THE CORE-PERIPHERY MODEL
WITH THREE REGIONS

SOFIA B. S. D. CASTRO[1]
JOÃO CORREIA-DA-SILVA[2]
PASCAL MOSSAY[3]

[1] FACULDADE DE ECONOMIA E CENTRO DE MATEMÁTICA
UNIVERSIDADE DO PORTO
[2] CEF.UP, FACULDADE DE ECONOMIA,
UNIVERSIDADE DO PORTO
[3] SCHOOL OF ECONOMICS, HENLEY BUSINESS SCHOOL,
UNIVERSITY OF READING
AND DEPARTAMENTO DE FUNDAMENTOS DEL ANALISIS ECONOMICO,
UNIVERSIDAD DE ALICANTE

U. PORTO

FEP FACULDADE DE ECONOMIA
UNIVERSIDADE DO PORTO

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Sofia B. S. D. Castro

sdcastro@fep.up.pt

Faculdade de Economia and Centro de Matemática. Universidade do Porto.

Rua Dr. Roberto Frias

4200-464 Porto, Portugal.

João Correia-da-Silva

joao@fep.up.pt

CEF.UP and Faculdade de Economia. Universidade do Porto.

Pascal Mossay

p.mossay@henley.reading.ac.uk

School of Economics. Henley Business School. University of Reading.

Departamento de Fundamentos del Análisis Económico. Universidad de Alicante.

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Abstract. We determine the properties of the core-periphery model with 3 regions and compare our results with those of the standard 2-region model. The conditions for the stability of dispersion and concentration are established. Like in the 2-region model, dispersion and concentration can be simultaneously stable. We show that the 2-region (resp. 3-region) model favours the dispersion (resp. concentration) of economic activity. We also exclude the partial agglomeration equilibrium as a possible stable outcome. Furthermore, we provide some results for the n-region model. We show that the stability of concentration of the 2-region model implies that of any model with an even number of regions. Numerical results also suggest that the larger the number of regions, the less stable the dispersion configuration.

1   Introduction

The New Economic Geography literature has emerged from the long-existing need to explain the spatial concentration of economic activity. The literature in the field provides a general equilibrium framework addressing the emergence of economic agglomerations as the result of a trade-off between increasing returns at the firm level and transportation costs related to the shipment of goods.

In this paper, we consider a standard New Economic Geography model involving n regions distributed along a circle. This model corresponds to the racetrack economy as studied by Fujita *et al.* (1999) and can be viewed as the extension of the core-periphery model of Krugman (1991) to the case of a spatial economy with n regions. Like in Krugman's original work, there are two sectors in the economy. While the agricultural sector employs farmers and produces a single homogeneous good under constant returns to scale, the manufacturing sector employs workers and produces differentiated goods which –unlike the agricultural good– are costly to transport across regions.

In the case of a spatial economy with 2 regions, the existence and uniqueness of short-run equilibrium have been established by Mossay (2006). Also, the number and stability of long-run equilibria have been determined by Robert-Nicoud (2005). If transportation costs are low, all the industrial activity locates in one region (concentration equilibrium). On the other hand, if transportation costs are high, the industrial activity gets dispersed equally across regions (dispersion equilibrium).

As stressed by Fujita *et al.* (1999), a theoretical analysis of economic geography must get beyond the 2-location framework. Interesting results in that direction have been obtained by Picard and Tabuchi (forthcoming) in the context of an agglomeration model with quadratic preferences. However, except for the work of Puga (1999), who considered a finite number of equidistant regions, no analytical result regarding the Krugman core-periphery model involving 3 regions or more has been derived so far. The aim of this paper is to contribute to fill this gap.

First we study the 3-region model. We establish the conditions for the stability of the

dispersion and concentration equilibria. As expected and already suggested by the standard core-periphery model, high (resp. low) transport costs favour the stability of the dispersion (resp. concentration) configuration. We prove the existence of a region in the parameter space where the dispersion and concentration configurations are simultaneously stable. This result generalizes the overlap interval already determined in the case of the standard core-periphery model by Robert-Nicoud (2005).

A detailed numerical analysis suggests that the partial concentration configuration –where the economic activity is equally concentrated in 2 of the 3 regions– is always unstable, regardless of the parameter values. Therefore, the model with 3 regions is intrinsically different from that with 2 regions: the dispersion configuration of the standard core-periphery model cannot be sustained as a stable equilibrium when a third location is available. By comparing the results of the 2- and 3-region models, we show that the 2-region (resp. 3-region) model favours the dispersion (resp. concentration) of economic activity.

Second, we obtain further results regarding the n-region model. We provide a simple sufficient condition for the stability of the concentration equilibrium, and show that the stability of concentration of the 2-region model implies that of any model with an even number of regions. Numerical results also suggest that the larger the number of regions, the less stable the dispersion configuration.

In Section 2 we describe the n-region core-periphery model and provide some general results regarding the steady states and the dynamics of the model. We derive the stability analysis of the various spatial configurations emerging in the 3-region model in Section 3. In Section 4 we compare our results with those of the standard core-periphery model. The equilibria emerging in the n-region model are studied in Section 5. Section 6 concludes.

2 The model

2.1 Economic environment

We consider a spatial economy with a finite number of regions, $i \in \{1, 2, ..., n\}$. The distance between any pair of regions is denoted by $d(i, j)$. Regions as evenly distributed along a circle meaning that successive firms are equidistant, see the racetrack economy in Krugman (1993) and Fujita *et al.* (1999). There are two sectors in the economy: the manufacturing sector, which exhibits increasing returns to scale, and the agricultural sector, which has constant returns. Agents at location i and time t enjoy a Cobb-Douglas utility from the two types of goods:

$$U_i(t) = C_M^{\mu}(i, t)C_A^{1-\mu}(i, t), \tag{1}$$

where C_A is the consumption of the agricultural good and C_M is the consumption of the manufactured aggregate, defined by

$$C_M(i, t) = \left[\sum_{j=1}^{n} \int_0^{v(j,t)} c_z(j, i, t)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $v(j, t)$ is the density of manufactured varieties available at location j, $c_z(j, i, t)$ is the consumption of variety z produced at j, and $\sigma > 1$ is the elasticity of substitution among manufactured varieties. From utility maximization, μ is the share of manufactured goods in expenditure.

There are two types of agents: workers and farmers. We normalize the total population of workers to 1, and denote the number of workers in each region i by $\lambda_i(t) \in [0, 1]$, with $\sum_{i=1}^{n} \lambda_i(t) = 1$. The number of farmers at any location i is constant and denoted by A.

