

**BASEL II CAPITAL REQUIREMENTS,  
FIRMS HETEROGENEITY, AND THE  
BUSINESS CYCLE**

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# Basel II Capital Requirements, Firms' Heterogeneity, and the Business Cycle

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## Abstract

This paper assesses the potential procyclical effects of Basel II capital requirements by evaluating to what extent those effects depend on the composition of banks' asset portfolios and on how borrowers' credit risk evolves over the business cycle.

By developing a heterogeneous-agent general equilibrium model, in which firms' access to credit depends on their financial position, we find that regulatory capital requirements, by forcing banks to finance a fraction of loans with costly bank capital, have a negative effect on firms' capital accumulation and output in steady state. This effect is amplified with the changeover from Basel I to Basel II, in a stationary equilibrium characterized by a significant fraction of small and highly leveraged firms. In addition, to the extent that it is more costly to raise bank capital in bad times, the introduction of an aggregate technology shock into a partial equilibrium version of the model supports the Basel II procyclicality hypothesis: Basel II capital requirements accentuate the bank loan supply effect underlying the bank capital channel of propagation of exogenous shocks.

*Keywords:* Business Cycles, Procyclicality, Financial Constraints, Bank Capital Channel, Basel II, Heterogeneity.

*JEL Classification Codes:* E44, E32, G28, E10

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# 1 Introduction

The increased sensitivity of a bank's regulatory capital requirements to the risk of its asset portfolio, which underlies the new Basel Capital Accord (Basel II hereafter), has been raising some concerns, at both academic and policy-making levels. In contrast with the bank capital regulation framework established by the Basel Accord of 1988 (Basel I hereafter), the risk weights used to compute bank capital requirements under Basel II are determined by both the institutional category and the riskiness of each particular borrower of the bank: the higher the credit risk of a given bank exposure, the higher the risk weight assigned to that exposure.<sup>1</sup> Consequently, if, during a recession, bank borrowers are downgraded by the credit risk models in use, minimum bank capital requirements will increase. To the extent that it is difficult or costly for banks to raise external capital in bad times, this co-movement in banks' assets credit risk and the business cycle may induce banks to further reduce lending during recessions, thereby amplifying the initial downturn.

The general concern is, thus, that the introduction of Basel II rules leads to the amplification of business cycle fluctuations by reinforcing the bank capital channel, according to which the introduction of bank capital requirements, for market or regulatory reasons, magnifies the real effects of monetary or other aggregate exogenous shocks:<sup>2</sup> if the value of bank capital is sufficiently low, because of loan losses or some other adverse shock, banks may be forced to reduce the supply of loans to satisfy the bank capital requirements, thereby contributing to a worsening of the initial downturn; under Basel II rules, not only will bank capital be lower in the aftermath of a tightening, but more will be needed to maintain capital adequacy, as risk-weighted assets increase. This potential procyclicality of Basel II may then render more difficult for policy makers to maintain macroeconomic stability.

The current financial crisis - being a perfect illustration of the procyclicality of financial systems, as recently argued by Rochet (2008) - called further attention to the potential procyclical effects of the Basel II framework, which, consequently, moved to the agendas of several international forums such as the G7, the G20, the Financial Stability Forum and the Basel Committee on Banking Supervision. A European Union working group was also set up to analyze the potential procyclical consequences of prudential and accounting standards and assess the range of policy responses that could help to mitigate undue potential procyclical effects of the financial system.

In this context, the present work contributes to clarify the role of financial factors on business cycle fluctuations by exploring whether and how the new regulation on bank capital may, in fact, accentuate the procyclical tendencies of banking. We address this question by developing a dynamic heterogeneous-agent model, in which firms differ in their access to bank credit depending on their financial position, that is, depending on their estimated credit risk. Heterogeneity is of interest in

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<sup>1</sup>Under Basel I only the borrower's institutional category is taken into account.

<sup>2</sup>See, for instance, Van den Heuvel (2002a,b).

our work because under Basel II the impact of aggregate shocks on firms' cost of funds is likely to be asymmetric, since, as mentioned before, the risk weights are also determined by the riskiness of each particular borrower and, thus, riskier firms are likely to be more affected. Compared with the representative-firm, a heterogeneous-firm framework thus allows a more accurate inference of the effects associated with the changeover from Basel I to Basel II capital requirements. By introducing risk-sensitive capital requirements into a model with heterogeneous firms, a new framework of analysis opens up, in which we may properly analyze to what extent the distribution of firms over risk contributes to the procyclical tendencies of banking under the new regulation.

Some empirical studies, aiming to infer the potential procyclicality of Basel II, have also motivated our modelization of the bank-borrower relationship under the new regulatory framework in the context of a heterogeneous-agent model. For instance, Kashyap and Stein (2004), who simulate the degree of capital charge cyclicity that would have taken place over the 1998-2002 interval, had the Basel II foundation Internal Ratings Based (IRB) approach been in use, show that Basel II capital requirements have the potential to create an amount of additional cyclicity in capital charges that may be quite large - depending on a bank's customer mix and the credit-risk models that it uses. Altman *et al.* (2005) argue that the procyclical effects of Basel II may be even more severe than expected if banks use their own estimates of loss given default to compute the capital requirements risk weights.

The Basel II procyclicality hypothesis should also depend on the timing of credit risk. According to Segoviano and Lowe (2002), one possible view is that the current performance of the economy can be taken as the best guess of its future performance (the "random walk view"). This view leads to risk being measured as low in an expansion and high in a recession, yielding higher regulatory capital requirements in a downturn than in a boom. As recently pointed out in *The Economist*, in the context of the current financial crisis, regulatory regimes based on market prices implicitly assume that risk goes down when the markets are doing well. However, regulators and market participants were all well aware that too much credit was being doled out too cheaply, although models showed that risk-weighted capital ratios were healthy.<sup>3</sup> In this context, an alternative view on the timing of credit risk, called the "predictability view", suggests that the forces that drive economic booms often contribute to future economic downturns by creating imbalances in both real and financial sectors. This view is thus consistent with the hypothesis that risk increases in the boom but materializes in the downturn, and opens the possibility of measured credit risk being relatively high when times are good, thus attenuating the potential procyclical effects of Basel II.<sup>4</sup>

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<sup>3</sup>The Economist, *Cycle Clips*, May 15th 2008.

<sup>4</sup>The literature identifies other factors that may influence the cyclical effects of Basel II: the approach adopted by banks to compute their minimum capital requirements - the standardized or the IRB approach -, the nature of the rating system used - through-the-cycle or point-in-time rating systems -, the capital buffers over the regulatory minimum held by the banking institutions, the improvements in credit risk management, and the supervisor and market intervention under Pillar 2 and 3 of Basel II. See, for instance, Carpenter *et al.* (2001), Amato and Furfine

Our model contributes to assess these potential procyclical effects, by evaluating to what extent they depend on the composition of banks' asset portfolios and on how borrowers' credit risk evolves over the business cycle. We take Bernanke *et al.* (1999)'s dynamic general equilibrium model as a starting point and we introduce decreasing returns to scale in the production function of each nonfinancial firm. Consequently, in contrast with Bernanke *et al.*, our economy is composed of heterogeneous firms with different levels of credit risk and, thus, facing different conditions when borrowing from banks. We also introduce banks that finance those nonfinancial heterogeneous firms using the funds of a representative household and that are constrained by a risk-based capital requirement according to which the ratio of bank capital to the risk-weighted nonfinancial loans cannot fall below 8%. Whereas under Basel I the capital requirements risk weights are constant and equal to one across all firms, under Basel II the risk weights depend positively on firms' credit risk. Finally, banks are limited in their lending to nonfinancial firms by the amount of bank capital that households are willing to hold, which, due to households' preferences for liquidity, is more expensive to raise than deposits.

In this context, we begin by developing the heterogeneous-agent general equilibrium model in steady state with uncertainty only at the firm level. In the stationary equilibrium firms undergo change both in size and leverage, leading capital requirements risk weights to evolve over time under the new Accord (but not under Basel I). The model allows us to conclude that regulatory capital requirements, by forcing banks to finance a fraction of loans with bank capital, increase banks' loan funding costs and, consequently, banks' lending rates, thereby leading to a lower aggregate amount of loans granted to firms and, thus, to lower physical capital accumulation and aggregate output. In a stationary equilibrium characterized by a significant fraction of high credit risk firms, this loan supply effect is stronger under Basel II than under Basel I. The model also predicts that small and more leveraged firms will lose more with the new risk-sensitive capital requirements, supporting the concerns that have been raised that the new regulation may raise the financing costs of small and medium-sized enterprises - due to banks' perception that these firms are riskier - and the special treatment given to these firms in Basel Committee on Banking Supervision (2004). Additionally, the effects of a permanent increase in the aggregate technology level on the stationary equilibrium indicate the existence of potential procyclical effects of Basel II.

In order to properly assess the likelihood of those procyclical effects, we proceed by simulating an aggregate technology shock and analyzing the transition dynamics. Due to the large number of state variables considered, we adopt a partial equilibrium version of the model, focusing on the bank-borrower relationship, in the absence of households. Assuming a countercyclical required return on bank capital, based on the evidence that raising bank capital tends to be more costly

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(2004), Ayuso *et al.* (2004), Catarineu-Rabell *et al.* (2005), and Drumond (2008) for a review.

during recessions, the model allows us to infer the relevance of the composition of banks' asset portfolios to the potential procyclicality of Basel II, which may be significant even when the perceived average credit risk behaves procyclically. To the extent that it is more costly to raise bank capital in bad times, a negative aggregate technology shock has a large effect on the cost of funds for highly leveraged firms, whose bank loans require more bank capital. In this context, if the loan portfolio of the banking system is characterized by a significant fraction of high credit risk firms, the introduction of an aggregate technology shock in the model supports the Basel II procyclicality hypothesis: the changeover from Basel I to Basel II capital requirements reinforces the loan supply effect which underlies the bank capital channel. Our results are thus in line with Kashyap and Stein (2004). In particular, our model predicts that the Basel II procyclical effect should be greater, the greater the fraction of firms who begin with relatively high leverage ratios, that is, with relatively high credit risk.

## Related Theoretical Literature

Since the introduction of the first Basel Accord some theoretical studies on the relationship between regulatory bank capital requirements and the business cycle have been developed. However, only a few focused on the potential macroeconomic effects of Basel II.

Tanaka (2002) extends a static IS-LM model, in the spirit of Bernanke and Blinder (1988), to introduce the new capital requirements rules by assuming that the risk weights used to compute capital requirements are a function of the mean probability of borrowers' default over the business cycle. The model predicts that an increase in credit risk raises the probability of banks facing a regulatory penalty, thus restricting banks' ability to lend. Therefore, if the credit risk varies with the business cycle, the new regulation may exacerbate macroeconomic fluctuations. The model also suggests that an expansionary monetary policy under Basel II may be less (more) effective during recessions (booms), when credit risk tends to be higher (lower): if banks become undercapitalized during a recession, bank loan supply becomes more insensitive to an expansionary monetary policy, since a lower capital-to-asset ratio restricts banks' ability to increase their risky asset holdings.

Also aiming at capturing the link between loan risk weights and borrowers' creditworthiness, Zicchino (2006) introduces capital requirements risk weights that vary with macroeconomic conditions, in the partial equilibrium model of Chami and Cosimano (2001): capital requirements risk weights depend on the macroeconomic activity, which, in turn, follows a first-order autoregressive stochastic process. Consequently, if banks face binding capital constraints, they will be able to increase their loan supply when times are good but they might be forced to reduce supply during a recession. Zicchino thus concludes that Basel II may lead to a greater reduction of credit following a negative macroeconomic shock: on the top of the loan demand fall, banks may be forced to reduce loan supply to satisfy tighter capital requirements. In order to avoid such an

eventuality, supervisors should, according to Zicchino, encourage banks to build a capital buffer during expansions above the one banks would choose voluntarily.

In fact, and as the model developed by Heid (2007) predicts, although capital buffers optimally set by banks should move procyclically under Basel II - helping to mitigate the impact of the volatility of capital requirements - a significant and stronger procyclical effect may exist, even when banks are not capital constrained, under the new Accord. A similar conclusion is drawn by Repullo and Suarez (2008). While they consider the possibility that banks optimally choose to keep capital buffers (when the value of the on-going lending relationships is large enough and the cost of bank capital is not very large), these capital buffers are insufficient to neutralize Basel II procyclicality: during a recession banks will significantly decrease the supply of credit to some of their dependent borrowers causing a credit crunch that would not occur under Basel I.

Our work differs from and adds to the existing literature by evaluating the potential procyclical effects of Basel II in the context of a heterogeneous-agent model: since one of the central changes of the new regulation is to introduce capital requirements risk weights that depend on the riskiness of each borrower, considering heterogeneous borrowers with different levels of credit risk and analyzing a representative bank's asset portfolio and how it varies with the business cycle is, in our opinion, essential to capture some of the potential cyclical effects of the new Basel Accord.

This paper is organized as follows. After this introduction, Section 2 develops and calibrates a heterogeneous-agent general equilibrium model in steady state. Three variants of the model are considered: the model assuming Basel II capital requirements rules, the model assuming Basel I capital requirements rules and the model without capital requirements. Section 3 simulates an aggregate technology shock under a partial equilibrium version of the model developed in the previous section. In order to analyze the potential procyclical effects of Basel II, we compare the effects of the technology shock under the three variants of the model. Section 4 offers current conclusions and summarizes the state of this research project.

## 2 The Model in Steady State

In this section we develop a dynamic general equilibrium model with no aggregate uncertainty and assuming three types of agents in the economy:

- Entrepreneurs, who own firms that need external finance to buy capital and produce output;
- Banks, which, using the funds of households, finance and monitor (*ex post*) the entrepreneurs;
- Households, who consume and allocate their savings to bank deposits and bank capital.

## 2.1 Entrepreneurs

At each point in time there is a continuum of heterogeneous firms, of total measure one, which have different access to credit depending on their financial position. Each firm is characterized by (i) the amount of physical capital held to produce output, (ii) the price paid per unit of capital, (iii) its net worth and (iv) its idiosyncratic productivity.

In each period each entrepreneur buys the entire capital stock for his firm in order to produce output in the next period, according to the following production function:

$$Y_t^j = \omega_t^j A (K_t^j)^\alpha, \quad (1)$$

where  $K_t^j$  represents the amount of capital bought by each entrepreneur of type  $j$  at time  $t - 1$  and used in production at time  $t$ ,  $A$  represents a common and constant productivity factor, and  $\omega_t^j$  is an idiosyncratic disturbance to the production function, representing the only source of uncertainty for firms in the steady state model. The idiosyncratic shock is independently and identically distributed (i.i.d.) across time and across firms. Following Bernanke, Gertler and Gilchrist (1999), BGG hereafter, we assume that  $\omega^j$  follows a log-normal distribution with  $E(\omega^j) = 1$ .

BGG also assume a constant returns to scale production function, which leads to all firms having the same leverage and, consequently, having a similar access to bank credit. However, and as mentioned by BGG, there is, in practice, considerable heterogeneity across firms, in particular in access to credit. Our model aims precisely to capture this heterogeneity by considering firms with different levels of credit risk and, consequently, facing different conditions when borrowing from banks. Hence, we assume decreasing returns to scale in the production function ( $\alpha < 1$ ), in contrast with BGG.

Finally, each entrepreneur's gross project output, at the end of each period, consists of the sum of his production revenues and the market value of his capital stock. As in Gertler *et al.* (2007), we assume that the idiosyncratic shock affects both the production of new goods and the market value of capital. The shock  $\omega_t^j$  may thus be considered a measure of the quality of the entrepreneur's overall capital investment. Each entrepreneur's gross project output, at the end of time  $t$ , is then given by

$$\omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j,$$

where  $\delta$  is the depreciation rate and  $Q_t^j$  is the price, at the end of time  $t$ , of a unit of capital held by each entrepreneur of type  $j$  (measured in units of household consumption).



## Firms' Demand for Capital and the Cost of Funds

At the end of period  $t$ , each entrepreneur has available net worth  $N_{t+1}^j$ , which he then uses to finance his expenditures on capital goods:  $Q_t^j K_{t+1}^j$ . To finance the difference between capital expenditures and the net worth, each entrepreneur borrows an amount  $L_{t+1}^j = Q_t^j K_{t+1}^j - N_{t+1}^j$  from the bank, which imposes a required return on lending, between  $t$  and  $t + 1$ , of  $R_{t+1}^{Fj}$ .

