Distributed causation and emergence in finite models

[António Machuco; Carlos Alves]

C. J. S. Alves
Departamento de Matemática
Instituto Superior Técnico

A. Machuco Rosa
Universidade Lusófona de Humanidades e Tecnologias

A. Nunes Antão
Universidade Lusófona de Humanidades e Tecnologias

Introduction.

In this article we expose some of the general properties of complex systems that can be symbolically represented by finite models. They are quite simple models, that consist in a certain number of elements that assume certain states and the evolution from one state to another is the result of the action of some law. Examples of this kind of models are genetic algorithms and cellular automata, for instance, the most known model is the Game of Life. We begin by presenting them here in some general context, as symbolic entities, because there exists common properties in a symbolic description that motivate an analysis not related to the reality they may intend to describe. Thus, through the paper, we refer to models considered in a scientific perspective, initially not connected to any particular object, without any intention of relating the model to a particular reality. However, when the abstraction is sufficiently general, it is natural that similar consequences arise from certain common characteristics. These analogies and interpretations will be presented in the final section.

To be more specific, in the first section we present some of the general properties of the finite models. We will often appeal to the reader's intuition, and therefore the second section will have as a main goal the computational synthesis of those properties. We will present computational experiments with a code conceived for that specific purpose, which will guide the discovery of complex systems and the analysis of the concept of *emergence*. In the third section, we will also make some computational experiments to which the reader is invited and where his intuition and action lead him to a particular finite model, the model of Schelling and to the segregation processes. Finally, in the last section, we will present what kind of substantive hypothesis about the empirical reality can be obtained from the symbolic description and the computational synthesis of the finite models.

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1. Finite Models

The finite models in discrete time processes here analysed are the models that describe the evolution of a system under successive application of a law. To be more precise, a model of that type consists in a finite number of elements that admit a finite number of states. A simple example is to consider that elements admit only two states, that we can call state 0 and state 1. To the set of states that the model presents in a certain moment we will call *configuration*. Obviously, instead of considering only two possible states, we can consider models where the elements can present a finite number of states. The following example shows these introductory definitions.

Example. *Consider a model with 10 elements, represented by small squares that may assume 4 different states represented by colours. The Fig. 1.1 presents two possible configurations of that model. The difference between them can be resumed in the change between the white and the blue colour in the two central elements. The arrow placed between the two configurations represents the evolution by the application of some law. In this example, that law caused a change of state in the central elements.*

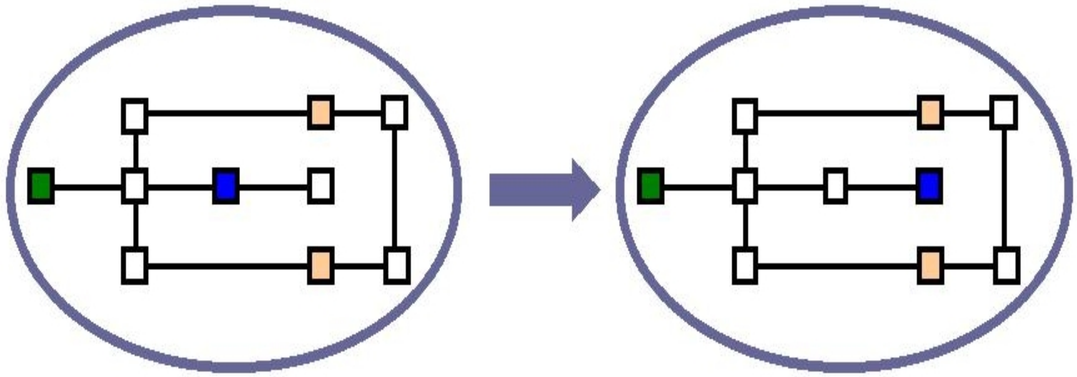

Figure 1.1. *Example of evolution of a certain configuration.*

Note that there are several possible interpretations for the evolution shown in Fig.1.1, and all these interpretations are equivalent. One can either consider that each element corresponds to the position of a static particle and, in that interpretation, the evolution was the colour change in the central elements, from blue to white and vice versa, or one can also see that change as a movement from a blue particle into a void space, represented by the white colour. The observer is the one responsible for this kind of interpretations, and they are usually motivated by a connection between the model and some reality, or is just an interpretation that allows an easier understanding. One should keep in mind that, in both situations, and with no connection to the interpretation, the configuration evolution is always the same. This is a characteristic of these models that we will address again.

The lines that are plotted connecting the small squares define a topology establishing a proximity relation between the elements. The elements and the lines connecting them define a *graph*, a structure that allows the formal analysis of the finite models as *nets*. That topological framework can be avoided, but it will be necessary when we need to distinguish between local and global phenomena, since neighbourhood notions are present. In the models presented in sections 2 and 3, being obvious what is the neighbourhood considered, we do not present the lines connecting the elements.

It is worth noting the enormous amount of possible configurations that exist even in simple examples, like the one presented in Fig.1.1. Having 4 colours for 10 positions, the number of different configurations is quite big, 4^{10} = 1 048 576, which shows the magnitude of these values even in models with a small number of elements (positions). For instance, bellow in the text, we will present simulations for a model with 400 positions and only two states, and this gives 2^{400} possible configurations - a number with 121 digits. We do not even consider finite models based on the pixels of a computer screen, since $1024*768$ *pixels* with 65536 possible colours, generates an astronomical number of combinations, with more than 3 million digits! This kind of magnitudes may lead to the illusion of almost infinite quantities, and to careless conclusions.

The evolution of finite models in discrete time, and the dynamics that leads from one configuration to the next one results by the application of some law. Each application of the law is called *iteration*, being possible to apply the same law to the new configuration, and so on. A characteristic of the evolution of the configurations on a finite model is the generation of *cycles* or of *fixed points*. When the law **determines only one possibility** and when after *n* iterations you obtain the same configuration, we get a *cycle* with length *n.* If the configuration does not change it is called a *fixed point.* Fixed points and cycles are types of what is called an *attractor*, and besides these two types one also considers *chaotic attractors*, that do not occur in finite models.

In closed finite models, when a well-determined law is applied, a simple and immediate consequence occurs:
After a finite number of iterations, the configurations are repeated and therefore cycles occur.
A justification of this fact is quite elementary. It suffices to think that the number of possible configurations is finite, equal to E^P, where P is the number of positions and E is the number of states. After a number of iterations greater than E^P there must be a repetition of a configuration and since the law determines the same evolution, a cycle is formed.

Below on the text we invite the reader to execute simulations where that conclusion is not immediate. Due to the enormous amount of possibilities, the viewer might think that the evolution leads always to different configurations. However, when a well-determined law is applied and the configurations are repeated, we are in the presence of a cycle or a fixed point. Only in the case of an open system, using external factors, for instance, random factors, that conclusion may not be true.

