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RESEARCH WORK IN PROGRESS

N. 337, Oct. 2009

# REGULATING A MONOPOLIST WITH UNKNOWN BUREAUCRATIC TENDENCIES

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### Regulating a monopolist with unknown bureaucratic tendencies\*

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Abstract. We determine the optimal contract for the regulation of a bureaucratic firm in the case in which the bureaucratic bias is firm's private information. We find that output is distorted upward when the bureaucratic bias is low, downward when it is high, and equals a reference output when it is intermediate (in this case, the participation constraint is binding). We also determine an endogenous reference output (equal to the expected output, which depends on the reference output), and find that the response of output to cost is null in the short-run (in which the reference output is fixed) whenever the managers' types are in the intermediate range and negative in the long-run (after the adjustment of the reference output to equal expected output).

**Keywords:** Procurement, Regulation, Adverse selection, Bureaucracy, Reservation utility. **JEL Classification Numbers:** D82, H42, H51, I11.

\* We are grateful to Inés Macho-Stadler and David Pérez-Castrillo for useful comments, and thank participants in the 2009 SAET Conference and the 3<sup>rd</sup> Economic Theory Workshop in Vigo. Ana Sofia Borges (040421002@fep.up.pt) thanks Fundação para a Ciência e Tecnologia for financial support (Ph.D. scholarship). João Correia-da-Silva (joao@fep.up.pt) acknowledges support from CEF.UP and research grant PTDC/ECO/66186/2006 from Fundação para a Ciência e Tecnologia and FEDER.

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### 1 Introduction

Behavior of managers within firms is more complex than profit maximization. Its immediate determinants are: salary, security, status, power, prestige, social service and professional excellence (Niskanen, 1971). Observed willingness to manage a firm without significant monetary incentives should be attributed to these non-monetary factors (Wilson, 1989).

The motivations of managers were synthesized and formalized by Williamson (1974), who concluded that, in addition to a preference for profit, the management also displays a preference for expenses in staff and emoluments. Such preference for expenditure may accentuate the business cycle, by inducing a systematic accumulation of staff and emoluments during prosperity, with divestment frequently becoming necessary during adversity.<sup>1</sup>

In general, the managerial discretion models (Maris, 1963; Williamson, 1974; Prendergast, 2007) aim to investigate implications of various hypotheses concerning the objective functions of managers, explicitly recognizing that these may not coincide with profit-maximization.<sup>2</sup> Although monetary incentives are relevant, the overwhelming evidence indicates that the managers' own preferences have the greatest effect on behavior (Brehm and Gates, 1997). It is, therefore, important to hire managers with the "right" preferences (Prendergast, 2007).

In the standard theory of regulation under asymmetric information (Baron and Myerson, 1982; Guesnerie and Laffont, 1984; Maskin and Riley, 1984; Laffont and Tirole, 1986), it is assumed that the managers' objective is the maximization of the monetary transfers that they receive, net of the monetary value of the effort that they exert. Recently, Borges and Correia-da-Silva (2008) generalized the model of Laffont and Tirole (1986) by allowing the manager of the firm to have a preference for high output (bureaucratic bias) that was known by the regulator. The intrinsic difficulty in the observation of preferences motivates us to study the case in which the bureaucratic bias of the manager is his private information.<sup>3</sup>

<sup>&</sup>lt;sup>1</sup>Baumol (1959) argued that some firms maximize sales subject to a profit constraint, thus producing more than if they operated as profit maximizers.

<sup>&</sup>lt;sup>2</sup>Empirical evidence of deviations from profit-maximization was provided by Chetty and Saez (2005) and by Brown, Liang and Weisbenner (2007), who studied the response of corporations to the 2003 dividend tax cut in the USA.

<sup>&</sup>lt;sup>3</sup>The preference for output is equivalent to the preference for staff consider by Williamson (1974).

We consider that the government (principal) offers a contract to a manager (agent) who has an unobservable preference for high output (bureaucratic bias) for the provision of a public good, with the objective of maximizing social welfare.<sup>4</sup> The contract is accepted by the manager if it provides a higher utility than his best outside option (managing a firm of some given reference size).

Since the utility that the manager attributes to the output of the firm (bureaucratic bias) is his private information, there is asymmetric information about the reservation utility of the manager. As a consequence, the informational rent may not be monotonic, that is, better types may not get higher rents (Jullien, 2000; Laffont and Martimort, 2002). In fact, interior types may have a null rent or be excluded from trade.<sup>5</sup>

We obtain results which are worthwhile to compare with the results of standard models of regulation and with the results of the managerial discretion models. Output turns out to be an increasing function of the manager's preference for output (bureaucratic bias). This result is in line with managerial discretion theory but results from a quite different mechanism. Instead of being the straightforward implication of managerial discretion over output, it follows from the manager's participation constraint: a larger output yields a larger utility to the manager who, then, becomes willing to accept a lower monetary transfer.

We also find that output is distorted above the first-best optimal level when the preference for output is low and above it when it is high. For intermediate values of the preference for output, output equals its reference level (associated with the outside option). In this case, unlike what is predicted by managerial discretion models, increases in marginal cost have no effect on output.

Finally we study the case in which the reference output (size of the firm associated with the outside option) is endogenous. More precisely, that it is equal to the expected output of the firm, with respect to the manager's distribution of types. Notice that this expected value depends on the reference output itself. This case may be useful to understand better the response of output to changes in cost. In the short-run (in which the reference output is

<sup>&</sup>lt;sup>4</sup>More precisely, the government offers an incentive compatible set of contracts, one for each type of manager.

<sup>&</sup>lt;sup>5</sup>The informational rent is the difference between the utility that the manager attains and his reservation utility (which is the attainable utility in the case of symmetric information).

fixed), output does not respond to small variations in cost, whenever the managers' types are in the "intermediate" range (over which the participation constraint is binding). In the long-run (after the adjustment of the reference output to equal expected output), the sensitivity of output to cost becomes negative.

The paper is organized as follows. Section 2 describes the model and section 3 analyzes the benchmark case of complete information. In section 4 we derive the optimal incentive scheme and provide the general results. Section 5 presents an illustration for quadratic social value functions and an uniform distribution over types (with an exogenous reference output and with an endogenous reference output). Finally, Section 6 offers some concluding remarks.

### 2 The model

We consider a model of procurement in which the government (principal) delegates to a firm (agent) the provision of a public good. The manager of the firm cares about profit, t, but also about the output level, q. The manager's utility function is:

$$U = t + \delta q$$

where  $\delta$ , the marginal utility of output to the manager, is a private information parameter, drawn according to a probability distribution over an interval  $\left[\underline{\delta}, \overline{\delta}\right]$ , with strictly positive density,  $f(\delta) > 0$ , and monotone hazard rates,  $\frac{d}{d\delta} \left[\frac{f(\delta)}{1 - F(\delta)}\right] > 0$  and  $\frac{d}{d\delta} \left[\frac{f(\delta)}{F(\delta)}\right] < 0$  (where  $F(\delta)$  denotes the cumulative distribution function).

We assume that the manager has a type-dependent outside option that is proportional to  $\delta$ . For the manager to accept the contract, expected utility has to be larger than  $\delta q_{ref}$  (the reservation utility). The output reference level,  $q_{ref}$ , is, for instance, the output level of the firm that he would manage upon rejecting the regulator's proposition:<sup>6</sup>

$$U(\delta) \ge \delta q_{ref}, \ \forall \delta \in \left[\underline{\delta}, \overline{\delta}\right].^7$$

<sup>&</sup>lt;sup>6</sup>In Section 5.2, we propose an endogenous determination of the reference level of output.

