

A Layerwise Model for Soft Core Sandwich Panels

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Abstract

The application of high damping materials, like high loss factor viscoelastic materials, in the core of sandwich plates can improve their dynamic response and reduce fatigue failure. However, the numerical simulation of the effects of this kind of damping treatment demands a cumbersome computational work, specially during the spatial modelling task of the layered structure. In this work, a layerwise model is developed and applied in the analysis of sandwich plates with thin viscoelastic cores, comparing the results achieved with those obtained with the usual modelling approach using a layered combination of standard plate and solid finite elements. The proposed model can correctly describe the high shear pattern developed inside the thin viscoelastic core, providing results similar to those obtained with the layered models.

Keywords: layerwise models, sandwich plates, viscoelastic damping.

1 Introduction

Light and thin structures, usually applied on spacecraft and aeronautical assemblies, are very sensitive to cycling or random loading, which promote high levels of mechanical vibration, noise and fatigue failure, leading to structural disturbance and premature failure [1]. Soft core sandwich panels (Figure 1), using thin viscoelastic layers, are usually applied on these structures in order to improve their dynamic behavior [2, 3]. A high damping soft viscoelastic core inserted inside a sandwich plate can improve its dynamic response due to the high level of energy dissipation that occurs in the viscoelastic layer as a result of the polymeric molecular chain reaction to the imposed cycling deformation.

Though its mechanical performance, low cost and damping efficiency, this kind of structures, with integrated viscoelastic treatments, demands a special and complex

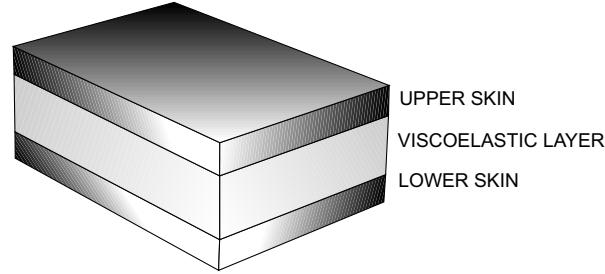


Figure 1: Integrated viscoelastic treatment

simulation task in order to properly determine the treatment parameters, like material type, thicknesses, number of layers, location and treatment coverage. Moreover, since the damping treatment efficiency is closely related with the shear deformation pattern that is developed inside the viscoelastic core of the sandwich plate, the finite element model must be able to accurately describe it.

The classical laminate plate theory cannot be used to simulate properly these structures since it is unable to describe accurately the high shear deformation of the viscoelastic layer [4]. Thus, it is usually applied a layered model, using standard plate and brick finite elements, where the soft core is modelled through a solid brick element in order to account for the shear deformation pattern developed in it [5, 6].

Figure 2 illustrates three of the layered models usually applied in the analysis of plates with constrained viscoelastic treatments and sandwich plates with a viscoelastic core.

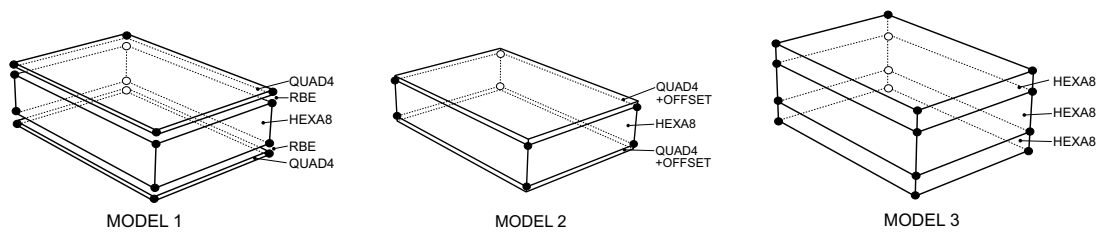


Figure 2: Layered models

Model 1 [7, 8] uses two plate elements to model the outside plates and one solid element to model the viscoelastic core. The translational degrees of freedom of the plate are connected to the brick ones by means of rigid links.

In Model 2, which is the commonly used layered model [4, 6, 9], the plate element nodes are localized by offset of half of the plate thickness to the plane in contact with the solid element, instead of remaining in the standard mid-plane. This results in coincident nodes and translational degrees of freedom for the plate and the adjacent

face of the solid element, which greatly simplify the modelling task, allowing some freedom in the redefinition of the thickness of the outside skins. Model 3 [5] uses solid elements in all the layers.