Examples. *Let us see two examples in which we obtain all the configurations in the system starting from one initial configuration. We consider a model with 3 elements (positions) and 2 states. This gives 8 different possible configurations. Initially we present a global cycle:*

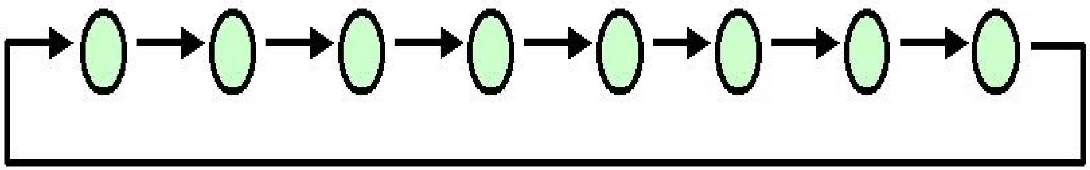

Figure 1.2. *Iterations on a global cycle. The arrows represent the well-determined evolution, one for each iteration. Notice that each ball represents a configuration (a set of elements with precise states) and not an element.*

In this first example (Fig.1.2), it is obvious that starting from any configuration we can arrive to another one after a finite number of iterations. This defines the length, or period, of the cycle. In other example (Fig.1.3), the initial configuration (and all the others in blue) are not repeated, because the evolution ends in a smaller cycle (configurations in green):

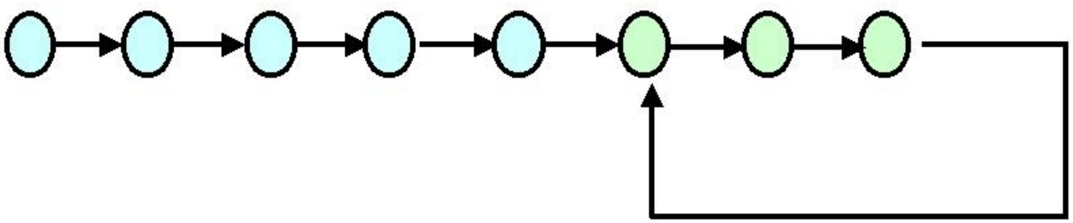

Figure 1.3. *Iteration that leads to a smaller cycle. Arriving at the last configuration, the system evolution returns to a previous configuration and enters in a cycle (configurations plotted in green).*

In these two examples one can see that the iterations may lead the system from one configuration to all the others. This is not always the case. In Fig. 1.4, we see that the system evolution may leave aside a part of the configurations:

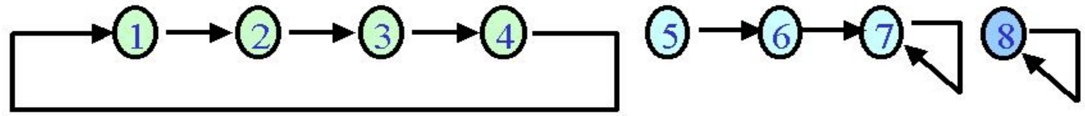

Figure 1.4. *Independent evolutions, cycles and fixed points.*

As shown in Fig.1.4, the configurations 1, 2, 3 and 4 are inside a cycle, therefore if we start the evolution in any of these configurations, it is clear that the configurations 5, 6, 7 or 8 will never occur in the evolution. On the other hand, if we start in any of the configurations 5, 6 or 7, the result, after a small number of iterations will always be the configuration 7... also, starting in the configuration 8, there will be no change. Configurations 7 and 8 are fixed points under the defined law.

There is a major difference between finite and infinite models. In the finite case, if the law is one to one, starting from a configuration that is not a fixed point it is not possible to attain a fixed point configuration. In the infinite case, this is perfectly possible with certain one to one laws. Thus, in a discrete law, we can only aim to attain fixed points starting from different configurations if the law that we apply is not one to one. This immediately implies the existence of *primordial configurations* for that law, i.e., the existence of configurations that do not result from the evolution of other configuration.

Example. *The existence of cycles and primordial configurations in a law that it is not one to one.*

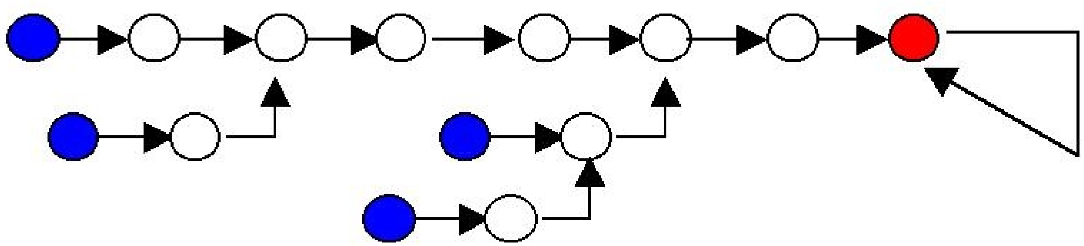

Figure 1.5. *Primordial configurations (in blue) and a fixed point (in red). Following the arrows, the reader may see that any initial configuration converges to a fixed point (in red).*
This leaves some initial configurations, plotted in blue, that are primordial configurations, meaning that they do not arise from a previous configuration, as the picture shows.
This defines several 'paths' *that, by the iteration of the law, lead from a primordial configuration (in blue) to the fixed point (in red).*

As pointed out before, even if fixed points do not exist, the result of the iteration of a well-determined law ends always in cycles. The configurations that are not included in those cycles are always preceded by primordial configurations. Therefore we can conclude that in a finite model the evolution of the configurations can be described by cycles or paths that end in cycles (these cycles may be fixed points).

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2. Emergence of patterns or laws

Previous conclusions were taken in a symbolic level, and at the same time they were used to introduce some basic concepts used on finite models and on the complex systems that they might represent. Their computational synthesis will be made by experiments (to which the reader is invited). By those experiments it will be possible to synthesise the local/global dialectic. The characteristics of these last two concepts will be revealed progressively, but we will be more close to their nature if we notice that a law operating in a finite model may present features not on the element level, but also at the level of a set of elements, i.e. at the level of a configuration. This level distinction, between a *micro* and a *macro* level will now assume a central role.

Consider a finite model in which the elements are 20x20 small squares (pixels). Each element may assume two possible states, red or white. In Fig.2.1 are represented two configurations for this model, where the configuration presented in the right picture was the result of the configuration on the left applying 5 times a prescribed law.

(In the simulations that follow, we use circular topology on the plotted square, in the sense that an element being in the north border is a neighbour of an element being in the south, and the same goes for the east and west borders of the square. Thus, when a particle is moving and arrives at the square border it may appear on the opposite side)

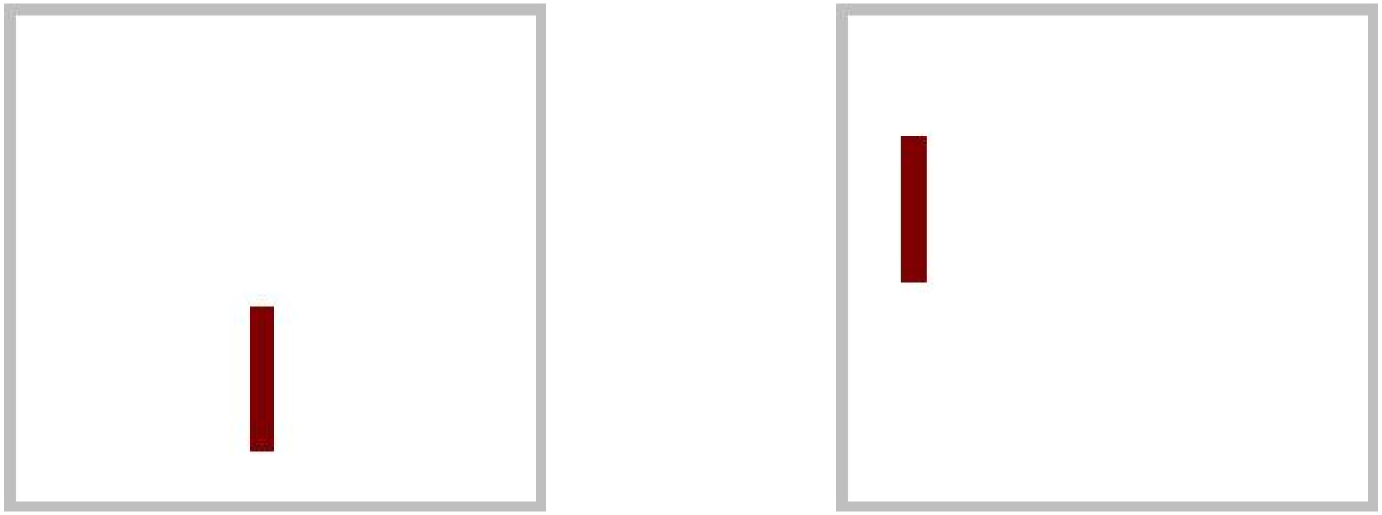

Figure 2.1. *Apparent movement of a segment on the model.*
We see that the vertical segment 'moves' in the northwest direction, keeping the shape.
EXECUTE SIMULATION

Suppose that the law that acts on the model is unknown. We will try to infer by abduction, using computational simulations, what is the law that explains the observed behaviours.