Each entrepreneur's decision on how much capital to buy,  $K_{t+1}^j$ , depends both on the expected marginal return to capital and on the marginal financing cost.

The expected marginal return to capital at the end of time  $t$ ,  $R_{t+1}^{Kj}$ , comprises both the expected marginal productivity of capital and the expected capital gains or losses:

$$R_{t+1}^{Kj} = \frac{A\alpha (K_{t+1}^j)^{\alpha-1} + E(Q_{t+1}^j)(1-\delta)}{Q_t^j}, \quad (2)$$

where  $E[\cdot]$  refers to expectations taken over the distribution of the idiosyncratic shock.

As in BGG, the relationship between the bank and each entrepreneur embodies an asymmetric information problem: only the entrepreneur observes costlessly the return of his project. In particular, we assume a costly state verification framework, according to which the bank must pay a monitoring cost in order to observe an individual borrower's realized return. This monitoring cost is assumed to equal a proportion  $\mu$  of the entrepreneur's gross project output (net of unexpected capital gains or losses):

$$\mu \left[ \omega_{t+1}^j A (K_{t+1}^j)^\alpha + E(Q_{t+1}^j)(1-\delta) \omega_{t+1}^j K_{t+1}^j \right],$$

where  $0 < \mu < 1$ .

At the end of time  $t$ , each entrepreneur (borrower) and the bank agree on a debt amount,  $L_{t+1}^j$ , and a borrowing rate,  $Z_{t+1}^j$ . At  $t + 1$ , the entrepreneur defaults if  $\omega_{t+1}^j$  is smaller than the default threshold,  $\bar{\omega}_{t+1}^j$ , defined by

$$\bar{\omega}_{t+1}^j \left[ A (K_{t+1}^j)^\alpha + E(Q_{t+1}^j)(1-\delta) K_{t+1}^j \right] = Z_{t+1}^j L_{t+1}^j. \quad (3)$$

To simplify the contracting problem, we assume that the unexpected capital gains or losses, which occur when  $Q_{t+1}^j$  differs from  $E(Q_{t+1}^j)$ , are borne by the entrepreneur.<sup>5</sup>

If  $\omega_{t+1}^j \geq \bar{\omega}_{t+1}^j$ , the borrower pays the lender the amount  $Z_{t+1}^j L_{t+1}^j$  and keeps the remaining. When  $\omega_{t+1}^j < \bar{\omega}_{t+1}^j$ , the borrower defaults while the bank monitors the borrower and receives,

$$(1 - \mu) \left[ \omega_{t+1}^j A (K_{t+1}^j)^\alpha + E(Q_{t+1}^j)(1-\delta) \omega_{t+1}^j K_{t+1}^j \right].$$

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<sup>5</sup>This holds as long as the entrepreneur has enough funds to cover the unexpected capital losses (see Appendix B for details). We are, thus, assuming that the contract is not contingent on the realization of  $Q_{t+1}^j$ .

The contract guarantees the bank an expected gross return on the loan equal to the required return  $R_{t+1}^{Fj}$  (taken as given in the contracting problem). That is, the loan contract established between each borrower and the bank must satisfy

$$\begin{aligned} & [1 - F(\bar{\omega}_{t+1}^j)] Z_{t+1}^j (Q_t^j K_{t+1}^j - N_{t+1}^j) + \\ & + (1 - \mu) \int_0^{\bar{\omega}_{t+1}^j} \left[ \omega_{t+1}^j A(K_{t+1}^j)^\alpha + E(Q_{t+1}^j) (1 - \delta) \omega_{t+1}^j K_{t+1}^j \right] f(\omega) d\omega = \\ & = R_{t+1}^{Fj} (Q_t^j K_{t+1}^j - N_{t+1}^j). \end{aligned}$$

where  $F(\omega)$  and  $f(\omega)$  are, respectively, the cumulative distribution function (c.d.f.) and the probability density function (p.d.f.) of  $\omega$ . Combining the former equation with equation (3) yields

$$[\Gamma(\bar{\omega}_{t+1}^j) - \mu\Theta(\bar{\omega}_{t+1}^j)] \left[ A(K_{t+1}^j)^\alpha + E(Q_{t+1}^j) (1 - \delta) K_{t+1}^j \right] = R_{t+1}^{Fj} (Q_t^j K_{t+1}^j - N_{t+1}^j) \quad (4)$$

where  $\Gamma(\bar{\omega}_{t+1}^j)$  is the expected gross share of profit going to the lender,

$$\Gamma(\bar{\omega}_{t+1}^j) \equiv \int_0^{\bar{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega) d\omega + \bar{\omega}_{t+1}^j \int_{\bar{\omega}_{t+1}^j}^\infty f(\omega) d\omega$$

and  $\mu\Theta(\bar{\omega}_{t+1}^j)$  the expected monitoring costs,

$$\mu\Theta(\bar{\omega}_{t+1}^j) \equiv \mu \int_0^{\bar{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega) d\omega.$$

Therefore,  $\Gamma(\bar{\omega}_{t+1}^j) - \mu\Theta(\bar{\omega}_{t+1}^j)$  represents the net share of profits going to the lender and  $[1 - \Gamma(\bar{\omega}_{t+1}^j)]$  the share going to the entrepreneur, as in BGG.

The contract determines the division of the expected gross project output between the borrower and the lender and results from the maximization of the borrower's expected payoff, with respect to  $K_{t+1}^j$  and  $\bar{\omega}_{t+1}^j$ , subject to equation (4). The first order conditions (FOCs) derived from the contracting problem yield, in turn, the following equations (see Appendix A for details):

$$\begin{aligned} & [1 - \Gamma(\bar{\omega}_{t+1}^j)] l_{t+1}^j - \frac{\Gamma'(\bar{\omega}_{t+1}^j)}{\Gamma'(\bar{\omega}_{t+1}^j) - \mu\Theta'(\bar{\omega}_{t+1}^j)} \frac{1}{k_{t+1}^j} + [1 - \Gamma(\bar{\omega}_{t+1}^j)] \alpha(1 - \alpha) A(K_{t+1}^j)^{\alpha-1} \frac{1}{R_{t+1}^{Fj} Q_t^j} + \\ & + \frac{\Gamma'(\bar{\omega}_{t+1}^j)}{\Gamma'(\bar{\omega}_{t+1}^j) - \mu\Theta'(\bar{\omega}_{t+1}^j)} [\Gamma(\bar{\omega}_{t+1}^j) - \mu\Theta(\bar{\omega}_{t+1}^j)] A(K_{t+1}^j)^{\alpha-1} (1 - \alpha)(\alpha - 1) \frac{1}{R_{t+1}^{Fj} Q_t^j} = 0 \quad (5) \end{aligned}$$

and

$$[\Gamma(\bar{\omega}_{t+1}^j) - \mu\Theta(\bar{\omega}_{t+1}^j)] l_{t+1}^j + [\Gamma(\bar{\omega}_{t+1}^j) - \mu\Theta(\bar{\omega}_{t+1}^j)] (1 - \alpha) A (K_{t+1}^j)^{\alpha-1} \frac{1}{R_{t+1}^{Fj} Q_t^j} - 1 + \frac{1}{k_{t+1}^j} = 0, \quad (6)$$

where  $l_{t+1}^j \equiv R_{t+1}^{Kj}/R_{t+1}^{Fj}$  (external finance premium faced by firms of type  $j$ , as defined by BGG) and  $k_{t+1}^j \equiv Q_t^j K_{t+1}^j/N_{t+1}^j$  (ratio of capital expenditures to net worth of type  $j$  firms). As we assume decreasing returns to scale, the cutoff value,  $\bar{\omega}$ , varies across firms, in contrast with BGG: borrowers have different ratios of capital expenditures to net worth and, consequently, different cutoff values for  $\omega$ . Figure 1 shows the (positive) relationship between these two variables, delivered by the financial contract calibrated for model analysis in 2.5: more leveraged firms face higher probability of default. In line with BGG, the financial contract also implies that, for a given price of capital and level of net worth, the external finance premium faced by leveraged firms increases with the capital stock. Figure 2 illustrates this relationship and shows that an increase in the firm's net worth, by improving the firm's financial position, causes a rightward shift in the external-finance-premium curve: an increase in net worth relative to the capital stock reduces the expected probability of default and, consequently, the external finance premium faced by the firm.

## Entrepreneurial Net Worth

As a technical matter, it is necessary to start entrepreneurs off with some net worth in order to allow them to begin operations. We assume that, in each period, each entrepreneur receives a transfer of net worth,  $W^e$ . The total net worth of entrepreneurs thus combines profits accumulated from previous capital investment and the endowment  $W^e$ .

To avoid the possibility that the entrepreneurial sector accumulates enough net worth to be fully self-financed, we assume that each entrepreneur consumes, in every period, a constant fraction  $(1 - \gamma)$  of his resources.<sup>6</sup> Therefore, the net worth ( $N_{t+1}^j$ ) and consumption ( $C_t^{ej}$ ) of each entrepreneur, at the end of time  $t$ , are defined as follows.<sup>7</sup>

a) If  $\omega_t^j \geq \bar{\omega}_t^j$ , the borrower pays the lender the amount  $Z_t^j L_t^j = \bar{\omega}_t^j \left[ A (K_t^j)^\alpha + E (Q_t^j) (1 - \delta) K_t^j \right]$  and keeps the remaining:

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<sup>6</sup>Alternatively, we could assume that entrepreneurs had finite horizons. We did not consider this possibility for simplicity, avoiding the exit and entry of firms.

<sup>7</sup>As a technical matter, under both hypotheses a) and b) we consider that  $N_{t+1}^j \geq \gamma W^e$ . Therefore, if, for instance,  $Q_t^j < E (Q_t^j)$  under b), we assume that the entrepreneur pays the bank  $\omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j$  and keeps the remaining  $(\gamma W^e)$ .

$$N_{t+1}^j = \gamma \left\{ \omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j + W^e - \bar{\omega}_t^j \left[ A (K_t^j)^\alpha + E (Q_t^j) (1 - \delta) K_t^j \right] \right\} \quad (7)$$

$$C_t^{ej} = (1 - \gamma) \left\{ \omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j + W^e - \bar{\omega}_t^j \left[ A (K_t^j)^\alpha + E (Q_t^j) (1 - \delta) K_t^j \right] \right\}. \quad (8)$$

b) If  $\omega_t^j < \bar{\omega}_t^j$ , the borrower pays the lender the amount  $\omega_t^j A (K_t^j)^\alpha + E (Q_t^j) (1 - \delta) \omega_t^j K_t^j$  and keeps the remaining:

$$N_{t+1}^j = \gamma \left\{ [Q_t^j - E (Q_t^j)] (1 - \delta) \omega_t^j K_t^j + W^e \right\} \quad (9)$$

$$C_t^{ej} = (1 - \gamma) \left\{ [Q_t^j - E (Q_t^j)] (1 - \delta) \omega_t^j K_t^j + W^e \right\}. \quad (10)$$

### Capital Producers

Following BGG, we specify each entrepreneur's investment decision under external capital adjustment costs. We depart, however, from BGG's model by introducing a specific capital producer for each entrepreneur (the capital producer is like a division within the manufacturer). In particular, an entrepreneur of type  $j$  sells his entire stock of capital,  $K_t^j$ , at the end of each period  $t$  to the capital producing firm associated with his firm. This capital producer also purchases raw output as an input and combines it with  $K_t^j$  to produce new capital goods via the production function  $\Xi \left( \frac{I_t^j}{K_t^j} \right) K_t^j$ , where  $\Xi(\cdot)$  is an increasing and concave function and  $I_t^j$  represents the entrepreneur's investment at time  $t$ . The new capital goods, jointly with the capital used to produce them, are then sold to the entrepreneur at the price  $Q_t^j$ . The capital stock of each firm of type  $j$  thus evolves according to:

$$K_{t+1}^j = \Xi \left( \frac{I_t^j}{K_t^j} \right) K_t^j + (1 - \delta) K_t^j, \quad (11)$$

and the FOC for investment for the capital producer yields

$$Q_t^j = \frac{1}{\Xi' \left( \frac{I_t^j}{K_t^j} \right)}. \quad (12)$$

Following Jermann (1998) and Boldrin *et al.* (2001), we assume that the capital adjustment cost function  $\Xi(\cdot)$  takes the following form

$$\Xi \left( \frac{I_t^j}{K_t^j} \right) = \frac{a_1}{1 - \frac{1}{\varphi}} \left( \frac{I_t^j}{K_t^j} \right)^{1 - \frac{1}{\varphi}} + a_2 \quad (13)$$

where  $\varphi (> 0)$  is the elasticity of the ratio of investment to the capital stock with respect to the price of capital, and  $a_1$  and  $a_2$  are two constants. Therefore, equation (12) may be rewritten as

$$Q_t^j = \frac{1}{a_1} \left[ \left( \frac{K_{t+1}^j}{K_t^j} - (1 - \delta) - a_2 \right) \frac{1 - \frac{1}{\varphi}}{a_1} \right]^{\frac{1}{\varphi-1}}. \quad (14)$$

## 2.2 Banks

Financial intermediation, consisting of collecting funds from households (deposits and bank capital) and granting loans to entrepreneurs, is assured by banks, which are legally subject to a risk-based regulatory capital requirement. The asset side of a bank's balance sheet includes loans granted to firms, whereas the liability side comprises deposits and bank capital. In line with the contract established between the representative bank and each entrepreneur, banks' assets and liabilities have the same, one period, maturity.

Following a simplified version of Basel II capital requirements rules, banks are required to raise at least a minimum amount of bank capital, determined by the amount of loans granted to firms and by the perceived credit risk of those firms. That is, we assume that the minimum amount of bank capital that each bank has to raise depends on the estimated credit risk of its loan portfolio, as specified by the following equation

$$S_{t+1} \geq 0.08 \int \alpha_{e_{t+1}}^j L_{t+1}^j d\Upsilon_{t+1}, \quad (15)$$

where  $S_{t+1}$  is the bank capital issued by the bank and held by households between  $t$  and  $t + 1$ ,  $L_{t+1}^j$  is the loan granted, at the end of time  $t$ , to firms of type  $j$ ,  $\alpha_{e_{t+1}}^j$  is the credit risk weight associated with type  $j$  firms, at the end of time  $t$ , and  $\Upsilon_{t+1}$  is the distribution of firms over the state space  $(N, K, Q, \omega)$ , at the end of time  $t$ .

Under Basel I,  $\alpha_{e_{t+1}}^j$  is constant and equal to one across all commercial and industrial loans. Under Basel II, the risk weights in the capital requirements constraint depend positively on the estimated credit risk of each exposure. According to our model, firms default on the loan if the idiosyncratic disturbance,  $\omega_{t+1}^j$ , turns out to be smaller than the cutoff value,  $\bar{\omega}_{t+1}^j$ . Therefore, the higher the cutoff value, the higher the probability of default -  $prob(\omega_{t+1}^j < \bar{\omega}_{t+1}^j)$ . The risk weights, under Basel II, should thus depend positively on the cutoff value,  $\bar{\omega}_{t+1}^j$ .

As mentioned in 2.1, the calibrated financial contract delivers a positive relationship between  $\bar{\omega}_{t+1}^j$  and the ratio of capital expenditures to net worth,  $k_{t+1}^j$ . Therefore, we consider that the Basel II risk weights ( $\alpha_{e_{t+1}}^j$ ) depend positively (and linearly, by assumption) on  $k_{t+1}^j$ , as follows:

$$\alpha_{e_{t+1}}^j = \begin{cases} a + bk_{t+1}^j, & \text{if } k_{t+1}^j > -\frac{a}{b} \\ 0, & \text{if } k_{t+1}^j \leq -\frac{a}{b} \end{cases} \quad (16)$$

where  $a$  and  $b$  are two constants (with  $b > 0$ ).

For simplicity, we assume that banks are allowed to issue bank capital at any time, on terms that depend on households' willingness to hold bank capital in addition to deposits. Since bank capital is more expensive to raise than deposits, due to households' preference for liquidity (as detailed below, in 2.3), the capital requirements constraint (15) is always binding.