Moreira and Rodrigues [5] present a comparison analysis between the three layered models, giving special attention to the convergence rate, accuracy and sensitivity to numerical instabilities, like shear locking phenomena.

Though this modelling approach can effectively describe accurately the dynamic behavior of integrated and constrained damping treatments, it requires a cumbersome modelling approach, specially for irregular shapes. Its application in the analysis of tridimensional geometries with curvature is not straightforward and it is not suitable for multiple viscoelastic layer analysis. Moreover, any attempt to modify the thickness of a single layer requires an arduous re-modelling task.

2 A proposed model based on the layerwise theory

In order to avoid the cumbersome spatial modelling task required by the layered model approach, a discrete layer model, based on the Zig-Zag model of the layerwise theory [10], is proposed.

Figure 3 represents a section of a multiple layer laminate where each layer is treated as a *Reissner-Mindlin* thick plate, imposing displacement continuity between the individual layers directly in the displacement field description.

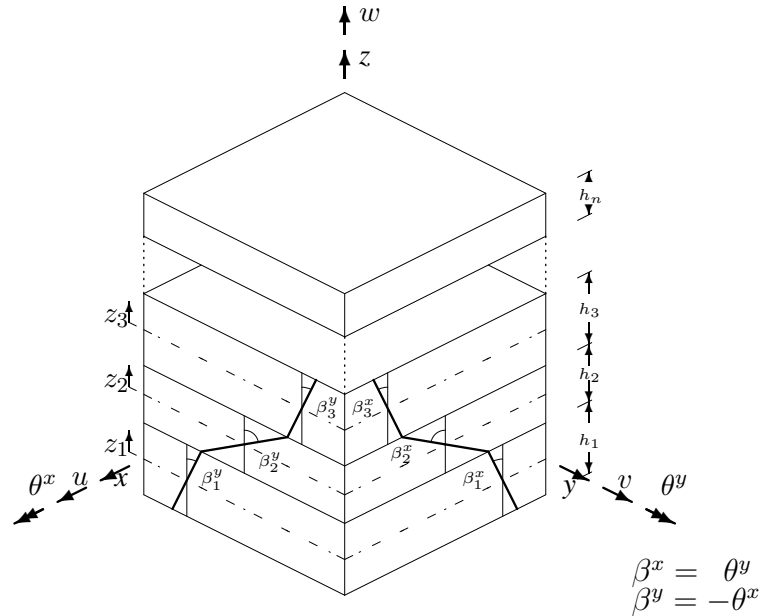


Figure 3: Layerwise theory - kinematic model

2.1 Kinematic model

Considering a generic layer k of the sandwich plate defined by:

$$\Omega_k = \{(x, y, z_k) \in \mathbf{R}^3 \mid z_k \in \left[-\frac{h_k}{2}, \frac{h_k}{2}\right], (x, y) \in A \subset \mathbf{R}^2\} \quad (1)$$

where Ω_k , A and h_k represent, respectively, the volume, area and thickness of the generic k -th layer, the displacement field $\{\mathbf{u}\}_k$ can be defined as:

$$\{\mathbf{u}\}_k = \begin{Bmatrix} u_k \\ v_k \\ w_k \end{Bmatrix} = \begin{Bmatrix} u_0 + \frac{h_1}{2}\beta_1^x + \sum_{j=2}^{k-1} h_j\beta_j^x + \frac{h_k}{2}\beta_k^x + z_k\beta_k^x \\ v_0 + \frac{h_1}{2}\beta_1^y + \sum_{j=2}^{k-1} h_j\beta_j^y + \frac{h_k}{2}\beta_k^y + z_k\beta_k^y \\ w_0 \end{Bmatrix} \quad (2)$$

where u_0 , v_0 and w_0 are the translations of the reference layer ($k = 1$) and β_k^x , β_k^y are the rotations of the normal about the y and x axes, respectively. A set of coupling terms guarantees the continuity of the displacement field along the laminate.