In the example presented in Fig.2.1, we can visually identify a vertical segment that seems to move in the northwest direction. Starting with the initial configuration given in the left picture, the reader is invited to press the button 'Iterate' on the dialog box of the online simulation. One sees that, in fact, the segment moves always in the northwest direction. Undoubtedly, the program viewer clearly identifies a vertical segment, but is that identification of its own responsibility? That grouping is even further justified when the viewer sees that by application of the law (pressing the button 'Iterate'), there is a similar structure that the only difference to the previous one is the fact that it has changed position. That is, with no further information the viewer might think that the only existing reality on the model is a segment that moves in a certain direction. The movement of the all block is therefore inferred. (In a certain way, one might say that this is the process that generates the illusion of animation. The identification of shapes by the association of pixels and the illusion of movement is a consequence of the viewer interpretation.)

It will be used the notion of *body* to designate any well-defined set of elements, like vertical or horizontal segments. When a body keeps its structure invariant under the action of some law, we will say that we have a *pattern*.

Notice that, at this stage, one is supposed to ignore what is the law that acts in the system. However, after some computational experiments, similar to the one presented in Fig. 2.1, for other vertical or horizontal segments, it might be quite tempting to admit that that law is simply: *bodies are moving northwest, keeping shape and constant 'speed'*. This tentative law holds perfectly if we only test the model with vertical and horizontal segments - the only bodies that we assume that might be tested/built at this stage. Notice that the process described is analogous to the method used on the elaboration of scientific theories. In both cases, one infers laws from the existing observables. In the computational simulations that follow, as in the scientific method, we will proceed with the construction of new observables.

The first of those simulations consists in testing the previous conjectured law using different bodies. In the example that follows we built segments that are almost diagonal, like the one shown in Fig.2.2A):

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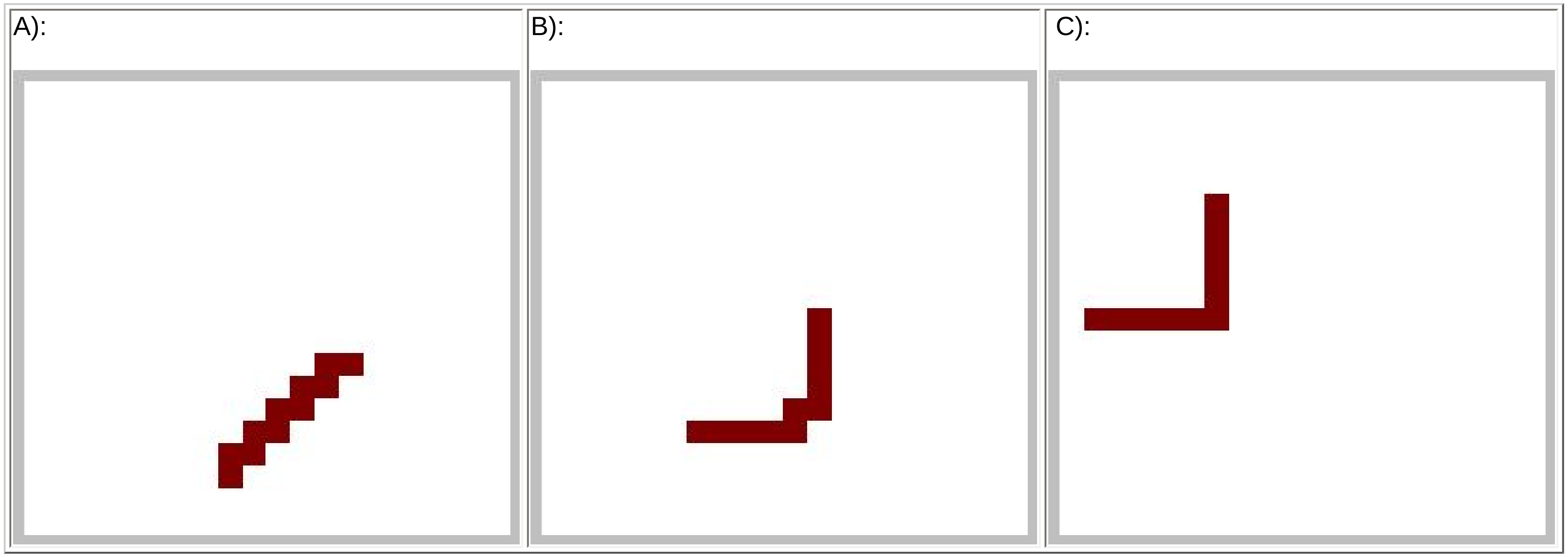

Figure 2.2. *Transformation of an almost diagonal segment.*
EXECUTE SIMULATION

Executing the suggested simulation the reader will see that initially the body will be transformed in a J-like shape, and when that transformation is concluded (Fig.2.2C):
, the body will be moving northwest, like the previous tested segments. However the previously conjectured law is not enough to explain the transformation from the almost diagonal shape to the J-like shape. Basing ourselves in the new observations, we might be tempted to introduce auxiliary hypothesis to that law, adding the assumption that almost diagonal bodies have a limited time of life, being decomposed in a new body which could be seen as the union of a vertical and an horizontal segment. This kind of auxiliary hypothesis is frequently used in science. But does this bring new advances in the theory that explains the evolution of the bodies in the system? Is one closer to the law that acts in the system? Let us see...

In a third phase, the new experiment consists in 'bombing' the almost diagonal segment with an elementary particle. Notice that this was not possible before introducing almost diagonal bodies, because all bodies were moving at the same speed, thus with no chocks. The result of the simulation is the following:

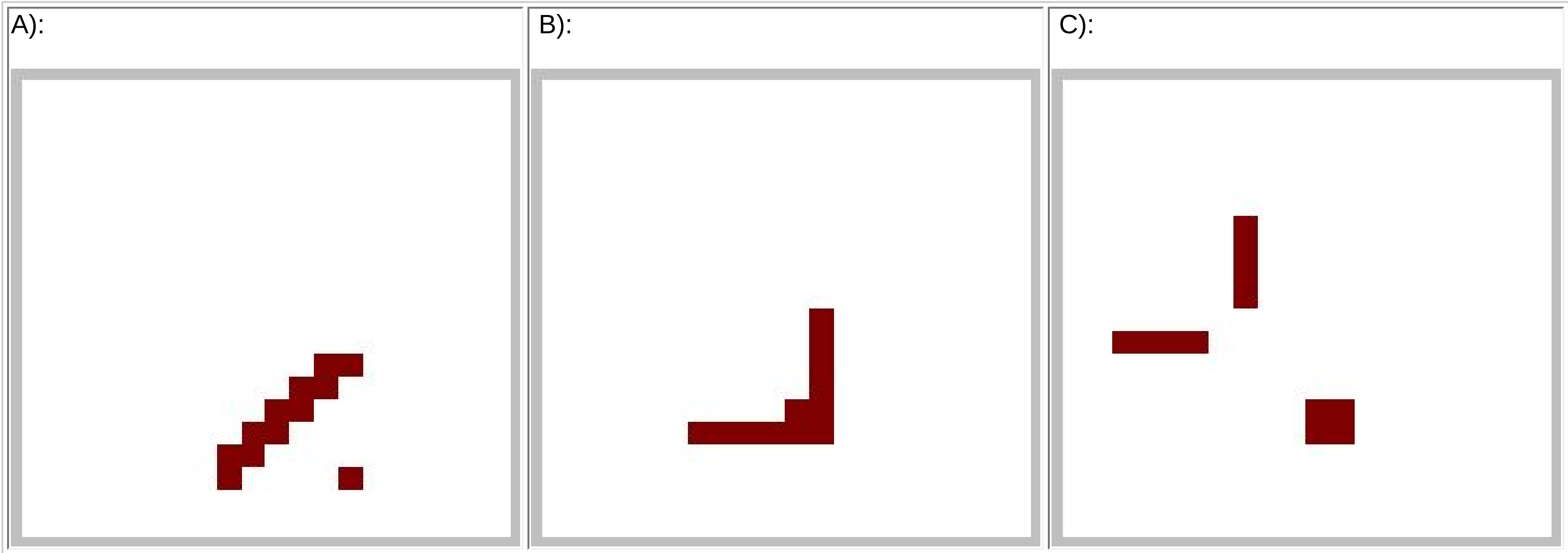

Figure 2.3. *Collision between an elementary particle and an almost diagonal particle.*
EXECUTE SIMULATION

In Fig.2.3A):
, we see that by simulating the chock between an elementary particle and a diagonal body, in the middle of the transformation process (Fig.2.3B):
, one obtains (Fig.2.3C):
the vertical and horizontal segments predicted before, but now they are disjoint. Additionally a new body was obtained - a 2x2 square. If the reader iterates further on, it can be seen that that square does not change its position, i.e. the new square is not affected by the 'northwest attraction law'! This is a bigger exception to that law, and new explanations were in need.

We will not proceed with a list of new experiments that could be made in order to find out the law that acts in the system and that is responsible by the several features of the tested bodies. We can say that with some effort, it could be possible to state the correct law or an equivalent one, which was simply:

Law: if an element has less than 3 neighbours of its own colour, he changes colour with a neighbour of opposite colour.
To choose among the possible opposite elements, he starts with the northwest one. If that one is not opposite, he chooses the west one, otherwise the southwest one.

(This law is applied starting with the northwest positions and finishing with southeast ones)

It is important to notice that this is a local law. Each element has short sight, since the evolution of his state is only determined by the states of its closest 8 neighbours. He does not see anything behind that horizon, he does not 'know' anything about the global configuration of the *net*. This type of law, defined using a neighbourhood with 8 elements, is usually used in cellular automata (from which this model is an example). An aspect of this law is shown in Fig.2.4.

Figure 2.4 *In the left picture one sees the red element rounded by its 8 white neighbours. Since that element has less than 3 neighbours of its own colour (in this case, it has none), the application of the law stated above is reflected on the change of position between the red and the northwest white element.*

Consider now the Fig.2.5. All coloured squares have 3 or more coloured neighbours, and this would imply no changes. However changes occur due to the fact that the white square in the centre has only two white neighbours. In fact, in this example, one can see two cross-shaped bodies glued together. The cross shape is the only one that enables immobility when we consider only 5 elementary particles, under the action of the stated law. One might expect that being cross shapes immobile under the action of the law, these two glued together would also be stable. However, since the white particle must change position, some of the coloured ones will also change.

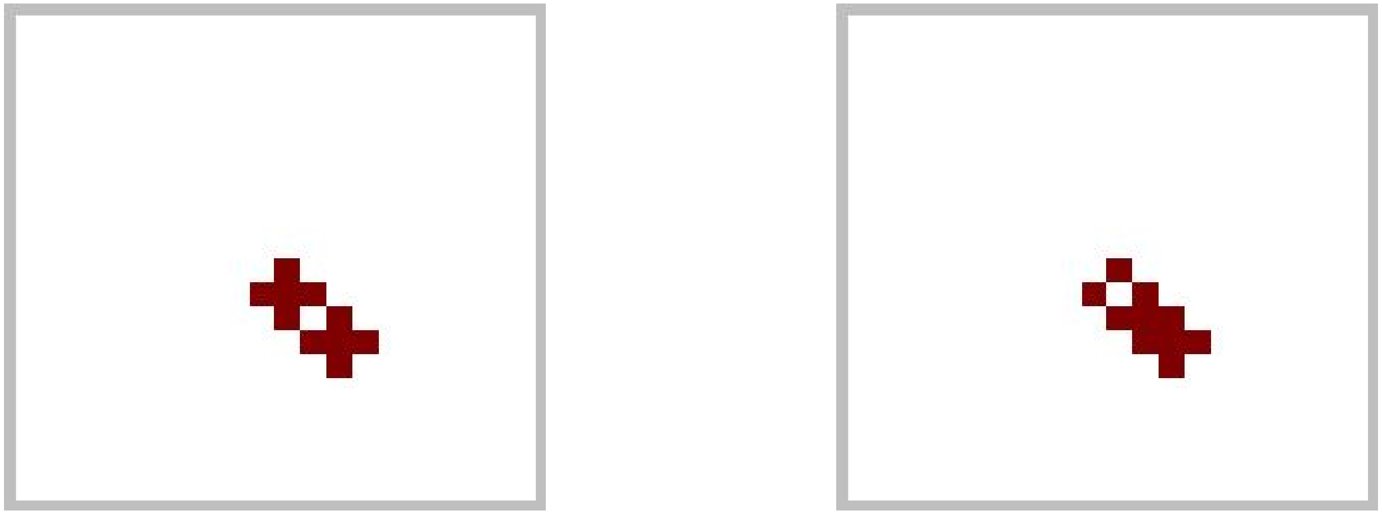

Figure 2.5. *A white square caught inside two stable cross-shaped bodies.*

What is mainly important to conclude from these simulations is that one can infer complex laws of behaviour, at a macroscopic level, from any law that determines the system evolution with local statements, applied to each element. Thus, the previously called 'northwest attraction law' is a completely true law for the observables available in the first stage of our experiment. It is a completely true law at that level – the macroscopic level, and we do not have to know the real underlying micro law that generates it. In subsequent stages one can infer other laws that are equally true and that have nothing relating them to the local law applied in the model. In that law, it was never said that one could never have an immobility situation (or a fixed point) with less than 4 coloured elements. One never stated that exactly with 5 coloured elements, the only immobility situation (and fixed point) would be cross shape. One never stated that the elements aligned in horizontal and vertical segments move together northwest, and so on. The law does not even predict the grouping of elements, it is only applied to each element in its neighbourhood.. Thus, and answering a previous question, one can say that the interpretation that the grouping of elements and patterns exist is made by the viewer under its own interpretation and responsibility.