The representative bank maximizes its expected profits, acting as a price (interest rate) taker. Its choice variables are loans, deposits and bank capital. The bank's objective is then given by:

$$\begin{aligned} & \max_{L_{t+1}^j, D_{t+1}, S_{t+1}} \int R_{t+1}^{Fj} L_{t+1}^j d\Upsilon_{t+1} - R_{t+1}^D D_{t+1} - R_{t+1}^S S_{t+1} \\ \text{s.t. } & \int L_{t+1}^j d\Upsilon_{t+1} = D_{t+1} + S_{t+1} \text{ (balance sheet constraint)} \\ & \frac{S_{t+1}}{\int \alpha_{e_{t+1}}^j L_{t+1}^j d\Upsilon_{t+1}} = 0.08 \text{ (capital requirements constraint)}, \end{aligned} \quad (17)$$

where  $D_{t+1}$  represents households' deposits from  $t$  to  $t+1$ ,  $R_{t+1}^{Fj}$  the required return on loans granted by the bank to firms of type  $j$ , between  $t$  and  $t+1$ ,  $R_{t+1}^D$  the gross return on deposits, and  $R_{t+1}^S$  the gross return on bank capital.

The comparison between Basel I and Basel II regulatory frameworks is straightforward:

**a)** Under Basel II,

$$\alpha_{e_{t+1}}^j = a + bk_{t+1}^j \Leftrightarrow \alpha_{e_{t+1}}^j = a + b \frac{Q_{t+1}^j K_{t+1}^j}{N_{t+1}^j} \Leftrightarrow \alpha_{e_{t+1}}^j = a + b \left( \frac{L_{t+1}^j}{N_{t+1}^j} + 1 \right).$$

Therefore, taking into account that  $k_{t+1}^j$  depends on the loan granted to the firm, the capital requirements constraint in the bank's objective may be rewritten as

$$S_{t+1} = 0.08 \int \left[ a + b \left( \frac{L_{t+1}^j}{N_{t+1}^j} + 1 \right) \right] L_{t+1}^j d\Upsilon_{t+1},$$

and the FOCs of the interior solution of problem (17) yield

$$R_{t+1}^{Fj} = [1 - 0.08 (a - b + 2bk_{t+1}^j)] R_{t+1}^D + 0.08 (a - b + 2bk_{t+1}^j) R_{t+1}^S. \quad (18)$$

The required return on loans by the bank is thus a weighted average of the gross return on deposits and the gross return on bank capital. The weights depend on firms' type: the higher the ratio of capital expenditures to net worth (that is, the higher the credit risk of the firm), the higher the weight associated with  $R_{t+1}^S$ , since a larger fraction of loans must be financed with bank

capital.

b) Under Basel I,  $\alpha_{e_{t+1}}^j = 1, \forall j, t$ , and the FOCs of the interior solution of problem (17) yield

$$R_{t+1}^{Fj} = (1 - 0.08)R_{t+1}^D + 0.08R_{t+1}^S, \forall j. \quad (19)$$

The required return on loans by the bank is, as before, a weighted average of the gross return on deposits and the gross return on bank capital. However, the weights are now constant and do not depend on firms' type. Consequently, all firms face the same required return on lending, in contrast with Basel II.

We also build a third variant of the model assuming no regulatory capital requirements. In this case, and since bank capital is more expensive to raise than deposits, the bank finances all loans with deposits:  $S_{t+1} = 0$  and  $R_{t+1}^{Fj} = R_{t+1}^D$ , for any  $j$ .

## 2.3 Households

The economy is composed of a continuum of infinitely lived identical risk averse households of length unity. Each household consumes and allocates its savings to assets which include deposits, that pay a riskless rate of return between  $t$  and  $t+1$  equal to  $R_{t+1}^D$ , and shares of ownership of banks in the economy, that pay  $R_{t+1}^S$ . For simplicity, labor is absent from our model. The representative household's instantaneous utility function is separable in consumption and liquidity (in the form of deposits) and given by:

$$U(C_t, D_{t+1}) = \frac{C_t^{1-\sigma}}{1-\sigma} + \alpha_0 \frac{D_{t+1}^{1-\beta_0}}{1-\beta_0}, \quad (20)$$

where  $C_t$  denotes household consumption at time  $t$  and  $D_{t+1}$  the deposits held by the household from  $t$  to  $t+1$ .

The level of deposits is included in the instantaneous utility function to indicate the existence of liquidity services from wealth held in the form of that asset. In short, we are assuming that, when compared to bank capital, deposits have an advantage in terms of liquidity, similarly to Poterba and Rotemberg (1987) and, in the context of the bank capital channel literature, Aguiar and Drumond (2007) and Van den Heuvel (2008).

The representative household chooses consumption and its asset portfolio to maximize the expected lifetime utility (appropriately discounted) subject to an intertemporal budget constraint. The household's optimization problem is then given by

$$\begin{aligned} \max_{C_t, D_{t+1}, S_{t+1}} E_t \sum_{k=0}^{\infty} \beta^k \left[ \frac{(C_{t+k})^{1-\sigma}}{1-\sigma} + \alpha_0 \frac{(D_{t+k+1})^{1-\beta_0}}{1-\beta_0} \right] \\ \text{s.t. } C_t = R_t^D D_t - D_{t+1} + R_t^S S_t - S_{t+1} + \Pi_t^B, \end{aligned} \quad (21)$$

where  $\beta \in (0, 1)$  is the subjective discount factor and  $\Pi_t^B$  are profits received from ownership of banks.

The FOCs with respect to  $D_{t+1}$  and  $S_{t+1}$  are the following:

$$\begin{aligned} (C_t)^{-\sigma} &= \beta R_{t+1}^D E_t [(C_{t+1})^{-\sigma}] + \alpha_0 D_{t+1}^{-\beta_0}, \\ (C_t)^{-\sigma} &= \beta \{ E_t [R_{t+1}^S (C_{t+1})^{-\sigma}] \}. \end{aligned}$$

In steady state there is no aggregate uncertainty and  $C_t = C_{t+1} = C$ . Therefore, assuming  $\sigma = \beta_0$ , the FOCs become,

$$1 = \beta R^D + \alpha_0 \left( \frac{C}{D} \right)^{\sigma}, \quad (22)$$

$$1 = \beta R^S. \quad (23)$$

Since  $\alpha_0 (C/D)^{\sigma}$  is strictly positive,  $R^S$  exceeds  $R^D$ , that is, the representative household, due to its preferences for liquidity, requires a liquidity premium,  $R^S - R^D$ , in order to hold bank capital in its portfolio.<sup>8</sup>

## 2.4 Equilibrium

A stationary equilibrium for this economy consists of:<sup>9</sup> decision rules  $C = C(D, S; \Upsilon)$ ,  $D = D(D, S; \Upsilon)$ ,  $S = S(D, S; \Upsilon)$  for the representative household; a decision rule  $K' = K(K, Q, N, \omega)$  for firms; a law of motion for firms' net worth,  $N' = N(K, Q, N, \omega)$ ; a decision rule  $Q' = Q(K, Q, N, \omega)$  for the capital producing division of each firm; equilibrium prices  $(R^K, R^F)$ , for each type of firm, and  $(R^D, R^S)$ ; and a stationary distribution  $\Upsilon(K, Q, N, \omega)$ , such that (i) the consumer decision rules solve problem (21); (ii)  $K' = K(K, Q, N, \omega)$  satisfies equation (2) and

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<sup>8</sup>As mentioned, we also consider a variant of the model in which banks do not face regulatory capital requirements and, thus, optimally choose to finance all the loans with deposits. In this case, there is no bank capital in the model and households allocate all their savings to deposits. Therefore, we set  $\alpha_0$  equal to zero and the Euler equation becomes  $1 = \beta R^D$ .

<sup>9</sup>A variable with the superscript ' refers to its end-of-period value. To simplify the notation we now drop the  $j$  superscript.



solves the contract problem (26); (iii)  $N' = N(K, Q, N, \omega)$  satisfies equations (7) and (9); (iv)  $Q' = Q(K, Q, N, \omega)$  satisfies equations (11) and (14); (v) the required return on lending by the bank,  $R^F$ , satisfies equation (18), under Basel II (or equation 19, under Basel I); (vi) the bank's balance sheet and the capital requirements constraint are satisfied; (vii) the markets clear,

$$\int L' d\Upsilon = \int (Q'K' - N') d\Upsilon,$$

$$Y + W^e + \int [Q'(1 - \delta)\omega K] d\Upsilon = C + C^e + \left[ \int (Q'K') d\Upsilon \right] + \text{Monitoring Costs}, \quad (24)$$

where  $Y$  denotes the aggregate output and  $C^e$  the aggregate entrepreneurial consumption; (viii) the stationary distribution  $\Upsilon(K, Q, N, \omega)$  is consistent with  $K(K, Q, N, \omega)$ ,  $N(K, Q, N, \omega)$ ,  $Q(K, Q, N, \omega)$  and the distribution of the idiosyncratic shock.<sup>10</sup>

## 2.5 Calibration

We calibrate the model assuming that a period is a quarter. The coefficient associated with deposits in the utility function (20),  $\alpha_0$ , is set such that, in steady state and under Basel I,  $R^D = 1.01(01)$ , as assumed in many other business cycle models, including BGG, for the riskless real rate of return, since it guarantees an average riskless interest rate of 4% per year.

In line with Levin *et al.* (2004), we consider a higher volatility of the idiosyncratic risk than BGG. We thus guarantee that, under decreasing returns to scale, all leveraged firms face an external finance premium higher than one.

The remaining parameters satisfy the following requirements.

(i) The fraction of wealth consumed by each entrepreneur at the end of each period,  $1 - \gamma$ , the common productivity factor,  $A$ , and the parameter  $\alpha$  in the production function (1) are set such that, in steady state:

- The fraction of self-financed firms is small (around 8% in steady state), as our model focuses on the behavior of leveraged firms;
- The firms' stationary distribution over net worth (size) is skewed to the right, that is, is skewed toward small firms, which, according to Cooley and Quadrini (2001, 2006), is an empirical regularity of the data;
- The average leverage ratio, measured by the average ratio of loans to capital expenditures, is close to 0.5, as the evidence reported by Rajan and Zingales (1995) suggests.

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<sup>10</sup>The aggregate consistency condition (24) is derived in Appendix B.

(ii) The capital adjustment costs parameters,  $a_1$  and  $a_2$ , in equation (14) are set such that when this equation is considered in aggregate terms and evaluated in the steady state,  $Q_t$  equals one and  $\Xi\left(\frac{I_{SS}}{K_{SS}}\right) = \frac{I_{SS}}{K_{SS}} = \delta$ . As mentioned by Jermann (1998), empirical studies do not offer precise evidence when it comes to calibrate the elasticity of the investment to capital ratio with respect to the price of capital ( $\varphi$ ). Jermann sets this parameter to 0.23, in order to maximize his model's ability to match a set of moments of interest. In a more recent paper, Christiano and Davis (2006) argue that data suggest  $\varphi$  of unity or less. Based on these studies, we assume two alternative values for  $\varphi$ :  $\varphi = 0.25$  and  $\varphi = 0.9$ . Since the conclusions do not change significantly, we only present the results with  $\varphi = 0.9$ .

(iii) The parameters underlying the relationship between the capital requirements risk weight ( $\alpha_e$ ) and the ratio of firm's capital expenditures to net worth ( $k$ ), under Basel II - see equation (16) - were calibrated such that a zero risk weight is assigned to firms with  $k = 1$  and a maximum risk weight of 2 is assigned to firms with  $k = 3$ . All firms with  $k > 3$  are assigned the maximum level of  $\alpha_e$  ( $= 2$ ), thus avoiding unrealistically high values of  $\alpha_e$ , according to rules specified by Basel II.<sup>11</sup>

For further details on the model's calibration see Appendix C. We then solve numerically the model, for the steady state, using the computational procedure also described in Appendix C.

## 2.6 Results

Before introducing an aggregate shock in the model to test the potential procyclical effects of Basel II, we first describe the firms' dynamics generated by the model in steady state. Three variants of the model are considered:

- Variant 1: the model assuming Basel II capital requirements rules;
- Variant 2: the model assuming Basel I capital requirements rules;
- Variant 3: the model without capital requirements.

Figure 3 plots the distribution of firms over size (net worth) and capital stock, showing that this economy, under the three variants of the model, is characterized by small and leveraged firms in steady state: having access to bank credit, firms are able to accumulate a significant amount of capital when compared to their size.<sup>12</sup>

Table 1 reports the steady state values of some key variables of the model. Comparing variant 3 with variants 1 and 2 allows us to conclude that regulatory capital requirements have a negative

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<sup>11</sup>See, for instance, the illustrative risk weights calculated for four asset classes types in the Annex 3 of Basel Committee on Banking Supervision (2004).

<sup>12</sup>Recall that the model was calibrated in order to generate a firms' stationary distribution over net worth skewed toward small firms.

effect on capital accumulation and, consequently, on firms' production - the steady state output in variant 1 (2) is 3.1% (1.9%) smaller than in variant 3. In both variants 1 and 2, banks are required to finance a fraction of loans with bank capital, which is more expensive to raise than deposits, due to households' preferences for liquidity ( $R^S > R^D$ ). This additional cost is passed on to firms through an increase in the average required return on lending across borrowers,  $R^F$ . The total amount of loans granted to firms, and, consequently, firms' capital accumulation and output are thus smaller in variants 1 and 2.

	<b>Variant 1</b> (Basel II)	<b>Variant 2</b> (Basel I)	<b>Variant 3</b> (No Capital Req.)
Aggregate Output	0.2251	0.2278	0.2323
Aggregate Capital Stock	2.4842	2.5175	2.5726
Aggregate Net Worth	1.2734	1.2781	1.2889
Average Leverage Ratio (borrowers)	0.5360	0.5394	0.5451
Average Ratio of Cap. Expend. to Net Worth	2.8563	2.8938	2.9540
Aggregate Loans	1.3041	1.3297	1.3737
% of Borrowers	0.9154	0.9160	0.9175
$R^D$	1.0090	1.01(01)	1.01(01)
$R^S$	1.0185	1.0185	—
Average $R^F$ (across borrowers)	1.0112	1.0108	1.01(01)
Average Capital Requirements Risk Weight	1.4206	1.0000	—
Aggregate Bank Capital/Aggregate Loans	0.1304	0.0800	—

Table 1: Key variables of the model in steady state

Table 1 also indicates that firms' net worth is smaller in variants 1 and 2. However, the differences across the three variants of the model are less significant with respect to this variable, when compared, for instance, with differences in output or in capital stock (see also Figure 3).

As described in 2.2, for a given amount of loans, the minimum amount of capital that banks must raise, in variant 1, is increasing in the capital requirements risk weights, which depend positively on borrowers' credit risk (proxied by the ratio of firms' capital expenditures to net worth). In contrast, under variant 2 the risk weights are constant and equal to one across all firms. The last two rows of Table 1 show that the average capital requirements risk weight in variant 1 is higher than in variant 2, leading, in turn, to a higher ratio of bank capital to loans, despite the decrease in borrowers' average leverage ratio with the changeover from Basel I to Basel II capital requirements rules. That is, the stationary distribution of firms in this economy is characterized by highly leveraged firms, thereby leading to a high level of average credit risk in steady state. Consequently, under Basel II, the representative bank must finance a higher proportion of loans with bank capital. As bank capital is more expensive to raise than deposits, the average financing cost faced by firms ( $R^F$ ) is higher under Basel II, leading to smaller steady state levels of aggregate loans, capital accumulation, and, consequently, aggregate output. This

result is in line with Zhu (2008), according to whom the impact of the changeover from Basel I to Basel II capital requirements may differ substantially across banks depending on the risk profile of their loan portfolios: Basel II leads to a higher ratio of bank capital to loans for small and also more risky banks.

## Firms' Dynamics in Steady State

Figures 4 and 5 show the key properties of the firms' dynamics generated by the calibrated steady state model. It is straightforward to conclude that, except for the required return on lending and the capital requirements risk weights, firms' dynamics in steady state do not vary significantly across variants 1, 2 and 3 (Basel II, Basel I and No Capital Requirements, respectively).