Equation (2) can be rewritten in the matricial form as:

$$\{\mathbf{u}\}_k = [\mathcal{N}]_k \{d\} \quad (3)$$

where:

$$\{d\} = \{u_0, v_0, w_0, \beta_1^x, \beta_1^y, \dots, \beta_j^x, \beta_j^y, \dots, \beta_n^x, \beta_n^y\}^T \quad (4)$$

represents the generalized displacement field and matrix $[\mathcal{N}]_k$ is defined as:

$$[\mathcal{N}]_k = \begin{bmatrix} 1 & 0 & 0 & \frac{h_1}{2} & 0 & \dots & h_j & 0 & \dots & \frac{h_k}{2} + z_k & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \frac{h_1}{2} & \dots & 0 & h_j & \dots & 0 & \frac{h_k}{2} + z_k & 0 & \dots \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 0 & \dots \end{bmatrix} \quad (5)$$

2.2 Potential energy

Potential energy of the whole set of layers is obtained by the sum of the potential energy of each individual layer, Π_k^P , defined as:

$$\Pi_k^P = \frac{1}{2} \int_{\Omega_k} \{\sigma\}_k^T \{\varepsilon\}_k d\Omega_k \quad (6)$$

where $\{\sigma\}_k$ and $\{\varepsilon\}_k$ represent, respectively, the tension field and the deformation field of the generic layer k .

The deformation field $\{\varepsilon\}_k$ can be related with the displacement field through a differential operator matrix $[\mathcal{L}]$ by the relation:

$$\{\varepsilon\}_k = [\mathcal{L}]\{\mathbf{u}\}_k \quad (7)$$

Recalling relation (3), Equation (7) can be rewritten as:

$$\{\varepsilon\}_k = [\mathcal{L}][\mathcal{N}]_k\{d\} = [\mathcal{B}]_k\{d\} \quad (8)$$

where $[\mathcal{B}]_k$ represents the deformation matrix. Tension field can be related with the deformation field by the constitutive set of equations as:

$$\{\sigma\}_k = [D]_k\{\varepsilon\}_k \quad (9)$$

where matrix $[D]_k$ represents the constitutive matrix, which, considering isotropic and linear material, corresponds to the plate elasticity matrix.

Replacing relations expressed in Equations (8)-(9), the potential energy equation (6) can be rewritten as:

$$\Pi_k^P = \frac{1}{2} \int_{\Omega_k} \{d\}^T [\mathcal{B}]_k^T [D]_k [\mathcal{B}]_k \{d\} \, d\Omega_k \quad (10)$$

2.3 Kinetic energy

The kinetic energy of the laminate is similarly obtained from the sum of the kinetic energies for the whole set of layers, being the kinetic energy of a generic layer k obtained from:

$$\Pi_k^K = \frac{1}{2} \rho_k \int_{\Omega_k} \{\dot{\mathbf{u}}\}_k^T \{\dot{\mathbf{u}}\}_k \, d\Omega_k \quad (11)$$

where $\{\dot{\mathbf{u}}\}_k$ represents the velocity field of the generic layer k and ρ_k is the material mass density.

The velocity field of a generic layer k is obtained from the relation:

$$\{\dot{\mathbf{u}}\}_k = [\mathcal{N}]_k \{\dot{d}\} \quad (12)$$

where $\{\dot{d}\}$ represents the time derivative of the generalized displacement field.

The kinetic energy of a generic layer k is, thus, obtained from:

$$\Pi_k^K = \frac{1}{2} \rho_k \int_{\Omega_k} \{\dot{d}\}^T [\mathcal{N}]_k^T [\mathcal{N}]_k \{\dot{d}\} \, d\Omega_k \quad (13)$$

2.4 Finite element model

In order to implement the presented layerwise theory, a low order isoparametric quadrilateral finite element is employed, using the usual bilinear shape functions to interpolate the geometry and the displacement field inside the finite element domain. The degrees-of-freedom vector, formed by three translations and two out-plane rotations per layer, is dynamically created, which generalizes the application of the formulation to multiple layer laminates.

2.4.1 Stiffness and mass matrices

The stiffness and mass matrices of the finite element that models the laminate can be evaluated from the stiffness and mass contributions of each individual layer.

Recalling the definition of the potential and kinetic energies of the laminate, Equations (10) and (13), the Hamilton's principle can be applied to derive the corresponding stiffness and mass matrices of one laminate finite element. Stiffness matrix is, thus, expressed as:

$$[K]^e = \sum_{k=1}^n \int_{\Omega_k^e} [B]_k^e{}^T [D]_k [B]_k^e d\Omega_k^e \quad (14)$$

where the finite element deformation matrix $[B]_k$ is evaluated from:

$$[B]_k^e = [\mathcal{L}][\mathcal{N}]_k [N]^e \quad (15)$$

and matrix $[N]^e$ represents the bilinear shape functions matrix of the finite element.