Farming is an activity that takes place under constant returns to scale. The agricultural output is:

$$Q_A(i, t) = A. \tag{3}$$

Manufacturing variety z involves a fixed cost and a constant marginal cost. The number

of workers employed in location i at time t to produce $Q_{M,z}(i,t)$ units of variety z is:

$$L_z(i,t) = \alpha + \beta Q_{M,z}(i,t). \tag{4}$$

Transport costs only affect manufactured goods and take Samuelson's iceberg form. More precisely, when the amount Z of some variety is shipped from locations j to i, then the amount X of that variety which is effectively available at location i is given by:

$$X = Ze^{-\tau d(i,j)}, \tag{5}$$

where τ is the transport cost per unit of distance.

There is a continuum of manufacturing firms. Each of them produces a single variety, and faces a demand curve with a constant elasticity σ. This will be confirmed below, see relation (14). The optimal pricing behaviour of any firm at location i and time t is therefore to set the price $p_z(i,t)$ of variety z at a fixed markup over marginal cost:

$$p_z(i,t) = \frac{\sigma}{\sigma - 1}\beta W(i,t), \tag{6}$$

where $W(i,t)$ is the worker wage rate prevailing in region i at time t.

Firms are free to enter into the manufacturing sector, so that their profits are driven to zero. Consequently, their output is given by:

$$Q_{M,z}(i,t) = \frac{\alpha}{\beta}(\sigma - 1). \tag{7}$$

Since all varieties are produced at the same scale, the density $v(i,t)$ of manufactured goods produced at each location is proportional to the density $\lambda_i(t)$ of workers at that location:

$$\lambda_i(t) = \int_0^{v(i,t)} L_z(i,t)dz = \alpha\sigma v(i,t). \tag{8}$$

Total income Y at location i and time t is given by:

$$Y(i,t) = Ap^A + \lambda_i(t)W(i,t), \tag{9}$$

where p^A is the price of the agricultural good.

Workers are not interested in nominal wages but rather in utility levels. To consume at location i one unit of variety z produced at location j, $e^{\tau d(i,j)}$ units must be shipped. The delivery price is, therefore, $p_z(j,t)e^{\tau d(i,j)}$.

The price index of the manufactured aggregate for consumers at location i, denoted by $\Theta(i,t)$, is obtained by computing the minimum cost of purchasing one unit of the manufactured aggregate $C_M(i,t)$:

$$\Theta(i,t) = \left[\sum_{j=1}^{n} \int_0^{v(j,t)} p_z(j,t)^{-(\sigma-1)} e^{-(\sigma-1)\tau d(i,j)} dz \right]^{-\frac{1}{\sigma-1}}. \tag{10}$$

By using the pricing rule (6) and relation (8), $\Theta(i,t)$ may be rewritten as:

$$\Theta(i,t) = \frac{\beta\sigma}{\sigma-1}(\alpha\sigma)^{\frac{1}{\sigma-1}} \left[\sum_{j=1}^{n} \lambda_j(t) W(j,t)^{-(\sigma-1)} e^{-(\sigma-1)\tau d(i,j)} \right]^{-\frac{1}{\sigma-1}} \tag{11}$$

The consumption of variety $z \in [0, v(j,t)]$ produced at j may be expressed for workers and farmers located at i as:

$$\begin{aligned} c_z^w(j,i,t) &= \mu W(i,t) p_z(j,t)^{-\sigma} e^{-(\sigma-1)\tau d(i,j)} \Theta(i,t)^{\sigma-1}; \\ c_z^a(j,i,t) &= \mu p^A p_z(j,t)^{-\sigma} e^{-(\sigma-1)\tau d(i,j)} \Theta(i,t)^{\sigma-1}. \end{aligned} \tag{12}$$

The total demand for variety z produced at j is then obtained by summing up the demand for that variety of all the consumers in the spatial economy:

$$\begin{aligned} Q_{M,z}^D(j,t) &= \sum_{i=1}^{n} [\lambda_i(t) c_z^w(j,i,t) + A c_i^a(j,i,t)] \\ &= \sum_{i=1}^{n} \mu[\lambda_i(t)W(i,t) + Ap^A] p_z(j,t)^{-\sigma} e^{-(\sigma-1)\tau d(i,j)} \Theta(i,t)^{\sigma-1}. \end{aligned} \tag{13}$$

By using the total income expression (9), we get:

$$Q_{M,z}^D(j,t) = \sum_{i=1}^{n} \mu Y(i,t) p_z(j,t)^{-\sigma} e^{-(\sigma-1)\tau d(i,j)} \Theta(i,t)^{\sigma-1}. \tag{14}$$

The market-clearing price for variety z produced at j is obtained by equating the demand $Q_{M,z}^D$ (14) and the supply $Q_{M,z}$ (7) of that variety:

$$p_z(j,t) = \left[\frac{\mu\beta}{\alpha(\sigma-1)} \sum_{i=1}^{n} Y(i,t)\Theta(i,t)^{\sigma-1} e^{-(\sigma-1)\tau d(i,j)} \right]^{\frac{1}{\sigma}}. \tag{15}$$

Because of the pricing rule (6), we get:

$$W(j,t) = \frac{\sigma-1}{\beta\sigma} \left[\frac{\mu\beta}{\alpha(\sigma-1)} \right]^{\frac{1}{\sigma}} \left[\sum_{i=1}^{n} Y(i,t)\Theta(i,t)^{\sigma-1} e^{-(\sigma-1)\tau d(i,j)} \right]^{\frac{1}{\sigma}}. \tag{16}$$

The manufacturing wage $W(j,t)$ is the wage prevailing at location j and time t such that firms at j break even.

The indirect utility $U_i(t)$ of a worker in location i is then obtained through expression (1):

$$\begin{aligned}
U_i(t) &= C_M^\mu(\Theta(i,t), W(i,t)) C_A^{1-\mu}(\Theta(i,t), W(i,t)) \\
&= (\mu W(i,t)/\Theta(i,t))^\mu [(1-\mu)W(i,t)/p^A]^{1-\mu} \\
&= \mu^\mu (1-\mu)^{1-\mu}(p^A)^{-(1-\mu)}\Theta(i,t)^{-\mu}W(i,t). \tag{17}
\end{aligned}$$

The adjustment dynamics postulates that workers migrate to regions where utility is higher:

$$\frac{d\lambda_i(t)}{dt} = k(U_i(t) - \overline{U}(t))\lambda_i(t),$$

where k denotes the adjustment speed and $\overline{U}$ denotes the average utility:

$$\overline{U}(t) = \sum_{i=1}^{n} \lambda_i(t)U_i(t).$$

2.2 Reduced system of equations

In the short-run, each region i is described by the variables $Y_i(t)$, $\theta_i(t)$, $W_i(t)$, and $U_i(t)$ which denote respectively the income level, the manufacturing price index, the nominal wage, and the indirect utility level.

