Market Equilibrium with search and computational costs

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Abstract: Although it is an empirical regularity that in the trading of homogeneous goods there are persistent price dispersion and competition between sellers, it is theoretically derived that when buyers are optimisers, in market equilibrium, there are neither price dispersion nor competition (the Diamond's paradox). This undesirable theoretical result induces the growth of doubt in the relevance of using optimisation in the study of human behaviour (the Hey's critique). In this work I demonstrate that it is not necessary to abandon the optimisation framework to overturn this pitfall if assumed that economic agents have computational restrictions. That is, I demonstrate that when optimiser buyers have search and computational costs in market equilibrium there is price dispersion, search and competition between sellers.

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1. Introduction

Dating back to Stigler (1961) there is in the economic literature an effort to justify the occurrence of persistent price dispersion on trade of homogeneous goods. This has been related to the fact that economic agents do not have perfect knowledge (e.g., Lippman and McCall, 1976).

However, within a theoretical framework where buyers are homogeneous and intend to acquire one product whose price is unknown, it results in a market equilibrium where there is neither price dispersion nor search. This result is valid either when buyers follow the optimal sequential sample strategy (Diamond, 1971) or the sub-optimal fixed sample size strategy (Vieira, 2004).

This undesirable theoretical result induces the growth of doubt in the relevance of using optimisation in the study of human behaviour. Hey (1981) ads to this result the conjecture that economic agents are unable to use optimal rules because it would be necessary an enormous computation effort. Nonetheless, in a search framework, economic agents payoffs are higher when they use a sub-optimal rule. But, in my opinion, if one wants economics science to overpass the metaphysical stage of just looking for a short-range cause-effect prevision being also a positive science that looks for the deep regularities of economic agents behaviour, the optimisation principle cannot be abandoned.

In this work I introduce micro-economic level computational limitation on buyers, over-passing Hey (1981)'s critic without abandoning the optimisation theoretical framework. Being that computation has a cost, I assume that buyers calculate the optimal strategy with an error component whose size results endogenous to the optimisation process.

As conclusion, I prove that the existence of both search and computational costs is a sufficient condition to the existence in market equilibrium of search, price dispersion and competition.

2. Assumption of the theory

The theory I build up is based on the next assumptions:

A1. $F(p)$ is the distribution function of market prices;

A2. Buyers search sequentially being c the marginal search cost;

A3. To compute the search optimal reservation price, buyers use computational intensity n that costs cc per unit used;

A4. With more computational intensity, buyers decrease the error e of the computation;

A5. The reduction of the error (an expected measure) is computed as a Monte-Carlo algorithm with no variance reduction technique: $e = e_0 / \sqrt{n}$;

A6. The expected expenditure increases with the distance between the optimal reservation price and the calculated one, being a linear function;

A7. Sellers have unlimited computational capacity;

A8. In sellers' perspective, buyers compute a reservation price obtaining as result a positive number extracted from the uniform distribution around the optimal reservation price *P**. Lets assume that $P^{+} \sim U[\max(0, P* - e), P* + e]$;

A9. All this assumptions are common knowledge.

3. Main results

Lemma 1. The optimal buyers strategy is to compute the reservation price with an error component.

Proof: It is known from the literature (McCall, 1965) that the optimal buyers strategy when there is no computation costs implies computing the reservation price *P** that corresponds to the minimum expected expenditure:

$$P^*:\ P^* F(P^*) = c + \int_0^{P^*} p\, f(p)\, dp \qquad (1)$$

When there are computational costs, reservation price is computed with error becoming the expected expenditure higher than the minimal value *P**. Assumed, with no generality loss, that the expected expenditure increases with the error in a linear functional form, it becomes:

$$E[V(n)] = P^* + e + n\, cc = P^* + e_0 \frac{1}{\sqrt{n}} + n\, cc \qquad (2)$$

Buyers minimize expression (2) by using an optimal computational intensity. This optimal intensity of computation is:

$$\frac{dE[V(n)]}{dn} = -\frac{1}{2} e_0 n^{-\frac{3}{2}} + cc = 0 \Leftrightarrow \frac{1}{2} e_0 n^{-\frac{3}{2}} = cc \Leftrightarrow n^* = \left(2 \frac{1}{e_0} cc\right)^{-\frac{2}{3}} \qquad (3)$$

That is a minimum because $\frac{d^2 E[V(n)]}{dn^2} = -\frac{3}{4} e_0\, n^{-\frac{5}{2}} < 0$.