However, we also notice that by stating that there is an interpretation made by the viewer, this does not mean that the macroscopic state is not real, that it is only an illusion! That state depends on the observer, but it is real, in the sense that it is an autonomous state to the microscopic dynamics underlying. We ask the reader to recall the simulation made in Fig.2.1. In the language of complex systems theory, one can say that the pattern (the segment moving northwest) *emerges*. It is a macroscopic pattern that emerges from the microscopic local law that, in some point of view, it is the only real dynamic acting on the model. As it was previously stated, in what concerns the law, it makes no sense talking about a vertical segment, or even of bodies. However, in the observer's mind, it makes sense to say that it exists and that it is real a macroscopic state that consists on grouping the elements, even if the pattern is generated by the local law. It is this subtle difference between a 'micro' and a 'macro' level that raises the interest on the emergence phenomena. The confirmation that these two levels exist and that they are real is that the pattern feature persists although the elements and their states are changing in each iteration. In the case of the simulation presented in Fig.2.1, we can consider that the segment in Fig.2.1B):
is made by different elements from the ones considered initially in Fig.2.1A):
. One can even say that this is the confirmation of the existence of a level that, being completely generated by a local macroscopic law, is however autonomous.

Another important conclusion concerns stability, the effect of small local changes in the global configuration. All the cases presented so far are deterministic, being perfectly possible to know the state of all elements after any number of iterations; however, if we are deprived of the knowledge of some part of the system, it may be not possible to predict the evolution in the known part. This fact is due to an interdependence of the local law in the neighbourhood, the behaviour of each one depends on the others. This interdependence among the elements of a model characterizes complex systems. This is illustrated in the example shown in Fig.2.6.

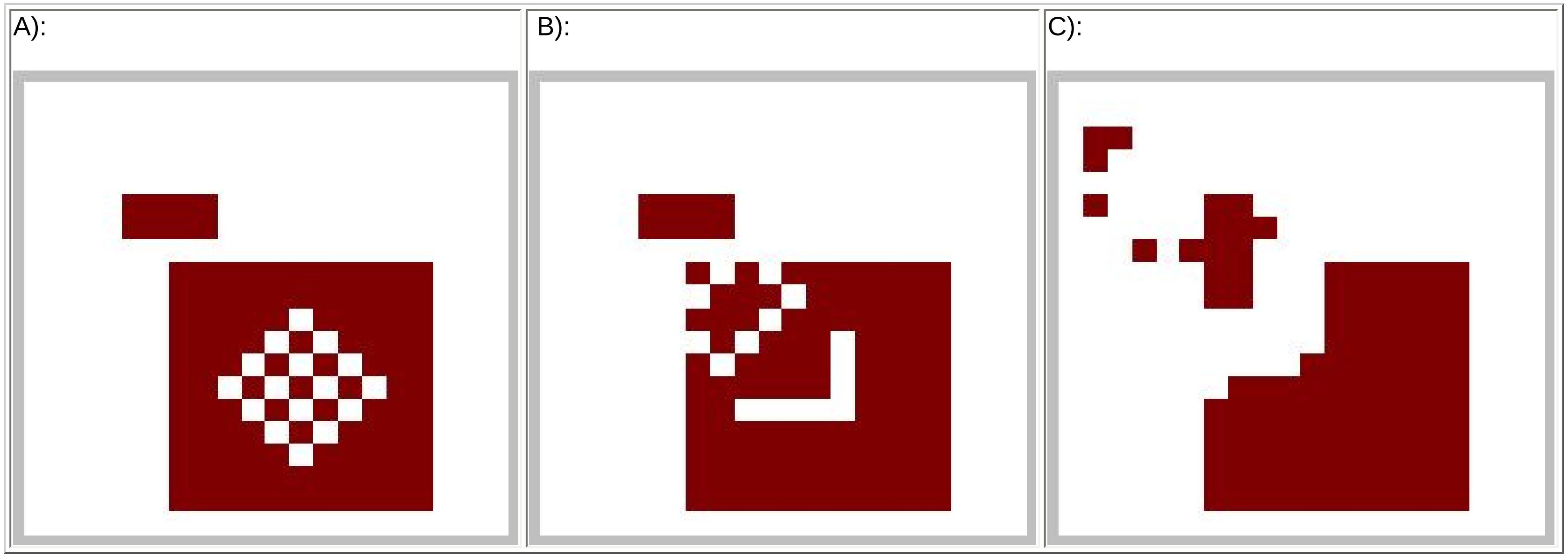

Figure 2.6. *Propagation of instability - impossibility of local prevision.*

In the situation shown in Fig.2.6A):
, the elements of the rectangle placed above the big chess-square seem stable according to the 3 neighbour law. In fact, making a prediction of the system evolution only using the elements that are in the neighbourhood of the rectangle, we hardly could suspect the evolution that will occur after a few iterations. The interior of the chess-square is not stable, because some of the elements with white states have only two white neighbours. This instability will be quickly propagated to other elements, generating a sort of

'explosion' of the chess-square. In Fig.2.6B):
it is already possible to predict that the instability generated inside the chess-square will produce the destruction of the northwest part of the square and will then change also the rectangle, previously considered stable (Fig.2.6C):
. It is interesting to notice that in this example, one could predict that no change could ever occur in the rectangle in the next two or three iterations, by an analysis of its neighbourhood. The instability verified inside the chess-square could never be propagated immediately to the rectangle. There is a maximum propagation speed of one element, by iteration, due to the fact that the local interactions, defined by the system law, apply only to the immediate neighbours. This is in some context analogous to what is assumed in the relativity theory, where a maximum speed of propagation of physical interactions is assumed - the speed of light. Notice that if the law acting in this system was not a local one, the last considerations could not be stated. In fact, finite models with local laws clearly verify the locality principle assumed in the physical models.

3. Schelling's Model

Having pointed out some analogies with Physics, we now state that the model that was presented is similar to a model introduced by Thomas Schelling in the seventies (Schelling, T. (1971). "Dynamic models of segregation". *Journal of Mathematical Sociology* 1, pp.143-186). It is probably the first complex system model with the objective of studying social phenomena. Schelling gave a particular interpretation to a model that is almost equivalent to the one presented above. Schelling was searching sufficient conditions to the existence of segregation processes (between two races, for instance). Its model will be present below. First, we ask the following question:
Let us suppose that the individuals in a certain population are tolerant persons, and do not feel the need to move even if they are in minority... for instance, suppose that they only move if they are surrounded by more than 62.5% of persons of another race.
This tolerant behaviour generates segregation phenomena?
Later on we will answer this question, first we state a formulation of the Schelling Model.

A formulation of Schelling's Model:
There exist elements (individuals) that may assume two states (they belong to one of two races), and the behaviour of each one of them is determined by the state of their neighbours (in the number of eight). The following law gives the evolution of the model:

- If an individual has at least three (among the eight) neighbours of its own state, it does not move. Otherwise it moves.

So far, the model is similar to the one previously presented. The difference is that we will not assume a privileged direction of movement, we will now assume that

- The movement is made in a random direction (thus, the ‘northwest tendency’ does not occur).

In what will follow, we will use the term *deterministic law* when the individual moves along a prescribed fixed direction (northwest, for instance), and we will speak of a *random law* when the individuals choose randomly one of the eight possible directions.

Starting with an initial random population like the one presented in Fig.3.1A):
, where each square represents an individual, we will first apply the deterministic law, considering (as before) that the movement is made in northwest direction.