We denote the transportation cost from locations i to j by $T_{i,j} = e^{\tau d(i,j)}$. Economic normalization leads to the following reduced system of equations describing the short-run equilibrium, see Fujita *et al.* (1999) or Mossay (2005). For simplicity of notation, we omit the time variable:

$$\begin{cases} Y_1 = \frac{1-\mu}{n} + \mu \, \lambda_1 \, W_1 \\ Y_2 = \frac{1-\mu}{n} + \mu \, \lambda_2 \, W_2 \\ ... \\ Y_n = \frac{1-\mu}{n} + \mu \, \lambda_n \, W_n \end{cases}$$

$$\begin{cases} \theta_1 = [\lambda_1 W_1^{-(\sigma-1)} + \lambda_2 (W_2 T_{2,1})^{-(\sigma-1)} + ... + \lambda_n (W_n T_{n,1})^{-(\sigma-1)}]^{-\frac{1}{\sigma-1}} \\ \theta_2 = [\lambda_1 (W_1 T_{1,2})^{-(\sigma-1)} + \lambda_2 W_2^{-(\sigma-1)} + ... + \lambda_n (W_n T_{n,2})^{-(\sigma-1)}]^{-\frac{1}{\sigma-1}} \\ ... \\ \theta_n = [\lambda_1 (W_1 T_{n,1})^{-(\sigma-1)} + \lambda_2 (W_2 T_{n,2})^{-(\sigma-1)} + ... + \lambda_n W_n^{-(\sigma-1)}]^{-\frac{1}{\sigma-1}} \end{cases}$$

$$\begin{cases} W_1 = \left[Y_1 \theta_1^{\sigma-1} + Y_2 \left(\frac{\theta_2}{T_{1,2}} \right)^{\sigma-1} + ... + Y_n \left(\frac{\theta_n}{T_{1,n}} \right)^{\sigma-1} \right]^{\frac{1}{\sigma}} \\ W_2 = \left[Y_1 \left(\frac{\theta_1}{T_{2,1}} \right)^{\sigma-1} + Y_2 \theta_2^{\sigma-1} + ... + Y_n \left(\frac{\theta_n}{T_{2,n}} \right)^{\sigma-1} \right]^{\frac{1}{\sigma}} \\ ... \\ W_n = \left[Y_1 \left(\frac{\theta_1}{T_{n,1}} \right)^{\sigma-1} + Y_2 \left(\frac{\theta_2}{T_{n,2}} \right)^{\sigma-1} + ... + Y_n \theta_n^{\sigma-1} \right]^{\frac{1}{\sigma}} \end{cases}$$

$$\begin{cases} U_1 = \theta_1^{-\mu} W_1 \\ U_2 = \theta_2^{-\mu} W_2 \\ ... \\ U_n = \theta_n^{-\mu} W_n. \end{cases}$$

The adjustment dynamics can then be rewritten as

$$\begin{cases} \dot{\lambda}_1 = (U_1 - \bar{U})\lambda_1 \\ \dot{\lambda}_2 = (U_2 - \bar{U})\lambda_2 \\ ... \\ \dot{\lambda}_n = (U_n - \bar{U})\lambda_3 \end{cases}, \tag{18}$$

where $\bar{U} = \lambda_1 U_1 + \lambda_2 U_2 + ... + \lambda_n U_n$.

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2.3 Equilibria and invariants

A simple symmetry argument establishes the existence of equilibria.

Lemma 2.1. *The configurations of dispersion, $(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})$, and concentration, $(1, 0, ..., 0)$ and its permutations, are equilibria.*

Proof. This is obtained by direct substitution in the system of differential equations describing the dynamics (18). □

We have the following invariant.

Lemma 2.2. *The boundary of the simplex is invariant for the dynamics.*

Proof. See Appendix A. □

Since the boundary of the simplex corresponds to a distribution that leaves one of the regions empty, this result asserts that if a region is initially deserted, then it will remain so over time unless there is some exogenous migration to that region.

3 The 3-region core-periphery model

In this Section, the spatial economy consists of 3 identical regions which are equally spaced along a circle. The distance between any two regions is equal to d and the corresponding transportation cost is $T = e^{\tau d}$.

The existing literature has provided numerical simulations of this core-periphery model. They suggest that only 2 spatial equilibria can emerge: the dispersion configuration, where the economic activity gets equally distributed across the 3 regions; and the concentration configuration, where the economic activity agglomerates in a single region, see e.g. Fujita *et al.* (1999). Our purpose is to support these numerical results, by providing further analytical results. We make clear the conditions under which dispersion and concentration

occur. In particular we show that these two equilibrium configurations can coexist in equilibrium and determine the region in the parameter space for which this actually happens. We also provide a numerical argument which clearly suggests that the partial concentration of the economic activity in 2 of the 3 regions can never be stable.

3.1 Equilibria and their stability

Lemma 3.1. *The configurations $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, $(\frac{1}{2}, \frac{1}{2}, 0)$ and $(0, 0, 1)$ are equilibria.*

Proof. Dispersion and concentration are equilibria by Lemma 2.1. The remaining result is obtained by direct substitution in the system of differential equations describing the dynamics (18). □

Note that the dispersion equilibrium is fully symmetric. The remaining two equilibria have partial symmetry: they are invariant by a reflection that swaps the first two regions (coordinates). Studying the stability of the above equilibria, provides the stability of $(\frac{1}{2}, 0, \frac{1}{2})$ and $(0, \frac{1}{2}, \frac{1}{2})$ from that of $(\frac{1}{2}, \frac{1}{2}, 0)$, and of $(1, 0, 0)$ and $(0, 1, 0)$ from that of $(0, 0, 1)$.

The stability of the three types of equilibria depends on the sign of the eigenvalues of the Jacobian matrix of the dynamical system (18). We evaluate them at each of the above equilibria.

Proposition 3.2. *Each of the following equilibrium is stable if its corresponding eigenvalues are negative as follows:*

- $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ *is stable when*

$$-\left(\frac{1 + 2T^{1-\sigma}}{3}\right)^{\frac{\mu}{\sigma-1}} \frac{T^\sigma - T}{\sigma - 1}.$$
$$\cdot \frac{T^\sigma(1-\mu)[(1-\mu)\sigma - 1] - T(-1 + \sigma + \mu^2\sigma - 2\mu + 4\mu\sigma)}{T^{2\sigma}(1-\mu) + T^2(1 + 2\mu + 3\sigma) + T^{1+\sigma}(6\sigma - 2 - \mu)} < 0;$$