<sup>&</sup>lt;sup>7</sup>It would be equivalent to consider  $U(\delta) = t + \delta(q - q_{ref})$ , with participation for  $U(\delta) \ge 0$ .

Of course that this covers the typical case in which  $U(\delta) \geq 0$  (setting  $q_{ref} = 0$ ).

The cost incurred by the firm is given by  $C = \beta q$ , where the firm's marginal cost,  $\beta > 0$ , is observable by the government. The social value of output is S(q), with marginal social value being strictly positive and decreasing, S'(q) > 0 and S''(q) < 0, for any  $q \in [0, \overline{q})$ . We also set S(0) = 0 and  $S''(\overline{q}) = 0$  (where  $\overline{q}$  can be interpreted as fully covering the needs of the population).

The government finances the public good provision using a distortionary mechanism (taxes, for example), so that the social cost of raising one unit of money is  $1 + \lambda$ , with  $\lambda > 0$ . The welfare of consumers is the difference between the social value of the public good and the cost of financing its provision,  $S(q) - (1 + \lambda)(t + C)$ . In the model of Laffont and Tirole (1986, 1993) and, more generally, in the regulation literature, the government is assumed to maximize the sum of the consumers' welfare with the utility of the firm. With the incorporation of a bureaucratic bias, it becomes natural to consider two possibilities:

- (i) social welfare,  $W(\delta)$ , is the sum of the consumer's welfare with the utility of the firm (this case corresponds to setting k = 0, below);
- (ii) social welfare,  $W(\delta)$ , is the sum of the consumer's welfare with the profit of the firm, while the bureaucratic component of the manager's utility is excluded (corresponds to setting k = 1, below).

In fact, we also allow for intermediate cases (any  $k \in [0, 1]$ , below).

The problem of the government (maximization of expected social welfare) is:

$$\max_{q,t} \int_{\delta}^{\overline{\delta}} S(q) - (1+\lambda)(t+C) + U - k\delta q \, d\delta \tag{1}$$

subject to

$$U \ge \delta q_{ref}. \tag{2}$$

### 3 The case of complete information

As a benchmark, we study the case in which there is no asymmetry of information between the government and the firm (the government is able to observe  $\delta$ ). Using the definitions of U and C, problem (1) can be written as:

$$\max_{q,U} \left\{ S(q) - (1+\lambda) \left(\beta - \delta\right) q - \lambda U - k\delta q \right\} \tag{3}$$

subject to

$$U \ge \delta q_{ref}. \tag{2}$$

The first order condition of problem (3) is:

$$S'(q) = (1 + \lambda)(\beta - \delta) + k\delta. \tag{4}$$

And the second order condition is: S''(q) < 0.

We make the following assumptions for the problem to be well-behaved.

### Assumption 1.

- (i)  $\lambda > 0$ ;
- (ii)  $\forall q \in [0, \overline{q}), S''(q) < 0;$
- (iii)  $S'(0) > (1 + \lambda) (\beta \underline{\delta}) + k\underline{\delta};$
- (iv)  $\beta > \frac{1+\lambda-k}{1+\lambda}\overline{\delta}$ .

Assumption (i) guarantees that the participation constraint is binding, (ii) ensures that the second order condition is satisfied, (iii) ensures a positive output level, and (iv) is necessary for the equilibrium output to be lower than  $\overline{q}$ .

Since the participation constraint is binding,  $U_c^*(\delta) = \delta q_{ref}$ , and the net transfer is:

$$t_c^*(\delta) = \delta \left[ q_{ref} - q_c^*(\delta) \right].$$

Observe that the net transfer,  $t_c^*$ , is positive (negative) if and only if the optimal output,  $q_c^*$ , is lower (higher) than the reference output,  $q_{ref}$ .

The optimality condition (4) equates the marginal benefit of output to consumers, S'(q), and the social marginal cost. Using this condition, the optimal output level,  $q_c^*$ , can be determined. It is clear that when the regulator cares about the non-monetary component of the manager's utility (k = 0), the level of output is higher and the net monetary transfer

is lower than when she does not care (k = 1). Observe also that the optimal output level is an increasing function of the bureaucratic bias, even when the regulator does not care at all about the utility that the manager derives from output (k = 1). This occurs because increasing the output implies an increase in the non-monetary component of the manager's utility, decreasing, therefore, the monetary transfer which is necessary to induce participation.

We also find that the social welfare,  $W_c^*(\delta)$ , is a convex function with respect to  $\delta$ . Applying the Envelope Theorem:<sup>8</sup>

$$\frac{dW_c^*}{d\delta} = \frac{\partial W_c^*}{\partial \delta} = (1 + \lambda - k)q_c^* - \lambda q_{ref}.$$

Since  $q_c^*$  is increasing in  $\delta$ :

$$\frac{d^2W^*}{d\delta^2} > 0.$$

With k=1, the social welfare is increasing (decreasing) with the bureaucratic bias,  $\delta$ , if and only if the optimal output,  $q_c^*$ , is higher (lower) than the reference output,  $q_{ref}$ .

In the following section, we determine the optimal procurement contract in the presence of an unobserved bureaucratic bias.

### 4 The optimal incentive scheme

In the asymmetric information case, the government does not know the firm's marginal utility of output,  $\delta$ . At the moment of contracting, the government only knows the prior probability distribution of  $\delta$ .

The government offers a contract to the firm,  $[q(\tilde{\delta}), t(\tilde{\delta}, C)]$ , specifying an output,  $q(\tilde{\delta})$  and a net payment,  $t(\tilde{\delta}, C)$ , that depend on the bureaucratic bias announced by the firm,  $\tilde{\delta}$ .

Thanks to the Revelation Principle, we can restrict (without loss of generality) our attention to incentive compatible direct revelation mechanisms.<sup>9</sup>

<sup>&</sup>lt;sup>8</sup>See, for example, Chiang and Wainwright (2005).

<sup>&</sup>lt;sup>9</sup>By the revelation principle (Myerson, 1979), given a Bayesian Nash equilibrium of a game of incomplete information, there exists a direct mechanism that has an equivalent equilibrium where the players truthfully

### 4.1 The firms's optimization problem

The government offers a contract to the firm such that the manager receives a net transfer  $t(\tilde{\delta})$  when he produces the output level  $q(\tilde{\delta})$ , and has to pay an extreme penalty  $t(\tilde{\delta}) = -\infty$  if he produces some output level different from  $q(\tilde{\delta})$ .

For any  $\delta \in [\underline{\delta}, \overline{\delta}]$ , truthful revelation must maximize the utility of the firm:

$$\delta \in \arg\max_{\tilde{\delta} \in [\underline{\delta}, \bar{\delta}]} \left\{ t(\tilde{\delta}) + \delta q(\tilde{\delta}) \right\}. \tag{5}$$

Let  $V(\delta)$  be the firm's value function (attainable utility as a function of  $\delta$ ):

$$V(\delta) = \max_{\tilde{\delta} \in [\underline{\delta}, \overline{\delta}]} \left\{ t(\tilde{\delta}) + \delta q(\tilde{\delta}) \right\} = t(\delta) + \delta q(\delta).$$

If announcing  $\tilde{\delta} \neq \delta$  is not profitable (5), then the output, transfer and value functions are almost everywhere differentiable (see Appendix A.1).

From the Envelope Theorem, we obtain the first order incentive compatibility constraint:

$$V'(\delta) = q(\delta), \tag{6}$$

which tells us that the derivative of the value function with respect to  $\delta$  is equal to the output level.