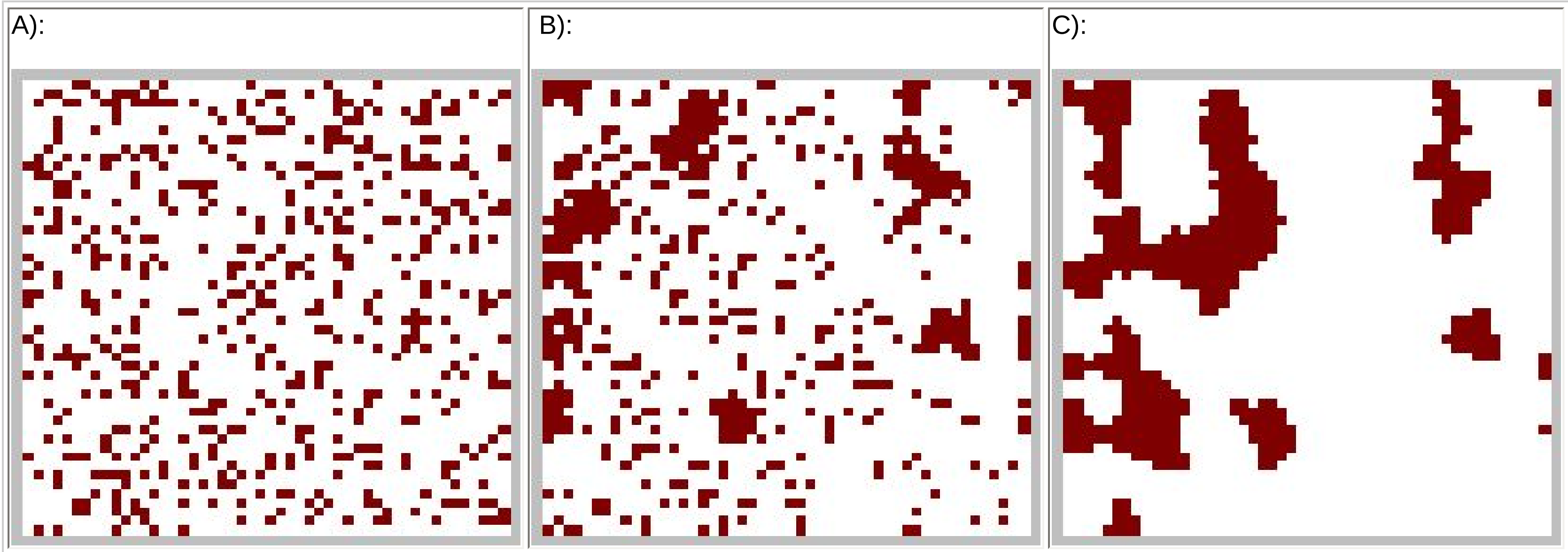

Figure 3.1. *Deterministic evolution to a fixed point, starting from an initial random state population.*
EXECUTE SIMULATION

Starting with the initial random states, after some iterations, we already see that there exist a concentration of individuals, forming groups, and only a few are remaining, that in their movement (along the northwest direction) will be fixed in the bigger groups already formed, increasing the dimension. In the picture Fig.3.1C):
we can see the final result, where the iteration was stabilized, which is a *fixed point* of the law. The reader may continue with new simulations and it will be seen that the model will always converge to a fixed point, which is an invariant that 'attracts' any initial configuration.

In the next simulations (Fig.3.2) we will present the result of evolutions made with a random direction law, and we will see again the generation of fixed points. Notice that in this random situation the law changes. The direction of the movement of the particles may be different even if at some iteration it was produced the same configuration. Thus cycles do not occur, only fixed points. When a particle has three neighbours it still does not move and therefore fixed point situations occur.

We begin with the simulation of law where the rule of the number of neighbours is decreased from 3 to 2. The reader may verify executing the simulation that the evolution is similar to the one presented in Fig.3.2B):
. That configuration is a fixed point to the 2 neighbours' law. Afterwards, the reader might change the option to the 3 neighbours' law and the previous structure is no longer stable... new movements occur and the system evolution leads to a configuration similar to the one presented in Fig.3.2C):
, which is now a fixed point to the 3 neighbours' law. It is clear that the segregation feature is more evident in the 3 neighbours' law, leading to the constitution of more compact groups.

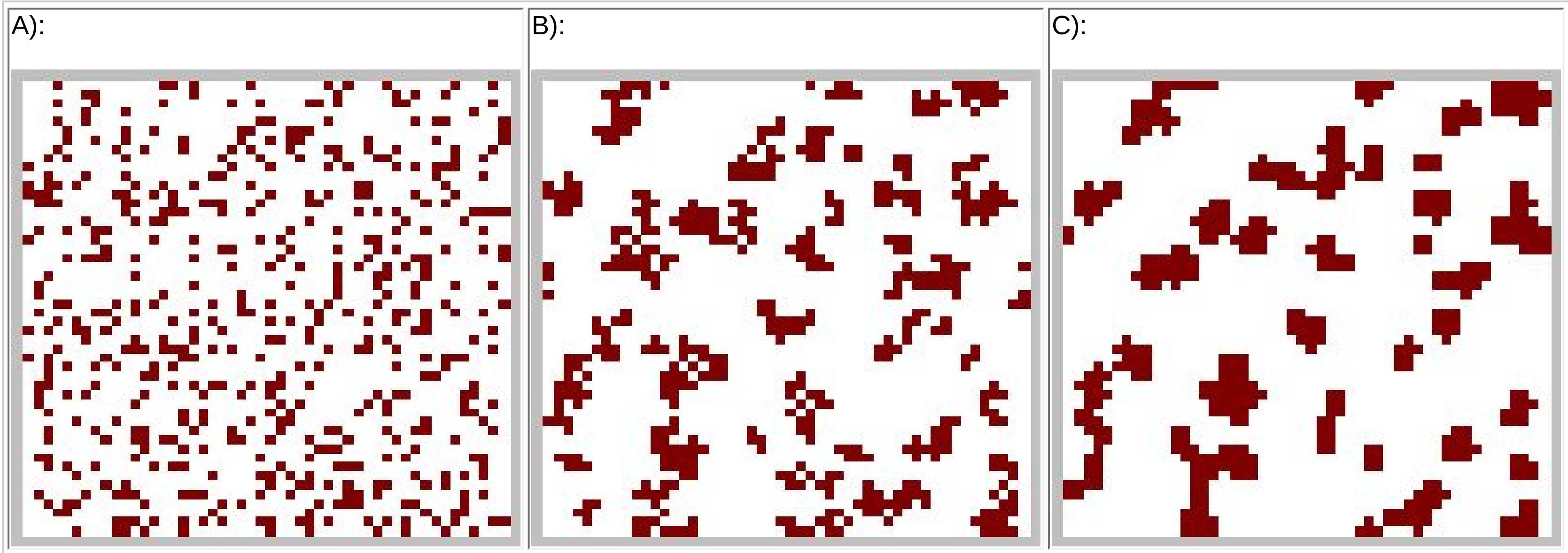

***Figure 3.2**. Evolution according to two different local laws. Fixed points for the 2 neighbours and 3 neighbours law.*
EXECUTE SIMULATION (2 neighbours' law) ---------- EXECUTE SIMULATION (3 neighbours' law)

Suppose again that the law is deterministic, prescribing the northwest tendency with the 3 neighbours law. As it was pointed out in Section 1, if there is not a convergence to a fixed point the evolution will lead to the generation of cycles. An example of that situation can be achieved with the initial configuration plotted in Fig. 3.3A):