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- $(\frac{1}{2}, \frac{1}{2}, 0)$ *is stable when*

$$\left(\frac{1+T^{1-\sigma}}{2}\right)^{\frac{\mu}{\sigma-1}} \frac{T^\sigma - T}{\sigma - 1} \cdot$$

$$\cdot \frac{T(2 + 4\mu - 7\mu\sigma - 2\sigma - 3\sigma\mu^2) + T^\sigma(1-\mu)[2(\sigma-1) - 3\mu\sigma]}{T^2[-2 - 4\mu - \sigma(1-\mu)] - T^{2\sigma}(1-\mu)(2+\sigma) - 2T^{1+\sigma}[5\sigma - 2 + \mu(\sigma-1)]} \quad < \quad 0;$$

and

$$-\left(\frac{1+T^{1-\sigma}}{2}\right)^{\frac{\mu}{\sigma-1}} + 3^{-\frac{1}{\sigma}}T^{-\mu}\left(\frac{T^{2\sigma}(1-\mu) + T^{1+\sigma}(1-\mu) + 2T^2(2+\mu)}{T(T+T^\sigma)}\right)^{\frac{1}{\sigma}} \quad < \quad 0;$$

- $(0, 0, 1)$ *is stable when*

$$-1 + T^{-\mu}\left(\frac{(1+T^{\sigma-1})(1-\mu) + (1+2\mu)T^{1-\sigma}}{3}\right)^{\frac{1}{\sigma}} < 0.$$

3.2 Stable equilibria

First we provide conditions under which dispersion and concentration are stable. Our results are obtained by studying the properties of the eigenvalues derived in Proposition 3.2. As it is usually assumed in the existing literature, we suppose that the "no-black-hole" condition, $\mu < (\sigma - 1)/\sigma$, holds, see Fujita *et al.* (1999).

Proposition 3.3. *The dispersion configuration $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is stable if and only if:*

$$T > T_d^* = \left(\frac{\sigma - 1 + \mu^2\sigma + 2\mu(2\sigma - 1)}{(1-\mu)[(1-\mu)\sigma - 1]}\right)^{\frac{1}{\sigma-1}}.$$

Proof. See Appendix B.

$\square$

This result means that the dispersion configuration is stable for high values of the transportation cost, as anticipated. Note that the "no-black-hole" condition guarantees that the critical value, T_d^*, is positive. If the "no-black-hole" condition were to fail, then dispersion would be unstable regardless of the value of transportation cost T. This latter scenario is not regarded as an interesting situation, see Fujita *et al.* (1999).

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Proposition 3.4. *The concentration configuration* $(0, 0, 1)$ *is stable if*

$$T < T_c^* = \left(\frac{1 + 2\mu}{1 - \mu} \right)^{\frac{1}{\sigma - 1}}.$$

Proof. See Appendix B.

$\square$

This Proposition means that the concentration configuration is stable for low values of the transportation cost, as anticipated. It is important to stress that the above result provides a sufficient stability condition only, meaning that concentration is stable for a wider range of parameter values than that given here.

We now address the possible coexistence of the above equilibria.

Proposition 3.5. *The concentration and dispersion equilibria can be simultaneously stable. This actually occurs for an open subset in the parameter space* (T, σ, μ).

Proof. See Appendix B.

$\square$

This result proves the co-existence of the concentration and dispersion configurations. The region in the parameter space for which this co-existence of equilibria actually occurs is depicted in Figure 1.
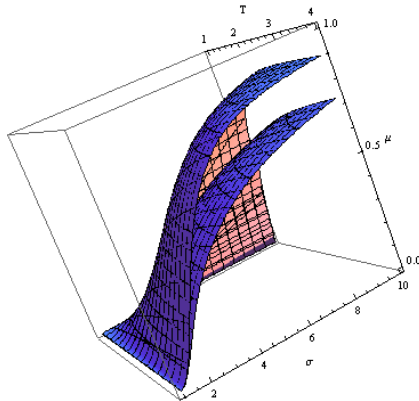
Figure 1: Critical stability surfaces where the dispersion (top surface) and concentration (bottom surface) equilibria change stability. Between the 2 surfaces, concentration and dispersion are simultaneously stable.

The following results about the stability of the partial concentration configuration $(\frac{1}{2}, \frac{1}{2}, 0)$ show that the model with three regions is intrinsically different from that with two regions: the dispersion configuration $(1/2, 1/2)$ cannot be sustained as a stable equilibrium when a third location is available.

Proposition 3.6. *For $\mu > \frac{2(\sigma-1)}{3\sigma}$, partial concentration is never stable.*

Proof. See Appendix B.

$\square$

Even though the expressions of the eigenvalues of the partial concentration provided in Proposition 3.2 prevent us from providing further analytical results, we present in Figure 2 a numerical representation of the surfaces in the parameter space for which these eigenvalues are zero. By a numerical inspection of how these eigenvalues change of sign, these eigenvalues are never simultaneously negative. This numerical argument suggests that the partial concentration equilibrium is never stable. This result supports the simulation results obtained by Fujita *et al.* (1999, Chapter 6).
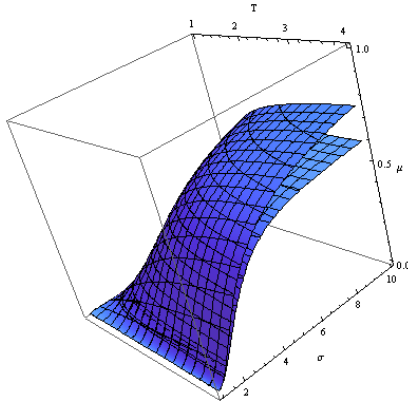
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Figure 2: Surfaces representing the eigenvalues determining the stability of partial concentration $(1/2, 1/2, 0)$. Numerical inspection shows that they are never simultanesously negative (i.e. between the 2 surfaces, both eigenvalues are positive; anywhere else, one eigenvalue is positive while the other one is negative).

4   Comparison between the equilibria of the 2- and 3-region models

In this section, we compare the set of parameters for which concentration and dispersion are simultaneously stable for the two models.

Lemma 4.1. *The stability region of the dispersion equilibrium of the 2-region model contains that of the 3-region model meaning that the 2-region model favours dispersion.*

Proof. See Appendix C. □

In other words, dispersion in an economy with three regions implies dispersion in an economy with only two regions (for the same parameter values). This is illustrated in Figure 3.
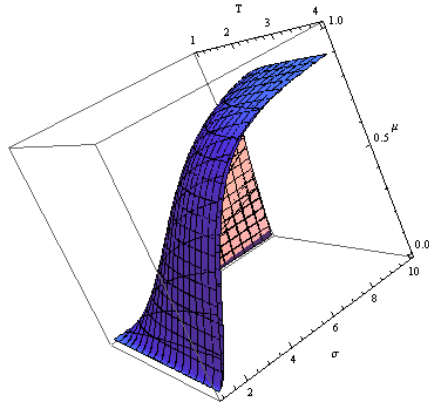
Figure 3: Critical stability surfaces where dispersion changes stability for the 2- and 3- region models. Below the surface, dispersion is stable. The top surface corresponds to the 2-region model.