Figure 4 reports typical decision rules for net worth and physical capital. We conclude that, for a given value of capital expenditures,  $N_{t+1}^j$  (firm's net worth at the end of time  $t$ ) is increasing in both  $N_t^j$  and the idiosyncratic shock,  $\omega_t^j$ . The capital stock is also increasing in the idiosyncratic shock (although this effect is imperceptible in the figure) and, due to capital adjustment costs, changes slowly over time. As expected (see equation 14), for a given value of  $N_t^j$  and  $Q_{t-1}^j$ , the price of capital at time  $t$  ( $Q_t^j$ ) is decreasing in  $K_t^j$  and increasing in the idiosyncratic shock.<sup>13</sup>

Figures 4 (d) and 5 report some unconditional moments, computed by averaging some key variables of the model using the firms' stationary distribution,  $\Upsilon$ . The main properties of firms' behavior may be summarized as follows:<sup>14</sup>

1. Small firms are more leveraged and face higher probability of default (as depicted in Figure 5, panels b and c);
2. Small firms face higher external finance premium (and higher required return on lending, under Basel II);
3. The growth rate of firms' net worth is decreasing in size.<sup>15</sup>

Figures 4 (d) and 5 (a and b) show that, although capital expenditures increase with firms' size, small firms take on more debt: small firms borrow more and, consequently, have higher ratios of capital expenditures to net worth. As depicted in Figure 4 (d), firms' capital expenditures increase with firms' size but at a slower pace than net worth.

A higher ratio of capital expenditures to net worth translates into a higher expected probability of default, as predicted by the contract established between each firm and the bank. Consequently, the capital requirements risk weights in variant 1 are, on average, higher for those firms - see Figure 5 (d). Finally, small firms, having higher probability of default, face higher external finance premia, as illustrated by Figure 5 (e).

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<sup>13</sup>For simplicity, we only plot the dynamics under Basel I in Figure 4 (c).

<sup>14</sup>These properties are in line with Cooley and Quadrini (2001)'s results.

<sup>15</sup>Not shown in the figure for brevity and because it is not essential for our purposes, but available upon request.

Figure 5 also reports the relationship between the average required return on lending by the bank,  $R^F$ , and the average ratio of capital expenditures to net worth,  $k$ . In variant 1, and in contrast with the other two variants of the model,  $R^F$  increases with  $k$ . Recall that under Basel II the required return on a loan granted to a particular firm is a weighted average of the return on deposits,  $R^D$ , and the return on bank capital,  $R^S$  (with  $R^S > R^D$ , due to households' preferences for liquidity) - see equation (18). The higher the leverage of the firms (which proxies for the credit risk), the higher the fraction of bank loans that must be financed with bank capital and, thus, the higher the weight associated with  $R^S$  and the higher the financing cost,  $R^F$ . Since small firms, in our model, are more leveraged, they face a higher  $R^F$ . That is,  $R^F$  increases with  $k$  and, consequently, decreases, on average, with firms' size.

Indeed, and as Figure 5 (d) illustrates, small firms are assigned higher capital requirements risk weights in variant 1 and, consequently, face a higher required return on lending,  $R^F$ . In variant 2 the required return on lending by the bank does not depend on firms' type (see equation 19), thus being independent of firms' leverage and size. Figure 5 (f) also allows us to conclude that only the less leveraged firms (those with a ratio of capital expenditures to net worth smaller than 2.1, approximately) benefit with the changeover from Basel I to Basel II rules: the required return on lending is smaller for those firms under the latter regulatory framework. The distribution of firms over leverage is thus essential to evaluate the effects of Basel II rules, as will become clearer in Section 3.<sup>16</sup>

## Changing the Common Productivity Factor in the Steady State Model

We also solved the model for the steady state assuming a higher value for the common productivity factor:  $A = 0.101$  (1% increase). Table 2 shows the impact of this change on the same variables reported in Table 1.

As expected, a higher common productivity factor leads to a higher level of aggregate steady state output (which increased by 4.3%, 4.2% and 3.7%, in variant 1, 2 and 3, respectively). The increase in output is due, not only to the higher common productivity factor, but also to the positive effect of  $A$  on capital accumulation. In fact, we find that a higher level of  $A$  implies a clear rightward shift of the stationary distribution of firms over the capital stock.<sup>17</sup>

The steady state firms' aggregate net worth also increases with  $A$ , but at a smaller extent than the aggregate capital expenditures. Therefore, firms become more leveraged, on average, in the new steady state.

The preceding results may be explained through the analysis of household and bank behavior,

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<sup>16</sup>The required return on lending faced by firms with  $k < 1.7$ , approximately, is smaller in variant 1 (Basel II) than in variant 3 (model with no capital requirements), since, according to our simulations, the steady state return on deposits,  $R^D$ , is also smaller in variant 1 (see Table 1).

<sup>17</sup>Evidence not shown here for brevity, but available upon request.

	<b>Variant 1</b> (Basel II)	<b>Variant 2</b> (Basel I)	<b>Variant 3</b> (No Capital Req.)
Aggregate Output	4.280	4.169	3.684
Aggregate Capital Stock	3.613	3.491	2.955
Aggregate Net Worth	1.482	1.466	1.276
Average Leverage Ratio (borrowers)	1.710	1.644	1.347
Average Ratio of Cap. Expend. to Net Worth	3.374	3.307	2.790
Aggregate Loans	5.411	5.279	4.421
% of Borrowers	0.249	0.246	0.202
$R^D$	-0.024	-0.019	0
$R^S$	0	0	—
Average $R^F$ (across borrowers)	-0.016	-0.018	0
Average Capital Requirements Risk Weight	1.297	0	—
Aggregate Bank Capital/Aggregate Loans	0.950	0	—

Table 2: Percentage deviations from the previous steady state values

as follows. As reported in Table 2, the return on deposits, in both variants 1 and 2, is smaller in the new steady state:<sup>18</sup> the increase in  $A$  leads to an increase in the steady state ratio of the household's consumption to deposits and, consequently, to a decrease in  $R^D$  (see equation 22 in 2.3). Therefore, the average required return on lending by the bank,  $R^F$ , is also smaller in both variants (see Table 2).<sup>19</sup> A smaller cost of financing leads, in turn, to a higher amount of loans granted to firms, stimulating capital accumulation. Finally, note that the increase in aggregate loans, capital accumulation and output is slightly stronger in variant 1 than in variant 2, indicating the existence of potentially stronger procyclical effects associated with Basel II capital requirements.

We may thus summarize the following conclusions from the steady state model:

- The introduction of regulatory capital requirements has a negative supply effect on bank lending. The financing cost is higher, on average, in the presence of capital requirements, leading to a smaller aggregate amount of loans granted to firms, which has a negative effect on firms' capital accumulation and output;
- In a steady state equilibrium characterized by a significant fraction of high credit risk firms, the former effect is stronger under Basel II capital requirements;
- The financing cost faced by small firms is higher, under Basel II, due to banks' perception that these firms are riskier and, hence, carry higher capital requirements than under Basel I;
- A higher common productivity factor has positive effects on steady state aggregate output, especially under the new regulatory framework.

<sup>18</sup>Since the discount factor,  $\beta$ , does not change with  $A$ ,  $R^S$  (in variants 1 and 2) and  $R^D$  (in variant 3) remain constant.

<sup>19</sup>Under Basel II, the effect of the decrease in  $R^D$  on  $R^F$  exceeds the effect of the increase in the average risk weight associated with  $R^S$  (caused by the increase in the average leverage ratio).

### 3 Introducing an Aggregate Technology Shock

We now introduce an aggregate technology shock in the model in order to analyze the effects on cyclical fluctuations of the changeover from Basel I to Basel II capital requirements rules. In particular, we aim to compare the impact of an aggregate technology shock across the three variants of the model considered before.

In contrast with the previous section, where the aggregate productivity factor  $A$  was assumed to be constant, we now introduce a temporary negative shock, which leads to a 1% decrease in  $A$ , at the beginning of period 1. This variable then gradually converges to its steady state value following the autoregressive process:

$$A_t = (1 - \rho_a)A + \rho_a A_{t-1}, \quad (25)$$

with  $\rho_a = 0.75$  and  $t = 1, 2, \dots, T$ .

It is well known that introducing an aggregate shock into a dynamic heterogeneous-agent model is not an easy computational task, since, by assuming a continuum of agents, the state of the economy, at any point in time, is an infinite-dimensional object. Specifically, in order to forecast prices (interest rates) accurately, agents need to keep track of the evolution of the distribution  $\Upsilon_{t+1}$ , which is an infinite dimensional object. One approach that renders these models computable was developed by Krusell and Smith (1998), who consider that agents only use a finite number of statistical moments, derived from the distribution, to predict future prices.

The large number of individual state variables considered in our model (the capital stock, the price of capital and firms' net worth) renders this methodology quite difficult to use. Besides, to analyze the consequences of Basel II capital requirements, we are interested in analyzing how the firms' distribution over the ratio of capital expenditures to net worth (which proxies for firms' probability of default and, thus, determines the capital requirements risk weights used under the new Basel Accord) evolves over time.

In this context, we follow Mendoza *et al.* (2007)'s approach.<sup>20</sup> In particular, after solving the model for the steady state, we choose a number of transition periods,  $T$ , taking into account the path of the common productivity factor  $A$ , given by equation (25). Assuming an initial shock of  $-1\%$ , the common productivity factor takes approximately 110 quarters to return to its steady state value. We thus consider  $T = 110$ . Using the first order conditions and the law of motion for the net worth, derived in Section 2 and properly modified in order to account for the aggregate productivity shock, we solve for the optimal choices backwards, starting from period  $T$  and taking into account that both  $A$  and the decision rules at  $T + 1$  are equivalent to those derived in the

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<sup>20</sup>The model developed by these authors does not consider an aggregate shock, but analyzes the transitional dynamics between two different steady states. We adjust their procedure in order to account for an aggregate shock.

steady state model. This procedure allows us to compute the decision rules at  $t = T, T - 1, \dots, 2, 1$ , which may then be used to find the sequence of firms' distribution over the state space  $(N, K, Q, \omega)$  at each point in time and to compute the aggregate variables of the model.

In contrast with Section 2, and due to the large number of state variables in the model, we consider a partial equilibrium framework, in which households are absent. In particular, we assume that both the return on deposits and the return on bank capital are exogenously set at their steady state values and do not change over the business cycle. Alternatively, we may interpret this economy as a small open economy, which takes interest rates as given.

### 3.1 Results

Figures 6 and 7 illustrate the impulse response functions of the key aggregate variables of the model under the three variants considered before, using the calibrated model economy, with each period equivalent to a quarter and the variables expressed as percentage deviations from their steady state values.

The decrease in the common productivity factor  $A$  triggers an immediate decline in output below its steady state value, after which it returns gradually to its steady state. Due to capital adjustment costs, the capital stock response is moderate, first decreasing and then gradually reverting to its steady state value. Therefore, and since labor is absent from our model, the response of output is essentially determined by the common productivity factor in the initial periods after the shock.<sup>21</sup>

As in BGG, the average price of physical capital, the aggregate capital expenditures and the aggregate firms' net worth are all procyclical. However, since the decrease in capital expenditures ( $QK$ ) is more amplified than the decrease in net worth ( $N$ ), firms' loans ( $QK - N$ ) also decrease after the shock - see Figure 6 (f).

As described in Section 2, under Basel I capital requirements, the amount of bank capital issued by the bank depends positively on the amount of loans granted to the firms. Therefore, the decrease in aggregate loans after the shock leads, necessarily, to a decrease in bank capital (see Figure 7, a).

Under Basel II, the minimum amount of bank capital that a bank must raise depends both on the total amount of loans granted by the bank and on the credit risk of its loan portfolio. As detailed in Section 2, firms' credit risk is proxied by the ratio of capital expenditures to net worth. Figure 7 (b) shows that the average value of this ratio decreases with the negative technology

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<sup>21</sup>A very simple growth accounting exercise shows that, in the second quarter after the shock, the capital stock explains around 0.5% of output (both variables expressed as percentage deviations from their steady state values), in all variants of the model. The role of capital then gradually increases, reaching 50% after 16 quarters (approximately).



shock, since the amount of loans granted to firms also decreases.<sup>22</sup> This effect is supported by the results obtained in 2.6, according to which a permanent decrease in the common productivity factor leads to a decrease in the steady state leverage ratio. Therefore, bank capital should not only be procyclical in variant 1, since both loans and the average ratio of capital expenditures to net worth decrease, but should also decrease by a larger extent than aggregate loans, after the negative shock.

However, as depicted in Figure 7, despite the decrease in bank capital under Basel II, the average ratio of bank capital to loans ( $S/L$ ) increases immediately after the shock, then decreasing below its steady state level, in the second quarter, and gradually reverting towards its equilibrium level from below after the fourth quarter. The average capital requirements risk weight ( $\alpha_e$ ) and the average required return on lending by the bank ( $R^F$ ) follow the same path, in variant 1. The analysis of the technology shock effects on the distribution of firms over the ratio of capital expenditures to net worth ( $k$ ) allows us to clarify this outcome, as follows.

First, note that according to our computational procedure (see Appendix C), all firms with  $k^j > 3$  face the same  $\alpha_e^j$  and  $R^{Fj}$  - see Figure 7 (h) and Figure 5 (f), respectively. That is, we assume that the highly leveraged firms are treated equally by the bank (face the same required return on lending, the same default threshold and the same external finance premium). As Figure 8 illustrates, the decrease in the average ratio of capital expenditures to net worth, after the shock, is mainly driven by a decrease in the fraction of highly leveraged firms in the economy (firms with  $k^j > 4$ ). Figure 8 also suggests that, immediately after the shock, some of those firms move to the preceding category ( $3 < k^j \leq 4$ ). This relocation affects negatively the average value of  $k$ , explaining the decrease in this variable after the shock, but does not affect the average capital requirements risk weight ( $\alpha_e$ ) and the average cost of financing ( $R^F$ ). In addition, firms with  $k^j$  between 1 and 2 in steady state, and which migrated to the two subsequent categories after the shock, as Figure 8 suggests, justify the initial increase in  $\alpha_e$  and  $R^F$ . The increase in  $\alpha_e$  explains, in turn, the increase in the average ratio of bank capital to loans in variant 1, as implied by the capital requirements constraint. In the second quarter after the shock, the fraction of firms with  $1 < k^j \leq 2$  and  $2 < k^j \leq 3$  increases and the fraction of firms with  $3 < k^j \leq 4$  and  $k^j > 4$  decreases, leading, simultaneously, to a decrease in  $k$  and in the perceived average credit risk. Consequently  $\alpha_e$ ,  $S/L$  and  $R^F$  also decrease (see Figure 7, panels c to e).

Figure 7 also shows that the impulse response functions of the average default threshold ( $\bar{\omega}$ ) and the average external finance premium (EFP) resemble the average capital requirements risk weight path in variant 1, also due to the response of firms' distribution over  $k$ . There is, however,

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<sup>22</sup>This result is in line with the predictability view, mentioned in the introduction of the paper, which opens the possibility of measured credit risk being relatively low (high) during recessions (expansions). Note that, since all loans mature in one period, the "build-up" and the "materialization" of firms' risk coincide. For evidence on the procyclicality of leverage of financially constrained firms see Korajczyk and Levy (2003).

an additional effect influencing the relationship between  $k$ ,  $\bar{w}$  and the EFP outside the steady state: in contrast with BGG, the common productivity factor,  $A$ , enters the first order conditions derived from the contracting problem under decreasing returns to scale (see equations 5 and 6, in 2.1). According to our simulations, a decrease in  $A$  triggers, everything else constant (including  $k$ ), an increase in the default threshold (that is, an increase in the expected probability of default), and an increase in the EFP faced by each firm. Therefore, the technology shock has two distinct effects which render the results derived from the contract outside the steady state more difficult to interpret than in BGG:

$$\Delta^- A \Rightarrow \begin{cases} \Delta^- k \Rightarrow \Delta^- \bar{w} \Rightarrow \Delta^- EFP \\ \Delta^+ \bar{w} \Rightarrow \Delta^+ EFP \end{cases} .$$

Figure 7 shows that the second effect (and the effect associated with the response of firms' distribution over  $k$ ) dominate the first effect, leading to an increase in the average cutoff value  $\bar{w}$  and in the average EFP immediately after the negative technology shock.