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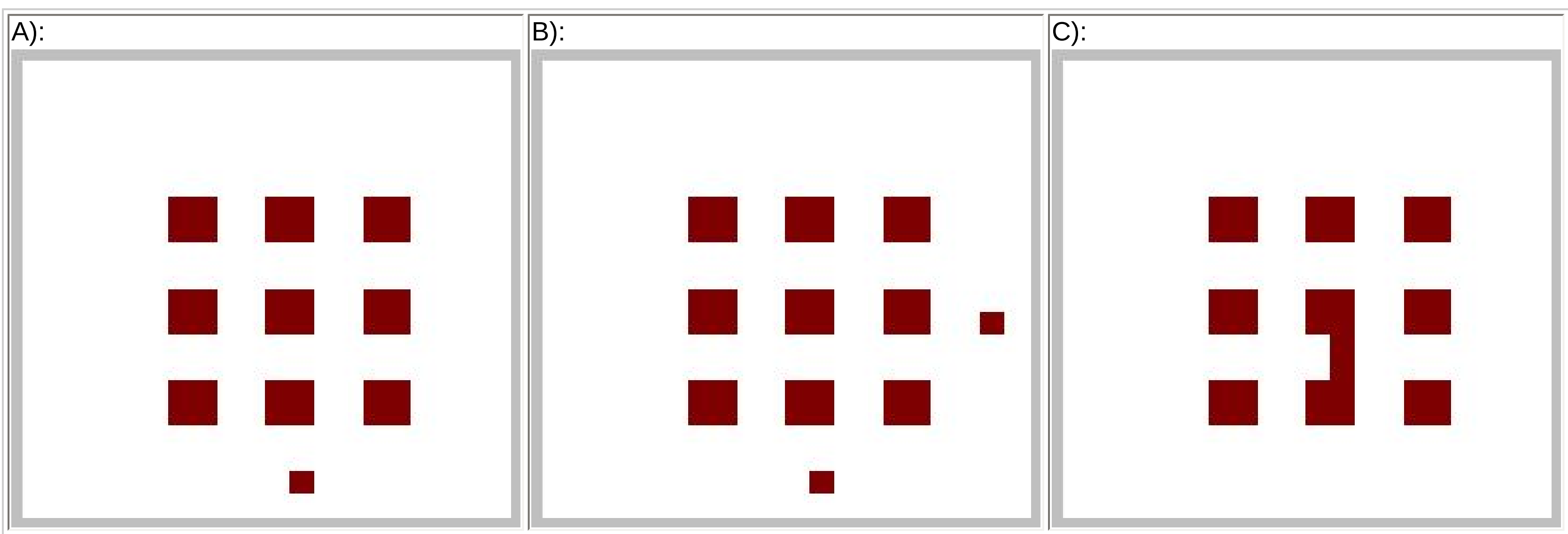

Figure 3.3. *An initial configuration case where local stability (almost everywhere) generates an extra difficulty for the global stability.*

The particle that is not aggregated to one of the 9 groups that we see in Fig.3.3A):
can never be placed in a stable situation, unless we change the position of the others. If the law is deterministic the evolution will enter in a cycle. The same situation occurs in Fig.3.3B):
if the evolution law is the one with the northwest tendency. The two isolated particles will enter in a cycle and they will never be with 3 red neighbours. Since we have a global vision of the model it would be easy to place the remaining 2 particles in a stable configuration, as presented in Fig.3.3C):
. This is also possible if the random direction law is prescribed. The configuration of Fig.3.3C):
corresponds to one of the possible stable situations and it was found after a large number of iterations. Notice that if there was only one particle like in Fig.3.3A):
, it would be impossible a stable configuration, even with the random direction law.

By introducing the random direction condition we are no longer with the deterministic case and it will be impossible to predict the system evolution. In the random case, the evolution of the same configuration may lead to different results and cycle situations will be no longer possible, the only type of attractors that may exist are fixed points. In fact, if one excludes initial configurations that can never converge to a fixed point (for instance, starting with an initial configuration like Fig.3.3A or with a configuration with less than 4 red elements), the system evolution will stop in a fixed point if one assumes that the distribution of probabilities is normal.

4. Complex Systems and Distributed Causation

The model proposed by Thomas Schelling is one of the first examples of *agent-based modelling*. It is a domain of research that is growing fast, and a considerable amount of information can be found in these *useful links*. A good web site to get software is Artificial Life Online, and one of the main research centers on the subject is http://www.santafe.edu/. As it was referred the model may be seen as a model of social dynamics of segregation. The initial idea of Schelling was that the model was to be interpreted as a sufficient condition of that kind of processes. Obviously, such a simple model can not by itself explain the dynamics of segregation, and it is not our intention to suggest otherwise. Nevertheless, we should note that its simplicity is really a virtue because it suggests a general type of explanation in the social sciences, which may be developed on the basis of the construction of models that have more and more empirical plausibility. This type of models is being proposed, and Sugarscape is the most ambitious project. It is important to notice that the general philosophy of these models is almost completely present in Schelling's Model, and therefore it may be a powerful heuristic tool that enables us to get common conclusions in any agent-based modelling. In fact, as it has been suggested on this paper, the computational model presented in the previous Sections exhibits generic properties. In the following, our aim will be to guide the formulation of substantive hypothesis about the reality using the model.

What are the sufficient conditions to the existence of segregation phenomena, racial, urbanistic or social ones? If such a question was addressed to someone that effectively belongs to a segregated group, probably the answer would be of this kind:
'I prefer to be in a neighbourhood where my group is in majority.'
Of course, this rule should be applied to everyone that wants to be in a same group. Below, we will analyse this type of answer, but we notice that the answer to this question obtained by an interpretation of Schelling's Model is *not* the same.

Recall that the law defining the evolution of the model is the following. Each individual counts the number of neighbours that have the same colour. If that number is less or equal to three (ie. more than 62.5% are of different colour), he moves in a random direction, otherwise he does not move.

We will now give an answer to the question formulated in Section 3, and recalled in the beginning of this section. This model states that even if the individuals have a tolerant behaviour in their neighbourhood, segregation phenomena occur. In fact, starting with any random distribution of individuals, we already saw what is the result of the consecutive application of that law. For instance, one of the possible results was presented in Fig.3.1C):

. The iteration led the system to an invariant final configuration, a fixed point. That configuration shows clearly a segregation situation: groups with red elements clearly separated by groups of white elements. Notice that, following the statement of the law, each individual does not mind to be in a minority situation, one can say that each one of them is non-segregationist, however the final invariant result of such a behaviour is a segregation configuration. If one thinks that the individuals follow such a rule one may conclude that they cause the global state, locally non intentional, of complete segregation.

One can go even further, and we suggest the reader to execute some iterations of this simulation, where the law was changed, increasing the criteria of the number of neighbours from 3 to 4. This means that each individual demands to be in majority in its neighbourhood (i.e. 5 in 9, including himself), otherwise he will move. This is the situation stated above; when we supposed that the answer would be that each member should be in a neighbourhood with a majority group. Notice that this is behaviour is not completely intolerant: each member admits the presence in its neighbourhood of members from another group. However, after a considerable number of iterations it is easy to conclude that the evolution of the system will not produce a stable configuration! The majority rule does not usually produce stable configurations... there is a generalized unhappiness! In fact, even with the more tolerant rule of the three neighbours, segregation configurations were already present, although each individual was not segregationist. This means that each one of the elements has no representation of the large-scale consequences that their local action is producing. In this example, the global state is even opposed to the premises of each individual.