Following the same procedure, mass matrix of the composite finite element is defined as:

$$[M]^e = \sum_{k=1}^n \int_{\Omega_k^e} [N]^e{}^T [J]_k [N]^e d\Omega_k^e \quad (16)$$

where the matrix $[J]_k$ represents the inertia matrix of each generic layer and is obtained from:

$$[J]_k = \rho_k [\mathcal{N}]_k^T [\mathcal{N}]_k \quad (17)$$

2.4.2 Stiffness formulation improvement

Since the main application of the analyzed models is related with very thin viscoelastic core sandwich plates, a special concern about transverse shear locking must be taken. Therefore, the plate bending component of the stiffness formulation is enriched with a shear locking stabilization procedure based on the *MITC* approach [11]. This locking stabilization procedure is easily introduced in the stiffness matrix computation using a $\bar{\mathbf{B}}$ method [12]. Additionally the membrane displacement field is enriched using a set of incompatible modes [13] which provide a stabilization procedure for the in-plane shear locking problem of the bilinear quadrilateral element. The correction issued by Taylor *et al.* [14] is also applied to overcome the mesh distortion sensitivity.

To provide a generalization of the plate formulation for facet-shell application, which suffers from rank deficiency and ill-conditioning problems when adjacent elements are coplanar, the drilling degrees-of-freedom, θ_k^z , are introduced in the membrane formulation of the plate using a fictitious stiffness stabilization matrix [15].

2.4.3 Mass formulation improvement

Lumped or diagonal mass matrices, usually obtained from the consistent mass matrix provided by Equation (16), are usually applied in dynamic analysis procedures mainly

due to its computational economy. However, in the layerwise model special care must be issued when selecting the proper lumping procedure due to the effect of the coupling terms, being impossible to apply the nodal quadrature using *Lobatto* integration technique [16] or the "row-sum" technique [16]. In the proposed model the "HRZ" procedure [17], which is based on a diagonal-scaling technique, is efficiently applied.

3 Models comparison analysis

A convergency and accuracy analysis of the layered models was already published in [5] where the three models illustrated in Figure 2 were analyzed and compared. Based on the results obtained in that study two major conclusions were identified. Firstly, the aspect ratio of the solid brick element plays an important role in the accuracy and behavior of the layered finite element, specially when using the third model (model 3). Secondly, since the deformation energy of the viscoelastic core is mainly due to shear deformation, and keeping in mind that the storage modulus of the viscoelastic materials typically applied in damping treatments is significantly lower than the modulus of the material applied in the outside plates of the sandwich structure, the shear locking that occur in the core finite elements do not produce any major effect on the global stiffness matrix of the laminate.

To get some conclusions about the aspect ratio sensitivity of the two modelling approaches and to compare their behavior for different core thicknesses, it is developed a numerical study where model 2, that is usually the layered model commonly applied in published work on the subject, and model 4, that represents the proposed model, are used to model the sandwich plate described in Table 1, which is simply supported in the two smaller and opposite sides and has a very soft core. Model 2 uses a standard 8-node solid brick finite element to model the viscoelastic core and two *Mindlin*-plate finite elements to model the sandwich skins, using the same stiffness matrix formulation applied in the generic layer of the proposed layerwise finite element, including the introduced improvements.

Five different core thicknesses and five finite element meshes with different spatial resolution are applied to extend the aspect ratio span of the finite elements that represent the viscoelastic core.

The models are used to compute the first natural mode of the simply supported sandwich plate, which corresponds to a bending mode. The natural frequencies are normalized by the frequency value obtained for a refined mesh (60x40 divisions) using model 2, which provided the convergence indicator of the analysis.

The graphic illustrated in Figure 4 represents the computed convergence indicator obtained for the five different core thicknesses and the five different mesh resolutions considered in the analysis. As depicted in the graphic, the results obtained with both models converge to the considered numerical solution. It is also clear that the convergence rate obtained with the proposed model, model 4, is higher than the one provide by model 2.