Finally, concerning the potential procyclical effects of bank capital requirements, we conclude that the impulse response functions are very similar across the three variants of the model, contradicting the procyclicality hypothesis. That is, the introduction of regulatory capital requirements in the model does not amplify the real effects of the aggregate shock. The bank capital channel, briefly described in the introduction, thus seems to be absent from the model. In fact, only  $\alpha_e$ ,  $S/L$  and, consequently,  $R^F$  respond in a distinct way under variant 1. However, as we are assuming that both the return on deposits and the return on bank capital are constant over the business cycle, the deviation of  $R^F$  from its steady state value is very small and not sufficient to generate significantly different responses of the remaining variables of the model under Basel I and II.

## **The Effects of a Technology Shock Assuming a Countercyclical Required Return on Bank Capital**

Thus far, we assume that the return on bank capital required by the representative household, in order to hold this asset in its portfolio, is constant throughout the business cycle, and does not vary with the changeover from Basel I to Basel II bank capital requirements rules. Thus, since the aggregate exogenous shock we introduced in the model is not sufficient to cause a significant change in firms' distribution over capital stock and leverage, it is not surprising that the aggregate shock does not render significantly different effects across the three variants of the model.

As a matter of fact, the procyclical effect of bank capital requirements, underlying the bank capital channel thesis, is usually associated with some specific cost in raising bank capital (*e.g.*, the information dilution costs introduced by Bolton and Freixas, 2006, or the countercyclical liquidity premium required by the households in order to hold bank capital, as considered by Aguiar and

Drumond, 2007).<sup>23</sup> In this line of thought, Gorton and Winton (2000) argue that bank capital is both privately and socially costly because a system-wide increase in bank capital reduces the aggregate amount of bank deposits, which are an efficient medium of exchange, forcing consumers to hold more information-sensitive bank equity, which is a poor liquidity hedge. Furthermore, Gorton and Winton's results suggest that the costs of raising additional capital tend to be highest in a recession.<sup>24</sup>

In this context, we consider now that, immediately after the negative technology shock, the cost of bank capital increases: in line with Gorton and Winton, during a downturn, the representative household demands a higher return on bank capital,  $R^S$  (and, consequently, a higher liquidity premium), in order to hold this asset and attenuate the decrease in consumption. In particular, we assume that, after the decrease in the common productivity shock,  $R^S$  increases by 0.5% (as in Aguiar and Drumond, 2007) gradually converging to its steady state value according to the following autoregressive process:

$$R_{t+1}^S = (1 - \rho_a)R^S + \rho_a R_t^S,$$

with  $\rho_a = 0.75$  and  $t = 1, 2, \dots T$ .

Figures 9 and 10 show the impact of the negative technology shock under the three variants of the model, assuming a countercyclical required return on bank capital.<sup>25</sup> As explained before, the response of output is mainly driven by the common productivity factor in the first quarters after the shock. Therefore, the impulse response functions of aggregate output are initially very similar across the three variants of the model. However, Figures 9 and 10 also show that the impact of the technology shock on the remaining economic and financial variables is visibly stronger in the presence of regulatory capital requirements: as in the previous experiment, in which  $R^S$  was assumed to be constant, the aggregate capital stock and its average price, the firms' net worth, the aggregate amount of loans and the average ratio of capital expenditures to net worth are all procyclical, but the effects of the technology shock on these variables are clearly amplified when capital requirements are introduced in the model. For instance, if we eliminate capital requirements from the model, that is, if we compare variant 3 with variants 1 and 2, the immediate impact of the technology shock on aggregate loans is reduced by 67.35% and 42.55% from variants 1 and

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<sup>23</sup>See also Markovic (2006), who develops a theoretical model that accounts for three distinct bank capital channels that trigger an increase in the required return on bank capital by shareholders, and thus an increase in the cost of bank capital, during an economic downturn.

<sup>24</sup>Calomiris and Wilson (2004) also argue that the cost of issuing new equity increases during a recession. In particular, focusing on the behavior of New York City banks during the interwar period, their study suggests that banks suffered much larger adverse-selection problems in equity markets (reflected in wider bid-ask spreads) during the 1930s than they had previously.

<sup>25</sup>Since bank capital is absent from variant 3, the results presented here, concerning this variant of the model, are the same as those reported in Figures 6 and 7.

2, respectively, to variant 3. Concerning the aggregate capital expenditures we find a reduction of 61.85% and 37.57% from variants 1 and 2, respectively, to variant 3. It is also straightforward to conclude that this amplification effect is stronger in variant 1, supporting the procyclicality hypothesis underlying the changeover from Basel I to Basel II capital requirements rules: the immediate impact of the technology shock on aggregate loans and aggregate capital expenditures is reduced by 43.17% and 37.9%, respectively, from variant 1 to variant 2.

The differences in the magnitude of this amplification effect between Basel I and Basel II may be explained through the analysis of bank behavior, as follows. In both variant 1 and variant 2, the average required return on lending by the bank,  $R^F$ , is a weighted average of the return on deposits and the return on bank capital. As derived in 2.2, the weights are constant under Basel I (see equation 19), whereas under Basel II the weights depend on firms' leverage (see equation 18).

In variant 2, the increase in  $R^S$  required by the households after the negative technology shock, is passed on to firms by the bank through an increase in the average required return on lending,  $R^F$ . The consequent decline in the aggregate amount of bank loans and in firms' capital expenditures under Basel I is thus more amplified than when  $R^S$  is assumed to be constant over the business cycle.

In variant 1, besides the former effect,  $R^F$  also depends on firms' credit risk, and its response to the technology shock is significantly stronger than under variant 2. Two effects contribute to this more amplified response.

First, and as in the previous experiment (with a constant  $R^S$ ), the adjustment in firms' distribution over  $k$ , immediately after the shock, leads to an increase in  $R^F$ .<sup>26</sup>

Second, the stationary equilibrium of this economy is characterized by a large fraction of highly leveraged firms (see 2.6) and, according to equation 18, the higher the leverage of the firm, the more sensitive is  $R^{Fj}$  to a change in  $R^S$  (see equation 18). Therefore, given the initial composition of the bank's asset portfolio, the response of the average financing cost ( $R^F$ ) to the exogenous shock is more amplified in variant 1: for highly leveraged firms, the increase in the required return on lending is stronger under Basel II than under Basel I.<sup>27</sup> Figure 11 supports this result: an increase in  $R^S$  causes a leftward shift in the loan supply function, and under Basel II the higher the leverage of the firm, the higher the increase in the firm's financing cost.

Finally, since the required return on lending by the bank increases more in variant 1 than in variant 2, the decrease in the aggregate amount of loans granted to firms (and, consequently, in

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<sup>26</sup>As before, due to the adjustment in the distribution of firms over  $k$ , the average capital requirements risk weight in variant 1, the average default threshold and the average EFP increase, immediately after the shock, despite the decline in the average ratio of capital expenditures to net worth: the decrease in this ratio is mainly driven by a shift of highly leveraged firms (with  $k^j > 4$ ) towards the preceding category ( $3 < k^j \leq 4$ ). We also find that the adjustments in firms' distribution over the ratio of capital expenditures to net worth are more amplified in variant 1 than in variant 2.

<sup>27</sup>And the decrease in the average leverage ratio is not sufficient to offset this effect.

firms' capital expenditures) is more amplified in the former variant.

We may thus conclude that, to the extent that it is more costly to raise bank capital in bad times and the representative bank's loan portfolio is characterized by a significant fraction of highly leveraged firms, the new bank capital requirements rules proposed by Basel II accentuate the procyclical tendencies of banking: the changeover from Basel I to Basel II capital requirements reinforces the loan supply effect which underlies the bank capital channel. Moreover, the Basel II procyclical effect should be greater, the greater the fraction of firms that begin with high leverage ratios, that is, with high credit risk. The distribution of firms over the leverage ratio, which in our model proxies for credit risk, is therefore crucial to understand the potential procyclical effects of the new bank capital requirements rules.

## 3.2 Some Additional Experiments

### Mimicking a Monetary Policy Shock

Although our model does not comprise a central bank and a monetary policy rule, we may still study the effects of a monetary policy shock assuming that the shock translates into an exogenous change in the return on deposits,  $R^D$  (in line with Meh and Moran, 2007, for instance). Specifically, we introduce an exogenous shock that leads to a 0.1% increase in  $R^D$ , which then gradually converges to its steady state value following an autoregressive process similar to the one previously defined for  $R^S$ .

For the same motives pointed out in 3.1, we also assume that, given the negative monetary policy shock, households require an increase in the return on bank capital,  $R^S$ , in order to hold this asset in their portfolios. In particular we consider that, simultaneously with the increase in  $R^D$ ,  $R^S$  increases by 0.5%, gradually converging to its steady state value. Consequently, the liquidity premium,  $R^S - R^D$ , increases after the shock.

Results, not shown here for brevity but available upon request, suggest that the response of both economic and financial variables in variant 1 is more pronounced than in variant 2, thus supporting the procyclicality hypothesis of Basel II. The aggregate output follows very closely the response of physical capital, as the common productivity factor is assumed to stay constant. Given the increase in the liquidity premium immediately after the shock and since the economy is characterized by a large fraction of highly leveraged firms, the rise in the financing cost is more amplified under Basel II, leading to a stronger decrease in the amount of loans granted to firms and, consequently, to a stronger decrease in firms' capital expenditures and output, after the shock. As before, the decrease in the average leverage ratio after the shock is insufficient to offset the effect associated with the high sensitivity of the financing cost of highly leveraged firms to changes in the required return on bank capital,  $R^S$ .

## Technology Shock Assuming Countercyclical Returns on Deposits and Bank Capital

We also test for the effects of the negative technology shock analyzed in 3.1 assuming that, immediately after the shock, households require an increase in both  $R^D$  and  $R^S$ .<sup>28</sup> In particular, we assume that, given the decrease in the common productivity shock,  $R^D$  and  $R^S$  increase by 0.1% and 0.5%, respectively, and then gradually converge to their steady state values. Therefore, the liquidity premium rises after the aggregate shock, as in the previous experiments. As expected, the asymmetry in the leftward shift in the loan supply of funds after the shock is attenuated: the increase in funding costs is more homogeneous across firms under Basel II (see Figure 12). However, more leveraged firms still face a higher increase in the financing cost, since the liquidity premium,  $R^S - R^D$ , increases after the negative aggregate shock. Consequently, the conclusions drawn before do not change: the amplification effect, associated with Basel II capital requirements, remains at work.<sup>29</sup>

## 4 Concluding Remarks

The banking sector is intrinsically procyclical, regardless of the design of capital requirements. In the presence of financial market frictions, concerns about loan quality and repayment probability lead banks to decrease lending in bad times, exacerbating the economic slowdown, as firms, that cannot easily substitute bank loans with alternative sources of funding, decrease their investment. In good times, banks tend to increase lending, possibly exacerbating the initial boom. The recent developments in the subprime mortgage and securitization markets are a good example of this. Despite the widely recognized effort of the new Basel Accord to deal with the shortcomings of the previous Accord, some concerns have been raised that Basel II may accentuate the procyclical tendencies of banking.

Our work, focusing on the relationship between the banking sector and credit constrained firms, provides a framework which may be used to evaluate the potential procyclicality of Basel II, by introducing a simplified version of the new capital requirements rules into a heterogeneous-agent model, in which firms have different access to bank credit depending on their financial position and, consequently, on their credit risk. It thus allows a fuller account of Basel II rules than other models in the literature, by considering that credit risk varies, not only over the business cycle, but also across firms.

The steady state general equilibrium model predicts that the introduction of regulatory capital requirements under both Basel I and Basel II has a negative supply effect on the economy's

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<sup>28</sup>In fact, the results obtained in Section 2 indicate that  $R^D$  varies negatively with  $A$  in both variants 1 and 2 (see the effects of an increase in  $A$  in the general equilibrium steady state model).

<sup>29</sup>Results not reported for brevity, but available upon request.

aggregate amount of loans. As households require a liquidity premium to hold bank capital in their portfolios, this asset is more costly to raise than deposits. Regulatory capital requirements, by forcing banks to finance a fraction of loans with bank capital, thus increase the banks' loan funding cost and, consequently, banks' lending rates, thereby leading to a lower aggregate amount of loans granted to firms and, thus, to lower physical capital accumulation and aggregate output.

In a stationary equilibrium characterized by a significant fraction of high credit risk firms, the former effect is stronger under Basel II than under Basel I. As minimum capital requirements become a function of each borrower's perceived credit risk, banks with a high risk asset portfolio must finance a higher fraction of loans with bank capital than under Basel I. Again, the resulting additional cost faced by those banks under the new Accord is passed on to borrowers through an increase in firms' financing costs, exacerbating the negative effects of the introduction of regulatory capital requirements on physical capital accumulation and on aggregate output.

Our model also predicts that small (and also more leveraged) firms lose more with the new risk-sensitive capital requirements, supporting the concerns that have been raised that the new regulation may raise the financing costs of small and medium-sized enterprises - due to banks' perception that these firms are riskier - and the special treatment given to these firms in Basel Committee on Banking Supervision (2004).

To the extent that it is more costly to raise bank capital in bad times and that the representative bank's loan portfolio is characterized by a significant fraction of highly leveraged firms, the introduction of an aggregate technology shock into a partial equilibrium version of the former heterogeneous-agent model supports the Basel II procyclicality hypothesis. Specifically, by considering that the liquidity premium required by the households moves countercyclically and it is, therefore, more costly for the bank to raise bank capital during an economic downturn, we embed a bank capital channel in a heterogeneous-agent model, and find that it accentuates the (countercyclical) response of firms' financing costs to an aggregate technology shock, leading to a more amplified response of firms' physical capital expenditures. This amplification effect, working through the loan supply side, is stronger under Basel II than under Basel I capital requirements: under Basel II it rests, not only on the countercyclical liquidity premium, but also on the risk profile of the representative bank's loan portfolio. In particular, the model predicts that the financing cost of highly leveraged firms under Basel II is very sensitive to changes in the required return on bank capital. As the economy's stationary equilibrium is characterized by a significant fraction of this type of firms, the average financing cost faced by firms responds more strongly to the aggregate technology shock under Basel II, leading to more amplified effects on capital accumulation and output. We may thus conclude that the amplification effect underlying Basel II tends to hold in an economy characterized by a significant fraction of small firms, which usually cannot easily substitute bank loans with alternative sources of funding and have higher perceived

credit risk than large firms.

This result supports Kashyap and Stein (2004)'s argument that Basel II capital requirements have the potential to create an amount of additional cyclical in capital charges that may be quite large, depending on a bank's customer mix, thus highlighting the possibility that differences among banks can affect market outcomes under capital requirements, as recently argued by VanHoose (2007). The Basel II procyclical effect should be greater, the greater the fraction of firms who begin with relatively high leverage ratios, that is, with relatively high credit risk. The distribution of firms underlying the bank's portfolio is therefore crucial to evaluate the potential procyclical effects of the new bank capital requirements rules. Besides, the Basel II procyclicality hypothesis holds even if the predictability view - which considers the possibility of measured credit risk being relatively high (low) when times are good (bad) - is confirmed. That is, the decrease in the average leverage ratio, after the negative aggregate shock, is insufficient to offset the former amplification effect.

Economic policy conclusions should be drawn carefully, however, since the model simplifies many important features of the economy. Our analysis has not been concerned with questions such as whether bank regulation is itself optimal and we abstract from risk and incentive issues that support the introduction of regulatory capital requirements. However, a clear lesson to be drawn is that the potential procyclical and across firms effects should be taken into account when designing a bank capital regulatory framework. Although not analyzed in this work, we believe that a well regulated, sounder and less prone to systemic risk banking system improves the financing of efficient firms across the economy. But it is no less true, as our work implies, that overregulation, leading to large and procyclical capital requirements, may counteract those positive aspects and, on top of that, may impose a stronger penalization to the financing of smaller and more leveraged firms, which, in many instances, coincide with the more dynamic and innovative segments of the economy.

Our work has taken a step towards evaluating the potential procyclical effects of Basel II. We leave for further research the introduction of the aggregate technology shock in a general equilibrium heterogeneous-agent model, in which the behavior of the required return on bank capital by the households, throughout the business cycle, is endogenously determined. Another positive way forward will be to introduce entry and exit of firms. This should avoid the possibility that the entrepreneurial sector accumulates enough net worth to be fully self-financed and permit to abandon the assumption that each entrepreneur consumes, in every period, a constant fraction of his resources. Finally, it may prove interesting to give more emphasis to the role of households' consumption, in which case it would be interesting to introduce labor in the model.