Integrating, we obtain:

$$V(\delta) = V(\underline{\delta}) + \int_{\delta}^{\delta} q(\gamma)d\gamma. \tag{7}$$

The second incentive compatibility constraint is derived from the second order condition of utility maximization (see Appendix A.2):<sup>10</sup>

$$q'(\delta) \ge 0, \ \forall \delta \in \left[\underline{\delta}, \overline{\delta}\right].$$
 (8)

It implies that the output level is increasing with the manager's bureaucratic bias.

report their types. A direct-revelation mechanism is said to be incentive compatible if, when each individual is expecting the others to be truthful, then he/she has interest in being truthful.

<sup>&</sup>lt;sup>10</sup>In Appendix A.3, it is shown that the first and second incentive compatibility constraints, (6) and (8), are equivalent to truth-telling (5).

### 4.2 The government's optimization problem

The objective of the government is to maximize expected social welfare,  $E_{\delta}[W(\delta)]$ :

$$\max_{q(\delta), V(\delta)} \int_{\delta}^{\overline{\delta}} \left\{ S\left[q(\delta)\right] - (1+\lambda)\left(\beta - \delta\right) q(\delta) - \lambda V(\delta) - k\delta q(\delta) \right\} f(\delta) d\delta, \tag{9}$$

subject to, for all  $\delta \in [\underline{\delta}, \overline{\delta}]$ ,

$$V(\delta) \ge \delta q_{ref},$$
 (10)

$$V'(\delta) = q(\delta), \tag{6}$$

$$q'(\delta) \ge 0. \tag{8}$$

We start by solving a relaxed problem in which the second order incentive compatibility condition (8) is ignored. Later, we will check that the solution of this relaxed problem is the solution of the general problem (in Appendix B.2).

The participation constraint can be incorporated in the objective function:<sup>11</sup>

$$\max_{q(\delta),V(\delta)} \int_{\underline{\delta}}^{\overline{\delta}} \left\{ S\left[q(\delta)\right] - (1+\lambda)(\beta-\delta)q(\delta) - \lambda V(\delta) - k\delta q(\delta) \right\} f(\delta) + \\
+ \eta(\delta)\lambda \left[V(\delta) - \delta q_{ref}\right] d\delta \tag{11}$$

subject to, for all  $\delta$ ,

$$V'(\delta) = q(\delta), \tag{6}$$

where  $\eta(\delta)$  satisfies  $\eta(\delta) \geq 0$ ,  $\eta(\delta)\lambda [V(\delta) - \delta q_{ref}] = 0$ . Notice that  $\eta(\delta)\lambda$  is the Lagrangian multiplier associated to the participation constraint of type  $\delta$ .<sup>12</sup>

We define  $\zeta(\delta) = \int_{\underline{\delta}}^{\delta} \eta(s) ds$ . It is shown in the proof of Proposition 1 that  $\eta$  is a probability distribution on  $[\underline{\delta}, \overline{\delta}]$ , and that, therefore,  $\zeta(\delta)$  is a c.d.f. on  $[\underline{\delta}, \overline{\delta}]$ .

### Lemma 4.1.

The output level is such that:

$$S'[q(\delta)] - (1+\lambda)(\beta - \delta) - k\delta + \lambda \frac{F(\delta) - \zeta(\delta)}{f(\delta)} = 0.$$
(12)

<sup>&</sup>lt;sup>11</sup>See Basov (2005), pages 124-126.

<sup>&</sup>lt;sup>12</sup>The participation constraint must be binding for some  $\delta$ , otherwise the government could increase expected social welfare by reducing the transfer to the manager (across all types) without violating the participation constraint.

Observe, from the incentive compatibility conditions (6) and (8), that  $V(\delta)$  is a convex function. Since the reservation utility is linear in  $\delta$ , it follows that the participation constraint is binding over an interval, which we denote by  $[\delta_0, \delta_1]$ .<sup>13</sup>

When the participation constraint is binding over an open interval,  $V(\delta) = \delta q_{ref}$ , from the first incentive compatibility constraint (6), we have  $q(\delta) = q_{ref}$  and  $t(\delta) = 0$ .

### **Proposition** 1 (Output function).

The participation constraint (10) either:

- (i) binds at  $\underline{\delta}$ , implying downward distortion,  $q(\delta) < q_c^*(\delta)$ , for all  $\delta > \underline{\delta}$ ;
- (ii) binds at  $[\underline{\delta}, \delta_1]$ , implying downward distortion,  $q(\delta) < q_c^*(\delta)$ , and a negative monetary transfer,  $t(\delta) < 0$ , for all  $\delta > \delta_1$  (for  $\delta \in [\underline{\delta}, \delta_1]$ ,  $q(\delta) = q_{ref}$  and  $t(\delta) = 0$ );
  - (iii) binds at  $\overline{\delta}$ , implying upward distortion,  $q(\delta) > q_c^*(\delta)$ , for all  $\delta < \overline{\delta}$ ;
- (iv) binds at  $[\delta_0, \overline{\delta}]$ , implying upward distortion,  $q(\delta) > q_c^*(\delta)$ , and a positive monetary transfer,  $t(\delta) > 0$ , for all  $\delta < \delta_0$  (for  $\delta \in [\delta_0, \overline{\delta}]$ ,  $q(\delta) = q_{ref}$  and  $t(\delta) = 0$ );
- (v) binds at an interval  $[\delta_0, \delta_1]$ , implying: upward distortion and negative monetary transfer for  $\delta \in [\underline{\delta}, \delta_0]$ ; and downward distortion and positive monetary transfer for  $\delta \in [\delta_1, \overline{\delta}]$  (for  $\delta \in [\delta_0, \delta_1]$ ,  $q(\delta) = q_{ref}$  and  $t(\delta) = 0$ ).

Proof. See Appendix B.3. 
$$\Box$$

The output level is strictly increasing with  $\delta$  over the two intervals  $[\underline{\delta}, \delta_0]$  and  $[\delta_1, \overline{\delta}]$ , being constant and equal to  $q_{ref}$  in  $[\delta_0, \delta_1]$ . When the output level is strictly increasing, the monetary transfer,  $t(\delta)$ , is strictly decreasing.

In the next section, we characterize the optimal contract in the case of a quadratic social value function of output and a uniform distribution over types.

<sup>&</sup>lt;sup>13</sup>This includes, for example, the case in which the participation constraint is only binding at one of the extremes of the interval ( $\delta_0 = \delta_1 = 0$  or  $\delta_0 = \delta_1 = 1$ ).

### 5 Characterizing the optimal contract

### 5.1 Exogenous reference output

In order to illustrate our general results, we now assume that the social value of output is  $S(q) = \alpha q - \frac{1}{2}q^2$  and that  $\delta$  is uniformly distributed on [0,1].

With complete information, the solution is:

$$q_c^* = \alpha - (1+\lambda)(\beta - \delta) - k\delta,$$
  

$$t_c^* = \delta(q_{ref} - q_c^*).$$

Assumption 1 (iii), which in this case is  $\alpha > (1+\lambda)\beta$ , ensures that the complete information output is always positive.

The participation constraint binds if and only if  $\delta \in [\delta_0, \delta_1]$ , with (Appendix C.2):

$$\delta_0 = \max \left\{ 0, \frac{-\alpha + q_{ref} + (1+\lambda)\beta}{1 + 2\lambda - k} \right\} \text{ and } \delta_1 = \min \left\{ \frac{\lambda - \alpha + q_{ref} + (1+\lambda)\beta}{1 + 2\lambda - k}, 1 \right\}.$$

For  $\delta \in [\delta_0, \delta_1]$ , bunching occurs at the reference output level  $(q^*(\delta) = q_{ref})$ . Outside the interval, there is no bunching.