Dimensions:		
Length [a]		300 mm
Width [b]		200 mm
Skin thickness [$h_{1/3}$]		1.0 mm
Core thickness [h_2]		0.2/0.6/1.0/1.4/1.8 mm
Material properties:		
- Aluminum (skins): (1 / 3)		
Young modulus [$E_{1/3}$]		72E9 Pa
Poisson ratio [$\nu_{1/3}$]		0.3
Mass density [$\rho_{1/3}$]		2700 Kg/m ³
- Viscoelastic material: (2)		
Young modulus [E_2]		1E6 Pa
Poisson ratio [ν_2]		0.49
Mass density [ρ_2]		1140 Kg/m ³

Table 1: Sandwich plate with viscoelastic core

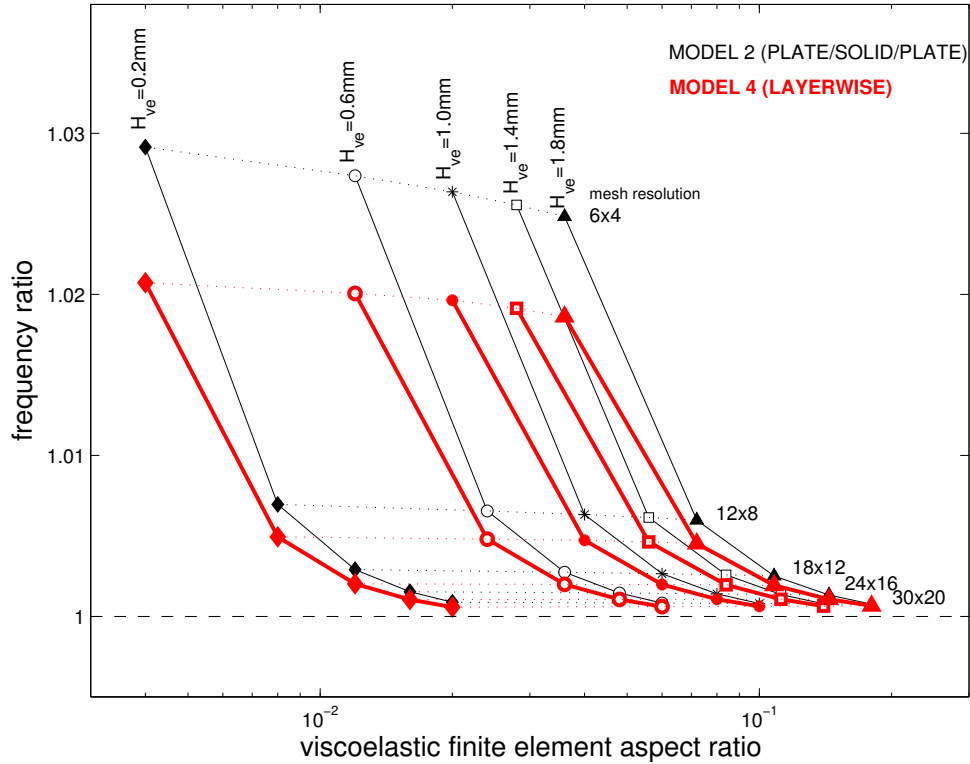


Figure 4: Aspect ratio/mesh resolution sensitivity analysis

Since the dotted lines, which identify the mesh resolution, are almost horizontal, we can conclude that the accuracy of the results is mainly defined by the mesh resolution and not by the aspect ratio sensitivity of the finite element. This behavior can be explained by the relatively low value of the core material and by the fact that most of the deformation energy developed inside the core is due to shear deformation. These conditions can reduce the effect of the aspect ratio sensitivity in the global stiffness matrix. Nevertheless, this sensitivity is still observed, specially for the coarse meshes.

4 Experimental validation

In order to compare the performance of the two selected models, model 2 and model 4, in the numerical simulation of the dynamic behavior of sandwich plates with a thin viscoelastic core, it was developed an experimental study with two sandwich plates with different core thickness.

The frequency response functions measured experimentally in the two specimens were directly correlated with those generated numerically using a direct frequency analysis [5] applying the select models. The viscoelastic material modulus, which is a complex entity and varies with frequency and temperature, is introduced in the analysis procedure through the application of the complex modulus approach [9, 5].

The experimental specimens are made of two aluminium plates, with 1mm of thickness each, bonded together with a thin (0.125 mm and 0.380 mm) 3M ISD112 [18] viscoelastic layer.

Figures 5 and 6 represent the direct frequency response functions, measured and numerically generated, for the two specimens. From the results we can observe that the frequency response functions obtained with both models are identical, providing the same description for the sandwich frequency response.

Since we can obtain the same results independently of the selected model, the modelling economy provided by the proposed model is a good and valuable reason to consider it as the right choice. Nevertheless, for thicker viscoelastic cores, which are not usually applied in damping treatments of light structures, it is necessary a correct description of the tridimensional deformation that occur in the soft core, which can only be well succeed if solid finite elements are used to model it.