# Appendices

## Appendix A: Contracting Problem

The contracting problem, which determines the division of the expected gross project output  $A (K_{t+1}^j)^\alpha + E (Q_{t+1}^j) (1 - \delta) K_{t+1}^j$  between the borrower and the lender, may be written as:

$$\begin{aligned} \max_{K_{t+1}^j, \bar{\omega}_{t+1}^j} \quad & [1 - \Gamma(\bar{\omega}_{t+1}^j)] \left[ A (K_{t+1}^j)^\alpha + E (Q_{t+1}^j) (1 - \delta) K_{t+1}^j \right] \\ \text{s.t.} \quad & [\Gamma(\bar{\omega}_{t+1}^j) - \mu\Theta(\bar{\omega}_{t+1}^j)] \left[ A (K_{t+1}^j)^\alpha + E (Q_{t+1}^j) (1 - \delta) K_{t+1}^j \right] = R_{t+1}^{Fj} (Q_t^j K_{t+1}^j - N_{t+1}^j). \end{aligned} \quad (26)$$

Taking equation (2) into account, the expected gross project output may be rewritten as

$$A (K_{t+1}^j)^\alpha + E (Q_{t+1}^j) (1 - \delta) K_{t+1}^j = R_{t+1}^{Kj} Q_t^j K_{t+1}^j + (1 - \alpha) A (K_{t+1}^j)^\alpha$$

and the contract problem becomes

$$\begin{aligned} \max_{K_{t+1}^j, \bar{\omega}_{t+1}^j} \quad & [1 - \Gamma(\bar{\omega}_{t+1}^j)] \left[ R_{t+1}^{Kj} Q_t^j K_{t+1}^j + (1 - \alpha) A (K_{t+1}^j)^\alpha \right] \\ \text{s.t.} \quad & [\Gamma(\bar{\omega}_{t+1}^j) - \mu\Theta(\bar{\omega}_{t+1}^j)] \left[ R_{t+1}^{Kj} Q_t^j K_{t+1}^j + (1 - \alpha) A (K_{t+1}^j)^\alpha \right] = R_{t+1}^{Fj} (Q_t^j K_{t+1}^j - N_{t+1}^j). \end{aligned}$$

Solving the contract, with respect to  $K_{t+1}^j$  and  $\bar{\omega}_{t+1}^j$ , renders the following first order conditions (FOCs):

$$\begin{aligned} K_{t+1}^j \quad & : \quad [1 - \Gamma(\bar{\omega}_{t+1}^j)] \left[ R_{t+1}^{Kj} Q_t^j + (1 - \alpha) \alpha A (K_{t+1}^j)^{\alpha-1} \right] + \\ & + \lambda^j \left\{ [\Gamma(\bar{\omega}_{t+1}^j) - \mu\Theta(\bar{\omega}_{t+1}^j)] \left[ R_{t+1}^{Kj} Q_t^j + (1 - \alpha) \alpha A (K_{t+1}^j)^{\alpha-1} \right] - R_{t+1}^{Fj} Q_t^j \right\} = 0; \\ \bar{\omega}_{t+1}^j \quad & : \quad \Gamma'(\bar{\omega}_{t+1}^j) - \lambda^j [\Gamma'(\bar{\omega}_{t+1}^j) - \mu\Theta'(\bar{\omega}_{t+1}^j)] = 0; \\ \lambda^j \quad & : \quad [\Gamma(\bar{\omega}_{t+1}^j) - \mu\Theta(\bar{\omega}_{t+1}^j)] \left[ R_{t+1}^{Kj} Q_t^j K_{t+1}^j + (1 - \alpha) A (K_{t+1}^j)^\alpha \right] - R_{t+1}^{Fj} (Q_t^j K_{t+1}^j - N_{t+1}^j) = 0 \end{aligned}$$

where  $\lambda^j$  is the Lagrange multiplier associated with the constraint that the bank earns its required rate of return in expectation. These FOCs yield, in turn, equations (5) and (6) in 2.1.

Following BGG, we made the following assumptions in order to solve equations (5) and (6):

$$\ln(\omega) \sim N(-0.5\sigma_{\ln\omega}^2, \sigma_{\ln\omega}^2).$$

Therefore,  $E(\omega) = 1$  and

$$E(\omega|\omega \geq \bar{\omega}) = \frac{1 - \Phi(z - \sigma_{\ln \omega})}{1 - \Phi(z)},$$

where  $\Phi(\cdot)$  is the c.d.f. of the standard normal,  $\phi(\cdot)$  is the p.d.f. of the standard normal, and  $z$  is related to  $\bar{\omega}$  through

$$z \equiv \frac{\ln(\bar{\omega}) + 0.5\sigma_{\ln \omega}^2}{\sigma_{\ln \omega}}.$$

Under these assumptions it is straightforward to compute  $\Gamma(\bar{\omega})$ ,  $\Theta(\bar{\omega})$ ,  $\Gamma'(\bar{\omega})$ ,  $\Theta'(\bar{\omega})$ .<sup>30</sup>

## Appendix B: Aggregate Consistency Condition

To derive the aggregate consistency condition - equation (24) in 2.4 - we first define the total amount of assets held by each entrepreneur, at the end of time  $t$ :

$$\mathcal{W}_{t+1}^j = \omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j + W^e.$$

Each entrepreneur's assets are allocated to consumption, to the payment to the bank, and to net worth, which is then used to buy capital ( $Q_t^j K_{t+1}^j$ ). Recall that, since  $N_{t+1}^j < Q_t^j K_{t+1}^j$ , the entrepreneur must borrow to buy capital:  $N_{t+1}^j + L_{t+1}^j = Q_t^j K_{t+1}^j$ .

For entrepreneurs that do not default at time  $t$  ( $\omega_t^j \geq \bar{\omega}_t^j$ ) we assume that,

- if  $\omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j \geq \bar{\omega}_t^j [A (K_t^j)^\alpha + E (Q_t^j) (1 - \delta) K_t^j]$ , the entrepreneur pays  $\bar{\omega}_t^j [A (K_t^j)^\alpha + E (Q_t^j) (1 - \delta) K_t^j]$  to the bank. Therefore, for this set of entrepreneurs (set  $A$ ),

$$\omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j + W^e = C_t^{ej} + \bar{\omega}_t^j [A (K_t^j)^\alpha + E (Q_t^j) (1 - \delta) K_t^j] + Q_t^j K_{t+1}^j - L_{t+1}^j; \quad (27)$$

- otherwise, the entrepreneur pays the bank  $\omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j$  (entrepreneurs of set  $B$ ):

$$\omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j + W^e = C_t^{ej} + \omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j + Q_t^j K_{t+1}^j - L_{t+1}^j. \quad (28)$$

For entrepreneurs that default at time  $t$  ( $\omega_t^j < \bar{\omega}_t^j$ ) we assume that,

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<sup>30</sup>Detailed derivation is available from the authors.

- if  $Q_t^j \geq E(Q_t^j)$ , the entrepreneur pays  $\omega_t^j \left[ A(K_t^j)^\alpha + E(Q_t^j)(1-\delta)K_t^j \right]$  to the bank. Therefore, for this set of entrepreneurs (set  $C$ ),

$$\omega_t^j A(K_t^j)^\alpha + Q_t^j(1-\delta)\omega_t^j K_t^j + W^e = C_t^{ej} + \omega_t^j \left[ A(K_t^j)^\alpha + E(Q_t^j)(1-\delta)K_t^j \right] + Q_t^j K_{t+1}^j - L_{t+1}^j; \quad (29)$$

- otherwise, the entrepreneur pays the bank  $\omega_t^j \left[ A(K_t^j)^\alpha + Q_t^j(1-\delta)K_t^j \right]$  - entrepreneurs of set  $D$ :

$$\omega_t^j A(K_t^j)^\alpha + Q_t^j(1-\delta)\omega_t^j K_t^j + W^e = C_t^{ej} + \omega_t^j \left[ A(K_t^j)^\alpha + Q_t^j(1-\delta)K_t^j \right] + Q_t^j K_{t+1}^j - L_{t+1}^j. \quad (30)$$

Aggregating equations (27), (28), (29) and (30) over firms, we get<sup>31</sup>

$$Y_t + W^e + \int Q_t^j(1-\delta)\omega_t^j K_t^j d\Upsilon_{t+1} = C_t^e + \int Q_t^j K_{t+1}^j d\Upsilon_{t+1} - L_{t+1} + \text{Bank Revenues}_t, \quad (31)$$

where  $\Upsilon_{t+1}$  is the distribution of firms over the state space  $(N, K, Q, \omega)$  at the end of time  $t$ ,  $Y_t = \int \omega_t^j A(K_t^j)^\alpha d\Upsilon_{t+1}$ ,  $C_t^e = \int C_t^{ej} d\Upsilon_{t+1}$ ,  $L_{t+1} = \int L_{t+1}^j d\Upsilon_{t+1}$  and

$$\begin{aligned} \text{Bank Revenues}_t &= \int_{j \in A} \bar{\omega}_t^j \left[ A(K_t^j)^\alpha + E(Q_t^j)(1-\delta)K_t^j \right] d\Upsilon_{t+1} + \\ &\int_{j \in B} \omega_t^j \left[ A(K_t^j)^\alpha + Q_t^j(1-\delta)K_t^j \right] d\Upsilon_{t+1} + \int_{j \in C} \omega_t^j \left[ A(K_t^j)^\alpha + E(Q_t^j)(1-\delta)K_t^j \right] d\Upsilon_{t+1} + \\ &\int_{j \in D} \omega_t^j \left[ A(K_t^j)^\alpha + Q_t^j(1-\delta)K_t^j \right] d\Upsilon_{t+1}. \end{aligned}$$

The realized bank's profits are given by

$$\Pi_t^B = \text{Bank Revenues}_t - R_t^D D_t - R_t^S S_t - \text{Monitoring Costs}_t.$$

Rearranging the preceding equation and substituting the  $\text{Bank Revenues}_t$  into equation (31) yields

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<sup>31</sup>Recall that we are assuming a continuum of firms, producers of manufactured goods, of total measure one.

$$\begin{aligned}
& Y_t + W^e + \left[ \int Q_t^j (1 - \delta) \omega_t^j K_t^j d\Upsilon_{t+1} \right] = \\
& = C_t^e + \left[ \int Q_t^j K_{t+1}^j d\Upsilon_{t+1} \right] - L_{t+1} + \Pi_t^B + R_t^D D_t + R_t^S S_t + \text{MonitoringCosts}_t.
\end{aligned}$$

Finally, using the bank's balance sheet constraint and the household's budget constraint we get equation (24):

$$Y_t + W^e + \int Q_t^j (1 - \delta) \omega_t^j K_t^j d\Upsilon_{t+1} = C_t + C_t^e + \int Q_t^j K_{t+1}^j d\Upsilon_{t+1} + \text{MonitoringCosts}_t.$$

## Appendix C: Calibration and Computational Procedure

To evaluate some of the model's parameters we follow BGG and Aguiar and Drumond (2007), who use U.S. data - see Table 3.

Depreciation rate	$\delta$	0.025
Monitoring costs parameter	$\mu$	0.12
Preference parameter	$\sigma$	1
Preference parameter	$\beta_0$	1
Discount factor	$\beta$	0.9818

Table 3: Calibration I

Each entrepreneur's endowment,  $W^e$ , is set to  $\frac{NVec(1)}{\gamma}$ , where  $NVec(1)$  represents the first grid point in the state space of firms' net worth.<sup>32</sup> This variable's law of motion, defined in 2.1, guarantees that each firm's net worth does not take values below  $\gamma W^e : N_{t+1}^j \geq \gamma W^e$ .

As mentioned in 2.5, we assume that all firms with  $k > 3$  are assigned the maximum level of  $\alpha_e$  (the maximum capital requirements risk weight). This is in line with the assumptions made concerning the financial contract established between the bank and each entrepreneur, as described below, in the computational procedure, and avoids unrealistically high values of  $\alpha_e$ .

These, and other parameters discussed in 2.5, are summarized in Table 4.

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<sup>32</sup>As detailed below, to solve the model we discretize the  $(N, K, Q, \omega)$  state space.

Entrepreneur's endowment	$W^e$	1
Elasticity of I/K with respect to the price of capital	$\varphi$	0.9
Capital adjustment costs parameter	$a_1$	0.0166
Capital adjustment costs parameter	$a_2$	0.25
Fraction of wealth consumed by each entrepreneur	$1 - \gamma$	0.5
Aggregate productivity factor	$A$	0.1
Production function parameter	$\alpha$	0.9
Standard Deviation of $\ln(\omega)$	$\sigma_{\ln \omega}$	0.6
Utility function parameter	$\alpha_0$	0.5084
Parameter of capital requirements' risk weights under Basel II	$a$	-1
Parameter of capital requirements' risk weights under Basel II	$b$	1

Table 4: Calibration II

### Computational Procedure<sup>33</sup>

1. Choose a discrete grid of points in the  $(K, N, Q, \omega)$  state space. We consider 30 grid points for  $K$  from  $[0.5, 30]$ , 30 grid points for  $N$  from  $[0.5, 30]$ , 10 grid points for  $Q$  from  $[0.5, 1.75]$ , and 18 for  $\omega$  from  $[0.1, 3.25]$ .
2. Using the grids defined in step 1, compute a grid for  $k_t = \frac{Q_{t-1}K_t}{N_t}$ , the ratio of capital expenditures to net worth at the end of time  $t - 1$  (which may generate values smaller than one: self-financed firms).
3. Compute, for each type of firm, the bank capital requirements weight  $\alpha_{e_t}$ . As mentioned in 2.5, we assume, under Basel II, that all firms with  $k > 3$  are assigned the maximum level of  $\alpha_e (= 2)$ , which is in line with the assumptions made concerning the contract established between the bank and each entrepreneur (see step 7, below) and avoids unrealistic values of  $\alpha_e$ . Self-financed firms are assigned  $\alpha_{e_t}$  equal to zero.
4. Given the calibrated discount factor, compute  $R^S$  using equation (23).
5. Guess an initial steady state value for  $R^D$ , the return on deposits.
6. Compute  $R_t^F$  for each type of firm. The existence of self-financed firms modifies the bank's optimization problem: we assume that when a firm's net worth exceeds its capital expenditures, the entrepreneur deposits the difference in the bank and receives, in the next period,  $(N - QK) R^D$ . Therefore, the bank's balance sheet constraint is now given by

$$\int_{j \in E} L_{t+1}^j d\Upsilon_{t+1} = D_{t+1} + D_{t+1}^e + S_{t+1}$$

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<sup>33</sup>To simplify the notation we drop the  $j$  superscript.

with  $D_{t+1}^e = \int_{j \in F} (N_{t+1}^j - Q_t^j K_{t+1}^j) d\Upsilon_{t+1}$  = amount of deposits held by self-financed entrepreneurs, from  $t$  to  $t + 1$ ,  $E$  = set of entrepreneurs that borrow from the bank and  $F$  = set of self-financed entrepreneurs. The FOCs of the bank's maximization problem yield the same results reported in 2.2.