The solution is fully characterized by the following propositions.

**Proposition** 2 (The intermediate reference output case).

When 
$$q_{ref} \in [\alpha - (1 + \lambda)\beta, \alpha + (1 + \lambda)(1 - \beta) - k]$$
:

- (i) for  $\delta \in [\delta_0, \delta_1] \subset [0, 1]$ , there is bunching, with  $q(\delta) = q_{ref}$ ,  $V(\delta) = \delta q_{ref}$  and  $t(\delta) = 0$ ;
- (ii) for  $\delta \in [0, \delta_0)$ , there is upward distortion  $(q(\delta) > q_c^*)$  and a positive monetary transfer  $(t(\delta) > 0)$ , with

$$q(\delta) = \alpha - (1+\lambda)\beta - k\delta + (2\lambda + 1)\delta, \tag{13}$$

$$V(\delta) = \left[\alpha - (1+\lambda)\beta\right]\delta + (2\lambda + 1)\frac{\delta^2}{2} - \frac{k\delta^2}{2} + \frac{\left[q_{ref} - \alpha + (1+\lambda)\beta\right]^2}{2(1+2\lambda - k)},\tag{14}$$

$$t(\delta) = -(2\lambda + 1)\frac{\delta^2}{2} + \frac{k\delta^2}{2} + \frac{[q_{ref} - \alpha + (1+\lambda)\beta]^2}{2(1+2\lambda - k)}.$$
 (15)

(iii) for  $\delta \in (\delta_1, 1]$ , there is downward distortion  $(q(\delta) < q_c^*)$  and a negative monetary transfer  $(t(\delta) < 0)$ , with

$$q(\delta) = \alpha - (1+\lambda)\beta - k\delta + (2\lambda + 1)\delta - \lambda, \tag{16}$$

$$V(\delta) = \left[\alpha - (1+\lambda)\beta - \lambda\right]\delta + (2\lambda + 1)\frac{\delta^2}{2} - \frac{k\delta^2}{2} + \frac{\left[q_{ref} + \lambda - \alpha + (1+\lambda)\beta\right]^2}{2(1+2\lambda - k)},\tag{17}$$

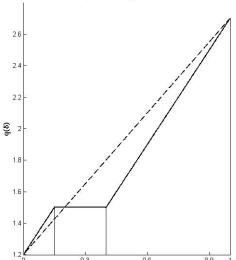
$$t(\delta) = -(2\lambda + 1)\frac{\delta^2}{2} + \frac{k\delta^2}{2} + \frac{[q_{ref} + \lambda - \alpha + (1+\lambda)\beta]^2}{2(1+2\lambda - k)}.$$
 (18)

*Proof.* See Appendices C.1, C.3 and C.4.

When  $\delta$  increases from 0 to 1, the output level first increases (until  $\delta_0$ ), being larger than the complete information output, then it is constant (between  $\delta_0$  and  $\delta_1$ ), and then increases again, being lower than the complete information optimal output (figure 1).

The intermediate reference output case

## The output in incomplete information The utility function in incomplete information



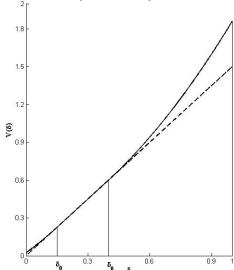


Figure 1: The output and the utility function of the firm with  $k=0, \alpha=3, \lambda=0.5, \beta=1.2,$   $q_{ref}=1.5, \delta_0=0.15, \delta_1=0.4.$ 

The following propositions describe what happens when the reference output  $q_{ref}$  is outside the interval defined in Proposition 2.

### **Proposition** 3 (The large reference output case).

When  $q_{ref} > \alpha + (1+\lambda)(1-\beta) - k$ , there is bunching at the top. Output, utility of the firm and monetary transfer are described by equations (13), (14) and (15) (upward distortion) when  $\delta \in [0, \delta_0)$ , and by  $q(\delta) = q_{ref}$ ,  $V(\delta) = \delta q_{ref}$  and  $t(\delta) = 0$  (bunching and binding participation constraint) when  $\delta \in [\delta_0, 1]$  (see figure 2).

### *Proof.* See Appendix C.3.

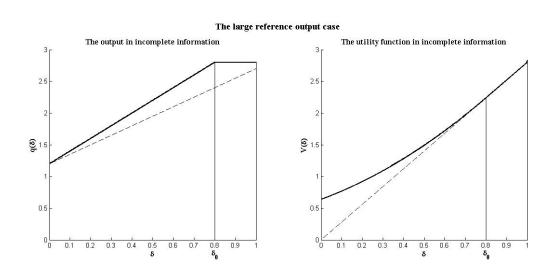


Figure 2: The output and the utility function of the firm with  $k=0, \alpha=3, \lambda=0.5, \beta=1.2,$   $q_{ref}=2.8, \delta_0=0.8.$ 

### **Proposition** 4 (The small reference output case).

When  $q_{ref} < \alpha - (1 + \lambda)\beta$  there is bunching at the bottom. Output, utility of the firm and monetary transfer are described by equations (16), (17) and (18) (downward distortion) when  $\delta \in (\delta_1, 1]$ , and by  $q(\delta) = q_{ref}$ ,  $V(\delta) = \delta q_{ref}$  and  $t(\delta) = 0$  (bunching and binding participation constraint) when  $\delta \in [0, \delta_1]$  (see figure 3).

### *Proof.* See Appendix C.4.

We illustrated the case in which the government incorporates the non-monetary component of the manager's utility in the welfare function (k = 0). If the government does not care

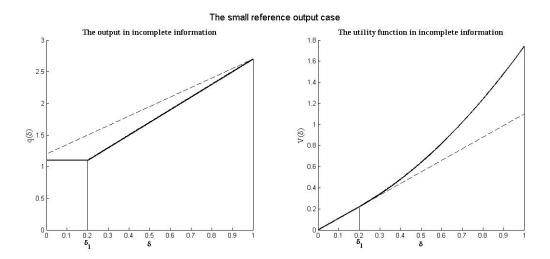


Figure 3: The output and the utility function of the firm with  $k=0, \alpha=3, \lambda=0.5, \beta=1.2,$   $q_{ref}=1.1, \delta_1=0.2.$ 

about the bureaucratic bias (k = 1), the interval over which the output is constant and the participation constraint of the manager is binding moves to the right and, outside this interval, the output and the manager's utility are lower.

We can use our results to study how the output responds to changes in cost, for a given reference output level. An increase in marginal cost,  $\beta$ , entails a reduction of the output level both when the manager's marginal utility is low  $(\delta \in [0, \delta_0])$  and when it is high  $(\delta \in [\delta_1, 1])$ . However, for intermediate values  $(\delta \in [\delta_0, \delta_1])$ , output does not change in response to (small) changes in cost.

Finally, we show that a sufficient condition for social welfare,  $W(\delta)$ , to be positive for all  $\delta \in [0, 1]$  (which implies that the regulator is happy to offer a contract to all possible types) is that the reference output is not larger than twice the optimal output in the absence of bureaucratic bias.

**Proposition** 5 (The government's participation constraint).

If  $q_{ref} \leq 2 \left[\alpha - (1+\lambda)\beta\right]$ , then the social welfare is positive,  $W(\delta) > 0$ , for all  $\delta \in [0,1]$ .

