 4\epsilon_5 \|e_T\| =$$

$$= -\tfrac{\epsilon_2}{2} \tfrac{p^{*j1} \cdot p^{*j2}}{\|p^{*j1}\| \|p^{*j2}\|} \|p^{*j1}\| + \tfrac{\epsilon_2}{2} \tfrac{p^{*j1} \cdot p^{*j1}}{\|p^{*j1}\| \|p^{*j1}\|} \|p^{j1}\| - \tfrac{\epsilon_2}{2}\gamma =$$

$$= \tfrac{\epsilon_2}{2}(1 - \tfrac{p^{*j1} \cdot p^{*j2}}{\|p^{*j1}\| \|p^{*j2}\|}) \|p^{*j1}\| - \tfrac{\epsilon_2}{2}\gamma \geq 0$$

In sum, we have found a displacement dx such that:

$$p^{kj1} \cdot x^{j2} - p^{kj1} \cdot x^{j1} > 0.$$

This consumption bundle, $x_i = x'_i + dx$, prevents x^k from being equilibrium in the economy k of the sequence, which is a contradiction.

Existence of a rational expectations equilibrium is, therefore, guaranteed if there is a lower bound on the prices of the contingent goods. For this bound to appear, we included an arbitrarily small, but perfectly informed trader in the economy.

Appendix: The endogenous incentive compatible set correspondence

The set of bundles satisfying the incentive compatibility restrictions depends on the prevailing prices. Consider the correspondence from prices to the set of incentive compatible bundles:

$$\Phi_i : \Delta_+^{\Omega l} \to \mathbb{R}_+^{\Omega l};$$

$$\Phi_i(p) = \left\{ x \in \mathbb{R}_+^{\Omega l} : \forall \omega_s,\ p^s \cdot x^s = \min_{t \in P_i(s)} \{p^s \cdot x^t\} \right\}.$$

If this correspondence were continuous, we could apply the theorem of existence of social equilibrium, yielding the result we seek: existence of rational expectations equilibrium in economies with uncertain delivery.

In finite dimensional Euclidean spaces, upper hemicontinuity of Φ_i at p_0 means that, given an arbitrary open set, V, containing $\Phi_i(p_0)$, there exists $\delta > 0$ such that for all $p \in B(p_0, \delta)$, we have $\Phi_i(p) \subseteq V$.

The correspondence is closed-valued since all the restrictions are inequalities which are not strict. With a compact range space, that is, in a bounded economy (for example, by the total initial endowments in the economy) a correspondence is upper hemicontinuous if and only if it has closed values. Therefore, Φ_i is upper hemicontinuous.

In finite dimensional Euclidean spaces, lower hemicontinuity of Φ_i at p_0 means that given an arbitrary open set, V, intersecting $\Phi_i(p_0)$, there exists $\delta > 0$ such that for all $p \in B(p_0, \delta)$, the image $\Phi_i(p)$ also intersects V.

The correspondence under study, Φ_i, is not lower hemicontinuous. Below is an example where this type of continuity fails.

Consider an economy with two goods, A and B, and two states of nature, s and t. Let $p_0 = (p_0^s, p_0^t) = (p_0^{As}, p_0^{Bs}; p_0^{At}, p_0^{Bt}) = (\frac{1}{4}, \frac{1}{4}; \frac{1}{4}, \frac{1}{4})$. The bundle $x_0 = (1, 0; 0, 1)$ belongs to the incentive compatible set, since:

$p_0^s \cdot x_0^s \leq p_0^s \cdot x_0^t \Leftrightarrow \frac{1}{4} \leq \frac{1}{4}$, and

$p_0^t \cdot x_0^t \leq p_0^t \cdot x_0^s \Leftrightarrow \frac{1}{4} \leq \frac{1}{4}$.

Delivering $(1, 0)$ in state s and $(0, 1)$ in state t does not violate incentive compatibility because both bundles have the same price in both states.

A small perturbation in prices can make $(0, 1)$ cheaper in state s and $(1, 0)$ cheaper in state t. Consider an open ball around x_0 with radius $0 < \epsilon < \frac{1}{10}$. After a perturbation in prices to $p = (\frac{1}{4} + \delta, \frac{1}{4} - \delta, \frac{1}{4} - \delta, \frac{1}{4} + \delta)$, this ball does not intersect the incentive compatible set.