7. Compute, for each  $k_t$ , the associated external finance premium required by the bank ( $l_t \equiv R_t^K / R_t^F$ ) and the cutoff value for the idiosyncratic risk ( $\bar{\omega}_t$ ), using the FOCs of the contractual arrangement problem between each firm and the bank. For self-financed firms, we assume  $\bar{\omega}_t = 0$  (no risk of default) and  $l_t = 1$  (no external finance premium required by the bank). Additionally, as the grids defined in step 1 also allow for highly leveraged firms, leading to very high values of  $l_t$ , we define upper bounds for  $l_t$  and for  $\bar{\omega}_t$  (derived from the contracting problem FOCs when  $k = \bar{k}$ ), and assume that those values hold for all firms with  $k \geq \bar{k}$  (we consider  $\bar{k} = 3$ ).<sup>34</sup> In sum, we assume that the bank treats equally all the firms with  $k \geq \bar{k}$ .
8. Guess an initial value of  $R_{t+1}^K$  for each type of firm in the state space  $(N, Q, K, \omega)$ .
9. Compute the decision rule for the capital stock,  $K_{t+1} = K(K_t, Q_{t-1}, N_t, \omega_t)$ , and for the price of capital,  $Q_t = Q(K_t, Q_{t-1}, N_t, \omega_t)$ , using equations (2), (11) and (14).
10. Compute the law of motion for the net worth  $N_{t+1} = N(K_t, Q_{t-1}, N_t, \omega_t)$ , as defined by equations (7) and (9). Taking into account the assumptions described above (see step 6), the net worth of each self-financed entrepreneur combines profits accumulated from previous capital investment, the endowment  $W^e$  and the return on deposits:

$$N_{t+1}^j = \gamma \left\{ \omega_t^j A (K_t^j)^\alpha + Q_t^j (1 - \delta) \omega_t^j K_t^j + W^e + (N_t^j - Q_{t-1}^j K_t^j) R^D \right\}.$$

11. Update the guess for  $R_{t+1}^K$ :
  - a) Using the decision rules for  $Q$  and  $K$  and the law of motion for  $N$ , compute  $k_{t+1}(\cdot) = \frac{Q_t K_{t+1}}{N_{t+1}}$ ;
  - b) Following the procedure described in steps 3 and 6, compute, for each type of firm,  $\alpha_{e_{t+1}}$  and  $R_{t+1}^F$ ;
  - c) Compute  $l_{t+1}$  and  $\bar{\omega}_{t+1}$ , following the procedure described in step 7;
  - d) Update the guess for  $R_{t+1}^K$  :  $R_{t+1}^K = l_{t+1} \times R_{t+1}^F$ .
  - e) Go back to step 9 until convergence.

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<sup>34</sup>Values of  $\bar{k}$  larger than 3 lead to unrealistic high values of the external finance premium.

12. Using the decision rules for  $K$  and  $Q$ , the law of motion for  $N$ , and the distribution of  $\omega$ , find the steady state distribution of firms over the state space  $(K, Q, N, \omega)$ :  $\Upsilon$ .
13. Compute bank's demand for the household's deposits:  $D = \int L^j d\Upsilon - D^e - S$ , with  $S = 0.08 \int (\alpha_e^j L^j) d\Upsilon$ .
14. Compute the amount of deposits held by the representative household, using the Euler equation (22) and the aggregate consistency condition (24).<sup>35</sup>
15. Update the guess for  $R^D$ , such that the bank's demand for the household's deposits equals the amount of deposits held by the representative household. Go back to step 6 until convergence.

## Figures

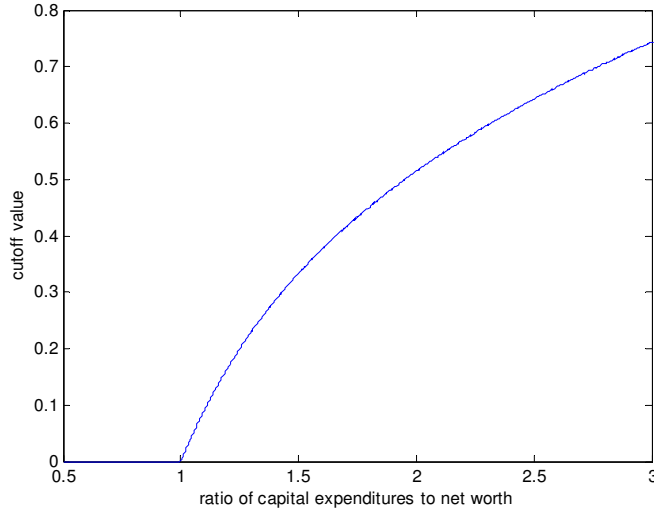


Figure 1: Relationship between the ratio of firm's capital expenditures to net worth ( $k_{t+1}^j$ ) and the cutoff value ( $\bar{\omega}_{t+1}^j$ ), derived from the financial contract under decreasing returns to scale.

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<sup>35</sup>The introduction of self-financed firms does not change this condition. Detailed derivation, similar to the one in Appendix B, is available upon request.

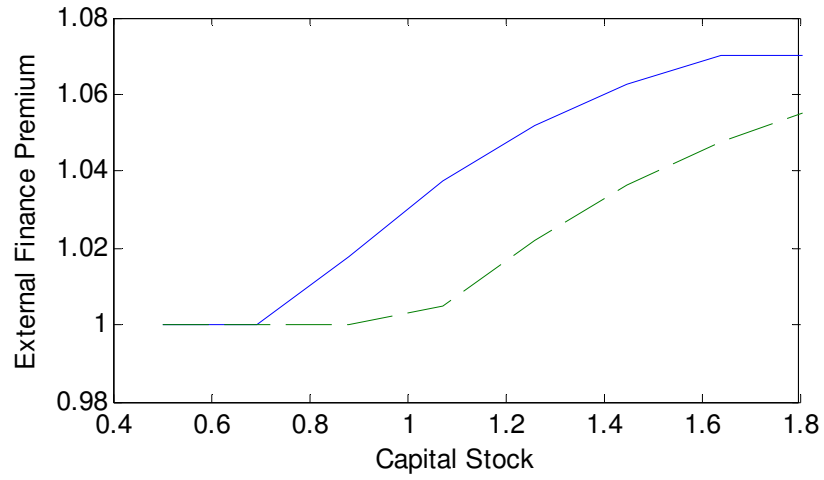


Figure 2: Relationship between the capital stock and the external finance premium. Solid line: Net Worth = 0.5; Dashed line: Net Worth = 0.7.

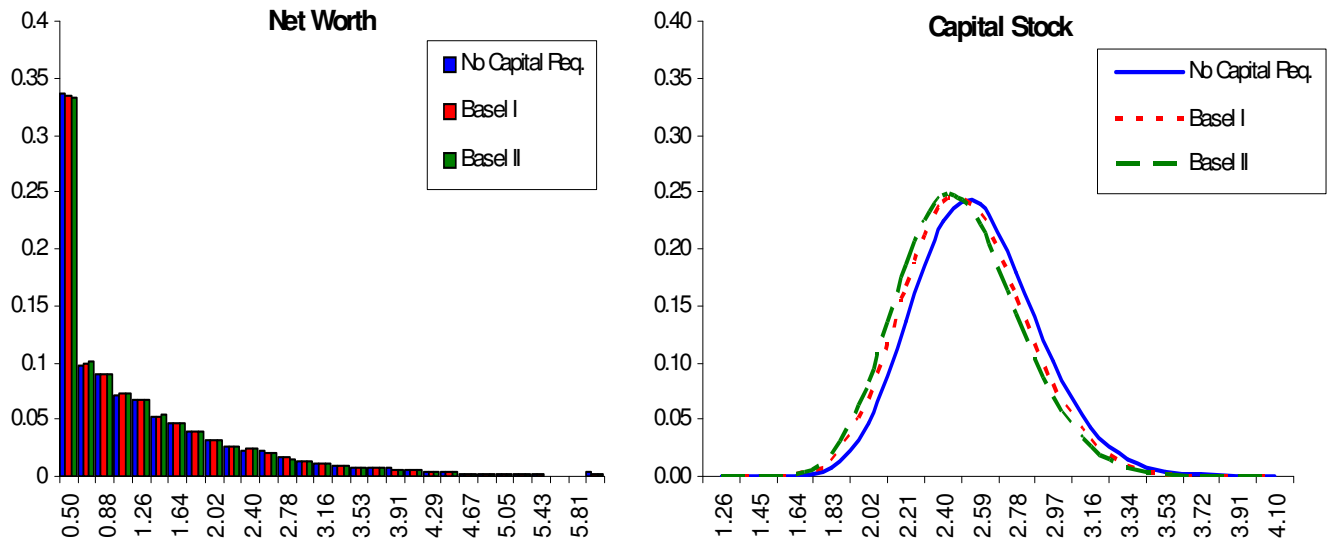


Figure 3: Stationary distribution of firms over net worth and capital stock



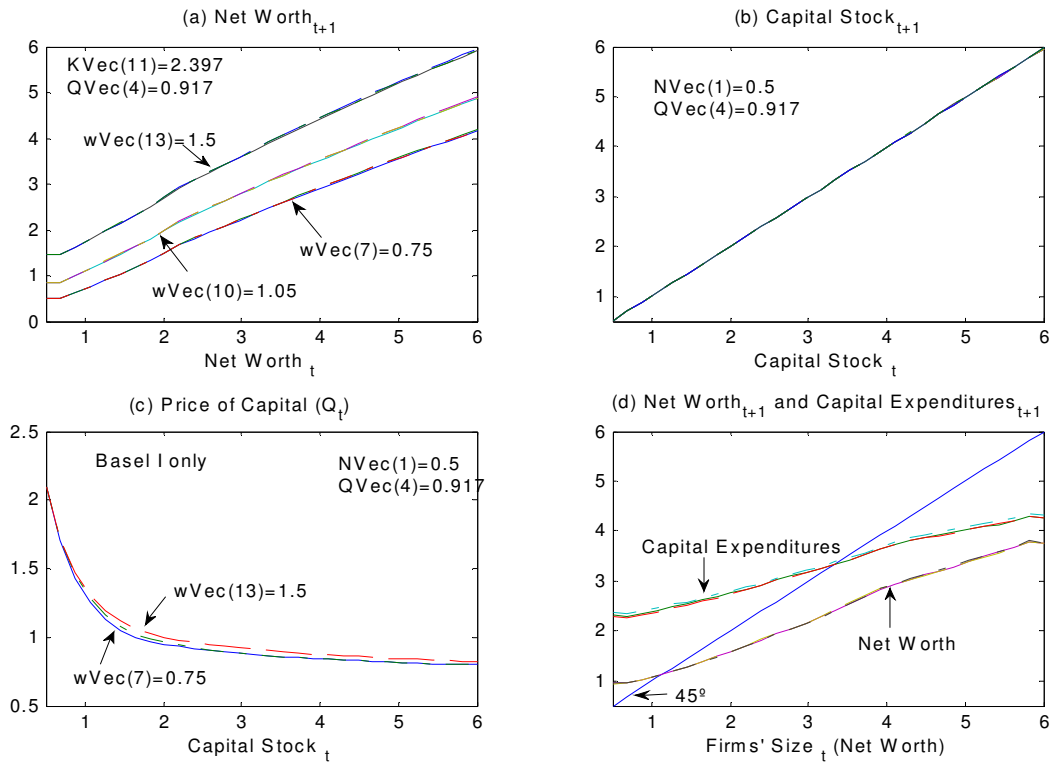


Figure 4: Firms' Dynamics in Steady State (I) - variant 1 (dashed line) - Basel II capital requirements; variant 2 (solid line) - Basel I capital requirements; variant 3 (dashed-dotted line) - no capital requirements.

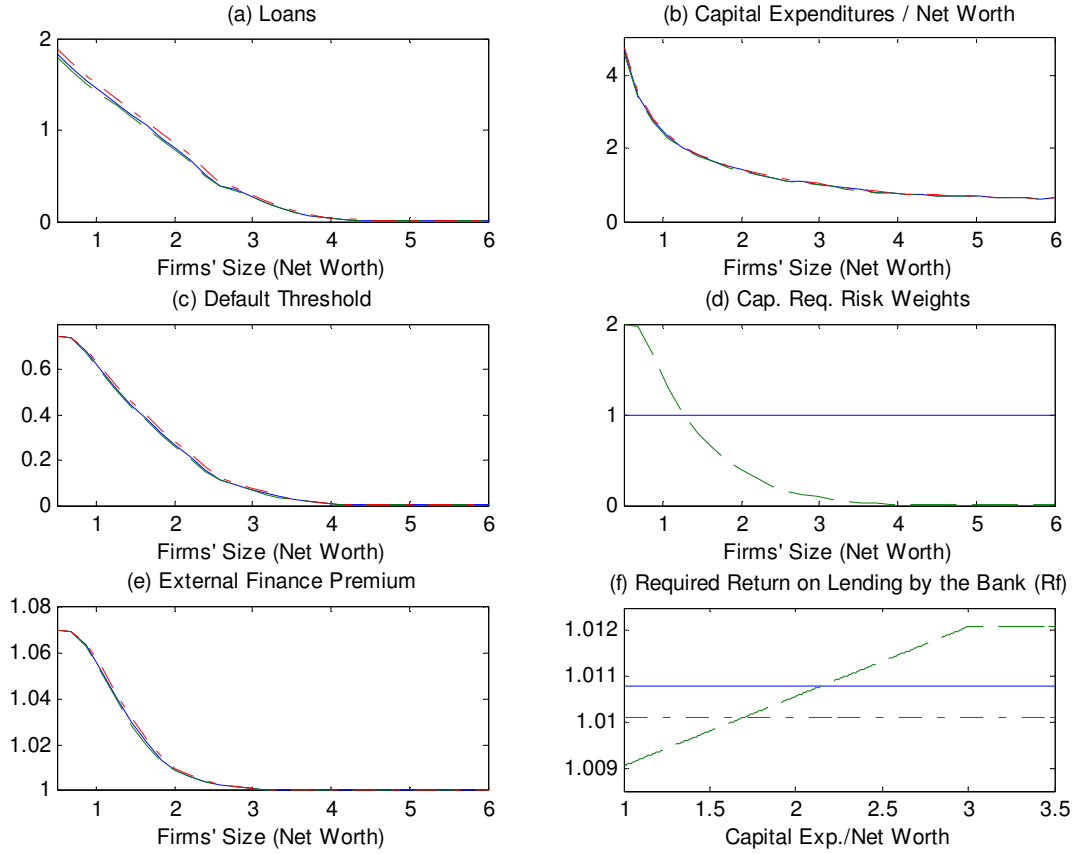


Figure 5: Firms' Dynamics in Steady State (II) - variant 1 (dashed line) - Basel II capital requirements; variant 2 (solid line) - Basel I capital requirements; variant 3 (dashed-dotted line) - no capital requirements.

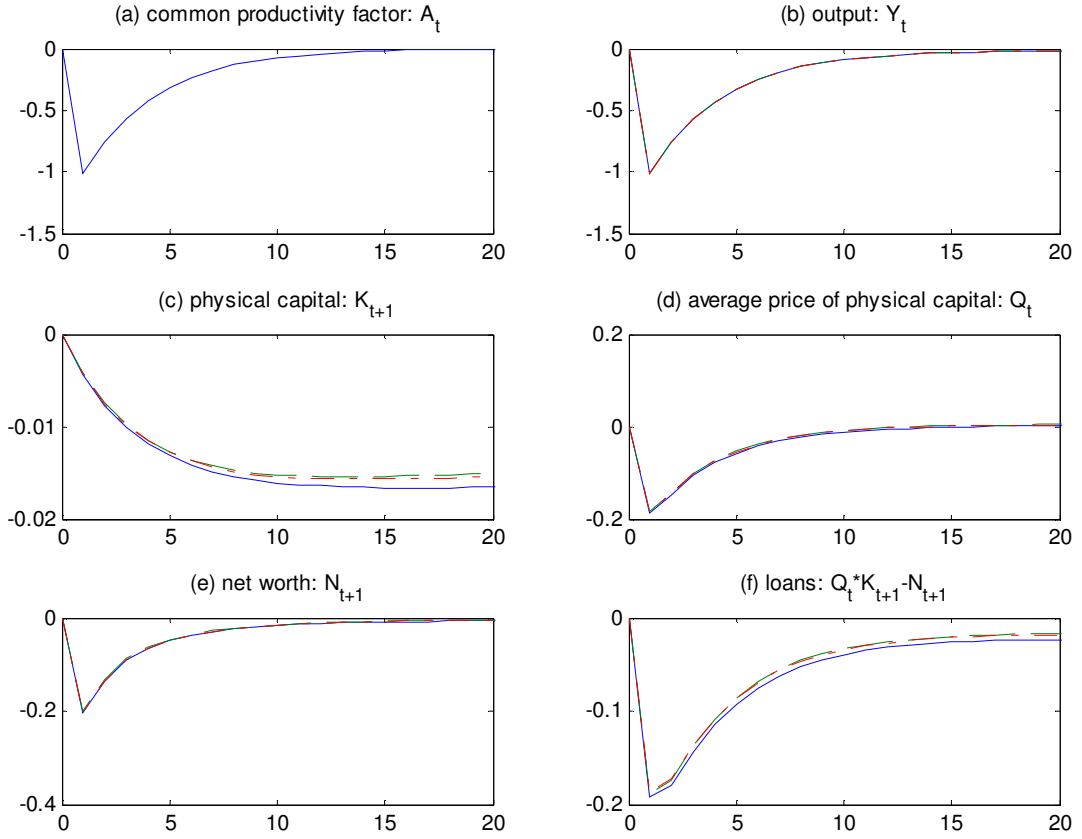


Figure 6: Response of economic activity to a negative technology shock: variant 1 (dashed line) - Basel II capital requirements; variant 2 (solid line) - Basel I capital requirements; variant 3 (dashed-dotted line) - no capital requirements.