*Proof.* See Appendix C.5.

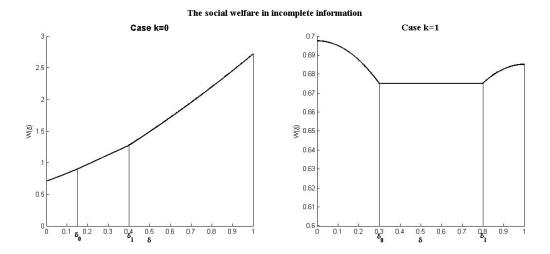


Figure 4: The social welfare with k=0 and k=1 in which  $\alpha=3, \lambda=0.5, \beta=1.2, q_{ref}=1.5.$ 

In the figure 4, we observe that, if the government cares about the bureaucratic bias (k = 0), the social welfare increases with  $\delta$ . If the government disregards the bureaucratic bias in the welfare function (precisely when k increases), the social welfare decreases in interval  $\delta \in [0, \delta_0)$ , is constant in  $\delta \in [\delta_0, \delta_1]$ , and increases in  $\delta \in (\delta_1, 1]$ .

### 5.2 Endogenous reference output

Up to now, we considered that the reference output was exogenous. Remember that the reference output determines the (type-dependent) reservation utility. It is natural to assume that this reference output is the expected output of the population of firms, with respect to the managers' types. Since this expected value depends on the reference output itself, the equilibrium value of the reference output is to be determined by stipulating that the reference output should equal the expected output,  $q_{ref} = E_{\delta}[q(\delta)]$ .

With quadratic social value of output and a uniform distribution over types, whenever the reference output is assumed to take an intermediate value (an assumption to be checked later), the expected output is given by:

$$E_{\delta}[q(\delta)] = \int_{0}^{\delta_{0}} \left[\alpha - (1+\lambda)\beta - k\delta + (2\lambda+1)\delta\right] d\delta + \int_{\delta_{0}}^{\delta_{1}} q_{ref} d\delta + \int_{\delta_{1}}^{1} \left[\alpha - (1+\lambda)\beta - k\delta + (2\lambda+1)\delta - \lambda\right] d\delta.$$

$$(19)$$

Equating (19) to  $q_{ref}$ , we obtain the unique equilibrium value of  $q_{ref}$  as

$$q_{ref}^* = \alpha - (1+\lambda)\beta + \frac{1}{2}(1+\lambda-k).$$
 (20)

Observing that  $q_{ref}^* \in [\alpha - (1 + \lambda)\beta, \alpha + (1 + \lambda)(1 - \beta) - k]$ , we validate, using Proposition 3, our initial assumption about the reference output taking an intermediate value.

We can now prove that this the only possible equilibrium. Indeed, if we assume that the reference output is large, we have

$$E_{\delta}[q(\delta)] = \int_{0}^{\delta_{0}} \left[\alpha - (1+\lambda)\beta - k\delta + (2\lambda+1)\delta\right] d\delta + \int_{\delta_{0}}^{1} q_{ref} d\delta, \tag{21}$$

and equating (21) to  $q_{ref}$ , we obtain as candidate equilibrium value  $q_{ref} = \alpha - (1 + \lambda)\beta + \frac{1-k}{2} + \lambda$ . We immediately observe that this value is lower than  $\alpha + (1 + \lambda)(1 - \beta) - k$ , contradicting our assumption that we were in the large reference output case.

Similarly, if we assume that the reference output is small, we have

$$E_{\delta}\left[q(\delta)\right] = \int_{0}^{\delta_{1}} q_{ref} d\delta + \int_{\delta_{1}}^{1} \left[\alpha - (1+\lambda)\beta - k\delta + (2\lambda+1)\delta - \lambda\right] d\delta, \tag{22}$$

and equating (22) to  $q_{ref}$ , we obtain as candidate equilibrium value  $q_{ref} = \alpha + (1 + \lambda)(1 - \beta) - k$ . We immediately observe that this value is larger than  $\alpha - (1 + \lambda)\beta$ , contradicting our assumption that we were in the small reference output case. This is summarized in Proposition 6 below.

### **Proposition** 6 (Endogenous reference output).

The unique endogenous reference output level is given by equation (20). It is such that only the "intermediate reference output case" occurs at equilibrium.

Making the reference output endogenous in this way leads us to reconsider what has been said at the end of the previous section. It is true that, for a given reference output, the firms may not respond to "small" changes in cost when their managers' types are intermediate. However, the (endogenous) reference output itself will adjust in response to changes in cost. Then, it makes sense to distinguish short-run and long-run output elasticity with respect to changes in cost. In the short-run, the reference output is given, for instance because the

contracts have already been signed between the regulators and the managers, and output can be insensitive to changes in costs or taxes for a whole category of firms. However, in the long run, as the old managers leave their jobs, new contracts are passed referring to the new reference output, and output converges to its new equilibrium value. It follows that the long-run output elasticity is going to be larger than the short-run one.

### 6 Concluding remarks and extensions

In this paper, we analyzed the optimal regulation of a bureaucratic firm when the bureaucratic bias (manager's utility of output) is private information and the reservation utility is type-dependent. We showed that the optimal output function is such that output is distorted upward when the manager has a low preference for output, downward when he/she has a high preference for output and equals his/her reference output when he/she has an intermediate preference for output and the individual rationality constraint is binding. When the government does not care about the non-monetary part of the manager's utility, the interval over which the output is constant and the participation constraint is binding moves to the right, while, outside this interval, equilibrium output is lower.

These results have implications for cost sensitivity of output. In the short-run, when the reference output can be considered as exogenous, the output of firms whose managers' types are intermediate are not going to respond to small variations in their unit cost of production, while the output of firms whose managers' types are either small or large will respond in the expected direction.<sup>14</sup> The overall sensitivity of output to cost variations depends on the relative numerical importance of these three groups of firms, or, using the terminology of this model, on the length of the interval  $[\delta_0, \delta_1]$ . In the long-run, the reference output adjusts progressively to its new equilibrium level, and so the output elasticity is larger.

For the sake of simplicity, we assumed throughout this paper that the government can observe the unit cost of production, and we simultaneously ignored the issue of moral hazard. Analyzing a model where both the marginal cost and the manager's utility for output are private information would be fruitful. This will be the subject of future research.

<sup>&</sup>lt;sup>14</sup>This looks like the result obtained by Sweezy (1939) in the very different framework of "the kinked oligopoly demand curve".

### A Appendix: Problem of the firm

### A.1 Differentiability of output, transfer, and utility functions

In this section, we prove that if (5) holds, then the output, transfer and value functions are almost everywhere differentiable.

**Lemma 1.**  $\tilde{\delta} < \delta \Rightarrow q(\tilde{\delta}) \leq q(\delta)$ .

*Proof.* If (5) holds, then:

$$t(\delta) + \delta q(\delta) \ge t(\tilde{\delta}) + \delta q(\tilde{\delta}),$$

$$t(\tilde{\delta}) + \tilde{\delta}q(\tilde{\delta}) \ge t(\delta) + \tilde{\delta}q(\delta).$$

Adding the two inequalities, we obtain:

$$\left(\delta - \tilde{\delta}\right) \left[q(\delta) - q(\tilde{\delta})\right] \ge 0.$$

Then,  $\delta - \tilde{\delta} > 0$  implies that  $q(\delta) - q(\tilde{\delta}) \ge 0$ .