Lemma 4.2. *The stability region of the concentration equilibrium of the 3-region model contains that of the 2-region model meaning that the 3-region model favours concentration.*

Proof. See Appendix C. □

In other words, concentration in an economy with two regions implies concentration in an economy with three regions. We illustrate this result in Figure 4.
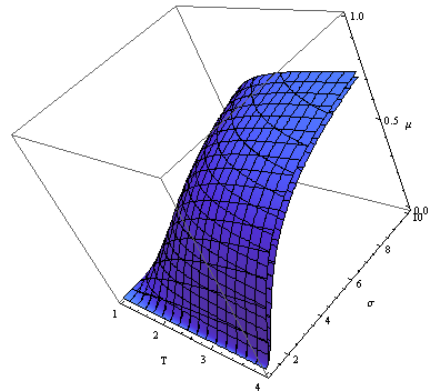

Figure 4: Critical stability surfaces where concentration changes stability for the 2- and 3-region models. Above the surface, concentration is stable. The top surface corresponds to the 2-region model.

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So far, when increasing the number of regions (from 2 to 3), we have been increasing the size of the circumference. However, it is also interesting to consider an alternative scenario where the perimeter of the circumference is kept fixed. To keep the transportation cost of a full lap around the circle constant, say $\mathcal{T}$, we should set the transportation cost T of the 2-region economy to $\sqrt{\mathcal{T}}$, and that of the 3-region economy to $\sqrt[3]{\mathcal{T}}$. In this alternative scenario, both Lemmas 4.1 and 4.2 still hold. This is because now the transportation cost of the 3-region economy has been reduced with respect to the initial scenario. As a consequence, the stability region of the dispersion equilibrium of the 3-region model shrinks, while that of concentration expands, meaning that dispersion may no longer be stable but concentration will surely remain stable.

5 On the n-region Model

The purpose of this section is to get an idea of how the stability of the dispersion and concentration equilibria behaves for different values of n. In order to ease the comparison, we consider n regions evenly distributed along a circumference with fixed perimeter as in the racetrack economy studied by Fujita *et al.* (1999). The transportation cost along a full lap around the circle is $\mathcal{T} = T^n$, where T remains the transportation cost between two adjacent regions, and the transportation cost between regions i and $j > i$ is $T_{i,j} = T^d$, where $d = \min\{j - i, n - j + i\}$.

First we provide results about the dispersion equilibrium of the 4-region model. By determining the analytical expression of the corresponding eigenvalues, we have compared numerically the stability region of the 4-region model with that of the 3-region model. This was done by representing the corresponding critical stability surfaces in the parameter space. It turns out that dispersion in the 4-region model is less stable than in the 3-region model. Because Lemma 4.1 holds (as discussed at the end of section 4), we have $2 \succ_d 3 \succ_d 4$, where the relation $n_1 \succ_d n_2$ means that dispersion in the n_1-region model is more stable than in the n_2-region model, see the illustration in Figure 5.
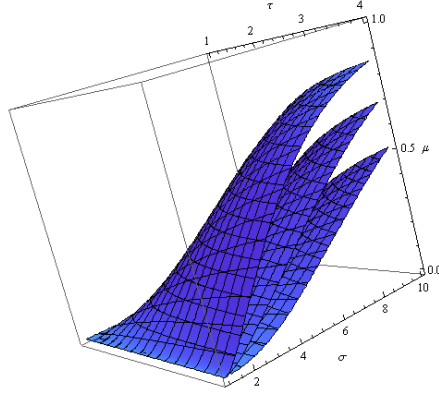
Figure 5: Critical stability surfaces where dispersion changes stability for the 4-, 3-, and 2-region models. Below the surface, dispersion is stable. The top surface corresponds to the 2-region model, the bottom one to the 4-region model.

This tends to suggest that the larger the number of regions in the model, the less stable the dispersion outcome.

Second the following result provides a characterization of the concentration equilibrium.

Proposition 5.1. *In an economy with an even number of regions, if the condition* $(1 + \mu)/2 \ \mathcal{T}^{(1-\sigma-\mu\sigma)/2} + (1-\mu)/2 \ \mathcal{T}^{(\sigma-1-\mu\sigma)/2} \leq 1$ *holds, then the concentration equilibrium* $(1, 0, \ldots, 0)$ *is stable.*

Proof. See Appendix D. □

Note that when $n = 2$, this sufficient condition turns out to be also necessary. This leads to the following Corollary.

Corollary 5.2. *If the concentration equilibrium is stable in the 2-region model, it will remain stable in an economy with any even number of regions.*

We illustrate this result by determining the critical stability surfaces of the concentration equilibrium for $n = 2, 4, 6$, and 8, see the illustration in Figure 6. This confirms that $2 \succ_c 4 \succ_c 6 \succ_c 8$ where the relation $n_1 \succ_c n_2$ means that concentration in the n_1-region model is more stable than in the n_2-model.
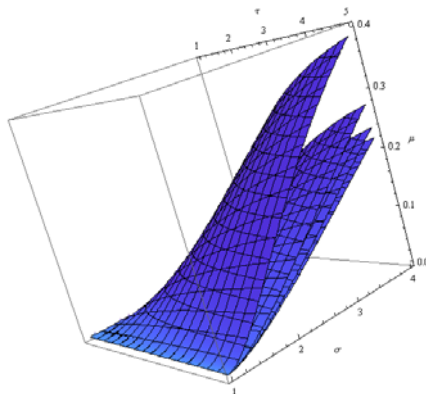
18

Figure 6: Critical stability surfaces of concentration. Above the surface, concentration is stable. From the top to the bottom, surfaces correspond respectively to $n = 2, 4, 6, 8$.

6   Concluding Remarks

We have provided results concerning the core-periphery model with more than two regions. Most results regarding the 3-region model are analytical. We have compared the stable outcomes of the 2- and 3-region models and have established that the 2-region model favours dispersion while the 3-region model favours concentration. The model with three regions is intrinsically different from that with two regions: the dispersion configuration in the standard core-periphery model cannot be sustained as a stable equilibrium when a third location is available. Furthermore, we have derived some results for the core-periphery model with more than 3 regions. The stability of concentration of the 2-region model implies that of any model with an even number of regions. Numerical results also suggest that the larger the number of regions, the less stable the dispersion configuration.

All the results which have been obtained complement those previously obtained by simulation in the existing literature.