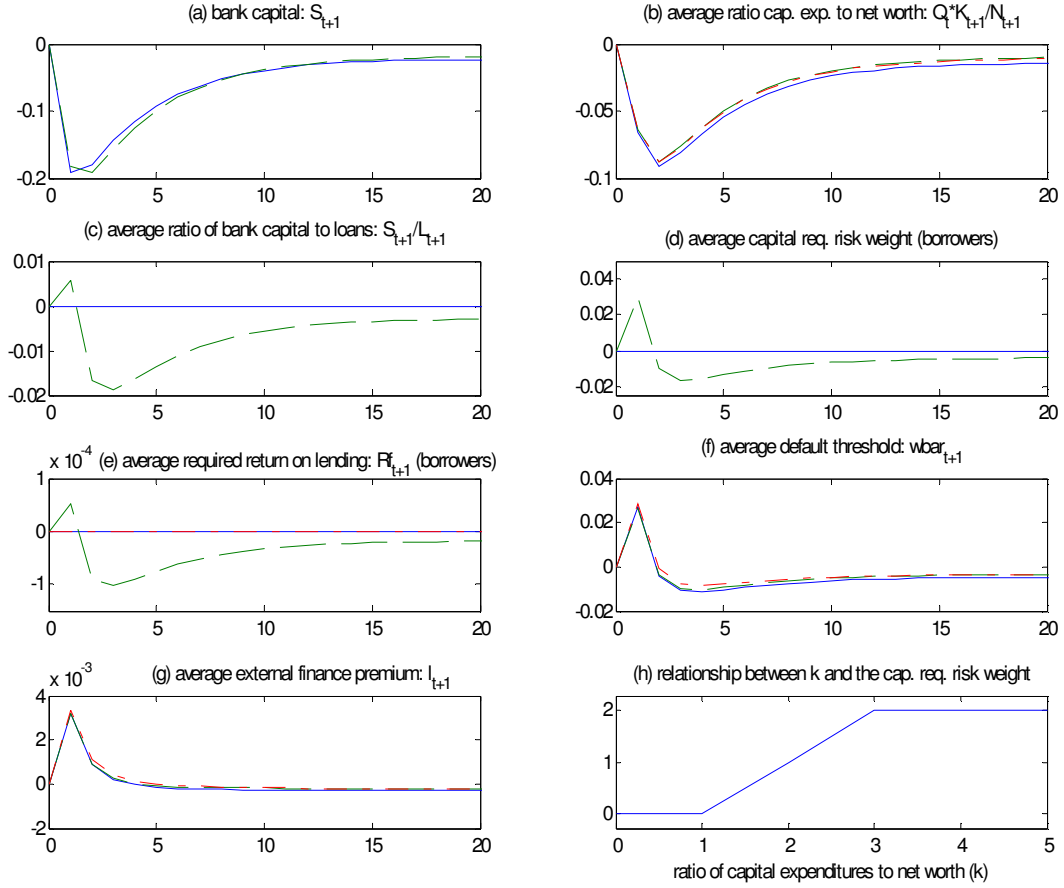


Figure 7: Response of financial variables to a negative technology shock: variant 1 (dashed line) - Basel II capital requirements; variant 2 (solid line) - Basel I capital requirements; variant 3 (dashed-dotted line) - no capital requirements.

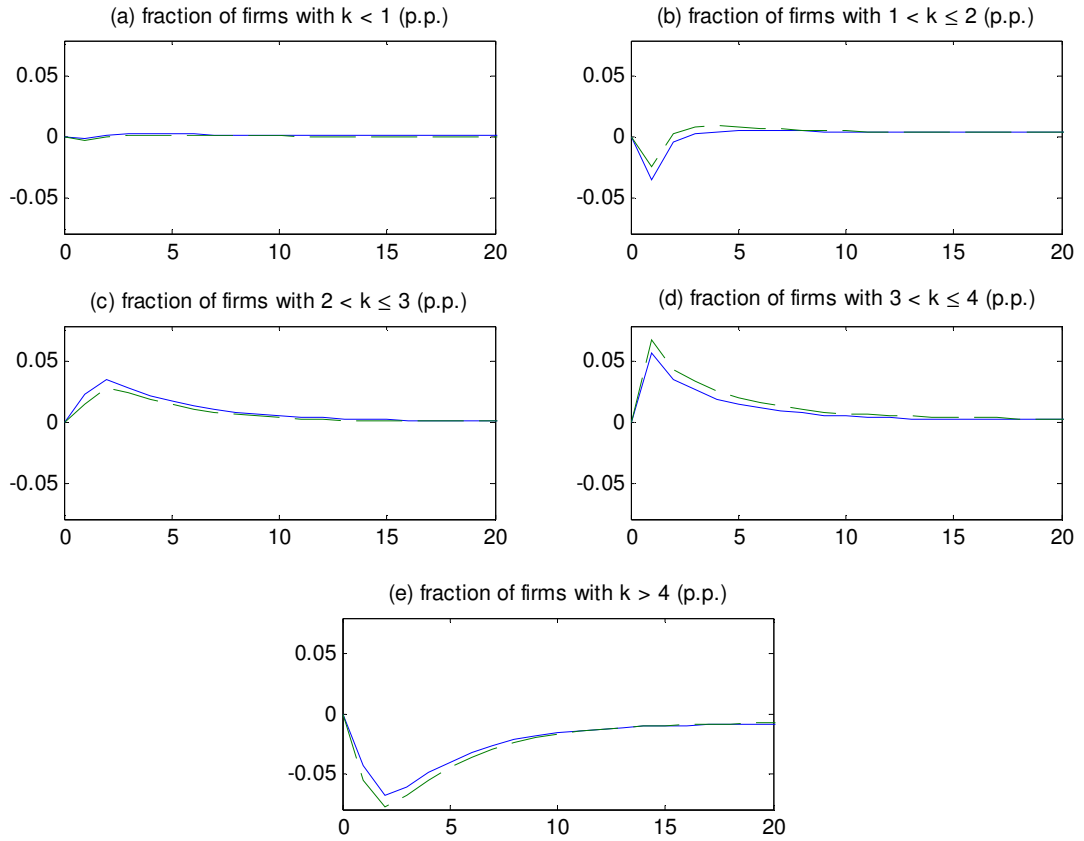


Figure 8: Response (in percentage points) of firms' distribution over the ratio of capital expenditures to net worth to a negative technology shock: variant 1 (dashed line) - Basel II capital requirements; variant 2 (solid line) - Basel I capital requirements.

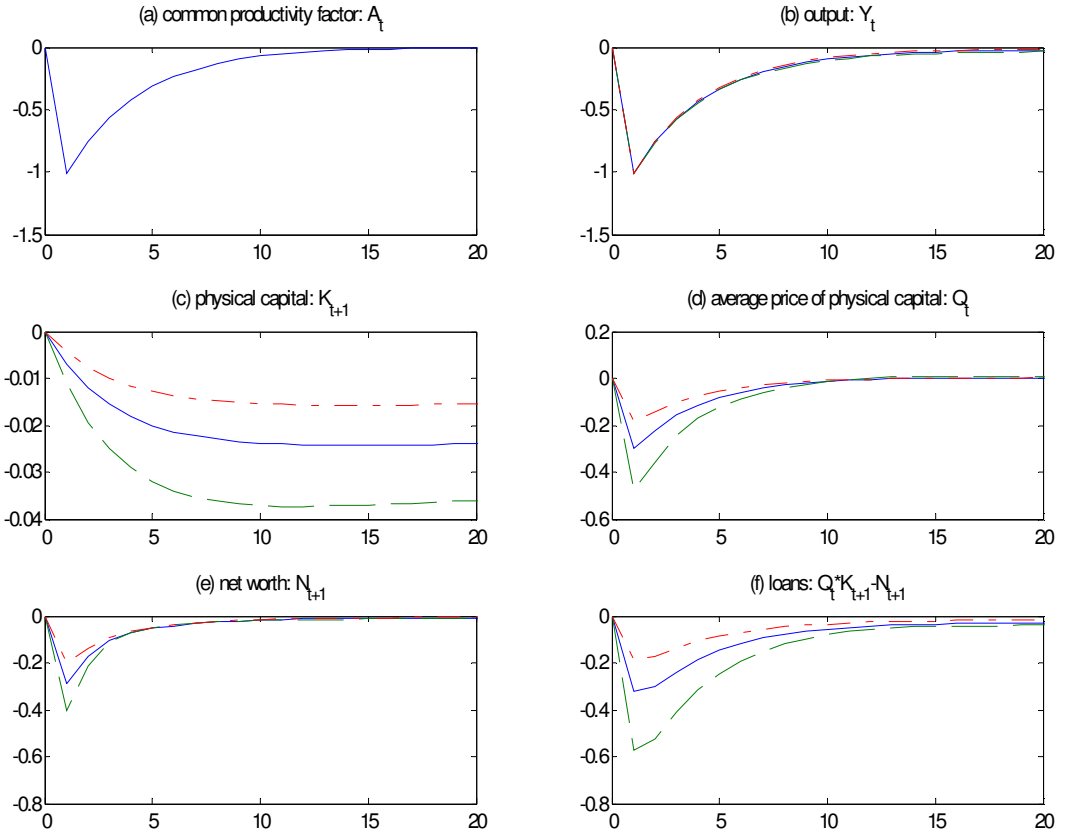


Figure 9: Response of economic activity to a negative technology shock and increase in  $R^S$ : variant 1 (dashed line) - Basel II capital requirements; variant 2 (solid line) - Basel I capital requirements; variant 3 (dashed-dotted line) - no capital requirements.

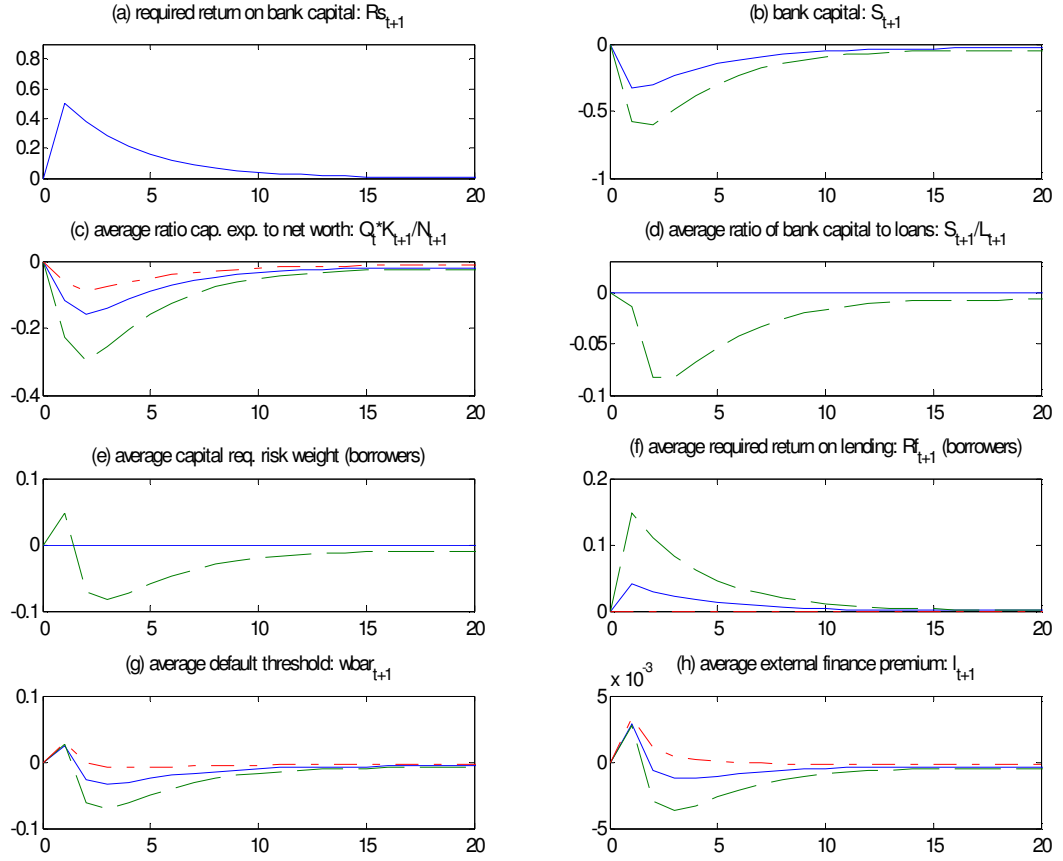


Figure 10: Response of financial variables to a negative technology shock and increase in  $R^S$ : variant 1 (dashed line) - Basel II capital requirements; variant 2 (solid line) - Basel I capital requirements; variant 3 (dashed-dotted line) - no capital requirements.

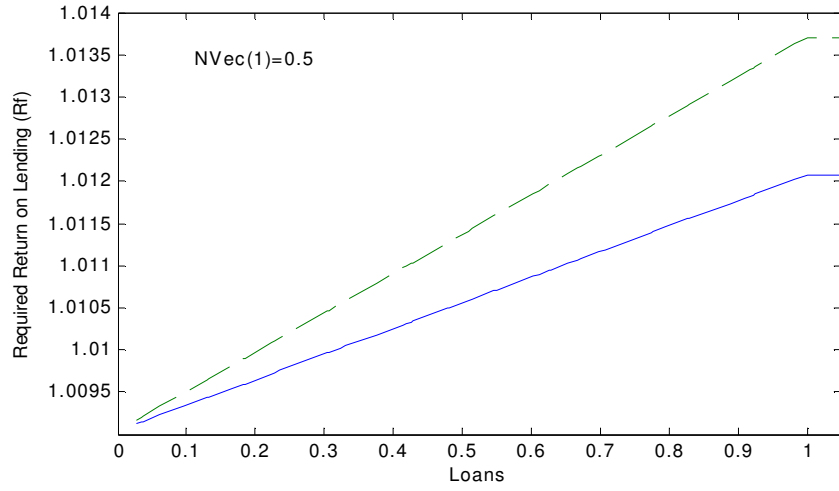


Figure 11: Response of the bank loan supply of funds to a 0.5% increase in  $R^S$ , for a given level of net worth, under Basel II (with  $R^D$  constant):  $R^S = 1.0185$  (solid line) *vs*  $R^S = 1.0236$  (dashed line).

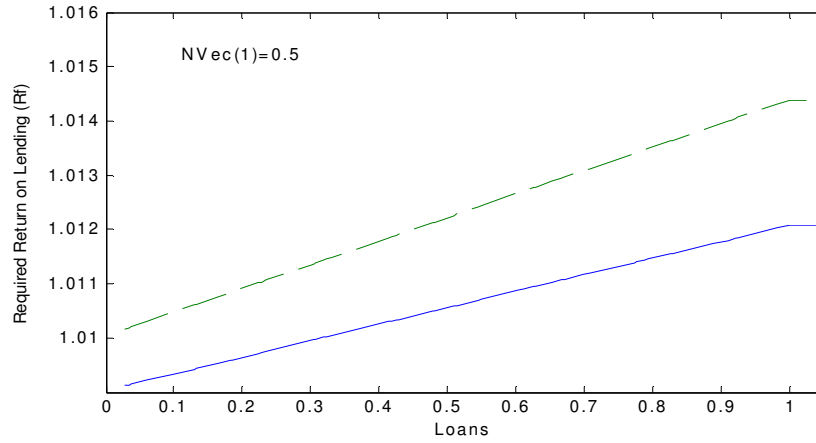


Figure 12: Response of the bank loan supply of funds to a 0.5% increase in  $R^S$  and a 0.1% increase in  $R^D$ , for a given level of net worth, under Basel II:  $R^S = 1.0185$  and  $R^D = 1.0092$  (solid line) *vs*  $R^S = 1.0236$  and  $R^D = 1.0102$  (dashed line).



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