**Lemma 2.**  $U(\tilde{\delta}, \delta)$ , as a function of  $\tilde{\delta}$ , is nondecreasing on  $[\underline{\delta}, \delta]$  and nonincreasing on  $[\delta, \overline{\delta}]$ .

*Proof.* Let us show monotonicity on  $[\underline{\delta}, \delta]$ . Assume that  $\tilde{\delta} < \delta' < \delta$  and, by way of contradiction,  $U(\tilde{\delta}, \delta) > U(\delta', \delta)$ , that is:

$$t(\tilde{\delta}) + \delta q(\tilde{\delta}) > t(\delta') + \delta q(\delta').$$

On the other hand, we know that a firm with a bureaucratic bias  $\delta'$  prefers to announce  $\delta'$  rather than announce  $\tilde{\delta}$ . Thus:

$$t(\delta') + \delta' q(\delta') \ge t(\tilde{\delta}) + \delta' q(\tilde{\delta}).$$

Adding the last two equations, we obtain:

$$(\delta - \delta') \left[ q(\tilde{\delta}) - q(\delta') \right] > 0 \implies q(\tilde{\delta}) - q(\delta') > 0.$$

Which is in contradiction with Lemma 1.

Monotonicity on  $\left[\delta, \bar{\delta}\right]$  can be proved in the same way.

**Lemma 3.**  $t(\delta) + \delta q(\delta)$  is nondecreasing.

*Proof.* Let  $\delta' < \delta$ . We need to show that  $t(\delta) + \delta q(\delta) > t(\delta') + \delta' q(\delta')$ .

By definition:

$$t(\delta') + \delta'q(\delta') = t(\delta') + \delta q(\delta') - (\delta - \delta')q(\delta') = U(\delta', \delta) - (\delta - \delta')q(\delta').$$

Using Lemma 2:

$$t(\delta') + \delta' q(\delta') \le U(\delta, \delta) - (\delta - \delta') q(\delta') \le t(\delta) + \delta q(\delta).$$

Lemmas 1 and 3 imply that the functions  $q(\delta)$  and  $V(\delta) = t(\delta) + \delta q(\delta)$  are a.e. differentiable. Hence,  $t(\delta)$  is also a.e. differentiable.

### A.2 Second incentive compatibility condition

The local second-order condition of the maximization program is:

$$\left. \frac{\partial^2 U(\tilde{\delta}, \delta)}{\partial \tilde{\delta}^2} \right|_{\tilde{\delta} = \delta} \le 0 \iff t''(\delta) + \delta q''(\delta) \le 0. \tag{23}$$

We want to show that (given the first-order condition) it is equivalent to  $q'(\delta) \geq 0$ .

Notice that, under the first incentive compatibility condition (5), the value function is:

$$V(\delta) = t(\delta) + \delta q(\delta). \tag{24}$$

Evaluating the derivative of (24) and equating to (6), we obtain:

$$t'(\delta) + \delta q'(\delta) = 0. \tag{25}$$

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The derivative of (25) is:

$$t''(\delta) + q'(\delta) + \delta q''(\delta) = 0.$$
(26)

Subtracting (26) from (23), the local second order condition becomes:

$$q'(\delta) \ge 0.$$

### A.3 The local second order condition implies the global one

**Lemma 4.** If the second incentive compatibility condition (8) is (strictly) satisfied,  $\forall \delta \in [\underline{\delta}, \overline{\delta}]$ , then  $\frac{\partial^2 U(\tilde{\delta}, \delta)}{\partial \delta \partial \tilde{\delta}}$  is also (strictly) positive,  $\forall \delta, \tilde{\delta} \in [\underline{\delta}, \overline{\delta}]$ .

*Proof.* Recall that:

$$U(\tilde{\delta}, \delta) = t(\tilde{\delta}) + \delta q(\tilde{\delta}).$$

The partial derivative with respect to  $\tilde{\delta}$  yields:

$$\frac{\partial U(\tilde{\delta}, \delta)}{\partial \tilde{\delta}} = t'(\tilde{\delta}) + \delta q'(\tilde{\delta}).$$

Therefore:

$$\frac{\partial^2 U(\tilde{\delta}, \delta)}{\partial \delta \partial \tilde{\delta}} = q'(\tilde{\delta}).$$

**Lemma 5.** If  $\frac{\partial U(\tilde{\delta}, \delta)}{\partial \tilde{\delta}}$  is increasing in  $\delta$ , then the local second order condition implies the global one.

*Proof.* The local second order condition implies that announcing the truth  $\tilde{\delta} = \delta$  gives a local maximum for the firm of type  $\delta$ . Is there another announcement,  $\delta' \neq \delta$ , that also satisfies the first order condition? That is, does there exist  $\delta' \neq \delta$  such that:

$$\frac{\partial U(\tilde{\delta}, \delta)}{\partial \tilde{\delta}} \Big|_{(\delta', \delta)} = \frac{\partial U(\tilde{\delta}, \delta)}{\partial \tilde{\delta}} \Big|_{(\delta, \delta)} = 0?$$

This would imply that

$$\frac{\partial U(\tilde{\delta}, \delta)}{\partial \tilde{\delta}} \Big|_{(\delta', \delta)} = \frac{\partial U(\tilde{\delta}, \delta)}{\partial \tilde{\delta}} \Big|_{(\delta', \delta')} = 0.$$

But this is inconsistent with the strict monotonicity of  $\partial U(\tilde{\delta}, \delta)/\partial \tilde{\delta}$  with respect to its second argument. Weak monotonicity implies that any announcement in the interval  $[\delta', \delta]$  is equally optimal. In any case, announcing  $\delta$  is optimal.

### B Appendix: Problem of the government

### **B.1** Necessary and Sufficient conditions

We first ignore the second order incentive compatibility condition (8) but we shall check later that the solution of the problem (11) is the solution of the general problem (9).

### Necessary conditions

The Hamiltonian is:

$$H = \{ S [q(\delta)] - (1+\lambda)(\beta - \delta)q(\delta) - \lambda V(\delta) - k\delta q(\delta) \} f(\delta) + \mu q(\delta) + \eta(\delta) [V(\delta) - \delta q_{ref}],$$
(27)

where  $\mu(\delta)$  is the co-state variable associated with (6) and  $\eta(\delta)$  is the multiplier associated with the participation constraint.

The first order conditions imply:

$$\mu'(\delta) = \lambda \left[ f(\delta) - \eta(\delta) \right], \tag{28a}$$

$$V'(\delta) = q(\delta), \tag{28b}$$

$$S'[q(\delta)] - (1+\lambda)(\beta - \delta) - k\delta + \frac{\mu(\delta)}{f(\delta)} = 0, \tag{28c}$$

$$\eta(\delta) \left[ V(\delta) - \delta q_{ref} \right] = 0, \tag{28d}$$

$$\mu(\underline{\delta}) = \mu(\overline{\delta}) = 0, \tag{28e}$$

$$\eta(\delta) \ge 0, \quad V(\delta) \ge \delta q_{ref}.$$
(28f)

Equation (28a) is the equation of motion of the co-state variable. Equation (28d) is the complementary slackness condition. Equation (28e) gives the transversality conditions.