Suppose that there existed a vector $dx = (\epsilon^{As}, \epsilon^{Bs}, \epsilon^{At}, \epsilon^{Bt})$ such that $x = (1 + \epsilon^{As}, \epsilon^{Bs}; \epsilon^{At}, 1 + \epsilon^{Bt})$ is inside that open ball and belongs to the incentive compatible set:

(1) $(\frac{1}{4} + \delta, \frac{1}{4} - \delta) \cdot (1 + \epsilon^{As}, \epsilon^{Bs}) \leq (\frac{1}{4} + \delta, \frac{1}{4} - \delta) \cdot (\epsilon^{At}, 1 + \epsilon^{Bt}) \Leftrightarrow$

$\Leftrightarrow (\frac{1}{4} + \delta)(1 + \epsilon^{As}) + (\frac{1}{4} - \delta)\epsilon^{Bs} \leq (\frac{1}{4} + \delta)\epsilon^{At} + (\frac{1}{4} - \delta,)(1 + \epsilon^{Bt}) \Leftrightarrow$

$\Leftrightarrow \frac{1}{4} + \frac{1}{4}\epsilon^{As} + \delta + \delta\epsilon^{As} + \frac{1}{4}\epsilon^{Bs} - \delta\epsilon^{Bs} \leq \frac{1}{4}\epsilon^{At} + \delta\epsilon^{At} + \frac{1}{4} + \frac{1}{4}\epsilon^{Bt} - \delta - \delta\epsilon^{Bt} \Leftrightarrow$

$\Leftrightarrow \frac{1}{4}(\epsilon^{As} + \epsilon^{Bs} - \epsilon^{At} - \epsilon^{Bt}) + \delta(\epsilon^{As} - \epsilon^{Bs} - \epsilon^{At} + \epsilon^{Bt}) \leq -2\delta;$

(2) $(\frac{1}{4} - \delta, \frac{1}{4} + \delta) \cdot (\epsilon^{At}, 1 + \epsilon^{Bt}) \leq (\frac{1}{4} - \delta, \frac{1}{4} + \delta) \cdot (1 + \epsilon^{As}, \epsilon^{Bs}) \Leftrightarrow$

$\Leftrightarrow (\frac{1}{4} - \delta)\epsilon^{At} + (\frac{1}{4} + \delta)(1 + \epsilon^{Bt}) \leq (\frac{1}{4} - \delta)(1 + \epsilon^{As} + (\frac{1}{4} + \delta)\epsilon^{Bs}) \Leftrightarrow$

$\Leftrightarrow \frac{1}{4}(\epsilon^{At} + 1 + \epsilon^{Bt} - 1 - \epsilon^{As} - \epsilon^{Bs}) + \delta(-\epsilon^{At} + 1 + 1 + \epsilon^{Bt} + \epsilon^{As} - \epsilon^{Bs} \leq 0 \Leftrightarrow$

$\Leftrightarrow \frac{1}{4}(\epsilon^{At} + \epsilon^{Bt} - \epsilon^{As} - \epsilon^{Bs}) + \delta(-\epsilon^{At} + \epsilon^{Bt} + \epsilon^{As} - \epsilon^{Bs}) \leq -2\delta.$

Adding the two inequalities, we obtain:

$(1 + 2) \quad \delta(\epsilon^{As} - \epsilon^{Bs} - \epsilon^{At} + \epsilon^{Bt}) \leq -2\delta \Leftrightarrow$

$\epsilon^{As} - \epsilon^{Bs} - \epsilon^{At} + \epsilon^{Bt} \leq -2.$

Which is impossible, because $\epsilon^{As} - \epsilon^{Bs} - \epsilon^{At} + \epsilon^{Bt} \geq -4\epsilon > -\frac{4}{10}$.

This is an example of the failure of lower hemicontinuity of the incentive compatible set correspondence. When prices are equal in two states, a small perturbation may induce a significant change in the incentive compatible set.

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