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Appendix A

Proof of Lemma 2.2:   The $(n-1)$-dimensional simplex is such that

$$\lambda_1 + \lambda_2 + \ldots + \lambda_n = 1$$

and its boundary satisfies

$$
\begin{aligned}
\lambda_i &= 0 \\
\lambda_1 + \ldots + \lambda_{i-1} + \lambda_{i+1} + \ldots + \lambda_n &= 1.
\end{aligned}
$$

Suppose $\lambda_i = 0$. Then

$$
\begin{aligned}
\dot{\lambda}_1 &= (U_1 - \bar{U})\lambda_1 \\
&\vdots \\
\dot{\lambda}_{i-1} &= (U_{i-1} - \bar{U})\lambda_{i-1} \\
\dot{\lambda}_i &= 0 \\
\dot{\lambda}_{i+1} &= (U_{i+1} - \bar{U})\lambda_{i+1} \\
&\vdots \\
\dot{\lambda}_n &= (U_n - \bar{U})\lambda_n
\end{aligned}
$$

and therefore, the boundary is invariant.

Appendix B

Proof of Proposition 3.3:   Consider the expression of the eigenvalue given in Proposition 3.2. Observe that the eigenvalue is negative if and only if the numerator of the large fraction is positive:

$$T^\sigma(1-\mu)[(1-\mu)\sigma - 1] - T(-1 + \sigma + \mu^2\sigma - 2\mu + 4\mu\sigma) > 0$$

$$\Leftrightarrow T^{\sigma-1}(1-\mu)[(1-\mu)\sigma - 1] > -1 + \sigma + \mu^2\sigma - 2\mu + 4\mu\sigma$$

$$\Leftrightarrow T^{\sigma-1} > \frac{\sigma - 1 + \mu^2\sigma + 2\mu(2\sigma - 1)}{(1-\mu)[(1-\mu)\sigma - 1]}.$$

where we used $(\mu - 1)[1 + (\mu - 1)\sigma] > 0$, given that the "no-black-hole" condition holds.

It remains to show that $T_d^* > 1$, otherwise dispersion would always be stable since $T > 1$. We have

$T_d^* = \left(\frac{\sigma - 1 + \mu^2\sigma + 2\mu(2\sigma - 1)}{(\mu - 1)(1 + (\mu - 1)\sigma)} \right)^{\frac{1}{\sigma - 1}} > 1$

$\Leftrightarrow \sigma - 1 + \mu^2\sigma + 2\mu(2\sigma - 1) > (\mu - 1)(1 + (\mu - 1)\sigma)$

$\Leftrightarrow \sigma - 1 + \mu^2\sigma + 2\mu(2\sigma - 1) > (\mu - 1)(1 + (\mu - 1)\sigma)$

$\Leftrightarrow \mu(6\sigma - 3) > 0$

which holds given that $\sigma > 1$ and $\mu \in (0, 1)$.

Proof of Proposition 3.4: From the expression in Proposition 3.2, concentration is stable if and only if

$$\left[(1 + T^{\sigma - 1})(1 - \mu) + (1 + 2\mu)T^{1 - \sigma} \right]^{\frac{1}{\sigma}} < 3^{1/\sigma}T^{\mu}. \tag{19}$$

Given that $T > 1$, we have $T^{\mu} > 1$ and $3^{1/\sigma}T^{\mu} > 3^{1/\sigma}$. Therefore, a *sufficient* condition for stability is that

$$
\begin{aligned}
\left[(1 + T^{\sigma - 1})(1 - \mu) + (1 + 2\mu)T^{1 - \sigma} \right]^{\frac{1}{\sigma}} &< 3^{1/\sigma} \\
\Leftrightarrow (1 - \mu)T^{\sigma - 1} + (1 - \mu) + (1 + 2\mu)T^{1 - \sigma} &< 3 \\
\Leftrightarrow (1 - \mu)T^{2(\sigma - 1)} + (1 - \mu)T^{\sigma - 1} + 1 + 2\mu &< 3T^{\sigma - 1} \\
\Leftrightarrow (1 - \mu)T^{2(\sigma - 1)} - (2 + \mu)T^{\sigma - 1} + 1 + 2\mu &< 0.
\end{aligned}
$$

By replacing $X = T^{\sigma - 1}$, we have

$$
\begin{aligned}
(1 - \mu)X^2 - (2 + \mu)X + 1 + 2\mu &< 0 \\
\Leftrightarrow X_- < X &< X_+,
\end{aligned}
$$

where X_- and X_+ are the roots of the polynomial in X, given by:

$$X_{\pm} = \frac{2 + \mu \pm 3\mu}{2(1 - \mu)}.$$

Since $X_- = 1$, a sufficient condition for the stability of concentration is that

$$T^{\sigma-1} < \frac{1+2\mu}{1-\mu}.$$

Proof of Proposition 3.5: It turns out that we never have $T_d^* < T_c^*$. This is because the condition for stability of concentration is only sufficient and therefore, too strong.

We prove the statement by showing that for some values of T for which dispersion is stable, the eigenvalue of the Jacobian matrix at concentration is negative.

By replacing T by ηT_d^* in the last condition of Proposition 3.2, where $\eta > 1$ is a real number ensuring that we are considering values of T for which dispersion is stable, we have

$$\left\{ \left[1 + (\eta T_d^*)^{\sigma-1} \right] (1-\mu) + (1+2\mu)(\eta T_d^*)^{1-\sigma} \right\}^{\frac{1}{\sigma}} < 3^{1/\sigma}(\eta T_d^*)^\mu \Leftrightarrow$$
$$(\eta T_d^*)^{\sigma-1} \left[1 + (\eta T_d^*)^{\sigma-1} \right] (1-\mu) + 1 + 2\mu < 3(\eta T_d^*)^{\mu\sigma+\sigma-1}.$$

Both sides of the above inequality are continuous functions of parameters σ, μ and η. So, if the inequality holds for a particular value of (σ, μ, η), it will also hold in an open set containing that particular value.

Choose $\sigma = 2$. The no-black-hole condition then requires that $\mu < 1/2$. Choose $\mu = 1/3$ and $\eta = 2$. It is trivial to check that the inequality holds for these parameter values.

Proof of Proposition 3.6: The first eigenvalue in Proposition 3.2 is negative if and only if the numerator of the large fraction is positive:

$T(2 + 4\mu - 7\mu\sigma - 2\sigma - 3\sigma\mu^2) + T^\sigma(1-\mu)[2(\sigma-1) - 3\mu\sigma] > 0$

$\Leftrightarrow 7\mu\sigma + 2\sigma + 3\sigma\mu^2 - 2 - 4\mu < T^{\sigma-1}(1-\mu)(2\sigma - 2 - 3\mu\sigma)$

For $\mu > \frac{2}{3}\frac{\sigma-1}{\sigma}$ (or equivalently $2\sigma - 2 - 3\mu\sigma < 0$), this is impossible, because the expression on the left-hand side is positive while the expression on the right-hand side is negative.