Integrating equation (28a) we obtain:

$$\mu(\delta) = \lambda \left[ F(\delta) - \zeta(\delta) \right], \tag{29}$$

and using (28e):

$$\mu(\underline{\delta}) = \lambda \left[ F(\underline{\delta}) - \zeta(\underline{\delta}) \right] \iff 0 = -\lambda \zeta(\underline{\delta}) \Leftrightarrow \zeta(\underline{\delta}) = 0, \tag{30}$$

$$\mu(\overline{\delta}) = \lambda \left[ F(\overline{\delta}) - \zeta(\overline{\delta}) \right] \iff 0 = \lambda \left[ 1 - \zeta(\overline{\delta}) \right] \Leftrightarrow \zeta(\overline{\delta}) = 1. \tag{31}$$

These last two equations imply that:

$$\int_{\delta}^{\overline{\delta}} \eta(\delta) d\delta = \int_{\delta}^{\overline{\delta}} f(\delta) d\delta = 1, \tag{32}$$

meaning that  $\eta(\cdot)$  is a probability distribution on  $[\underline{\delta}, \overline{\delta}]$ . Denote the corresponding c.d.f. by  $\zeta(\delta) = \int_{\underline{\delta}}^{\delta} \eta(s) ds$ .

Replacing equation (29) into (28c), we find that output is such that:

$$S'[q(\delta)] - (1+\lambda)(\beta - \delta) - k\delta) + \lambda \frac{F(\delta) - \zeta(\delta)}{f(\delta)} = 0.$$
 (33)

The only difference with respect to the complete information case is the last term. It should be clear that  $\zeta(\delta) = 1$  implies downward distortion and  $\zeta(\delta) = 0$  implies upward distortion.

Note that if the participation constraint (6) is not binding on some open interval  $(\delta_a, \delta_b)$  then  $\zeta(\cdot)$  is constant on it, since  $\zeta'(\delta) = \eta(\delta) = 0$ .

### Sufficient condition

The second order derivative of the Hamiltonian (27) is:  $\frac{\partial^2 H}{\partial q^2} = S''[q(\delta)] < 0$ .

# B.2 The solution of the unconstrained problem is the solution of the general problem

We now check that the condition which was omitted in the problem (11),  $q'(\delta) \geq 0$ , is satisfied. Differentiating equation (12) with respect to  $\delta$ , we obtain:

$$q^{*'}(\delta) = -\frac{1 + \lambda - k + \lambda \left(1 - \frac{\eta(\delta)}{f(\delta)}\right) - \lambda \frac{f'(\delta)[F(\delta) - \zeta(\delta)]}{f(\delta)^2}}{S''[q(\delta)]}.$$

The assumption of monotone hazard rates are used here. Observe that:

$$\frac{d}{d\delta} \left[ \frac{f(\delta)}{F(\delta)} \right] < 0 \iff \frac{f'(\delta)F(\delta)}{f(\delta)^2} < 1$$

and

$$\frac{d}{d\delta} \left[ \frac{f(\delta)}{1 - F(\delta)} \right] > 0 \iff \frac{f'(\delta) \left[ F(\delta) - 1 \right]}{f(\delta)^2} < 1.$$

For  $\delta < \delta_0$ , we have  $\eta(\delta) = 0$  and  $\zeta(\delta) = 0$ , implying that:

$$q^{*'}(\delta) = -\frac{1 + 2\lambda - k - \lambda \frac{f'(\delta)F(\delta)}{f(\delta)^2}}{S''[q(\delta)]} > -\frac{1 + \lambda - k}{S''[q(\delta)]} > 0.$$

For  $\delta > \delta_1$ , we have  $\eta(\delta) = 0$  and  $\zeta(\delta) = 1$ , implying that:

$$q^{*'}(\delta) = -\frac{1 + 2\lambda - k - \lambda \frac{f'(\delta)[F(\delta) - 1]}{f(\delta)^2}}{S''[q(\delta)]} > -\frac{1 + \lambda - k}{S''[q(\delta)]} > 0.$$

When the participation constraint is binding,  $q^*(\delta) = q_{ref}$  and, therefore,  $q^{*'}(\delta) = 0$ .

We have found that  $q^{*'} \geq 0$ , which implies that (8) is verified. The solution of the unconstrained problem (11) is also the solution of the general problem (9).

### B.3 Characterization of the output function

*Proof.* In the main text, we have explained why one of these three situations must occur.

In case (i), since  $\zeta(\delta) = 1, \forall \delta \in (\underline{\delta}, \overline{\delta}]$ , the output, for  $\delta > \underline{\delta}$ , is given by:

$$S'[q(\delta)] - (1+\lambda)(\beta - \delta) - k\delta) = \lambda \frac{1 - F(\delta)}{f(\delta)} > 0.$$

The left term is null in the case of complete information, therefore we have  $q(\delta) < q_c^*(\delta)$ .

In case (ii), since  $\zeta(\delta) = 1, \forall \delta \in (\delta_1, \overline{\delta}]$ , the output, for  $\delta > \delta_1$ , is given by:

$$S'[q(\delta)] - (1+\lambda)(\beta - \delta) - k\delta) = \lambda \frac{1 - F(\delta)}{f(\delta)} > 0,$$

which, for the same reason, implies that  $q(\delta) < q_c^*(\delta)$ .

The first incentive compatibility constraint,  $V'(\delta) = q(\delta)$ , and the binding participation constraint,  $V(\delta) = \delta q_{ref}$ , imply that, for  $\delta \in (\underline{\delta}, \delta_1)$ , we have  $q(\delta) = q_{ref}$  and, thus,  $t(\delta) = 0$ .

The first incentive compatibility constraint is equivalent to  $t'(\delta) + \delta q'(\delta) = 0$ . Then, the second incentive compatibility constraint implies that, for  $\delta > \delta_1$ , we have  $t(\delta) < 0$ .

Cases (iii), (iv) and (v) can be analyzed similarly.

### C Appendix: The linear-quadratic case

We first characterize the optimal output level and the manager's equilibrium utility over the interval  $[\delta_0, \delta_1]$ , where the participation constraint is binding. We determine the precise values of  $\delta_0$  and  $\delta_1$  and we also study the two other situations over the two intervals where the participation constraint is not binding:  $\delta \in [0, \delta_0)$  and  $\delta \in (\delta_1, 1]$ .

### C.1 When $\delta \in [\delta_0, \delta_1]$

When  $\delta \in [\delta_0, \delta_1]$  the participation constraint is binding  $V(\delta) = \delta q_{ref}$ .

From the first incentive compatibility constraint (6) we have

$$q(\delta) = q_{ref}, \ \forall \ \delta \in [\delta_0, \delta_1].$$

### C.2 The values of $\delta_0$ and $\delta_1$

From Proposition 1 (iii) we know that:

$$\zeta(\delta) = \frac{1}{\lambda} \left[ \alpha - q_{ref} - (1 + \lambda) \beta - k \delta \right] + \left( 2 + \frac{1}{\lambda} \right) \delta.$$
 (34)

Since  $\zeta(0) = 0$  and  $\eta(\delta) = 0$  for all  $\delta < \delta_0$ , it must be that  $\zeta(\delta_0) = 0$ , replacing in equation (34), we get the value of  $\delta_0$ :

$$\delta_0 = \max\left\{0, \frac{-\alpha + q_{ref} + (1+\lambda)\beta}{1 + 2\lambda - k}\right\}. \tag{35}$$

When  $q_{ref} \ge \alpha - (1 + \lambda)\beta - k\delta = q^*(0)$ , we obtain  $\delta_0 \ge 0$ , whereas, when  $q_{ref} < \alpha - (1 + \lambda)\beta - k\delta$ , bunching occurs at the bottom of the interval, i.e.  $\delta_0 = 0$ .

