ADAPTIVE REDUCED BIAS ESTIMATION OF FINANCIAL LOG-RETURNS[7]

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Abstract: Jointly with a set of classical extreme value index (EVI) estimators, we suggest the consideration of associated second-order corrected-bias estimators, and propose the use of resampling-based methods for an asymptotically consistent choice of the *thresholds* to use in an adaptive EVI-estimation of financial log-returns.

1. CVRB EVI-estimators under study

We shall deal with the estimation of a positive *extreme value index* (EVI), denoted γ, the primary parameter in *Statistics of Extremes*. Apart from the classical Hill (Hill, 1975) and moment (Dekkers *et al.*, 1989) semi-parametric EVI-estimators, based on the largest k top order statistics and denoted $H(k)$ and $M(k)$, respectively, we shall also consider associated classes of second-order reduced-bias estimators, in the lines of Gomes *et al.* (2011). These classes are based on the adequate estimation of a "scale" and a "shape" second-order parameters, $\beta \neq 0$ and $\rho < 0$, respectively, are valid for a large class of heavy-tailed underlying parents and are appealing in the sense that we are able to reduce the asymptotic bias of a classical estimator without increasing its asymptotic variance. We shall call these estimators "*classical-variance reduced-bias*" (CVRB) estimators. The CVRB class associated with $H(k)$ was introduced in

[7]Research partially supported by FCT/OE, POCI 2010 and PTDC/FEDER.

Caeiro *et al.* (2005) and it is given by

$$\overline{H}(k) := H(k)\big(1-\hat{\beta}(n/k)^{\hat{\rho}}/(1-\hat{\rho})\big),$$

being a *minimum-variance reduced-bias* (MVRB) class of EVI-estimators. Associated with $M(k)$, we have the CVRB EVI-estimator,

$$\overline{M}(k) := M(k)\big(1-\hat{\beta}\,(n/k)^{\hat{\rho}}\,/(1-\hat{\rho})\big) - \hat{\beta}\;\hat{\rho}\,(n/k)^{\hat{\rho}}\,/(1-\hat{\rho})^2.$$

Let us generally denote C any of the classical H and M estimators, and $\overline{C}(k)$ any of the reduced-bias estimators. Under the validity of adequate third-order conditions, and adequate estimation of (β,ρ) (see Caeiro *et al.*, 2009; Gomes *et al.*, 2011), $\overline{C}(k)$ outperforms $C(k)$, $\forall k$.

In Section 2, we briefly refer an adaptive EVI-estimation based on bootstrap methods, similar in spirit to the bootstrap adaptive classical EVI-estimation in Gomes and Oliveira (2001), and references therein, and to the bootstrap adaptive MVRB estimation in Gomes *et al.* (2009). In Section 3, we refer partial results of a Monte-Carlo simulation related with the behaviour of the non-adaptive estimators. Finally, in Section 4, we provide an application to the analysis of log-returns of a financial time series.

2. Adaptive classical and CVRB EVI-estimation

With AMSE standing for “asymptotic mean square error (MSE)”, $\hat{\gamma}$ denoting either C or $\overline{C}$, and with $k_0^{\widehat{\gamma}}(n) := \arg\min_k MSE(\widehat{\gamma}(k))$, we again get $k_{0|\widehat{\gamma}}(n) := \arg\min_k AMSE(\widehat{\gamma}(k)) = k_0^{\widehat{\gamma}}(n)(1+o(1))$, and a double bootstrap based on subsamples of size $n_1 = o(n)$ and $n_2 = [n_1^2/n]$ enabled Gomes *et al.* (2011) to consistently estimate the optimal sample fraction of $\overline{C}(k)$, on the basis of a consistent estimator of $k_{0|\widehat{\gamma}}(n)$. Such a double bootstrap leads to a k_0-estimate $\hat{k}^*_{0\hat{\gamma}}$ and to an adaptive EVI-estimate, $\widehat{\gamma}^* := \widehat{\gamma}(\hat{k}^*_{0\hat{\gamma}})$. In order to obtain a final adaptive EVI-estimate on the basis of one of the estimators under consideration, we still suggest the estimation of the MSE of any of the EVI-estimators at the bootstrap k_0-estimate, denoted $\widehat{MSE}(\hat{k}^*_{0\hat{\gamma}}|\hat{\gamma}^*)$, and the choice of the estimate $\hat{\gamma}^{**} := \arg\min_{\hat{\gamma}^*}\widehat{MSE}(\hat{k}^*_{0\hat{\gamma}}|\hat{\gamma}^*)$.