Appendix C

Proof of Lemma 4.1: Denote the critical value of the 3-region model obtained in Proposition 3.3 by T_{d3}^*. We now turn to the 2-region model. Calculating the eigenvalue of the Jacobian matrix at $(1/2, 1/2)$, we find that dispersion is stable if and only if

$$- \left(2^{\frac{1}{\sigma-1}} (1 + T^{\sigma-1})^{\frac{1}{1-\sigma}} \right)^{-\mu} (T^\sigma - T).$$

$$\cdot \frac{T^\sigma(\mu - 1)(1 + (\mu - 1)\sigma) - T(1 + \mu)(-1 + \sigma + \mu\sigma)}{(\sigma - 1)(-T^{2\sigma}(\mu - 1) + T^2(1 + \mu) + T^{1+\sigma}(4\sigma - 2))} \quad < \quad 0.$$

Simplifying the above expression and taking into account that the expression $- \left(2^{\frac{1}{\sigma-1}} (1 + T^{\sigma-1})^{\frac{1}{1-\sigma}} \right)^{-\mu}$ $(T^\sigma - T)$ has a constant negative sign, we conclude that dispersion is stable if and only if

$$\frac{T^{\sigma-1}(\mu - 1)(1 + (\mu - 1)\sigma) - (1 + \mu)(\sigma - 1 + \mu\sigma)}{(1 - \mu)T^2(\sigma - 1) + 2T^{\sigma-1}(2\sigma - 1) + 1 + \mu} > 0.$$

It is easy to see that the denominator is positive and therefore, the stability of dispersion takes place when

$$0 \quad < \quad T^{\sigma-1}(\mu - 1)(1 + (\mu - 1)\sigma) - (1 + \mu)(\sigma - 1 + \mu\sigma) \Leftrightarrow$$

$$T^{\sigma-1} \quad > \quad \frac{(1 + \mu)(\sigma - 1 + \mu\sigma)}{(\mu - 1)(1 + (\mu - 1)\sigma)} = (T_{d2}^*)^{\sigma-1}.$$

We conclude the proof by showing that $T_{d3}^* > T_{d2}^*$. Because the denominators are equal, we have

$$T_{d3}^* \quad > \quad T_{d2}^* \Leftrightarrow$$

$$\sigma - 1 + \mu^2\sigma + 2\mu(2\sigma - 1) \quad > \quad (1 + \mu)(\sigma - 1 + \mu\sigma) \Leftrightarrow$$

$$2\mu\sigma - \mu \quad > \quad 0,$$

which is always the case.

Proof of Lemma 4.2: In the 2-region model, concentration is stable if and only if

$$f_2(T) = -1 + T^{-\mu} \left(\frac{T^{\sigma-1}(1 - \mu) + T^{1-\sigma}(1 + \mu)}{2} \right)^{1/\sigma} < 0.$$

We have seen in Proposition 3.2 that concentration in the 3-region model is stable if and only if

$$f_3(T) = -1 + T^{-\mu} \left(\frac{(1+T^{\sigma-1})(1-\mu) + (1+2\mu)T^{1-\sigma}}{3} \right)^{1/\sigma} < 0.$$

We conclude the proof by showing that $f_3(T) < f_2(T)$. Given the above expressions, we have to check that

$$
\begin{aligned}
T^{-\mu} \left(\frac{(1+T^{\sigma-1})(1-\mu) + (1+2\mu)T^{1-\sigma}}{3} \right)^{1/\sigma} &< T^{-\mu} \left(\frac{T^{\sigma-1}(1-\mu) + T^{1-\sigma}(1+\mu)}{2} \right)^{1/\sigma} \Leftrightarrow \\
\frac{(1+T^{\sigma-1})(1-\mu) + (1+2\mu)T^{1-\sigma}}{3} &< \frac{T^{\sigma-1}(1-\mu) + T^{1-\sigma}(1+\mu)}{2} \Leftrightarrow \\
T^{2(\sigma-1)} - 2T^{\sigma-1} + 1 &> 0 \Leftrightarrow \\
(T^{\sigma-1} - 1)^2 &> 0,
\end{aligned}
$$

which is always the case.

Appendix D

Proof of Proposition 5.1: We start by characterizing the concentration equilibrium in region 1 ($\lambda_1 = 1$).

From the reduced system of equations in Section 2.2, it is straightforward to show that $W_1 = 1$ and, then, to obtain the remaining variables.

The incomes of the different regions are:

$$
\begin{cases}
Y_1 = \frac{1-\mu}{n} + \mu \\
Y_2 = \frac{1-\mu}{n} \\
\ldots \\
Y_n = \frac{1-\mu}{n}.
\end{cases}
$$

The manufacturing price indexes are:

$$
\begin{cases}
\theta_1 = [W_1^{-(\sigma-1)}]^{-\frac{1}{(\sigma-1)}} = 1 \\
\theta_2 = [(W_1 T_{1,2})^{-(\sigma-1)}]^{-\frac{1}{(\sigma-1)}} = T_{1,2} \\
... \\
\theta_n = [(W_1 T_{1,n})^{-(\sigma-1)}]^{-\frac{1}{(\sigma-1)}} = T_{1,n}.
\end{cases}
$$

The utility of workers in each region is given by:

$$
\begin{cases}
U_1 = 1 \\
U_2 = T_{1,2}^{-\mu} W_2 \\
... \\
U_n = T_{1,n}^{-\mu} W_n.
\end{cases}
$$

The nominal wages in each region are:

$$
\begin{cases}
W_1 = 1 \\
W_2 = \left[Y_1 \left(\frac{1}{T_{2,1}} \right)^{\sigma-1} + Y_2 T_{1,2}^{\sigma-1} + Y_3 \left(\frac{T_{1,3}}{T_{2,3}} \right)^{\sigma-1} + ... + Y_n \left(\frac{T_{1,n}}{T_{2,n}} \right)^{\sigma-1} \right]^{1/\sigma} \\
... \\
W_n = \left[Y_1 \left(\frac{1}{T_{n,1}} \right)^{\sigma-1} + Y_2 \left(\frac{T_{1,2}}{T_{n,2}} \right)^{\sigma-1} + ... + Y_n T_{1,n}^{\sigma-1} \right]^{1/\sigma}.
\end{cases}
$$

Is it possible that workers prefer to migrate from region 1 to another region? To address that issue, consider region $d+1$, which is d steps away from region 1. Without loss of generality, let $1 \leq d \leq n/2$. Wages are given by

$$
\begin{aligned}
W_{d+1}^{\sigma} &= Y_1 \left(\frac{1}{T_{d+1,1}} \right)^{\sigma-1} + Y_2 \left(\frac{T_{1,2}}{T_{d+1,2}} \right)^{\sigma-1} + ... + Y_{d+1} T_{1,d+1}^{\sigma-1} + ... + Y_n \left(\frac{T_{1,n}}{T_{d+1,n}} \right)^{\sigma-1} \\
&= Y_1 \mathcal{T}^{\frac{1-\sigma}{n}d} + \frac{1-\mu}{n} \mathcal{T}^{\frac{\sigma-1}{n}d} + \frac{1-\mu}{n} \sum_{j \notin \{1,d+1\}} \left(\frac{T_{1,j}}{T_{d+1,j}} \right)^{\sigma-1}.
\end{aligned}
$$