Then, it becomes optimal that $e = \left(2\, e_0{}^2\ cc\right)^{1/3} > 0$ QED

Although the existence of heterogeneity in the reservation price is considered in the literature connected with exogenous buyers' heterogeneity, (e.g., Axell, 1977) in this work it is for the first time associated with computational restrictions.

Lemma 2. No seller will affix prices out of the domain [max (0, *P** – *e*); *P** + *e*].

Proof: First, a buyer with reservation price P^+ will search the good until he/she finds a price smaller than or equal to that reservation price. Being so, the probability that the buyer acquires the good with a price smaller than or equal to *p* is (Axell, 1977):

$$W(p \mid P^+) \quad = \text{Prob}\left(\text{acq. price} \leq p \mid P^+\right) \quad = \begin{cases} F(p)/F(P^+), & p \leq P^+ \\ 1 & , p > P^+ \end{cases} \tag{4}$$

Expanding expression (4) to all possible reservation prices, one obtains, in expected terms, the market distribution function of the price at which buyers acquire the good:

$$\begin{aligned} W(p) &= \int_0^\infty W(p \mid P^+) u(P^+) dP^+ = \int_0^p u(P^+) dP^+ + \int_p^\infty \frac{F(p)}{F(P^+)} u(P^+) dP^+ \\ &= U(p) + F(p) \int_p^\infty \frac{1}{F(P^+)} u(P^+) dP^+ \end{aligned} \tag{5}$$

Assuming, without loss, that the number of buyers and sellers are normalised to one, then the expected profit function of a seller is the derivative of expression (5) divided by the quantity of sellers that affix price *p*, *f*(*p*), and multiplied by *p*:

$$E[p(p)] = p \; E[q(p)] = p \; \frac{w(p)}{f(p)} = p \int_p^\infty \frac{u(P^+)}{F(P^+)} dP^+ \; . \tag{6}$$

Simplifying, one obtains:

$$E[p(p)] = p \frac{1}{2e*} \begin{cases} \int_{P*-e*}^\infty \frac{1}{F(P^+)} dP^+, & p < P* - e* \\ \int_p^\infty \frac{1}{F(P^+)} dP^+, & P* - e* \leq p \leq P* + e* \\ 0, & p > P* + e* \end{cases} \tag{7}$$

It results that the expected profit function (7) is negative if price is negative, increasing with price until max(0, *P** – *e*), positive (and constant, see Lemma 3) for all prices affixed in the interval [max (0, *P** – ε); *P** – ε] and zero elsewhere, so it is not optimal that firms affix prices out of the interval [max (0, *P** – ε); *P** – ε]. QED

Lemma 3. The market equilibrium price distribution function has the form $F(p) = p^2 \frac{1}{(P*+e)^2}$ with $p \in$ [*max*(*0, P*–e*)*; P*–e*].

Proof: In order to market be in a Nash equilibrium situation, the expected profit function must be horizontal for all prices affixed (in [max (0, *P** – ε); *P** – ε] by Lemma 2). Being so, from expression (7) and Lemma 2, it results that:

$$p\frac{1}{2e}\int_{p}^{\infty}\frac{1}{F(P^{+})}dP^{+}=K \tag{8}$$

Deriving both members, one obtains:

$$F(p)=p^{2}\frac{1}{2e\ K}\Rightarrow F(p)=p^{2}\frac{1}{\left(P^{*}+e\right)^{2}}\Rightarrow$$

$$f(p)=\begin{cases}0 & p<\max(0,P^{*}-e)\\ \frac{p^{2}}{\left(P^{*}+e\right)^{2}}, & p=\max(0,P^{*}-e)\\ \frac{2p}{\left(P^{*}+e\right)^{2}}, & \max(0,P^{*}-e)\leq p\leq P^{*}+e\\ 0, & p>P^{*}+e\end{cases} \tag{9}$$

QED

Theorem1. Reservation price decreases with the increase in computation costs and the decrease in search costs.

Proof. Sellers compute the reservation price substituting expression (9) in (1).

$$P^{*}=\frac{c+\frac{(P^{*}-e)^{2}}{(P^{*}+e)^{2}}(P^{*}-e)+\int_{P^{*}-e}^{P^{*}}2p^{2}\frac{1}{(P^{*}+e)^{2}}dp}{\frac{(P^{*})^{2}}{(P^{*}+e)^{2}}} \tag{10}$$

Simplifying this expression one obtains the following implicit function:

$$0=\begin{cases}\frac{1}{3}P^{*3}-\frac{c}{(P^{*}+e)^{2}} & P^{*}-e<0\\ (P^{*})^{2}e-P^{*}e^{2}+\frac{1}{3}e^{3}-\frac{c}{(P^{*}+e)^{2}} & P^{*}-e\geq 0\end{cases} \tag{11}$$

The properties of the reservation price result from this implicit function. From (11) and (3), it is straightforward to see that $\frac{dP^{*}}{dcc}<0$ and $\frac{dP^{*}}{dc}>0$.