We emphasize that we do not state that Schelling's model is a necessary and sufficient condition capable of explaining the empirical reality of segregation. The emergence of the state of segregation in this model is not even generated by social behaviour constraints, it is not only a consequence of the local law but also of the topological/geometrical features that characterise the connections between the elements. This topological constraint exists in any model in which the interactions occur and is determined by the number of local connections that were established, defining a neighbourhood. Thus, Schelling’s model has generic properties that are shared by any dynamic based in local action principles. We can render this idea more plausible if we point out some of the generic situations of Schelling’s model.

Suppose that a model similar to Schelling's model (perhaps more sophisticated, with other parameters - that is not important here) is a model of the empirical reality, i.e. the individuals are really myopes and they only act based in some local rules. In the model this situation was analysed in Section 2 with the example in Fig.2.6. Analysing the system locally, one could not predict the evolution, due to the interdependence of all elements. The real agents, that the model may represent, they also do not have the perception of all interactions, and since they are myope they only represent isolated parts of the system. Therefore, one concludes that a real agent does not have the capability of anticipation of the local actions produced, and one can only predict the future configuration after an evaluation of the whole. Schelling's model reveals its heuristic capability of thinking some phenomena. It points to one fundamental hypothesis that each individual is in fact myope, has an extremely bounded rationality, and does not have the perception of the global state to which he is contributing in a non-intentional way.

Such a simple model can also suggest other heuristic lines of thought. In fact, it raises epistemological questions such as the need of rethinking a fundamental category of thought, the *category of causation.*

We now formulate the question of segregation differently. What was the cause of the segregation feature exhibited in Schelling's model? It is certain that it is not any of the four great causes mentioned by Aristotle. If we leave aside the *formal cause*, with an interpretation that is not always very clear, we are left with the following causes:
Material causation – that from which one thing comes and that makes it persist, i.e. the material from which one thing is made.
Efficient causation – the primary cause of rest and change, i.e. the thing or agent responsible by the change in the form of a certain body, as in the case of the sculptor and the statue. This definition does not necessarily mean that the change must occur by direct physical contact.
Final causation – the purpose for which one thing is made, as when a knife is used to cut some desired food.

These causes may be resumed in the conception of an individual (considered almost in isolation) that is the cause that modifies some object (efficient causation), eventually as a mean to an end (final causation). Notice that, among the three causes, the efficient causation is the primordial type of causation, and it is a kind of *local* causation.

So, again, which type of causation is present in Schelling’s model? Which one is responsible by the final state of segregation that can be mathematically described as a fixed point? In a certain sense, at least in the Aristotelian concept of causation, that cause does not exist. The real cause is a distributed property that it is not present in any isolated part of the system. We will name it: *distributed causation.* It is no more than the result of the multiple non-linear interactions between the elements of the system. As it was shown in the simulations, it is not present in any individual taken separately, and therefore it can not be represented or identified by any of the individuals.

In such hypothesis, what is the condition of the myopic individuals that we are? Our condition is a *given* condition, with unknown origin and historic evolution. The past and future are both beyond our horizon of accessibility. We emphasize again that these statements do not contain any metaphysics of history, and their intelligibility is instantly rendered clear by the simple models studied here. In fact, they are the *leitmotiv* of a very precise research project in sociology, as documented by the great work of Michael Mann .

New ideas may now arise. The individuals live in realities that are given, and the evolution, the dynamic, is beyond accessibility. None of the individuals has the perception of the whole distributed causation. However, it is known that intelligibility is always searched. How? A detailed answer is not possible here, but we suggest that a certain sort of intelligibility is obtained when the distributed causation is replaced by 'rational' causes of control, in particular by the efficient causation and by the final causation. Obviously, we are not saying that this is made in a conscious way. We have said that the individuals can not represent a distributed causation, because it is beyond the horizon of accessibility. As a matter of fact, it is not really a replacement but rather the quest for explanations using a type of causation that is accessible to an individual. The form of the single causes: some entity that is the absolutely first and radiant center responsible by the dynamic (following the efficient causation), which drives towards a certain final result (following the final causation). In other words, we are arguing that in one hand the linear causation, as an independent sum of the parts, is intelligible. On the other hand, the non-linear causation can not be in the perception of an individual tangled in a web of non-linear relationships.

We could give many examples supporting the last statement, with the help of different branches of knowledge such as the theory of financial markets (see http://www.unifr.ch/econophysics/) or the mythology. Nevertheless, we can present the main idea using again Schelling's model. In the beginning of this Section we raised the question about the conditions thought to be sufficient for a segregation process. Let us suppose again that a local and distributed causation led to a state of segregation.
To the ones attached to a cohesive group, segregation appears as given. We could ask a question:
'What do you think it was in the origin of this segregation?'. Surely the reader will agree that it will be implausible an answer of the type:*'It seems to have been generated by the accumulation of many interactions of individuals, all non-segregationist'*.
It is more plausible to consider other answers: *'our community as decided so'*, or *'each one of us has decided so because we don't like them around and they don't like us'.* Notice that the last answer assumes a 'majority rule' which in fact was not present. We also point out here the use of the pronouns *'our', 'we'*.
In the first case this shows the replacement of a local and distributed causation (which no one can experience) by a single and global cause (the 'colectivity', in this case) which completely satisfies the intelligibility, because that cause works in the same way that a local cause works, i.e. as an efficient causation. In the second case the answer shows the replacement of non-linear interactions by an explanation based on linear ones, using the independent sum of segregation behaviours.

Thus, two processes of searching intelligibility are:
(i) The use of a single global cause.
This is in the origin of myths.

(ii) The use of linear independent local causes.
This is a classical scientific approach.

In fact, when a non-intelligible cause is replaced by a single global cause, this satisfies each individual with an intelligible explanation. Explaining the segregation by a community decision (a single and efficent act of will) gives the idea of a central and coordinated order obeyed by everyone. This could be a plausible justification to the final invariant state, but it could hardly explain the whole evolution. However, since the individuals would not have access to that detailed information, the use of a single global cause could be kept as an explanation to the final state.

On the other hand, in the second case, the local law produces a global state from which one may infer a segregational behaviour. The projection of this global behaviour to a local law simplifies the model using an intelligible explanation. In Schelling's model this would correspond to consider a different number of admissible neighbours, in such a way that the majority rule would hold. Curiously, such a deduction would lead to instable situations, as it was seen in the 4-neighbour law simulation. The non-intelligible cause is replaced by the sum of independent local causes, simplifying the rule, assuming that the final result is in fact the result of a similar local law. In the case of Schelling's model, this would not even justify the stability on the border, where some individuals are in minority, but this could be seen as an exception to the rule, for instance, saying that some of the individuals (the ones that are on the border) are more tolerant than others.

(i) Why do we say that the first one is in the origin of myths?
In fact, arguing on the existence of one unique central cause is in the genesis of most of the myths. The evolution to the final state is no longer important, the result is what matters. There is also the possibility that a class of final states can be attributed to a cause, and other classes to other causes. This induces the generation of several distinct myths. However it is perfectly clear that we can easily substitute the several different causes by a single cause.

(ii) Why do we say that the second one is a scientific approach?
From the observation of global states a local law may be inferred. To check the validity of the law, one planifies an experiment. Isolating a part of a system, controlling it, seeing how it works, corresponds to an experiment that it is intelligible.We then argue that the same experiment can be repeated with similar results. If we then assume that the observed evolution is the result of the sum of all these parts, one may get an approximate model of what happens in more complex situations. This model is intelligible, but it only constitutes a good approximation in some favorable conditions. Usually, in transition situations these explanations (given by the sum of independent actions) have some serious exceptions. These exceptions are often at the origin of new breakthroughs on science development.