3. A Monte-Carlo simulation

Comparatively with the behaviour of the classical EVI-estimators $H(k)$ and $M(k)$, we next illustrate, in Figure 2.6, the finite-sample behaviour of the

CVRB EVI-estimators, $\overline{H}(k)$ and $\overline{M}(k)$, providing the patterns of mean values (E) and root mean square errors (RMSE) of the estimators, as a function of $h = k/n$, for an underlying Fréchet parent, and sample sizes $n = 500$. Similar results have been obtained for other simulated models. Note the clear reduction in bias achieved by any of the reduced-bias estimators. Such a bias reduction leads to lower mean square errors for the CVRB estimators, comparatively with the associated classical EVI-estimators.

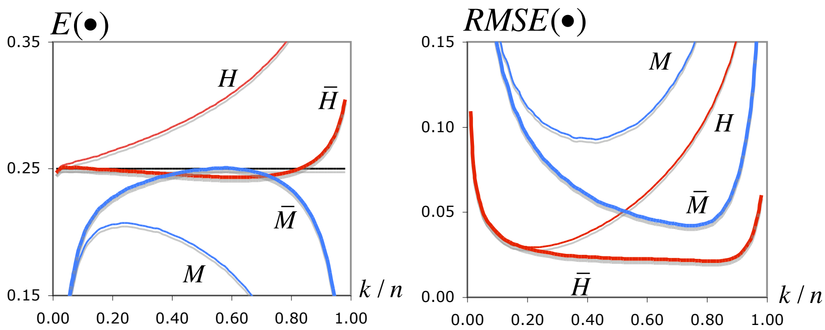

Figure 2.6: Patterns of mean values (*left*) and root mean square errors (*right*) of the classical estimators H and M, jointly with the associated CVRB estimators, as functions of k/n, for an underlying Fréchet parent with $\gamma = 0.25$ ($\rho = -1$).

4. An application to financial data

For the daily log-returns of IBM, collected from January 4, 1999, until November 17, 2005 (with a size $n = 1762, n^+ = 881$), we show in Figure 2.7, the sample paths of $C(k)$ and $\overline{C}(k)$, for $C = H$ and M, jointly with the bootstrap adaptive EVI-estimates described in Section 2. The results clearly favour the $\overline{C}$-estimators. We have been led to the estimate $\overline{H}^{**} = 0.364$.

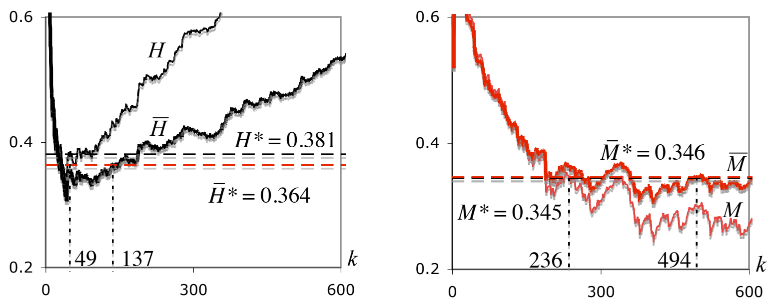

Figure 2.7: Estimates of γ, through the EVI-estimators under consideration, for the IBM log-returns.

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