In the above expression, each region's specific term depends only on the difference between

the distances to region 1 and to region $d+1$. By a careful evaluation[1], we have

$$
\sum_{j\notin\{1,d+1\}}\left(\frac{T_{1,j}}{T_{d+1,j}}\right)^{\sigma-1}=\sum_{j=2}^{d}\left(\frac{\mathcal{T}^{(j-1)/n}}{\mathcal{T}^{(d+1-j)/n}}\right)^{\sigma-1}+\sum_{j=d+2}^{n/2+1}\left(\frac{\mathcal{T}^{(j-1)/n}}{\mathcal{T}^{(j-d-1)/n}}\right)^{\sigma-1}+
$$

$$
+\sum_{j=n/2+2}^{d+n/2}\left(\frac{\mathcal{T}^{(n+1-j)/n}}{\mathcal{T}^{(j-d-1)/n}}\right)^{\sigma-1}+\sum_{j=d+1+n/2}^{n}\left(\frac{\mathcal{T}^{(n+1-j)/n}}{\mathcal{T}^{(n+d+1-j)/n}}\right)^{\sigma-1}
$$

$$
=\sum_{j=2}^{d}\mathcal{T}^{\frac{\sigma-1}{n}(2j-d-2)}+\sum_{j=d+2}^{n/2+1}\mathcal{T}^{\frac{\sigma-1}{n}d}+\sum_{j=n/2+2}^{d+n/2}\mathcal{T}^{\frac{\sigma-1}{n}(n-2j+d+2)}+\sum_{j=d+1+n/2}^{n}\mathcal{T}^{\frac{1-\sigma}{n}d}
$$

$$
=2\sum_{j=2}^{d}\mathcal{T}^{\frac{\sigma-1}{n}(2j-d-2)}+\sum_{j=d+2}^{n/2+1}\mathcal{T}^{\frac{\sigma-1}{n}d}+\sum_{j=d+1+n/2}^{n}\mathcal{T}^{\frac{1-\sigma}{n}d}
$$

$$
=2\sum_{j=2}^{d}\mathcal{T}^{\frac{\sigma-1}{n}(2j-d-2)}+\frac{n-2d}{2}\mathcal{T}^{\frac{\sigma-1}{n}d}+\frac{n-2d}{2}\mathcal{T}^{\frac{1-\sigma}{n}d}.
$$

Therefore, we get

$$
\begin{aligned}
W_{d+1}^{\sigma} &= Y_1\mathcal{T}^{\frac{1-\sigma}{n}d}+\frac{1-\mu}{n}\mathcal{T}^{\frac{\sigma-1}{n}d}+\frac{(1-\mu)(n-2d)}{2n}\left(\mathcal{T}^{\frac{\sigma-1}{n}d}+\mathcal{T}^{\frac{1-\sigma}{n}d}\right)+ \\
&\quad +\frac{2(1-\mu)}{n}\sum_{j=2}^{d}\mathcal{T}^{\frac{\sigma-1}{n}(2j-d-2)} \\
&= \mu\mathcal{T}^{\frac{1-\sigma}{n}d}+\frac{(1-\mu)(n-2d+2)}{2n}\left(\mathcal{T}^{\frac{\sigma-1}{n}d}+\mathcal{T}^{\frac{1-\sigma}{n}d}\right)+\frac{2(1-\mu)}{n}\sum_{j=2}^{d}\mathcal{T}^{\frac{\sigma-1}{n}(2j-d-2)}.
\end{aligned}
$$

With more manipulation,

$$
\begin{aligned}
W_{d+1}^{\sigma} &= \mu\mathcal{T}^{\frac{1-\sigma}{n}d}+\left[\frac{1-\mu}{2}-\frac{(1-\mu)(d-1)}{n}\right]\left(\mathcal{T}^{\frac{\sigma-1}{n}d}+\mathcal{T}^{\frac{1-\sigma}{n}d}\right)+ \\
&\quad +\frac{2(1-\mu)}{n}\sum_{j=2}^{d}\mathcal{T}^{\frac{\sigma-1}{n}(2j-d-2)} \\
&= \mathcal{T}^{\frac{1-\sigma}{n}d}\left[\frac{1+\mu}{2}-\frac{(1-\mu)(d-1)}{n}\right]+\mathcal{T}^{\frac{\sigma-1}{n}d}\left[\frac{1-\mu}{2}-\frac{(1-\mu)(d-1)}{n}\right]+ \\
&\quad +\frac{2(1-\mu)}{n}\sum_{j=2}^{d}\mathcal{T}^{\frac{\sigma-1}{n}(2j-d-2)}.
\end{aligned}
$$

---

[1] Ignore the summation terms whenever the subscript is higher than the superscript.

Since $\mathcal{T}^x + \mathcal{T}^{-x}$ is an increasing function of x, we can get rid of the summation and get

$$W_{d+1}^{\sigma} < \frac{1+\mu}{2}\mathcal{T}^{\frac{1-\sigma}{n}d} + \frac{1-\mu}{2}\mathcal{T}^{\frac{\sigma-1}{n}d}.$$

The welfare in region $d+1$ is such that:

$$U_{d+1} = \mathcal{T}^{-\frac{\mu d}{n}}W_{d+1} < \left(\frac{1+\mu}{2}\mathcal{T}^{\frac{1-\sigma-\mu\sigma}{n}d} + \frac{1-\mu}{2}\mathcal{T}^{\frac{\sigma-1-\mu\sigma}{n}d}\right)^{\frac{1}{\sigma}}.$$

This means that workers won't surely move to region $d+1$ if:

$$\frac{1+\mu}{2}\mathcal{T}^{\frac{1-\sigma-\mu\sigma}{n}d} + \frac{1-\mu}{2}\mathcal{T}^{\frac{\sigma-1-\mu\sigma}{n}d} \leq 1.$$

By differentiation with respect to d, we obtain that if the above condition is satisfied for $d = n/2$, then it is satisfied for any d, and concentration is stable. Hence, a sufficient condition for the stability of concentration is:

$$\frac{1+\mu}{2}\mathcal{T}^{\frac{1-\sigma-\mu\sigma}{2}} + \frac{1-\mu}{2}\mathcal{T}^{\frac{\sigma-1-\mu\sigma}{2}} \leq 1.$$

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