5 Conclusions

In this work it is proposed a layerwise model to simulate the dynamic response of sandwich structures with thin an soft cores like those provided by an integrated viscoelastic damping.

The usual modelling approach, using a layered model obtained from combination of standard plate and solid finite elements, requires a time-consuming modelling task that restricts its application to simple and planar structures. The proposed model

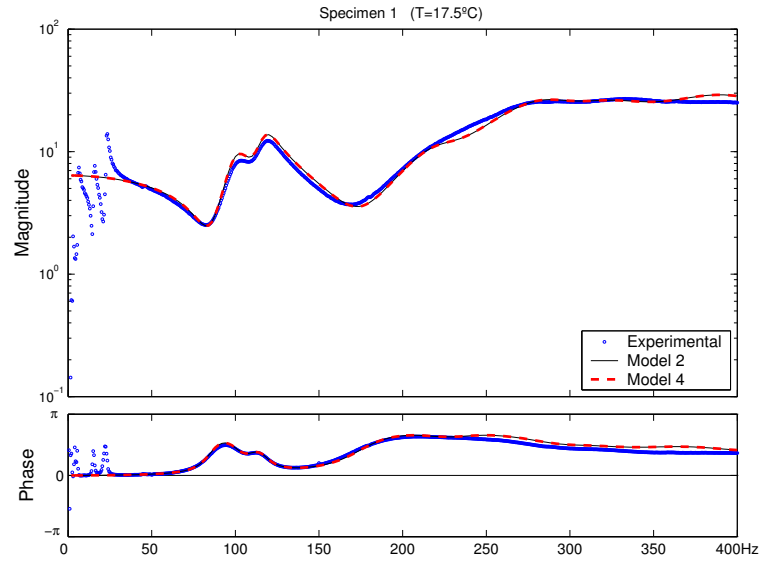


Figure 5: Direct frequency response function—experimental vs. numerical

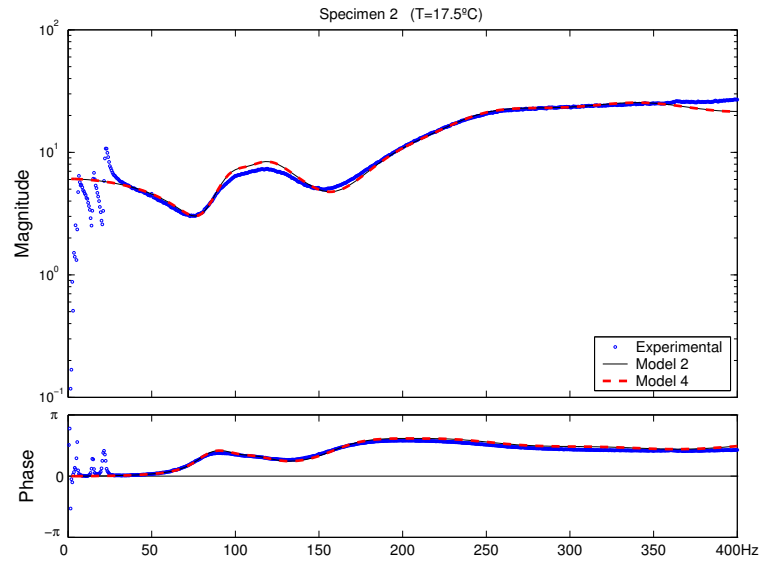


Figure 6: Direct frequency response function—experimental vs. numerical

avoids this restriction using a standard quadrilateral mesh generator and, since the laminate description (materials properties, thicknesses, and finite element formulation parameters) are introduced in the solving process by an auxiliary data table, any modification to the laminate is straightforward and does not demand the entire re-modelling task that the layered models require.

It is presented a comparison analysis between the proposed model and the usual layered models, where both modelling approaches are applied in the dynamic analysis of damped sandwich plates with soft elastic cores. The proposed model is compared with the usually applied layered model, model 2, obtaining some information about the sensitivity of their formulations to the aspect ratio of the finite element and also some indicators on the convergence rate of the results. The results showed that the accuracy and convergence rate of the proposed model are identical, or even better, to those obtained with model 2.

To validate the applicability of both modelling approaches the analyzed models were applied in a direct frequency analysis to generate the frequency response functions of two experimental specimens with integrated viscoelastic treatments. The numerically generated functions obtained with the proposed model agree well with the experimental ones and are identical to those obtained using the layered models.

Acknowledge

The authors gratefully acknowledge 3M company who provided the viscoelastic layers used in the experimental work hereby presented.

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