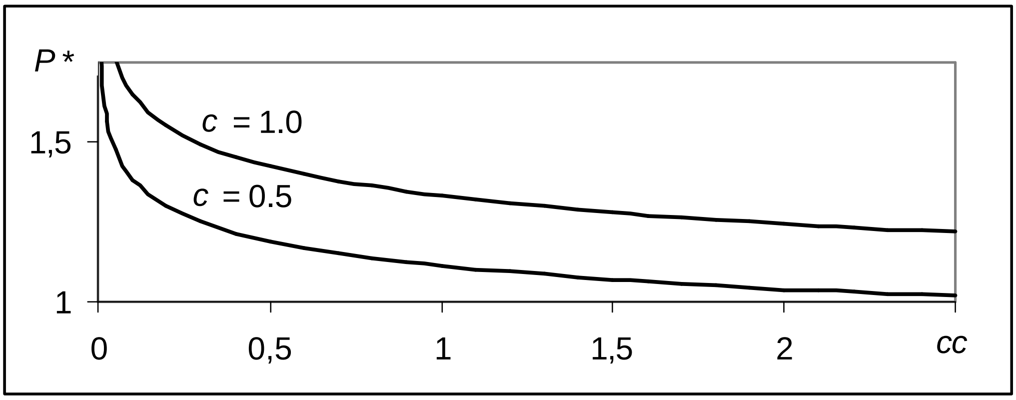

Fig. 1 – Evolution of the reservation price with search and computational costs

QED

Corollary 1. When computation costs tend to zero and search cost maintains positive the solution approximates the Diamond result (the reservation price tends to the monopoly price and price dispersion tends to zero).

Proof. From expression (3) when computation costs tend to zero, the computation intensity increases and the error tends to zero. From Lemma 2, then there is no price dispersion. From expressions (11) and (3), the equilibrium price is the monopoly price as $\lim_{cc\to 0}\left[(P^*)^2 e - P^* e^2 + \frac{1}{3}e^3 - \frac{c}{(P^*+e)^2} = 0\right] \Rightarrow P^* = \infty$. QED

4. Conclusion

In this work I study the influence of computational costs in the behaviour of buyers when they search for an unknown price. Specifically, I study whether that individual behaviour changes are sufficient to surpass the Diamond (1971)'s paradox. I introduce computational limitations on buyers' side but not, contrary to Hey (1981), without abandoning the optimisation theoretical framework. My conclusion is affirmative: when buyers have both search costs and computational costs, then, in market equilibrium there is price dispersion, search and competition between sellers.

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Appendix A

Algebraic steps from expression (10) to (11)

From expression (10), when $P^* - e < 0$, there will not be a mass point

$$P^* = \frac{\frac{c}{(P^*+e)^2} + \int_0^{P^*} 2p^2 dp}{(P^*)^2} \Leftrightarrow (P^*)^3 = \frac{c}{(P^*+e)^2} + \int_0^{P^*} 2p^2 dp \Leftrightarrow$$

$$(P^*)^3 = \frac{c}{(P^*+e)^2} + \frac{2}{3}(P^*)3 \Leftrightarrow \frac{1}{3}(P^*)^3 - \frac{c}{(P^*+e)^2} = 0$$

When $P^* - e \geq 0$, there will be a mass point

$$P^* = \frac{\frac{c}{(P^*+e)^2} + (P^*-e)^3 + \int_{P^*-e}^{P^*} 2p^2 dp}{(P^*)^2} \Leftrightarrow$$

$$(P^*)^3 = \frac{c}{(P^*+e)^2} + (P^*-e)^3 + \int_{P^*-e}^{P^*} 2p^2 dp \Leftrightarrow$$

$$(P^*)^3 = \frac{c}{(P^*+e)^2} + (P^*-e)^3 + \frac{2}{3}(P^*)^3 - \frac{2}{3}(P^*-e)^3 \Leftrightarrow$$

$$0 = \frac{c}{(P^*+e)^2} - \frac{1}{3}(P^*)^3 + \frac{1}{3}\left(P^{*3} - 3P^{*2}e + 3P^*e^2 - e^3\right) \Leftrightarrow$$

$$P^{*2}e - P^*e^2 + \frac{1}{3}e^3 - \frac{c}{(P^*+e)^2} = 0$$

Resulting the expression (11)