Since  $\zeta(1) = 1$  and  $\eta(\delta) = 0$  for all  $\delta > \delta_1$ , it must be that  $\zeta(\delta_1) = 1$ , replacing in equation (34), we get the value of  $\delta_1$ :

$$\delta_1 = \min \left\{ \frac{\lambda - \alpha + q_{ref} + (1 + \lambda)\beta}{1 + 2\lambda - k}, 1 \right\}. \tag{36}$$

When  $q_{ref} \leq \alpha + (1 + \lambda)(1 - \beta) - k\delta$ , we obtain  $\delta_1 \leq 1$ , whereas, when  $q_{ref} > \alpha + (1 + \lambda)(1 - \beta) - k\delta$  bunching occurs at the top of interval, i.e,  $\delta_1 = 1$ .

### C.3 When $\delta_0 > 0$ and $\delta \in (0, \delta_0)$

When  $\delta_0 > 0$  and  $\delta \in (0, \delta_0)$ , the participation constraint is not binding then  $\zeta(\delta)$  is constant on this interval, since  $\eta(\delta) = 0$ . From Proposition 1 (ii) we obtain:

$$q(\delta) = \alpha - (1+\lambda)\beta - k\delta + (2\lambda+1)\delta.$$

Integrating the equation (6) we get the firm's utility function:

$$V(\delta) = \int_0^{\delta} q(s)ds = \left[\alpha - (1+\lambda)\beta\right]\delta - \frac{k\delta^2}{2} + (2\lambda+1)\frac{\delta^2}{2} + C,$$

where C is an integration constant. To determine C, we use the continuity of V at  $\delta = \delta_0$ . From point C.1, we know that  $V(\delta_0) = \delta_0 q_{ref}$ . To determine C is:

$$C = \delta_0 q_{ref} - [\alpha - (1+\lambda)\beta] \,\delta_0 + \frac{k\delta_0^2}{2} - (2\lambda + 1)\frac{\delta_0^2}{2},$$

and  $\delta_0$  is given by equation (35), we obtain  $C = \frac{\left[q_{ref} - \alpha + \beta(1+\lambda)\right]^2}{2(1+2\lambda-k)}$ . The firm's utility is:

$$V(\delta) = [\alpha - (1+\lambda)\beta] \, \delta + (2\lambda + 1) \frac{\delta^2}{2} - \frac{k\delta^2}{2} + \frac{[q_{ref} - \alpha + (1+\lambda)\beta]^2}{2(1+2\lambda - k)}.$$

### C.4 When $\delta_1 < 1$ and $\delta \in (\delta_1, 1)$

When  $\delta_1 < 1$  and  $\delta \in (\delta_1, 1)$ , the participation constraint is not binding then  $\zeta(\delta)$  is constant on this interval, since  $\eta(\delta) = 0$ . From Proposition 1 (i) we obtain:

$$q(\delta) = \alpha - (1+\lambda)\beta - k\delta + (2\lambda + 1)\delta - \lambda.$$

Integrating the equation (6) we get the firm's utility function:

$$V(\delta) = \int_{\delta}^{1} q(s)ds = \left[\alpha - (1+\lambda)\beta - \lambda\right]\delta - \frac{k\delta^{2}}{2} + (2\lambda + 1)\frac{\delta^{2}}{2} + C,$$

where C is an integration constant. To determine C, we use the continuity of V at  $\delta = \delta_1$ . From point C.1 we know that  $V(\delta_1) = \delta_1 q_{ref}$ . To determine C is:

$$C = \delta_1 q_{ref} - [\alpha - (1+\lambda)\beta - \lambda] \, \delta_1 + \frac{k\delta_1^2}{2} - (2\lambda + 1) \frac{\delta_1^2}{2},$$

and  $\delta_1$  is given by equation (36), we obtain  $C = \frac{\left[q_{ref} + \lambda - \alpha + \beta(1+\lambda)\right]^2}{2(1+2\lambda-k)}$ . The firm's utility is:

$$V(\delta) = \left[\alpha - (1+\lambda)\beta - \lambda\right]\delta + (2\lambda + 1)\frac{\delta^2}{2} - \frac{k\delta^2}{2} + \frac{\left[q_{ref} + \lambda - \alpha + (1+\lambda)\beta\right]^2}{2(1+2\lambda - k)}.$$

### C.5 The government's participation constraint

The regulator's welfare when the agent's type is  $\delta$  is:

$$W(\delta) = S[q(\delta)] - (1+\lambda)(\beta - \delta)q(\delta) - k\delta q(\delta) - \lambda V(\delta).$$

Differentiating and accounting for the incentive compatibility condition (6) we obtain:

$$W'(\delta) = \{ S'[q(\delta)] - (1+\lambda)(\beta-\delta) - k\delta \} q'(\delta) + (1-k)q(\delta). \tag{37}$$

By the condition (12) we know that:

$$S'[q(\delta)] - (1+\lambda)(\beta - \delta) - k\delta = -\lambda \frac{F(\delta) - \zeta(\delta)}{f(\delta)}.$$

Notice that here  $F(\delta) = \delta$ . Replacing in (37), we obtain:

$$W'(\delta) = (1 - k)q(\delta) - \lambda \left[\delta - \zeta(\delta)\right] q'(\delta).$$

Let us consider the three possible cases:

- 1.  $\delta \in [\delta_0, \delta_1]$ , it easy to check that  $W'(\delta) = (1 k)q_{ref}$ .
- **2.** When  $\delta \in [0, \delta_0]$ , we know that  $\zeta(\delta) = 0$ ,  $q(\delta) = \alpha (1 + \lambda)\beta + (1 + 2\lambda k)\delta$  (see equation (22)) and  $q'(\delta) = 1 + 2\lambda k$ . It follows that

$$W'(\delta) = (1-k)[\alpha - (1+\lambda)\beta + (1+2\lambda - k)\delta] - \lambda\delta(1+2\lambda - k).$$

This is strictly negative when k = 1. When k = 0, this simplifies to  $W'(\delta) = \alpha - (1 + \lambda)\beta + \delta(1 + 2\lambda)(1 - \lambda)$  which is strictly positive under Assumption 2 if  $\lambda < 1$ .

**3.** When  $\delta \in [\delta_1, 1]$ , we know that  $\zeta(\delta) = 1$ ,  $q(\delta) = \alpha - (1 + \lambda)\beta + (1 + 2\lambda - k)\delta - \lambda$  (see equation (25)) and  $q'(\delta) = 1 + 2\lambda - k$ . It follows that

$$W'(\delta) = (1-k)[\alpha - (1+\lambda)\beta + (1+2\lambda - k)\delta - \lambda] + \lambda(1-\delta)(1+2\lambda - k).$$

This is always strictly positive.

Now, (i) When k=1,  $W(\delta)$  takes its minimum value for  $\delta \in [\delta_0, \delta_1]$ , where it equals  $\alpha q_{ref} - \frac{1}{2}q_{ref}^2 - \beta(1+\lambda)q_{ref}$ . In order to guarantee that the participation constraint of the principal is always satisfied it is then enough to assume that  $q_{ref} \leq 2y$  where  $y = \alpha - \beta(1+\lambda)$ .

(ii) When k=0, assuming  $\lambda < 1$  and  $\alpha - (1+\lambda)\beta > 0$  (from Assumption 2),  $W(\delta)$  takes its minimum value for  $\delta = 0$  where it equals  $\frac{1}{2+4\lambda} \left[ -\lambda q_{ref}^2 + y(2\lambda q_{ref} + (1+\lambda)y) \right]$ . It is enough to suppose that  $q_{ref} \leq 2y$  to guarantee that this value is strictly positive and, consequently, that the principal's participation constraint is never binding.

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